Q1)

$$\left[\begin{array}{ccccc|cc|c} 1 & 1 & 2 & 3 & 5 & 1 & 0 & 19 \\ 2 & 4 & 3 & 2 & 1 & 0 & 1 & 57 \\ \hline -10 & -24 & -20 & -20 & -25 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccccc|c} 0.2 & 0.2 & 0.4 & 0.6 & 1 & 0.2 & 0 & 3.8 \\ 1.8 & 3.8 & 2.6 & 1.4 & 0 & -0.2 & 1 & 53.2 \\ \hline -5 & -19 & -10 & -5 & 0 & +5 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccccc|c} - & 0 & - & - & 1 & - & - & 1 \\ - & 1 & - & - & 0 & - & - & 14 \\ \hline 4 & 0 & 3 & 2 & 0 & 4 & 5 & \end{array}\right]$$

$x^* = (0, 14, 0, 0, 1)$

$F_{max} = 361$

Q2)

$$\left[\begin{array}{ccccc|c} 2 & -1 & -1 & -1 & 0 & 3 \\ 1 & -1 & 1 & 0 & -1 & 2 \\ \hline -2 & 1 & 0 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -0.5 & -0.5 & -0.5 & 0 & 1.5 \\ 0 & -0.5 & 1.5 & 0.5 & -1 & 0.5 \\ \hline 0 & 0 & -1 & -1 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & -2/3 & 0 & -1/3 & -1/3 & 5/3 \\ 0 & -1/3 & 1 & 1/3 & -2/3 & 1/3 \\ \hline 0 & -1/3 & 0 & -2/3 & -2/3 & \end{array}\right]$$

$x^* = \left(\frac{5}{3}, 0, \frac{1}{3}\right)$

$F_{min} = 10/3$

Dual Problem

$2y_1 + y_2 \leq 2$

$-y_1 - y_2 \leq -1$

$-y_1 + y_2 \leq 0$

$y_1, y_2 \geq 0$

Solution $\longrightarrow \left(\frac{2}{3}, \frac{1}{3}\right)$

(y_1, y_2)

Q3) Introduce Artificial Variables A_1 A_2 A_3

Phase-1

$$x_1 + 2x_2 - x_3 + x_4 + A_1 + \quad + \quad = 0$$
$$2x_1 - 2x_2 + 3x_3 + 3x_4 + \quad + A_2 \quad = 9$$
$$x_1 - x_2 + 2x_3 - x_4 + \quad + \quad + A_3 = 6$$
$$x_i, A_i \geq 0$$

$$\left[\begin{array}{ccccccc|c} -1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 2 & -2 & 3 & 3 & 0 & 1 & 0 & 9 \\ 1 & -1 & 2 & -1 & 0 & 0 & 1 & 6 \\ \hline 2 & -5 & 6 & 1 & 0 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccccc|c} -1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 5 & 4 & 0 & 6 & -3 & 1 & 0 & 9 \\ 3 & 3 & 0 & 1 & -2 & 0 & 1 & 6 \\ \hline 8 & 7 & 0 & 7 & -6 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccccc|c} 0 & -1.2 & 1 & 0.2 & 0.4 & 0.2 & 0 & 1.8 \\ 1 & 0.8 & 0 & 1.2 & -0.6 & 0.2 & 0 & 1.8 \\ 0 & 0.6 & 0 & -2.6 & -0.2 & -0.6 & 1 & 0.6 \\ \hline 0 & 3.5 & 0 & -2.6 & -1.2 & -1.6 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccccc|c} 0 & 0 & 1 & -5 & 0 & -1 & 2 & 3 \\ 1 & 0 & 0 & 14/3 & -1/3 & 1 & -4/3 & 1 \\ 0 & 1 & 0 & -13/3 & -1/3 & -1 & 5/3 & 1 \\ \hline 0 & 0 & 0 & 0 & -1 & -1 & -1 & \end{array}\right]$$

Phase-2

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & -5 & 3 \\ 1 & 0 & 0 & 14/3 & 1 \\ 0 & 1 & 0 & -13/3 & 1 \\ \hline 0 & 0 & 0 & -97/3 & \end{array}\right] \rightarrow (x_1, x_2, x_3, x_4) = (1, 1, 3, 0)$$

Q4)

a)
$$\left[\begin{array}{cccccc|c} 3 & 1 & -2 & 1 & 1 & 0 & 2 \\ 1 & 3 & 0 & -1 & 0 & 1 & 2 \\ \hline -18 & -12 & -2 & -6 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 8/3 & 0 & -2 & 4/3 & 1 & -1/3 & 4/3 \\ 1/3 & 1 & 0 & -1/3 & 0 & 1/3 & 2/3 \\ \hline -14 & 0 & -2 & -10 & 0 & 4 & \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -0.75 & 0.5 & 3/8 & -1/8 & 1/2 \\ 0 & 1 & 0.25 & -0.5 & -1/8 & 3/8 & 1/2 \\ \hline 0 & 0 & -12.5 & -3 & +5.25 & +2.25 & \end{array}\right]$$

$F^* = 15$

$x^* = (1/2, 1/2, 0)$

b) Dual - max. $2y_1 + 2y_2$

subject $3y_1 + 2y_2 \leq 18$ - (1)

$y_1 + 3y_2 \leq 12$ - (2)

$-2y_1 \leq 2$ - (3)

$y_1 - y_2 \leq 6$ - (4)

$y_1, y_2 \geq 0$ - (5)

$(y_1, y_2) = \left(\frac{30}{7}, \frac{18}{7}\right)$

c) $x_1, x_2 \neq 0$ at optimality

$\Rightarrow$ they constitute the active constraints

Column 1, Column 2

For the dual problem, we see that (1), (2) are active constraints

Q5) max. $30x_1 + 20x_2 + 40x_3 + 25x_4 + 10x_5$

$$2x_1 + x_2 + 3x_3 + 3x_4 + x_5 \leq 700$$
$$3x_1 + 2x_2 + 2x_3 + x_4 + x_5 \leq 1000$$
$$x_i \geq 0 \;\; \forall i \in \{1,2,3,4,5\}$$

$$\left[\begin{array}{ccccccc|c} 2 & 1 & 3 & 3 & 1 & 1 & 0 & 700 \\ 3 & 2 & 2 & 1 & 1 & 0 & 1 & 1000 \\ \hline -30 & -20 & -40 & -25 & -10 & 0 & 0 & \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccccccc|c} 2/3 & 1/3 & 1 & 1 & 1/3 & 1/3 & 0 & 700/3 \\ 5/3 & 4/3 & 0 & -1 & 1/3 & -2/3 & 1 & 1600/3 \\ \hline -10/3 & -20/3 & 0 & 15 & 10/3 & 40/3 & 0 & \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccccccc|c} 0.25 & 0 & 1 & 1.25 & 0.25 & 0.5 & -0.25 & 100 \\ 1.25 & 1 & 0 & -0.75 & 0.25 & -0.5 & 0.75 & 400 \\ \hline 5 & 0 & 0 & 10 & 5 & 10 & 5 & \end{array}\right]$$

$x^* = (0, 400, 100, 0, 0)$

$F_{max} = 12000$

Q9)

	1	2	3	4	5	6	7	8	9	10	11	12
a	0	0	0	0	0	0	0	0	0	0	0	0
b	2	2	2	2	2	2	2	2	2	2	2	2
c	3	3	3	3	3	3	3	3	3	3	3	3
d	1	1	1	1	1	1	1	1	1	1	1	1
e	–	3	3	3	3	3	3	3	3	3	3	3
f	–	4	4	4	4	4	4	4	4	4	4	4
g	–	–	5	5	5	5	5	5	5	5	5	5
h	–	–	–	–	–	–	9	9	9	9	9	9
i	–	–	–	–	–	8	8	7	7	7	7	7
j	–	–	–	–	5	5	5	5	5	5	5	5
k	–	–	–	–	6	6	6	6	6	6	6	6
l	–	–	–	–	–	–	–	7	7	7	7	7
m	–	–	–	–	–	–	–	7	7	7	7	7
n	–	–	–	–	–	–	–	–	–	10	10	10

	13	14
a	0	0
b	2	2
c	3	3
d	1	1
e	3	3
f	4	4
g	5	5
h	9	9
i	7	7
j	5	5
k	6	6
l	7	7
m	7	7
n	10	10

(a) a - n $\longrightarrow$ 10

(b) a - j $\longrightarrow$ 5

Q10) Initial Matrix →

0	1	1	-	-	-	-	-	-	-	-
1	0	2	-	3	-	-	-	-	-	-
1	2	0	2	-	5	-	-	-	-	-
-	-	2	0	2	5	-	-	-	-	-
-	3	-	2	0	1	-	8	-	-	-
-	-	5	5	1	0	6	-	3	-	-
-	-	-	-	-	6	0	3	4	1	-
-	-	-	-	8	-	3	0	-	3	5
-	-	-	-	-	3	4	-	0	6	-
-	-	-	-	-	-	1	3	6	0	3
-	-	-	-	-	-	-	5	-	3	0

Final Matrix →

	a	b	c	d	e	f	g	h	i	j	k
a	0	1	1	3	4	5	11	12	8	12	$\boxed{15}$
b	1	0	2	4	3	4	10	11	7	11	14
c	1	2	0	2	4	5	11	$\boxed{12}$	8	12	15
d	3	4	2	0	2	3	9	10	6	10	13
e	4	3	4	2	0	1	7	8	4	8	11
f	5	4	5	3	1	0	6	9	3	7	10
g	11	10	11	9	7	6	0	3	4	1	4
h	12	11	12	10	8	9	3	0	7	3	5
i	8	7	8	6	4	3	4	7	0	5	8
j	12	11	12	10	8	7	1	3	5	0	3
k	15	14	15	13	11	10	4	5	8	3	0

(a) a - k → 15

(b) c - h → 12

Q6) min. $3500\left(\frac{x_{11}}{0.3} + \frac{x_{12}}{0.2} + \frac{x_{13}}{0.3}\right) + 3000\left(\frac{x_{21}}{0.3} + \frac{x_{22}}{0.4} + \frac{x_{23}}{0.2}\right)$

$x_{11} + x_{21} \geq 900,000$
$x_{12} + x_{22} \geq 800,000$
$x_{13} + x_{23} \geq 500,000$
$x_1, x_2 \geq 0$

Solving using simplex method, we get

$(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}) = (0, 0, 5\times10^5, 9\times10^5, 8\times10^5, 0)$

$\min z = ₹\frac{625}{3}\times10^8$

Q7) Supply = 14K is greater than Demand = 10K, we add a slack variable

a)

	1	2	3	4
1	3	1	1	0
2	6	2	7	0
3	1	9	12	0

$x_{11} + x_{12} + x_{13} + x_{14} = 2$
$x_{21} + x_{22} + x_{23} + x_{24} = 6$
$x_{31} + x_{32} + x_{33} + x_{43} = 6$
$= \quad 5 \quad 3 \quad 2 \quad 4$

NW Algorithm $\longrightarrow$

$$\begin{matrix} 2 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 \\ 0 & 0 & 2 & 4 \end{matrix}$$

Using Simplex Multipliers $\longrightarrow$

$$\begin{matrix} 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 3 \\ 5 & 0 & 0 & 1 \end{matrix} \longrightarrow z_{min} = 13$$

b) How should every location sell oil so that total profit made is maximal.

Q8) $\text{minimize}\begin{pmatrix} 1\cdot sp_{11} + 2\cdot sp_{12} + 4\times PM_{11} + 2\times PM_{12} + PM_{13} \\ + 2\cdot sp_{21} + 1.5 sp_{22} + 3\times PM_{21} + 4 PM_{22} + 2 PM_{23} \end{pmatrix} 100$

	Demand		
Supply	4	3	2
	4.5	4	3

$x_{11} + x_{12} + x_{13} = 10$

$x_{21} + x_{22} + x_{23} = 15$

$= 8 \quad 14 \quad 3$

NW Principle $\rightarrow$

8	2	0
0	12	3

Final solution with all simplex multipliers non-zero

0	10	0
8	4	3

$Z_{min} = 9100$ rupees.