# Model of thrust/Load−Torque in terms of geometric parameters of the blade for design and simulation of small scale propellers used in miniature UAVs

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# Model of Thrust/Load-Torque in terms of Geometric Parameters of the Blade for Design and Simulation of Small Scale Propellers used in Miniature UAVs

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***Abstract*—Autonomous flight of miniature UAVs demands high performance actuators satisfying the requirements of modern control algorithms. Specially for quadrotors, where lift and control forces are the result of a synchronized action of the four BLDC/propeller actuators, the form of their propellers are fundamental to attain a fast and reliable dynamic response. In this paper, we present a mathematical model of the thrust and the Load-Torque produced by a small scale propeller used in miniature quadrotors in terms of the geometric properties of the blade assuming an operation in hover mode. This model is intended to provide a useful expression for simulation analysis with design purposes. The model is based on the Blade Element Theory and the Euler equation for fluids, and describes the Lift on a thin and curved airfoil with a known angle of attack, as the normal acceleration between the deflected streamlines of an incompressible fluid. This allows the calculation of the thrust and the Load-Torque of a turning propeller in terms of the geometric parameters of its blades, i.e. radial and cross section length, curvature and angle of attack, but preserving the conventional results relating the thrust to a square function of the rotational speed of the propeller. As a reference, we include the conventional model based on experimental coefficients of Lift and Drag before introducing our approach, and then, we illustrate the advantage of our approach with an example based on a propeller with blades of fixed and constant pitch angle and constant width, and finally, we briefly discuss about the evidence, based on experimental and simulation data obtained in the literature, that validates our approach.**

## I. Introduction

Unmanned Aerial Vehicles (UAVs), also known as drones, are aircraft without pilot on board, used in civilian and military applications, specially in dangerous or inaccessible environments for humans. In general, UAVs are teleoperated systems provided with a certain degree of autonomy to assist the operator during the flight, and in the cases where a full autonomy is available, it is limited to be attain only for short periods, letting the total control of the aircraft in human hands. Among the multiple configuration of UAVs, miniature quadrotor, a small size multi-rotor rotary wing aircraft ( $10^{-2}$m), receives special attention of researchers and engineers because of their economic potential in common life activities; delivering, supervising, or realizing recreational activities.

Autonomous flight of miniature quadrotors has already caught the attention of control engineers who are attracted by their small size and simple architecture, capable of a static flight as well as aggressive manoeuvres, but characterized of having a complex dynamics. Quadrotors have four, coplanar

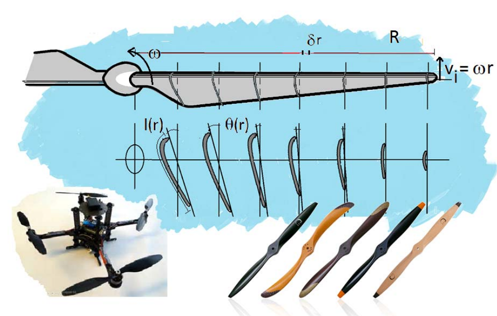


Fig. 1. Small scale propellers used in miniature Quadrotors.

and parallel, thrusters symmetrically arranged in a diamond configuration around a main axis, with two sets of opposite and counter-rotating rotor pairs, whose joint action produce the lift force, for the flight itself, and the control forces for pitch, roll and raw, that is, describing a coupling of lift and control forces that, besides of their fast and unstable dynamics, imposes a interesting challenge for the modern control approaches [1]. Even more, given their small size, for practical applications the payload capacity and the autonomous energy supply are very limited, imposing new constraint on the controllers regarding the energy consumption. In any case, controllers are designed upon the assumption of counting with high performance actuators [2][3], motivating the study of small scale actuators under their real operational conditions [4], in particular, arousing an interest in propellers [6] [5] and their effects on the general dynamic of the aircraft and their impact in the energy consumption [7].

In general, the dynamic properties of propellers are analysed and modelled using two main approaches[8],[9],[10],[11]. In the first one, based on Bernoulli's law considering the airflow, before and after the impulse produced by the propeller, far enough from the plane described by its rotation, the thrust

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is described as a function of the square of the induced velocity of the air and proportional to the area described by the turning propeller. And, in the second approach, the blade element theory, where the lifting force of a propeller is calculated from a differential airfoil on the blade and integrated along its radio, thrust is also calculated as a square function of the speed but in terms of experimental coefficients, Lift and Drag, expressed in terms of the angle of attack [4]. In both cases, the geometric properties of the propeller are implicit in the calculation of the thrust, however, these expressions do not show the simultaneous effects of these properties in the resulting lifting force and they are not straightforward useful for optimal designing purposes.

Lifting force on an airfoil produced by an air flow is a very complex phenomenon that has been explained using at least 5 different models: Bernoulli's law, Conservation of Momentum, Circulation, flow-turning or stream-curvature, and 3D vortex-shedding [15] [16]. Among them, Bernoulli's law, regarding the flow around the airfoil, and the Newtonian description of the deflected air by the airfoil, are the commonly used approaches to describe the forces produced by the airflow providing a quite good enough approximation [13] [14]. In particular, when the airfoil is composed of a curved surface without camber, as the cross section of the small scale propellers used in miniatures quadrotors (Fig. 1), the forces producing the thrust by the deflected airflow can be described in terms of its geometric parameters using the Euler equation for fluids and then, if we combine this with the blade element theory, we assume that it is possible to obtain a simple expression of thrust in term of geometric parameters of the blade. In this paper, in order provide a useful expression for simulation analysis and designing purposes, we present a simple mathematical model of the thrust and the Drag produced by a turning small scale propeller used in miniature quadrotor. Thus, assuming a small size propeller with a curve cross section and non-camber surface, and the validity of the Euler equation of the normal acceleration between streamlines of incompressible fluids when describe the surfaces of a curved profile, the model expresses these forces as a square function of the angular speed of the propeller, as in conventional models, but it introduces as a novelty an expression based on the geometric parameters of the propeller. Of course, this approach do not consider swirl effects in the deflected airflow by the turning propeller reducing the effective thrust, but it provides an approximation that can be very useful in numerical simulation for analysis and design.

## II. Conventional Model of Thrust using Blade Element Theory

The thrust produced by a turning propeller and Load-Torque opposing its movement in a hover mode, can be calculated using the Blade Element Theory [11]. In this approach, the total forces on the propeller are the result of adding along its radio, the forces exerted by an incompressible airflow on a differential surface of the blade with known cross-section (Figure 2).

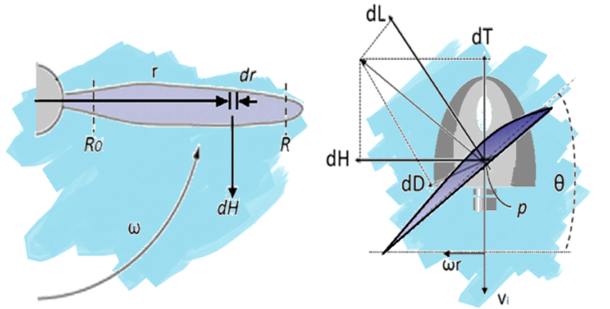


Fig. 2. Aerodynamic forces, Lift and Drag, acting on the propeller described by the Blade Element Theory on a cambered airfoil profile.

### A. Differential Lift/Drag using Experimental Coefficients

The Lift and Drag Forces on a differential section of wing, respectively $dT$ and $dH$, are described as in [9] [11] [12],

$$\begin{aligned} dT &= \tfrac{1}{2}\rho c(r) C_L(\alpha_i) u^2 dr \\ dH &= \tfrac{1}{2}\rho c(r) C_D(\alpha_i) u^2 dr \end{aligned} \tag{1}$$

where $c(r)$ is the cross section length of the blade in $r$, $\alpha_i = \alpha_i(\theta, v_i)$, is the induced angle of attack as a function of the cord, $\theta(r)$, the induced velocity, $v_i$, such that $u^2 = v_i^2 + \omega^2 r^2$, with the rotor speed, $\omega_r$, and the coefficient of Lift, $C_L$, and Drag, $C_D$ [4]. Observe that for moderate angle of attack ($\alpha(v_i) \leq 15°$), these coefficient can be expressed using experimental data obtained as a function of the angle of attack for a given profile (see for instance the Figure 3 showing the experimental coefficients $C_L$ and $C_D$ as a function of the angle of attack for an airfoil section NACA 0012 [17]),

$$\begin{aligned} C_L(\alpha_i) &= l_0 + l_1\alpha_i \\ C_D(\alpha_i) &= d_0 + d_1\alpha_i + d_2\alpha_i^2, \end{aligned} \tag{2}$$

where $C_{DL} = d_1$ and $C_{DL^2}$ are respectively, the coefficients of parasite drag and lift-induced drag [7], and the constants $d_0$, $d_1$, $d_2$ can be calculated to approach the curve of experimental data as a function of $\alpha_i$, for example, if the cross blade airfoil corresponds to a NACA 0012 profile, its graph of experimental data are showed in Figure 3.

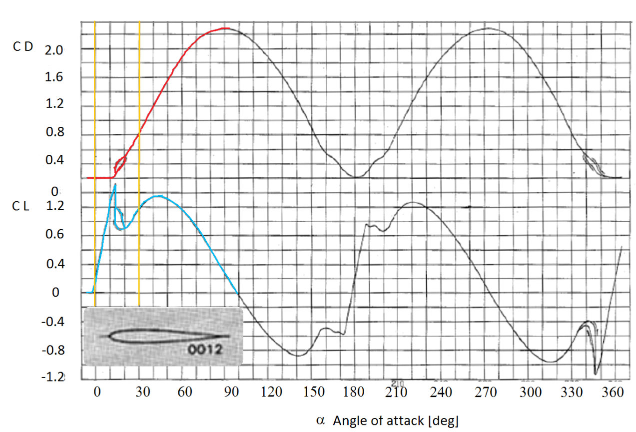


Fig. 3. Coefficients of Lift and Drag on an airfoil NACA 0012 [17].

Thus, recalling that $dT_L = rdH$ and integrating along the radius $r$ of the blade, from $R_0$ to $R_h$, thrust, $T$, and Load Torque, $T_L$, of a turning propeller with $N$ number of blades can be expressed as

$$\begin{array}{rl} T = N & \int_{R_0}^{R_h} \frac{1}{2}\rho c(r) C_L(\alpha_i) u^2 dr \\ T_L = N & \int_{R_0}^{R_h} r\frac{1}{2}\rho c(r) C_D(\alpha_i) u^2 dr. \end{array} \tag{3}$$

### B. Example 1: Propeller with constant cross section blade

In order to illustrate the application of this approach, assume as in [4], a propeller with $N$ small symmetrical blades such that $c(r) = c$, $\theta(r) = \theta$, $\alpha(v_i) = \theta - arctan(\frac{v_i}{\omega r})$, thrust, $T$, and Load Torque, $T_L$, can be expressed as

$$\begin{aligned} T = \; & N\frac{1}{2}\rho c \Bigg\{ \frac{1}{3}\frac{l\,\theta\left(v_i^2+\omega^2 r^2\right)^{\frac{3}{2}}}{\omega} - \frac{l\,v_i \omega\, r^2}{2} \\ & -\frac{1}{2}\left[ r\sqrt{v_i^2+\omega^2 r^2} + \frac{v_i^2 \ln\left(\omega r + \sqrt{v_i^2+\omega^2 r^2}\right)}{\omega} \right] \\ & \left(d_0 + d_2\theta^2\right) v_i + 2 d_2 \theta v_i^2 r \\ & - \frac{d_2 v_i^3 \ln\left(\omega\, r + \sqrt{v_i^2+\omega^2 r^2}\right)}{\omega} \Bigg\}_{R_0}^{R_h}, \end{aligned}$$

$$\begin{aligned} T_L = \; & N\frac{1}{2}\rho c \Bigg\{ \frac{1}{3}\frac{(v_i^2+\omega^2 r^2)^{3/2} l\theta v_i}{\omega^2} - \frac{1}{2} r^2 l \theta v_i^2 \\ & + \frac{1}{4}\frac{d_0 r (v_i^2+\omega^2 r^2)^{\frac{3}{2}}}{\omega} - \frac{1}{8}\frac{d_0 v_i^2 r\sqrt{v_i^2+\omega^2 r^2}}{\omega} \\ & - \frac{1}{8}\frac{d_0 v_i^4 \ln\left(\omega r + \sqrt{v_i^2+\omega^2 r^2}\right)}{\omega^2} \\ & + \frac{1}{3} d_2 \theta^2 \omega r^3 - \frac{2}{3} d_2 \theta v_i \omega r^3 \\ & + \frac{1}{2}\frac{d_2 v_i^2 r \sqrt{v_i^2+\omega^2 r^2}}{\omega} \\ & - \frac{1}{2}\frac{d_2 v_i^4 \ln\left(\omega r + \sqrt{v_i^2+\omega^2 r^2}\right)}{\omega^2} \Bigg\}_{R_0}^{R}. \end{aligned} \tag{4}$$

## III. Model of Thrust using the Euler Equation of Fluids and Blade Element Theory

The Lift force exerted on an airfoil by an airflow, has been described traditionally by the laws of Bernoulli and of Conservation of Momentum, according to the convenience given the type of available data to characterize a particular flow pattern. Both concepts are right and interrelated, but they represent a simplification of the real phenomenon which is intuitive and closer to reality when both, the angle of attack and the camber of the airfoil, are small (i.e. in a condition with zero vorticity).

The force exerted on the airfoil is a consequence of the pressure gradient produced by the moving air passing through the airfoil and not only by the reaction force of the deflected airflow on its lower side; the low pressure on its upper side is produced by the enlargement of the cross section of the streamline when it is deviated from its inertial straight line trajectory. The lower pressure on the upper side of the surface accelerates any volume of air approaching to it in the streamline direction, attracting them to follow the surface and increasing its velocity. Thus, the Lift force can be explained regarding the radial acceleration that deviates the streamlines along the surfaces of the airfoil, producing a gradient of pressure and then, an upward force on the airfoil (Figure 4). This acceleration can be determined at any point of the streamline by its velocity and its radius of curvature, and calculated using the Euler's equation for fluids. Specially, when the airfoil profile is thin and non-cambered, with a constant curvature, the Euler's equation provides a very simple analytical expression to calculate the total lift exerted on it. In particular, the most common propellers used in miniature quadrotors (see for instance [5]), verify these geometric properties and also the fact of having a profile with almost continuous curvature, allowing a simple application of the Euler's equation for fluids, in order to calculate the lift on the airfoil, thus, the thrust of the propeller, using the Blade Element approach.

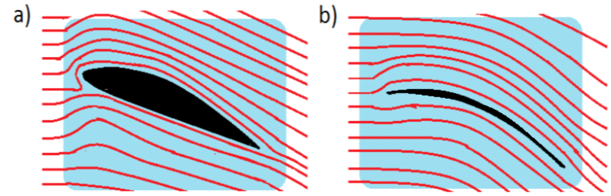


Fig. 4. Ideal airflow deviated by an airfoil with a small angle of attack: a) cambered profile commonly used in wings, b) thin, non-cambered, and continuous curved cross section.

### A. Euler Equation of Fluids

The Euler Equation of Fluids, is a formula that calculates the gradient of pressure generated in the airflow by the deflected streamlines. This equations can be obtained by a direct calculation using the conservation of mass and the balance of energy and momentum in a flow of non-viscous flow, or as en special case of the Navier-Stokes equations for adiabatic and inviscid flows.

The Euler Equation is given by

$$\frac{dP}{dR_c} = \rho \cdot \frac{v^2}{R_c}, \tag{5}$$

where $P$ is the pressure, $R_c$, the radius of curvature, and $v^2$ the stream velocity as showed in Figure 5.

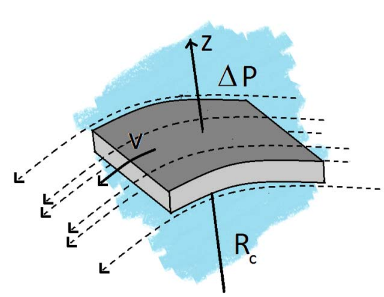


Fig. 5. Principe of the Euler Equation describing the gradient of Pressure produced in a laminar airflow by a curved surface.

### B. Thrust based on the Euler's equation and the Blade Element approach

In order to calculate the thrust produced by a turning propeller using the Euler's equation and the Blade Element approach, we assume the cross section of the propeller blade has a thin, non-cambered, and constant curved profile, as the one showed in in Figure 6, where $l$, $R_c$, and $\theta$, are the geometric properties length of the profile, the radius of curvature $R_c$, the angle of attack with respect to a horizontal plane, respectively, and $dr$, the differential radial length, $\omega$, the angular speed of the propeller, and $v_i = \omega r$, the induced velocity of the air by the turning propeller at the point of radius $r$.

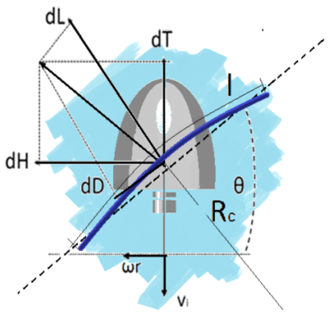


Fig. 6. Lift and Drag forces acting on a thin, curved and non cambered airfoil profile described by the Blade Element Theory

It is important to remark that in our approach, we are interested in the gradient of pressure $\Delta P$ produced in a laminar airflow deviated by the airfoil, where the streamlines experienced a radial acceleration parallel to the normal vector of the airfoil surfaces as illustrated in Figure 7.

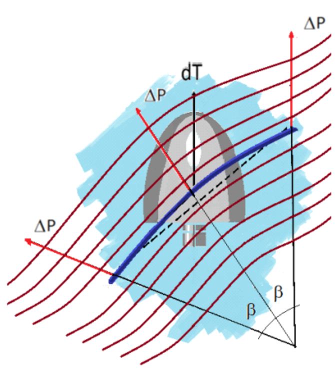


Fig. 7. Gradient of pressure created by the deflected airflow assuming a laminar condition.

In this case, the Lift, $dT$, and Drag, $dH$, forces on a differential section of the blade at the point $r$, are calculated as the vertical and horizontal components of the total force $dL_r$ exerted on the airfoil by the radial gradient of pressure $\Delta P$, i.e.,

$$\begin{aligned} dT = &\quad dL_r.v \\ dH = &\quad dL_r.h \end{aligned} \tag{6}$$

where the vertical force, $dL_r.v$, as well as the horizontal force, $dL_r.h$, are calculated by integrating, along the length of the airfoil profile, the vertical and the horizontal projections of the punctual force, $dL_r$, excerpted on a differential surface of the airfoil,

$$\begin{aligned} dL_r.v = &\quad \int_0^l cos(\theta_r) \cdot dL_{\theta.r} \\ dL_r.h = &\quad \int_0^l sin(\theta_r) \cdot dL_{\theta.r}. \end{aligned} \tag{7}$$

The punctual force, $dL_r$, is the radial gradient of pressure acting on a differential surface, $dA = dldr$, on the airfoil, that can be expressed in terms of the rotational speed of the propeller using the Euler equation for fluids (Eq.5), and recalling that $dl = R_c d\theta_r$ and $v_i = \omega r$, thus

$$\begin{aligned} dL_r = &\quad \frac{P}{R_c}|_0 \, dA \\ = &\quad \rho\frac{v_i^2)}{R_c} dldr \\ = &\quad \rho\frac{(\omega r)^2}{R_c} R_c d\theta_r dr \\ = &\quad \rho\omega^2 r^2 d\theta_r dr. \end{aligned} \tag{8}$$

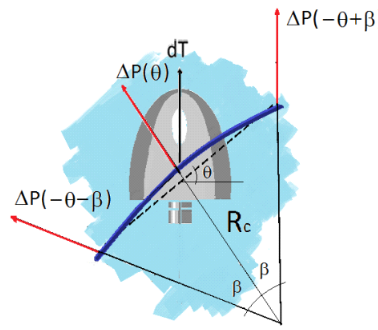


Fig. 8. Geometric parameters of a thin and curved profile: transversal superficial length and its arc angle, and its radius of curvature.

Now, the differential Lift (Eq. 6) on the profile can be determined by

$$\begin{aligned} dT = &\quad \rho\omega^2 r^2 \int_{\theta-\beta}^{\theta+\beta} cos(\theta_r) d\theta_r dr \\ = &\quad \rho\omega^2 r^2 sin(\theta_r) \,|_{\theta-\beta}^{\theta+\beta} \, dr \\ = &\quad \rho\omega^2 r^2 (sin(\theta+\beta) - sin(\theta-\beta))dr \\ = &\quad \rho\omega^2 r^2 (2sin(\beta)cos(\theta))dr \\ = &\quad 2\rho\omega^2 r^2 sin(\beta)cos(\theta)dr, \end{aligned} \tag{9}$$

where $\theta$ and $\beta$, are the directional angle of the airfoil profile, and the angles that determine the angular sector of the lengths $l$ on the curved profile, as shown in Figure 8. And, in the same way, it is possible to calculate the differential Drag (Eq. 6), i.e.,

$$\begin{aligned} dH = &\quad \rho\omega^2 r^2 \int_{\theta-\beta}^{\theta+\beta} sin(\theta_r) d\theta_r dr \\ = &\quad \rho\omega^2 r^2 (-cos(\theta_r)) \,|_{\theta-\beta}^{\theta+\beta} \, dr \\ = &\quad \rho\omega^2 r^2 (cos(\theta-\beta) - cos(\theta-\beta))dr \\ = &\quad \rho\omega^2 r^2 (-2sin(\beta)sin(\theta))dr \\ = &\quad -2\rho\omega^2 r^2 sin(\beta)sin(\theta)dr, \end{aligned} \tag{10}$$

Thus, integrating along the radius $r$ of the blade, from $R_0$ to $R_h$, and recalling that $dT_L = rdH$, thrust, $T$, and Load Torque, $T_L$, can be finally expressed for a $N$ blades propeller as

$$\begin{aligned} T = &\quad N \int_{R_0}^{R_h} 2\rho\omega^2 r^2 sin(\beta)cos(\theta)dr \\ T_L = &\quad N \int_{R_0}^{R_h} 2\rho\omega^2 r^2 sin(\beta)sin(\theta)dr \end{aligned} \tag{11}$$

Observe that the general form of the Equations (10) and (11), agrees with the conventional models expressing the thrust as a function of the square rotational speed, but the novelty is the expression showing explicitly the geometric parameters of the propeller.

### C. Example 2: Propeller with constant cross section blade

We illustrate the practical form of our approach calculating the thrust and the Load Torque of the same propeller as in the previous example. We assume a thin and uniform curved propeller with a constant cross section $l = c$ (or in equivalent manner $\beta(r) = \beta$), and a constant pitch $\theta(r) = \theta$, then the forces of thrust, $T$, and Load Torque, $T_L$, using the Equation 11 can be expressed as

$$\begin{aligned} T = &\quad N\tfrac{2}{3}\rho\omega^2 r^3 sin(\beta)cos(\theta) \\ T_L = &\quad N\tfrac{1}{2}\rho\omega^2 r^4 sin(\beta)sin(\theta). \end{aligned} \tag{12}$$

Observe, that these expression are simpler than those obtained with the traditional approach (Equation 4) and they describe the total thrust and the Load Torque in a very convenient form for simulation analysis or designing methods. However, their validity if founded on the assumption of a laminar airflow, imposing constrains in the selection of the parameters $l$ and $\theta$ to assure the condition of a small angle of attack at the leading edge.

## IV. Discussion

The model of the differential thrust and Load Torque obtained using the Blade Element Theory and the Euler Equation of Fluids (Eq.9, Eq.10) is a very simple expression in terms of geometric parameters of the blade. It is given in terms of geometric parameters of the propeller and this could be useful for simulation analysis but specially, in the designing methods under optimization conditions. In these equations, compared with the expressions obtained using de conventional approach, associate the Lift and Drag coefficients (Eq.2) to sinusoidal functions of the angle of attack (Fig.6), i. e., proportional to $cos(\theta)$ and to $sin(\theta)$. In a general matter, this description agree with experimental data reported in the literature, see for instance, the graph showed in Figure 3, where the Lift and Drag coefficients, on a profile NACA 0012, are presented as a function of the angle of attack [17]. Both coefficient along the full range of $\theta$, behaves qualitatively similar to the predicted sinusoidal functions, even if the profile NACA 0012 is completely different of the thin and continuously curved profile assumed in our approach. Nevertheless, our approach is based on the existence of a laminar flow determined by the parameter $l$ and $R_c$ of the propeller, and then imposing a geometrical constrain on the possible range of the angle $\theta$.

The existence of a laminar flow requires the transversal length of the blade element of being long enough to assure a continuous path of the streamlines when it gets in contact with it, i.e., $l$ is such that the tangent surface of the airfoil at the leading edge present a small angle of attack with respect to the incoming airstream in such a way that the effects of vorticity can be neglected. Otherwise, the cross section of the blade behaves as an aifoil with a large angle of attack as illustrated in Figure 9, positive or negative according to the geometric dimension of the blade, in any case, the movement of the propeller induces more Drag than Lift, affecting its performance.

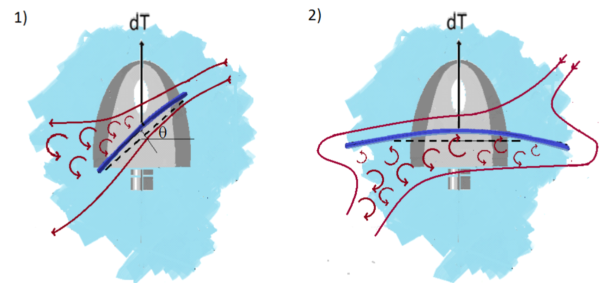


Fig. 9. Example of undesired situations on the blade element: a)$l$ is too small given $\theta$; b)$l$ is large and the radius $R_c$ is mall with $\theta = 0$

The angle of attack of the airfoil surface at the leading edge, $\theta_{le}$ is a critical parameter rather than the general orientation of the airfoil, $\theta$, to get the laminar condition. When the angle $\theta_{le}$ is small, streamlines are deviated by the surface of the airfoil and induce on them a radial acceleration that redirect the speed of the airflow without modifying its magnitude, initially given by the induced velocity of the turning blade element $v_i = \omega r$. This speed is modified with a vertical component when $\theta_{le}$ is not small enough and the magnitude of the induced velocity is $v_i = \omega r cos(\theta_{le})$. Observe that, in this case, the vertical and horizontal components of the speed are related by geometrical constrains and it can be shown that given its direction in a defined point of the blade, it is independent of the rotary speed of the blade. The assumption of laminar

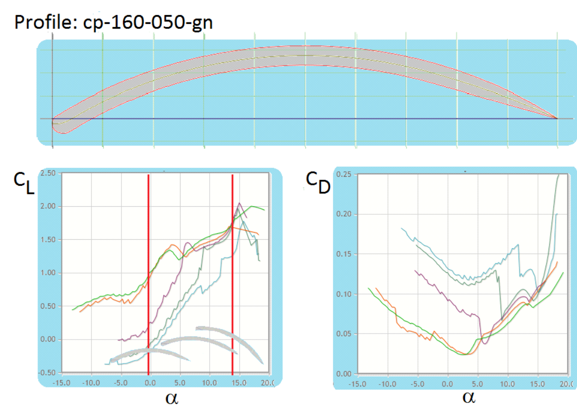


Fig. 10. Simulation data of Lift and Drag coefficient with respect to variation of the angle of attack at different induced velocity with Reynols cyan $5E4$, darkgreen $1E5$, purple $2E5$, orange $5E5$ and green $1E6$.

airflow in a constant-curvature thin aifoil facing small variation of the angle of attack at the leading edge can be appreciated in simulation data of the coefficient of Lift and Drag with respect to the angle of attack $\theta$. When the laminar condition is attain, the induced velocity is almost constant and presents small variation proportional to the angle of attack, this is reflected in the behaviour of the Lift and Drag coefficients. The Lift coefficient tends to be constant or with a slight slope, and the Drag one, with an abrupt diminution when the mode of the laminar flow is reached. This phenomenon can be appreciated in simulation data, showed in Figure 9, obtained using a profile cd-160-050-gn with variation of the angle of attack from -5 to 20 degrees, for different range of velocity from 300 rpm to 7000 rpm, in air at 10 C [18]. It is important to remark that this behaviour is very sensible to the radius of curvature presented by the airfoil, when it is almost constant, the laminar airflow can be appreciate on the simulation data even if the profile is not uniformly width (for example profiles GOE 531 or GOE 804 -EA8-), but when the radius of curvature is large and changes along the airfoil, it is not very clear to appreciate it (for example using the profiles GOE 417A, NACA M25, davissm, CH10-48-13). In any case, further research is required to stablish design criteria to determine the parameters length $l$, curvature radius $R_c$, and twist to assure the optimal performance of the propeller.

## V. Conclusion

In this paper, we presented a model of the thrust and Load-Torque produced by a turning small scale propeller used in miniature quadrotors in hover mode. The model, expressed in terms of the angular speed of the rotor, is characterized by its simplicity and by the inclusion of geometric parameters of the cross section of the blade instead of experimental coefficients (radius of curvature, transversal superficial length, angle of attack), and it is intended to provide a simulation tool for design an control purposes of high performance actuator for UAVs.

The model is obtained using the Blade Element Theory but instead of the conventional approach based on the coefficient of Lift and Drag, it innovates with the application of the Euler Equation of Fluids to calculate the forces acting along the differential airfoils of the blade. It takes advantage of the propellers generally used in quadrotors, thin and convex, that can be designed in a configuration that favours the appearance of a laminar airflow condition, which is appropriated for a simple description using the Euler Equation.

This approach is consistent with the traditional description explaining the forces on the airfoil in a Bernoulli's equation basis or in a Newtonian approach of the conservation of momentum, but imposes geometric conditions on the propeller profile in order to attain a laminar airflow where the vorticity can be neglected; These conditions, relates the length $l$ and the radius of curvature $R_c$ with the angle of attack $\theta$ in a trade-off concerning the general performance of the propeller. This relation are very useful to determine designing criteria to improve its performance, for example, combining the cross-section length and the twist in a blade to increase the thrust and reduce the drag and the sensibility to crosscurrents.

Even if we found some evidence, in experimental and numerical simulation data equivalent to the operational range of a small propeller used in miniature UAV, that laminar airflow mode is attainable supporting the assumptions of the model, further experimental research is needed, specially considering the update of the traditional designing criteria to cover the case of the small scale propellers.

## References

[1] G.M. Hoffmann, H.Huang, S.L. Waslander and C.J. Tomlin, *Quadrotor Helicopter Flight Dynamics and Control: Theory and Experiment*, Proceedings of the AIAA Guidance, Navigation and Control Conference, 2007.

[2] O. Fritsch, B. Henze and B. Lohmann *Fast and Saturating Attitude Control for a Quadrotor Helicopter*, Proceeding of the European Control Conference (ECC) 2013.

[3] T. Wang, L. Wang, J. Liang, Y. Chen *Autonomous Control and Trajectory Tracking of Quadrotor Helicopter*, IEEE International Conference on Computer Science and Automation Engineering (CSAE) 2012

[4] R. Martinez-Alvarado, F. J. Ruiz-Sanchez, A. Sanchez-Orta, O. Garcia-Salazar *Dynamic Response of BLDC-Thruster for Small Scale Quadrotors under Aerodynamic Load Torque*, Reunion of Power, Electronics and Computing IEEE (ROPEC) 2014.

[5] J. B. Brandt and M. S. Selig, *Propeller Performance Data at Low Reynolds Number*, Proceedings of the AIAA Aerospace Science Meeting, 2011.

[6] Guo-qing Zhang, Shu-xing Yang, Youg Xu, *Investigation of Propeller/Wing Interaction Base on the Sliding Mesh Technology*, Proceedings of the 2nd International Conference on Modelling, Simulation and Visualization Methods, 2010.

[7] P. J. Bristeau, Ph. Martin, E. Salatin, N. Petit, *The Role of Propeller Aerodynamics in the Model of a Quadrotor UAV*, Proceedings of the European Control Conference 2009.

[8] A. Gessow and G. C. Myers, *Aerodynamics of the Helicopter*, Frederick ungar Publishing Co., 1967.

[9] D. I. Bazov, *Helicopter Aerodynamics*, Transport Press Moscow, 1969.

[10] G. D. Padfield, *Helicopter Flight Dynamics: The Theory and Applications of Flying Qualities and Simulation Modeling*, Blackwell Publishing, Second Edition, 2007.

[11] J. Seddon, *Basic Helicopter Aerodynamics*, BSP Professional Books, 1990.

[12] M. Heene, *Aerodynamic Propeller Model for Load Analysis*, Dissertation M.Sc. in Scientific Computing, Royal Institute of Technology, 2012.

[13] Charles N. Eastlake, *An Aerodynamicist View of Lift, Bernoulli and Newton*, The Physics Teacher, Vol. 40, March 2002, pp 166-173.

[14] K. Weltner, *A comparison of explanations of the aerodynamic lifting force*, American Journal of Physics, 55 (1), January 1987.

[15] W. Beaty, *Airfoil Lifting Force Misconception Widespread in K-6 Textbooks*, Science Hobbyist, http://amasci.com/wing/airfoil.html,2016.

[16] Robert S. Merrill, *Nonlinear Aerodynamic Corrections to Blade Element Momentum Module with Validation Experiments*, M.Sc. Dissertation, Mechanical and Aerospace Engineering Department, Utah State University, 2011.

[17] National Advisory Committee for Aeronautics, *Aeronautic Characteristics of NACA 0012 Airfoil Section at Angles of Attack from 0? to 180?*, Technical note 3361, Langley Aeronautical Laboratory, 1955.

[18] Camberade plate c=16% T=5% R=0.86 (cp-160-050-gn), *Airfoil Tools*, http://www.airfoiltools.com/airfoil/details?airfoil=cp-160-050-gn, 2016.