

Move

Name	Dharini Baskaran
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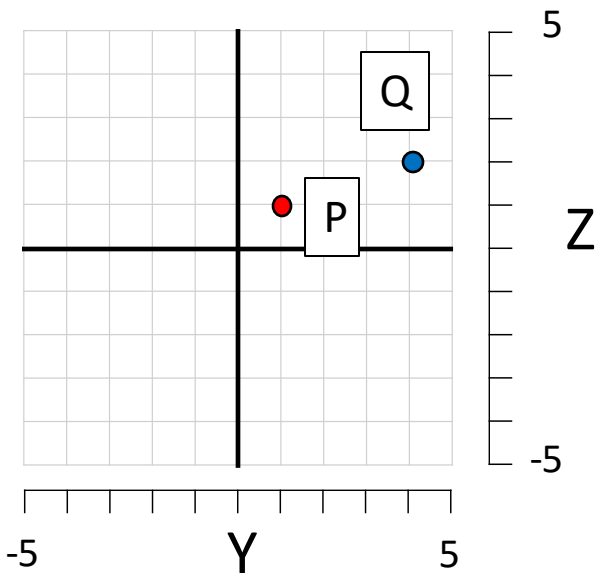
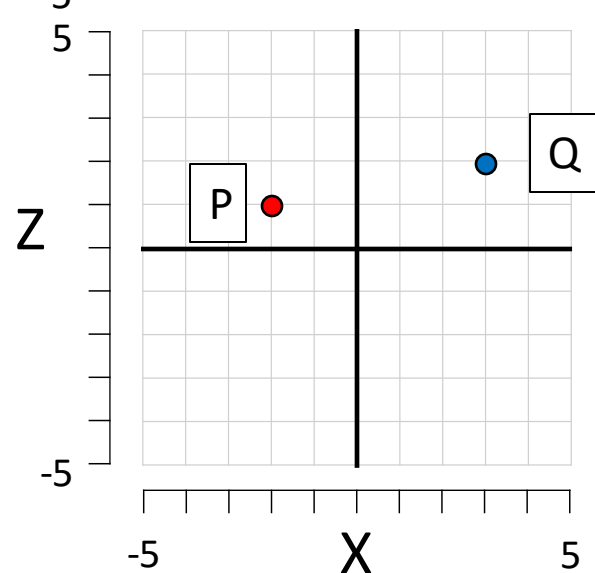
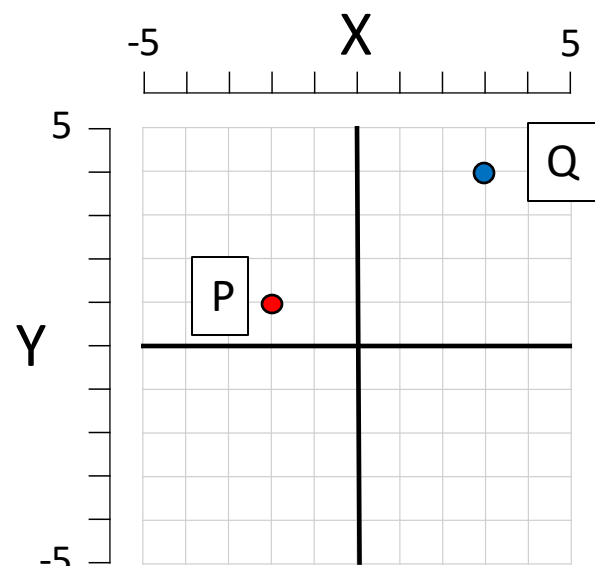
	Level	Completed	Goal	
	Beginner	9	4722	14
	Intermediate	8	5722	16
	Advanced	1	Total Completed	
	Expert	0	18	

1 ☒ ☐ Calculate T

$P(-2, 1, 1)$

$Q(3, 4, 2)$

$\Delta = (5, 3, 1)$

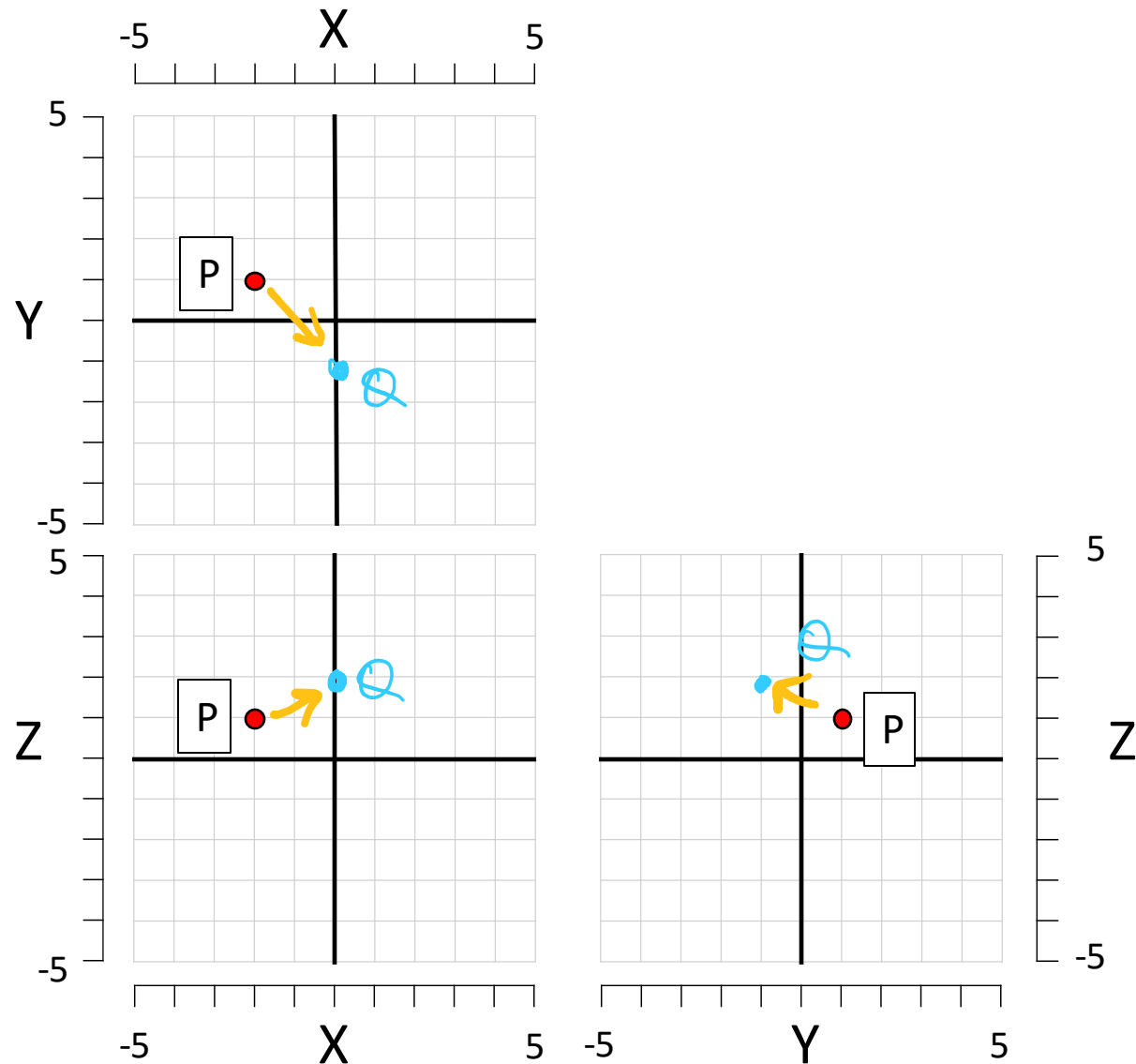


P
-2
1
1
1

T			
1			5
	1		3
		1	1
			1

T * P → Q
3
4
2
1

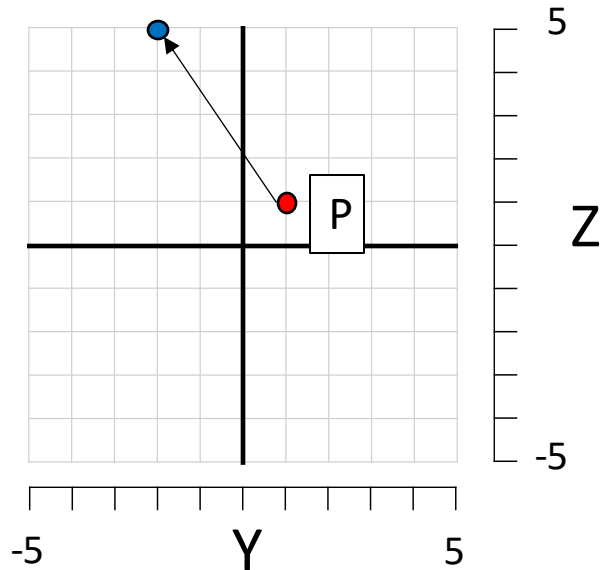
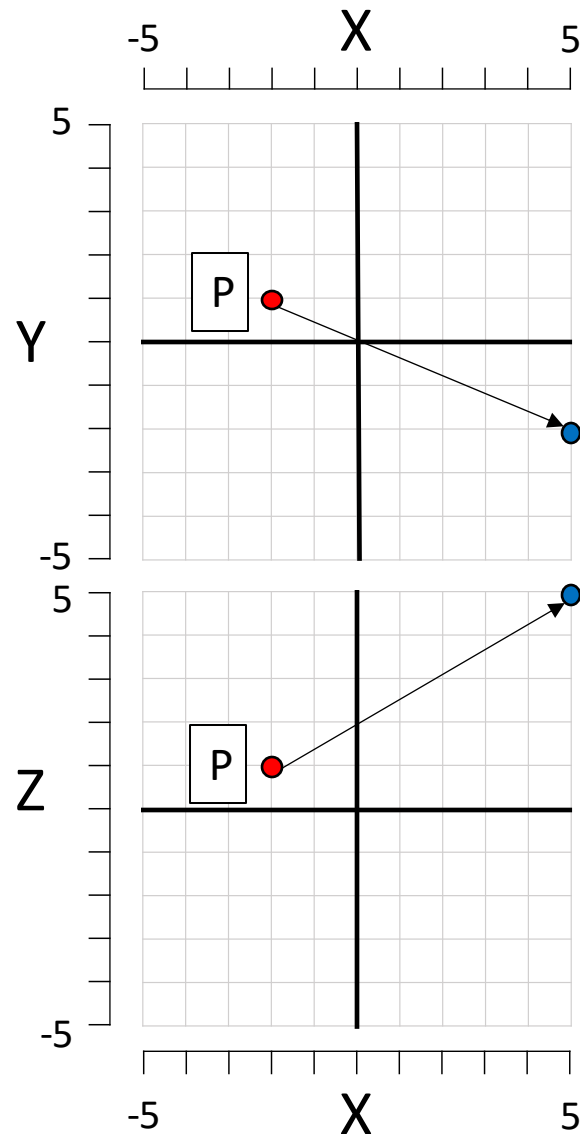
2 ☒ ☐ Calculate and Draw Q



P
-2
1
1
1

T				T * P → Q	
1			2	0	-2 + 2
	1		-2	-1	1 - 2
		1	1	2	1 + 1
			1	1	1

3 ☒ ☐ Calculate T and Q



$$P(-2, 1, 1)$$

$$Q(5, -2, 5)$$

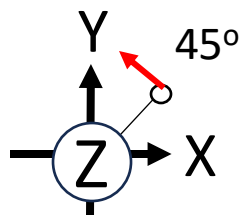
$$\Delta = (7, -3, 4)$$

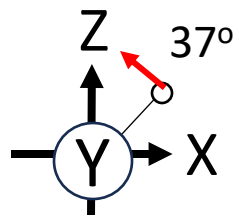
P	
-2	
1	
1	
1	

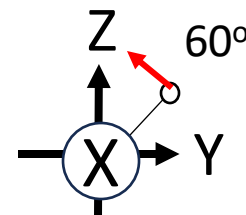
T			
1			7
	1		-3
		1	4
			1

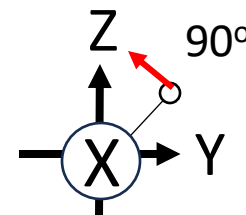
T * P → Q	
5	-2 + 7
-2	1 - 3
5	1 + 4
1	1

4 ☒ 3D rotation notations → Matrices



$$\begin{matrix} & \begin{matrix} X \\ Y \\ Z \end{matrix} \\ \begin{matrix} X \\ Y \\ Z \end{matrix} & \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix}$$


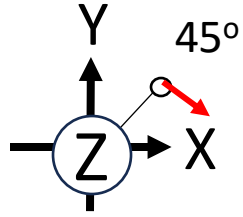
$$\begin{matrix} & \begin{matrix} X \\ Y \\ Z \end{matrix} \\ \begin{matrix} X \\ Y \\ Z \end{matrix} & \begin{bmatrix} \frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & 1 & 0 \\ \frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix} \end{matrix}$$


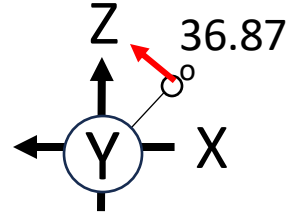
$$\begin{matrix} & \begin{matrix} X \\ Y \\ Z \end{matrix} \\ \begin{matrix} X \\ Y \\ Z \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \end{matrix}$$


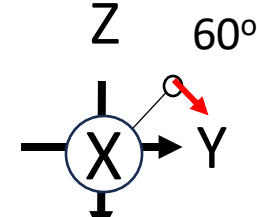
$$\begin{matrix} & \begin{matrix} X \\ Y \\ Z \end{matrix} \\ \begin{matrix} X \\ Y \\ Z \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

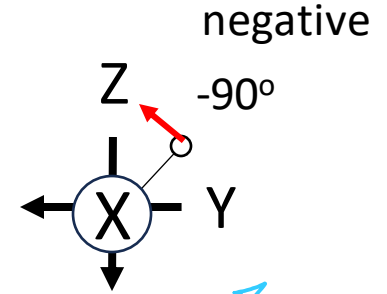
degrees	30	37	45	53	60	90
$\sin(\theta)$	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos(\theta)$	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0

5 ☒ ☐ 3D rotation notations → Matrices



$$\begin{matrix} & \begin{matrix} x & y \end{matrix} \\ \begin{matrix} c & s \end{matrix} & \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \\ -s & c \end{bmatrix} \end{matrix}$$


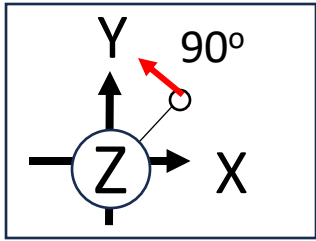
$$\begin{matrix} & \begin{matrix} c & s \end{matrix} \\ \begin{matrix} x & z \end{matrix} & \begin{bmatrix} 4/5 & -3/5 \\ 3/5 & 4/5 \end{bmatrix} \end{matrix}$$


$$\begin{matrix} & \begin{matrix} c & s \end{matrix} \\ \begin{matrix} y & z \end{matrix} & \begin{bmatrix} 1 & 0 \\ 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} \end{matrix}$$


$$\begin{matrix} & \begin{matrix} c & s \end{matrix} \\ \begin{matrix} y & z \end{matrix} & \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix} \end{matrix}$$

degrees	30	37	45	53	60	90
sin(θ)	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
cos(θ)	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0

6 ☒ ☐



Calculate and Draw Q

$P(2, -4, -1)$

degrees	30	37	45	53	60	90
$\sin(\theta)$	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos(\theta)$	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0

P

2
-4
-1
1

Q

4
2
-1
1

R

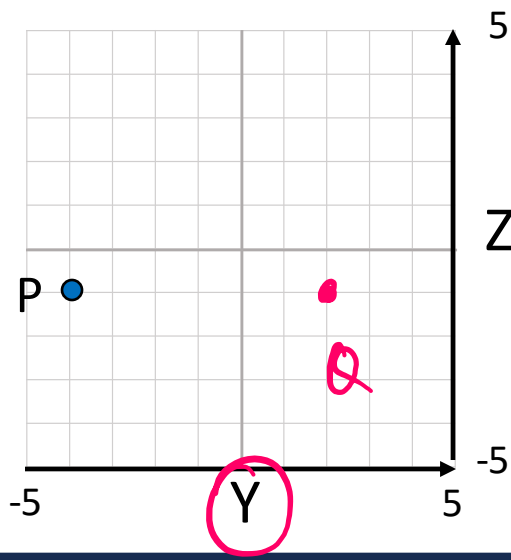
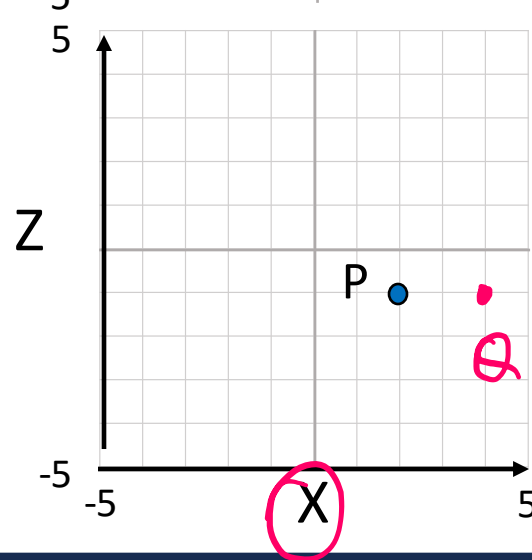
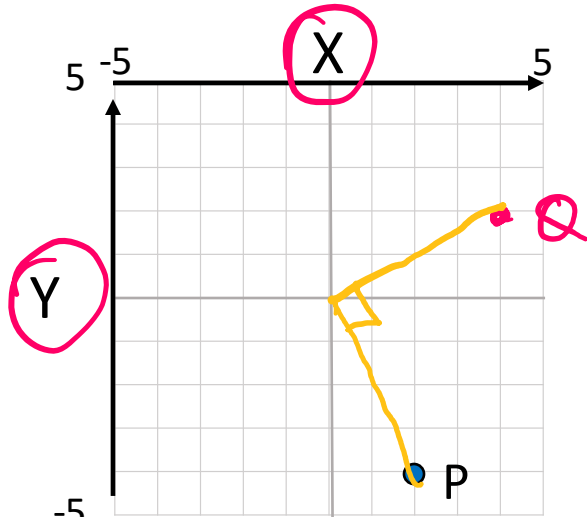
-5

R*P

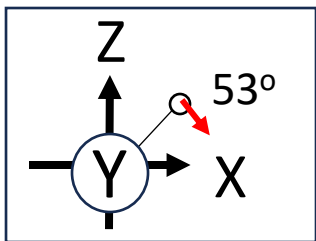
C

S

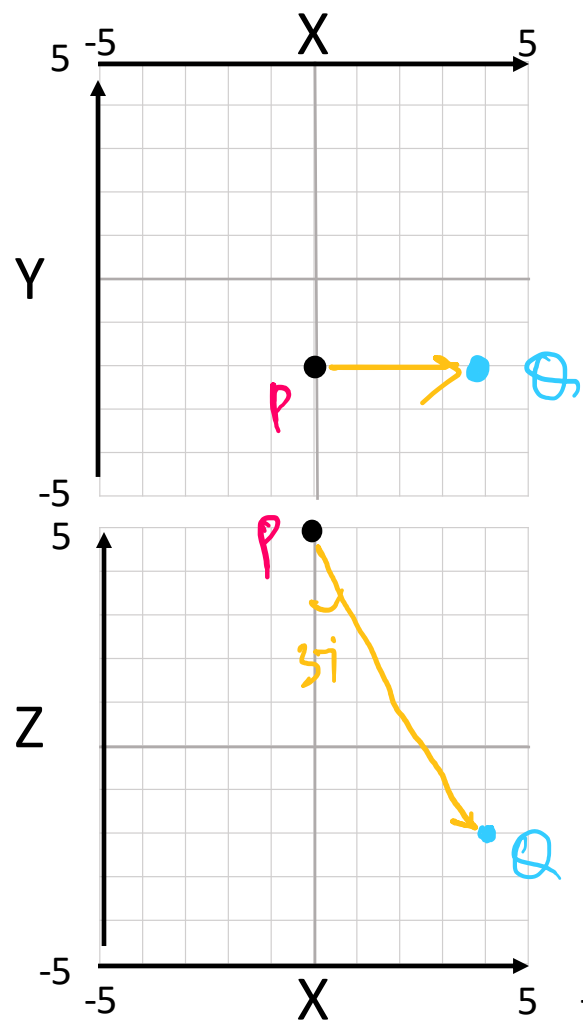
0	-1		0
1	0		0
	C	1	0
			1



7 ☒ ☐

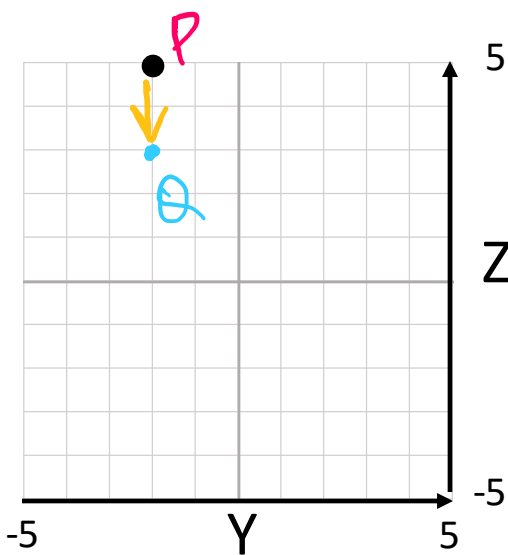


Rotate p along Y by (-53°)
Calculate and draw Q



degrees	30	37	45	53	60	90
$\sin(\theta)$	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos(\theta)$	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0

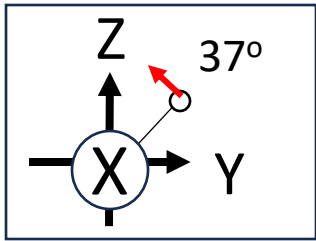
$P(0, -2, 5)$



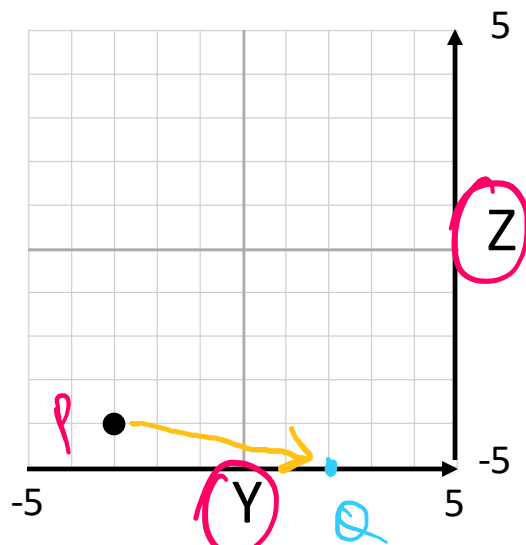
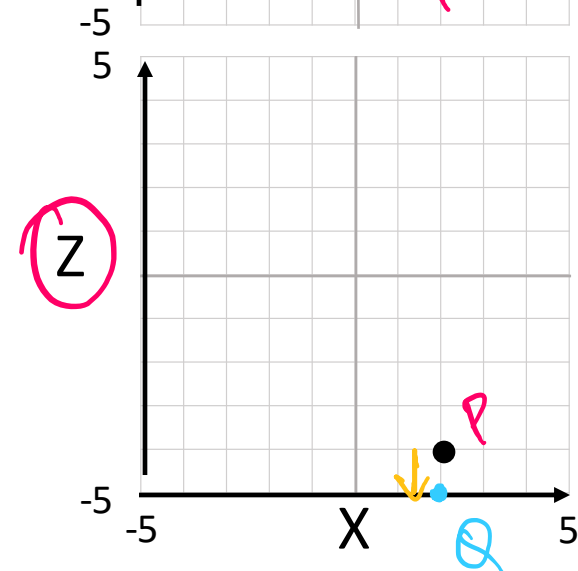
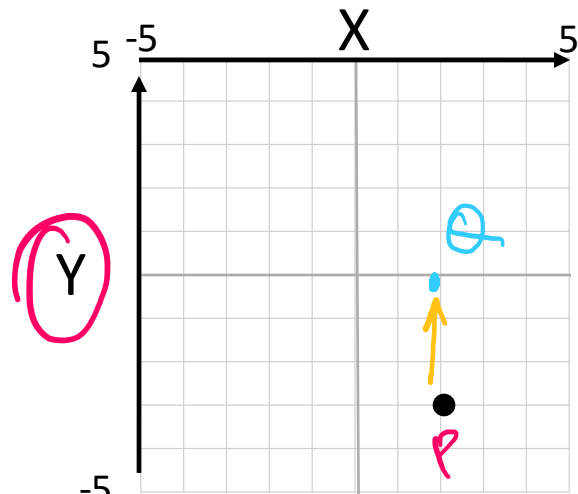
	R	x	S	R*P	Q
C	3/5		4/5	0	4
		1		0	-2
Z	-4/5		3/5	0	3
			C	1	1

P	0
	-2
	5
	1

8 ☒ ☐



Rotate p along X by (37°)
Calculate and draw Q



degrees	30	37	45	53	60	90
$\sin(\theta)$	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos(\theta)$	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0

R

1	c	-s	0
	$4/5$	$-3/5$	0
	$3/5$	$4/5$	0
	s	c	1

Z

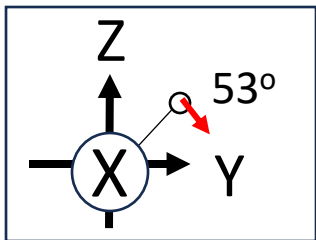
R*P

2
-3
-4
1

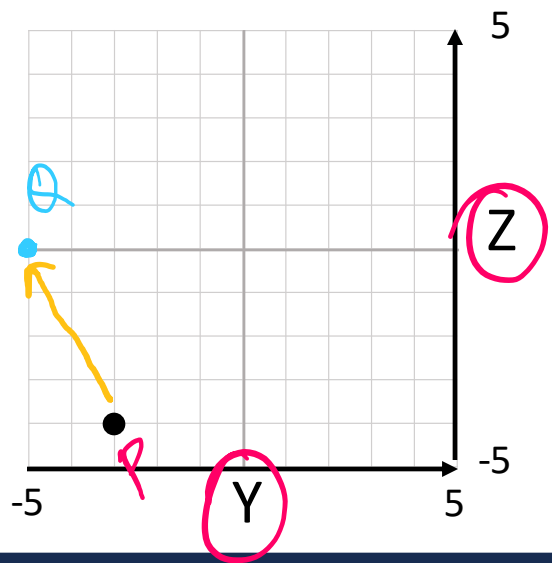
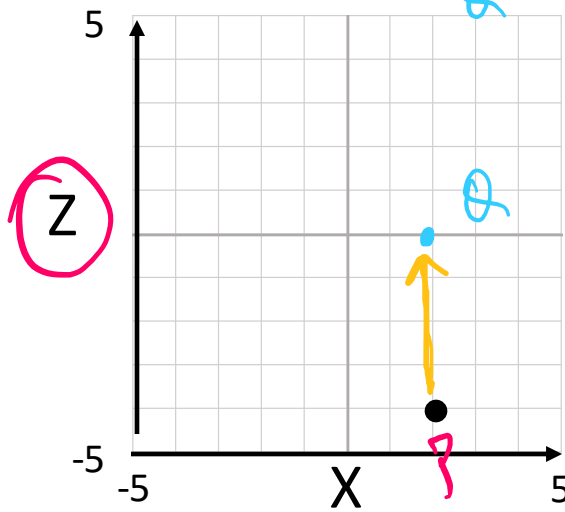
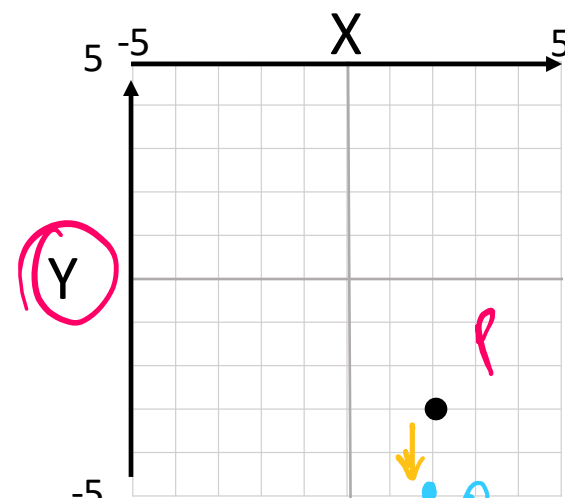
Q

2
0
-5
1

9 ☒ ☐



Rotate p along X by (-53°)
Calculate and draw Q



degrees	30	37	45	53	60	90
$\sin(\theta)$	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos(\theta)$	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0

R				y		R*P	
1	c	s	0	2		2	
	$3/5$	$4/5$	0	-5		-3	
	$-4/5$	$3/5$	0	0		-4	
	-s	c	1	1		1	

Q

P	2
	-3
	-4
A * P	1

10 ☒ Order of rotations

Given a set of rotations, R_1 , R_2 , and R_3 to apply to a point p . Consider two different sequence of rotations as follows:

$$q_1 = (R_1 * (R_2 * (R_3 * p)))$$

$$q_2 = (R_3 * (R_2 * (R_1 * p)))$$

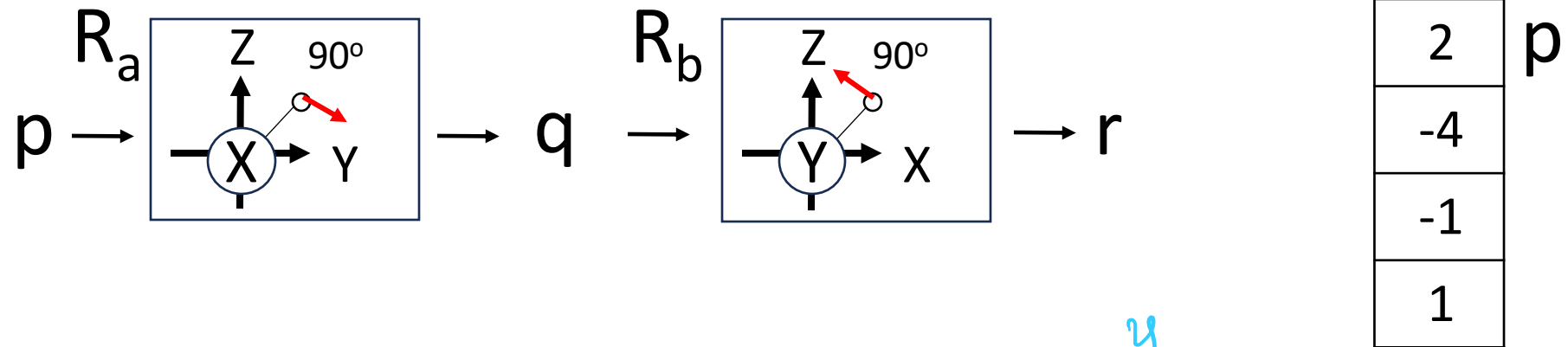
In general, which two statements are true

☐ A. $q_1 = q_2$ in 3D

☒ B. $q_1 \neq q_2$ in 3D

☒ C. $q_1 = q_2$ in 2D

☐ D. $q_1 \neq q_2$ in 2D



degrees	30	37	45	53	60	90
$\sin(\theta)$	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos(\theta)$	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0

The diagram illustrates the process of finding the reduced row echelon form (RREF) of matrix A .

Matrix A:

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -5 & 2 & 1 \end{bmatrix}$$

Row Operations:

- $R_2 \leftrightarrow R_3$ (Swap rows 2 and 3)
- $R_4 + 5R_3$ (Add 5 times row 3 to row 4)
- $R_1 - 2R_3$ (Subtract 2 times row 3 from row 1)
- $R_2 + R_3$ (Add row 3 to row 2)
- $R_4 - R_3$ (Subtract row 3 from row 4)

Resulting RREF:

$$R_{\text{a}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12

degrees R_1 R_2^y

p

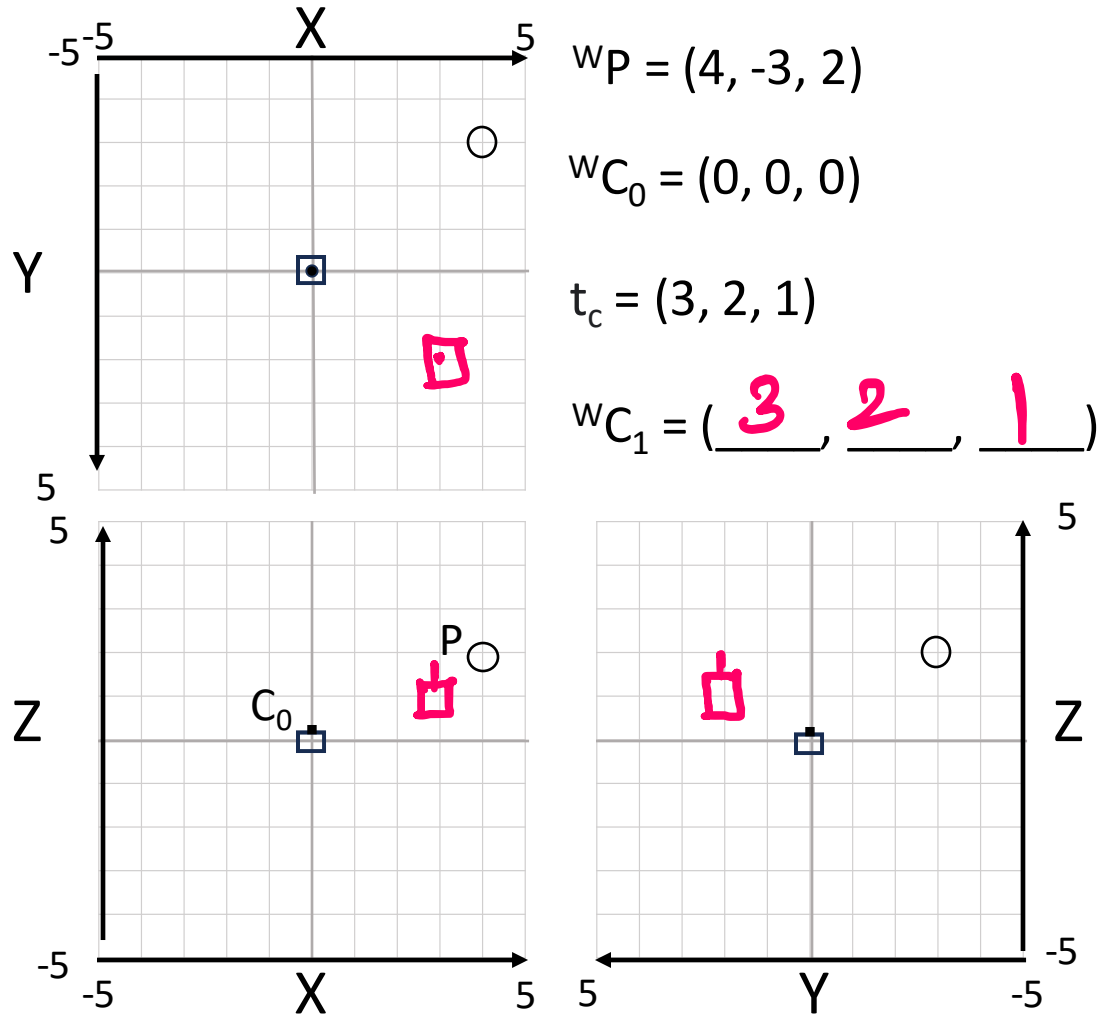
q

r

13 ☒ ☐ Move the camera in the world; Calculate and draw WC_1

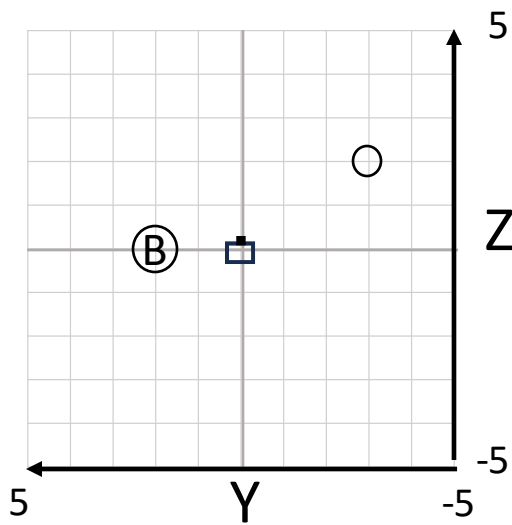
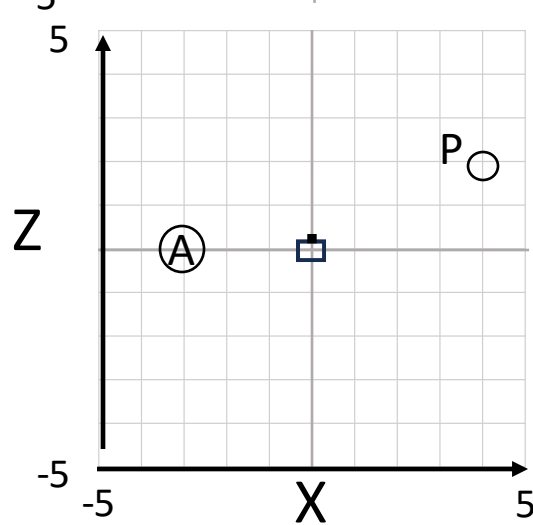
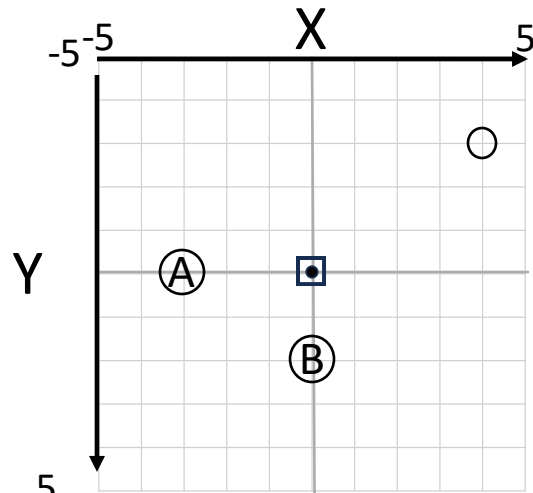
World Coordinate System

Reminder: In the world coordinate, Y's direction is downward in "axial", and left in "sagittal"



14 ☒ ☐ P moves in the Camera Coordinate system the Calculate and draw C_1P

World Coordinate System

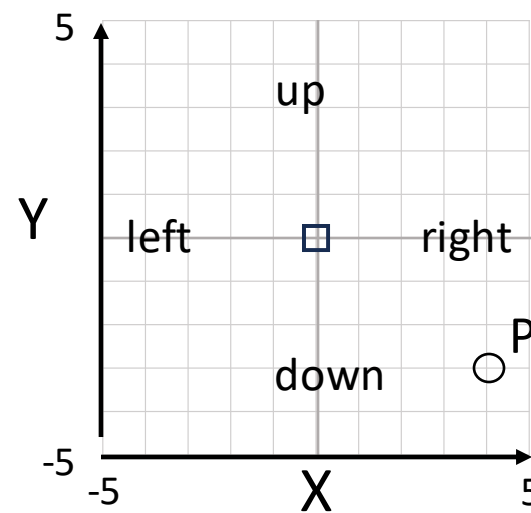


In the world, if the camera moves to A,
then in the camera view, P moved { left | right | up | down }

In the world, if the camera moves to B,
then in the camera view, P moved { left | right | up | down }

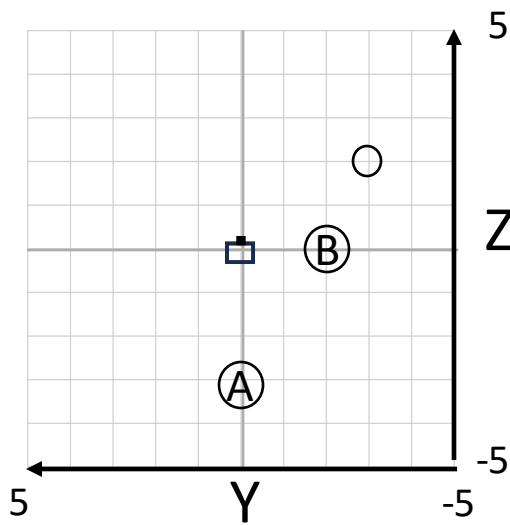
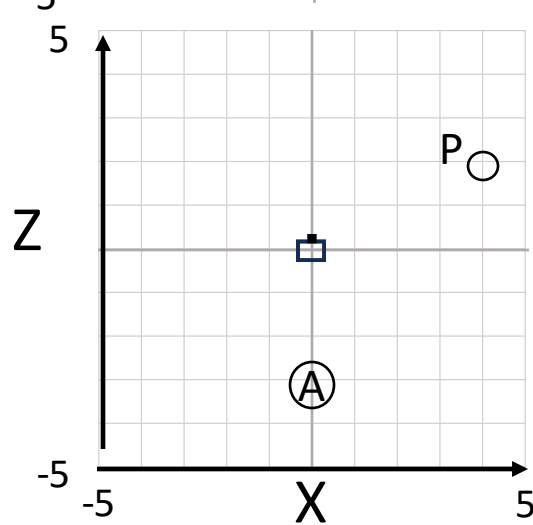
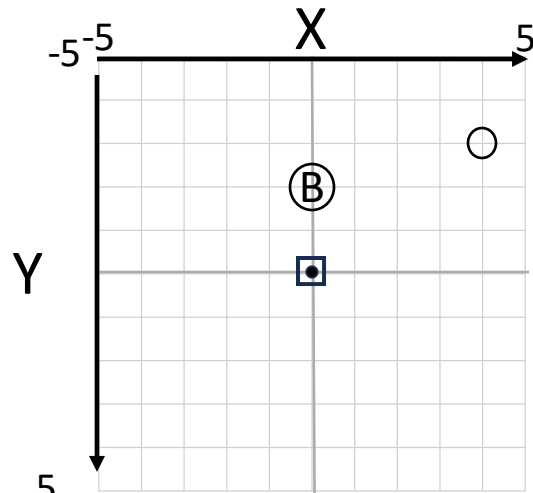
(Circle the direction. Hint: draw the body)

Camera Coordinate System
X-Y plane (coronal) only



15 ☒ ☐ P moves in the Camera Coordinate system the Calculate and draw C^1P

World Coordinate System

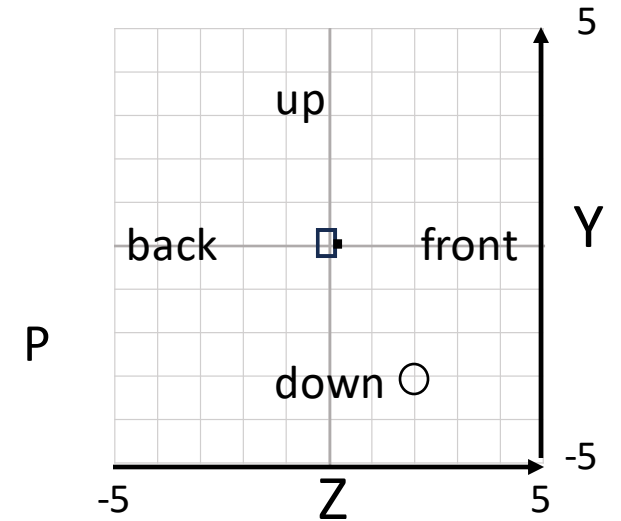


In the world, if the camera moves to A,
then in the camera view, P moved { front | back | up | down }

In the world, if the camera moves to B,
then in the camera view, P moved { front | back | up | down }

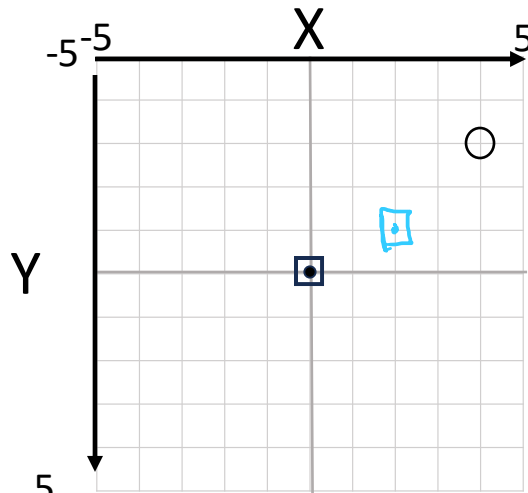
(Circle the direction. Hint: draw the body)

Camera Coordinate System
Z-Y plane (sagittal) only



16 ☒ ☐ Infer camera movement from point correspondence ${}^{C_0}p$ and ${}^{C_1}p$. Draw WC_1

World Coordinate System

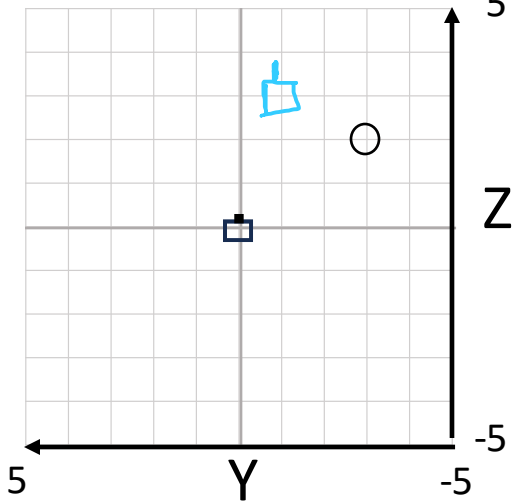
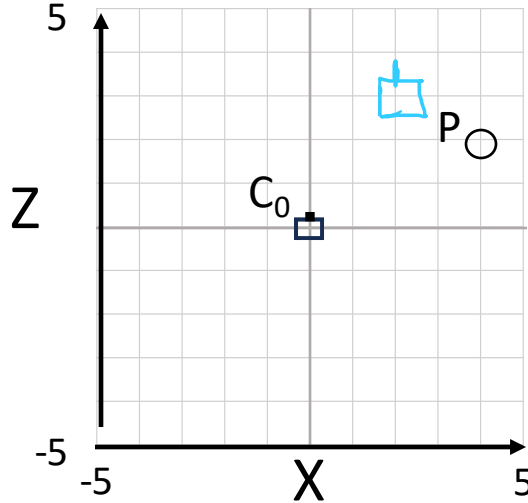


$${}^Wp = (4, -3, 2)$$

$${}^WC_0 = (0, 0, 0)$$

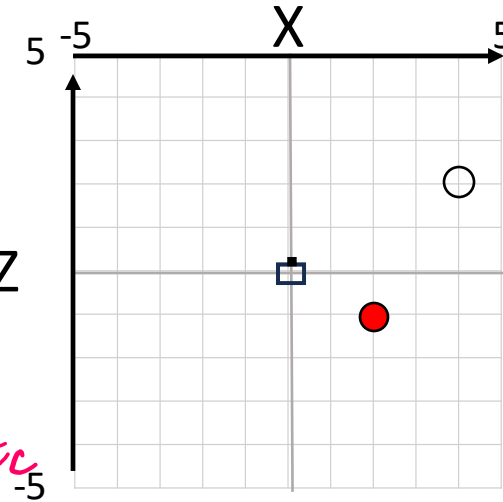
$$t_c = (\underline{2}, \underline{-1}, \underline{3}) \quad t_c = -t_p Z$$

$${}^WC_1 = (\underline{2}, \underline{-1}, \underline{3}) \quad {}^WC_1 = {}^WC_0 + t_c$$



Camera Coordinate System

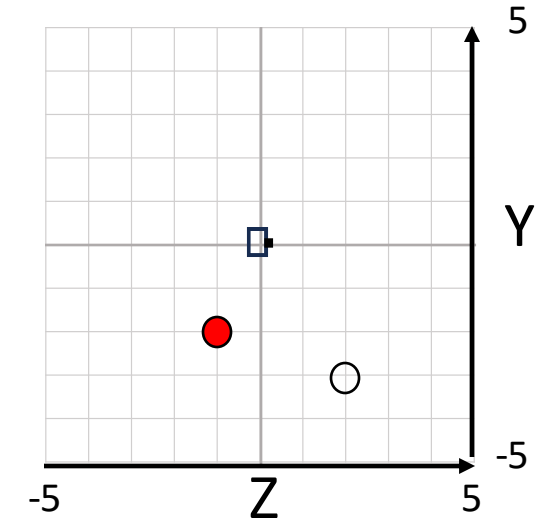
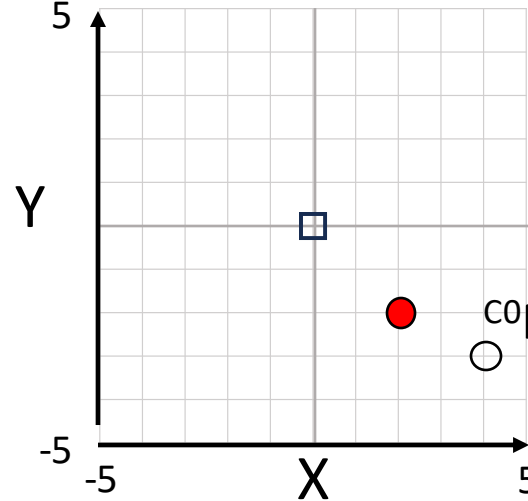
$${}^{C_1}p = {}^{C_0}p + t_p$$



$${}^{C_0}p = (4, -3, 2)$$

$$t_p = (\underline{-2}, \underline{1}, \underline{-3})$$

$${}^{C_1}p = (\underline{2}, \underline{-2}, \underline{-1})$$



17 ☒ ☐Calculate camera's movement: $T_c = (\underline{-1}, \underline{-4}, \underline{6})$

				P1	P2	P3
Tp	5	3	2			
	-4	0	1			
	4	5	5			
	1	1	1			
1	0	0	1	6	4	3
0	1	0	4	0	4	5
0	0	1	-6	-2	-1	-1
0	0	0	1	1	1	1

$$t_c = -t_p$$

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Calculate camera's movement: $T_c = (\underline{-2}, \underline{-5}, \underline{5})$

The measurements of point correspondences P and P' may not be perfect. Thus, you may derive a different T_p from each point correspondence.

Apply a simple "majority vote" to determine the T_p to solve this problem.

P1	P2	P3	P4	P5
5	5	3	3	2
-4	-3	0	1	1
4	4	5	5	5
1	1	1	1	1

T_p

1	0	0	2
0	1	0	5
0	0	1	-5
0	0	0	1

P1'	P2'	P3'	P4'	P5'
7	7	5	5	4
1	1	5	5	6
-1	-1	0	0	0
1	1	1	1	1