

# Filter

Name	Dharini Baskaran
Identity Key	dhba5060

	Level	Completed	Goal	
	Beginner	11	4722	13
	Intermediate	5	5722	15
	Advanced	1	Total Completed	
	Expert	0	17	

# Filter

CSCI 5722/4722: Computer Vision

Spring 2024

Dr. Tom Yeh

Dr. Mehdi Moghari

# 1D Filters

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# 1D Signal

$I[i]$	1	4	5	-2	3	7	9	6	8	-1	2
--------	---	---	---	----	---	---	---	---	---	----	---

# Element-wise Operation

I[i]	1	4	5	-2	3	7	9	6	8	-1	2
*	2										
							18	12	16	-2	4

# Cross Correlation

$I[i]$ 

1	4	5	-2	3	7	9	6	8	-1	2
---	---	---	----	---	---	---	---	---	----	---

$H[u]$ 

1	0	-1
---	---	----

1	0	-1
---	---	----

--	--	--	--	--	--	--	--	--	--	--

# Neighborhood

--	--	--	--	--	--	--	--	--	--	--

1	4	5	-2	3	7	9	6	8	-1	2
---	---	---	----	---	---	---	---	---	----	---

--	--	--	--	--	--	--	--	--	--	--



# Neighborhood

Neighbor

2 to the left

			a				b			
X	X	7	3	2	4	0	0	7	3	2

7	3	2	4	0	0	7	3	2	4	0
---	---	---	---	---	---	---	---	---	---	---

2 to the right

2	4	0	0	7	3	2	4	0	X	X
---	---	---	---	---	---	---	---	---	---	---

c



$\Sigma a = 7$ ;  $\Sigma b = 7$ ;  $\Sigma c = 12$



# Cross Correlation as Matrix Multiplication

Neighbor

**L1**

--	--	--	--	--	--	--

**C**

1	4	5	2	3	7	9
---	---	---	---	---	---	---

**R1**

--	--	--	--	--	--	--

--	--	--

--	--	--	--	--	--	--

--	--	--

--	--	--	--	--	--	--

--	--	--

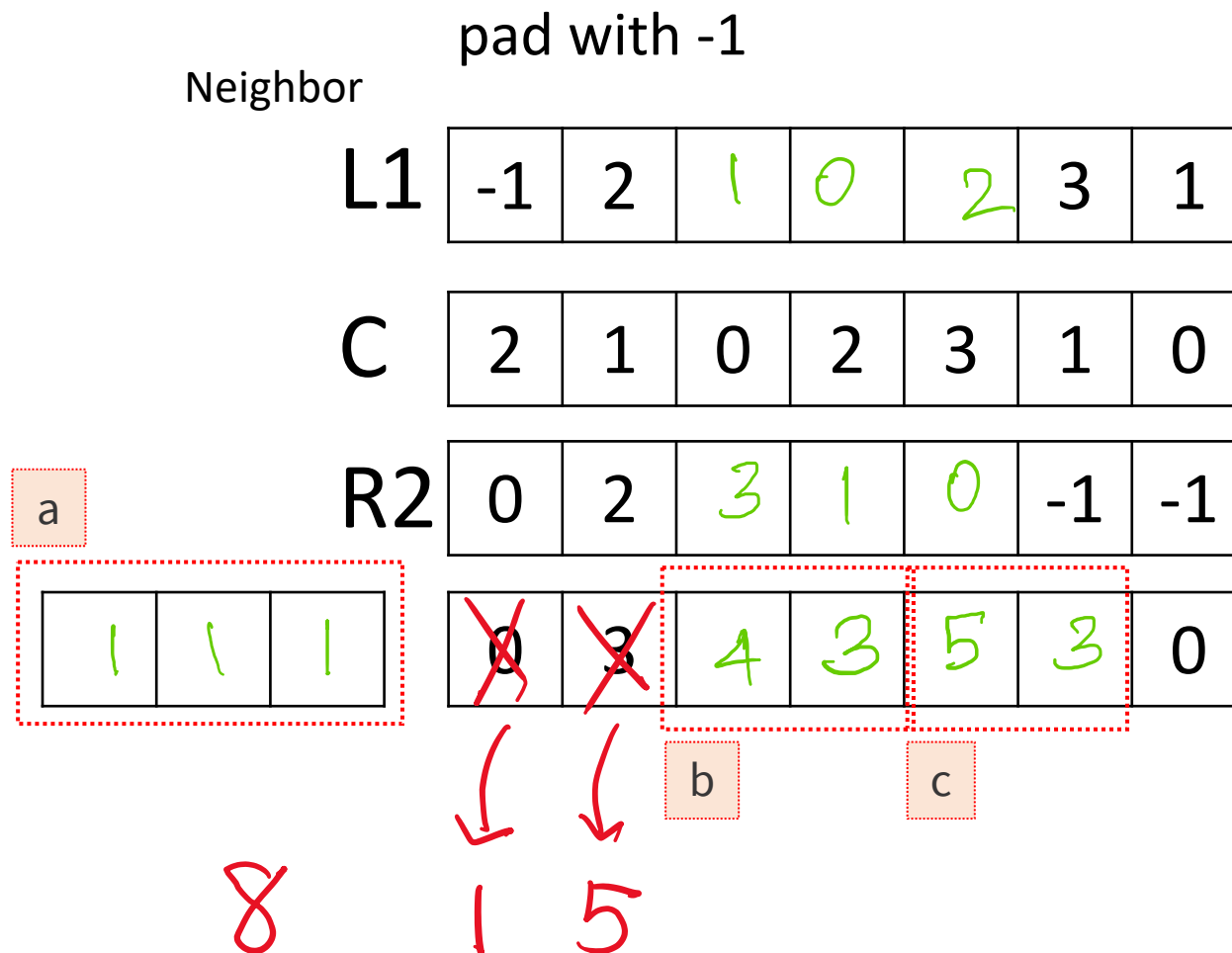
--	--	--	--	--	--	--

--	--	--

--	--	--	--	--	--	--



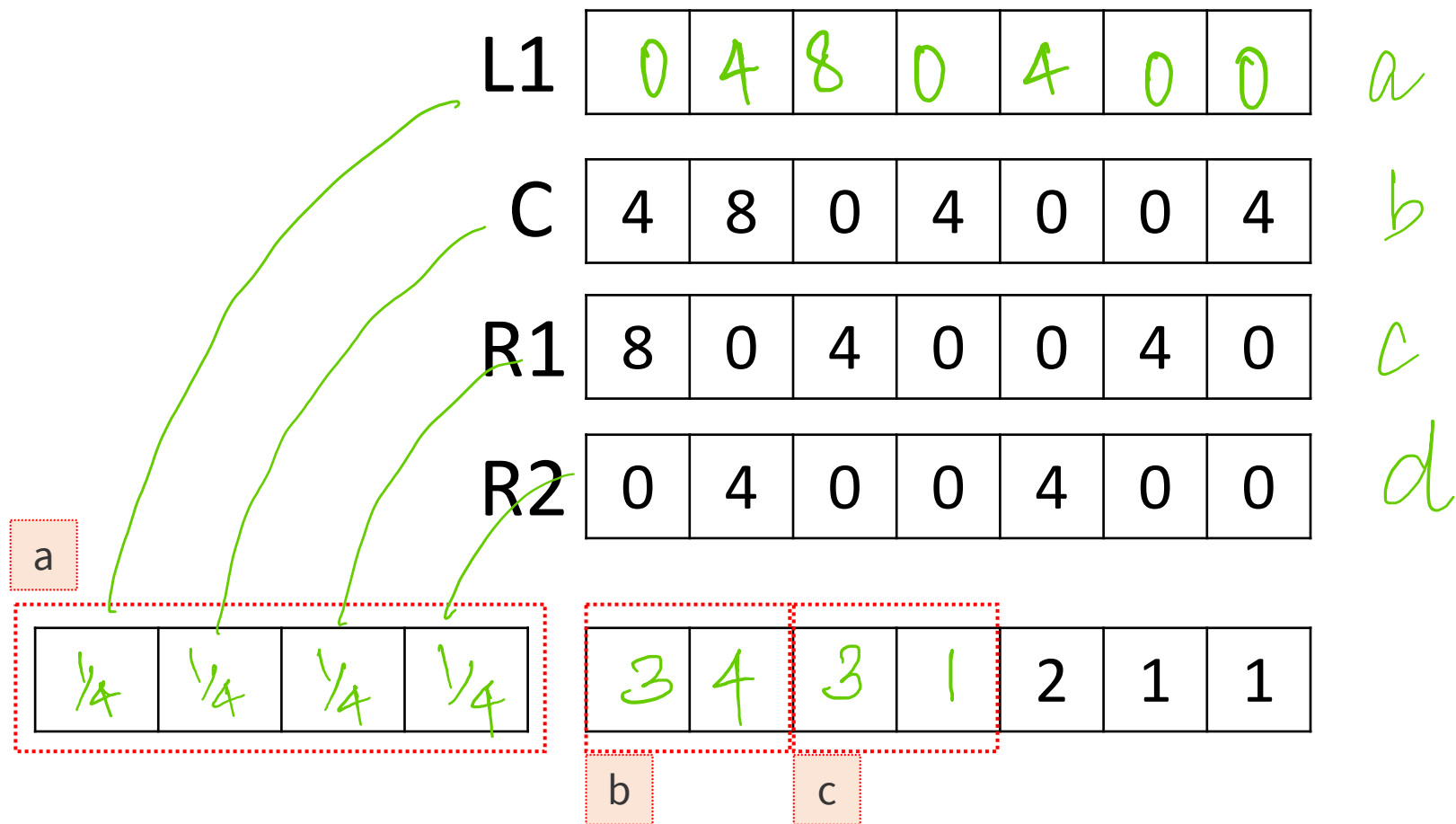
# Box filter



sum(a) = 3; sum(b) = 7; sum(c) = 7

# ☒ ☐ Mean filter

pad with 0



🔑  $\text{sum}(a) = 1; \text{sum}(b) = 7; \text{sum}(c) = 4$

# Up or Down?

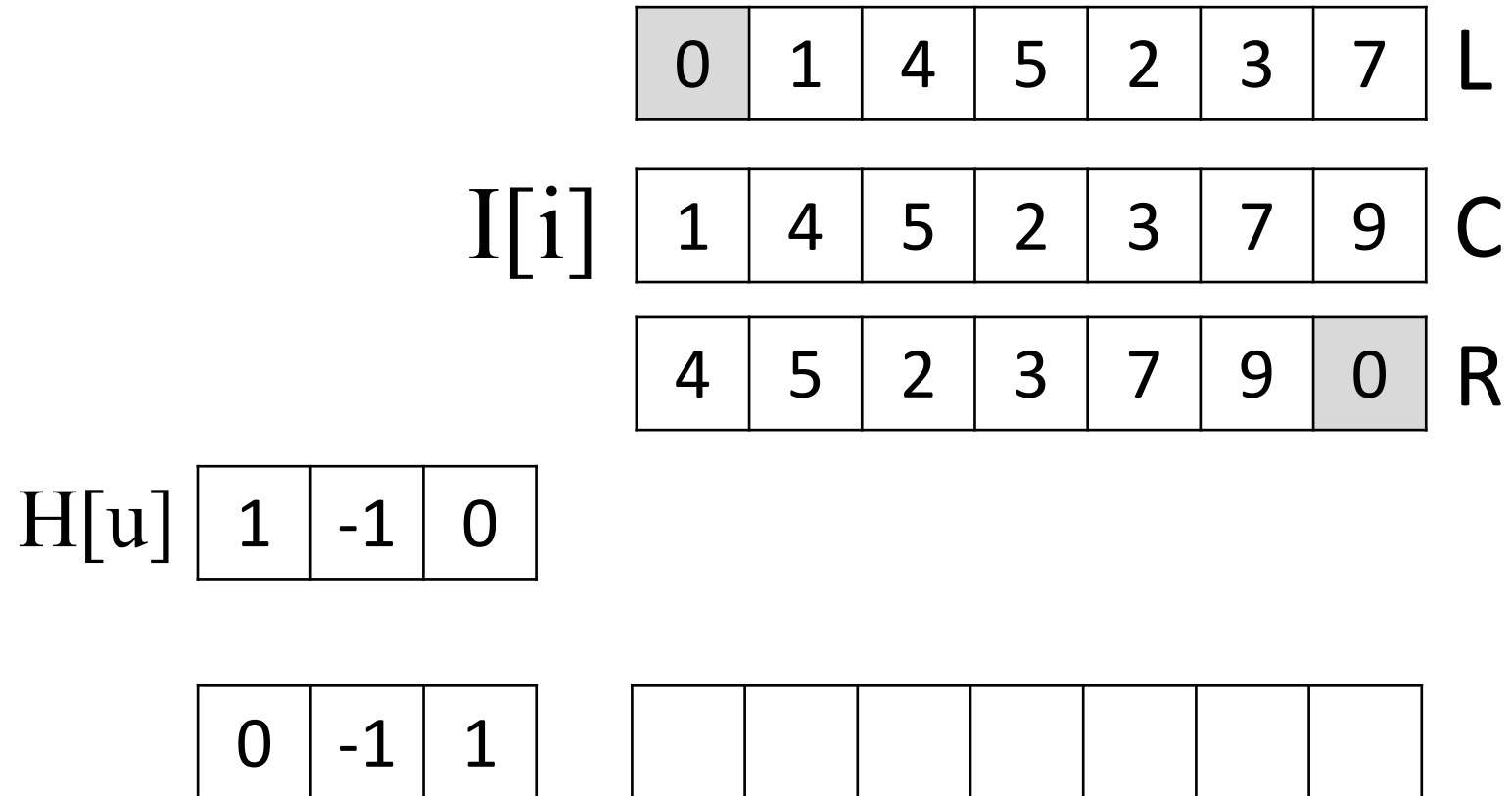
L	X	1	2	6	8	12	7	5	2	-2
C	1	4	6	8	12	7	5	2	-2	-5
R	4	6	8	12	7	5	2	-2	-5	X



# Math: Cross-Correlation $H \otimes F$

Diagram illustrating the state of the arrays  $L$ ,  $C$ , and  $R$  after the third iteration of the merge sort algorithm. The array  $L$  contains the elements  $[0, 1, 4, 5, 2, 3, 7]$ . The array  $C$  contains the elements  $[1, 4, 5, 2, 3, 7, 9]$ . The array  $R$  contains the elements  $[4, 5, 2, 3, 7, 9, 0]$ , with the element  $0$  highlighted in grey. Below these arrays, the array  $H[u]$  is shown with the first three elements  $[1, -1, 0]$  and the rest empty.

# Math: Convolution $H * F$





# Cross Correlation $\otimes$ vs. Convolution $*$

H[u] 

1	0	-1	0
---	---	----	---

0	1	2	0	2	1	0
---	---	---	---	---	---	---

 L1

I[i] 

1	2	0	2	1	0	2
---	---	---	---	---	---	---

 C

2	0	2	1	0	2	0
---	---	---	---	---	---	---

 R1

0	2	1	0	2	0	0
---	---	---	---	---	---	---

 R2

H[u] 

1	0	-1	0
---	---	----	---

0	-1	0	1
---	----	---	---

-2	1	0	-1	2	-1	0
----	---	---	----	---	----	---

 $H \otimes I$

-1	0	1	-2	1	0	-2
----	---	---	----	---	---	----

 $H * I$

🔑  $\text{sum}(a) = 1; \text{sum}(b) = 0$

# Properties

$$H \otimes F = F \otimes H$$

$$H * F = F * H$$

$$\begin{aligned} F \otimes H_1 \otimes H_2 &= \\ F \otimes (H_1 \otimes H_2) \end{aligned}$$

$$\begin{aligned} F * H_1 * H_2 &= \\ F * (H_1 * H_2) \end{aligned}$$



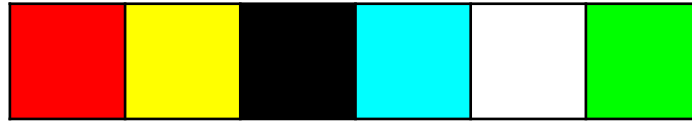
# Multiple Channels

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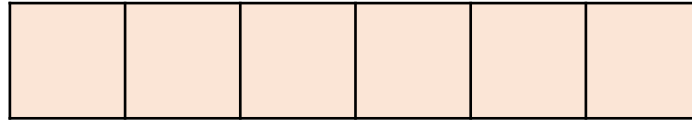


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# Color: floating value representation

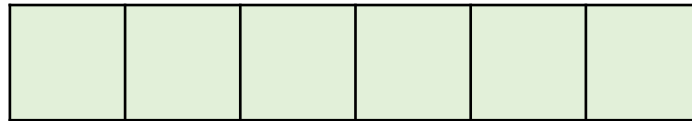


Red



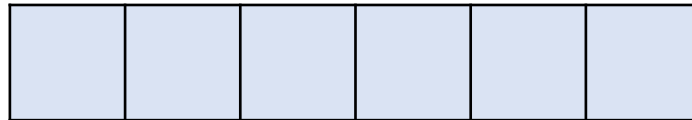
Range: [0, 1]

Green



Range: [0, 1]

Blue



Range: [0, 1]

# RGB → Grayscale



Red

1	1	0	0	1	0
---	---	---	---	---	---

Range: [0, 1]

Green

0	1	0	1	1	1
---	---	---	---	---	---

Range: [0, 1]

Blue

0	0	0	1	1	0
---	---	---	---	---	---

Range: [0, 1]

--	--	--

--	--	--	--	--	--

# RGB: Float $\rightarrow$ Integer



Red	1	1	0	0	1	0	Range: [0, 1]
Green	0	1	0	1	1	1	
Blue	0	0	0	1	1	0	
.	*						

Red							Range: [0, 255]
Green							
Blue							

# Scaling 1D Filtering

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# Single Channel

1	1	0
0	1	1

0	1	2	3						
1	2	3							
2	3								


# Two Channels

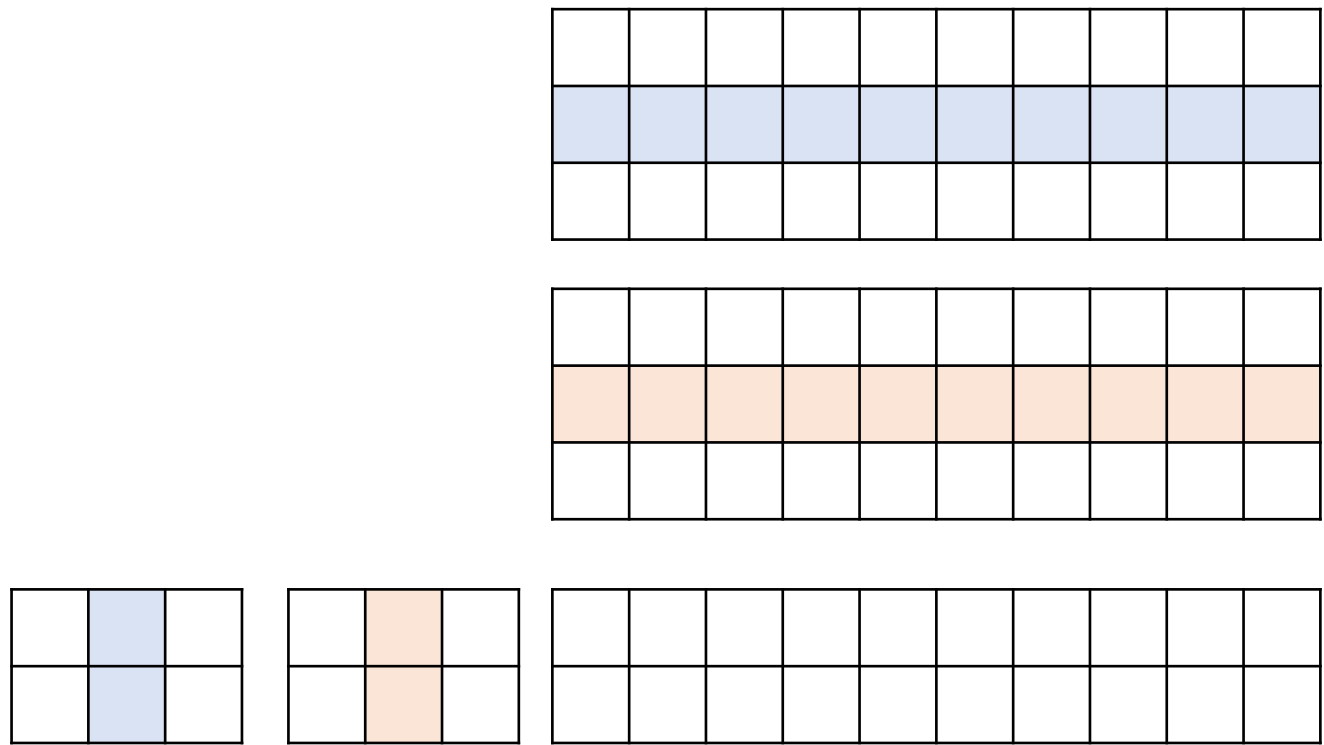
0		-1							
	-1								
-1									

0	1	2	3						
1	2	3							
2	3								

0	0	1
1	0	0

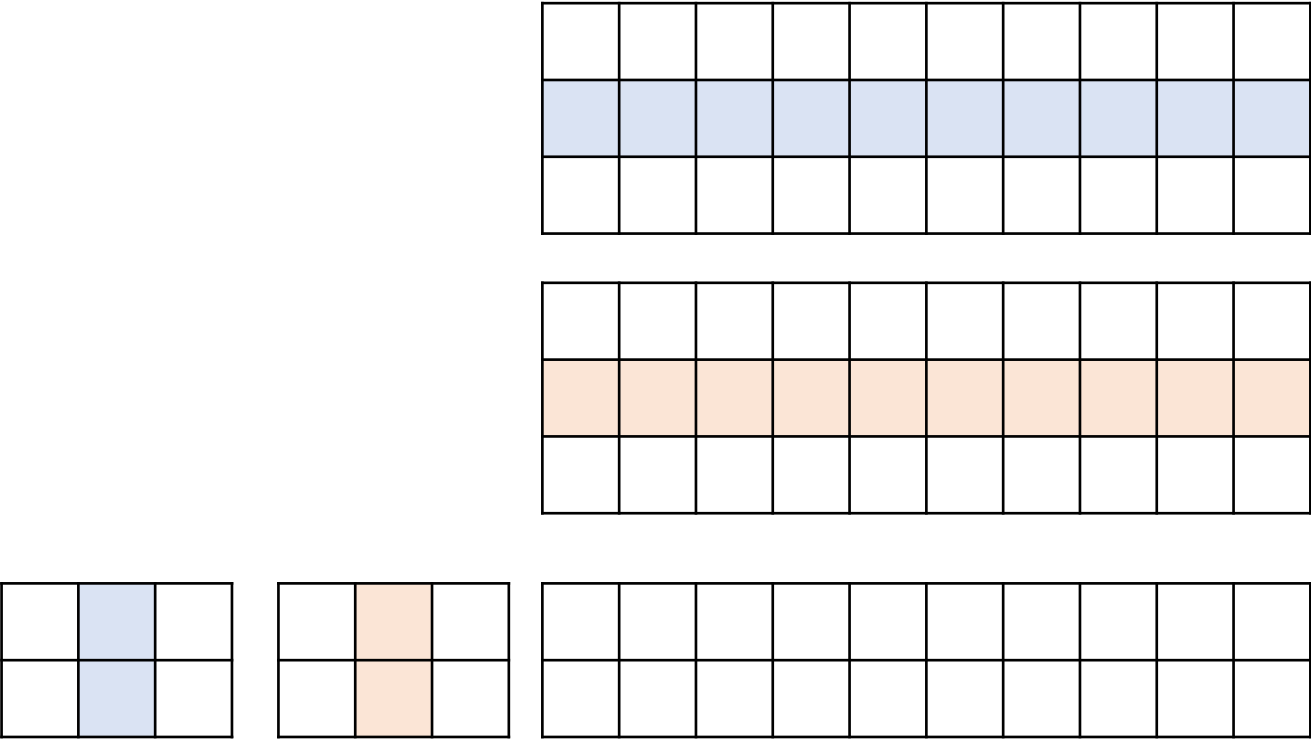
1	1	0
0	0	1


# Add One Channel

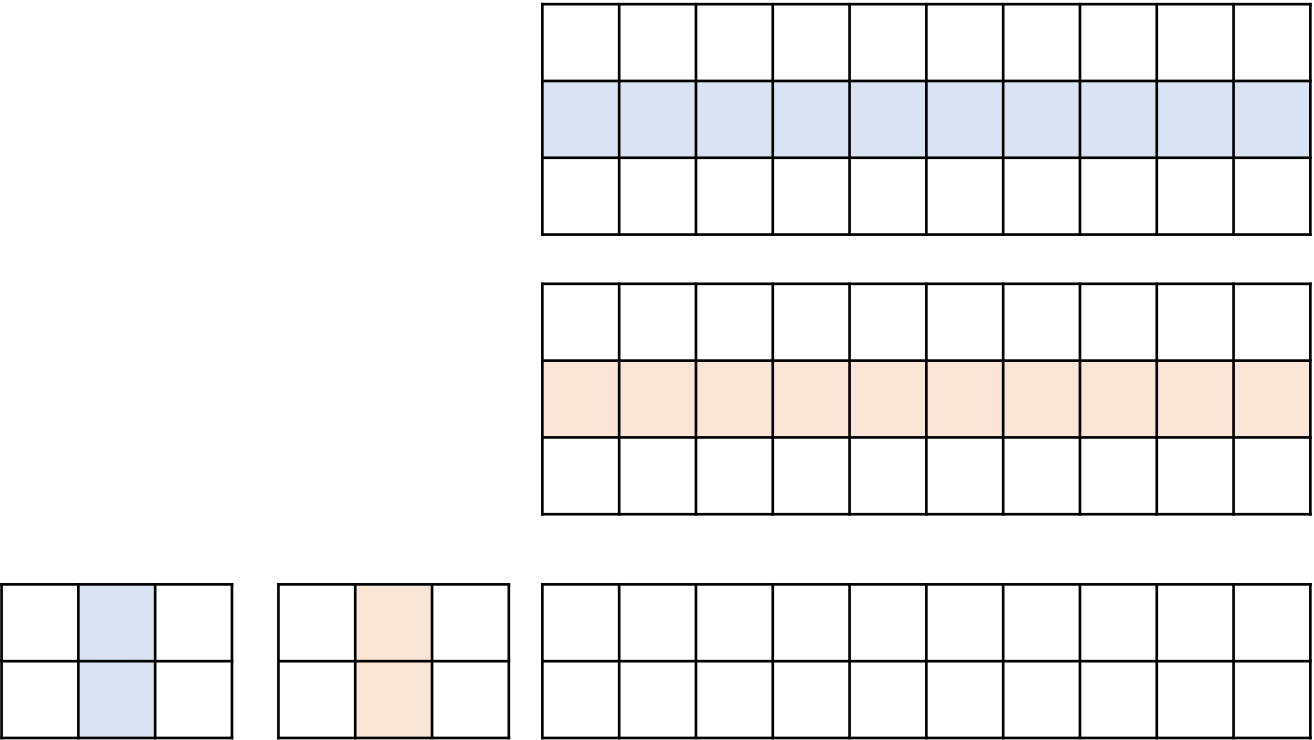




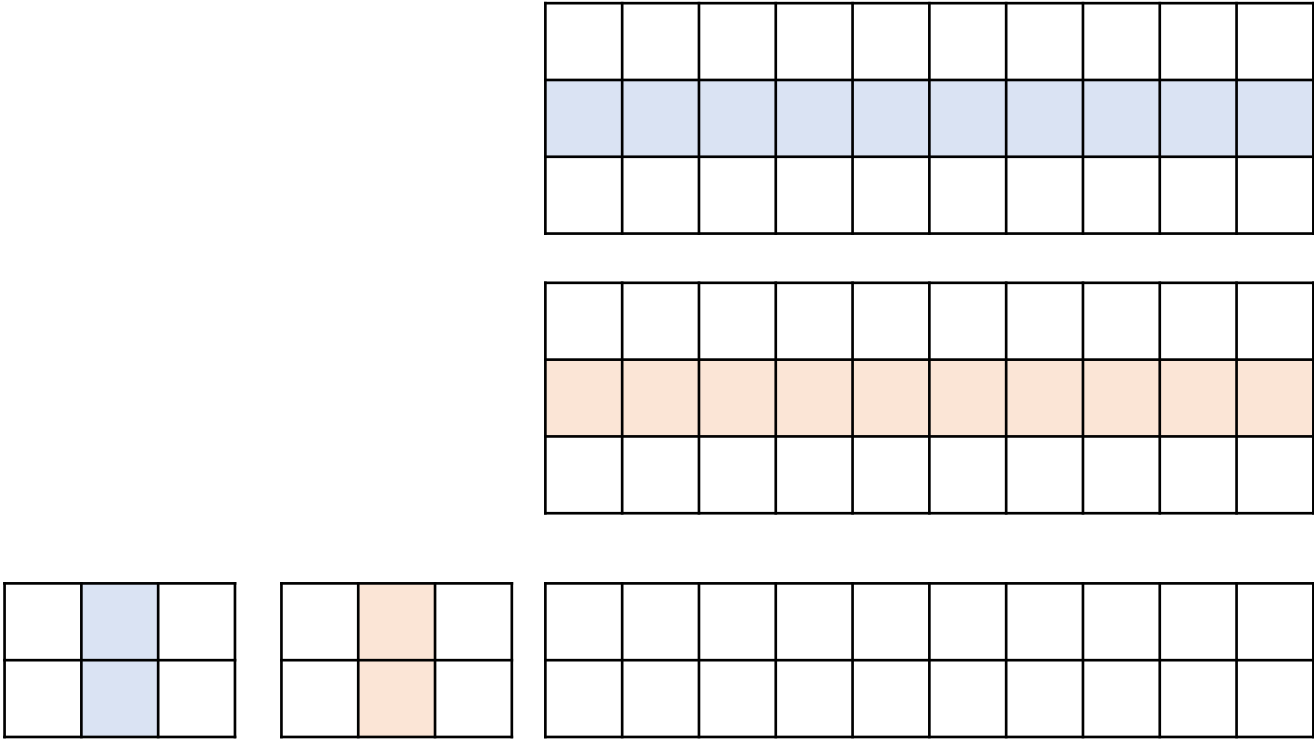
# Add One Filter



# Add One Neighbor



# Add One Input



# Add 1 Neighbor

Copy, add rows and columns  
Show your work

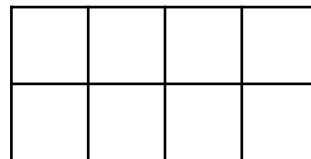
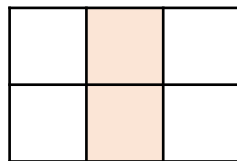
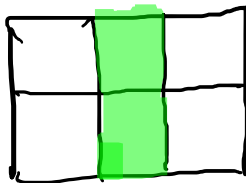
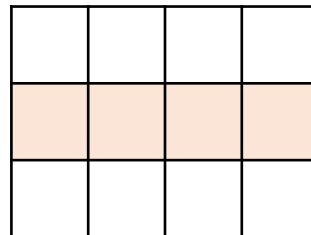
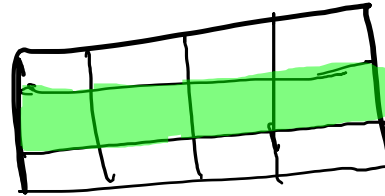



Number of new cells added = \_\_\_\_\_



# Add 1 Channel

Copy, add rows and columns  
Show your work



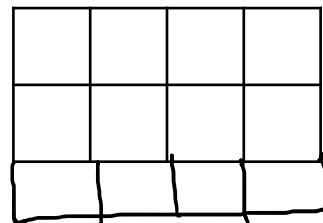
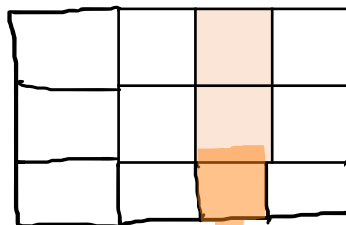
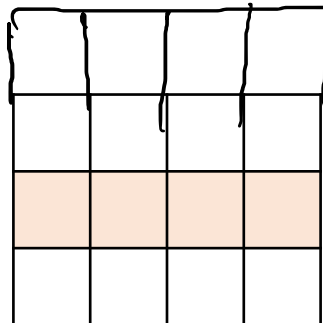
$$a \% 4 = 2$$

Number of new cells added = <sup>a</sup> 18



# Add 1 Neighbor + 1 Filter

Copy, add rows and columns  
Show your work



a

Number of new cells added = 14



$$a \% 6 = 2$$



# Add 2 inputs

Copy, add rows and columns  
Show your work




a

Number of new cells added = 10

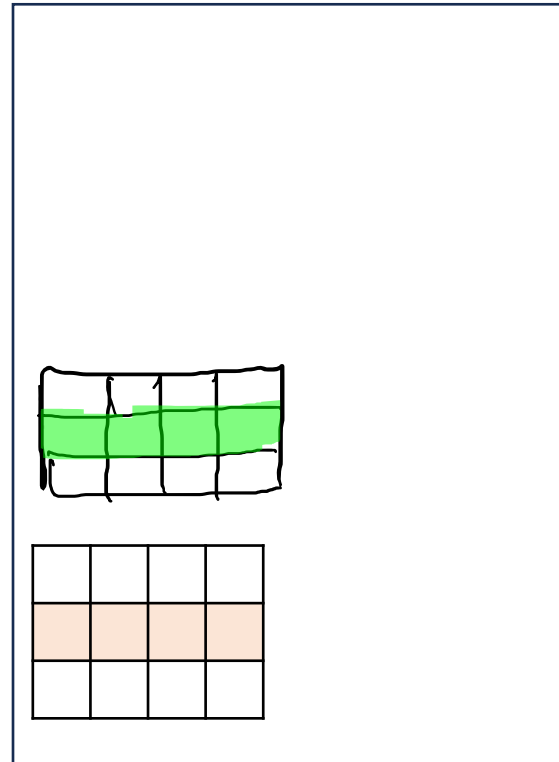


$a \% 3 = 1$

# ☒ ☐ Scale to 2 channels, 3 filters

Copy, add rows and columns  
Show your work

K



X

Z

	a	b
shape(K) = (	3	6
shape(X) = (	6	4
shape(Z) = (	3	4

🔑 sum(a)=12; sum(b)=14

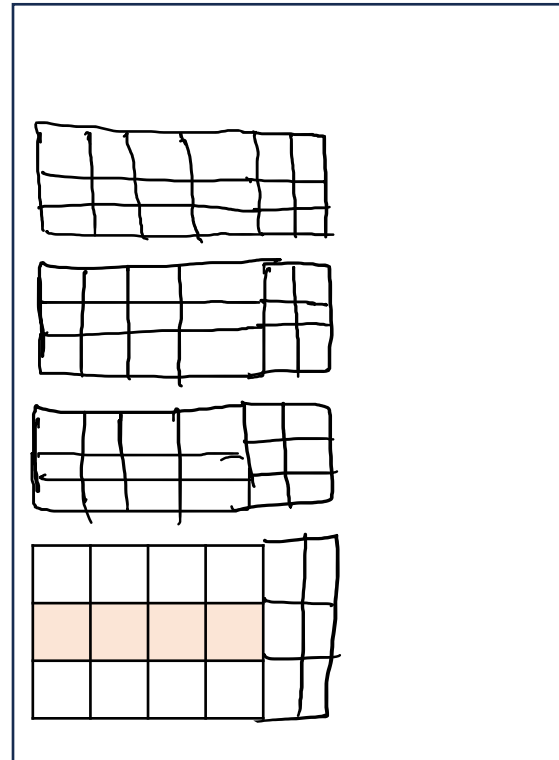
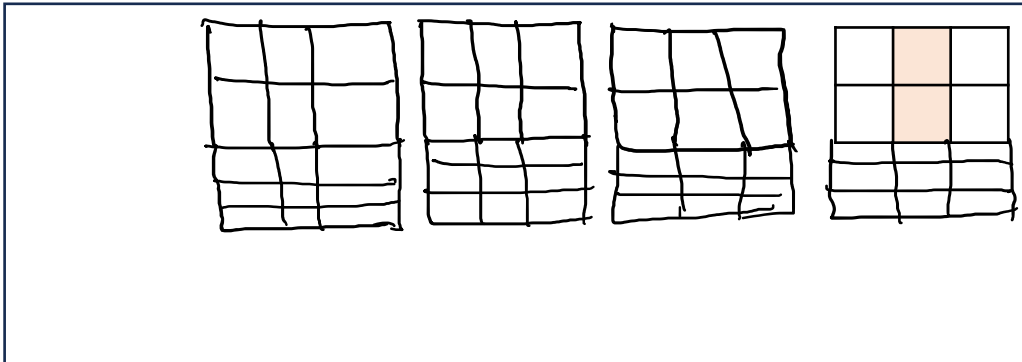




# Scale to 4 channels, 5 filters, 6 inputs

Copy, add rows and columns  
Show your work

K



X

Z

	a	b
shape(K) = (	5	12
shape(X) = (	12	6
shape(Z) = (	5	6

key sum(a)=22; sum(b)=24

# 2D Filters

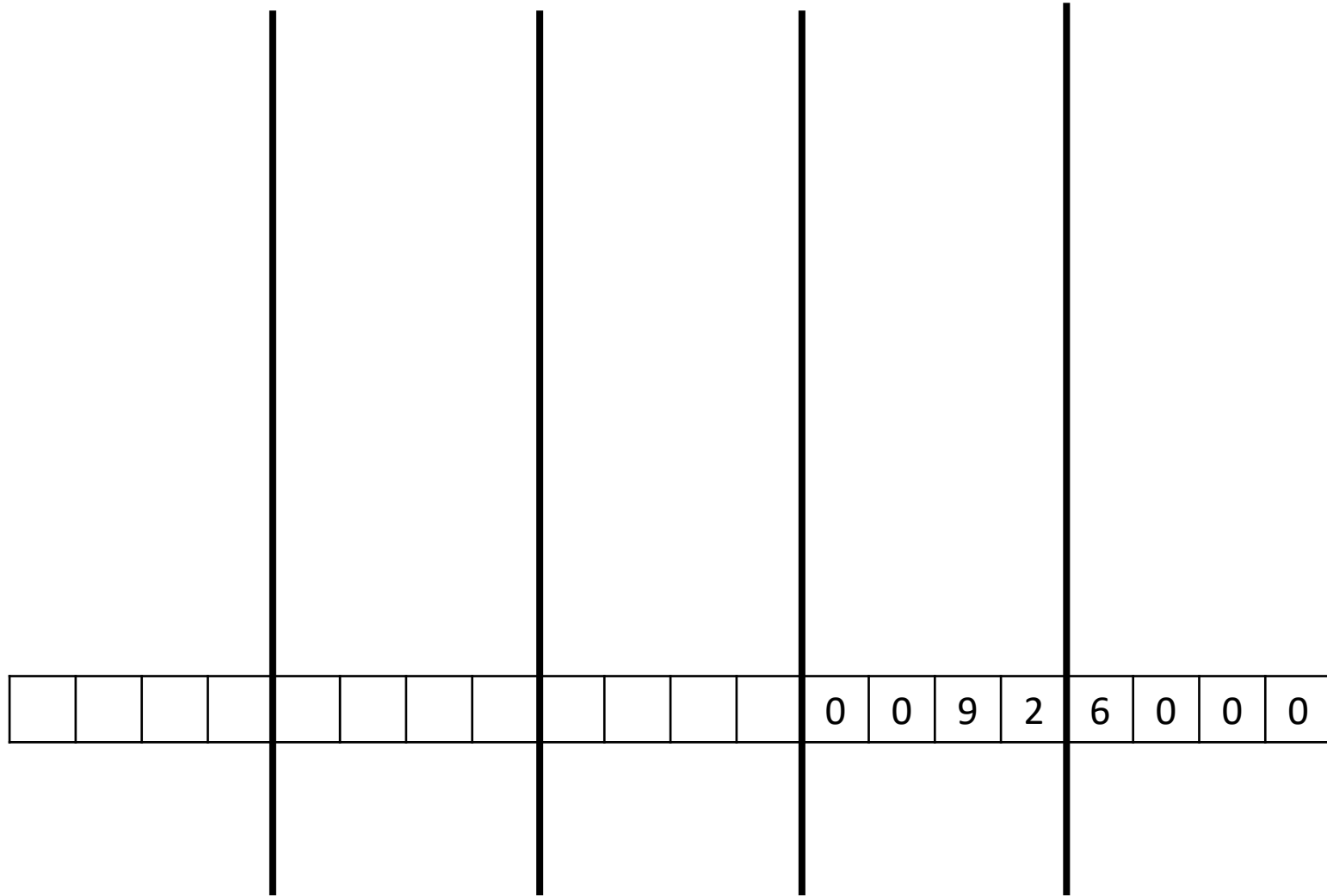
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# Flatten

0	2	3	0
0	0	4	0
3	5	0	0
0	0	9	2
6	0	0	0



# Center (Self)

0	2	3	0
0	0	4	0
3	5	0	0
0	0	9	2
6	0	0	0

	C	

C=3

C	0	2	3	0	0	0	4	0	3	5	0	0	0	0	9	2	6	0	0	0

# Left/Right Neighbors

0	2	3	0
0	0	4	0
3	5	0	0
0	0	9	2
6	0	0	0

L	C	R

R=-1

L=1

C=3

R

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

L

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

C

0	2	3	0	0	0	4	0	3	5	0	0	0	0	9	2	6	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

--	--	--

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

# Above/Below Neighbors

0	2	3	0
0	0	4	0
3	5	0	0
0	0	9	2
6	0	0	0

	A	
L	C	R
	B	

A=-2  
 B=2  
 R=-1  
 L=1  
 C=3

--	--	--	--	--

A								0	0	4	0	3	5	0	0					
B								0	0	9	2	6	0	0	0					
R	2	3	0	x	0	4	0	x	5	0	0	x	0	9	2	x	0	0	0	x
L	x	0	2	3	x	0	0	4	x	3	5	0	x	0	0	9	x	6	0	0
C	0	2	3	0	0	0	4	0	3	5	0	0	0	0	9	2	6	0	0	0

# Corner Neighbor

0	2	3	0
0	0	4	0
3	5	0	0
0	0	9	2
6	0	0	0

Q	T	
L	C	R
	B	

Q=-3  
A=-2  
B=2  
R=-1  
L=1  
C=3

--	--	--	--	--	--

Q												x	3	5	0	x	0	0	9	
A	x	x	x	x	0	2	3	0	0	0	4	0	3	5	0	0	0	0	9	2
B	0	0	4	0	3	5	0	0	0	0	9	2	6	0	0	0	x	x	x	x
R	2	3	0	x	0	4	0	x	5	0	0	x	0	9	2	x	0	0	0	x
L	x	0	2	3	x	0	0	4	x	3	5	0	x	0	0	9	x	6	0	0
C	0	2	3	0	0	0	4	0	3	5	0	0	0	0	9	2	6	0	0	0



# Left/Right Neighbors

0	2	3	0
0	0	4	0
0	3	5	0
2	0	9	0
0	0	0	6

sum(a)=5; sum(b)=4

Calculate a, b. Pad with -1 instead of X.

L	C	R

R=1

L=-1

C=1

1	-1	1
---	----	---

2	3	0	-1	0	4	0	-1	3	5	0	-1	0	9	0	-1	0	0	6	-1
-1	0	2	3	-1	0	4	0	-1	0	3	5	-1	2	0	9	-1	0	0	0
0	2	3	0	0	0	4	0	0	3	5	0	2	0	9	0	0	0	0	6
3	5	1	-4	1	4	0	-1	4	8	2	-6	3	7	9	-10	1	0	6	5

a

b





# Corner Neighbors

0	2	3	0
0	0	4	0
0	3	5	0
2	0	9	0
0	0	0	6



sum(a)=17; sum(b)=~~10~~; sum(c) = -2

↓  
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Calculate a, b, c. Pad with -1 instead of X.

A		
	C	
		B

A=-1  
B=1  
C=2

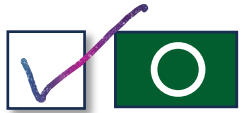
-1	1	2
----	---	---

A	-1	-1	-1	-1	-1	0	2	3	-1	0	0	4	-1	0	3	5	-1	2	0	9
B	0	4	0	-1	3	5	0	-1	0	9	0	-1	0	0	6	-1	-1	-1	-1	
C	0	2	3	0	0	0	4	0	0	3	5	0	2	0	9	0	0	0	6	
	1	9	7	0	4	5	6	-4	1	9	10	-8	3	0	21	-6	0	-3	-1	2
	a				b												c			

# Math: Cross-Correlation vs. Convolution in 2D

$$H \quad F[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

$$H \quad F[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$



# Math: Cross-Correlation vs. Convolution in 2D

Complete the equations by replacing ? with the right expressions. Adjust the sizes of the font or textbox if necessary.

$$H \otimes F[x, y] = \sum_{p=-k}^k \sum_{q=-k}^k H[x+p] F[y+q]$$

$$H * F[x, y] = \sum_{p=-k}^k \sum_{q=-k}^k H[x-p] F[y-q]$$

# NumPy by Hand 🖋️

## Broadcast

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1.  $a = I * 2$

I	0	1	2	3	4	5				

# 1D

1.  $a = I * 2$
2.  $b = a + 3$
3.  $c = \text{np.ones}(2)$
4.  $d = b - c$

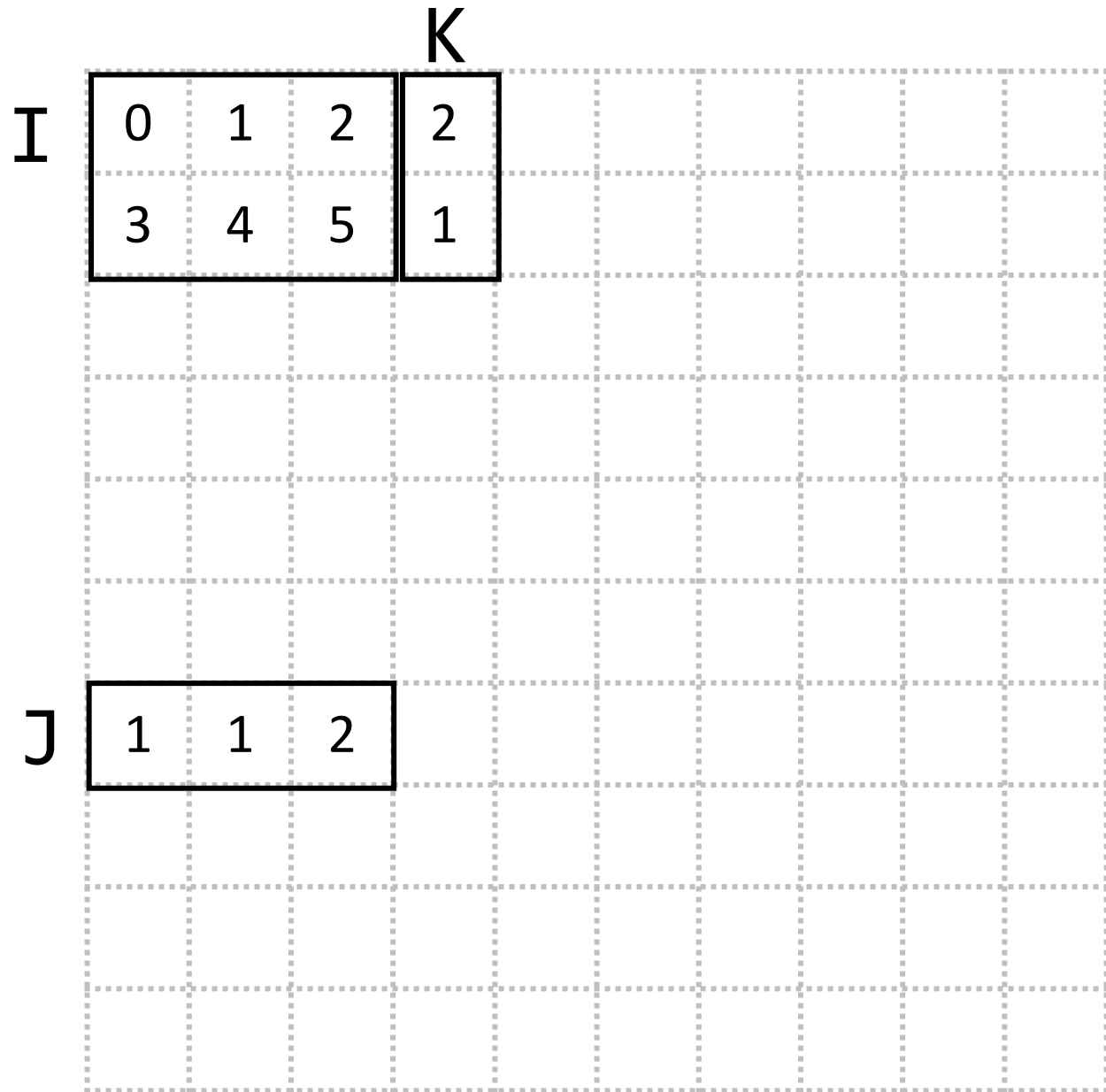
I	0	1	2	3	4	5				

# 2D

1.  $a = I * 2$

2.  $b = a + J$

3.  $c = I + K$



2D

**I**

0	1	2							
3	4	5							




  Execute by 

1.  $a = I * 3$

2.  $b = a + J$

3.  $c = I - K$

	K		C	
I	0	1	2	-2 -1
	3	2	1	2 1
*	3			
a	0	3		
	9	6		
J	1	2		
	1	2		
b	1	5		
	10	8		

  $\text{sum}(b)=24; \text{sum}(c)=0$

# NumPy by Hand 🖍️

## Compare

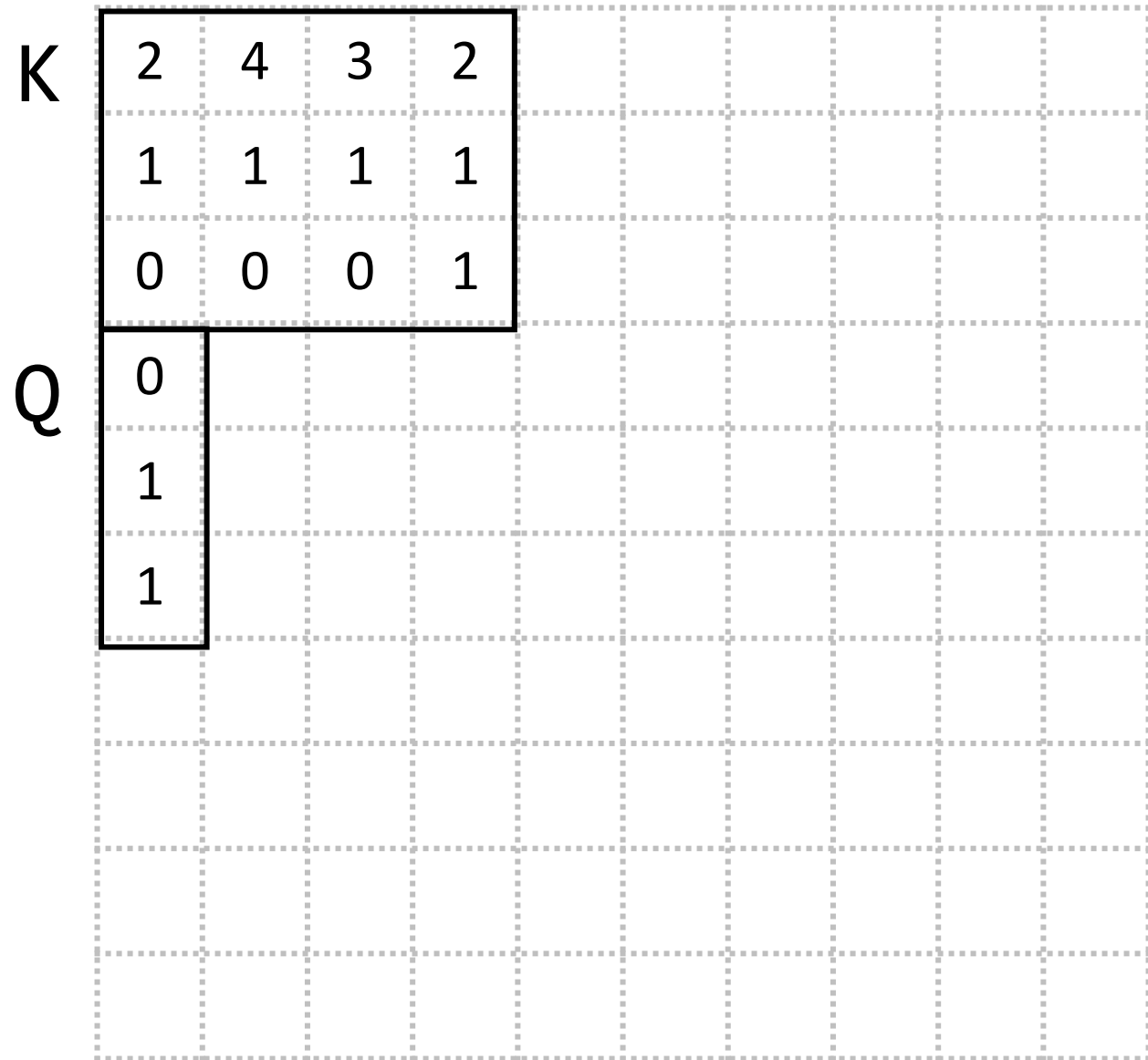
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# L1

1. `a = K - Q`
2. `b = np.abs(a)`
3. `c = np.sum(b, axis=0)`





# Compute L1

1.  $a = K - Q$
2.  $b = \text{np.abs}(a)$
3.  $c = \text{np.sum}(b, \text{axis}=0)$

K	1	4	3	3
	2	2	1	2
Q	1	1	1	1
	3	3	3	3
a	0	3	2	2
	-1	-1	-2	-1
b	0	3	2	2
	1	1	2	1
C	1	4	4	3




$\text{sum}(c)=12$

# Compute L1

1.  $a = K - Q$
2.  $b = \text{np.abs}(a)$
3.  $c = \text{np.sum}(b, \text{axis}=1)$

	Q				C			
K	1	3	1	2				1
	3	2	1	2				2
	1	1	1	2				1
	2	2	1	2				1
a	0	1	0	1				
	2	0	2	0				
	0	-1	0	1				
	1	0	1	0				
			(b)					

  $\text{sum}(c)=5$

# Code with ChatGPT [Filtering]

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Ask ChatGPT to generate python code to convert your “travel” photo to a grayscale image. Submit the results below.

### Code from ChatGPT

{Replace this code block, adjust the font size if necessary to fit}

```
import numpy as np
from PIL import Image

def flip_image_vertically(input_path, output_path):
    # Open the image and convert it to a numpy array
    # with Image.open(input_path) as img:
    img_array = np.array(img)

    # Flip the image array vertically
    flipped_array = np.flipud(img_array)

    # Convert the flipped array back to an image
    flipped_image = Image.fromarray(flipped_array)

    # Save the flipped image
    flipped_image.save(output_path)

# Example usage
flip_image_vertically('path/to/your/image.jpg',
'path/to/save/flipped_image.jpg')
```

### RGB



### Grayscale





Ask ChatGPT to generate python code to resize a photo to (200,200) and apply a "Gaussian" filter to it. Use the code to process your travel photo with kernel sizes 5 and 15. Submit the results below.

### Code from ChatGPT

{Replace this code block, adjust the font size if necessary to fit}

```
import numpy as np
from PIL import Image

def flip_image_vertically(input_path, output_path):
    # Open the image and convert it to a numpy array
    with Image.open(input_path) as img:
        img_array = np.array(img)

    # Flip the image array vertically
    flipped_array = np.flipud(img_array)

    # Convert the flipped array back to an image
    flipped_image = Image.fromarray(flipped_array)

    # Save the flipped image
    flipped_image.save(output_path)

# Example usage
flip_image_vertically('path/to/your/image.jpg',
'path/to/save/flipped_image.jpg')
```

### Kernel size = 5



### Kernel size = 15

