Epipolar

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	Level	Completed
	Beginner	12
	Intermediate	7
	Advanced	1
	Expert	

Goal	
4722	16
5722	18
Total Completed	
20	

Apply the intrinsic matrix

${}^{C}P = (4, 2, 2)$
${}^{P}P = (16, 4)$
$f = 2$

Flip y: Yes
2 pixels / unit x
2 pixels / unit y

Offset:
(8, 8)

$O_x = 8$
$O_y = 8$

$S_x =$ 2
$S_y =$ -2
$S_x f =$ 4
$S_y f =$ -4

$$\underbrace{\begin{bmatrix} 4 & & 8 \\ & -4 & 8 \\ & & 1 \end{bmatrix}}_{M^I} \underbrace{\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}}_{{}^{C}P} \quad \begin{bmatrix} 32 \\ 8 \\ 2 \end{bmatrix} \quad \underbrace{\begin{bmatrix} 16 \\ 4 \\ 1 \end{bmatrix}}_{{}^{P}P}$$

Apply the intrinsic matrix

$^{C}P = (2, 4, 2)$

$^{P}P = (16, 0)$

$f = 4$

Flip y: Yes

2 pixels / unit x

4 pixels / unit y

Offset:

(8, 32)

$S_x =$ 2

$S_y =$ −4

$S_x f =$ 8

$S_y f =$ −16

$$\begin{array}{ccc} M^I & {}^{C}P & {}^{P}P \\ \begin{bmatrix} 8 & & 8 \\ & -16 & 32 \\ & & 1 \end{bmatrix} & \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 32 \\ 0 \\ 2 \end{bmatrix} & \begin{bmatrix} 16 \\ 0 \\ 1 \end{bmatrix} \end{array}$$

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Apply the intrinsic matrix

${}^{C}P = (2, 4, 4)$

${}^{P}P = (2, 2)$

$f = (4, 2)$

Flip y: Yes

2 pixels / unit x

4 pixels / unit y

Offset:

(-2, 8)

$S_x =$ 2

$S_y =$ -4

$S_x f =$ 8

$S_y f =$ -8

$${}^{C}P = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$M^{I} = \begin{bmatrix} 8 & & -2 \\ & -8 & 8 \\ & & 1 \end{bmatrix} \qquad \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix} \qquad {}^{P}P = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

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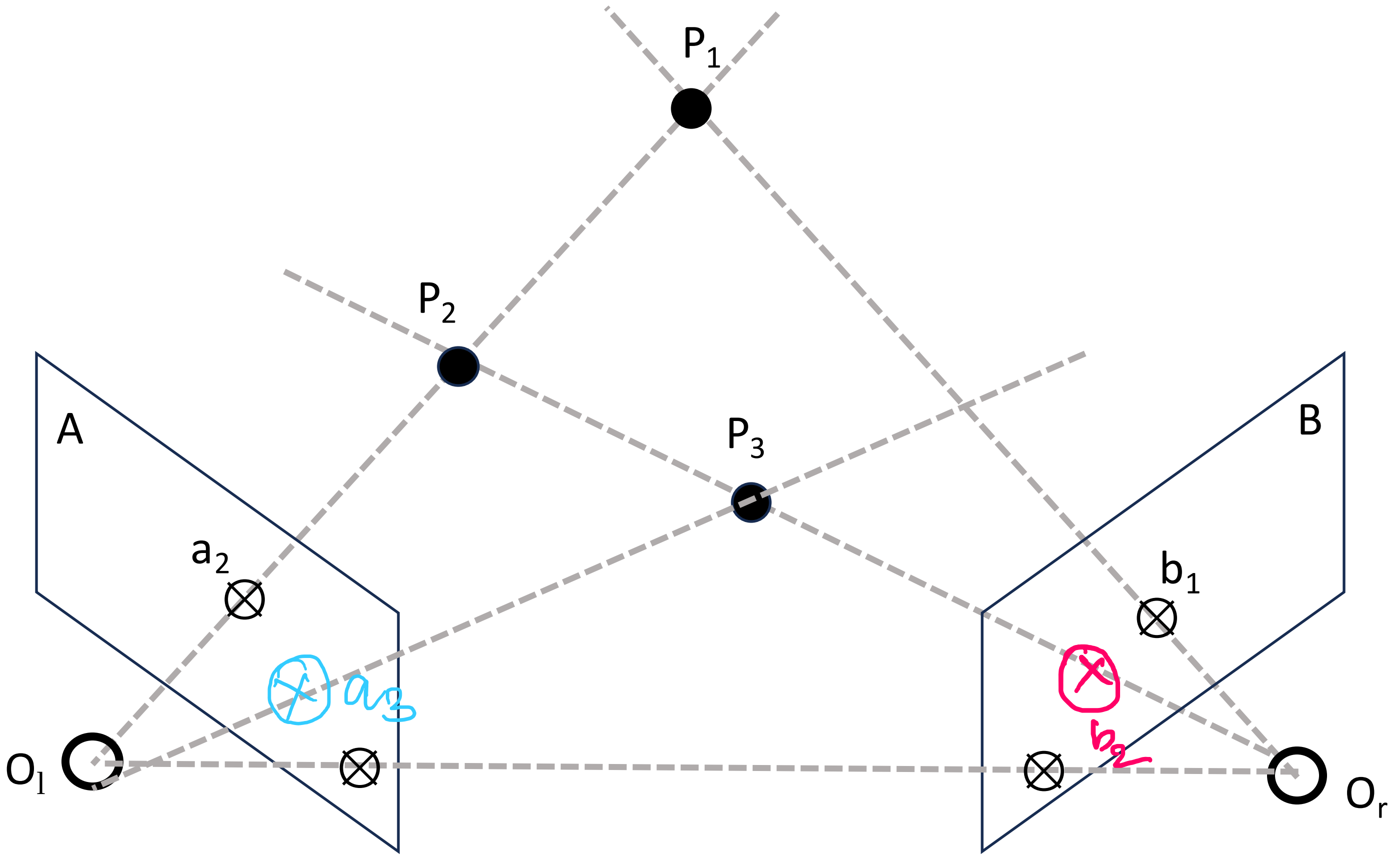
Projections on virtual image planes

Suppose P_1, P_2, P_3 are co-planar

- ☐ Draw $b_2 = P_2 \downarrow B$
- ☐ Draw $a_3 = P_3 \downarrow A$



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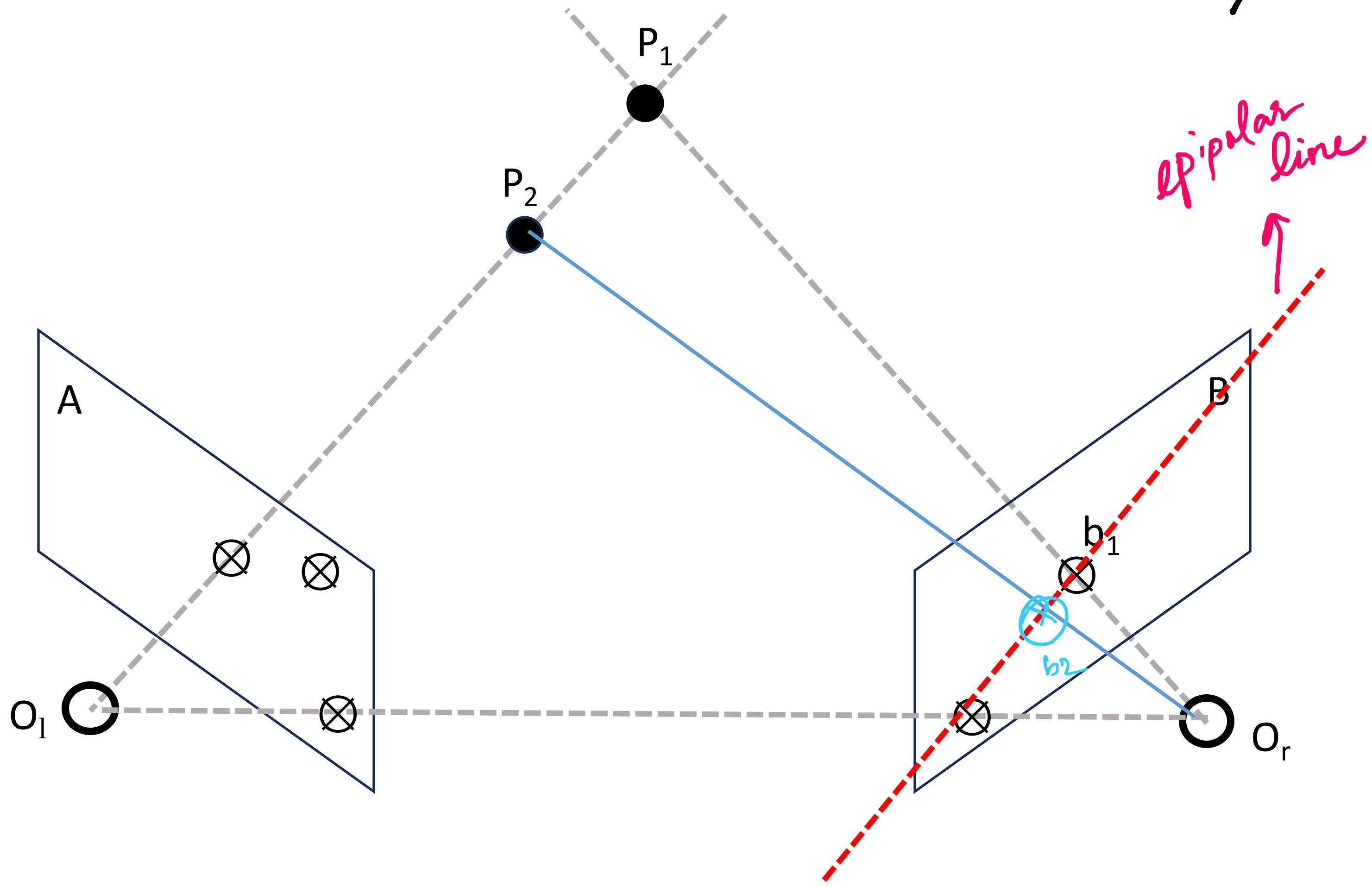
Draw epipolar lines

You can either draw directly or drag this dotted line

- [x] Draw the epipolar line on B corresponding to P_1, P_2
- [x] Draw $b_2 = P_2 \downarrow B$



Hint: Epipolar lines must go through the epipole.

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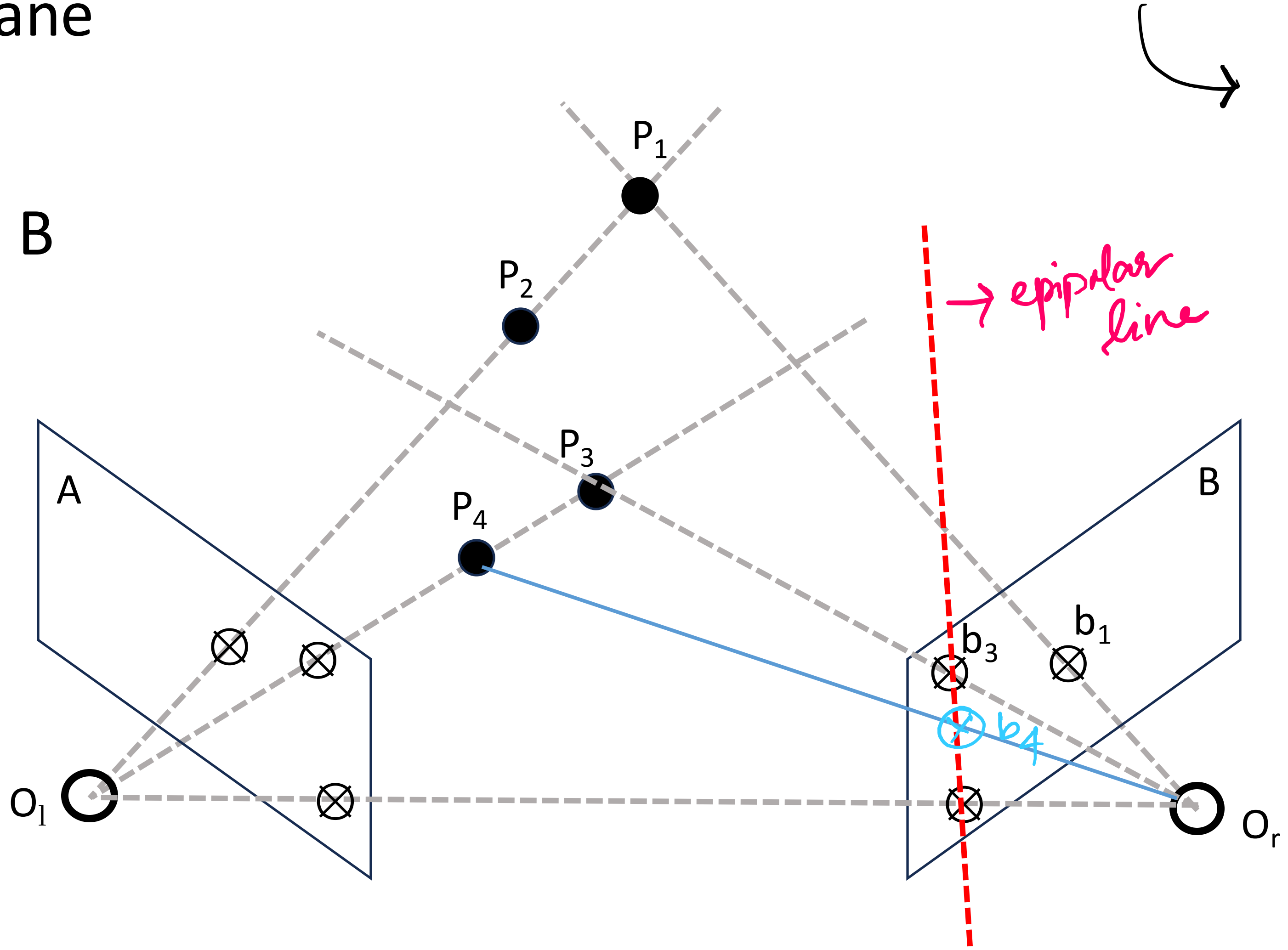
Draw epipolar lines

Note that P_3 and P_4 are a plane "above" P_1 and P_2

- ☑ Draw the epipolar line on B corresponding to P_3, P_4
- ☑ Draw $b_4 = P_4 \downarrow B$

You can either draw directly or drag this dotted line



Hint: Epipolar lines must go through the epipole.

Translation → Cross-product Matrix

$T = [\ t_x, t_y, t_z\ ]$

$$T_\times = \begin{bmatrix} & -t_z & t_y \\ t_z & & -t_x \\ -t_y & t_x & \end{bmatrix}$$

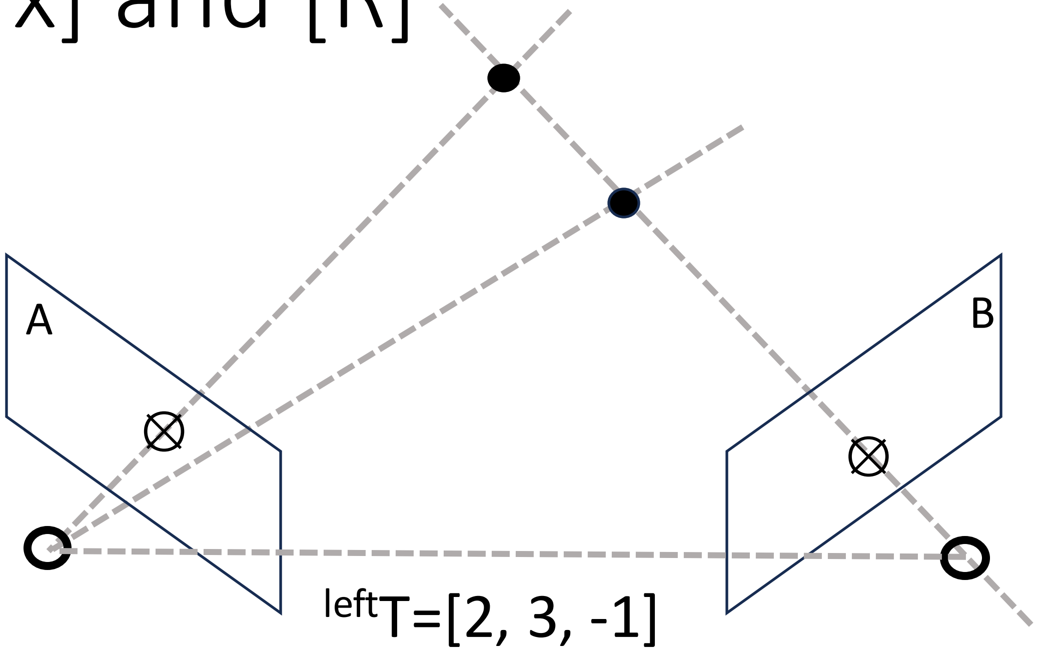
$T = [\ 2, 3, -1\ ]$

$$T_\times = \begin{bmatrix} & 1 & 3 \\ -1 & & -2 \\ -3 & 2 & \end{bmatrix}$$

$T = [\ 5, -2, 4\ ]$

$$T_\times = \begin{bmatrix} & -4 & -2 \\ 4 & & -5 \\ 2 & 5 & \end{bmatrix}$$

Derive [Tx] and [R]



Camera Rotates 90^o (X->Z)

$$^{Left}Tx = \begin{bmatrix} & 1 & 3 \\ -1 & & -2 \\ -3 & 2 & \end{bmatrix}$$

$$^{Left}R = \begin{array}{c c} & \begin{array}{ccc} & & z \end{array} \\ \begin{array}{c} x \\ \\ \\ \end{array} & \begin{bmatrix} & & -1 \\ & 1 & \\ 1 & & \end{bmatrix} \end{array}$$

$\sin 90^\circ = 1$

$\cos 90^\circ = 0$

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Epipolar Constraint ${}^{left}P_1 \rightarrow {}^{right}P_2$

Given ${}^{left}P_1 = (2, 3, 4)$ and ${}^{right}P_2 = (2, 2, 1)$.
Verify that the epipolar constraint is satisfied
by multiplying from left to right.

$$
\begin{bmatrix} & -1 & \\ 1 & & -2 \\ & 2 & \end{bmatrix}
\qquad
\begin{bmatrix} & -1 & \\ 1 & & \\ & & 1 \end{bmatrix}
\qquad
\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}
\qquad
\begin{bmatrix} 3 & 6 & -6 \end{bmatrix}
\qquad
\begin{bmatrix} 6 & -3 & -6 \end{bmatrix}
\qquad
\begin{bmatrix} 0 \end{bmatrix}
$$

$12 - 6 - 6 = 0$

10 Epipolar Constraint ${}^{left}P_1 \leftarrow {}^{right}P_2$

Given ${}^{left}P_1 = (2, 3, 4)$ and ${}^{right}P_2 = (2, 2, 1)$.
Verify that the epipolar constraint is satisfied by multiplying from right to left (i.e, transposed from top down).

$$
{}^{r}P_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} & -1 & \\ 1 & & \\ & & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} & -1 & \\ 1 & & -2 \\ & 2 & \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ 4 \end{bmatrix}
$$

$$
{}^{l}P_1 = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ -4 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}
$$

$$
-4-12+16 = 0
$$

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Calculate the Essential Matrix (E)

Between the two cameras:

Rotation: 90° (Y->Z)
Translation: [-2, 3, 1]

1. Calculate TxR

Hint: Verify $^{l}P * E * {}^{r}P = 0$

R

1		
		-1
	1	

Tx

	-1	3
1		2
-3	-2	

E = TxR

0	3	1
1	2	0
-3	0	2

^{r}P

1
1
-1

^{l}P

1	1	1

-2	5	3

0

-2+5-3
=0

Calculate the Essential Matrix (E)

Between the two cameras:

Rotation: 90° (X->Y)
Translation: [2, 0, 1]

1. Calculate Tx
2. Calculate R
3. Calculate TxR

Hint: Verify $^{l}P * E * ^{r}P = 0$

R

		y	
x		-1	
	1		
			1

Tx

	-1	0
1		-2
0	2	

E = TxR

-1	0	0
0	-1	-2
2	0	0

^{r}P

1
-1
1

^{l}P

1	1	1

1	-1	-2

0

1+1-2
= 0

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Derive M_l and M_l^{-1}

Hint: Verify $P * M_l * M_l^{-1} = P$

Left Camera:
f = 2
Sx = 2
Sy = 1

$${}^{l}P_1 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

$${}^{left}M_l = \begin{bmatrix} 4 & & \\ & 2 & \\ & & 1 \end{bmatrix} \qquad {}^{A}P_1 = \begin{bmatrix} 12 \\ 10 \\ 1 \end{bmatrix}$$

$${}^{left}M_l^{-1} = \begin{bmatrix} \frac{1}{4} & & \\ & \frac{1}{2} & \\ & & 1 \end{bmatrix} \qquad {}^{l}P_1 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

Right Camera:
f = 2
Sx = 1
Sy = 2

$${}^{r}P_1 = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

$${}^{right}M_l = \begin{bmatrix} 2 & & \\ & 4 & \\ & & 1 \end{bmatrix} \qquad {}^{B}P_1 = \begin{bmatrix} 2 \\ 32 \\ 1 \end{bmatrix}$$

$${}^{right}M_l^{-1} = \begin{bmatrix} \frac{1}{2} & & \\ & \frac{1}{4} & \\ & & 1 \end{bmatrix} \qquad {}^{r}P_1 = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

Infer Sx and Sy and Derive M_l^{-1}

Hint: Verify $P * M_l * M_l^{-1} = P$

Left Camera:

f = 4

Sx = 3

Sy = 5

$^{l}P_1 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$

$^{left}M_l = \begin{bmatrix} 12 & & \\ & 20 & \\ & & 1 \end{bmatrix}$ $\quad ^{A}P_1 = \begin{bmatrix} 36 \\ 100 \\ 1 \end{bmatrix}$

$^{left}M_l^{-1} = \begin{bmatrix} \frac{1}{12} & & \\ & \frac{1}{20} & \\ & & 1 \end{bmatrix}$ $\quad ^{l}P_1 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$

Right Camera:

f = 3

Sx = 2

Sy = 3

$^{r}P_1 = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$

$^{right}M_l = \begin{bmatrix} 6 & & \\ & 9 & \\ & & 1 \end{bmatrix}$ $\quad ^{B}P_1 = \begin{bmatrix} 6 \\ 72 \\ 1 \end{bmatrix}$

$^{right}M_l^{-1} = \begin{bmatrix} \frac{1}{6} & & \\ & \frac{1}{9} & \\ & & 1 \end{bmatrix}$ $\quad ^{r}P_1 = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$

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Calculate the Fundamental Matrix (F)

Left Camera:
f = 2
Sx = 1
Sy = 1

Right Camera:
f = 2
Sx = 1
Sy = 1

E

-8		
	-8	16
	16	

${}^{right}M_I^{-1}$

1/2		
	1/2	
		1

${}^{left}M_I^{-1}$

1/2		
	1/2	
		1

-4		
	-4	8
	16	

F

-2		
	-2	8
	8	0

${}^{B}P$

2
-3
1

${}^{A}P$

1	2	1

-2	4	16

0

-4
-12
+16
0

Hint: Verify ${}^{A}P * F * {}^{B}P = 0$

Calculate the Fundamental Matrix (F)

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Left Camera:
f = 2
Sx = 1 2
Sy = 2 4

Right Camera:
f = 2
Sx = 2 4
Sy = 1 2

E

-8		
	-8	16
	16	

${}^{right}M_I^{-1}$

1/4		
	1/2	
		1

${}^{left}M_I^{-1}$

1/2		
	1/4	
		1

-4		
	-2	4
	16	

F

-1		
	-1	4
	8	

${}^{B}P$

1
-1
1

-2
-6
+8
0

Hint: Verify ${}^{A}P * F * {}^{B}P = 0$

${}^{A}P$

2	2	1

-2	6	8

0

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Epipolar Line

Given a pixel location in the left image:

(3, 4)

The epipolar line on the right image is

__4__ u + __11__ v + __15__ = 0

^{A}P

3	4	1

Fundamental Matrix

	3	5
1		
	2	
4	11	15

^{B}P

u
v
1
0

Hint: You may not need to use all the cells to calculate.

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Epipolar Line

^{B}P

2
3
1

Fundamental Matrix

	3	5	14
1			2
	2		6

^{A}P

u	v	1	0

Given a pixel location in the right image:

(2, 3)

The epipolar line on the left image is

$\underline{\ 14\ }\ u + \underline{\ 2\ }\ v + \underline{\ 6\ } = 0$

Hint: You may not need to use all the cells to calculate.

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Estimate F's parameter

Suppose the pixel location (2, 7) in the left image corresponds to the pixel location (2, 4) in the right image. Estimate the missing parameter of the Fundamental Matrix.

Hint: Solve one linear equation involving *a*

^{B}P

2
4
1

Fundamental Matrix

0	**a**	3	4a+3
1	0	0	2
0	-1	0	-4

^{A}P

2	7	1	0

$8a + 6 + 14 - 4 = 0$

$8a = -16$

$a = -2$

Answer:

a = -2

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Estimate F's parameters

Suppose we know two pairs of pixel-to-pixel correspondences between the left and right images. Estimate the two missing parameters of the Fundamental Matrix.

Hint: Solve a system of two linear equations involving *a* and *b*

$^BP_1$	$^BP_2$
1	1
1	-1
1	1

Fundamental Matrix

0	**a**	**b**
1	0	0
0	3	0

a+b	b-a
1	1
3	-3

$^AP_1$	1	-1	1
$^AP_2$	1	3	1

0	
	0

$a+b-1+3=0$

$a+b=2$

$b-a+3-3=0$

$a=b$

$a=1,\ b=1$

Answer:

a = 1

b = 1