

Transform

Name	Dharini Baskaran
Identity Key	dhba5060

	Level	Completed	Goal	
	Beginner	17	4722	18
	Intermediate	6	5722	22
	Advanced	2	Total Completed	
	Expert	0	25	

Transform

CSCI 5722/4722: Computer Vision

Spring 2024

Dr. Tom Yeh

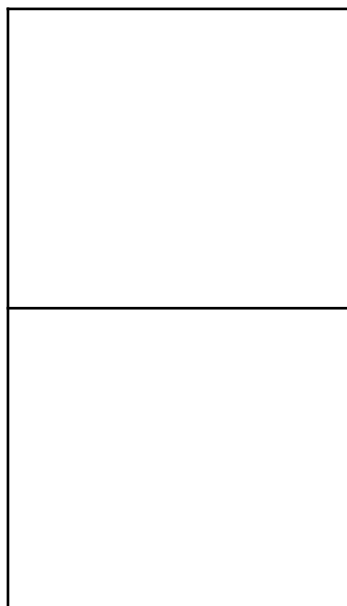
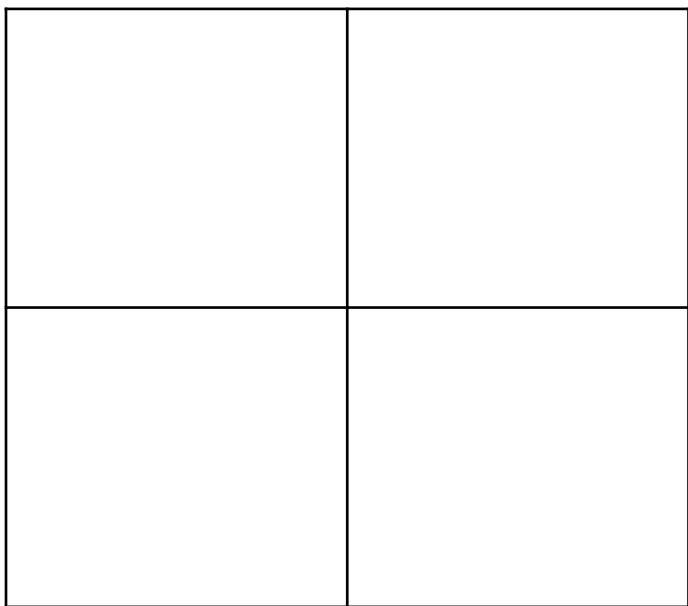
Dr. Mehdi Moghari

Linear Transform

CSCI 5722/4722 Computer Vision



University of Colorado
Boulder



2	7
3	5

1
-1

2	7
3	5

1	1	0	1
-1	0	1	1

1	1	0	1
-1	0	1	1

2	7
3	5

1	0	0	0
0	0	1	0
0	1	0	1
0	0	0	1
1	0	1	0

4	5	1
2	2	4
7	4	2
9	3	8

4	5	1
2	2	4
7	4	2
9	3	8

1	0	0	0
0	0	1	0
0	1	0	1
0	0	0	1
1	0	1	0



Linear Transform (i.e., Multiply Matrices)

1	1	0	1
-1	0	1	1

4	2
5	1

2	4	2	6
4	5	1	6

☒ ☐ Linear Transform (i.e., Multiply Matrices)

4	5	-1
-2	2	4
7	-4	2
9	3	8

1	0	0	0
0	0	1	0
0	1	0	1
0	0	0	1
1	0	1	0

4	5	-1
7	-4	2
7	5	12
9	3	8
11	1	1



What are the missing values?

99	53	24
173	21	74
21	54	36

0	1	0
1	0	0
0	0	1

173	21	74
99	53	24
21	54	36

☒  What are the values of A and B?

			3	7
			2	8
			1	-1
A	1	0	-1	1
0	2	B	9	11

$$A = \underline{\hspace{1cm}} - 1 \underline{\hspace{1cm}}$$

$$B = \underline{\hspace{1cm}} 5 \underline{\hspace{1cm}}$$

Homogeneous Coordinate

CSCI 5722/4722 Computer Vision



University of Colorado
Boulder

Convert to Homogeneous Coordinates

4
2

3
5
2

Convert to Homogeneous Coordinates

2	1	2	0
4	2	3	1

Equality in Homogeneous Coordinates

p_0

4
2
1

$p_1 \equiv p_2 \equiv p_3$

8	12	-8

$q_1 \equiv q_2$

8	16

Which point is not equivalent to the rest in the homogenous coordinate? Cross it out!

A	B	C	D
3	9	-6	3
2	6	-4	2
1	3	-2	2



Which point is not equivalent to the rest in the homogenous coordinate? Cross it out!

A	B	C	D
2	2	-2	4
5	5	-5	10
1	2	-1	2



Suppose these points are all equivalent in the homogenous coordinates. Fill in the missing values.

A	B	C	D
4	8	2	12
2	4	1	6
1	2	0.5	3

Homogenous Transformation

CSCI 5722/4722 Computer Vision

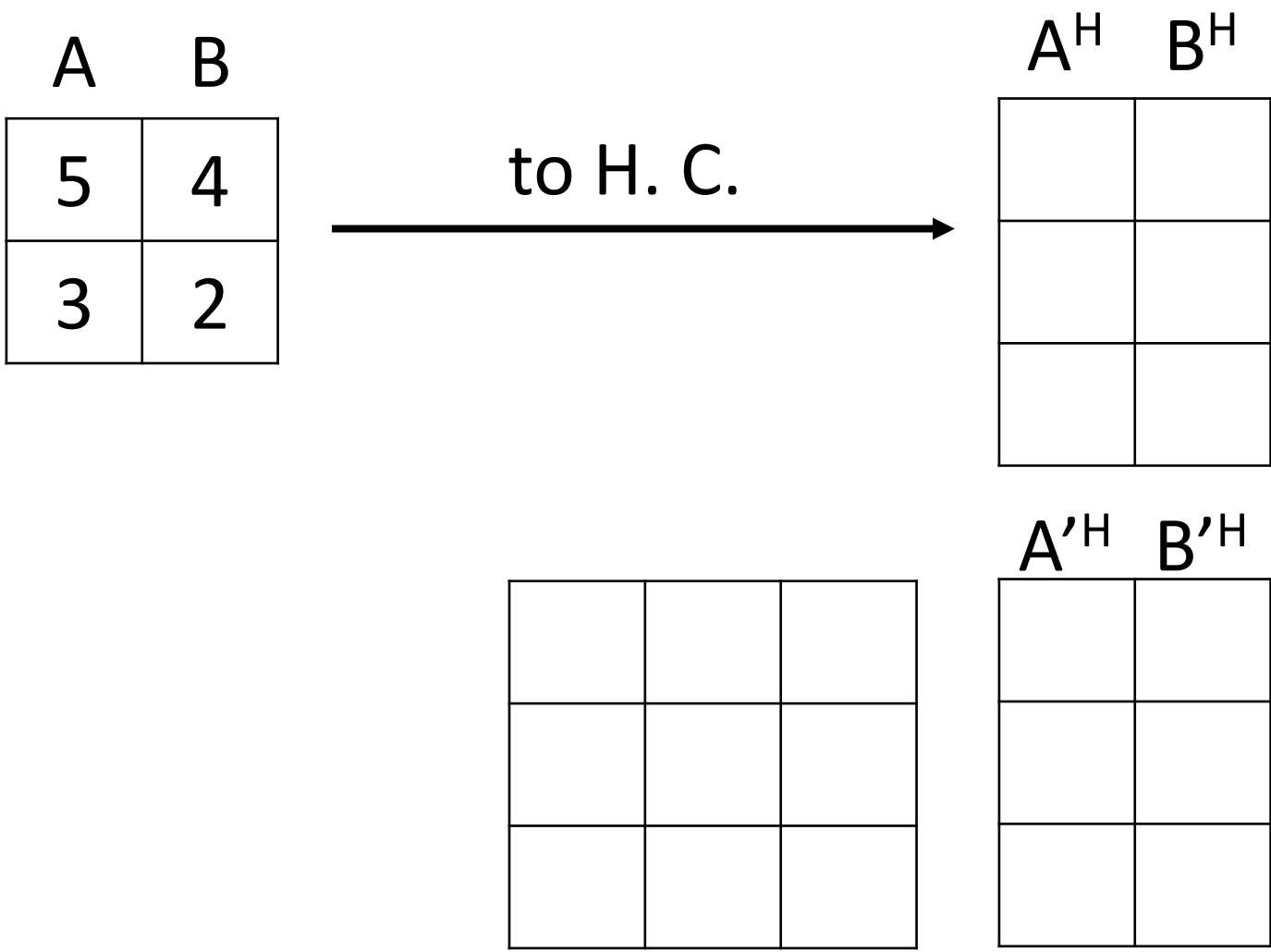


University of Colorado
Boulder

Translation Matrix

x
y

Translate by $[-3, 2]$



Scaling Matrix

x
y

Scale by $[3, -2]$

A	B
2	1
3	4

to H. C.



A^H	B^H

A'^H	B'^H

☒ ☐ Translate by [2, 3]

A	B
4	-2
1	5

to H. C.



A^H	B^H
4	-2
1	5
1	1

1	0	2
0	1	3
0	0	1

A'^H	B'^H
6	0
4	8
1	1



Scale by [2, 3]

A	B
3	1
2	-1

to H. C.



A^H	B^H
3	1
2	-1
1	1

2	0	0
0	3	0
0	0	1

A'^H	B'^H
6	2
6	-3
1	1



Suppose P is translated to P' . What are the values of the transformation matrix M ?

			P		
			5		
			8		
			1		
M			P'		
1	0	-3	2		
0	1	6	14		
0	0	1	1		



Suppose P is scaled to P' . What are the values of the transformation matrix M ?

P

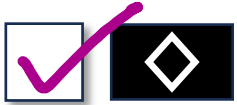
5
8
1

M

3	0	0
0	-0.25	0
0	0	1

P'

15
-2
1



Suppose a group of points are scaled by a common matrix M . Which point does not belong to this group? Cross it out!

P1 P2 P3 P4

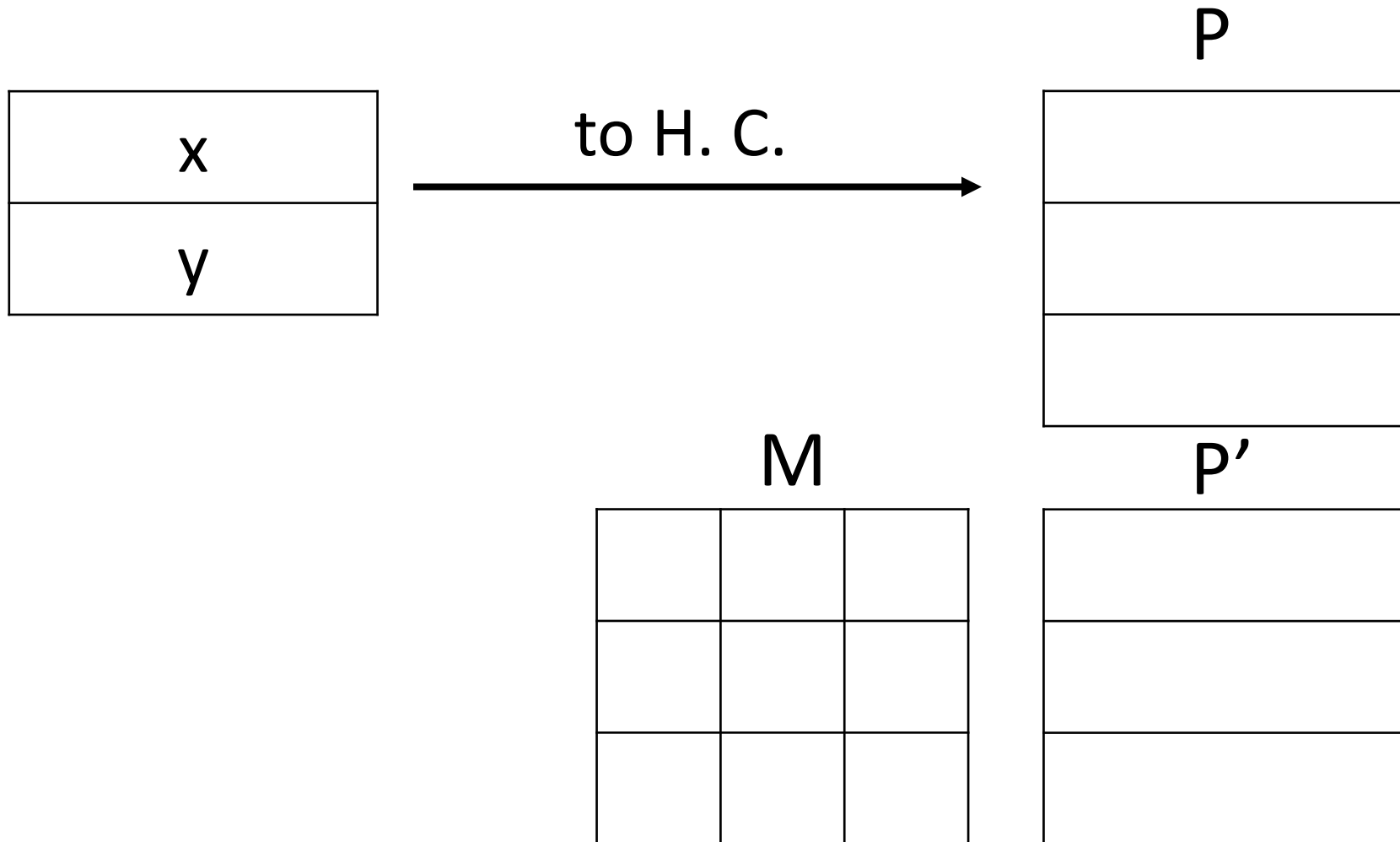
2	4	3	5
3	-1	2	-2
1	1	1	1

M

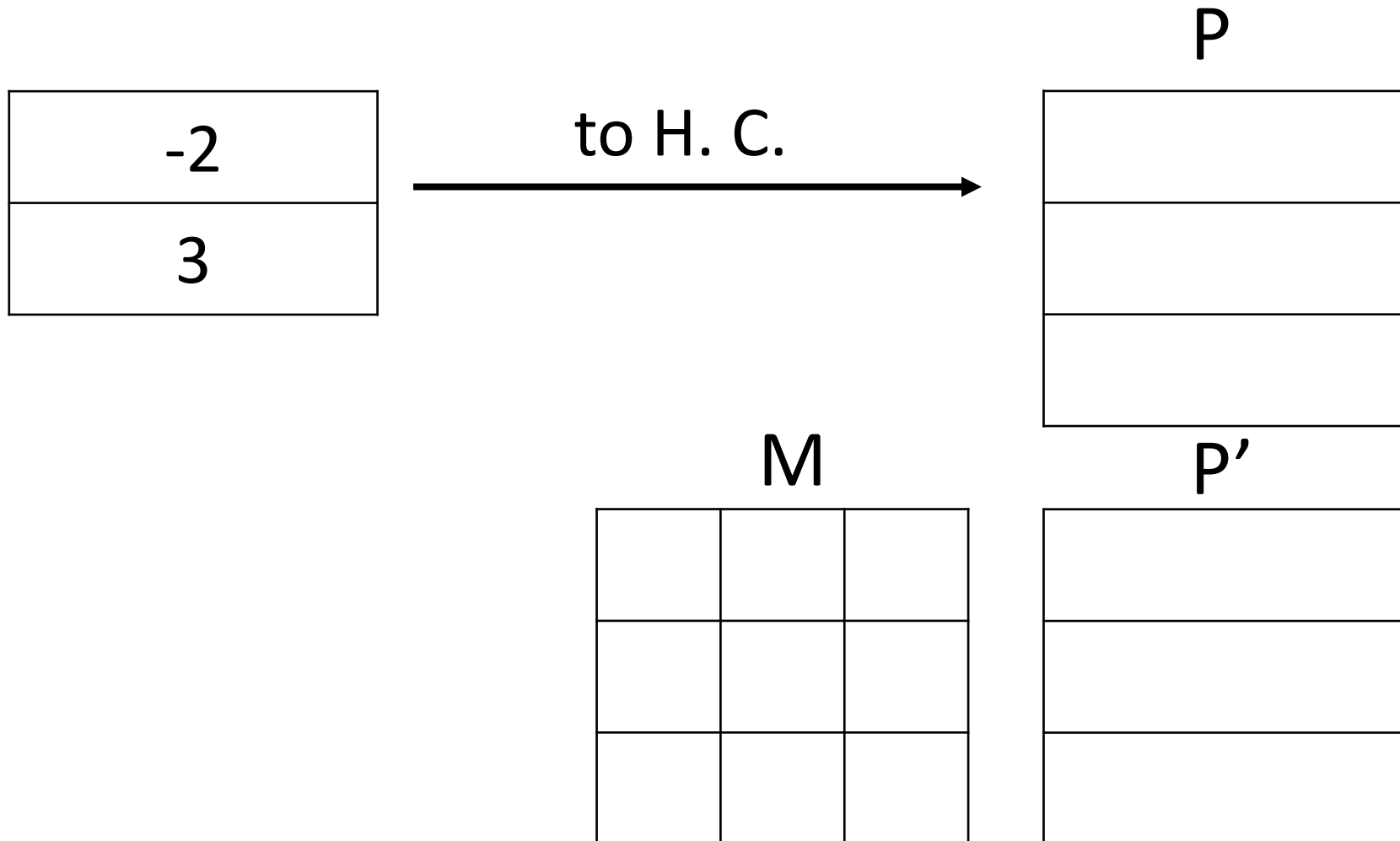
2	0	0
0	-2	0
0	0	1

4	8	6	10
-6	3	-4	4
1	1	1	1

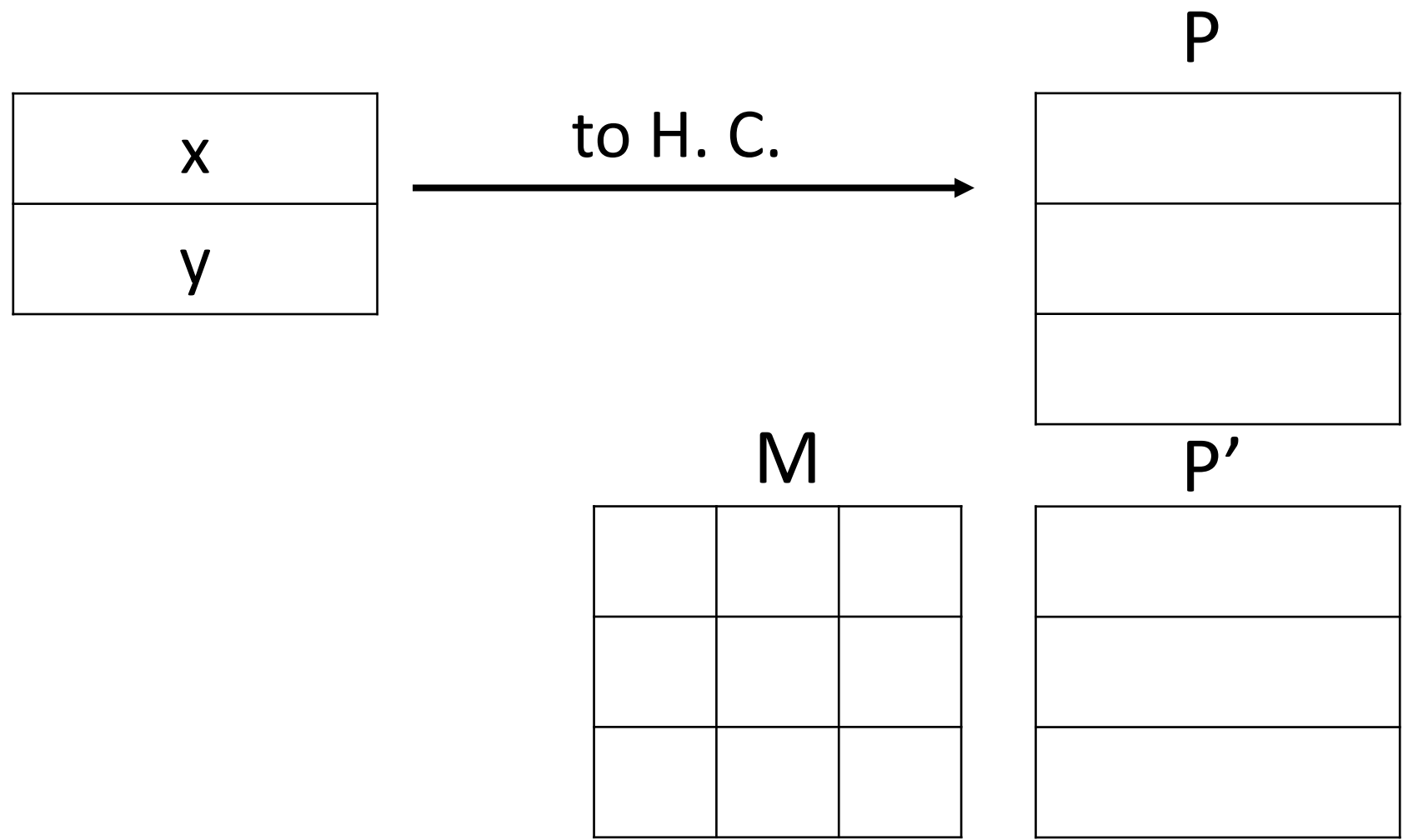
Scale by $[s_x, s_y]$ then Translate by $[t_x, t_y]$



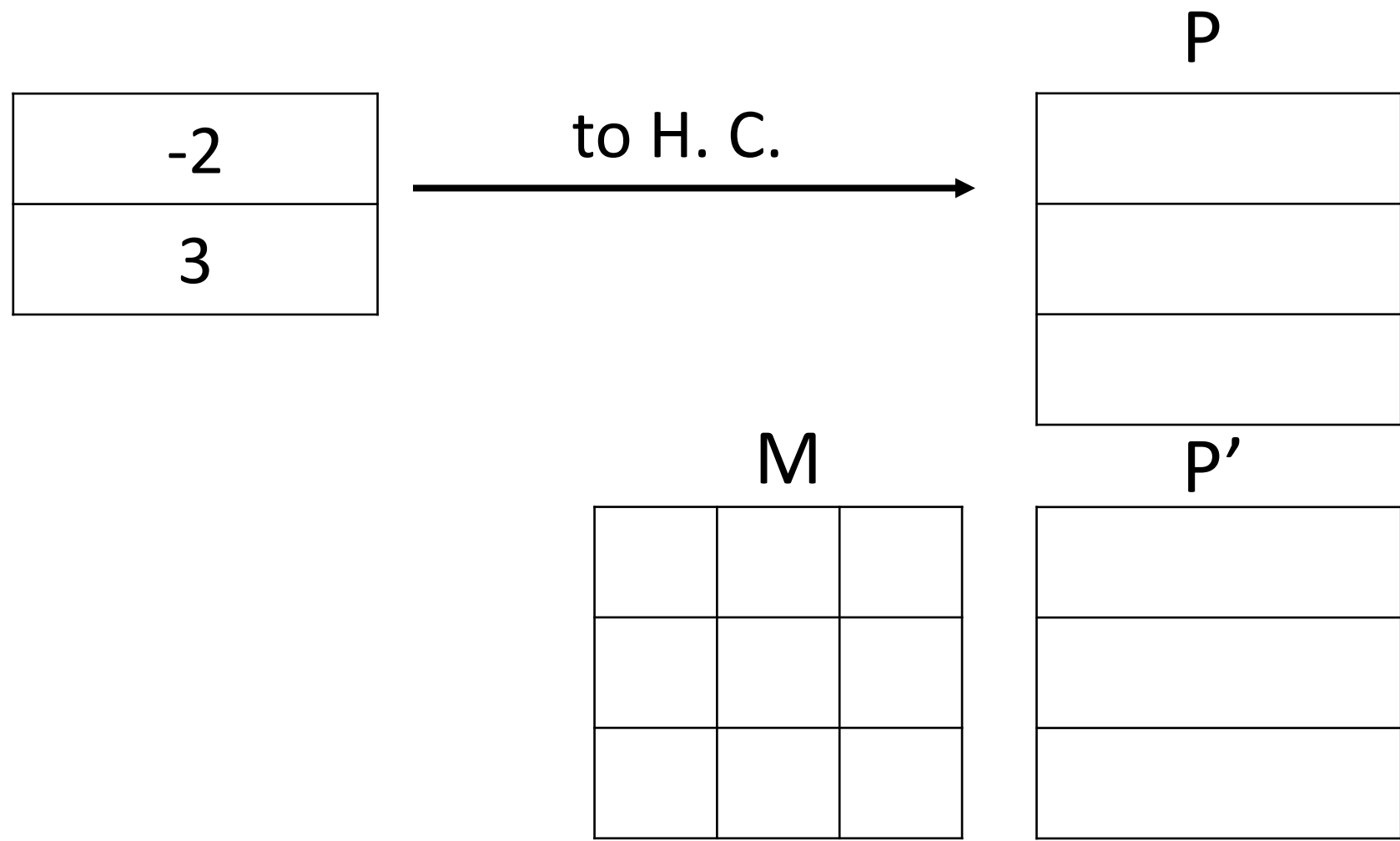
Scale by $[2, 5]$ then Translate by $[-3, 10]$



Translate by $[t_x, t_y]$ then Scale by $[s_x, s_y]$



Translate by [-3, 10] then Scale by [2, 5]



☒ ☐ Scale by $[3, 2]$ then Translate by $[4, -3]$

4
-2

to H. C.



P

4
-2
1

M

3	0	4
0	2	-3
0	0	1

P'

16
-7
1

☒ ☐ Translate by $[4, -3]$ then Scale by $[3, 2]$

4
-2

to H. C.



P

4
-2
1

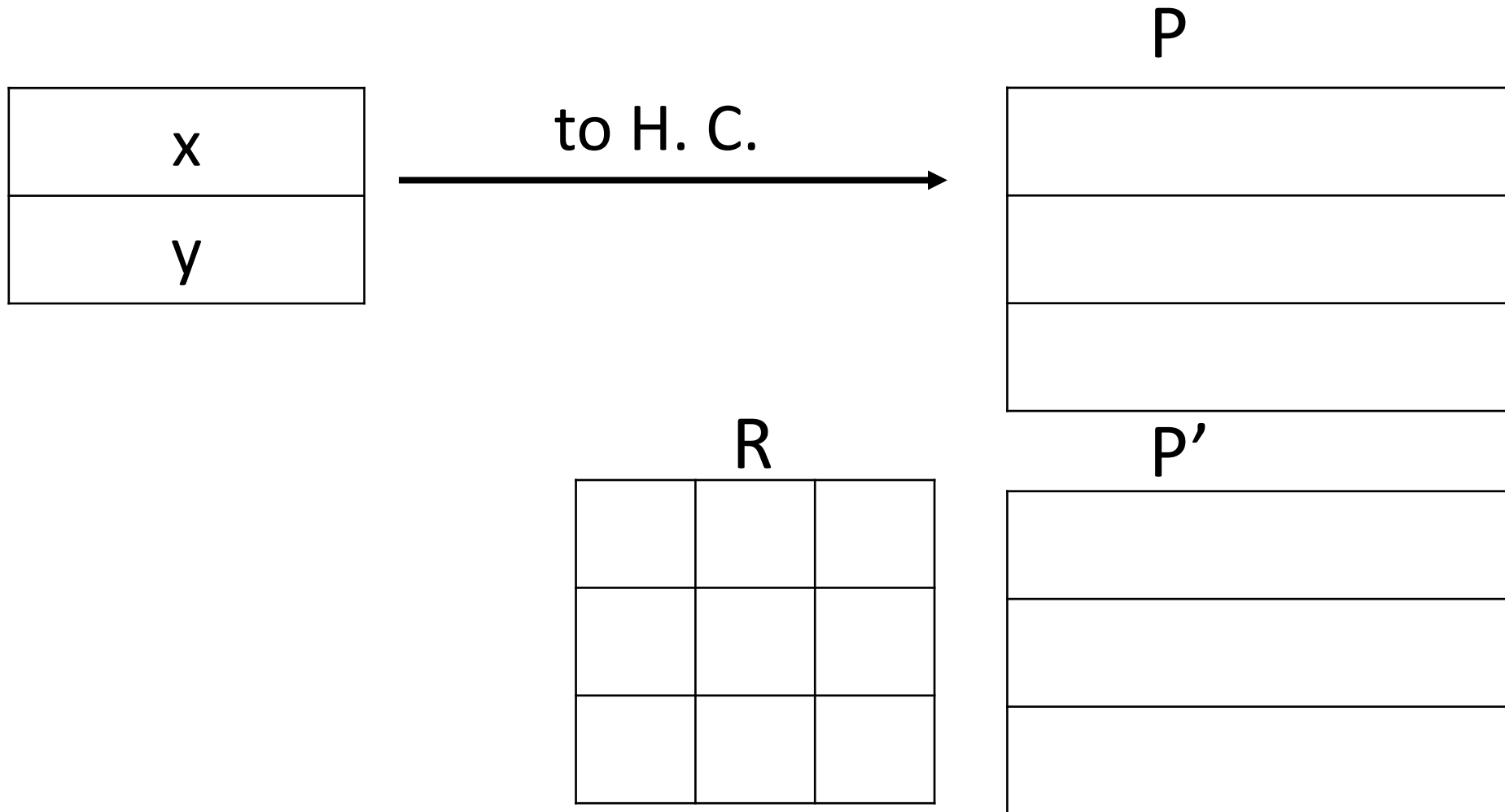
M

3	0	12
0	2	-6
0	0	1

P'

24
-10
1

Rotation Matrix



Rotate by 45° (CCW)

2
4

to H. C.



P

R

P'

$$\cos(45^\circ) = \underline{\hspace{2cm}}$$

$$\sin(45^\circ) = \underline{\hspace{2cm}}$$

☒ ☐ Rotate by 37° (CCW)

10
25

to H. C.



P

10
25
1

$$\cos(37^\circ) = \frac{4}{5}$$

$$\sin(37^\circ) = \frac{3}{5}$$

R

4/5	-3/5	0
3/5	4/5	0
0	0	1

P'

-7
26
1

☒ ☐ Rotate by 90° (CCW)

8
5

to H. C.



P

8
5
1

R

0	-1	0
1	0	0
0	0	1

P'

-5
8
1

$$\cos(90^\circ) = \underline{\underline{0}}$$

$$\sin(90^\circ) = \underline{\underline{1}}$$

Scale, Translate, Rotate

$$P = (3, 2)$$

Scale by $[4, 2]$

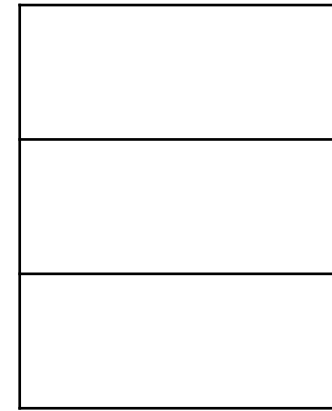
Translate by $[3, 6]$

Rotate CCW by 37°

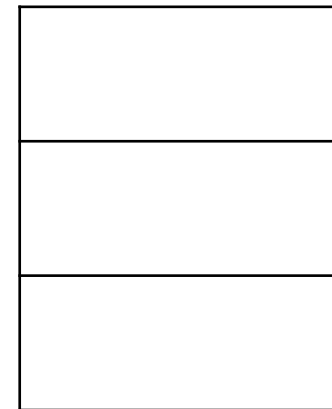
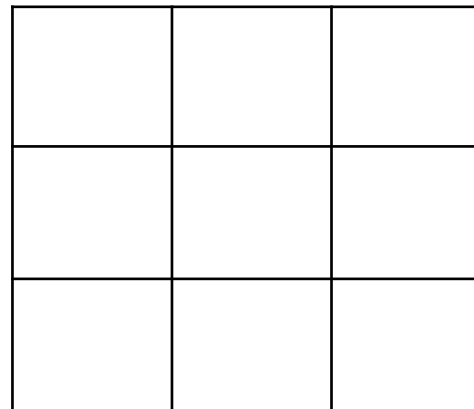
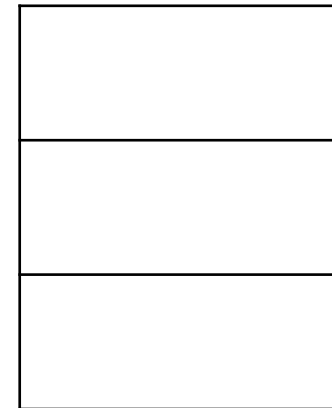
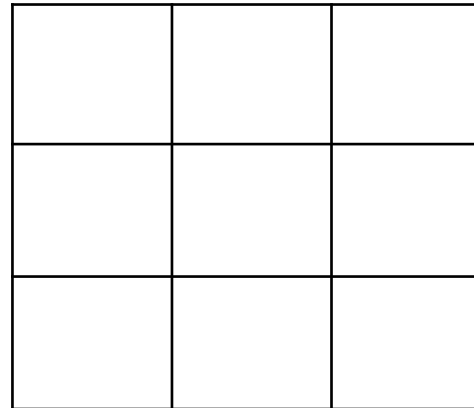
$$P' = ?$$

$$\cos(37^\circ) = \frac{4}{5}$$

$$\sin(37^\circ) = \frac{3}{5}$$



P



P'



Scale, Translate, Rotate

$$P = (2, 3)$$

Scale by $[3, -4]$

Translate by $[-1, 2]$

Rotate CCW by 53°

$$P' = ?$$

$$\cos(53^\circ) = \frac{3}{5}$$

$$\sin(53^\circ) = \frac{4}{5}$$

2
3
1

P

3	0	-1
0	-4	2
0	0	1

5
-10
1

$\frac{3}{5}$	$-\frac{4}{5}$	0
$\frac{4}{5}$	$\frac{3}{5}$	0
0	0	1

11
-2
1

P'

NumPy by Hand 🖋️

Basics

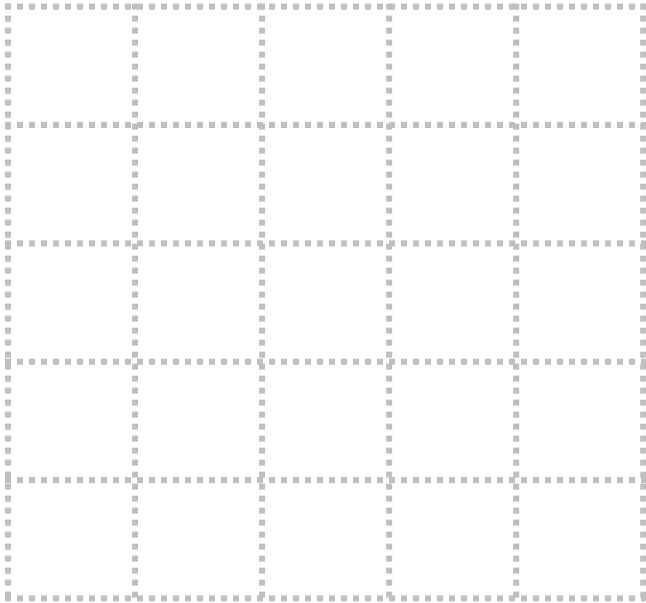
CSCI 5722/4722 Computer Vision



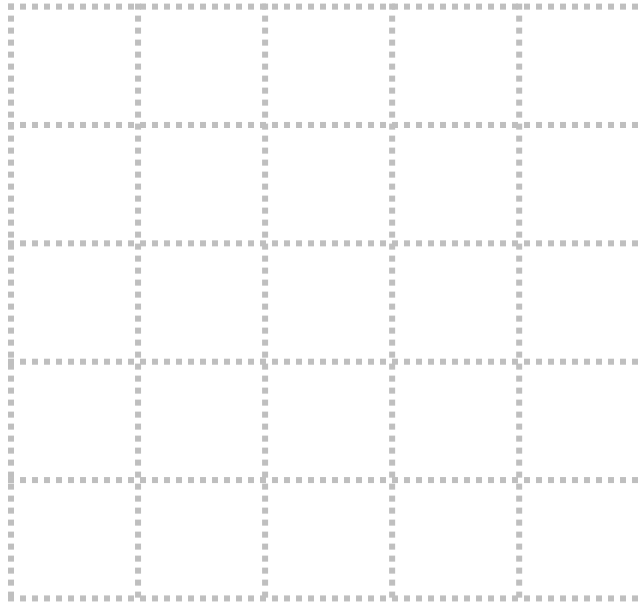
University of Colorado
Boulder

np.zeros

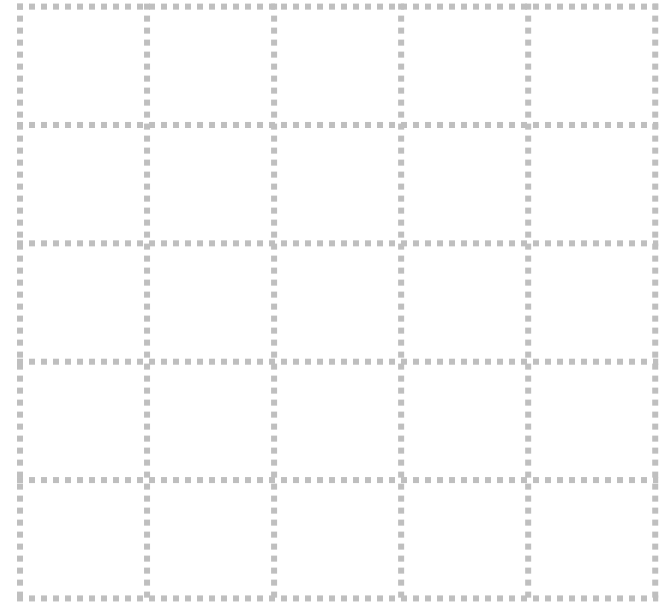
`np.zeros(3)`



`np.zeros((3, 2))`



`np.zeros((2, 4))`



np.ones

`np.ones(3)`



`np.ones((2, 1))`

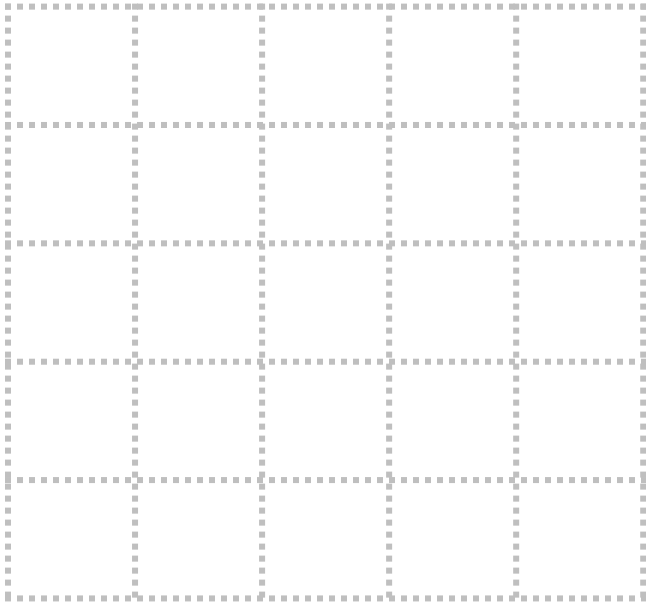


`np.ones((2, 3))`



`np.arange(stop)`

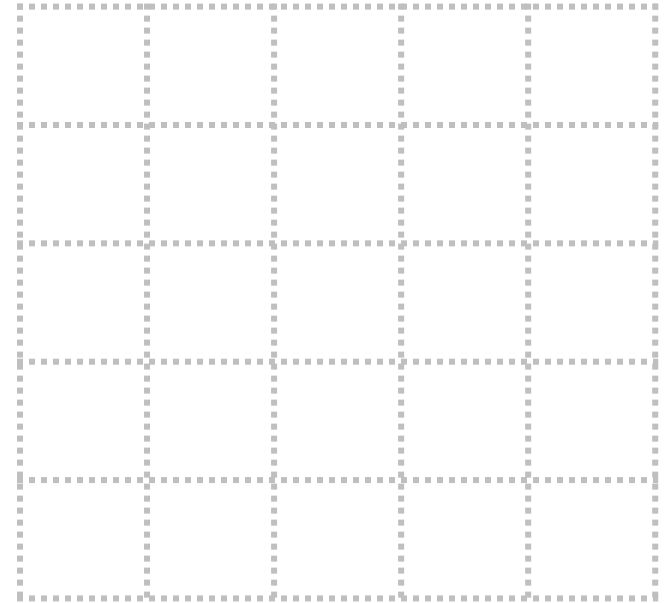
`np.arange(2)`



`np.arange(3)`

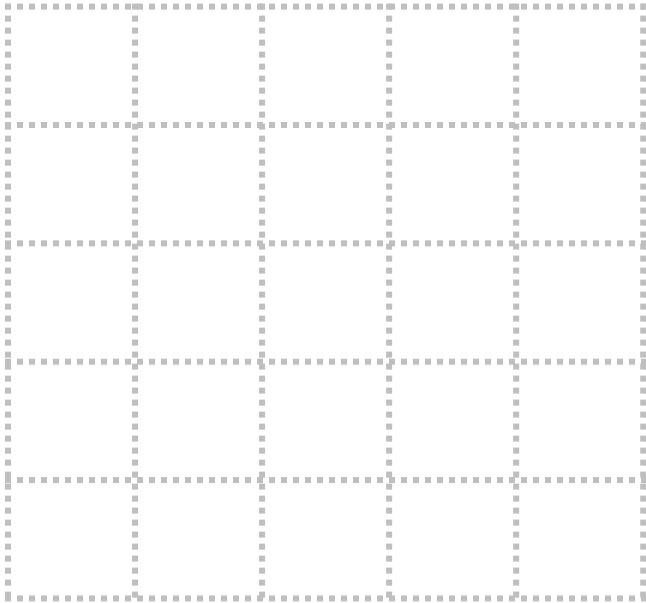


`np.arange(4)`



`np.arange(start, stop)`

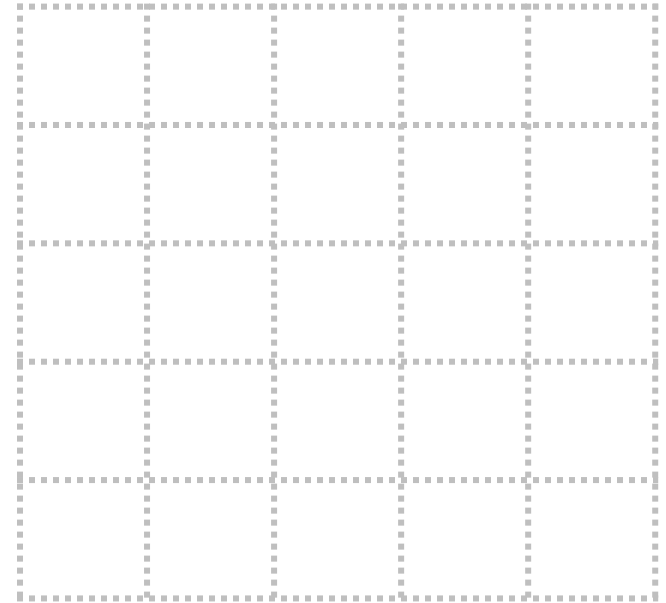
`np.arange(1,4)`



`np.arange(2,6)`

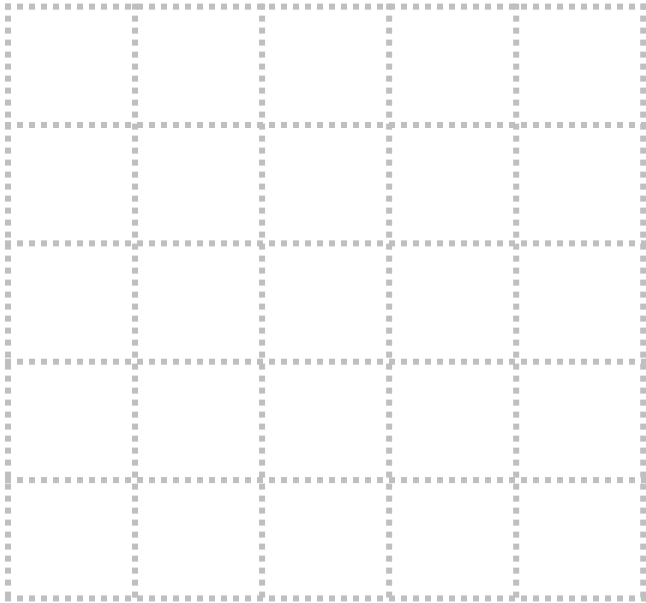


`np.arange(2, 2)`

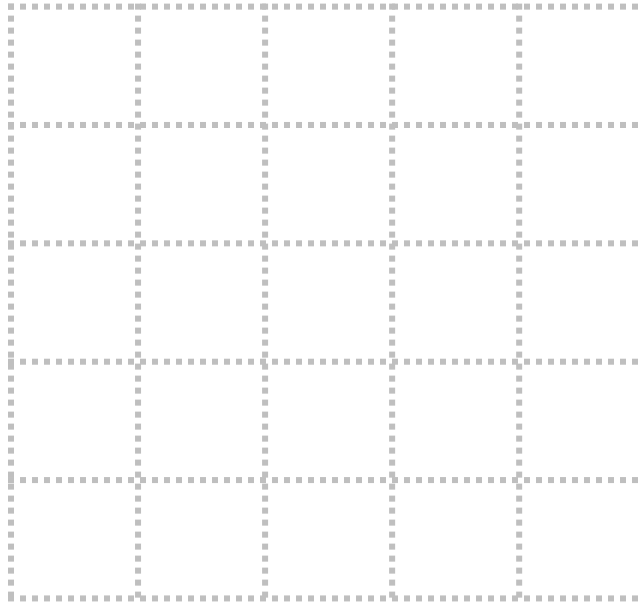


np.eye

`np.eye(1)`



`np.eye(2)`

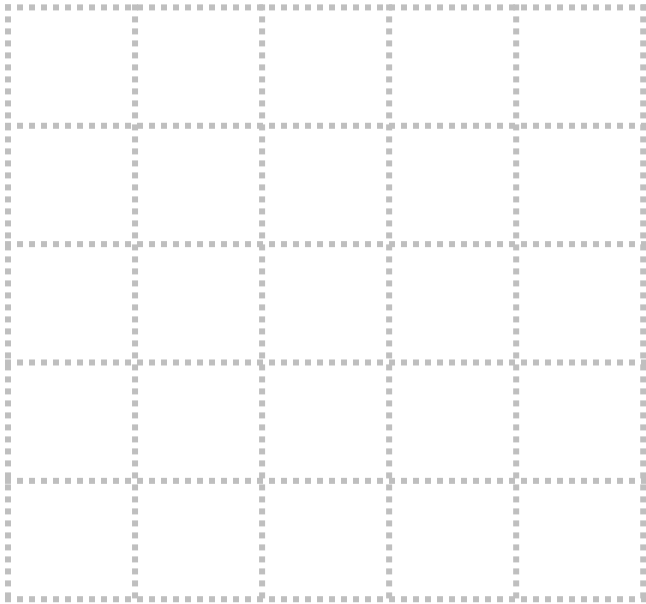


`np.eye(3)`



np.vstack

```
a = np.ones((2,3))  
b = np.zeros((3,3))  
np.vstack((a,b))
```



```
a = np.ones((2,2))  
b = np.zeros((1,2))  
np.vstack((a,b))
```

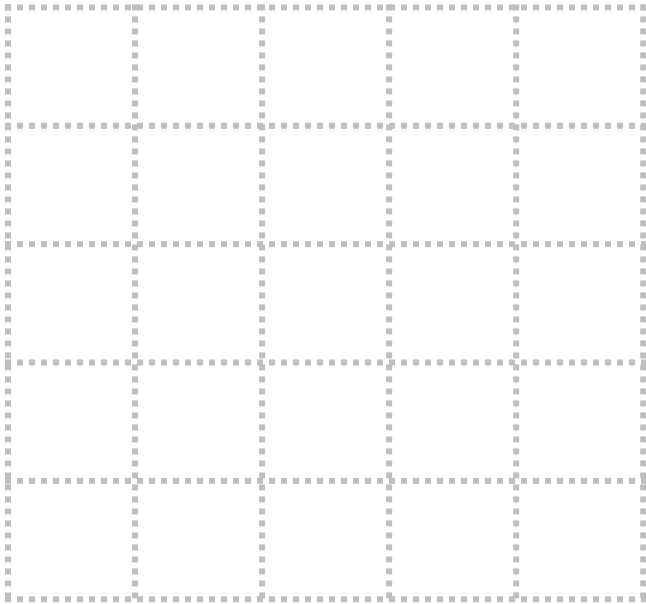


```
a = np.ones((2,3))  
b = np.zeros((1,3))  
np.vstack((b,a,b))
```

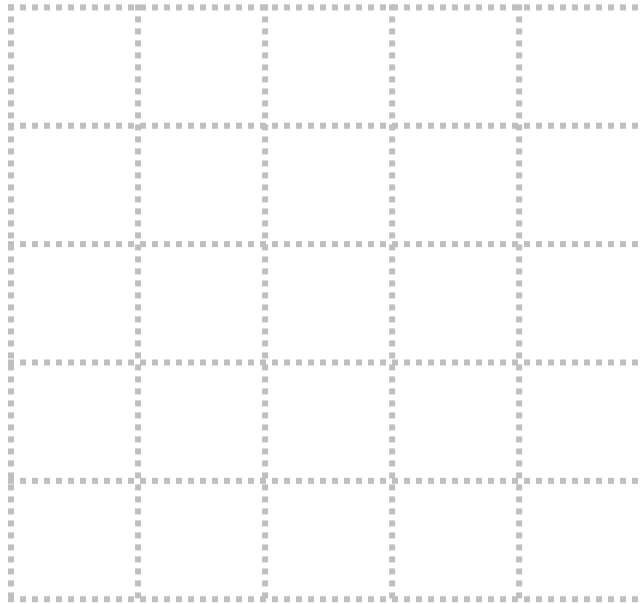


np.hstack

```
a = np.ones((2,2))  
b = np.zeros((2,2))  
np.hstack((a,b))
```



```
a = np.ones((3,2))  
b = np.zeros((3,1))  
np.hstack((a,b))
```



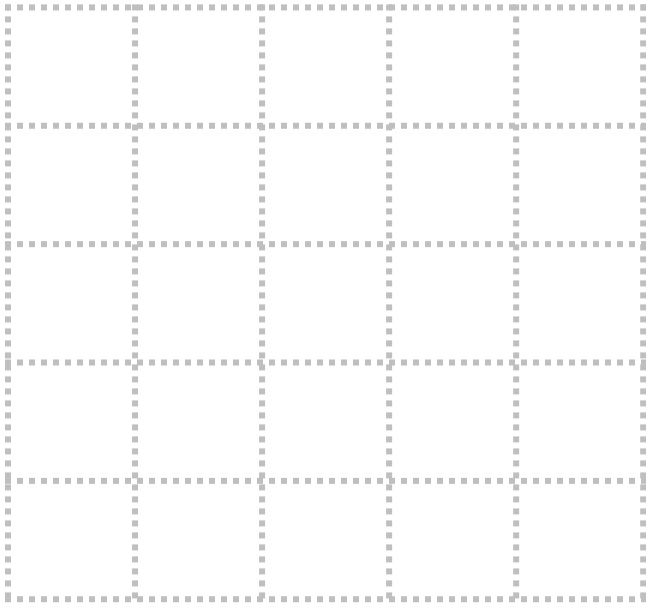
```
a = np.ones((4,1))  
b = np.zeros((4,1))  
np.hstack((a,b,a,b))
```



hstack vs. vstack

```
a = np.arange(1,3)  
b = np.zeros(2)
```

```
np.vstack((a,b))
```



```
np.hstack((a,b))
```





np.vstack

```
a = np.eye(2)
b = np.zeros((2,2))
np.vstack((a,b))
```

1	0			
0	1			
0	0			
0	0			

```
a = np.arange(3)
b = np.zeros(3)
np.vstack((a,b))
```

0	1	2		
0	0	0		

```
a = np.arange(2,5)
b = np.arange(1,4)
np.vstack((b,a,b))
```

1	2	3		
2	3	4		
1	2	3		



np.hstack

```
a = np.ones((3,1))  
b = np.zeros((3,1))  
np.hstack((a,b))
```

1	0			
1	0			
1	0			

```
a = np.eye(2)  
b = np.zeros((2,2))  
np.hstack((a,b))
```

1	0	0	0	
0	1	0	0	

```
a = np.eye(3)  
b = np.ones((3,1))  
np.hstack((b,a,b))
```

1	1	0	0	1
1	0	1	0	1
1	0	0	1	1

1D Slices

	0	1	2	3	4	5	6	7	8
data	5	3	-2	4	8	2	4	-1	2

data[0:2]	5	3	-2	4	8	2	4	-1	2
-----------	---	---	----	---	---	---	---	----	---

data[3:6]	5	3	-2	4	8	2	4	-1	2
-----------	---	---	----	---	---	---	---	----	---

data[5:]	5	3	-2	4	8	2	4	-1	2
----------	---	---	----	---	---	---	---	----	---

2D Slices

data

1	3	2	4
1	4	3	2
5	2	1	3

data[:,0]

1	3	2	4
1	4	3	2
5	2	1	3

data[:,1]

1	3	2	4
1	4	3	2
5	2	1	3

data[:,1:2]

1	3	2	4
1	4	3	2
5	2	1	3

data[:,2:4]

1	3	2	4
1	4	3	2
5	2	1	3

2D Slices

data

1	3	2	4
1	4	3	2
5	2	1	3

`data[0:2,0:2]`

1	3	2	4
1	4	3	2
5	2	1	3

`data[1:3,1:3]`

1	3	2	4
1	4	3	2
5	2	1	3

`data[1:3,1:4]`

1	3	2	4
1	4	3	2
5	2	1	3

`data[1:,1:]`

1	3	2	4
1	4	3	2
5	2	1	3



2D slices

data

1	3	2	4
1	4	3	2
5	2	1	3

data[:,0:2]

1	3	2	4
1	4	3	2
5	2	1	3

data[:,1:3]

1	3	2	4
1	4	3	2
5	2	1	3

data[:,1:4]

1	3	2	4
1	4	3	2
5	2	1	3

data[:,0:3]

1	3	2	4
1	4	3	2
5	2	1	3



2D slices

data

1	3	2	4
1	4	3	2
5	2	1	3

`data[1:2,0:2]`

1	3	2	4
1	4	3	2
5	2	1	3

`data[0:1,1:3]`

1	3	2	4
1	4	3	2
5	2	1	3

`data[1:3,1:4]`

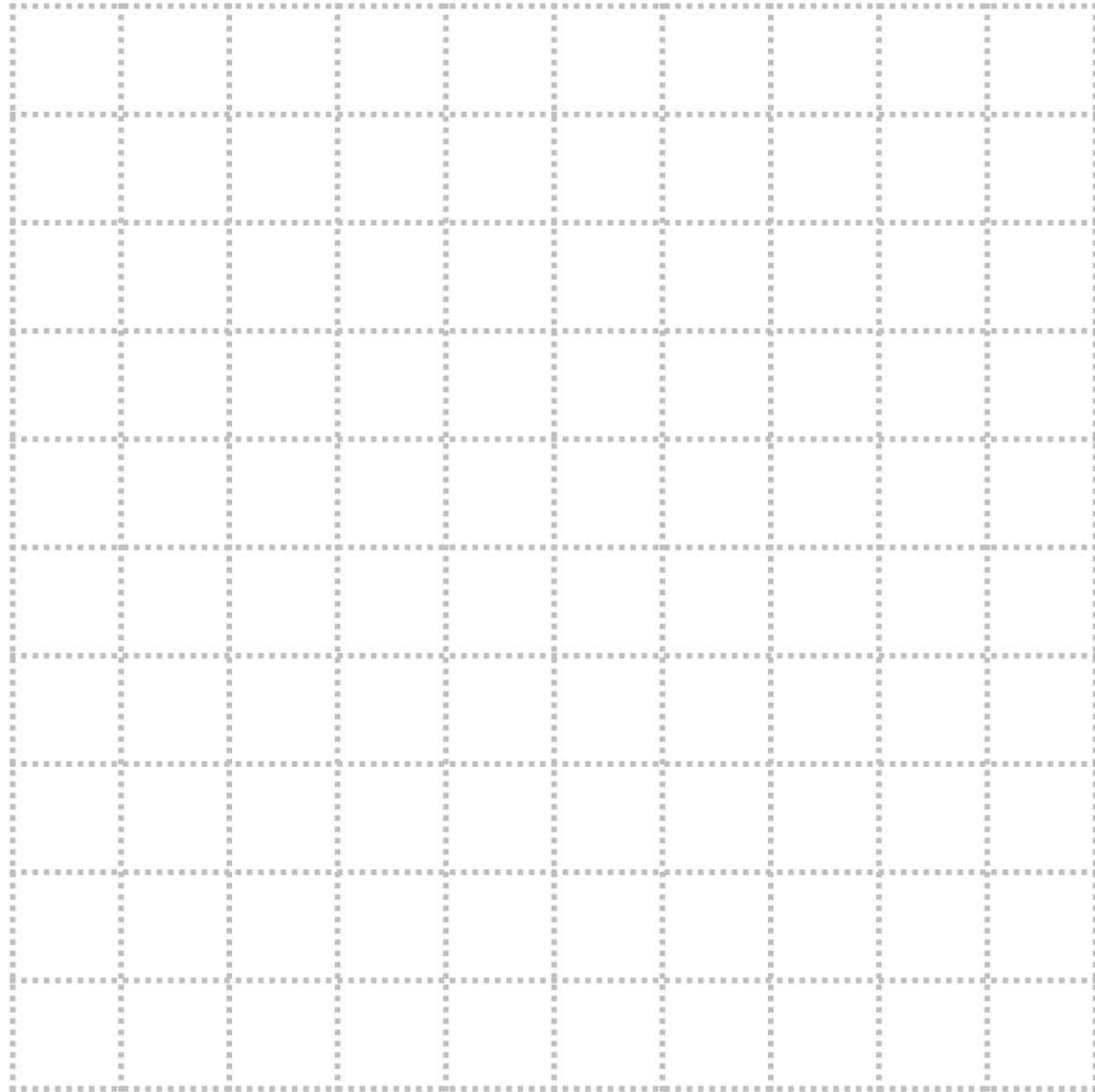
1	3	2	4
1	4	3	2
5	2	1	3

`data[1:,1:4]`

1	3	2	4
1	4	3	2
5	2	1	3

Draw an image

1. `I = np.zeros((5,5))`
2. `I[2,2] = 1`
3. `I[4,:] = 2`
4. `I[1:3, 0:2] = 3`

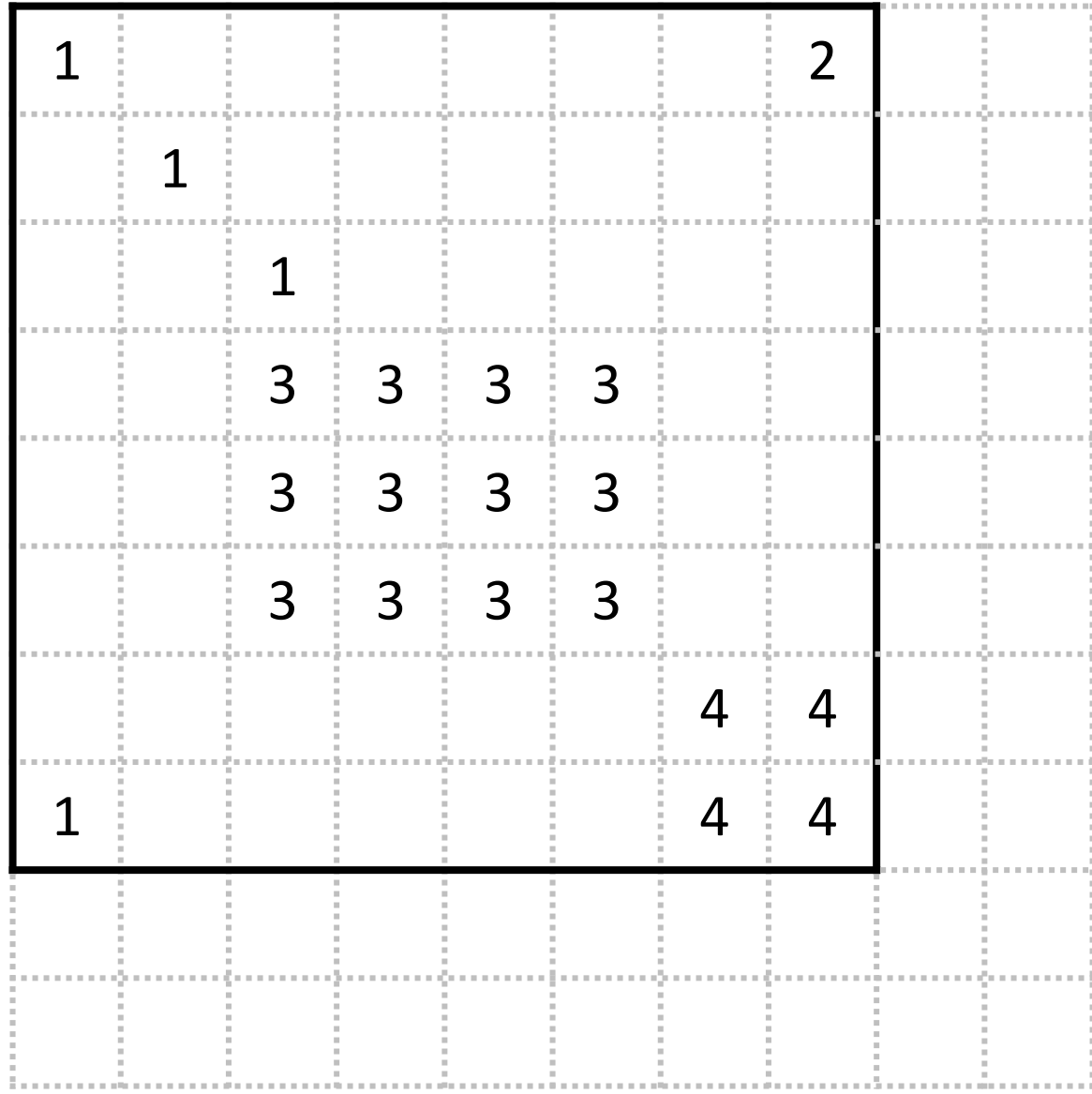




Draw an image

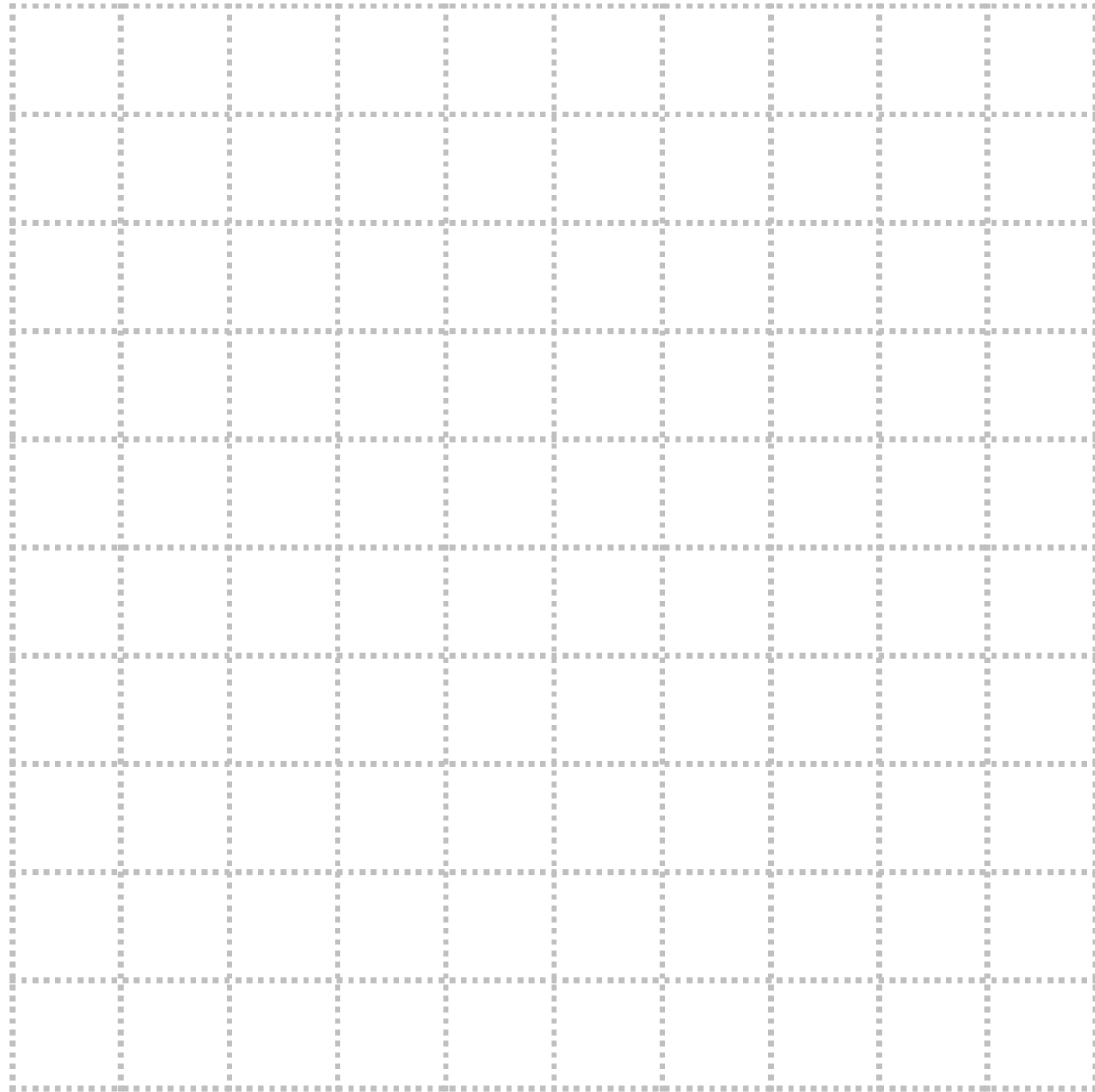
I

1. `I = np.zeros((8,8))`
2. `I[:,0] = 1`
3. `I[0,:] = 2`
4. `I[3:6, 2:5] = 3`
5. `I[6:8, 6:8] = 4`



Reference vs Copy

```
1. I = np.eye((3,3))  
2. J = I  
3. J[0,2] = 2  
4. K = I.copy()  
5. K[0,1] = 3  
6. I[2,0] = 4
```





Reference vs. Copy

```
1. I = np.zeros(8)
2. J = I
3. J[1] = 1
4. I[2] = 2
5. K = I.copy()
6. J[3] = 5
7. K[4] = 6
8. I[5] = 7
```

I J
K

0	1	2	5	0	7	0	0
0	1	2	0	6	0	0	0



Reference vs Copy

K I

1. `I = np.eye((4,4))`
2. `K = I`
3. `K[0,1:3] = np.arange(2)`
4. `J = K.copy()`
5. `J[3,2] = 3`
6. `I[2,3] = 4`
7. `K[2,1] = 5`

J

1	0	1	4
1	1	1	1
5	1	1	1
1	1	1	1

Logic Operation

```
1. I = np.arange(5)
2. a = I < 2
3. b = I == 3
4. c = I[a]
5. d = I[b]
```

I	0	1	2	3	4							

Logic Operation

1. `a = I < 5`
2. `J = np.zeros((3,3))`
3. `J[a] = 2`
4. `J[I==9] = -1`

I

1	2	3							
4	5	6							
7	8	9							



Logic Operation

1. `I = np.arange(5)`
2. `a = I > 3`
3. `b = I == 1`
4. `c = np.zeros(5)`
5. `c[a] = 5`
6. `c[b] = 7`

I
a
b
c

0	1	2	3	4
F	F	F	F	T
F	T	F	F	F
0	7	0	0	5



Logic Operation

1. $a = I > 7$
2. $b = I < 3$
3. $J = \text{np.ones}((3,3))$
4. $J[a] = 6$
5. $J[b] = 9$

			J						
I	1	2	3	9	9	1			
	4	5	6	1	1	1			
	7	8	9	1	6	6			
a	F	F	F						
	F	F	F						
	F	T	T						
b	T	T	F						
	F	F	F						
	F	F	F						