Electronics I Test I Formulae

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Circuit Fundamentals

[Voltages and currents, resistors, inductors, transistors, DC analysis, AC analysis]

$$\textbf{Newton's Second Law:} \quad \vec{F} = m \cdot \vec{a} \quad | \quad \left[\vec{F}\right] = 1\, kg\, m\, s^{-2}.$$

$$\textbf{Incremental work:} \quad dW = \vec{F} \cdot d\vec{s} \quad | \quad [dW] = 1\, kg\, m^2\, s^{-2}. = 1\, J.$$

$$\textbf{Power:} \quad P = \frac{dW}{dt} \quad | \quad [P] = 1\, kg\, m^2\, s^{-3}.$$

$$\textbf{Electric current:} \quad I = \frac{dQ}{dt} \quad | \quad [I] = 1\, C\, s^{-1} = 1\, A.$$

Some constants:

- Charge of an electron $q_e = -1.602 \times 10^{-19}\, C$.
- Possble values of charge: $Q = \pm n \cdot q_e$, $n \in \mathbb{N}$.
- $1\, C$ of charge $\equiv 6.28 \times 10^{18}$ electrons.

Electric Field

Coulomb's Law:

$$\vec{F} = k \cdot \frac{q_1 q_2}{|\, r\,|^2}$$

Electric field:

$$\vec{E} = \frac{\vec{F}}{Q} = k \cdot \frac{\vec{q}\,Q}{|\, \vec{r}\,|^2} \cdot \frac{1}{Q} = k \frac{\vec{q}}{|\, \vec{r}\,|^2}.$$

Voltage

Potential difference:

$$\text{Work} \quad \vec{F} = Q \cdot \vec{E} \implies dW = \vec{F} \cdot d\vec{s} = Q \cdot \vec{E} \cdot d\vec{s}$$

$$\text{Voltage} \quad dV = \frac{1}{Q}\, dW = \frac{1}{Q} Q \cdot \vec{E} \cdot d\vec{s} \quad \therefore dV = \vec{E} \cdot d\vec{s}.$$

$$E = -\frac{dV}{ds} \quad | \quad [E] = 1\, V\, m^{-1}$$

, where $\vec{E}$ points in the direction of decreasing battery, i.e. from +ve to −ve.

For plate capacitor:

$$\vec{E} = -\frac{V}{d} \text{ ; } d \text{ is the distance between the plates.}$$

Current

$$I = \frac{dQ}{dt} \quad | \quad [I] = 1\,C\,s^{-1} = 1\,A.$$

Ohm's Law

$$\text{Power} \quad P = \frac{dW}{dt} = \frac{dW}{dQ} \cdot \frac{dQ}{dt} \iff \text{Ohm's Law} \quad P = VI.$$

$$\text{Resistivity} \quad R = \rho\frac{\ell}{A}\,, \quad \text{where } \textit{conductance} \;\; G = \frac{1}{R}$$

Joule's Heating Law

$$\text{Joule's Power law:} \quad P = VI = \frac{V^2}{R} = I^2 R = V^2 G$$

$$\text{where the thermal energy is} \quad dW = P\,dt = I^2 R\,dt.$$

Root mean squared values

$$\text{in AC circuits} \quad V_{rms} = \frac{V_p}{\sqrt{2}} \implies E = \frac{V_{rms}^2}{R}t$$

RLC circuits

$$\text{Capacitance} \quad Q = cV$$

$$\text{Exponential decay function [RC]:} \quad I_0 = \frac{V}{R}e^{-t/\tau}\,, \quad \text{where } \tau := RC.$$

$$\textbf{RC voltage:} \quad V_c(t) = A + Be^{-t/\tau}.$$

$$\text{Exponential decay function [RL]:} \quad V_i = I_0 R_i e^{-t/\tau}\,, \quad \text{where } \tau := \frac{L}{R}.$$

$$\text{Voltage dividor rule:} \quad V_c^i(t) = \sum_i \frac{V_s}{R_{tot}} R_i$$

$$\text{Superposition theorem:} \quad V_c^i(t) = \sum_i \frac{V_s}{R_E} R_i \;\; \text{or resolve by shorting each S (better).}$$

$$\textbf{RLC circuit:} \quad I_L(t) = Ae^{s_1 t} + Be^{s_2 t}. \qquad V_L(t) = L\frac{d}{dt}I(t).$$

$$\text{Damping factor} \;\; \alpha = \frac{1}{2RC}\,, \quad \text{Natural frequency} \quad \omega_0^2 = \frac{1}{LC}.$$

$$\text{Characteristic equation:} \quad s^2 + s\frac{1}{RC} + \frac{1}{LC} = 0 \;\; \text{where} \;\; s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha \pm \omega_d.$$

$$\textbf{Standard integral for DEs:} \quad \int e^{at}\sin(pt)\,dt = \frac{e^{at}}{a^2+p^2}\left[a\sin(pt) - p\cos(pt)\right] + C.$$

$$V_c(t) = \frac{1}{C}\int I(t)\,dt. \qquad I_L(t) = e^{at}\left[A\sin(\omega t) + B\cos(\omega t)\right].$$

Kirchhoff's Laws

Sources: Battery, power supply, potential rises.
Sinks: Resistors, potential drops.

Take a T-circuit with two loops $\gamma_{1,2}$ and nodes n:

$$\text{Kirchhoff's voltage law:} \quad \sum_\gamma V = 0 \iff V_B = \sum V_{R_i} \quad \forall i$$

$$\text{Kirchhoff's current law:} \quad \sum_n I_n = 0 \iff I_B = \sum_i I_{R_i} \quad \forall i$$

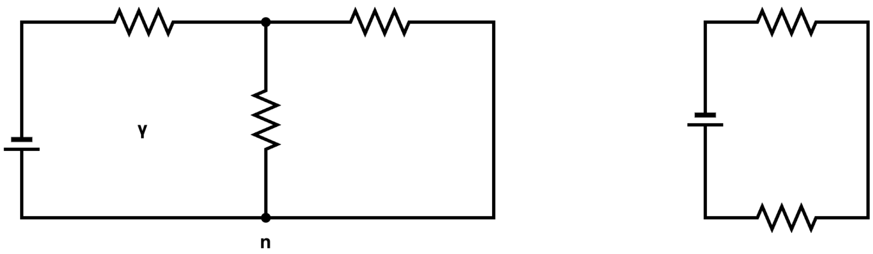

Figure 1: Kirchhoff's laws [(a) KVL; (b) KIL]

Alternatively, we can say that voltage remains constant across parallel units and current remains constant over series units, or voltage divides along series units and current divides along parallel units. This alternate view of circuits gives the following:

$$R_E = \sum R_s \quad ; \quad R_s = \sum_i R_i \quad , \quad R_p = \left(\sum_i \frac{1}{R_i} \right)^{-1} .$$

Results, Theorems

- **Source equivalence:** If two sources produce the same short-circuit current (I_s) when $R_L = 0$ and $R_L = \infty$ [i.e. two sources produce the same v-i characteristics at their terminals], then the sources are equivalent.
- **Circuit transformations:** Two v-i character equivalent sources can be transformed from current sources to equivalent voltage source, and vice versa.
- **Superposition:** Circuit theory is a linear analysis, giving rise to the principle of superposition. It states that in a circuit with more than one source present, the voltage or current anywhere in the circuit can be obtained by first finding the repose due to one source acting alone, then the second source acting alone, and so forth.
- **Thévenin's Theorem** states that for any arbitrary one-port network of arbitrary complexity, connected to a load resistor, the one-port can be replaced by a series combinations of an ideal voltage source V_{th}, and a resistance R_{th}, where V_{th} is the open-circuit voltage of the one-port and R_{th} is the ratio of the open-circuit voltage to the short-circuit current of the one-port.

$$R_L \equiv R_{E-op} \equiv R_{th} \equiv R_{in}$$

- **Norton's Theorem** states that the equivalent circuit for a one-port can also be a practical current source, hence positing a dual to Thévenin's theorem.
- **Maximum power transfer theorem** states that maximum power is transferred from the source to the load when the load resistance R_L is equal to the internal resistance of the source R_i.

Circuit Transformation Case Study

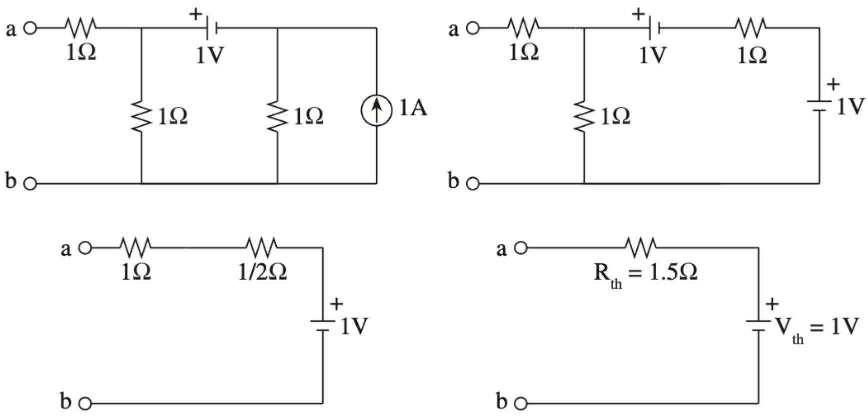

Figure 2: Thévenin's equivalent circuit reduction

$$\max P = \frac{V_{th}^2}{R_{th}} = \frac{1}{R_L}\left(\frac{1}{2}V_L\right)^2 \quad \because V_{th} \equiv \frac{1}{2}V_L.$$