Probability and Statistics Final Exam Formula Sheet

Chapter 6: Continuous Random Variables

1. Uniform Distribution

- **Density Function:** $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$
- **Mean:** $\mu = \frac{a+b}{2}$
- **Variance:** $\sigma^2 = \frac{(b-a)^2}{12}$
- **Probabilities:**

$$P(X < c) = \int_a^c f(x)dx, \quad P(X > c) = \int_c^b f(x)dx, \quad P(c < X < d) = \int_c^d f(x)dx$$

2. Normal Distribution

- **Standard Normal (Z):** $f(z) = \frac{1}{\sqrt{2\pi}}e^{-z^2/2}$
- **Normal (X):** $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$
- **Mean and Variance:** μ_X, σ_X^2 (for X); $\mu_Z = 0, \sigma_Z^2 = 1$ (for Z)
- **Z-Transform:** $Z = \frac{X-\mu}{\sigma}$
- **Normal Approximation to Binomial:** $Z = \frac{X-n\pi}{\sqrt{n\pi(1-\pi)}}$
- **Continuity Correction:** Add or subtract 0.5 to the bounds of X

3. Using the Normal Table

- $P(Z < a)$: Area under the standard normal curve to the left of a
- $P(Z > a)$: $1 - P(Z < a)$
- $P(a < Z < b)$: $P(Z < b) - P(Z < a)$
- Critical Values: Z_α: Value such that $P(Z > Z_\alpha) = \alpha$

Chapter 7: Functions of Random Variables

1. Transformations

- $Y = f(X)$: Use Theorems 7.1-7.4 to find the distribution of Y

2. Moments and Moment Generating Functions

- r-th Moment: $E(X^r)$
- Moment Generating Function (MGF): $M_X(t) = E(e^{tX})$
- Relation: $E(X^r) = \left.\frac{d^r M_X(t)}{dt^r}\right|_{t=0}$

Chapter 8: Fundamental Sampling Distributions

1. Central Limit Theorem (CLT)

- $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ as $n \to \infty$
- Sampling Distribution of the Difference of Means:

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

2. Chi-Square Distribution (χ^2)

- $f(x;v) = \frac{1}{2^{v/2}\Gamma(v/2)} x^{v/2-1} e^{-x/2}$, $x > 0$
- Degrees of Freedom: v
- Critical Value: χ^2_α: $P(\chi^2 > \chi^2_\alpha) = \alpha$

3. t-Distribution

- $f(t;v) = \frac{\Gamma((v+1)/2)}{\sqrt{v\pi}\Gamma(v/2)} \left(1 + \frac{t^2}{v}\right)^{-(v+1)/2}$
- Degrees of Freedom: v
- Critical Value: t_α: $P(T > t_\alpha) = \alpha$

4. F-Distribution

- $f(F;v_1,v_2) = \frac{\sqrt{\left(\frac{v_1 F}{v_1 F + v_2}\right)^{v_1} \left(\frac{v_2}{v_1 F + v_2}\right)^{v_2}}}{F\, B(v_1/2, v_2/2)}$
- Degrees of Freedom: v_1, v_2
- Critical Value: F_α: $P(F > F_\alpha) = \alpha$

Chapter 9: One and Two Sample Estimation Problems

1. Confidence Intervals

- Known σ^2:

$$\mu \in \left[\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

- Unknown σ^2:

$$\mu \in \left[\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}\right]$$

- Confidence Interval for Variance (σ^2):

$$\sigma^2 \in \left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right]$$

2. Difference of Two Means

- Known Variances:

$$\mu_1 - \mu_2 \in \left[\bar{X}_1 - \bar{X}_2 \pm Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$$

- Unknown Equal Variances:

$$\mu_1 - \mu_2 \in \left[\bar{X}_1 - \bar{X}_2 \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\right]$$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$$

3. Ratio of Two Variances

- Confidence Interval:

$$\frac{\sigma_1^2}{\sigma_2^2} \in \left[\frac{s_1^2/s_2^2}{F_{\alpha/2}}, \frac{s_1^2/s_2^2}{F_{1-\alpha/2}}\right]$$