Interferometry

Lab Report # 01

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Abstract

This experiment aimed to measure the wavelength of light emitted by a He-Ne laser using a Michelson interferometer and to calculate the index of refraction of air. The interference theory was applied by observing the fringe patterns produced when splitting and recombining laser beams, while varying the optical path length using a movable mirror. Part I involved calculating the laser's wavelength from the displacement of the mirror and fringe count. MATLAB was used to plot the collected data and derive the wavelength, yielding a value of 660 nm, which closely matches the accepted wavelength of 632.82 nm for He-Ne lasers. The percentage error, fractional uncertainty, and other statistical measures such as R-squared were calculated to assess the quality of the results. The experiment demonstrated the effectiveness of the interferometer in precise measurements, though potential improvements in alignment and environmental control were noted.

Part II focused on using similar techniques to measure the index of refraction of air by analyzing how pressure variations affect the fringe patterns. However, it could not be performed due to faulty equipment.

Copyright Statement

Contents

1 Introduction

An interferometer is an investigative tool or device which works on the principles of superposition of waves. Its operational mechanism involves splitting a beam of light into two orthogonal paths, which bounce back from a mirror, and converge. The effects of this procedure is finally observed on a viewing screen, placed after the point of convergence.

The interferometer is a helpful tool in the investigations of physical laws and properties of light, and the media in which it may travel. The purpose of this laboratory is twofold:

- Firstly, it is to calculate the wavelength of light produced by the laser source by measuring the distance between two consecutive crests of the interferometer-produced interference pattern on the viewing screen. The measured quantities in this portion of the experiment are the displacement of the micrometer knob from index zero, and the prefixed number of fringes passing a reference point. The calculated value is the wavelength of light. The physical principle used is interference theory.

$$\lambda = \frac{2d_m}{m} \tag{1}$$

- Secondly, it is to calculate the index of refraction of air, using the interferometer to measure the wavelength of light in a medium with varying air pressure. The measured quantities in this portion of the experiment are the initial and final pressure values in the vacuum gauge, and the number of fringes passing a reference point. The calculated value is the refractive index of air. The physical principle used is the principle of refraction of light.

$$\frac{n_i - n_f}{P_i - P_f} = \frac{\Delta m\, \lambda_0}{2d\,(P_i - P_f)} \tag{2}$$

The purpose of this laboratory experiment was to measure two key physical quantities using an interferometer: the wavelength of light from a laser source and the index of refraction of air. These measured quantities allowed us to apply fundamental wave and optical theories to determine the desired physical properties accurately.

2 Theory

2.1 Physical Principles involved

The Michelson interferometer operates based on the **principle of light wave interference**, where light beams from the same coherent source superimpose, producing patterns of maxima (bright fringes) and minima (dark fringes) due to constructive and destructive interference. The foundational concept of interference relies on the **superposition principle**, where the oscillating electric and magnetic fields of individual light waves combine vectorially at each point in space. The resulting field depends on the relative phase and amplitude of the individual waves.

When two light beams meet, the outcome of their superposition depends on their phase relationship. If the beams originate from separate sources, they do not have a fixed phase relationship, and their interference results in a rapidly fluctuating intensity, which is averaged by the human eye into a uniform light pattern. However, if both beams come from the same source, they maintain a fixed phase relationship, producing distinct interference patterns of alternating bright and dark fringes.

The Michelson interferometer, devised by A.A. Michelson in 1881, leverages this interference principle to measure the wavelength of light and extremely small displacements. Michelson's design splits a beam of light into two separate beams using a **beam splitter**, a device that reflects half the light and transmits the other half. The two beams then travel along different paths, one towards a fixed mirror and the other towards a movable mirror. Upon reflection from the mirrors, the beams recombine at the beam splitter and interfere with each other. Depending on the difference in the optical path lengths, constructive or destructive interference occurs, creating a pattern of light and dark fringes on a viewing screen.

2.2 Interference and Path Length Difference

Since the two beams originate from the same coherent light source, they were initially in phase when split. The phase difference between them at the point of recombination is determined by the **optical path length difference** (OPLD), which is defined as the difference in the total distances traveled by the two beams. The interference condition at any point on the viewing screen is directly related to this path difference:

- **Constructive interference** (bright fringes) occurs when the path difference is an integer multiple of the wavelength, $\Delta L = m\lambda$, where m is an integer.
- **Destructive interference (dark fringes)** occurs when the path difference is an odd multiple of half the wavelength, $\Delta L = (m + \frac{1}{2})\lambda$, where m is an integer.

The optical path difference is varied by moving the **movable mirror (M2)**. As the mirror is shifted by a distance d_m, the light beam traverses this path twice (once towards the mirror and once back). Therefore, the total path length changes by $2d_m$. By carefully measuring d_m, the number of fringes m observed, and using the interference condition, the wavelength of light can be calculated as:

$$\lambda = \frac{2d_m}{m}$$

This derivation provides the formula to measure the wavelength of the laser light source by observing the number of fringes that shift as the mirror moves by a known distance.

2.3 Application of Interference Theory

Michelson's interferometer was originally designed to test the hypothesis of the **luminiferous ether**, a theoretical medium in which light was believed to propagate. The experiment aimed to detect the relative motion of matter through the ether by comparing the speed of light along different paths. However, the negative results of the experiment provided key evidence against the existence of the ether, leading to the development of **Einstein's theory of relativity**.

In modern applications, the Michelson interferometer has become a precise tool for measuring **wavelengths of light**, **index of refraction changes**, and **minute displacements**. When used to measure wavelengths, the movable mirror is displaced by a known distance, and the number of fringe shifts is counted to compute the wavelength using the equation derived above. Conversely, if the wavelength of the light source is known, the Michelson interferometer can measure extremely small distances with high accuracy.

2.4 Interference Pattern

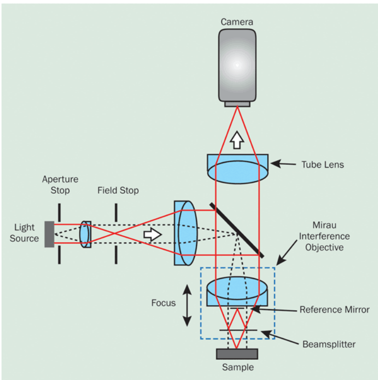
When a lens is placed between the laser and the beam splitter, the light beam diverges, creating a broader interference pattern of concentric bright and dark rings. The pattern can be explained by considering the varying path length difference across the beam's cross-section. At different points on the viewing screen, the path difference between the two beams changes, resulting in different interference conditions. Bright and dark rings represent points of constructive and destructive interference, respectively.

As the movable mirror is displaced by a distance of one-quarter of the wavelength ($\lambda/4$), the optical path changes by $\lambda/2$. This shift causes the interference pattern to invert: the positions of the maxima (bright rings) and minima (dark rings) switch places. When the mirror is moved further by an additional $\lambda/4$, the pattern is restored to its original configuration. By continuously shifting the mirror and counting the number of times the pattern is restored to its original state, one can accurately measure the wavelength of the light.

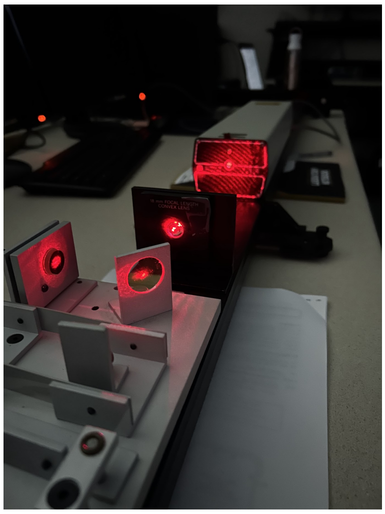
3 Experimental Procedure

3.1 Experimental setup

The optics bench was leveled to the table using adjustment screws on the bench. The interferomenter was placed on the smaller partition of the optics bench with one screw at the end. The laser was placed on the longer partition of the optics bench, opposing the interferometer. The laser was made flush against against the alignment rail on the edge of the longer partition of the optics bench, and the interferometer was also leveled accordingly. The laser was turned on and directed such that the laser beam reflected off of the movable mirror and directly into the aperture of the laser. The beam-splitter was rotated so it was at an angle of approximately 45°with the incident beam from the laser, producing two sets of laser spots on the viewing screen - one for each path followed. The angle of the other mirror was adjusted until the two sets of laser spots superimpose on the viewing screen. The 18 mm focal length was then placed in front of the laser beam and aligned to produce circular fringes of the interference pattern.



(a) Diagram

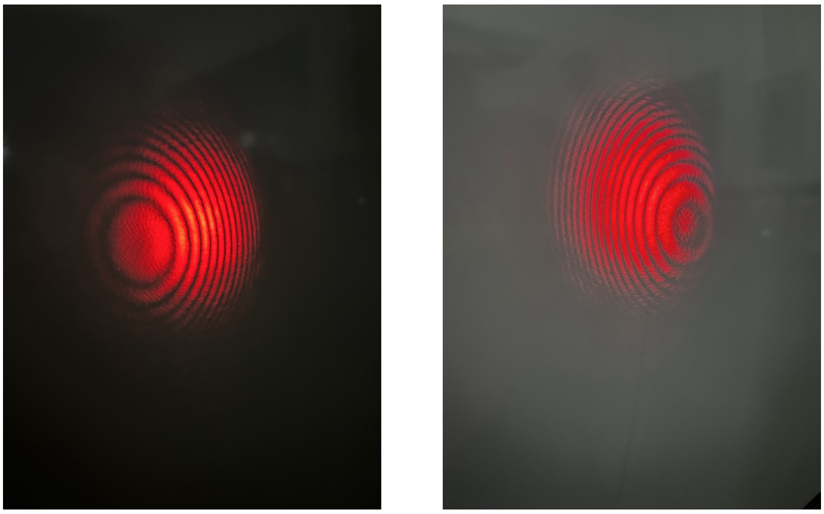


(b) Actual

Figure 1: Experimental setup

3.2 Part I: Measuring the wavelength of light

The micrometer knob on the interferometer was adjusted so the lever arm was parallel to the edge of the width of the interferometer. The knob was turned counterclockwise for over one full rotation until the "0" value on the knob was aligned with the index mark. Two marks were drawn with a black dry erase pen to distinguish one fringe of the interference pattern on the viewing screen. The micrometer knob was slowly rotated counterclockwise. A specific number of fringes (over 20) was predetermined a the fringes were counted with reference to the black marks. The knob was rotated until the predetermined number of fringes pass through the mark. The displacement of the knob in microns was recorded, with each division of the micrometer knob corresponding to one micron ($10^{-6}\, m$) of mirror movement. The procedure was then repeated 34 more times, with ranging number of fringes, and the data for 35 successful trials was recorded.



(a) Darker (b) Lighter

Figure 2: Viewing screen

3.3 Part II: Measuring the index of refraction of air

The experiment could not be conducted because the vacuum cell was broken and a replacement was not available.

4 Data

4.1 Collected data from part I

Table 1: Experimental data from part I

	Number of fringes passed	**Displacement of micrometer knob (microns)**
1	20	6.0
2	21	6.9
3	21	6.9
4	21	6.9
5	21	7.3
6	22	7.4
7	22	7.5
8	22	7.5
9	22	7.2
10	22	7.0
11	23	7.8
12	23	7.8
13	24	8.0
14	24	8.0
15	24	7.1
16	25	7.8
17	25	7.8
18	25	8.0
19	25	8.2
20	26	8.9
21	26	8.8
22	26	8.5
23	27	8.1
24	27	8.1
25	27	8.1
26	27	9.0
27	28	9.1
28	28	9.9
29	28	9.1
30	28	8.8
31	29	9.8
32	29	9.8
33	29	9.5
34	30	10
35	30	10.3

4.2 Data plot of part I

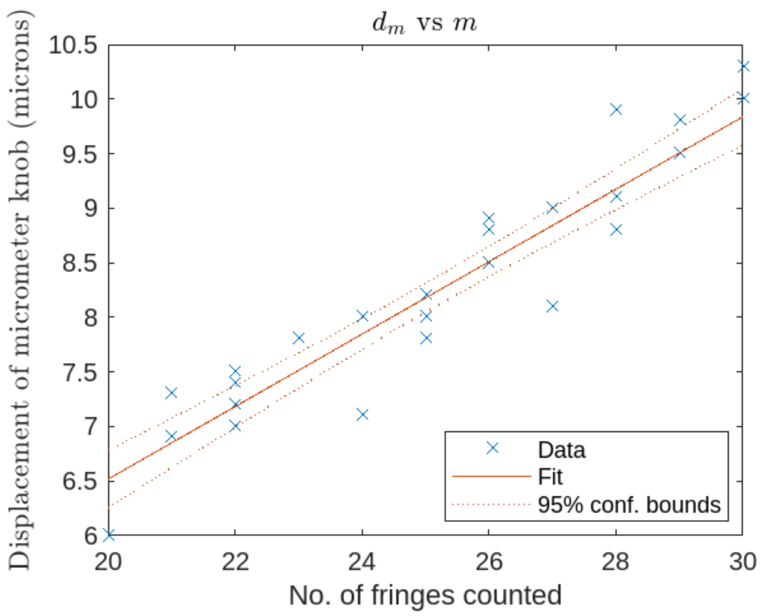


Figure 3: Data plot and linear regression

4.3 Calculated values for part I

```
    Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate       SE        tStat       pValue
                   ________    ________    ________    __________

    (Intercept)    -0.12247     0.56036    -0.21856       0.82834
    x1              0.33203    0.022213      14.947    2.9871e-16
```

5 Calculations and Results

5.1 Description of results

The relationship in equation (3) was used to estimate the value of the wavelength of the laser, using MATLAB to plot the acquired data points and produce a line of best fit. A linear regression algorithm was used to find the slope of the line, which is equivalent to the value of the experimental wavelength. A slope value of $3.32 \times 10^{-11}\,m$ was observed. Using the aforementioned relationship, we acquire the value $\lambda = 6.64 \times 10^{-11}\,m \approx 660\,nm$. The He-Ne laser used has an actual wavelength of $\lambda_0 = 632.82\,nm$ [2]. These values were used to calculate percentage error, fractional uncertainty, and percentage uncertainty. Furthermore, MATLAB produced the other measures recorded in the following subsection. Section (6) talks about the error analysis in more detail.

5.2 Measurements of errors

- Percentage Error: 4.3%
- Fractional Uncertainty from Standard Error (SE): ± 0.236 microns = $\pm 263\,nm$
- Percentage Uncertainty: 39.6%
- F-statistic: 223 ($\nu = 35, 33$, $P_f = 1.93\%$)
- t-test: 14.947 (p-value: 2.9871×10^{-16})
- Root-mean-squared Error: 0.383
- R-squared/Adjusted R-squared: 0.871/0.867

6 Error Analysis

- Percentage Error:

$$|\frac{\lambda_x - \lambda_0}{\lambda_0}| \times 100\% = |\frac{664 - 632.82}{632.82}| \times 100\% = 4.3\%$$

The percentage error is calculated to compare the experimental wavelength obtained from the experiment to the actual wavelength of the He-Ne laser. The percentage error of 4.93% indicates a small but notable deviation between the experimental and actual wavelengths. This suggests the experiment was reasonably accurate, although there may have been slight errors in measurement or calibration of the interferometer.

- Fractional Uncertainty from Standard Error (SE):

The fractional uncertainty measures the uncertainty relative to the experimental value. To calculate the uncertainty in the slope, we use the standard error (SE) of the slope. The standard error formula for a linear regression slope is given by:

$$SE = \frac{\sigma}{\sqrt{n}};$$

where σ is the standard deviation of the residuals (errors) and n is the sample size (number of data points).

The uncertainty in the wavelength $\Delta\lambda$ can be calculated from the uncertainty in the slope Δm using the relationship:

$$\frac{\Delta m}{m} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta\lambda}{2m} \implies \Delta\lambda = 2\Delta m = 2\sqrt{n}SE;$$

where m is the slope of the line of best fit and the factor of 2 accounts for the path length doubling in the interferometer. Substituting the given values:

$$\Delta\lambda = 2 \times \sqrt{35} \times 0.0222 = \pm 0.236 \text{ microns} = \pm 263\, nm$$

The fractional uncertainty is then calculated as:

$$\frac{\Delta\lambda}{\lambda} 100\% = \frac{263}{664} 100\% = 39.6\%$$

The fractional uncertainty of 39.6% represents a large uncertainty relative to the measured wavelength. This large uncertainty suggests that there was significant variability in the data or errors in the experimental setup. Since we know that the precision of a He-Ne laser is extremely high, we can infer that the sources of error is limited to the interferometer or experimental setup/procedure.

- F-statistic:

  The F-statistic is a measure of how well the linear regression model fits the data. It is calculated as the ratio of the mean squared regression (MSR) to the mean squared error (MSE). The formula for the F-statistic is:

$$F = \frac{MSR}{MSE} = \frac{\frac{\sum(\hat{y_i}-\bar{y})^2}{p}}{\frac{\sum(y_i-\hat{y_i})}{n-p-1}} \; ;$$

  where $\hat{y_i}$ are the predicted values from the regression model, y_i are the observed data points, $\bar{y}$ is the mean of the observed values, p is the number of predictors, and n is the number of data points. Given an F-statistic of 223 and degrees of freedom $= 35, 33$, the p-value for the F-test is given as $P_f = 1.93\%$, meaning there is a 1.93% chance that the observed relationship is due to random noise rather than a true effect. A high F-statistic (223) with a low p-value (1.93%) strongly suggests that the linear regression model provides a statistically significant fit to the data, meaning the relationship between the variables is not due to random chance.

- t-test:

  The t-test evaluates the significance of individual coefficients in the regression model. For the slope, the t-statistic is calculated as:

$$t = \frac{m}{SE} \; ;$$

  where m is the slope and SE is the standard error of the slope. Given the t-statistic of 14.947, we can compare this value to the critical value from the t-distribution with 34 degrees of freedom. The associated p-value of 2.9871×10^{-16} is extremely small, indicating a highly significant result. The large t-value and extremely small p-value show that the slope of the regression line is significantly different from zero. This suggests that the experimental data provides a strong basis for calculating the wavelength, with a high confidence level.

- Root-mean-squared Error:

  The RMSE is a measure of the standard deviation of the residuals (prediction errors). It quantifies how spread out the errors are from the regression line. The formula is:

$$RMSE = \frac{\sum(y_i - \hat{y_i})^2}{n} \; ;$$

  where y_i are the actual values, and $\hat{y_i}$ are the predicted values from the regression. Given an RMSE of 0.383, this suggests that the typical error in the predicted wavelength is 0.383%, which is significantly small compared to the overall wavelength scale. The RMSE value of 0.383 indicates that the model's predictions are fairly accurate, with errors that are small compared to the overall range of measurements.

- R-squared/Adjusted R-squared:

  The R-squared value represents the proportion of the variance in the dependent variable that is explained by the independent variable. It is given by:

  $$R^2 = \frac{\sum (y_i - \hat{y_i})^2}{\sum (y_i - \bar{y})^2} \ ;$$

  where $\hat{y_i}$ are the predicted values from the regression model, y_i are the observed data points, and $\bar{y}$ is the mean of the observed values. The Adjusted R-squared accounts for the number of predictors in the model and is given by:

  $$R^2_{adj} = 1 - \left(\frac{(1 - R^2)(n-1)}{n-p-1} \right) .$$

  For this experiment $R^2 = 0.871$ and $R^2_{adj} = 0.867$, indicating that approximately 87% of the variation in the experimental data is explained by the linear regression model. The high R-squared and adjusted R-squared values suggest that the model fits the data well, explaining most of the variability in the experimental results.

Overall, the experimental results are valid on the basis of numerous metrics. Despite the relatively high variance in data points, the regression line converges on the correct slope over numerous iterations.

7 Conclusion

In this experiment, the wavelength of a He-Ne laser was measured using a Michelson interferometer. The experimentally determined wavelength was $\lambda = 664\,nm$, while the accepted value for the He-Ne laser wavelength is $\lambda_0 = 632.82\,nm$. The percentage error between the experimental and accepted values was calculated as 4.93%. This small percentage error suggests that the experimental process was fairly accurate, and the results are in reasonable agreement with the accepted value of the laser wavelength.

However, the fractional uncertainty from standard error was found to be 39.6%, which is a significant source of concern regarding the precision of the measurements. The large uncertainty can likely be attributed to experimental factors such as slight misalignment of the interferometer mirrors, errors in the micrometer measurement, or environmental conditions affecting the interference pattern.

Statistical analysis using linear regression, including a high R-squared value of 0.871, confirms that the linear model fits the data well, indicating that the relationship between the displacement of the mirror and the number of fringes counted is valid. Furthermore, the low p-value and high t-statistic strongly suggest that the observed data is not due to random fluctuations but reflects a true underlying relationship between the variables.

While the experiment provided an experimentally measured wavelength reasonably close to the accepted value, the high uncertainty suggests that improvements in precision are necessary. The results are consistent with the wave theory of light and the principles of interferometry, where interference fringes are directly related to the wavelength of light. Therefore, despite the uncertainties, the experiment successfully demonstrated the principles of interferometry and provided a fairly accurate measurement of the He-Ne laser wavelength.

To improve the accuracy and precision of the experiment, several modifications can be made. First, better alignment of the interferometer components, particularly the mirrors, would reduce measurement errors caused by misalignment. Using a more precise micrometer to control the movement of the movable mirror would help minimize errors in distance measurements. Additionally, conducting the experiment in a controlled environment with minimized vibrations and air currents would improve the stability of the interference pattern, leading to more accurate fringe counting. Finally, averaging multiple trials and refining the data analysis methods, such as reducing noise in MATLAB plots, would further enhance the accuracy of the measured wavelength, especially given the high precision of the He-Ne laser.

Appendix A List of Equipment

- Pasco Interferometer (model OS-8501)
- Interferometer base with built-in micrometer and leveling feet
- Pasco Optics Track that is partitioned into two sections to adjust the laser height relative to the interferometer
- Vacuum cell (included with the Pasco Interferometer set)
- Mityvac vacuum hand pump XMXT with a Mityvac barrometer attached (serial no.: 81001810)
- 18 mm focal length convex lens and magnetic stand (included in the Pasco Interferometer set)
- Spectra-Physics Inc. He-Ne 0.95 mW laser (model 155-1, serial no.: 308066, manufactured: June 1987)
- Dry erase whiteboard as a viewing screen

Appendix B MATLAB Diary

```
A = [20 21 21 21 21 22 22 22 22 22 23 23 24 24 24 25 25 25 25 26 26 26 27 27 27 27 28
     28 28 28 29 29 29 30 30]

A =

  Columns 1 through 16

    20    21    21    21    21    22    22    22    22    22    23    23    24    24
          24    25

  Columns 17 through 32

    25    25    25    26    26    26    27    27    27    27    28    28    28    28
          29    29

  Columns 33 through 35

    29    30    30

B = [6.0 6.9 6.9 6.9 7.3 7.4 7.5 7.5 7.2 7.0 7.8 7.8 8.0 8.0 7.1 7.8 7.8 8.0 8.2 8.9
     8.8 8.5 8.1 8.1 8.1 9.0 9.1 9.9 9.1 8.8 9.8 9.8 9.5 10.0 10.3]

B =

  Columns 1 through 9

    6.0000    6.9000    6.9000    6.9000    7.3000    7.4000    7.5000    7.5000
         7.2000

  Columns 10 through 18

    7.0000    7.8000    7.8000    8.0000    8.0000    7.1000    7.8000    7.8000
         8.0000
```

```
  Columns 19 through 27

    8.2000    8.9000    8.8000    8.5000    8.1000    8.1000    8.1000    9.0000
         9.1000

  Columns 28 through 35

    9.9000    9.1000    8.8000    9.8000    9.8000    9.5000   10.0000   10.3000

mdl = fitlm(A,B)

mdl =


Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   <strong>Estimate</strong>       <strong>SE</strong>
                        <strong>tStat</strong>        <strong>pValue</strong>
                   <strong>________</strong>    <strong>________</strong>
                        <strong>________</strong>    <strong>__________</strong>

    <strong>(Intercept)</strong>    -0.12247     0.56036    -0.21856       0.82834
    <strong>x1         </strong>     0.33203    0.022213      14.947    2.9871e-16


Number of observations: 35, Error degrees of freedom: 33
Root Mean Squared Error: 0.383
R-squared: 0.871,  Adjusted R-Squared: 0.867
F-statistic vs. constant model: 223, p-value = 2.99e-16
plot(mdl)
title(sprintf('$d_{m}\,vs\,m$, Interpreter, latex'))
[Warning: Escaped character '\,' is not valid. See 'doc sprintf' for supported special
     characters.]
title(sprintf('$d_{m}$ vs $m$, Interpreter, latex'))
title(sprintf('$d_{m}$ vs $m$', Interpreter, latex))
{Unrecognized function or variable 'Interpreter'.
}
title(sprintf('$d_{m}$ vs $m$'), 'Interpreter', 'latex'))
 title(sprintf('$d_{m}$ vs $m$'), 'Interpreter', 'latex'))
                                                  [U+FFFD]
{Invalid expression. When calling a function or indexing a variable, use parentheses.
     Otherwise,
check for mismatched delimiters.
}
title(sprintf('$d_{m}$ vs $m$'), 'Interpreter', 'latex')
xlabel('No. of fringes counted')
ylabel(sprintf('Displacement of micrometer knob ($10^{-6}\,m$)'), 'Interpreter',
     'latex')
[Warning: Escaped character '\,' is not valid. See 'doc sprintf' for supported special
     characters.]
[Warning: Error in state of SceneNode.
String scalar or character vector must have valid interpreter syntax:
Displacement of micrometer knob ($10^{-6}
]
ylabel(sprintf('Displacement of micrometer knob (microns)'), 'Interpreter', 'latex')
diary save
```

References

[1] Zygo Corporation (2015). *Interferometry: Measuring with Light.* www.photonics.com. https://www.photonics.com/Articles/Interferometry_Measuring_with_Light/a25128

[2] Duaret, F.J. (2003). 3.2.3 Continuous Wave Lasers in *Tunable Laser Optics* (pp. 182), Academic Press.