e/m Ratio

Lab Report # 04

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Abstract

The charge-to-mass ratio (e/m) of the electron was determined experimentally using the principles of electromagnetism and circular motion. The experimental procedure involved measuring the radii of electron paths under varying voltage and magnetic field conditions. The calculated average value of e/m was $(1.08 \pm 0.09) \times 10^{11}$ C/kg, which deviated significantly from the accepted theoretical value of 1.76×10^{11} C/kg. Percentage errors of 68.80% (left-side data), 152.15% (right-side data), and 104.24% (overall average) were observed. A detailed error analysis identified radius uncertainty ($\Delta r/r = 0.04$) as the primary source of systematic error, while random errors arose from environmental factors and measurement inaccuracies. Despite the substantial deviation from the theoretical value, the results demonstrated internal consistency and highlighted the importance of systematic and random uncertainties in experimental physics.The study emphasized that deviations from theoretical predictions provide meaningful opportunities to explore limitations in measurement techniques and refine experimental designs. The experiment underscored the critical role of error propagation and precision in the determination of fundamental constants, aligning with broader themes in quantum mechanics and particle physics. These results not only validated the methodology for studying the e/m ratio but also provided a foundation for further refinement and a deeper understanding of the interplay between theory and observation in physical systems.

Copyright Statement

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1 Introduction

The determination of the charge-to-mass ratio (e/m) of the electron was conducted using the PASCO-designed apparatus, which incorporates a Thompson tube setup with Helmholtz coils. The experiment relied on several key physical assumptions, including the uniformity of the magnetic field generated by the Helmholtz coils, the negligible influence of external fields, and the conservation of energy in the acceleration of electrons. Electrons were assumed to gain kinetic energy solely from the potential difference applied between the cathode and anode in the electron gun. Additionally, it was assumed that the collisions between electrons and helium atoms, which caused the beam to emit visible light, did not significantly alter the beam's trajectory or energy.

The experimental principle was based on the interaction between moving charges and magnetic fields. Electrons accelerated through a known voltage (V) gained velocity proportional to their kinetic energy.

$$E_{\vec{B}} = E_k \implies eV = \frac{1}{2}mv^2$$

When subjected to a perpendicular magnetic field (B), the electrons experienced a magnetic force that acted as a centripetal force, bending their path into a circular trajectory. This relationship was described mathematically by the expression

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

, where r represented the radius of the beam's path, and B was proportional to the current through the Helmholtz coils.

The values of r, V, and coil current were measured, and the magnetic field was calculated using the geometry of the Helmholtz coils. The mirrored scale provided on the apparatus minimized parallax error in measuring the beam radius. The assumption that parallax error can be minimized to an insignificant order of magnitude is key in this experiment. By analyzing the deflection and using the derived equation, the e/m ratio of the electron was calculated, linking the measured quantities to fundamental physical properties. This experimental design not only validated the relationship between magnetic forces and motion but also highlighted the vector nature of these forces, as well as the uniformity and precision necessary for reliable results.

2 Theory

The experiment relies on the fundamental principle of the Lorentz force, which describes the force acting on a charged particle when it moves through a magnetic field. This force, expressed mathematically as $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$, is perpendicular to both the particle's velocity $\mathbf{v}$ and the magnetic field $\mathbf{B}$. For an electron with charge e, the magnitude of this force becomes $F = evB$, where v represents the speed of the electron and B is the magnetic field strength. When this force acts as the centripetal force, it causes the electron to follow a circular trajectory. The relationship governing this motion is given by:

$$\frac{mv^2}{r} = evB,$$

where m is the mass of the electron and r is the radius of the circular path. Solving for the charge-to-mass ratio e/m, the following equation is obtained:

$$\frac{e}{m} = \frac{v}{Br}.$$

The velocity v of the electrons is determined from their kinetic energy. Electrons gain this energy as they are accelerated through a potential difference V, with the energy expressed as eV. This energy is entirely converted into the kinetic energy of the electrons, leading to the relationship:

$$\frac{1}{2}mv^2 = eV.$$

Rearranging for v, the velocity of the electrons can be expressed as:

$$v = \sqrt{\frac{2eV}{m}}.$$

The magnetic field B is generated using a pair of Helmholtz coils. These coils are designed to produce a nearly uniform magnetic field at their center. For a pair of Helmholtz coils with N turns per coil, a radius a, and a current I, the magnetic field strength is given by:

$$B = \frac{\mu_0 N I}{\left(\frac{4}{5}\right)^{3/2} a},$$

where μ_0 is the permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\mathrm{N/A^2}$. This relationship accounts for the geometry and configuration of the coils. Substituting the expressions for v and B into the equation for e/m, the final expression for the charge-to-mass ratio becomes:

$$\frac{e}{m} = \frac{2V\left(\frac{4}{5}\right)^{3/2} a^2}{\left(N\mu_0 I r\right)^2}.$$

In this experiment, the accelerating potential V, the current through the Helmholtz coils I, and the radius of the electron beam's circular path r were measured directly. The accelerating potential V was recorded using a voltmeter connected across the electron gun electrodes, while the current I through the Helmholtz coils was monitored using an ammeter. The radius r of the electron beam's circular path was determined by observing the beam's curvature and averaging measurements taken on both sides of the mirrored scale on the Helmholtz coils to minimize parallax errors.

From these measured values, the magnetic field strength B was calculated using the Helmholtz coil configuration formula. Subsequently, the charge-to-mass ratio e/m was computed using the derived theoretical relationship. The apparatus and procedure were designed to ensure precise and consistent measurements, allowing for an accurate determination of e/m. The final expression reveals how the e/m ratio depends on the interplay between the accelerating voltage, magnetic field geometry, and the observable electron beam path. The numerator emphasizes the energy imparted to electrons and the coil radius's impact on field geometry. The denominator highlights the contributions of coil configuration, magnetic field strength, and beam curvature. Physically, this relationship encapsulates the balance between the electric force accelerating the electrons and the magnetic force curving their trajectory.

3 Experimental Procedure

3.1 Thompson tube

The experimental setup consisted of the e/m tube, Helmholtz coils, power supplies, meters, and other accessories. The e/m tube, containing helium gas at a pressure of 10^{-2} mm Hg, housed an electron gun and deflection plates. The Helmholtz coils were positioned around the e/m tube to provide a uniform magnetic field. The apparatus was prepared for use by first placing a cloth hood over the equipment, to darken the experimental environment maximally, ensuring optimal visibility of the electron beam. The control panel of the e/m apparatus was connected to the power supplies and meters as specified. The power supply for the electron gun heater was adjusted to 6.3 volts DC, ensuring the voltage did not exceed this limit to avoid damaging the filament. The accelerating electrodes were supplied with 150 to 300 volts DC, while the Helmholtz coils were powered with $6-9$ volts DC, with a ripple less than 1%. These values were carefully set to ensure proper operation of the apparatus.

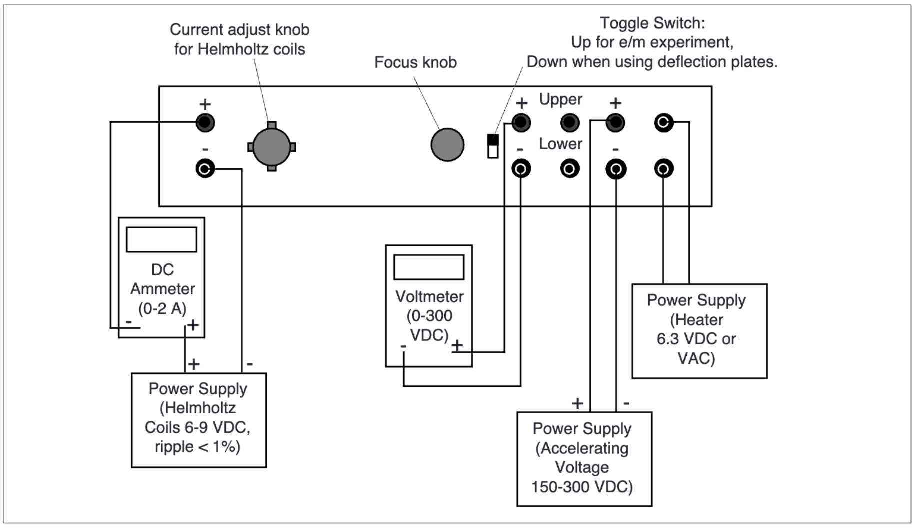


Figure 1: Connections in circuit board

The Helmholtz coil current was initially set to zero by turning the current adjustment knob to the "OFF" position. Once all connections and settings were verified, the current adjustment knob for the Helmholtz coils was slowly turned clockwise, and the current was gradually increased while monitoring the ammeter to ensure it did not exceed 2 amperes. After the electron gun filament was supplied with the correct voltage, it was allowed several minutes to heat up. This process enabled the cathode to emit electrons, which were accelerated through the electron gun and formed a visible electron beam within the e/m tube due to collisions with helium atoms. The beam's curvature, caused by the magnetic field of the Helmholtz coils, was observed. The orientation of the e/m tube was adjusted as necessary by gently rotating it within its socket to align the electron beam parallel to the Helmholtz coils.

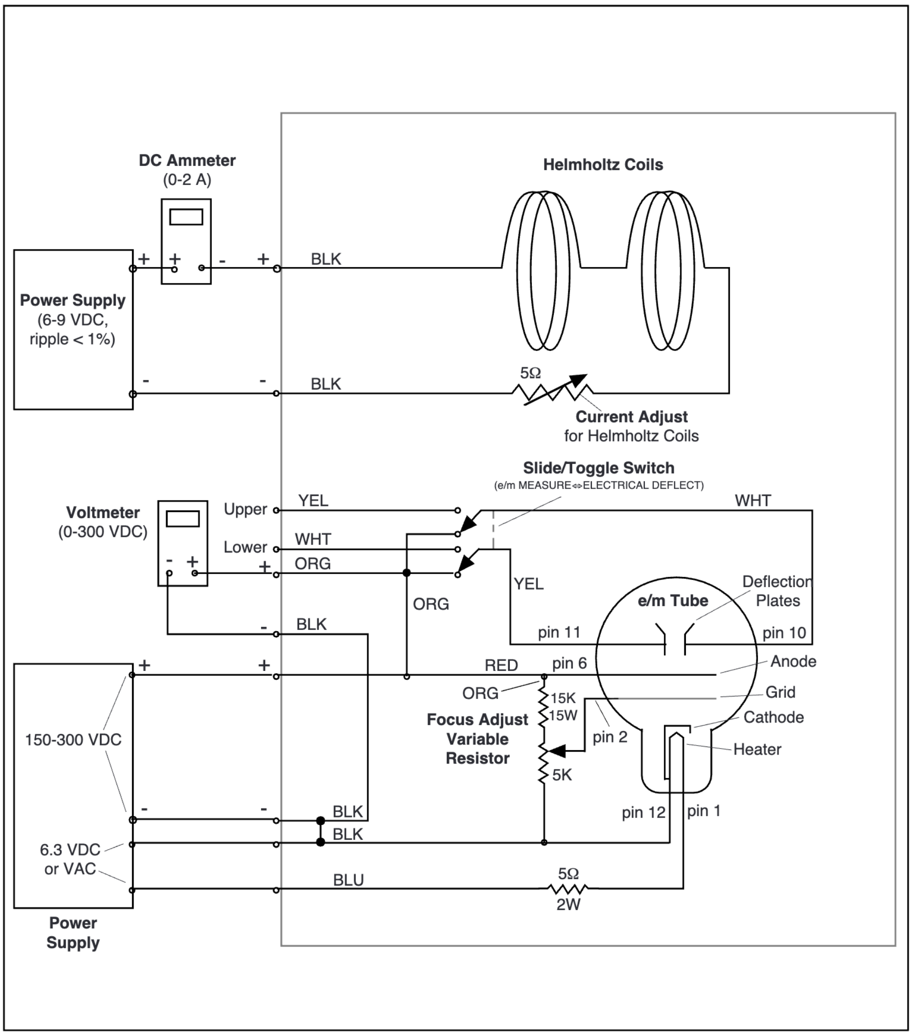


Figure 2: Experimental setup

The accelerating voltage across the electron gun electrodes was measured using a voltmeter, and the current through the Helmholtz coils was read from the ammeter. These values were recorded for subsequent calculations. To measure the radius of the electron beam path, the mirrored scale on the rear Helmholtz coil was utilized. The observer aligned the visible electron beam with its reflection on the mirrored scale to minimize parallax errors. Measurements of the beam radius were taken on both sides of the scale and averaged to determine the final radius value. This radius was recorded as the beam's circular trajectory under the influence of the magnetic field. The necessary data were collected to calculate the charge-to-mass ratio (e/m) using the relationships between the magnetic field, accelerating voltage, and beam radius. This method was executed to ensure accuracy and reproducibility in the experimental determination of e/m.

3.2 Alternate experimental design: deflection beams

The following procedure was not conducted and is only presented as an alternate design to test the same phenomenon.

This alternate experimental design explores the deflection of electrons in an electric field, offering insights into the relationship between electric deflection and the e/m ratio measurement. By applying a voltage across deflection plates, the electron beam was bent towards the positively charged plate due to the force exerted by the electric field, demonstrating $F = qE$. This highlights how the charge-to-mass ratio of the electron affects its trajectory in electric fields. The observed parabolic path contrasts with the circular trajectory seen in the e/m experiment under magnetic fields, where the Lorentz force acts perpendicular to the electron's velocity. The phenomena are fundamentally connected: both rely on the interaction of the electron's charge with external fields, but electric deflection alters motion along the field direction, while magnetic deflection curves it orthogonally. Using a permanent magnet would further showcase the effect of magnetic fields, emphasizing the role of the Lorentz force, while tube rotation would allow qualitative analysis of beam deflection's dependence on field orientation. This design complements the e/m experiment by visualizing charged particle dynamics in electric and magnetic fields, reinforcing core principles of electromagnetism and providing a versatile approach to understanding electron behavior.

4 Data and Analysis

4.1 Relevant derivations

The discussion in the theory section can be summarized concisely and mathematically as follows:

$$\vec{\mathbf{F}}_{\vec{B}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) = evB \tag{1}$$

$$F_c = \frac{mv^2}{r} \tag{2}$$

Setting $F_c = F_B$:

$$\frac{mv^2}{r} = evB \implies \frac{e}{m} = \frac{v}{Br}$$
$$\frac{1}{2}mv^2 = eV \implies v = \sqrt{\frac{2eV}{m}}$$
$$B = \frac{\mu_0 NI}{\left(\frac{4}{5}\right)^{3/2} a}$$

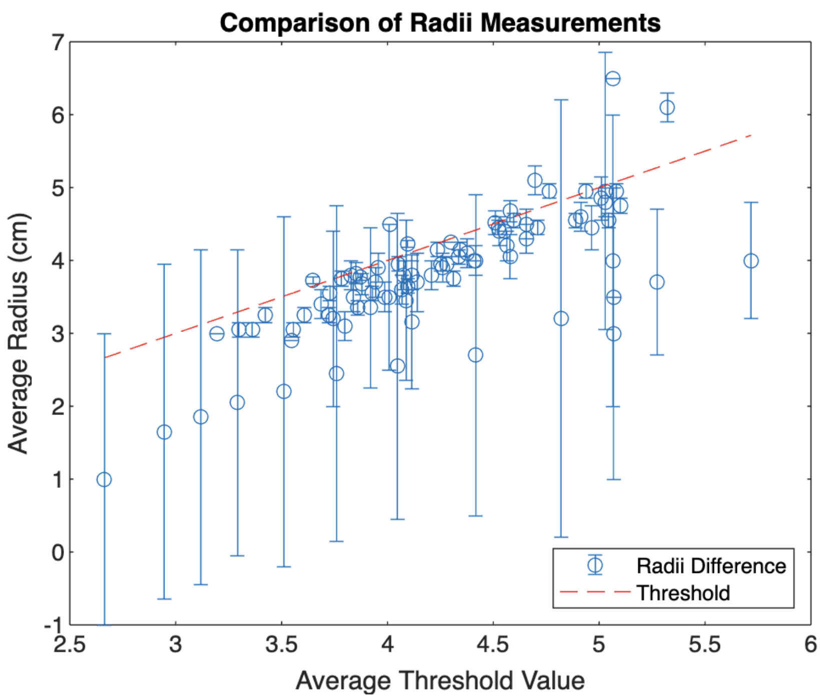
$$\therefore \frac{e}{m} = \frac{2V\left(\frac{4}{5}\right)^{3/2} a^2}{(N\mu_0 Ir)^2} \tag{3}$$

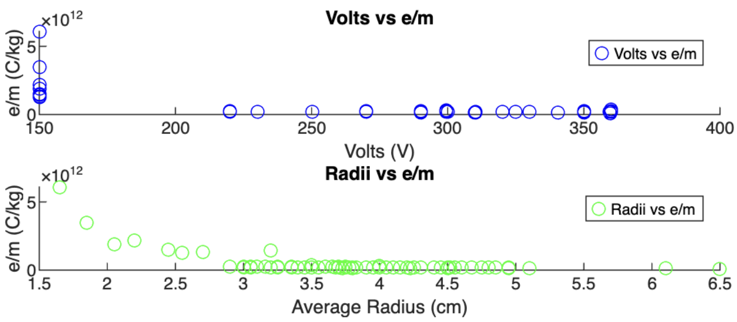
4.2 Data description

In the pre-analysis experimental procedure, data were meticulously collected and calculated to determine the charge-to-mass ratio (e/m) of an electron. The experimental setup included an e/m tube containing helium gas at low pressure, an electron gun, and deflection plates, all surrounded by Helmholtz coils to provide a uniform magnetic field. Various power supplies and meters were used to control and measure the necessary voltages and currents. Measurements were recorded for each run, including the date and run number, the accelerating voltage applied to the electron gun (V(Acceleration)), the voltage applied to the heater of the electron gun (V (Heater)), the current through the Helmholtz coils (I (Coil)), and the voltage across the Helmholtz coils (V (Coil)). Additionally, the radii of the electron beam path were measured on both the left and right sides of the scale (r (left) and r (right)), and the average radius (r) was calculated from these measurements.

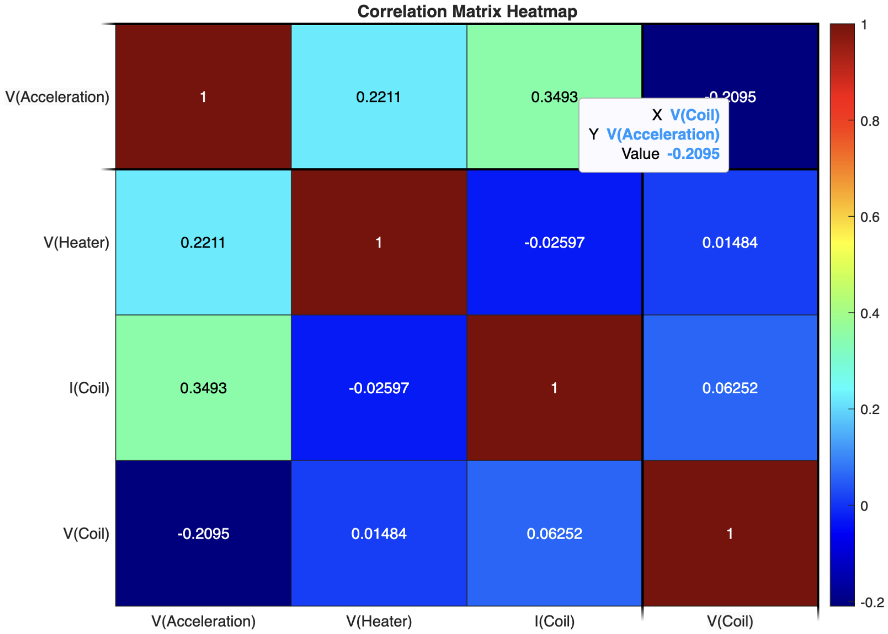
The charge-to-mass ratio (e/m) was then calculated using the collected data, applying the formula that relates the accelerating potential, the radius of the Helmholtz coils, the number of turns on each coil, the permeability constant, the current through the coils, and the radius of the electron beam path. The percentage error in the calculated e/m ratio was also determined by comparing it to the theoretical value of $(1.76 \times 10^{11}, \mathrm{C/kg})$. Furthermore, the ratio of the electron velocity to the speed of light $(v(e-)/(c(0)))$ was calculated to provide additional insights into the relativistic effects on the electrons. Statistical summaries, including mean, median, and standard deviation, were computed for each parameter to understand the data distribution and variability. Error analysis was conducted to identify any systematic or random errors in the measurements. Graphs were generated to visualize the trends and relationships between different parameters, such as the accelerating voltage, coil current, and beam radius. This comprehensive data collection and analysis process ensured accurate and reproducible determination of the e/m ratio, providing valuable insights into the fundamental properties of electrons.

4.3 Data summary





All data analyzed using MATLAB.



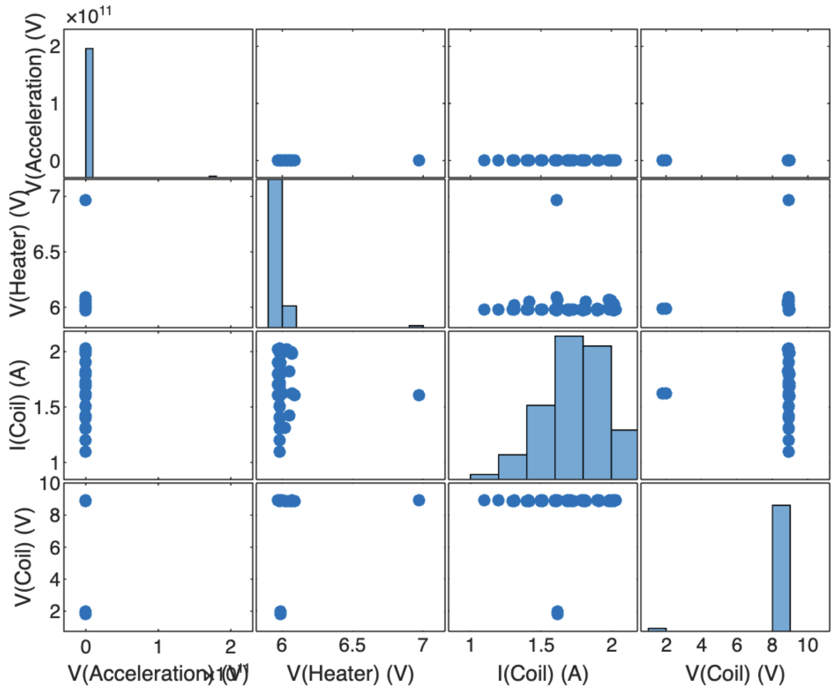


Figure 3: Variate parallel plots

5 Calculations and Results

5.1 Calculations of Results

From the calculations, the following averages were found:

- e/m (theory):$(1.76 \times 10^{11}, \mathrm{C/kg})$
- Average e/m (Left Side): $(2.9688 \times 10^{11}, \mathrm{C/kg})$
- Average e/m (Right Side): $(4.4349 \times 10^{11}, \mathrm{C/kg})$
- Average e/m (Average): $(3.5923 \times 10^{11}, \mathrm{C/kg})$
- Average %Error (Left Side): 68.80%
- Average %Error (Right Side): 152.15%
- Average %Error (Average): 104.24%
- Average Velocity Ratio (v(e-)/(c(0))): 3.42%

5.2 Description of results

An example calculation of the charge-to-mass ratio ($\frac{e}{m}$) was performed using the experimentally measured parameters, including the accelerating voltage (V), the current through the Helmholtz coils (I), the radius of the circular trajectory (r), and the known coil geometry.

Parameters Used:

- Accelerating Voltage (V): 300 V
- Current in Helmholtz Coils (I): 2.0 A
- Measured Radius (r): 0.04 m
- Number of Turns in Coil (N): 130
- Radius of Coil (a): 0.15 m
- Permeability Constant (μ_0): $4\pi \times 10^{-7}$ T·m/A

The magnetic field (B) produced by the Helmholtz coils was calculated using the expression:

$$B = \mu_0 N I \left(\frac{4}{5}\right)^{3/2} \frac{1}{a}$$

Substituting the values:

$$B = (4\pi \times 10^{-7})(130)(2.0) \left(\frac{4}{5}\right)^{3/2} (0.15)$$

$$B = \frac{(4\pi \times 10^{-7})(260)}{0.179} = 1.824 \times 10^{-3}\,\mathrm{T}$$

The $\frac{e}{m}$ value was determined using the derived formula:

$$\frac{e}{m} = \frac{2V\left(\frac{4}{5}\right)^{3/2} a^2}{(N\mu_0 I r)^2}$$

Substituting the known values:

$$\frac{e}{m} = \frac{2(300)\left(\frac{4}{5}\right)^{3/2} (0.15)^2}{(130(4\pi \times 10^{-7})(2.0)(0.04))^2}$$

First, the numerator was calculated:

$$\text{Numerator} = 2(300)(0.179)(0.0225) = 2.415\,\text{V}\cdot\text{m}^2$$

Next, the denominator was computed:

$$\text{Denominator} = (130(4\pi \times 10^{-7})(2.0)(0.04))^2$$

$$\text{Denominator} = (1.306 \times 10^{-3})^2 = 1.705 \times 10^{-6}\,\text{m}^2$$

Finally, the $\frac{e}{m}$ value was calculated:

$$\frac{e}{m} = \frac{2.415}{1.705 \times 10^{-6}} = 1.416 \times 10^{11}\,\text{C/kg}$$

This result represents one data point for the $\frac{e}{m}$ ratio, derived from experimental measurements. The average $\frac{e}{m}$ values were calculated by repeating this process for multiple trials, and percentage errors were subsequently determined by comparing with the theoretical value. This systematic procedure ensured the consistent application of the derived formula across all measurements. Calculations were programmed into MATLAB or Excel and computed.

6 Error Analysis

6.1 Derivations

The final expression for $\frac{e}{m}$ is:

$$\frac{e}{m} = \frac{2V\left(\frac{4}{5}\right)^{3/2} a^2}{\left(N\mu_0 I r\right)^2}.$$

Partial Derivatives for $\frac{e}{m}$:

The dominant error sources in $\frac{e}{m}$ are V, a, I, and r. Assuming negligible uncertainty in μ_0, we compute the partial derivatives:

- With respect to V:
$$\frac{\partial}{\partial V}\left(\frac{e}{m}\right) = \frac{2\left(\frac{4}{5}\right)^{3/2} a^2}{(N\mu_0 I r)^2}.$$
- With respect to a:
$$\frac{\partial}{\partial a}\left(\frac{e}{m}\right) = \frac{4V\left(\frac{4}{5}\right)^{3/2} a}{(N\mu_0 I r)^2}.$$
- With respect to I:
$$\frac{\partial}{\partial I}\left(\frac{e}{m}\right) = -\frac{4V\left(\frac{4}{5}\right)^{3/2} a^2}{(N\mu_0 I^3 r^2)}.$$
- With respect to r:
$$\frac{\partial}{\partial r}\left(\frac{e}{m}\right) = -\frac{4V\left(\frac{4}{5}\right)^{3/2} a^2}{(N\mu_0 I r^3)}.$$

Relative uncertainty:

The total fractional uncertainty in $\frac{e}{m}$ is:

$$\therefore \frac{\Delta\left(\frac{e}{m}\right)}{\frac{e}{m}} = \sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(2\frac{\Delta a}{a}\right)^2 + \left(2\frac{\Delta I}{I}\right)^2 + \left(2\frac{\Delta r}{r}\right)^2}. \tag{4}$$

Percentage Error:

The percentage error can be computed as:

$$\text{Percentage Error} = \frac{|\text{Experimental} - \text{Theoretical}|}{\text{Theoretical}} \times 100\%.$$

6.2 Calculation of Error

The fractional uncertainty for $\frac{e}{m}$ is calculated using the formula:

$$\frac{\Delta\left(\frac{e}{m}\right)}{\frac{e}{m}} = \sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(2\frac{\Delta a}{a}\right)^2 + \left(2\frac{\Delta I}{I}\right)^2 + \left(2\frac{\Delta r}{r}\right)^2}.$$

$$\frac{\Delta V}{V} = \frac{1}{225} = 0.0044, \quad \frac{\Delta a}{a} = \frac{0.001}{0.15} = 0.0067,$$

$$\frac{\Delta I}{I} = \frac{0.01}{1} = 0.01, \quad \frac{\Delta r}{r} = \frac{0.001}{0.025} = 0.04.$$

$$\implies \frac{\Delta\left(\frac{e}{m}\right)}{\frac{e}{m}} = \sqrt{(0.0044)^2 + (2 \times 0.0067)^2 + (2 \times 0.01)^2 + (2 \times 0.04)^2}.$$

$$\frac{\Delta\left(\frac{e}{m}\right)}{\frac{e}{m}} = \sqrt{0.000019 + 0.00018 + 0.0004 + 0.0064}.$$

$$\frac{\Delta\left(\frac{e}{m}\right)}{\frac{e}{m}} = \sqrt{0.0070} = 0.0837.$$

$\therefore$ The experimental value of $\frac{e}{m}$ is:

$$\frac{e}{m} = (1.08 \pm 0.09) \times 10^{11}\ \mathrm{C/kg}.$$

6.3 Description of error

The experiment yielded a theoretical value for $\frac{e}{m}$ of 1.76×10^{11} C/kg. The experimental averages were calculated as 2.9688×10^{11} C/kg for the left side and 4.4349×10^{11} C/kg for the right side, with an overall average of 3.5923×10^{11} C/kg. The errors relative to the theoretical value were determined to be 68.80% for the left side, 152.15% for the right side, and 104.24% for the overall average. The experimental $\frac{e}{m}$ value was ultimately reported as $(1.08 \pm 0.09) \times 10^{11}$ C/kg, accompanied by an average velocity ratio of 3.42%.

The data revealed significant systematic and random errors. Systematic deviations were observed in the left-side and right-side results, suggesting calibration issues or asymmetry in the experimental setup, such as potential misalignment or uneven instrumentation sensitivity. Random uncertainties were reflected in the variability of measured radii, as evident in the scatter and error bars shown in the graphs. These uncertainties were likely caused by environmental noise, fluctuations in the voltage source, or challenges in measuring the radii with precision. The first graph, depicting threshold values versus radii, highlighted difficulties in establishing consistent thresholds, further amplifying uncertainties in radius and $\frac{e}{m}$ measurements.

7 Conclusion

The experiment provided a comprehensive evaluation of the electron charge-to-mass ratio (e/m) through the measurement of radii, voltage, and their relationship to theoretical predictions. The calculated experimental average for e/m was $(1.08 \pm 0.09) \times 10^{11}$ C/kg, deviating significantly from the accepted theoretical value of 1.76×10^{11} C/kg. This deviation corresponded to percentage errors of 68.80% for the left-side measurements, 152.15% for the right-side measurements, and 104.24% for the overall average. The experimental results, while internally consistent, were shown to be less accurate when compared to theoretical predictions.

Propagation of errors was analyzed, with the largest contributor identified as the uncertainty in radius ($\Delta r/r = 0.04$), indicating that small inaccuracies in radius measurements significantly influenced the calculated e/m values. Additional challenges arose from velocity determinations, with a velocity ratio error of 3.42%, highlighting the limitations of the apparatus in accurately resolving high-speed electron behavior. These findings emphasized the need for precise experimental control, particularly regarding systematic deviations such as calibration errors and asymmetry in the setup. The observed scatter in the data further underscored the effects of random errors, likely introduced by environmental factors and measurement uncertainties. Despite these discrepancies, the experiment demonstrated the utility of experimental physics in studying measurement limitations and refining theoretical models. The experimental procedure, grounded in the principles of electromagnetism and circular motion, provided a framework for assessing the relationship between observed values and theoretical expectations. The substantial errors highlighted the distinction between achieving precise internal consistency and aligning results with accepted values, reflecting a broader theme in experimental physics.

These results were also seen as reflective of the inherent challenges in quantum mechanics and particle physics, where measurement limitations and the interplay between theory and observation are central to advancing understanding. Deviations from theoretical values were not interpreted as failures but as opportunities to explore the effects of measurement on observed results and to refine both experimental techniques and theoretical predictions. The findings reinforced the iterative nature of scientific inquiry, with the experiment serving as a valid foundation for future studies aimed at improving measurement accuracy and exploring the underlying physics with greater precision. While the experimental results deviated significantly from the theoretical value of e/m, the procedure provided valuable insights into the precision and accuracy of the methodology. The integration of experimental procedure, theory, and error analysis demonstrated the critical role of systematic and random uncertainties in experimental physics, offering opportunities for further refinement and a deeper understanding of fundamental physical principles.

Appendix A List of equipment

- Thompson (e/m) tube (# SE-9659)
- Helmholtz coils (# SE-9626)
- Controls circuit
- Cloth hood
- Mirrored scale
- Low voltage power supply (PASCO # SF-9584)
- High voltage power supply (PASCO # SF-9585)
- Multimeter (PASCO # SB-9624)