CT216

LAB GROUP 1| PROJECT GROUP 2

POLAR CODES

Assigned by: Professor Yash Vasavada

Honor Code

We declare that
→ The work that we are presenting is our own work.
→ We have not copied the work (the code, the results, etc.) that someone else has done.
→ Concepts, understanding and insights we will be describing are our own.
→ We make this pledge truthfully. We know that violation of this solemn pledge can carry grave consequences

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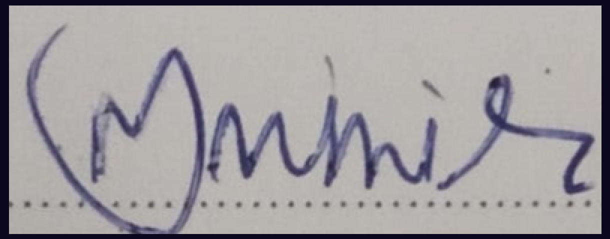

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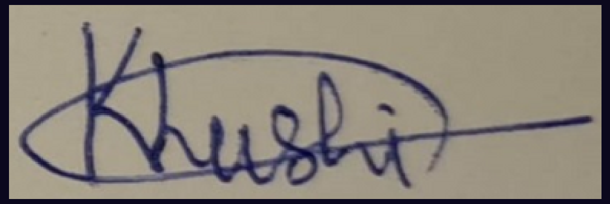

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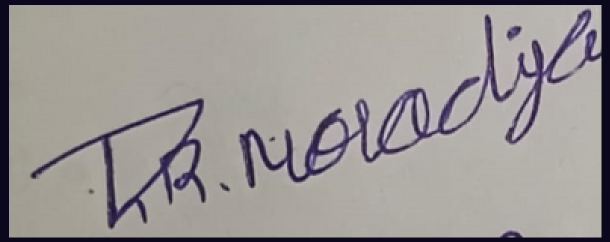

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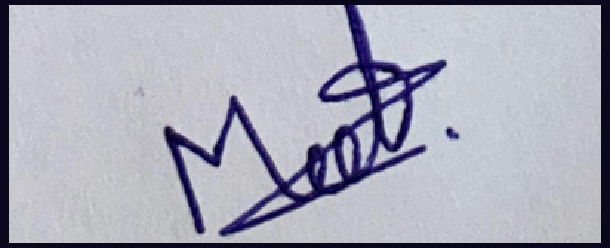

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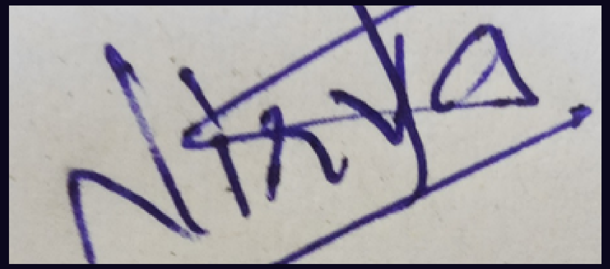

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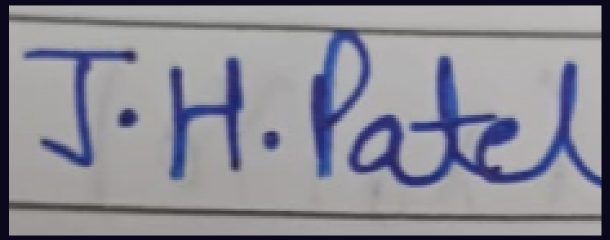

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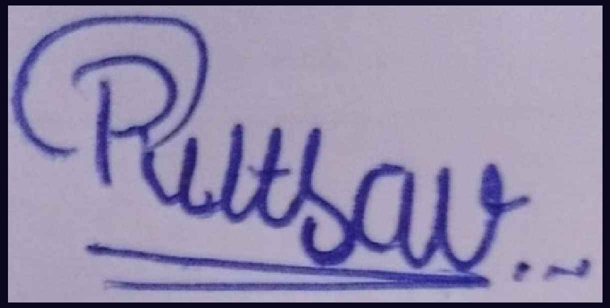

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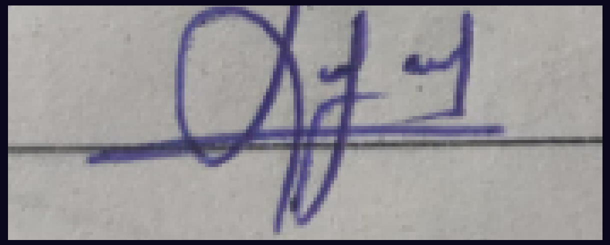

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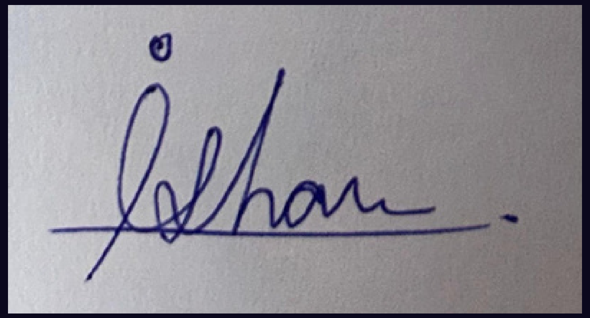

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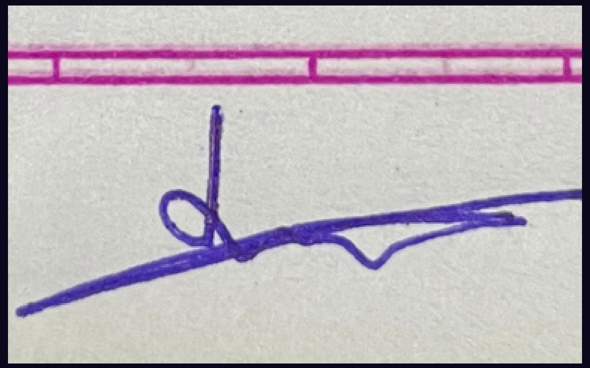

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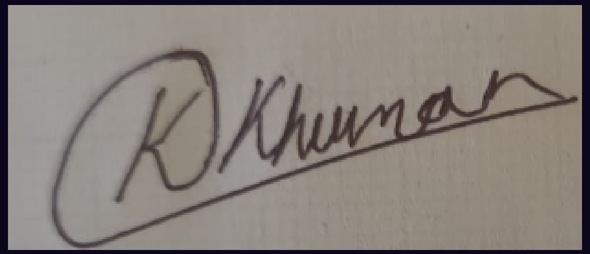


CONTENTS

1) Introduction

- Invented by Erdal Arikan 2008
- Based on the idea of channel polarization
- First codes to have explicit proof for approaching capacity
- Included as codes for the control channels in the 5G standard
- Sequential in nature
- Defined using generator matrix in a recursive (Kornecker product) definition

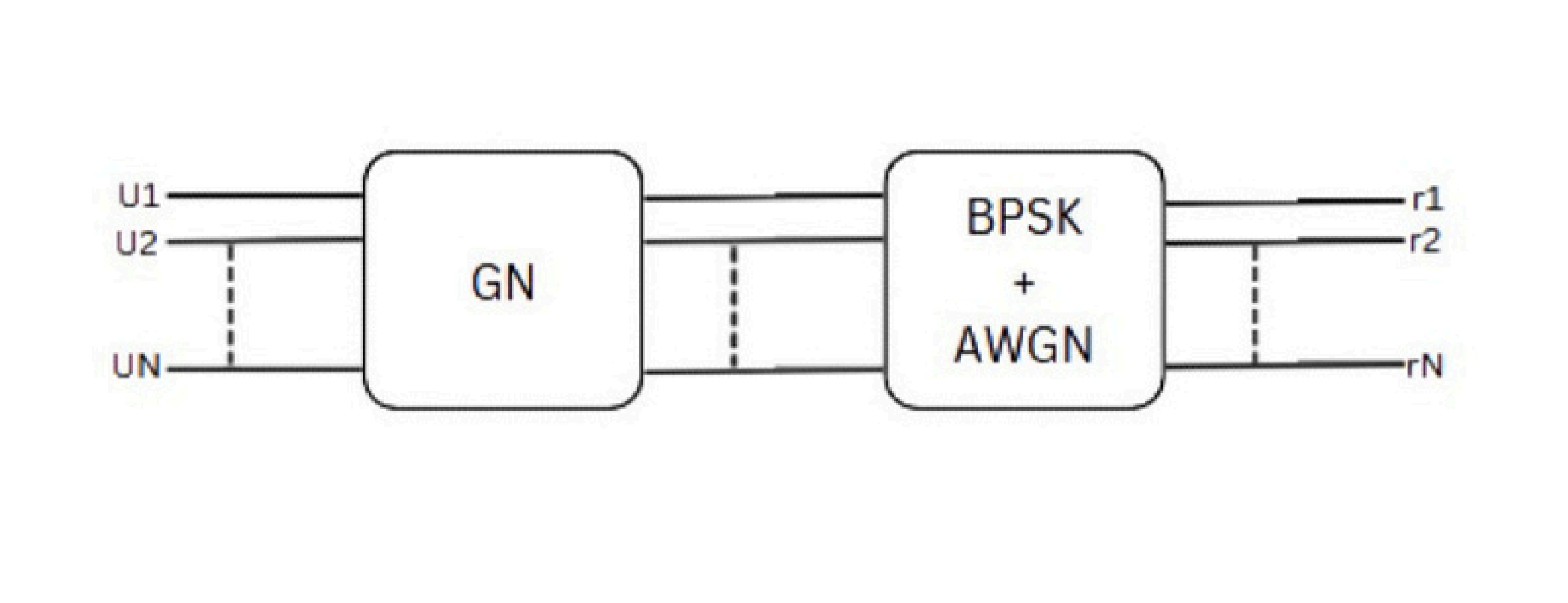
img source: https://www.bilkent.edu/bilkent/erdal-arikan-receives-awards/

2) Applications of Polar Codes

Polar codes have several important applications across different areas of communication and information theory like:

- 5G wireless communication - polar codes are used in the data and control channels of 5G because of their ease of decoding and capacity achieving capabilities.
- Internet of things (IOT) - polar codes are used to deliver dependable data transport and effective error correction.
- Satellite communication - polar codes are used for reliable error correction to maintain the data integrity over noisy satellite links.

AWGN AND BPSK:



AWGN:

Additive White Gaussian Noise

- noise over spectral density over all frequencies.
- It follows normal distribution with mean 0 and standard deviation σ
- It is additive in nature

BPSK:

Binary Phase Shift Keying

- It is a form of digital modulation where binary information is represented by different phases of the carrier signal.
- Less spectral efficient compared to higher order modulation schemes.
- It is sensitive to noise.
- if(r=1) x' = -1
- if(r=0) x' = 1

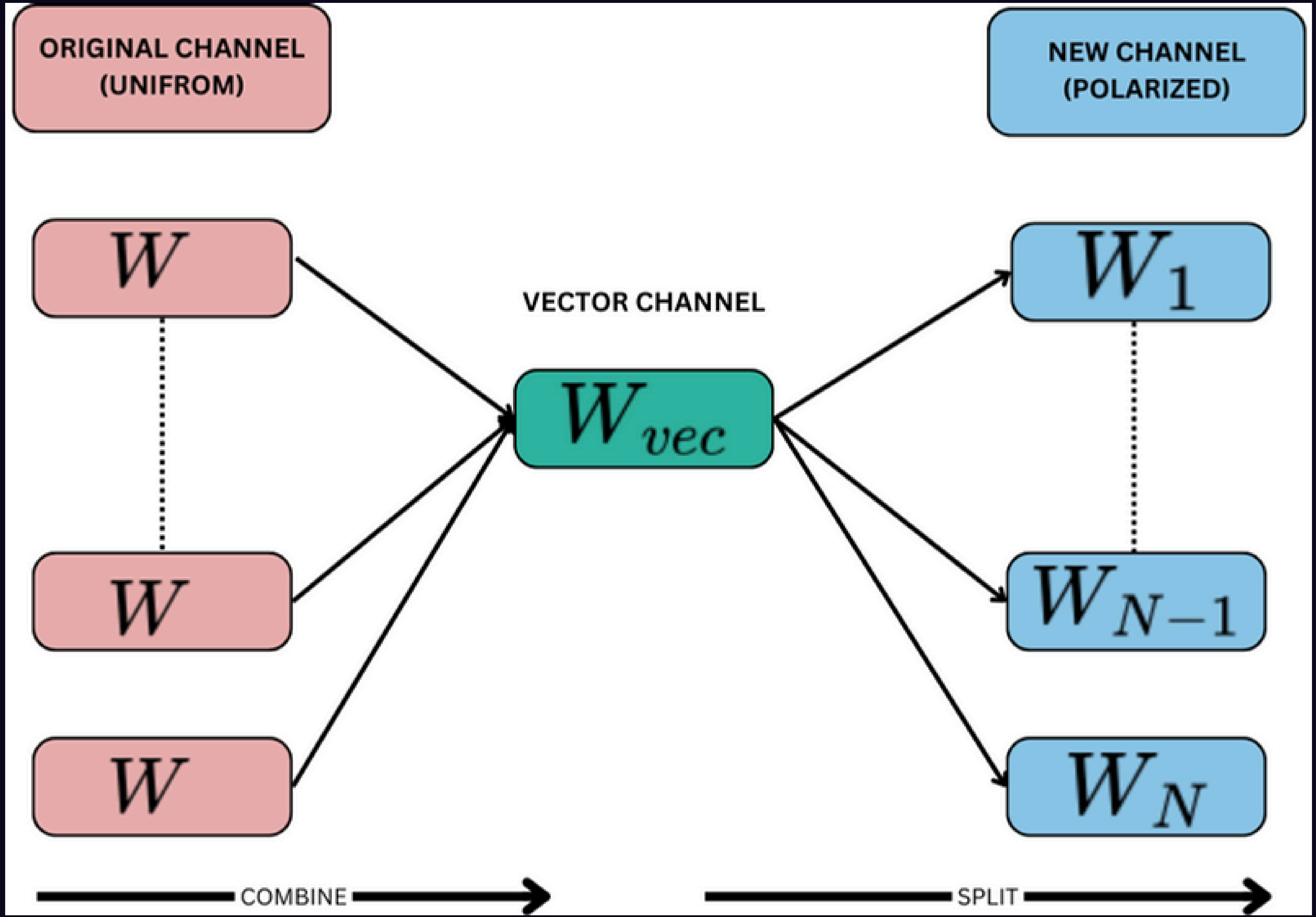
HOW DOES POLAR CODES WORK?

Polar Codes use the concept of Polarization to polarize the channels.

Channel polarization

Let a Binary-DMS channel with capacity $0 <= C <= 1$. When a code word is G x in N channel uses(when tends to infinity), the channel polarization converts,
C fraction of the N bit channels as noiseless (i.e their capacity ≈ 1)
(1-C) remaining as extremely noisy (i.e their capacity ≈ 0)

How does channel get polarised?



Which channel is more informative than the other?

Here we refer to the reliability sequence given by the 5G standard. (This is an experimental computation of entropy of each channel and sequence is obtained from least reliable($I(W) \approx 0$) moving to most reliable($I(W) \approx 1$)

Q=[0 1 2 4 8 16 32 3 5 64 9 6 17 10 18 128 12 33 65 20 256 34 24 36 7 129 66 512 11 40 68 130 ... 19 13 48 14 72 257 21 132 35 258 26 513 80 37 25 22 136 260 264 38 514 96 67 41 144 28 69 42 516 49 74 272 160 520 288 528 192 544 70 44 131 81 50 73 15 320 133 52 23 134 384 76 137 56 27 97 39 259 84 138 145 261 29 43 98 515 88 140 30 146 71 262 265 161 576 45 100 640 51 148 46 75 266 273 517 104 162 53 193 152 77 164 768 268 274 518 54 83 57 521 112 135 78 289 194 85 276 522 58 168 139 99 86 60 280 89 290 529 524 196 141 101 147 176 142 530 321 31 200 90 545 292...

Let’s understand with an example of N=2

Let’s consider N=2 (it means transmitting 2 bits)

$$I(X_1, X_2; Y_1Y_2) = I(X_1; Y_1) + I(X_2; Y_2) = 2I(W)$$

$$2I(W) = I(U_1U_2; Y_1Y_2) = I(U_1; Y_1Y_2) + I(U_2; Y_1Y_2|(U_1)$$

$$= I(U_1; Y_1Y_2) + I(U_2; Y_1Y_2U_1)$$

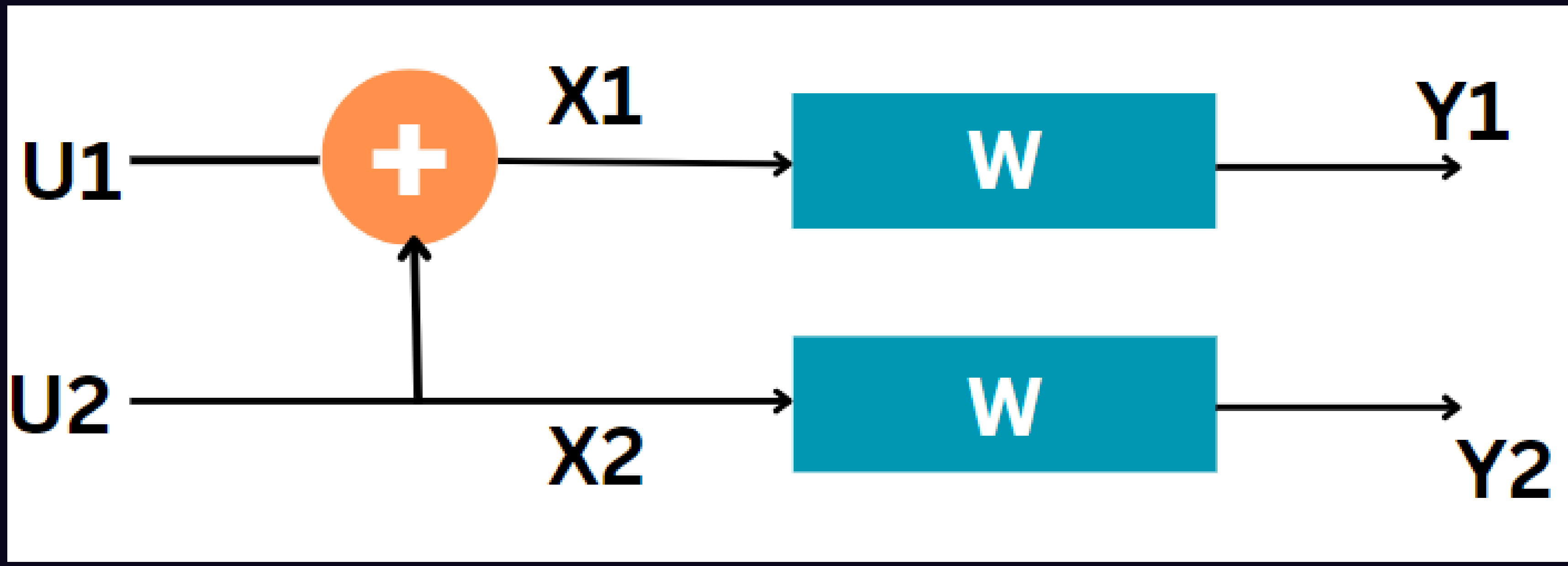
$$= I(W^-) + I(W^+)$$

$$I(W^+) = I(U_2; Y_1Y_2U_1) \geq I(U_2; Y_2)$$

$$I(U_2; Y_2) = I(X_2; Y_2) = I(W)$$

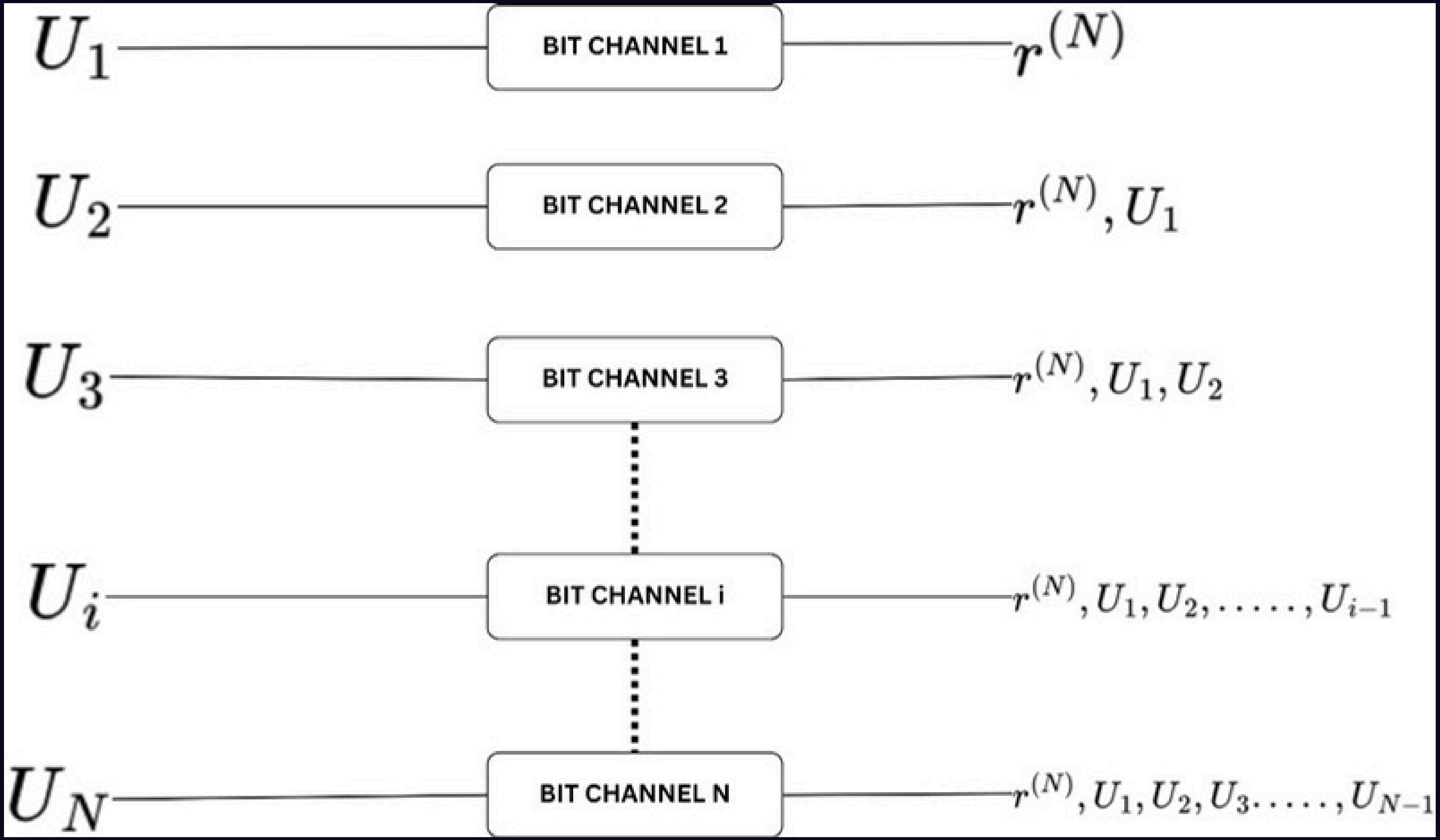
$I(W^+) \geq 1(W)$ ---1

$2I(W) = I(W^+) + I(W^-)$ ---2



From 1 & 2, $I(W) \geq I(W^-)$

So, $I(W^-) \leq I(W) \leq I(W^+)$


U_1
BIT CHANNEL 1
$r^{(N)}$
U_2
BIT CHANNEL 2
$r^{(N)}, U_1$
U_3
BIT CHANNEL 3
$r^{(N)}, U_1, U_2$
U_i
BIT CHANNEL i
$r^{(N)}, U_1, U_2, \ldots\ldots, U_{i-1}$
U_N
BIT CHANNEL N
$r^{(N)}, U_1, U_2, U_3 \ldots\ldots, U_{N-1}$

We can extend this explanation to any N

As N-->infinity

Some channels become highly informative and some very noisy. The number of noisy channels plus the number of highly informative channels approximates to total number of channels.

4) Encoding Polar codes

For encoding Polar codes we are using Kronecker Product and Generator Matrix :

The basic Generator Matrix of 2 bits to 2 bits : G_2

$$G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Now while generating G_4 we have to use Kronecker Product :

$$G_4 = G_2 \otimes G_2$$

Kronecker Product

Generating G Matrix

$$G_4 = G_2 \otimes G_2$$

$$G_4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

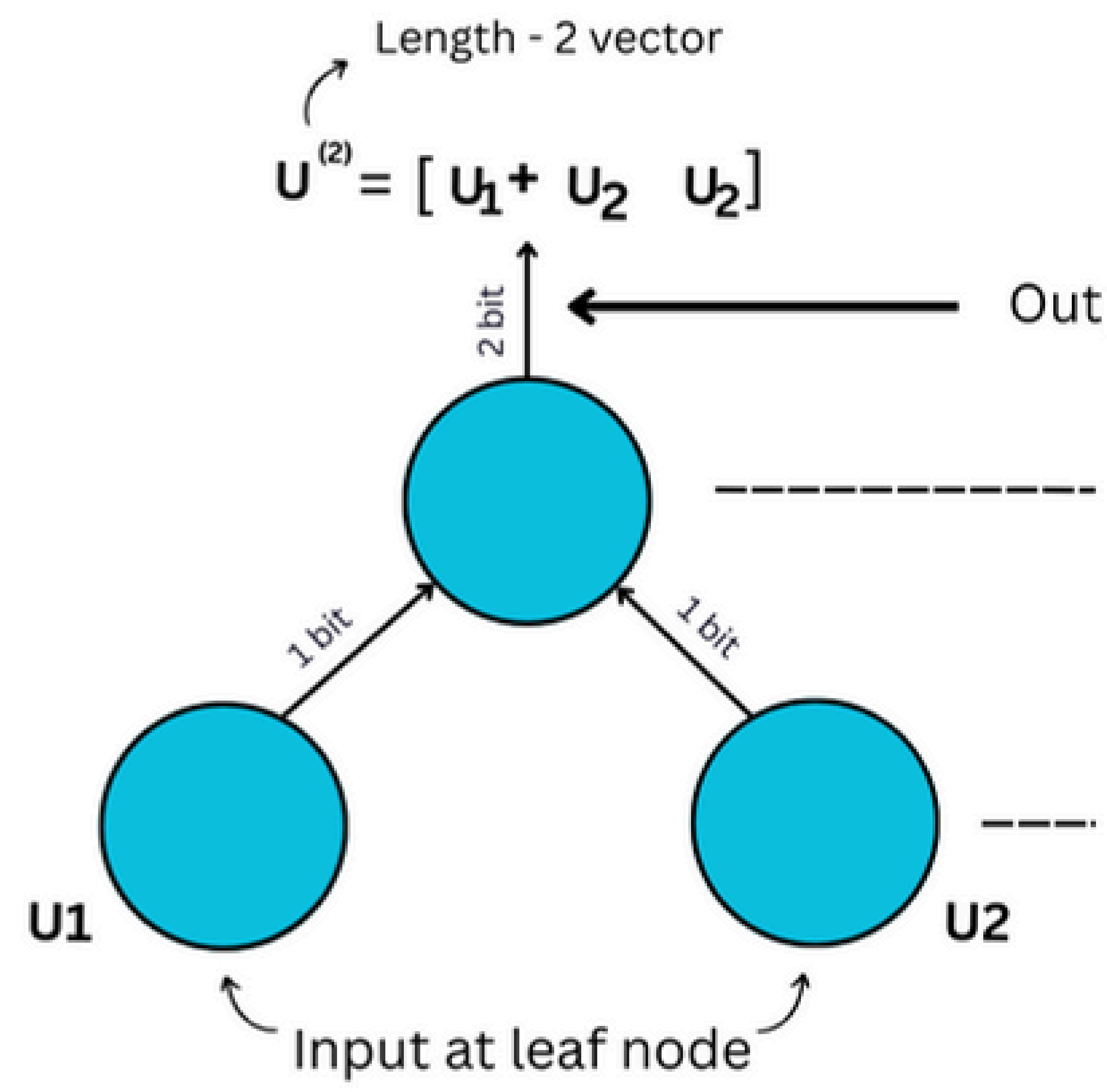
$$G_4 = \begin{bmatrix} 1 \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} & 0 \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\ 1 \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} & 1 \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{bmatrix}$$

$$G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Final G_4 Matrix.

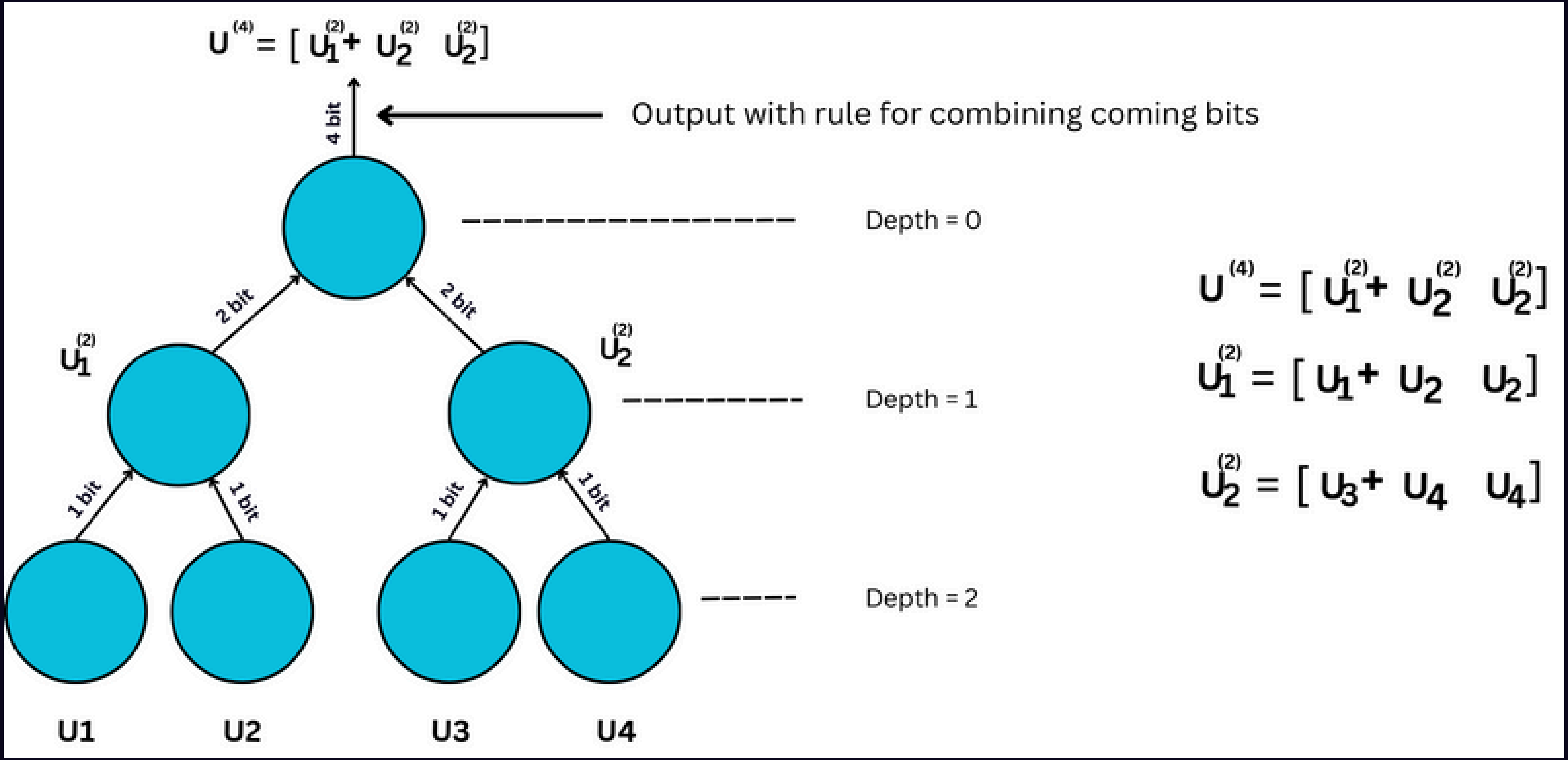
Encoding with binary tree representation

(explain for N=2)



Note : here "+" is showing modulo 2 operation or XOR operation

(explain for N=4)



General Picture

$$G_{2^n} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{\otimes n}$$

G matrix with 2^n bits to 2^n bits.

- $N = 2^n$
- G_N: N x N matrix
- Binary tree representation :

  Depth : n

  $U^{(N)} = UG_N$: Evaluated on tree with U at bottom and $U^{(N)}$ at top

  Polar Transform
- 5G uses up to n = 10.

Encoding steps

(1) $N = 2^n$

(2) Message (Information Bits) = K bits

(3) From a vector U of length N

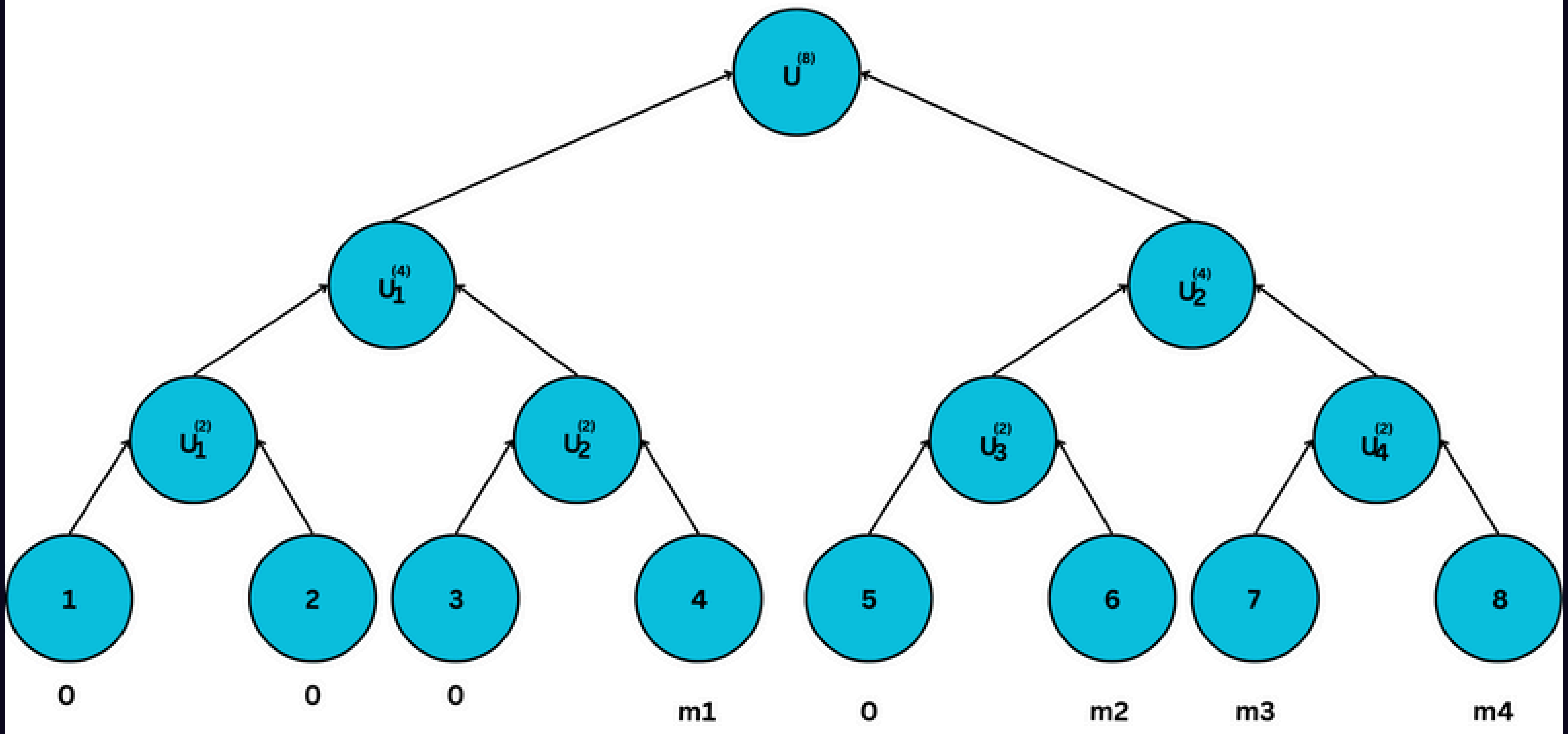
 (i) Find (N-K) least reliable channels from reliability sequence and set those positions to zero (Frozen Position)

 (ii) Set remaining position the information bits.

(4) Codeword : UG_N

- Reliability Sequence : 1 2 3 5 4 6 7 8
  Frozen : 1 2 3 5
  Message : 4 6 7 8

$$U^{(8)} = [\, U_1^{(4)} + U_2^{(4)} \quad U_2^{(4)} \,]$$



Values

$$U_1^{(2)} = [\ 0 \quad 0\ ]$$

$$U_2^{(2)} = [\ m1 \quad m1\ ]$$

$$U_3^{(2)} = [\ m2 \quad m2\ ]$$

$$U_4^{(2)} = [\ m3+m2 \quad m4\ ]$$

$$U_1^{(4)} = [\ m1 \quad m1 \quad m1 \quad m1\ ]$$

$$U_2^{(4)} = [\ m2+m3+m4 \quad m2+m4 \quad m3+m4 \quad m4\ ]$$

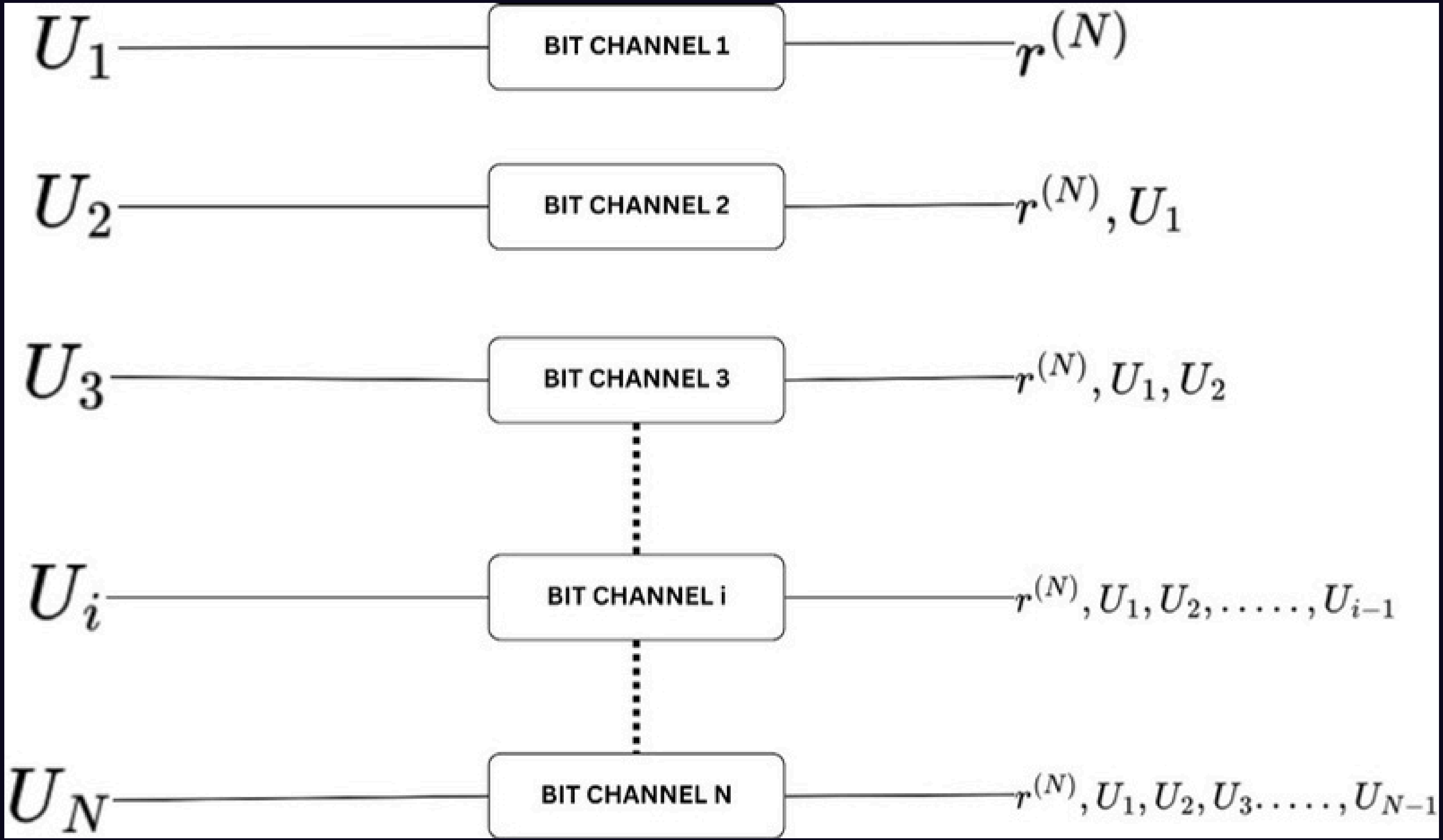
$$U^{(8)} = [\ m1+m2+m3+m4 \quad m1+m2+m4 \quad m1+m3+m4 \quad m1+m4 \quad m2+m3+m4 \quad m2+m4 \quad m3+m4 \quad m4\ ]$$

5) Decoding Polar codes

These are various algorithms:

1) Successive Cancellation.
2) Successive Cancellation List.

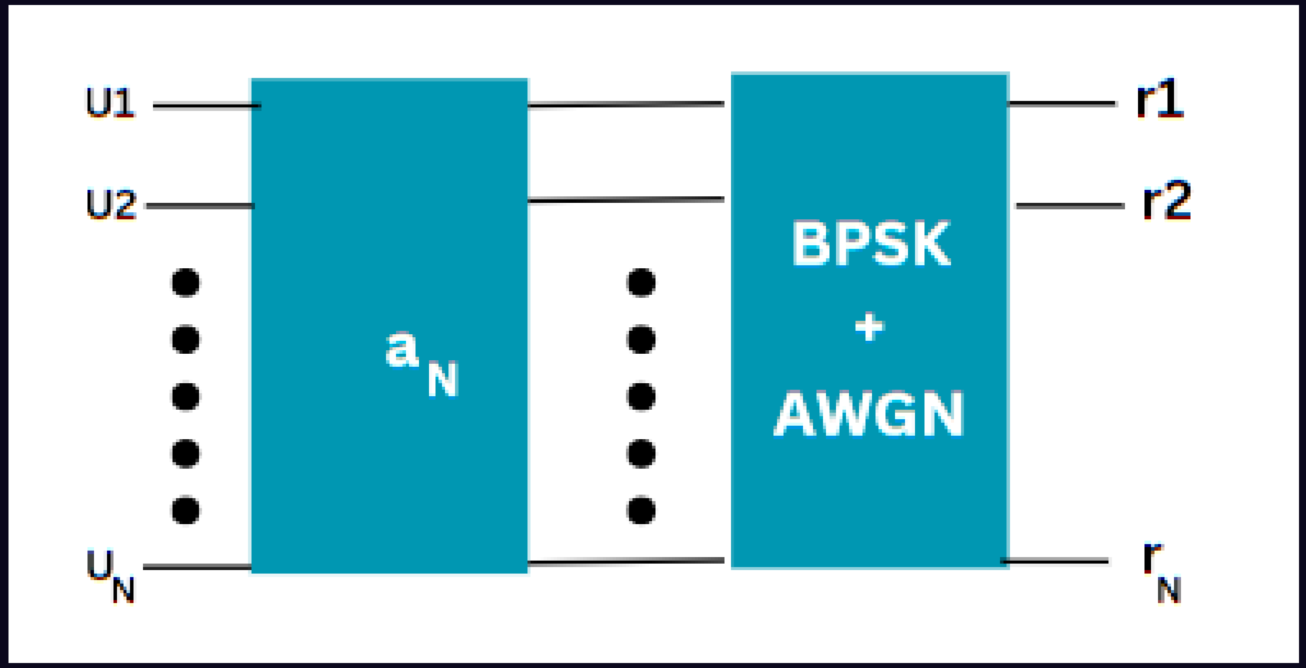
SUCCESSIVE CANCELLATION DECODER:



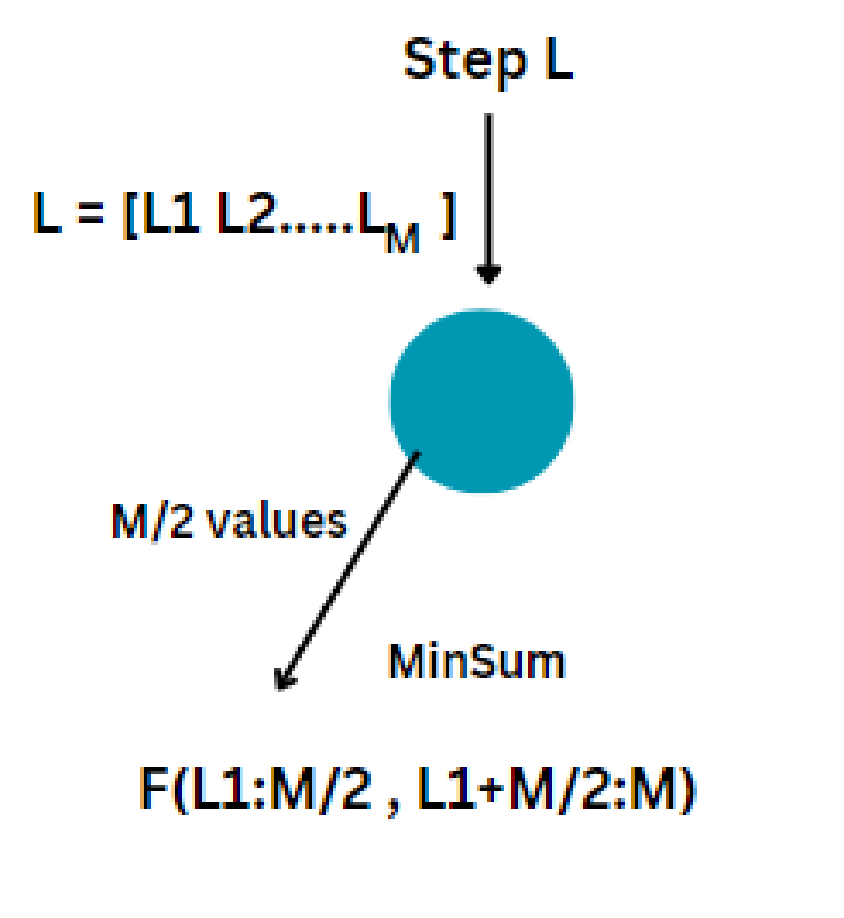
Steps:

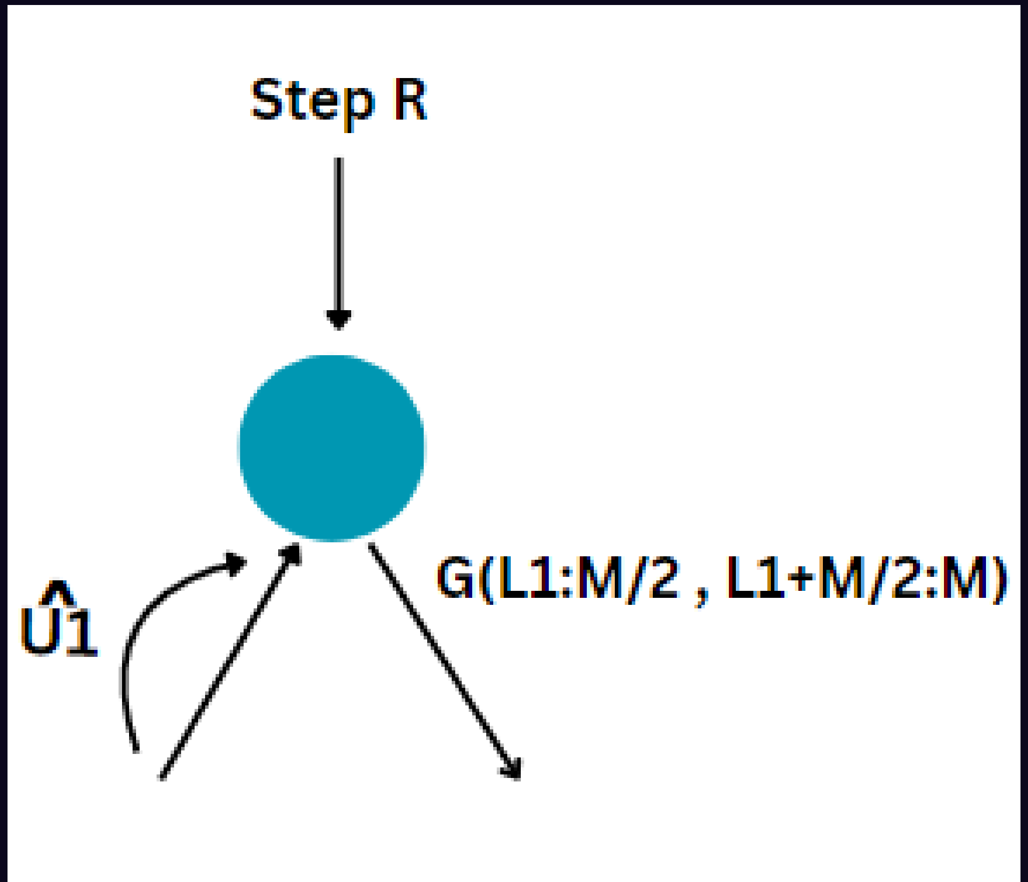
1) Starts at root
2) at every node
    if not leaf
        1) Do Step L and go to left child.
        2) When decision is received from left child, do Step R and go to the right child.
        3) When decision is received from right child, do step U and go to parent.
3) If leaf node,
    make decision and go to parent

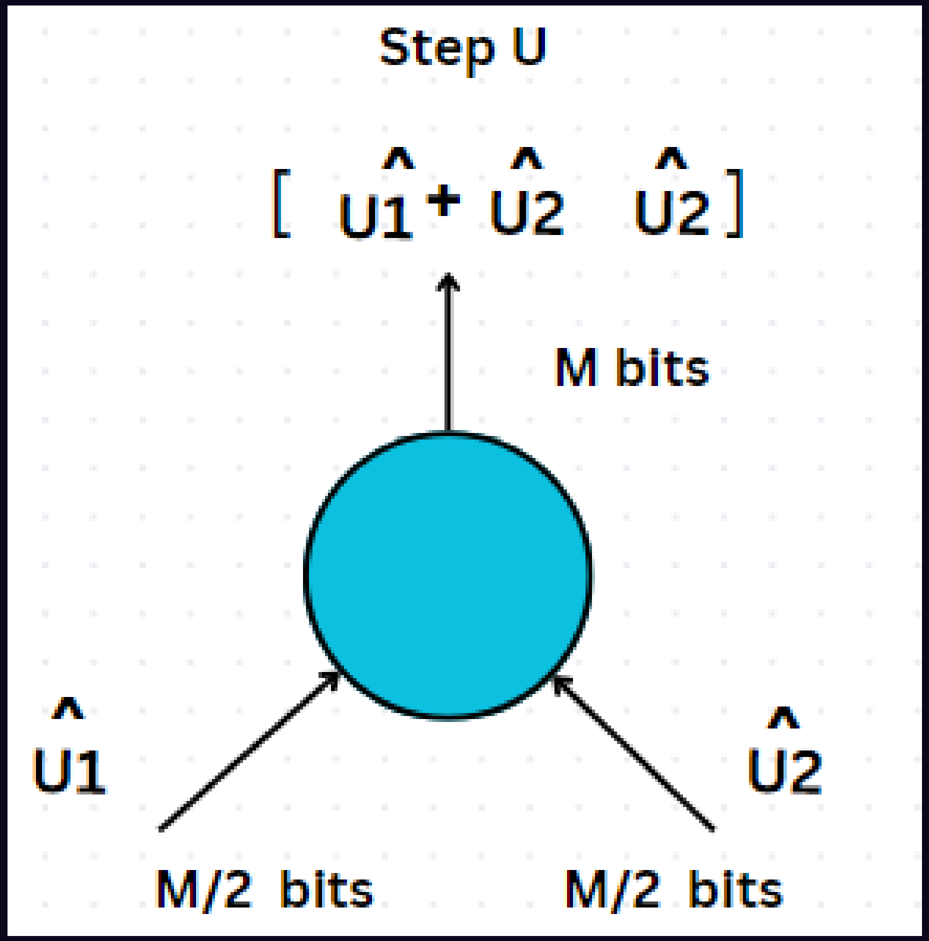
SC decoder:



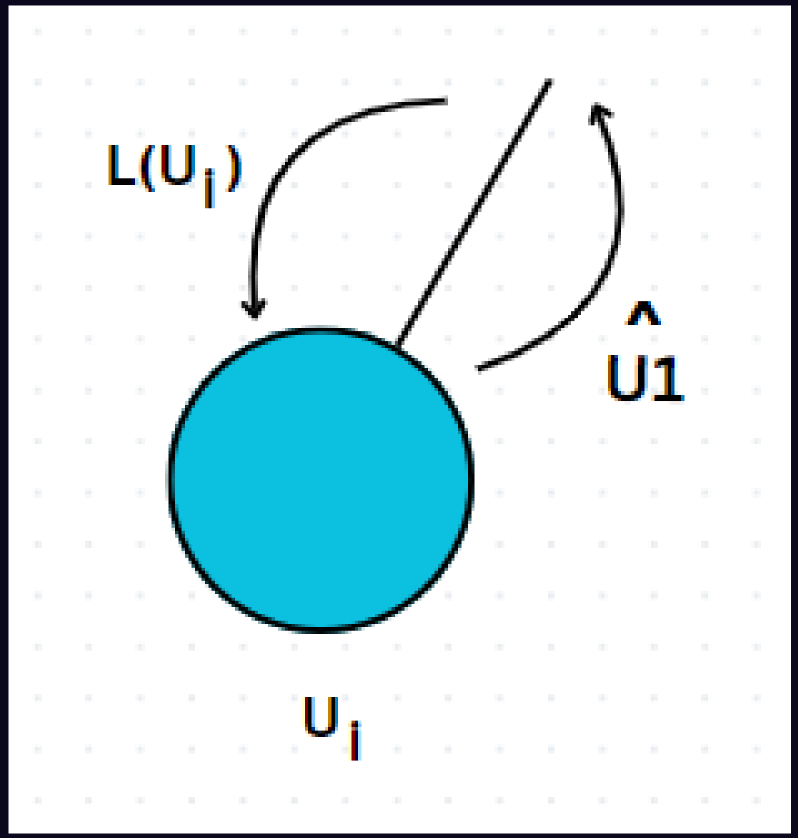
$r^{(N)}$ = [r1 r2 r3.....r_N] : received vector







Left Leaf Node:



if(i is frozen) $\hat{u}i=0$

else $\quad L(ui) >= 0 \ \hat{u}i=0$

$\quad L(ui) < 0 \ \hat{u}i=1$

Right Leaf Node:

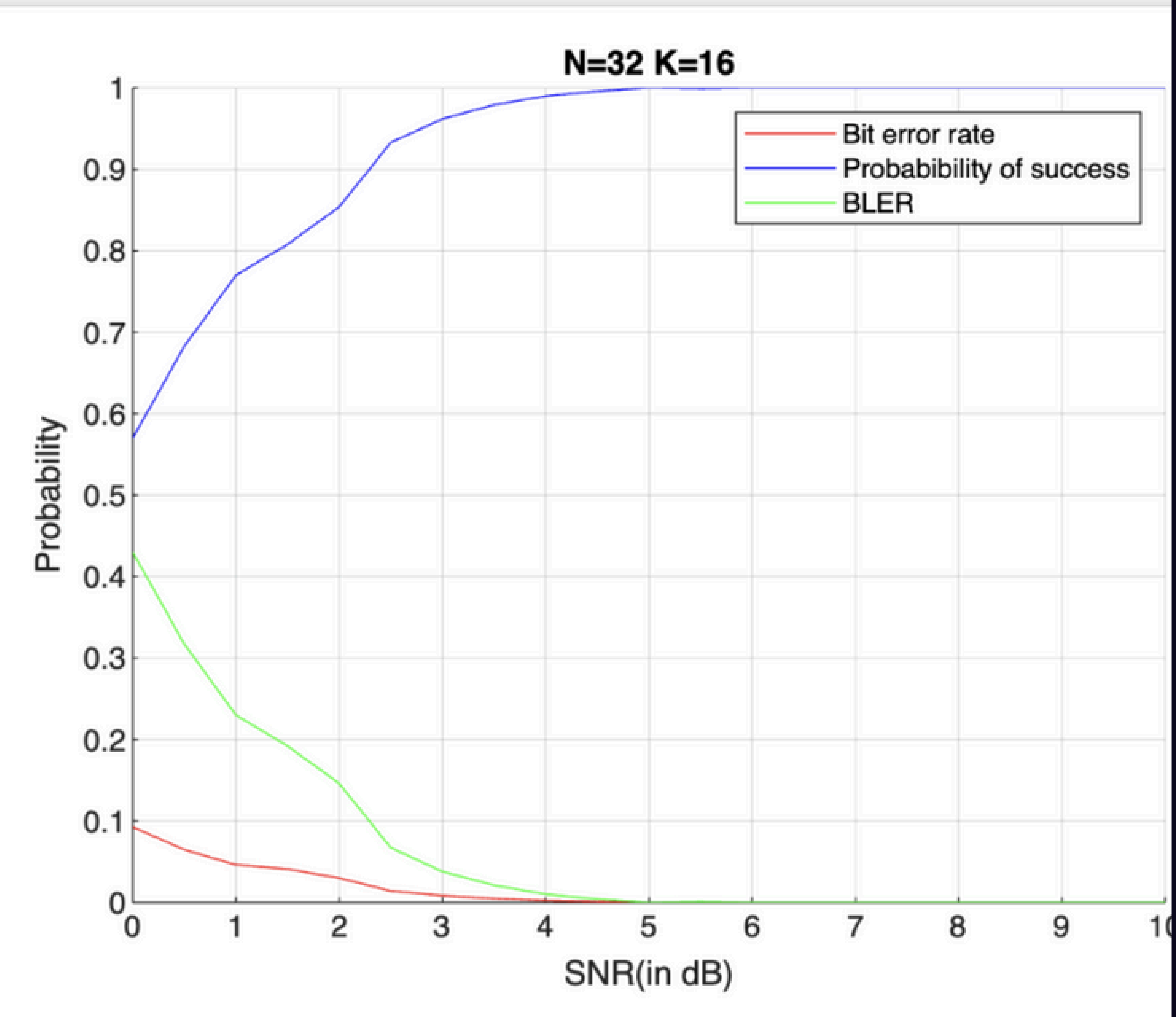
Given $\hat{U}1$ decodes U2 (Rep)

$\hat{U}1 = 0 \quad L(U2) = r2+r1 \quad (X = [U2\ U2])$

$\hat{U}1 = 1 \quad L(U2) = r2-r1 \quad (X = [\bar{U}2\ U2])$

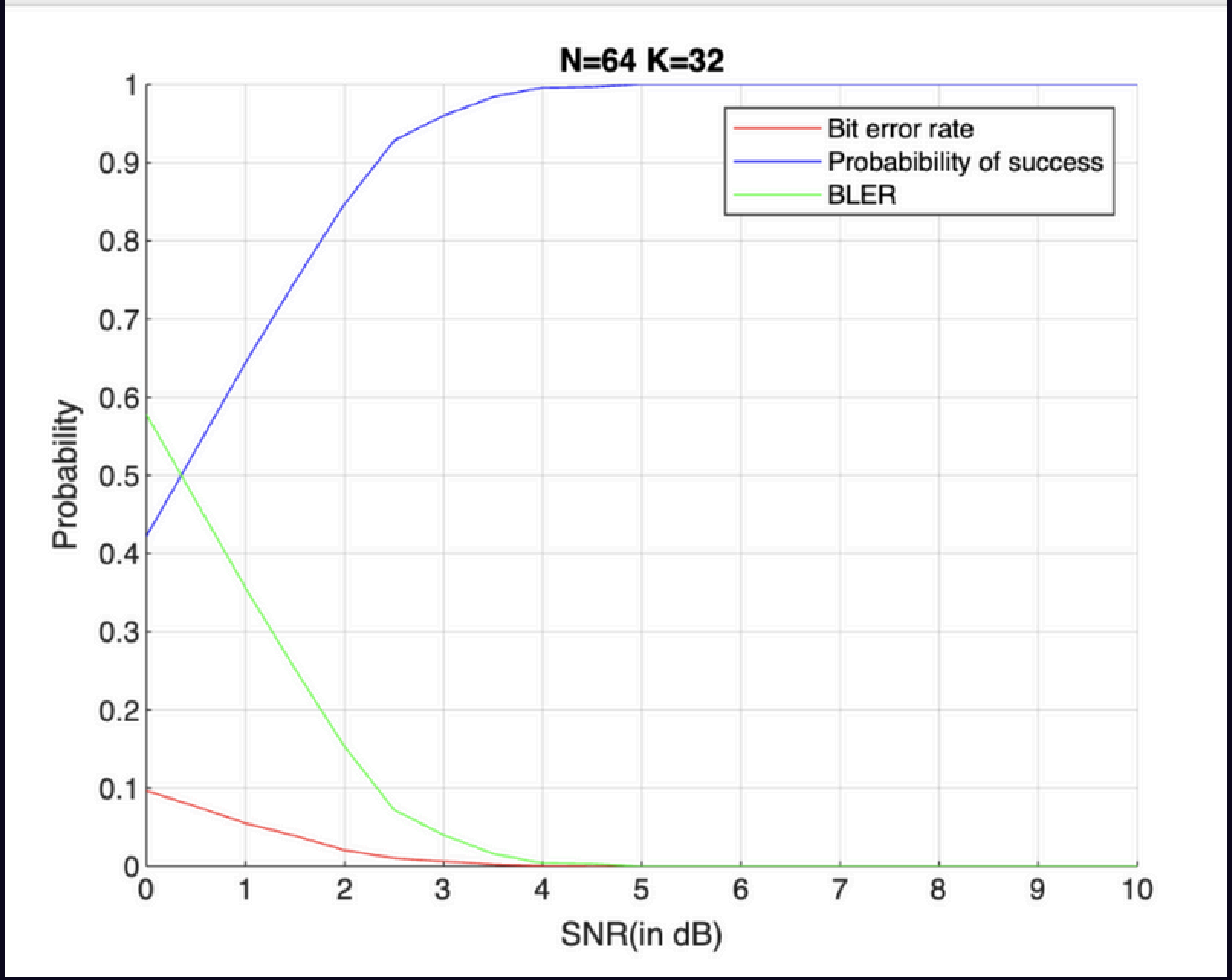
Results obtained from graphs (SC)

N=32



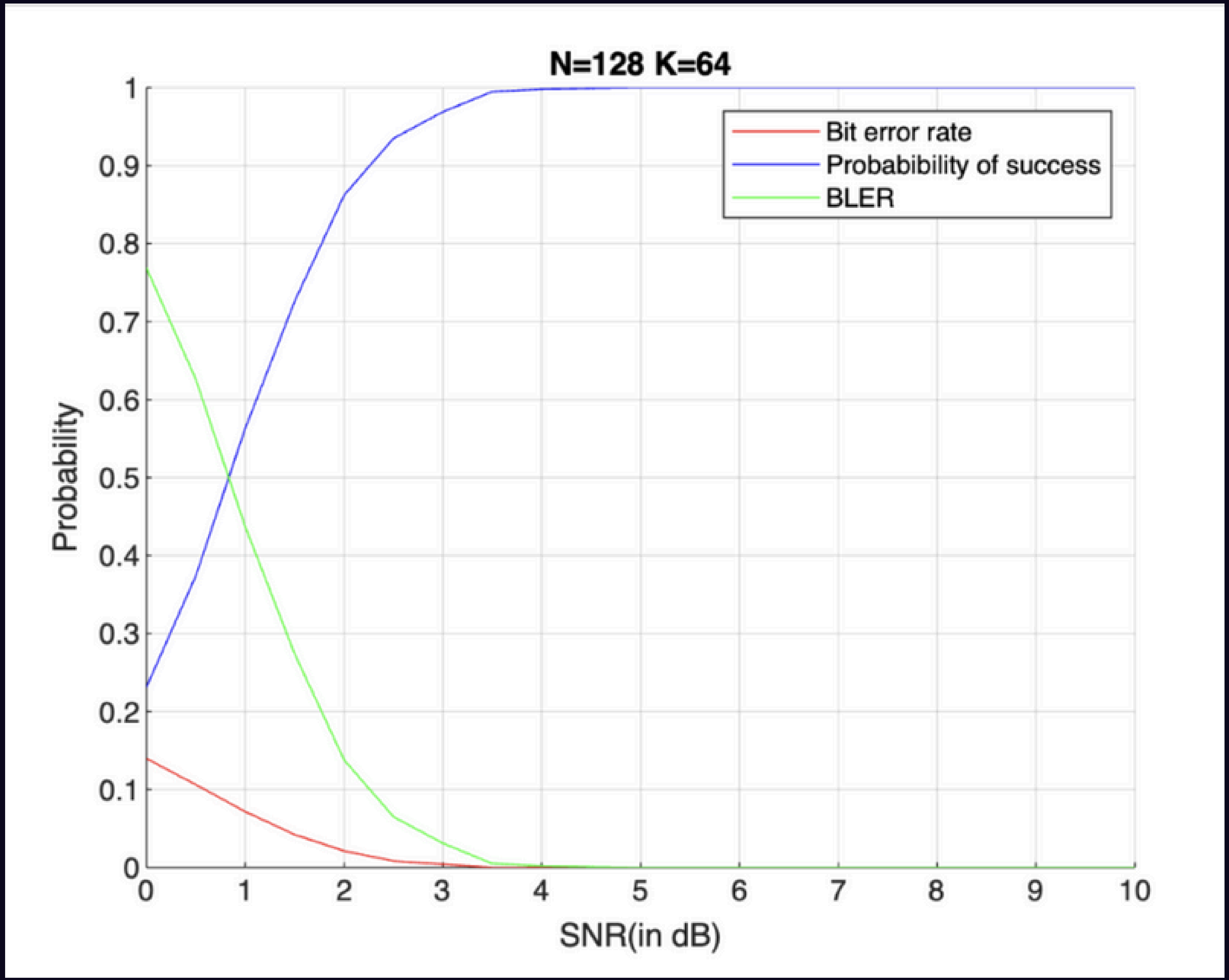
Results obtained from graphs (SC)

N=64



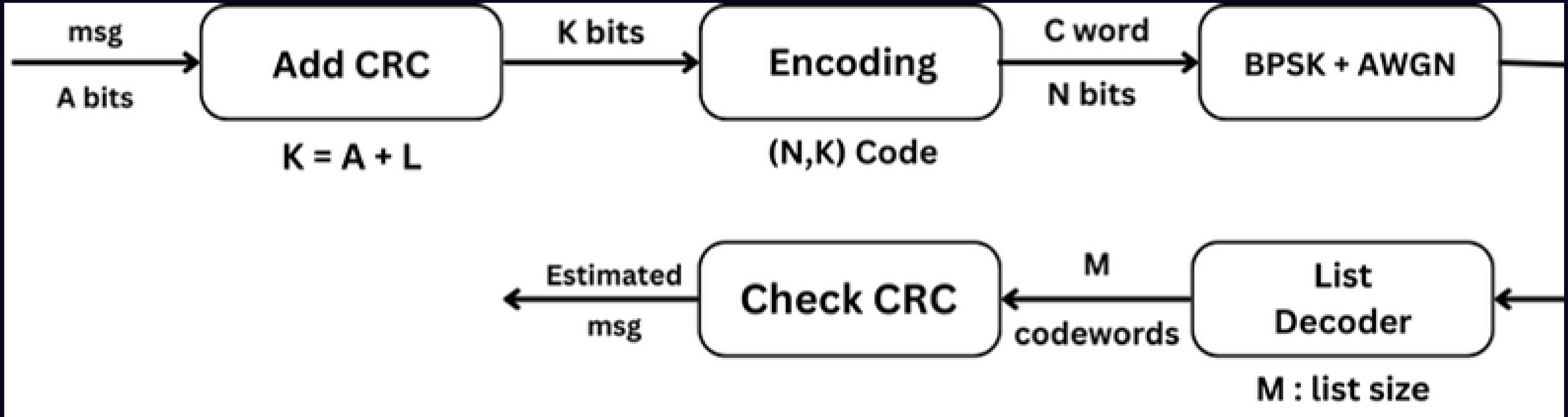
Results obtained from graphs (SC)

N=128



Due to inefficiency in decoding we move on to Successive Cancellation List Decoding!!!

SUCCESSIVE CANCELLATION LIST DECODING



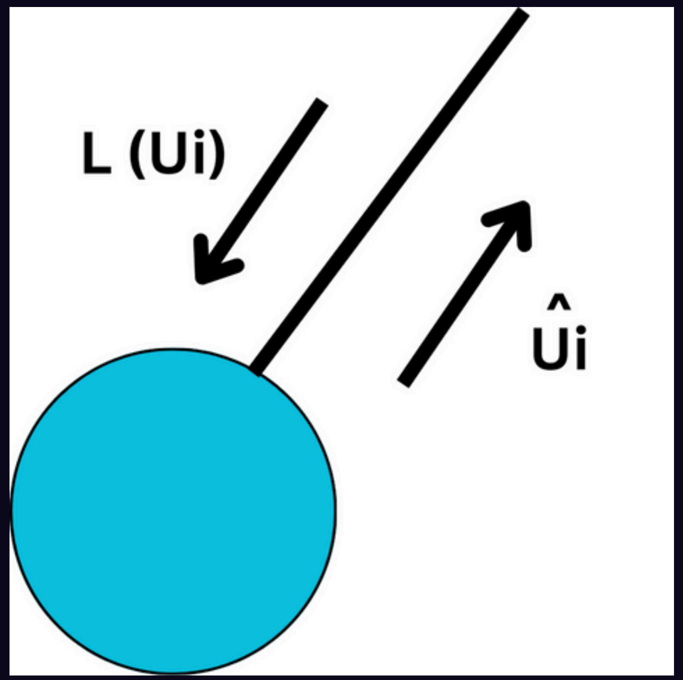
(conflict : If no decoder match CRC, pick the codeward with least path metric.)

How to produce multiple codewords at decoder?

- Polar SC decoder : consider both decision for each bit.
  assign decision metric (DM)

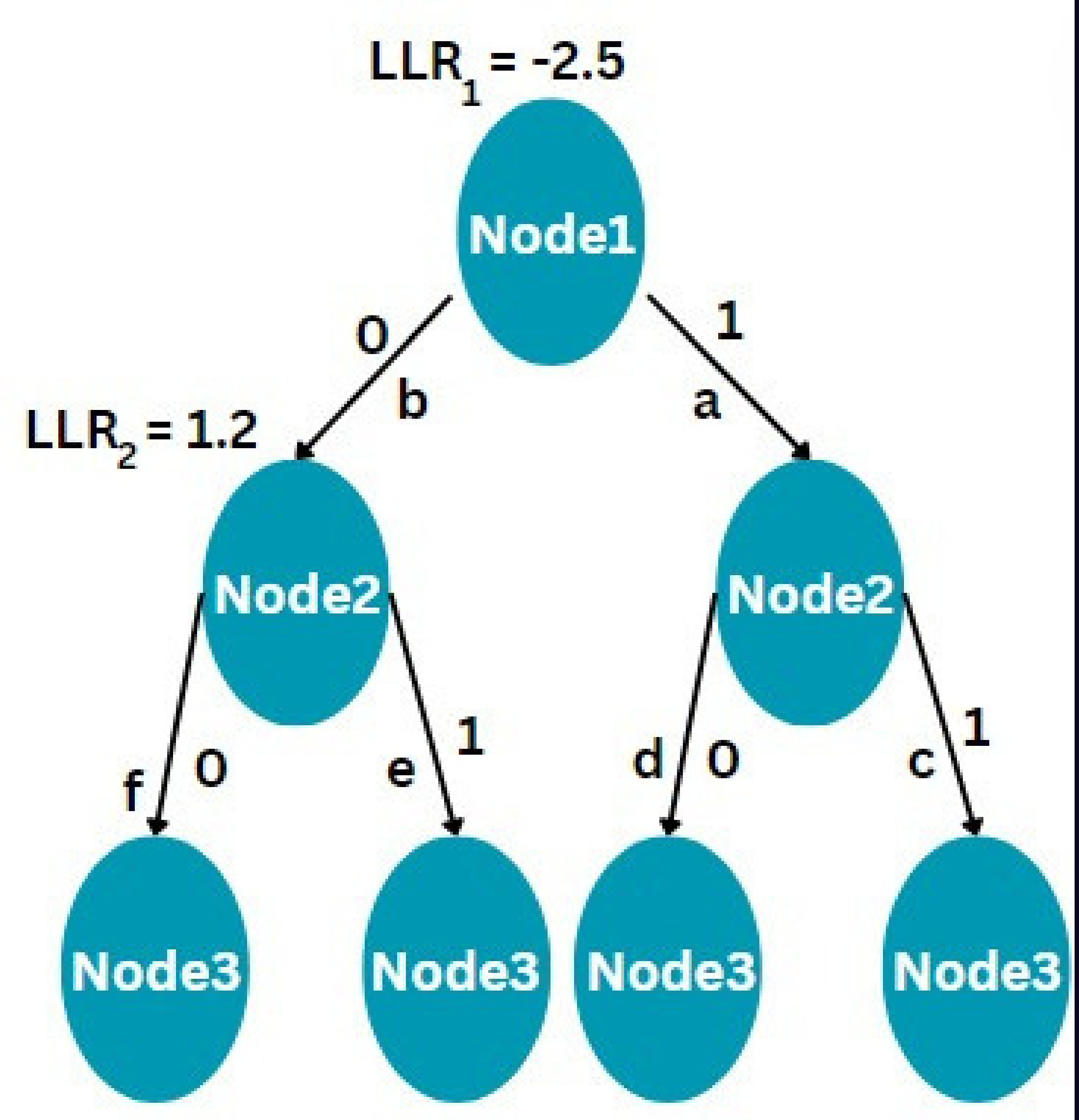
Steps:

Step 1: Create a binary message of length A.
Step 2: Choose a polynomial for CRC error detection.
Step 3: Divide the padded message by the CRC polynomial and find the remainder.
Step 4: Modify the padded message by replacing the zeros with the CRC remainder.
Step 5: Determine the LLRs for each received bit, representing the likelihood of it being a 0 or a 1.
Step 6: Unlike traditional SC decoding where each non-frozen bit is decoded as either 0 or 1, SCL decoding considers the possibility of multiple values.
Step 7: maintain a limit on the number of paths considered during decoding.
Step 8: Rank the paths based on their penalties.
Step 9: If the number of paths exceeds the limit, retain only the top paths and discard the rest.
Step 10: Divide each decoded message by the CRC polynomial. If the remainder is zero, consider the decoded message as the final output. If not, move to the next decoded message on the list and repeat the process.



- if L(Ui) >= 0 : Ûi = 0 has DMi = 0, Ûi = 1 has DMi = | L(Ui) |
  if L(Ui) < 0 : Ûi = 1 has DMi = 0, Ûi = 0 has DMi = | L(Ui) |
- imp : DM assigned even if 'i' is frozen

  i -> frozen than only one decision Ui = 0
- metric : sum of decision metrics on path of choices.

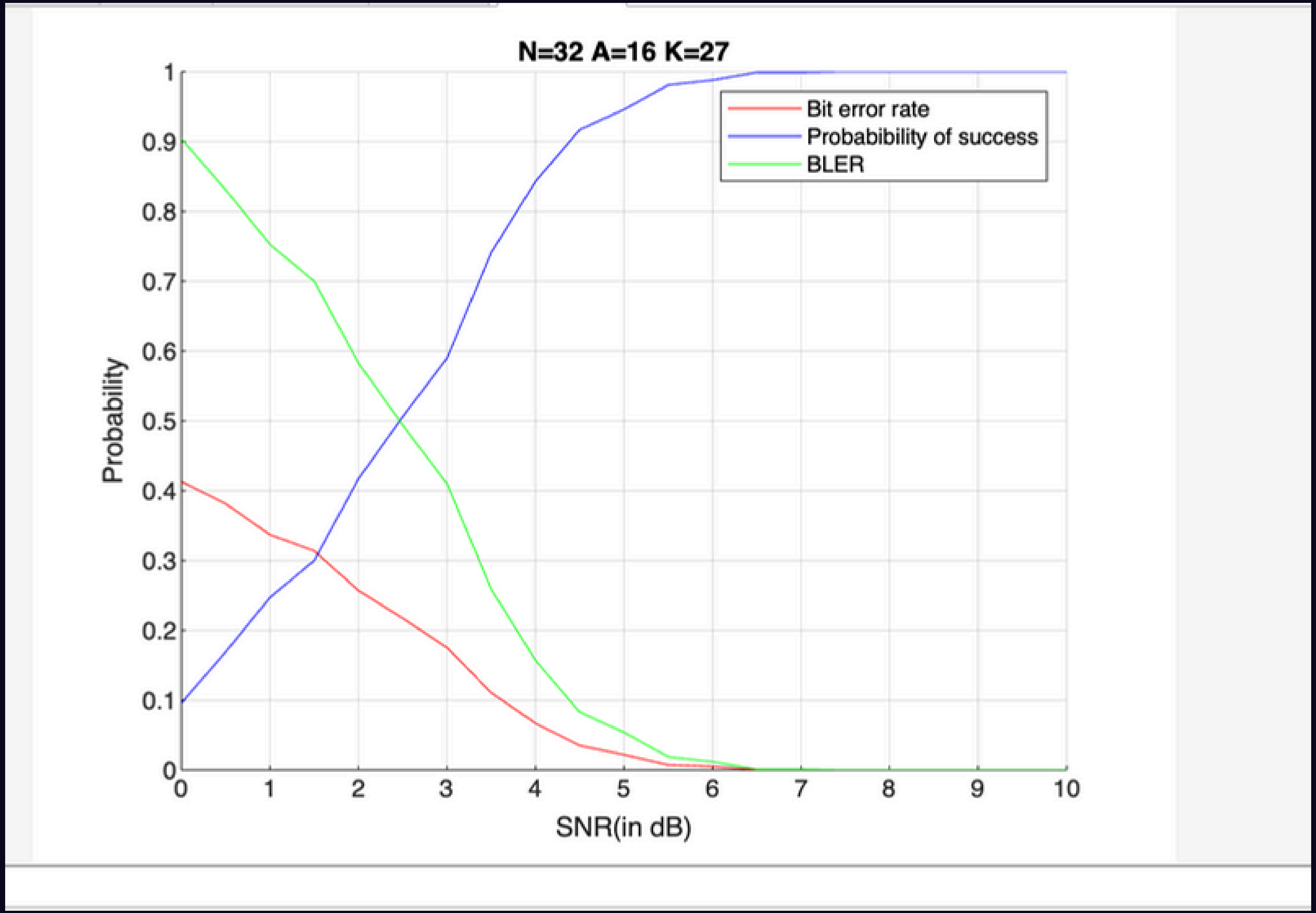


Here in this example,,we have possible 4 paths

Paths	Penalty
a->c	1.2
a->d	0
b->e	3.7
b->f	2.5

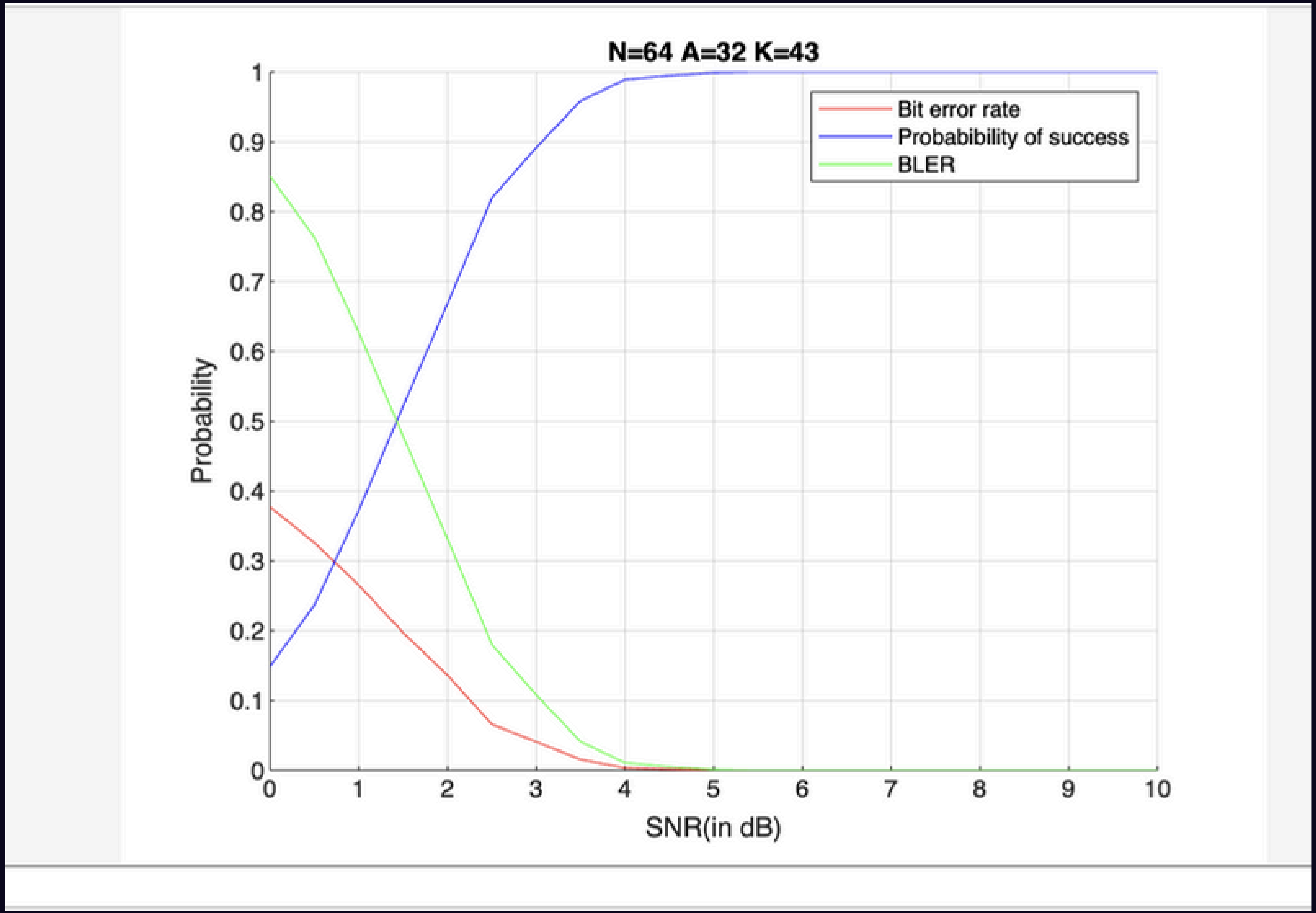
Results obtained from graphs (SCL)

N=32



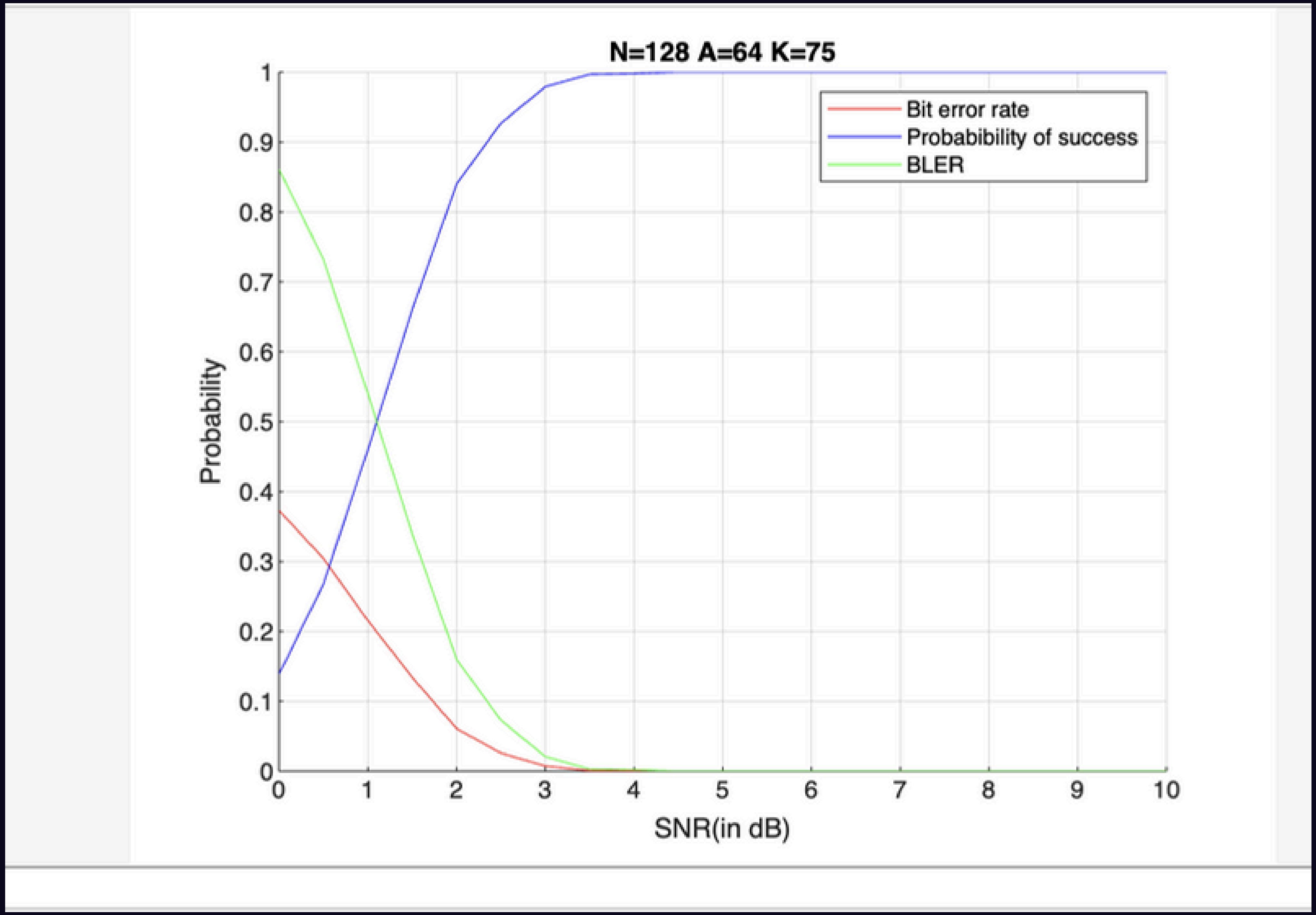
Results obtained from graphs (SCL)

N=64



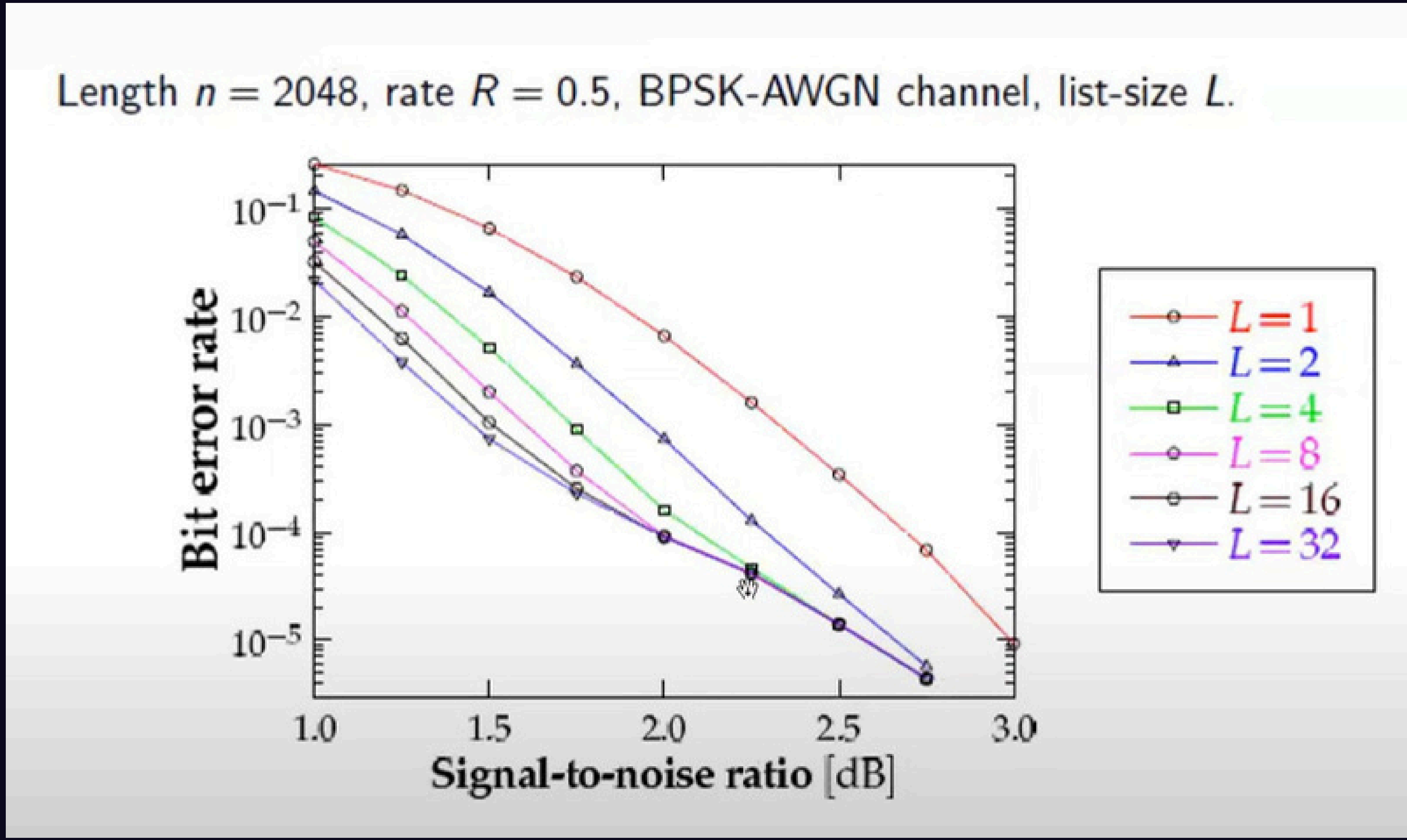
Results obtained from graphs (SCL)

N=128



Final Conclusion from List Decoding.........

As the size of list increases, accuracy of decoding increases



SC V/S SCL

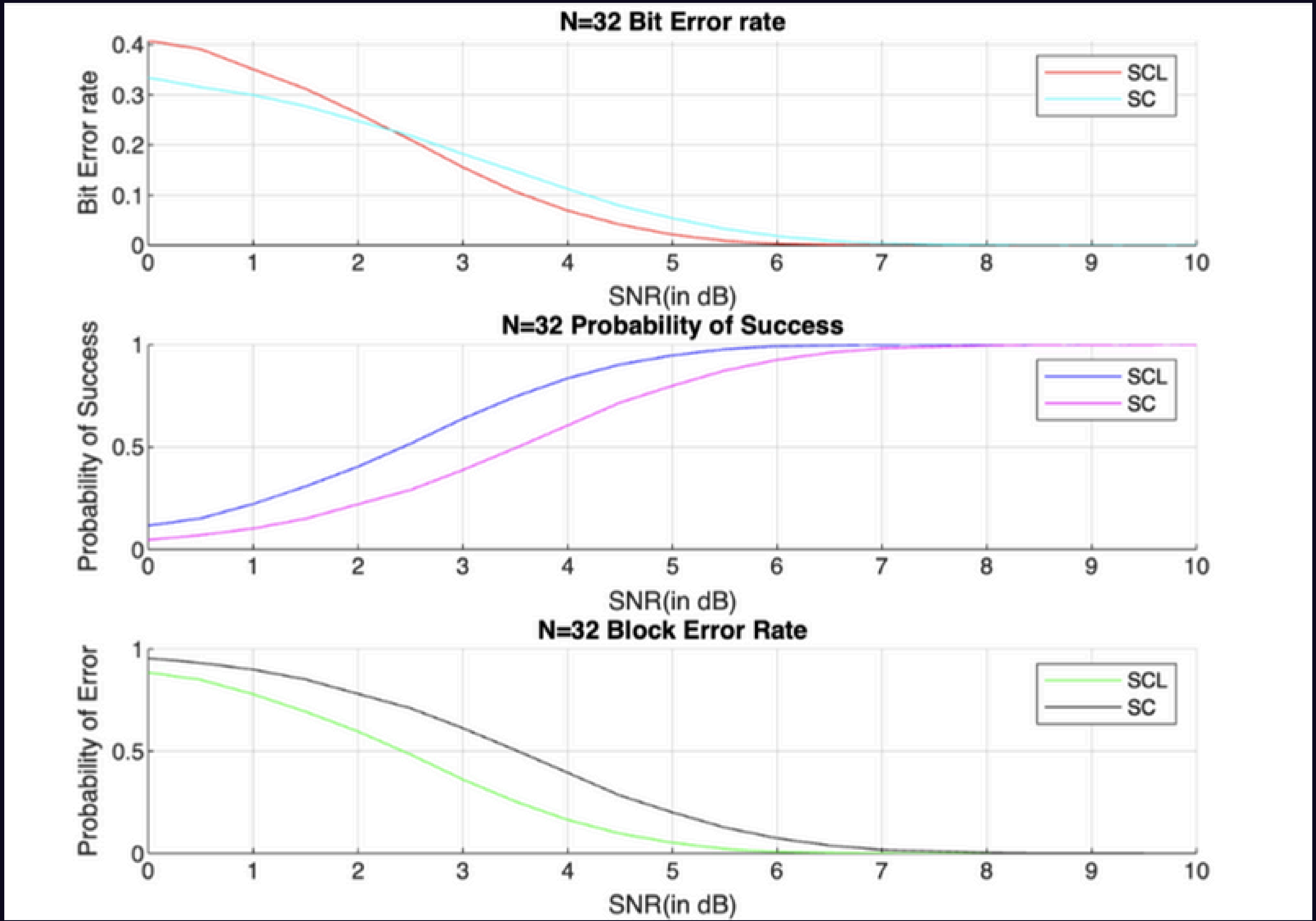
SUCCESSIVE CANCELLATION DECODING:

- Simplest decoding algorithm
- Each bit decision affects the subsequent bit decisions Performance is slightly less as compared to SCL, but less complex in nature
- Error Probability is more as it gives only one codeword at the end

SUCCESSIVE CANCELLATION LIST DECODING:

- Extention to SC algorithm
- It explores all possible paths in parallel and stores information of most likely candidates
- It provides with multiple codewords which reduces the error probbility

Graphical Interpretation of SC v/s SCL Decoding



7) Summary

Given channel w, N=2^n Polar codes (N,k) with information bits k can be constructed using the concept of polarization.
Encoding complexity: O (N logN)
Successive Cancellation decoding complexity: O(N logN)

$$P_e(N,R) \approx 2^{-\sqrt{N}}$$

As $N \to \infty$ $P_e(N,R)$ approximates to 0.

Polar codes approximates to Shannons' Channel Capacity

Successive Cancellation List Decoding is more efficient as compared to SC.

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[9]Information Theory,Lecture 9: Polar Codes,Mert Pilanci,Stanford University,February 5, 2019
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[11]https://www.youtube.com/playlist?list=PLyqSpQzTE6M81HJ26ZaNv0V3ROBrcv-Kc

THANK YOU!!!