October 27, 2015

Problems for the next seminar

Symmetric groups

For Friday: October 30

Problem 1. Consider the Hecke algebra $H(n)$ and introduce $t_i = s_i + 1/(y_i - y_{i+1})$. Show that $t_i t_{i+1} t_i = t_{i+1} t_i t_{i+1}$.

Problem 2. Let X_i be the Jucys-Murphy elements. Find the minimal polynomials of X_i, $i = 1, 2, 3, 4$, acting by the left multiplication in $\mathbb{C}S_4$.

Problem 3. Recall that a skew Young diagram λ/μ with k boxes determines a representation $V_{\lambda/\mu}$ of the Hecke algebra $H(k)$. We have $\mathbb{C}S_k \subset H(k)$. Determine the decomposition into irreducibles of the module $V_{\lambda/\mu}$ as a $\mathbb{C}S_k$-module for

a) $\lambda = (2, 2)$, $\mu = (1)$.
b) $\lambda = (2, 2, 1)$, $\mu = (1, 1)$.
c) $\lambda = (3, 2, 1)$, $\mu = (2, 1)$.

Problem 4. Recall that given an inclusion of groups $G \subset H$ and a G module V we have the induced module Ind_G^H. The induced module is the space $\mathbb{C}H \otimes_{\mathbb{C}G} V$ with H-action given by the left multiplication in the first factor. Compute the decomposition into irreducibles of the induced modules

a) $\mathrm{Ind}_{S_2}^{S_3} V$, for all irreducible S_2 modules V.
b) $\mathrm{Ind}_{S_3}^{S_4} V$, for all irreducible S_3 modules V.
b) $\mathrm{Ind}_{S_2 \times S_2}^{S_4} (V_1 \otimes V_2)$, for all irreducible S_2 modules V_1, V_2.

Problem 5. Let $V = \mathbb{C}^2$. The group S_4 acts on $V \otimes V \otimes V \otimes V$ by permutation of factors. Compute the decomposition into irreducibles.

Problem 6. Show that the subalgebra of $H(3)$ generated by y_1, y_2, y_3 is maximal commutative. Show that the center of $H(3)$ consists of symmetric polynomials in y_1, y_2, y_3.

Problem 7. a) Show that there exists an action of $H(2)$ in $\mathbb{C}[y_1, y_2]$ such that y_1, y_2 act as the multiplication operators and $s \cdot 1 = 1 \in \mathbb{C}[y_1, y_2]$. Find an explicit formula for the action of s.

b) Prove the same result for $H(3)$.

Problem 8. a) Consider the polynomial representation of $H(2)$ described in Problem 7. Show that for any $a, b \in \mathbb{C}$ the subalgebra $A_{a,b}$

of $\mathbb{C}[y_1, y_2]$ generated by $y_1 + y_2 - a$ and $y_1 y_2 - b$ is an $H(2)$-submodule. Describe the $H(2)$-module $C[y_1, y_2]/A_{a,b}$.

b) Do the same problem for $H(3)$.

Problem 9. Classify all irreducible representation of $H(3)$.