

Logistic Regression:

- It comes under supervised learning
- Logistic regression is classification algorithm used to predict the probability of categorical target variable.

Assumption of logistic Regression

- No or Little multicollinearity
- The output should be binary (0/1), (True/False), (diabetic/non-diabetic)
- It requires huge amount of data.

Types of classification

- ① Binary classification: Binary classification algorithms are used when target column has two categories
ex:- diabetic (0) non diabetic (1)
- ② Multi classification: Multi classification algorithm used when we have two or more categories in target column,
ex:- Iris

How Logistic Regression Works?

marks	Result	Linear Regression
20	Fail (0)	Fit a line
30	Fail	$m = ? \Rightarrow m = 0.5$
45	pass (1)	$C = ? \Rightarrow C = 20$
78	pass	$y = 0.5x + 20$
92	pass	$y = 0.5x + 20$
50	?	$y = 45$

Why Linear Regression fails on classification data.

- It fits a straight line by finding slope and intercept then we use that line to make predictions we get continuous values which is violating the assumption of logistic regression.
- Linear regression fails because it gives values other than 0 & 1.

O How to overcome the issue caused by linear regression?

A. Sigmoid function:

Sigmoid function is a mathematical function which transform continuous data into a range of 0 to 1.

$$\text{h}_0(z) = \frac{1}{1+e^{-z}}$$

$z = mx+c$.

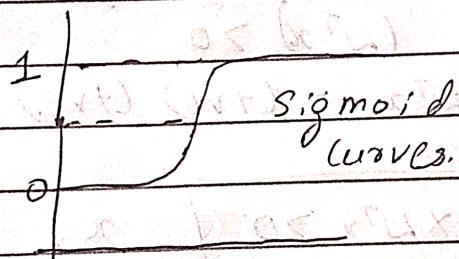
Marks	Result	\hat{y}	$h_a(x)$	\hat{y}
20	Fail	-100	0.2	0
30	Fail	-20	0.3	0
45	Pass	100	0.6	1
78	Pass	200	0.7	1
92	Pass	300	0.9	1

Threshold:

$$h_a(x) < 0.5, \hat{y} = 0$$

$$h_a(x) \geq 0.5, \hat{y} = 1$$

Graph:



$$\begin{array}{l|l} Y = m_1 x_1 + m_2 x_2 + m_3 x_3 + C & W^T = \begin{bmatrix} m_1 & m_2 & m_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ Y = W^T x + C. \end{array}$$

Distance b/w point and plane.

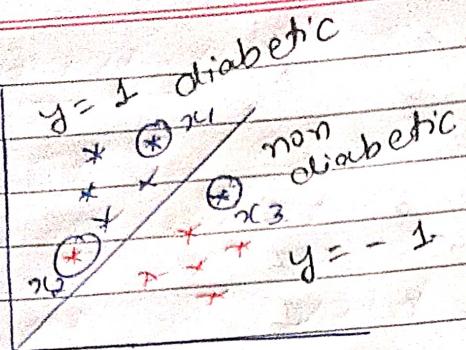
$$d = \frac{|W^T x + C|}{\sqrt{W^T W}}, \text{ assume } |W| = 1 \rightarrow \text{unit vector}$$

$$d = |W^T x + C|$$

The observation is correctly classified only if

$$Y \times W^T x > 0.$$

Ex:-



- The positive classes are denoted by $y = 1$
- the negative classes are denoted by $y = -1$
- The distance above the plane is always positive $w^T x > 0$

Case - 1 :-

x_1 belongs to positive classes, so $y = 1$
 x_1 is above the plane.
 So, $w^T x_1 > 0$.
 $y \times w^T x = (+ve) (+ve) = (+ve)$

$\boxed{y \times w^T x > 0}$ x_1 is correctly classified

Case - 2 :-

x_2 is belong to negative, so $y = -1$
 x_2 is above the plane, $w^T x_2 > 0$

$$y \times w^T x = (-ve) (+ve) = -ve$$

$y \times w^T x < 0 \rightarrow x_2$ is wrongly classified.

Case - 3 :-

x_3 is belong to positive, $y = 1$
 x_3 is below the plane, ~~so~~ $w^T x < 0$

$$y \times w^T x = (1) (-1) = (-ve) (+ve) = -ve$$

$y \times w^T x < 0 \rightarrow x_3$ is wrongly classified.

Evaluating the model.

(1) Confusion matrix:

A confusion matrix is a table that summarizes the performance of a classification model by comparing predicted & actual class labels.

		Prediction →	
		Non diabetic (0)	Diabetic (+ve) (1)
Actual ↓	(-ve) Non diabetic.	T N	F P
	(+ve) diabetic.	F N	T P

		Prediction →	
		(-ve) 0	(+ve) 1
Actual ↓	(-ve) 0	T N	F P
	(+ve) 1	F N	T P

↓

TYPE 2

$$\text{correct classification} = TN + TP$$

$$\text{wrong classification} = FP + FN.$$

actual	Pred.	
0	0.	T N
1	1	T P
0	1	F P
1	0	F N

(2) Accuracy:

- It is used to measure the strength of the model.
- It is valid only when data is balanced.

Accuracy = $\frac{\text{correct predictions}}{\text{total prediction}}$

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

(3) Precision:

$$\text{Precision} = \frac{TP}{TP + FP}$$

Out of positive prediction, what percentage is true positive.

(4) Recall:

$$\text{Recall} = \frac{TP}{TP + FN}$$

out of actual positive, how many are true positives

(5) F₁ Score:

It is the harmonic mean of Recall & precision.

$$F_1 \text{ score} = 2 \times \frac{\text{Recall} \times \text{Precision}}{\text{Recall} + \text{Precision}}$$

- Accuracy is good when data is balanced.
- F₁-score is good when data is Imbalanced.

(6) ROC-AUC curve:

It is one of the metric used to evaluate binary classification problem

ROC → Receiver operator or characteristics

It is used to evaluate binary classification model at various thresholds.

y	\hat{y}	$h_0(x) = 0.5$	0, 2	0, 7
0	0.2	0	0	0
1	0.4	0	1	0
0	0.3	0	1	0
1	0.7	1	1	1
1	0.8	1	1	1
1	0.9	1	1	1
0	0.5	1	1	0
1	0.6	1	1	0

TPR → True positive rate

$$\text{TPR} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$

$FPR \rightarrow$ False positive rate

$$FPR = \frac{FP}{FP + TN}$$

at 0.5

$$TPR = \frac{TP}{TP + FN} = \frac{4}{4+1} = \frac{4}{5} = 0.8$$

$$TP = [1 - 1] = 1 - 1 = 0 \quad \text{in table 4 so, } \uparrow \\ FN [10 - 0] = 1$$

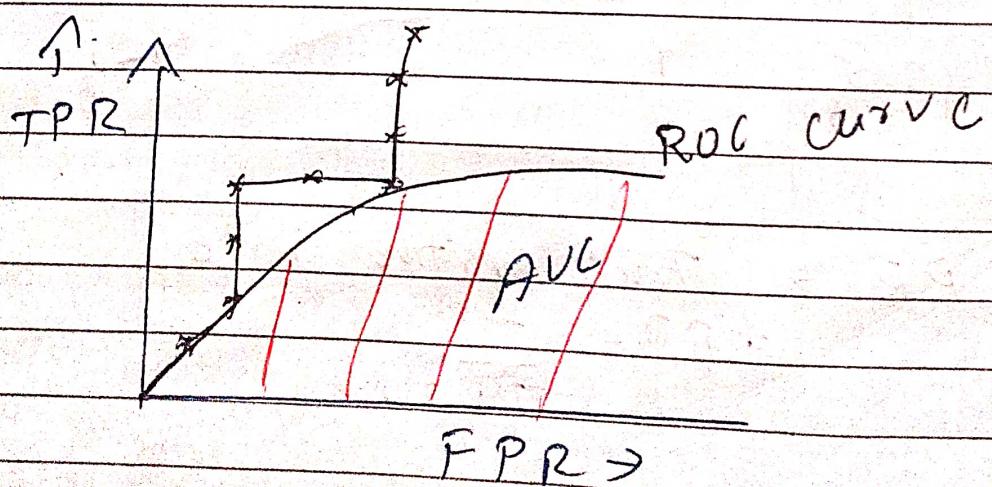
~~TPR~~

$$FPR = \frac{FP}{FP + TN} = \frac{1}{1+2} = \frac{1}{3}$$

$$TN = [0 - 0] = 2$$

$$FP = [0 - 1] = 1$$

- ROC will find TPR and FPR for various thresholds
- Plot graph of TPR vs FPR



- $AUC \rightarrow$ Area under the curve
 - AUC tells about strength of the model by measuring area below the curve
- Range of $AUC = [0, 1]$
- more the area below the curve then model is good
 - If area below the curve is less, than model is bad.