Linear Regression using Stats model

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

import statsmodels
import statsmodels.api as sm
import sklearn
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
from sklearn.metrics import r2_score
from sklearn.linear_model import LinearRegression
```

Step1: Reading the data

In [2]:
```
data = pd.read_csv("advertising.csv")
data.head()
```

Out[2]:

	TV	Radio	Newspaper	Sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	12.0
3	151.5	41.3	58.5	16.5
4	180.8	10.8	58.4	17.9

In [3]:
```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 200 entries, 0 to 199
Data columns (total 4 columns):
 #   Column     Non-Null Count  Dtype  
---  ------     --------------  -----  
 0   TV         200 non-null    float64
 1   Radio      200 non-null    float64
 2   Newspaper  200 non-null    float64
 3   Sales      200 non-null    float64
dtypes: float64(4)
memory usage: 6.4 KB
```

In [4]:
```
data.describe()
```

Out[4]:

	TV	Radio	Newspaper	Sales
count	200.000000	200.000000	200.000000	200.000000
mean	147.042500	23.264000	30.554000	15.130500
std	85.854236	14.846809	21.778621	5.283892
min	0.700000	0.000000	0.300000	1.600000
25%	74.375000	9.975000	12.750000	11.000000
50%	149.750000	22.900000	25.750000	16.000000

	TV	Radio	Newspaper	Sales
75%	218.825000	36.525000	45.100000	19.050000
max	296.400000	49.600000	114.000000	27.000000

In [5]:
```
data.corr()
```

Out[5]:

	TV	Radio	Newspaper	Sales
TV	1.000000	0.054809	0.056648	0.901208
Radio	0.054809	1.000000	0.354104	0.349631
Newspaper	0.056648	0.354104	1.000000	0.157960
Sales	0.901208	0.349631	0.157960	1.000000

Step 2: Training the model

In [6]:
```
# Create X and Y
X = data['TV'] # Capital X matches documentation of libraries like scikit-learn, Ten
y = data['Sales']
```

In [7]:
```
# train and test split
X_train, X_test, y_train, y_test = train_test_split(X,y, train_size=0.7, random_stat
# with random_state You'll always get the same 70% training and 30% test data split
```

- Statsmodel library doesn't include the constant c(intercept), it only include the coefficient of the predicted variable so we will add the constant to statsmodel library explicitly to train the model because we can't exclude the constant unless we are sure that it is zero.

In [8]:
```
# training the model
X_train_sm =sm.add_constant(X_train)
X_train_sm.head()
```

Out[8]:

	const	TV
74	1.0	213.4
3	1.0	151.5
185	1.0	205.0
26	1.0	142.9
90	1.0	134.3

general equation is y = c + m1.X1 # the above equation for statsmodel will be y = c.X0 + m1.X1 where X0 = 1

In [9]:
```
# fitting the model,  Objective of OLS: Find β values that minimize the total square
lr = sm.OLS(y_train, X_train_sm)
lr_model = lr.fit()
lr_model.params
```

Out[9]:
```
const    6.948683
TV       0.054546
dtype: float64
```

- Objective of OLS: Find β values that minimize the total squared error between predicted and actual values:
- OLS comes with rich statistical inference tools: p-values, R-squared, confidence intervals, F-statistics
- OLS provides the Best Linear Unbiased Estimator (BLUE) meaning among all linear unbiased estimators, OLS has the lowest variance.

- the above values are the constant and the coefficient of TV so the equation will be Sales = 6.91 + 0.05*TV
- This is not the actual model unless we verify from summary that it is statistically significant.

In [10]:
```
lr_model.summary() # only Statsmodel give you such summary
```

Out[10]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.816
Model:	OLS	**Adj. R-squared:**	0.814
Method:	Least Squares	**F-statistic:**	611.2
Date:	Mon, 30 Jun 2025	**Prob (F-statistic):**	1.52e-52
Time:	20:06:25	**Log-Likelihood:**	-321.12
No. Observations:	140	**AIC:**	646.2
Df Residuals:	138	**BIC:**	652.1
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	6.9487	0.385	18.068	0.000	6.188	7.709
TV	0.0545	0.002	24.722	0.000	0.050	0.059

Omnibus:	0.027	**Durbin-Watson:**	2.196
Prob(Omnibus):	0.987	**Jarque-Bera (JB):**	0.150
Skew:	-0.006	**Prob(JB):**	0.928
Kurtosis:	2.840	**Cond. No.**	328.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [11]:
```
y_train_pred = lr_model.predict(X_train_sm)
```

Step 3: Residual analysis

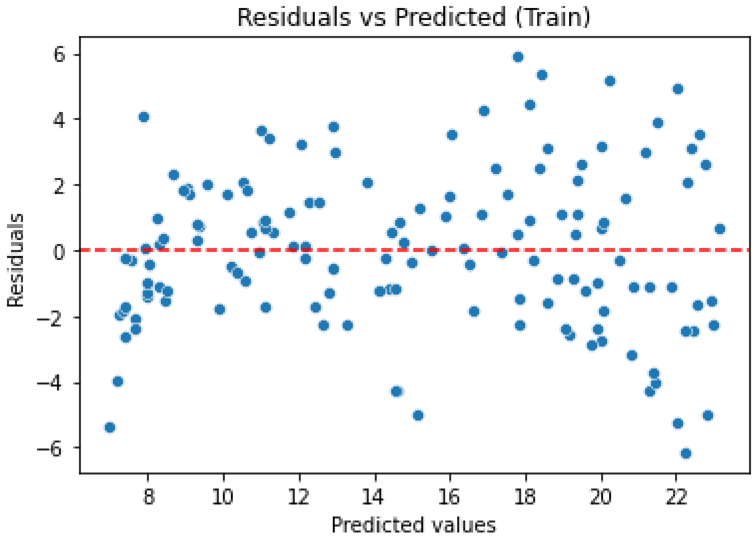
- Linearity: the residuals should be randomly scattered around zero — no curve or trend.

- Homoscedasticity: Means the variance of residuals is constant across all levels of predicted values. Spread of residuals should be even — not getting wider or narrower like a cone.
- Histogram of Residuals (Normality): Assumes the residuals follow a normal distribution (bell curve). A smooth, bell-shaped curve (especially with the KDE line) suggests normality.

In [12]:
```
residuals_train = y_train - y_train_pred
```

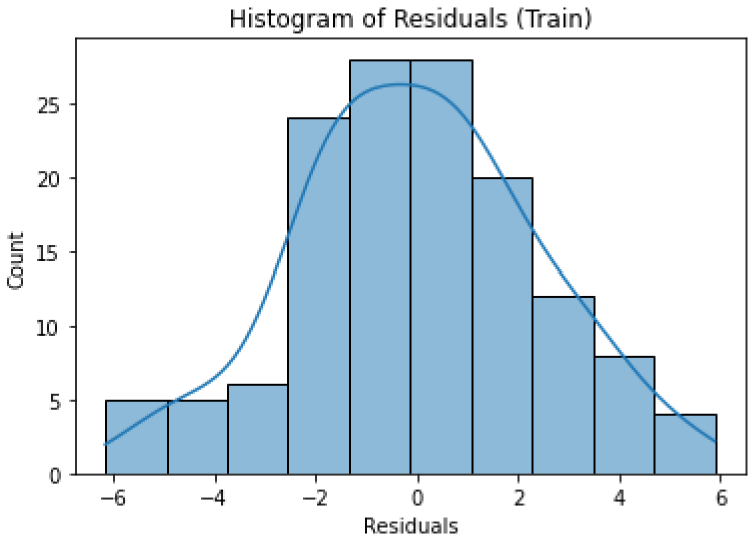
Residual Plot (Linearity & Homoscedasticity)

In [13]:
```
plt.figure(figsize=(6,4))
sns.scatterplot(x=y_train_pred, y=residuals_train)
plt.axhline(0, color='red', linestyle='--')
plt.xlabel("Predicted values")
plt.ylabel("Residuals")
plt.title("Residuals vs Predicted (Train)")
plt.show()
```



Histogram of Residuals (Normality)

In [14]:
```
sns.histplot(residuals_train, kde=True)
plt.title("Histogram of Residuals (Train)")
plt.xlabel("Residuals")
plt.show()
```



In [15]:
```
r_train = r2_score(y_true=y_train, y_pred=y_train_pred)
r_train
```

Out[15]: 0.8157933136480389

Step 4: Predicting and evaluating on the test set

In [16]:
```
# add a contant/intercept to test
X_test_sm = sm.add_constant(X_test)

# Prediction on the test
y_test_pred = lr_model.predict(X_test_sm)
```

In [17]:
```
# evaluate the model, R-squared, on the test
r_test = r2_score(y_true=y_test, y_pred=y_test_pred)
r_test
```

Out[17]: 0.7921031601245658

Linear regression using SKlearn

In [18]:
```
# train and test split
X_train, X_test, y_train, y_test = train_test_split(X,y, train_size=0.7, random_stat
```

In [19]:
```
# reshape X_train to (n, 1) to ensure the input array has the correct 2D shape, espe
X_train_lm = X_train.values.reshape(-1, 1)
X_test_lm = X_test.values.reshape(-1, 1)
```

In [20]:
```
# Steps in sklearn model building:

# 1. create an object of linear regression
lm = LinearRegression()

# 2. fit the model
lm.fit(X_train_lm, y_train)
```

Out[20]: LinearRegression()

In [21]:
```
# 3. View coefficients
print(lm.coef_)
print(lm.intercept_)
```

```
[0.05454575]
6.9486832000013585
```

In [22]:
```
# make predictions
y_train_pred = lm.predict(X_train_lm)
y_test_pred = lm.predict(X_test_lm)
```

In [23]:
```
# 4. evaluate the model
print(r2_score(y_true=y_train, y_pred=y_train_pred))
print(r2_score(y_true=y_test, y_pred=y_test_pred))
```

```
0.8157933136480388
0.792103160124566
```

Multiple Regression

Step 1: Reading the Data

In [24]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

import sklearn
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import MinMaxScaler
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor

from sklearn.metrics import r2_score

import warnings
warnings.filterwarnings('ignore')
```

In [25]:
```
hous = pd.read_csv('Housing.csv')
hous.head()
```

Out[25]:

	price	area	bedrooms	bathrooms	stories	mainroad	guestroom	basement	hotwaterheatin
0	13300000	7420	4	2	3	yes	no	no	n
1	12250000	8960	4	4	4	yes	no	no	n
2	12250000	9960	3	2	2	yes	no	yes	n
3	12215000	7500	4	2	2	yes	no	yes	n
4	11410000	7420	4	1	2	yes	yes	yes	n

In [26]:
```
hous.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 545 entries, 0 to 544
Data columns (total 13 columns):
 #   Column            Non-Null Count  Dtype 
---  ------            --------------  ----- 
 0   price             545 non-null    int64 
 1   area              545 non-null    int64 
 2   bedrooms          545 non-null    int64 
 3   bathrooms         545 non-null    int64 
 4   stories           545 non-null    int64 
 5   mainroad          545 non-null    object
 6   guestroom         545 non-null    object
 7   basement          545 non-null    object
 8   hotwaterheating   545 non-null    object
 9   airconditioning   545 non-null    object
 10  parking           545 non-null    int64 
 11  prefarea          545 non-null    object
 12  furnishingstatus  545 non-null    object
dtypes: int64(6), object(7)
memory usage: 55.5+ KB
```

Step 2: Data Preparation

In [27]:
```
# # Encode binary variables
varlist = ['mainroad', 'guestroom', 'basement', 'hotwaterheating', 'airconditioning'
hous[varlist] = hous[varlist].apply(lambda x: x.map({'yes': 1, 'no': 0}))
hous[varlist].head()
```

Out[27]:

	mainroad	guestroom	basement	hotwaterheating	airconditioning	prefarea
0	1	0	0	0	1	1
1	1	0	0	0	1	0
2	1	0	1	0	0	1
3	1	0	1	0	1	1
4	1	1	1	0	1	0

Dummy variable

In [28]:
```
# Creating dummy variable for furnishing status
status = pd.get_dummies(hous['furnishingstatus'], drop_first=True)
hous = pd.concat([hous, status], axis=1)
hous.drop('furnishingstatus', axis=1, inplace=True)
hous.head()
```

Out[28]:

	price	area	bedrooms	bathrooms	stories	mainroad	guestroom	basement	hotwaterheatin
0	13300000	7420	4	2	3	1	0	0	
1	12250000	8960	4	4	4	1	0	0	
2	12250000	9960	3	2	2	1	0	1	
3	12215000	7500	4	2	2	1	0	1	
4	11410000	7420	4	1	2	1	1	1	

Scaling is important because we interpret coefficients.

In [29]:
```
# Scaling numeric variables
scaler = MinMaxScaler()
num_vars = ['area', 'bedrooms', 'bathrooms', 'stories', 'parking', 'price']
hous[num_vars] = scaler.fit_transform(hous[num_vars])
```

Split into train and test set

In [30]:
```
df_train, df_test = train_test_split(hous, train_size=0.7, random_state=100)
```

Step 3: Training the Model

In [31]:
```
# X_train, y_train
y_train = df_train.pop('price')
X_train = df_train
```

In [32]:
```
# 1. Add constant
X_train_sm = sm.add_constant(X_train)
#fit the model
```

```
lr_model = sm.OLS(y_train, X_train_sm).fit()
print(lr_model.summary())
```

```
                            OLS Regression Results                            
==============================================================================
Dep. Variable:                  price   R-squared:                       0.681
Model:                            OLS   Adj. R-squared:                  0.670
Method:                 Least Squares   F-statistic:                     60.40
Date:                Mon, 30 Jun 2025   Prob (F-statistic):           8.83e-83
Time:                        20:06:52   Log-Likelihood:                 381.79
No. Observations:                 381   AIC:                            -735.6
Df Residuals:                     367   BIC:                            -680.4
Df Model:                          13                                         
Covariance Type:            nonrobust                                         
===================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------
const               0.0162      0.021      0.768      0.443      -0.025       0.058
area                0.3005      0.039      7.795      0.000       0.225       0.376
bedrooms            0.0467      0.037      1.267      0.206      -0.026       0.119
bathrooms           0.2862      0.033      8.679      0.000       0.221       0.351
stories             0.1085      0.019      5.661      0.000       0.071       0.146
mainroad            0.0504      0.014      3.520      0.000       0.022       0.079
guestroom           0.0304      0.014      2.233      0.026       0.004       0.057
basement            0.0216      0.011      1.943      0.053      -0.000       0.043
hotwaterheating     0.0849      0.022      3.934      0.000       0.042       0.127
airconditioning     0.0669      0.011      5.899      0.000       0.045       0.089
parking             0.0607      0.018      3.365      0.001       0.025       0.096
prefarea            0.0594      0.012      5.040      0.000       0.036       0.083
semi-furnished      0.0009      0.012      0.078      0.938      -0.022       0.024
unfurnished        -0.0310      0.013     -2.440      0.015      -0.056      -0.006
==============================================================================
Omnibus:                       93.687   Durbin-Watson:                   2.093
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              304.917
Skew:                           1.091   Prob(JB):                     6.14e-67
Kurtosis:                       6.801   Cond. No.                         15.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
ified.
```

In [33]:

```
# Calculate VIF
vif = pd.DataFrame()
vif['Features'] = X_train.columns
vif['VIF'] = [variance_inflation_factor(X_train.values, i) for i in range(X_train.sh
vif = vif.sort_values(by='VIF', ascending=False)
print(vif)
```

```
           Features       VIF
1          bedrooms  7.373262
4          mainroad  6.085302
0              area  5.010193
3           stories  2.701669
11   semi-furnished  2.187446
9           parking  2.121678
6          basement  2.014989
12      unfurnished  1.823967
8   airconditioning  1.771997
2         bathrooms  1.666097
10         prefarea  1.506383
5         guestroom  1.468821
7   hotwaterheating  1.135913
```

In [34]:

```
# Drop high VIF or high p-value variables and rebuild model
X_train = X_train.drop(['bedrooms', 'semi-furnished'], axis=1)
X_train_sm = sm.add_constant(X_train)
lr_model = sm.OLS(y_train, X_train_sm).fit()
print(lr_model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.680
Model:                            OLS   Adj. R-squared:                  0.671
Method:                 Least Squares   F-statistic:                     71.31
Date:                Mon, 30 Jun 2025   Prob (F-statistic):           2.73e-84
Time:                        20:06:54   Log-Likelihood:                 380.96
No. Observations:                 381   AIC:                            -737.9
Df Residuals:                     369   BIC:                            -690.6
Df Model:                          11
Covariance Type:            nonrobust
===================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------
const               0.0319      0.015      2.142      0.033       0.003       0.061
area                0.3006      0.038      7.851      0.000       0.225       0.376
bathrooms           0.2947      0.032      9.132      0.000       0.231       0.358
stories             0.1178      0.018      6.654      0.000       0.083       0.153
mainroad            0.0488      0.014      3.423      0.001       0.021       0.077
guestroom           0.0301      0.014      2.211      0.028       0.003       0.057
basement            0.0239      0.011      2.183      0.030       0.002       0.045
hotwaterheating     0.0864      0.022      4.014      0.000       0.044       0.129
airconditioning     0.0665      0.011      5.895      0.000       0.044       0.089
parking             0.0629      0.018      3.501      0.001       0.028       0.098
prefarea            0.0596      0.012      5.061      0.000       0.036       0.083
unfurnished        -0.0323      0.010     -3.169      0.002      -0.052      -0.012
==============================================================================
Omnibus:                       97.661   Durbin-Watson:                   2.097
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              325.388
Skew:                           1.130   Prob(JB):                     2.20e-71
Kurtosis:                       6.923   Cond. No.                         13.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
ified.
```

In [35]:

```python
vif = pd.DataFrame()
vif['Features'] = X_train.columns
vif['VIF'] = [variance_inflation_factor(X_train.values, i) for i in range(X_train.sh
vif = vif.sort_values(by='VIF', ascending=False)
print(vif)
```

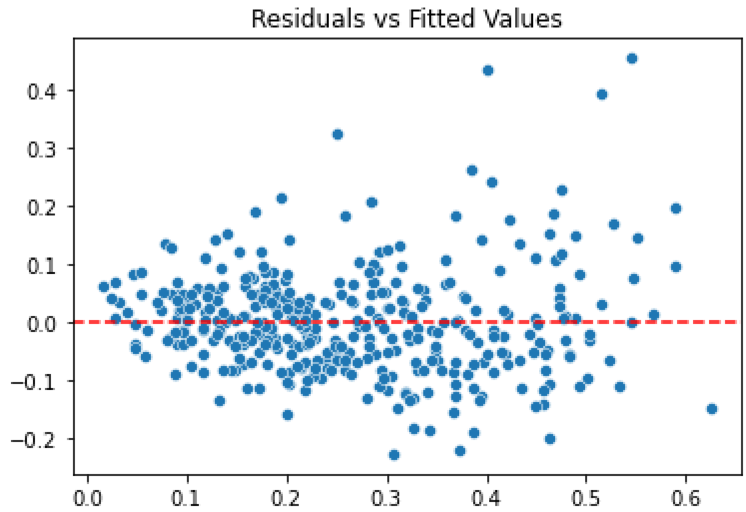
```
           Features       VIF
3          mainroad  4.920987
0              area  4.842720
2           stories  2.226136
8           parking  2.104392
5          basement  1.872522
7   airconditioning  1.766005
1         bathrooms  1.609070
9          prefarea  1.502837
4         guestroom  1.462664
10      unfurnished  1.334898
6   hotwaterheating  1.125136
```

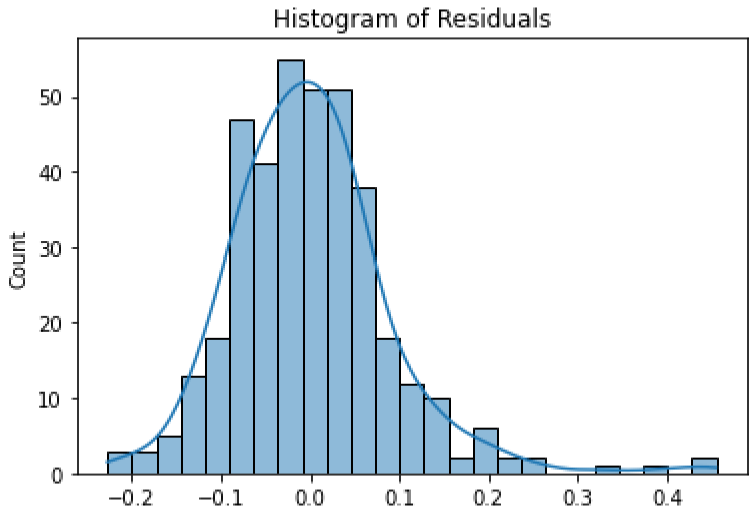
Step 4: Residual Analysis

In [36]:

```python
# Residual analysis
y_train_pred = lr_model.predict(X_train_sm)
res = y_train - y_train_pred
sns.scatterplot(x=y_train_pred, y=res)
plt.axhline(0, color='red', linestyle='--')
plt.title('Residuals vs Fitted Values')
plt.show()
sns.histplot(res, kde=True)
```

```
plt.title('Histogram of Residuals')
plt.show()
```





In [37]:

```
# R² score on train
print('Train R²:', r2_score(y_train, y_train_pred))
```

```
Train R²: 0.6800930630265903
```

Prediction Evaluation on the Test Set

In [38]:

```
# Prepare test set
y_test = df_test.pop('price')
X_test = df_test.drop(['bedrooms', 'semi-furnished'], axis=1)
X_test_sm = sm.add_constant(X_test)
X_test_sm = X_test_sm[lr_model.model.exog_names]
```

In [39]:

```
# Predict and evaluate on test
y_test_pred = lr_model.predict(X_test_sm)
print('Test R²:', r2_score(y_test, y_test_pred))
```

```
Test R²: 0.6713505684480789
```

Multiple Regression with RFE

In [40]:

```
hous = pd.read_csv('Housing.csv')
hous.head()
```

Out[40]:

	price	area	bedrooms	bathrooms	stories	mainroad	guestroom	basement	hotwaterheatin
0	13300000	7420	4	2	3	yes	no	no	n
1	12250000	8960	4	4	4	yes	no	no	n
2	12250000	9960	3	2	2	yes	no	yes	n
3	12215000	7500	4	2	2	yes	no	yes	n
4	11410000	7420	4	1	2	yes	yes	yes	n

In [41]:

```
from sklearn.feature_selection import RFE
from sklearn.linear_model import LinearRegression
```

In [42]:

```
lm = LinearRegression()
```

In [43]:

```
rfe = RFE(estimator=lm, n_features_to_select=8)
rfe.fit(X_train, y_train)
```

Out[43]:

```
RFE(estimator=LinearRegression(), n_features_to_select=8)
```

In [44]:

```
top_features = X_train.columns[rfe.support_]
print("Selected Features by RFE:", top_features.tolist())
```

```
Selected Features by RFE: ['area', 'bathrooms', 'stories', 'mainroad', 'hotwaterheati
ng', 'airconditioning', 'parking', 'prefarea']
```

In [45]:

```
X_train_rfe = X_train[top_features]
X_train_rfe_sm = sm.add_constant(X_train_rfe)
lr_model_rfe = sm.OLS(y_train, X_train_rfe_sm).fit()
print(lr_model_rfe.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.658
Model:                            OLS   Adj. R-squared:                  0.651
Method:                 Least Squares   F-statistic:                     89.63
Date:                Mon, 30 Jun 2025   Prob (F-statistic):           5.43e-82
Time:                        20:07:05   Log-Likelihood:                 368.46
No. Observations:                 381   AIC:                            -718.9
Df Residuals:                     372   BIC:                            -683.4
Df Model:                           8
Covariance Type:            nonrobust
===================================================================================
                      coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------
const               0.0207      0.014      1.499      0.135      -0.006       0.048
area                0.3135      0.039      7.995      0.000       0.236       0.391
bathrooms           0.3216      0.033      9.838      0.000       0.257       0.386
stories             0.1087      0.018      6.167      0.000       0.074       0.143
mainroad            0.0565      0.015      3.873      0.000       0.028       0.085
hotwaterheating     0.0943      0.022      4.269      0.000       0.051       0.138
airconditioning     0.0741      0.012      6.438      0.000       0.051       0.097
parking             0.0614      0.018      3.345      0.001       0.025       0.097
prefarea            0.0693      0.012      5.829      0.000       0.046       0.093
==============================================================================
Omnibus:                       92.306   Durbin-Watson:                   2.127
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              310.202
Skew:                           1.062   Prob(JB):                     4.37e-68
Kurtosis:                       6.876   Cond. No.                         12.5
==============================================================================

Notes:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
ified.
```

In [46]:

```python
vif_rfe = pd.DataFrame()
vif_rfe['Features'] = X_train_rfe.columns
vif_rfe['VIF'] = [variance_inflation_factor(X_train_rfe.values, i) for i in range(X_
vif_rfe = vif_rfe.sort_values(by='VIF', ascending=False)
print(vif_rfe)
```

```
          Features       VIF
0             area  4.753624
3         mainroad  4.384834
2          stories  2.119970
6          parking  2.075213
5  airconditioning  1.735078
1        bathrooms  1.558539
7         prefarea  1.446108
4  hotwaterheating  1.120494
```

In [47]:

```python
df_train, df_test = train_test_split(hous, train_size=0.7, random_state=100)
df_test[num_vars] = scaler.transform(df_test[num_vars])  # re-scale
y_test = df_test['price']
X_test_rfe = df_test[top_features]
X_test_rfe_sm = sm.add_constant(X_test_rfe)
```

In [48]:

```python
y_train_pred_rfe = lr_model_rfe.predict(X_train_rfe_sm)
print("Train R² with RFE-selected features:", r2_score(y_train, y_train_pred_rfe))
```

```
Train R² with RFE-selected features: 0.6584118638839078
```

In this notebook, I developed and compared multiple linear regression models using both manual feature selection and RFE (Recursive Feature Elimination). Initially, I cleaned the dataset by encoding categorical variables and scaling numeric ones. I built a manual model by iteratively removing features with high p-values and high VIFs to ensure both statistical significance and low multicollinearity. This model achieved a Train R^2 of 0.68 and a Test R^2 of 0.67, with all predictors being interpretable and statistically sound.

To validate and possibly refine the selection, I implemented RFE to automatically select the top 8 predictive features. The resulting RFE model produced a Train R^2 of 0.658, slightly lower than the manual model. However, the RFE approach offered simplicity and speed, confirming the importance of core features like area, bathrooms, stories, and airconditioning.

Ultimately, the manual model offered better interpretability and slightly better performance, making it the more suitable choice in this case. This project demonstrates a balanced approach to model building—combining statistical rigor with machine-driven optimization—ensuring both robustness and explainability in predictive modeling.

In [ ]: