☆ Exclusive-OR & Equivalence fn

- denoted by $\oplus$ & $\odot$, respectively.

$$x \oplus y = xy' + x'y$$

$$x \odot y = xy + x'y'$$

- two operations are complements of each other.
- both are commutative & associative.

i.e $(A \oplus B) \oplus C = A \oplus (B \oplus C)$
$= A \oplus B \oplus C$

- useful in arithmetic operation & error detection & correction.

- n-variable ex-OR is having $2^n/2$ minterms with binary numbers have an odd number of 1's.

- n-variable ex-NOR is having $2^n/2$ minterms with binary no. have an even number of 0's.

- 2 - input X-OR & X-NOR gate

- XOR

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

$$\boxed{A \oplus B = A'B + AB'}$$

- $2^n/2$ minterms have odd no of 1's

- $2^2/2 = 4/2 = 2$ terms.

- X-NOR

A	B	$A \odot B$
0	0	1
0	1	0
1	0	0
1	1	1

$A \odot B = AB + A'B'$

- $2^n/2$ minterms have even no of. 0's

- X

$2^2/2 = 4/2 = 2$ terms

Note :-
* when the no of input n is even the ex-OR & ex-NOR are complement of each other
* when n is odd both are same.

ex-OR , i/s $n=3$

A	B	C	$A \oplus B \oplus C$	$A \odot B \odot C$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$C \oplus A \oplus B = \Sigma(1,2,4,7)$

$C \odot A \odot B = \Sigma(1,2,4,7)$

A \ BC	00	01	11	10
0		1		1
1	1		1	

$\boxed{\text{Ex-OR} - \text{odd nos of 1's}}$

$\boxed{\text{Ex-NOR} - \text{even nos of 0's}}$

$\boxed{A \oplus B \oplus C = A \odot B \odot C}$

- when the minterms of a f^n with an odd no of variables have even no of 1's (or, odd nos of 0's), the f^n can be expressed as the complement of either an ex-OR or an ex-NOR.

i.e $(A \oplus B \oplus C)' = A \oplus B \odot C$

$(A \odot B \odot C)' = A \odot B \oplus C$

A \ BC	00	01	11	10
0	1		1	
1		1		1

$= A'B'C' + A'BC + AB'C + ABC'$

$= A'(B'C' + BC) + A(B'C + BC')$

$= A'(B \oplus C)' + A(B \oplus C)$

$F = A \oplus B \odot C$

$= A \odot B \oplus C$

$\boxed{F = A \odot (B \oplus C)}$

- EX-OR & EX-NOR (for - 4 inputs)

- EX-OR

AB \ CD	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

$F = A \oplus B \oplus C \oplus D$

= odd no of 1's

- EX-NOR

AB \ CD	00	01	11	10
00	1		1	
01		1		1
11	1		1	
10		1		1

$F = A \odot B \odot C \odot D$

= even no of 0's.

* Parity generator & Checker

→ Parity generator

* The circuit that generates the parity bit in the transmitter is called a parity generator

* 3-bit parity generator

→ odd parity

x	y	z	parity bit generated (P)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$P = \Sigma(0, 3, 5, 6)$

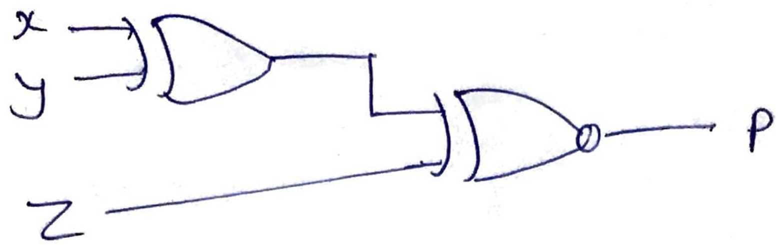
x \ yz	00	01	11	10
0	1		1	
1		1		1

- even 1's & odd 0's

$P = x \oplus y \odot z$

or

$P = x \odot y \oplus z$



3-bit odd parity generator

* 4-bit even parity generator

w	x	y	z	even parity bit P
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$\Sigma(1, 2, 4, 7, 8, 11, 13, 14)$

→ odd no 1's

$P = w \oplus x \oplus y \oplus z$

⇒ parity checker

- The circuit that checks the parity in the receiver is called parity checker

⇒ odd parity checker

* 3-bit message with parity bit are transmitted to their destination & applied to a parity checker circuit.

* An error occurs during transmission if the parity of the four bits received is even.

parity - error check

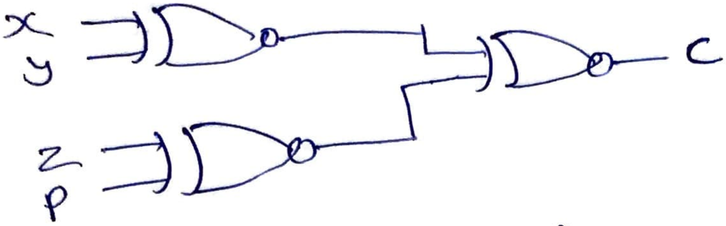
→

X	Y	Z	P	C
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

$C = \Sigma(0, 3, 5, 6, 9, 10, 12, 15)$

$\boxed{\text{seven 0's.}}$

$\boxed{C = X \odot Y \odot Z \odot P}$



4- bit odd parity checker.