b. Heat flows through a uniform bar of length l which has its side insulated and the temperature at the ends kept at zero. If the initial temperature at the interior points of the bar is given by x, $0<x<l$, find the temperature distribution in the bar at time t. | 8 | 3 | 3 | 1,2

24\. a. Find the Fourier transform of $\frac{x}{x^2+a^2}$. | 8 | 3 | 4 | 1,2

(OR)

b. Evaluate $\int_0^\infty \frac{dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$ using Fourier transforms. | 8 | 3 | 4 | 1,2

25\. a. Find $Z^{-1}\left[\frac{z^2+2z}{z^2+2z+4}\right]$ using long division method. | 8 | 2 | 5 | 1,2

(OR)

b. Find the Z-transform of $\frac{1}{(n+1)(n+2)}, n>0$. | 8 | 2 | 5 | 1,2

PART – C (1 × 15 = 15 Marks)
Answer **ANY ONE** Question

Marks | BL | CO | PO

26\. Determine the first 3 harmonics of the fourier series for the following data. | 15 | 4 | 2 | 1,2

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
y	2.34	2.2	1.6	0.83	0.51	0.88	2.34

27\. A rod 30 cm long has its ends A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function $u(x,t)$ taking $x=0$ at A. | 15 | 4 | 3 | 1,2

* * * * *

Reg. No.

B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, MAY 2023
Third Semester

21MAB201T – TRANSFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted from the academic year 2021 - 2022 & 2022 - 2023)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours Max. Marks: 75

PART – A (20 × 1 = 20Marks)
Answer **ALL** Questions

Marks | BL | CO | PO

1\. The complete integral of $z = px + qy + 2pq$ is | 1 | 1 | 1 | 1,2
(A) $z = ax + by + 2ab$ (B) $z = a(x+y) + 2ba$
(C) $z = ax + by + 2c$ (D) $z = ax - by + a$

2\. The complementary function of $\left(D^2 + DD' - 2D'^2\right)z = x^2y$ | 1 | 1 | 1 | 1,2
(A) $z = \phi_1(y-x) + \phi_2(y-2x)$ (B) $z = \phi_1(y+x) + \phi_2(y+2x)$
(C) $z = \phi_1(y-x) + \phi_2(y+2x)$ (D) $z = \phi_1(y+x) + \phi_2(y-2x)$

3\. The particular integral of $\left(D^2 - 2DD'\right)z = e^{2x}$ | 1 | 1 | 1 | 1,2
(A) $e^{-2x}/2$ (B) $xe^{-2x}/4$
(C) $x^2e^{2x}/2$ (D) $e^{2x}/4$

4\. The complete integral of $\sqrt{p} + \sqrt{q} = 1$ | 1 | 1 | 1 | 1,2
(A) $z = ax + by$ (B) $z = a(x+y) + b$
(C) $z = ax + \left(1-\sqrt{a}\right)^2 y + c$ (D) $z = ax - by + a$

5\. $\tan x$ is periodic function with period | 1 | 1 | 2 | 1,2
(A) π (B) $\pi/2$
(C) 2π (D) 4π

6\. Which of the following is an even function? | 1 | 1 | 2 | 1,2
(A) $\sin x$ (B) x
(C) e^x (D) x^2

7\. The RMS value of $f(x) = x$ in $-1 \le x \le 1$ is | 1 | 1 | 2 | 1,2
(A) 1 (B) 0
(C) $\frac{1}{\sqrt{3}}$ (D) -1

8. If $f(x)$ is discontinuous at $x=a$, then the fourier series at $x=a$ is 1 1 2 1,2

(A) $\left[f(a^-)-f(a^+)\right]/2$ (B) $f(a^-)-f(a^+)$
(C) $\left[f(a^-)-f(a^+)\right]/3$ (D) $\left[f(a^-)+f(a^+)\right]/2$

9. One dimensional heat equation is used to find 1 1 3 1,2

(A) Density (B) Temperature
(C) Time (D) Displacement

10. How many initial and boundary conditions are required to solve $\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ 1 1 3 1,2

(A) Two (B) Three
(C) Five (D) Four

11. One dimensional wave equation is 1 1 3 1,2

(A) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (B) $\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2}$
(C) $\frac{\partial u}{\partial t}=\alpha\frac{\partial^2 u}{\partial x^2}$ (D) $\frac{\partial^2 u}{\partial x^2}=\alpha\frac{\partial^2 u}{\partial t^2}$

12. Heat flows from _______temperature. 1 1 3 1,2

(A) Higher to lower (B) Uniform
(C) Lower to higher (D) Stable

13. The Fourier transform of a function $f(x)$ is 1 1 4 1,2

(A) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{ist}dt$ (B) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$
(C) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{isx}dx$ (D) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(s)e^{isx}dx$

14. Fourier sine transform of $f(x)$ is 1 1 4 1,2

(A) $\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(s)\sin sx\,dx$ (B) $\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin sx\,dx$
(C) $\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin x\,dx$ (D) $\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos sx\,dx$

15. $F\left[f(x-a)\right]=$ 1 1 4 1,2

(A) $e^{ias}F(a)$ (B) $e^{iax}F(x)$
(C) $e^{iax}F(a)$ (D) $e^{ias}F(s)$

16. $F^{-1}\left[F(s).G(s)\right]=$ 1 1 4 1,2

(A) $f(x).g(x)$ (B) $f(x)+g(x)$
(C) $f(x)*g(x)$ (D) $f(x)-g(x)$

17. $Z^{-1}\left[\frac{1}{z-a}\right]=$ 1 1 5 1,2

(A) na^{n-1} (B) a^{n+1}
(C) a^{n-1} (D) na^n

18. $Z\left[f(n)*g(n)\right]=$ 1 1 5 1,2

(A) $F(z)G^{-1}(z)$ (B) $F^{-1}(z)G^{-1}(z)$
(C) $F(z).G(z)$ (D) $F^{-1}(z).G(z)$

19. $Z^{-1}\left[\frac{z}{z-a}\right]$ 1 1 5 1,2

(A) a^{n+1} (B) a
(C) a^n (D) a^{n-1}

20. The poles of $\phi(z)=\frac{z^n}{(z-1)(z-2)}$ are 1 1 5 1,2

(A) $z=1, z=0$ (B) $z=1, z=2$
(C) $z=0, z=2$ (D) $z=0$

PART – B (5 × 8 = 40 Marks)
Answer ALL Questions

		Marks	BL	CO	PO
21. a.	Solve $x(y-z)p+y(z-x)q=z(x-y)$.	8	2	1	1,2
	(OR)				
b.	Solve $\left(D^3-7DD'^2-6D'^3\right)z=\sin(x+2y)$.	8	2	1	1,2
22. a.	Express $f(x)=(\pi-x)^2$ as a Fourier series in $0\le x\le 2\pi$.	8	3	2	1,2
	(OR)				
b.	Find the half range sine series of $f(x)=kx(x-l)$ in $0\le x\le l$.	8	3	2	1,2
23. a.	A tightly stretched string of length 'l' has its ends fastened at $x=0$, $x=l$. At $t=0$, the string is in the form $f(x)=x^3$ and then released. Find the displacement at any point on the string at a distance 'x' from one end and at any time $t>0$.	8	3	3	1,2
	(OR)				