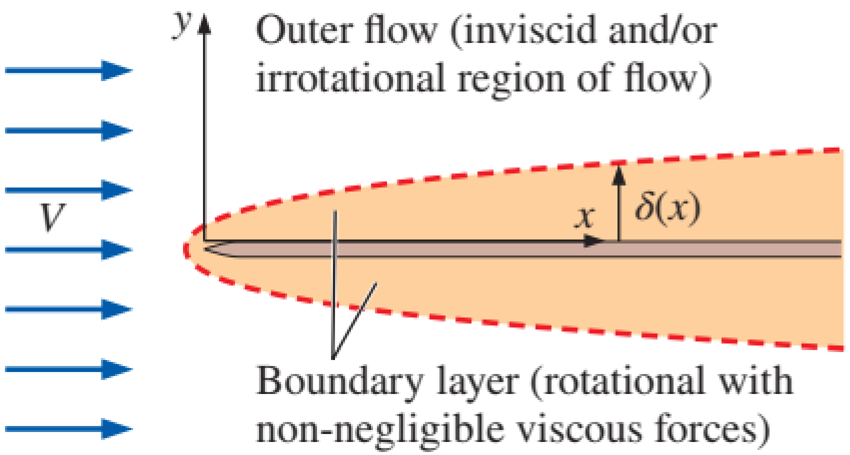
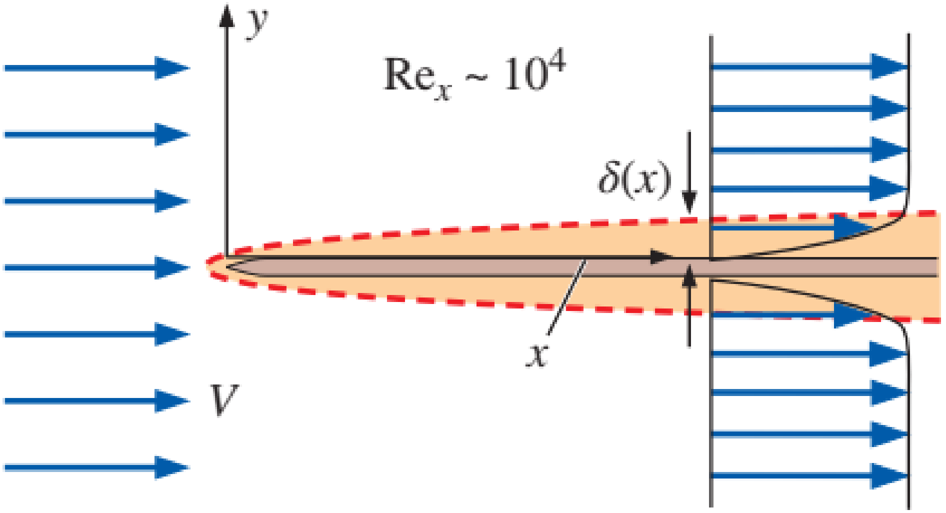
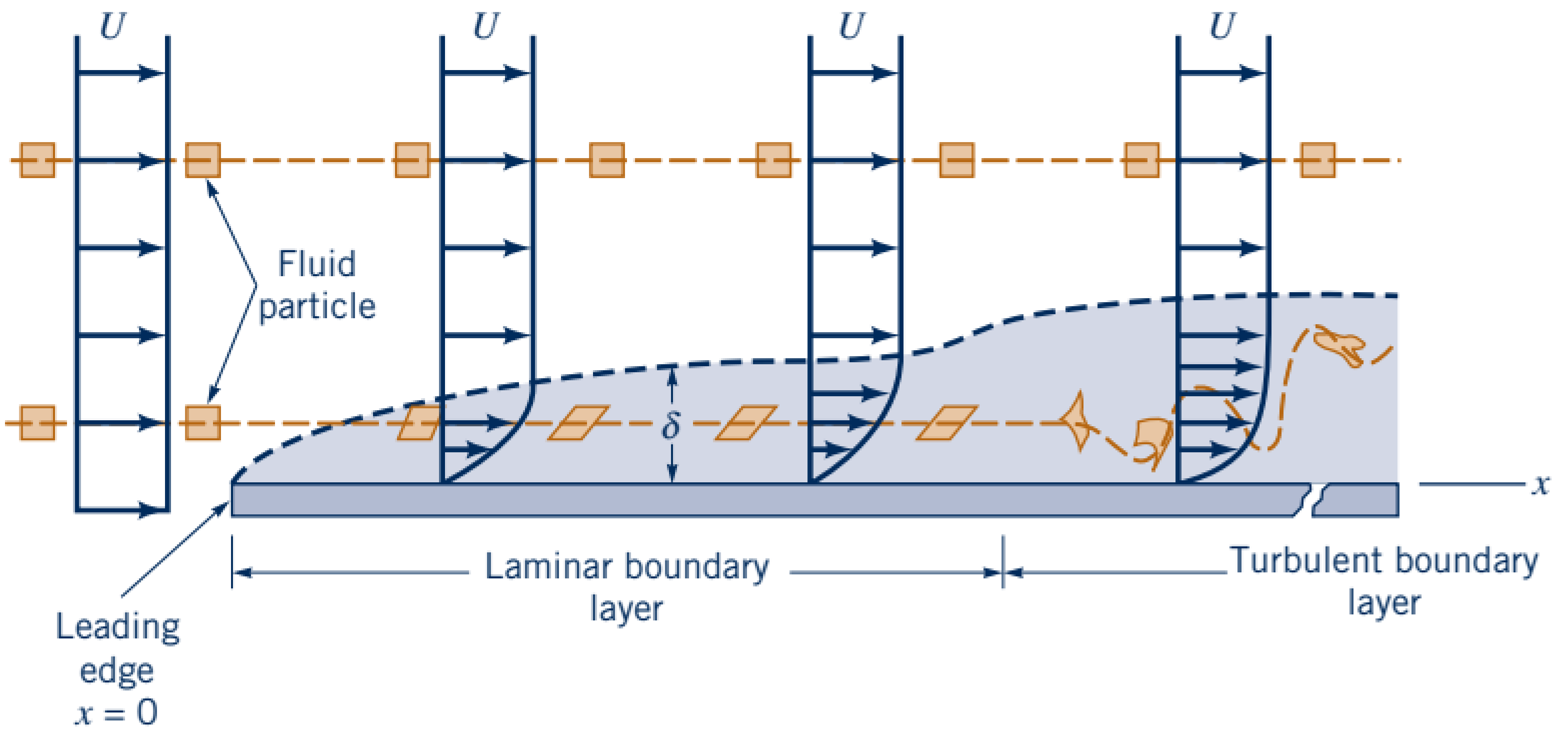
Boundary Layer Theory

- The concept of boundary layer was first introduced by a German engineer, Prandtl in 1904.
- According to Prandtl theory, when a real fluid flows past a stationary solid boundary, the flow will be divided into two regions.
  a) A thin layer adjoining the solid boundary where the viscous force and rotation cannot be neglected.
  b) An outer region where the viscous force is very small and can be neglected. The flow behavior is similar to the upstream flow.





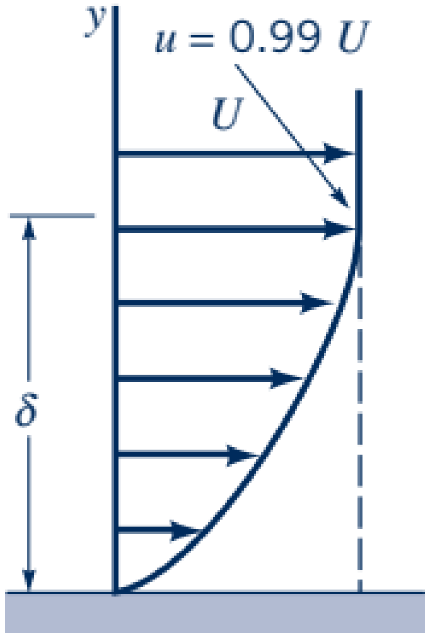
Boundary Layer Definitions and Characteristics



- Inside the boundary layer *the viscous force effect is* significant. Because of fluid viscosity, the first layer of fluid sticks to the solid surface – No Slip Condition.
- This retarded layer causes further retardation for the adjacent layer, thereby developing a thin region where the flow velocity increases from zero at the solid boundary and approaches the velocity of the main stream.

Boundary layer thickness, δ

The boundary layer thickness (δ) is defined as the distance from the boundary of solid body measured in y direction to the point where the flow velocity is approximately 99 percent of the velocity of the free stream.



$$\delta = y \qquad \text{where} \qquad u = 0.99U$$

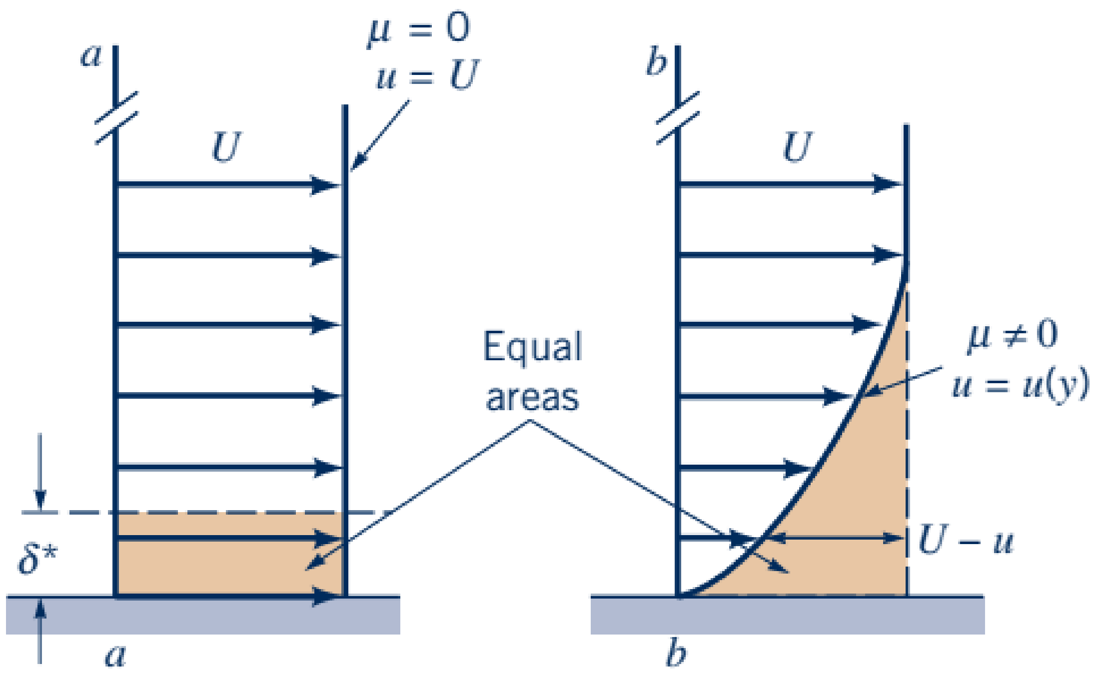
The other definitions of boundary layer are the

- Boundary layer displacement thickness, δ^*
- Boundary layer momentum thickness, θ
- Boundary layer energy thickness, δ^{**}

Boundary Layer Displacement Thickness, δ*

It is defined as the distance, measured perpendicular to the boundary of solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.

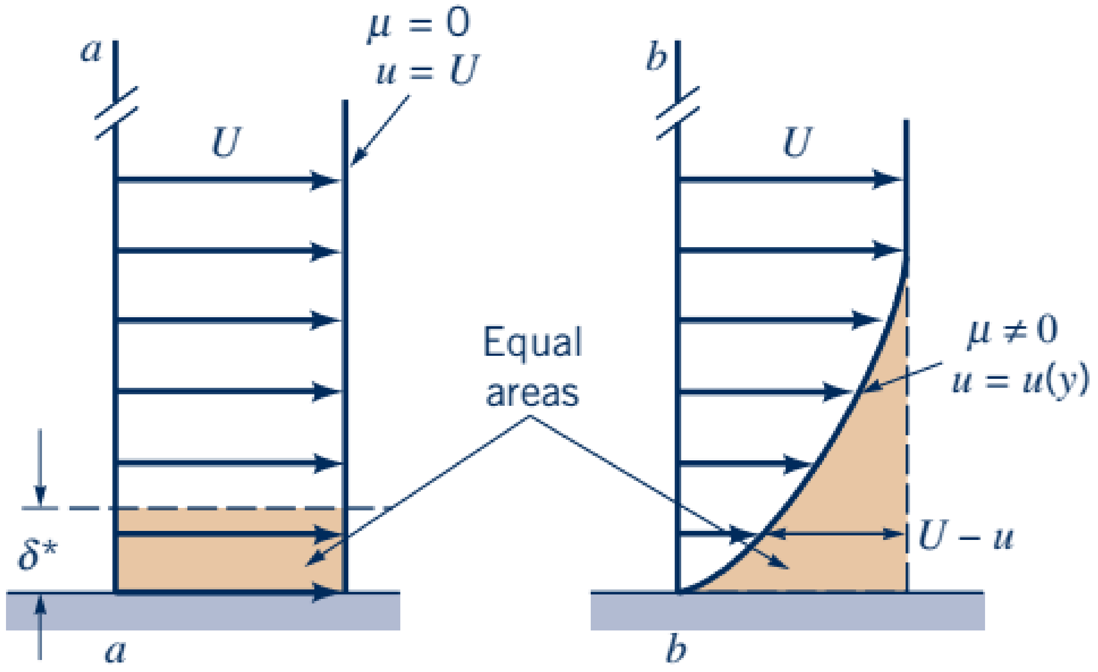
$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$



Because of the velocity deficit within the boundary layer, the flowrate across section *b–b* is less than that across section *a–a.* However, if we displace the plate at section *a–a* by an appropriate amount the *boundary layer displacement thickness*, the flowrates across each section will be identical

Boundary Layer Displacement Thickness, δ*

It is defined as the distance, measured perpendicular to the boundary of solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.



$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

Boundary Layer Momentum Thickness, θ

It is defined as the distance, measured perpendicular to the boundary of solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.

$$\boxed{\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$$

Boundary Layer Energy Thickness, δ**

It is defined as the distance, measured perpendicular to the boundary of solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation.

$$\boxed{\delta^{**} = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)dy}$$