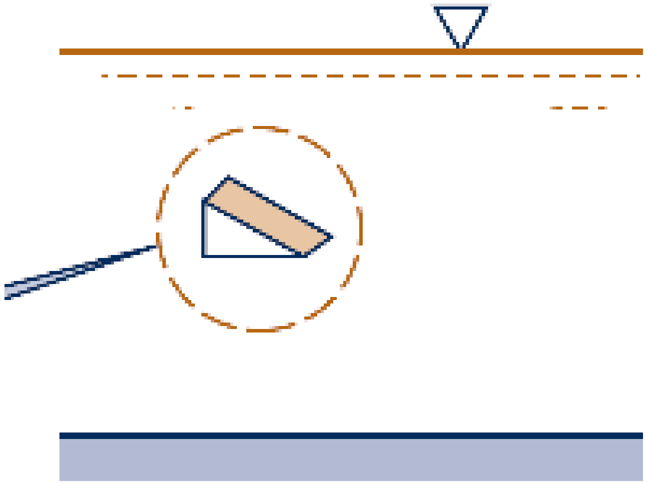
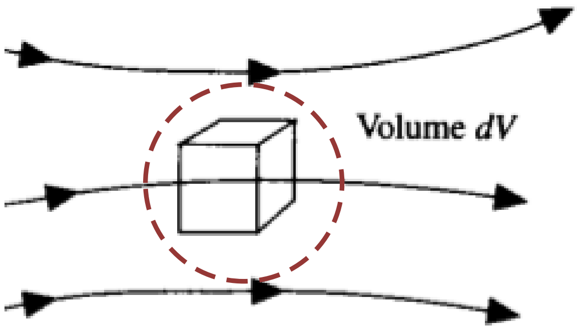
- *Fluid statics*
- *Pascal's law*
- *Hydrostatic law*

Forces on fluid elements

Fluid element:

Fluid element can be defined as an infinitesimal region of the fluid continuum in isolation from its surroundings.

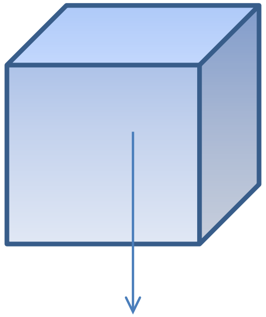




Infinitesimal fluid element fixed in space with the fluid moving through it

Two types of forces exist on fluid elements

1. Body force
2. Surface force



$$W = mg = \rho \forall g$$

Body Force:

It is distributed over the entire mass or volume of the element.

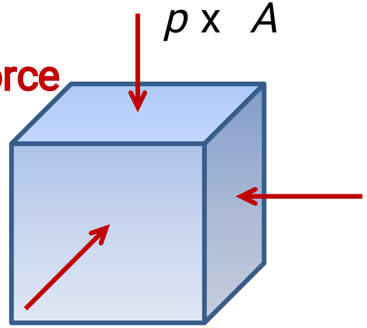
Eg.: Gravitational Force, Electromagnetic force fields etc.

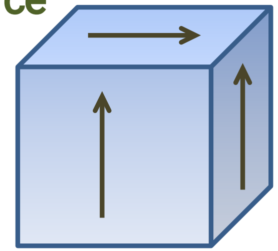
Surface Force:

Forces exerted on the fluid element by its surroundings through direct contact at the surface.

Surface force has two components:

- Normal Force:*along the normal to the area*
- Shear Force:*along the plane of the area.*





Fluid statics

- Fluid is at rest i.e., *Static*

- Fluid is at rest (or) no relative motion between adjacent fluid particles - there will be no shearing stresses in the fluid, i.e. $\tau = 0$

- The only forces that develop on the surfaces of the particles will be due to the pressure.

Important applications

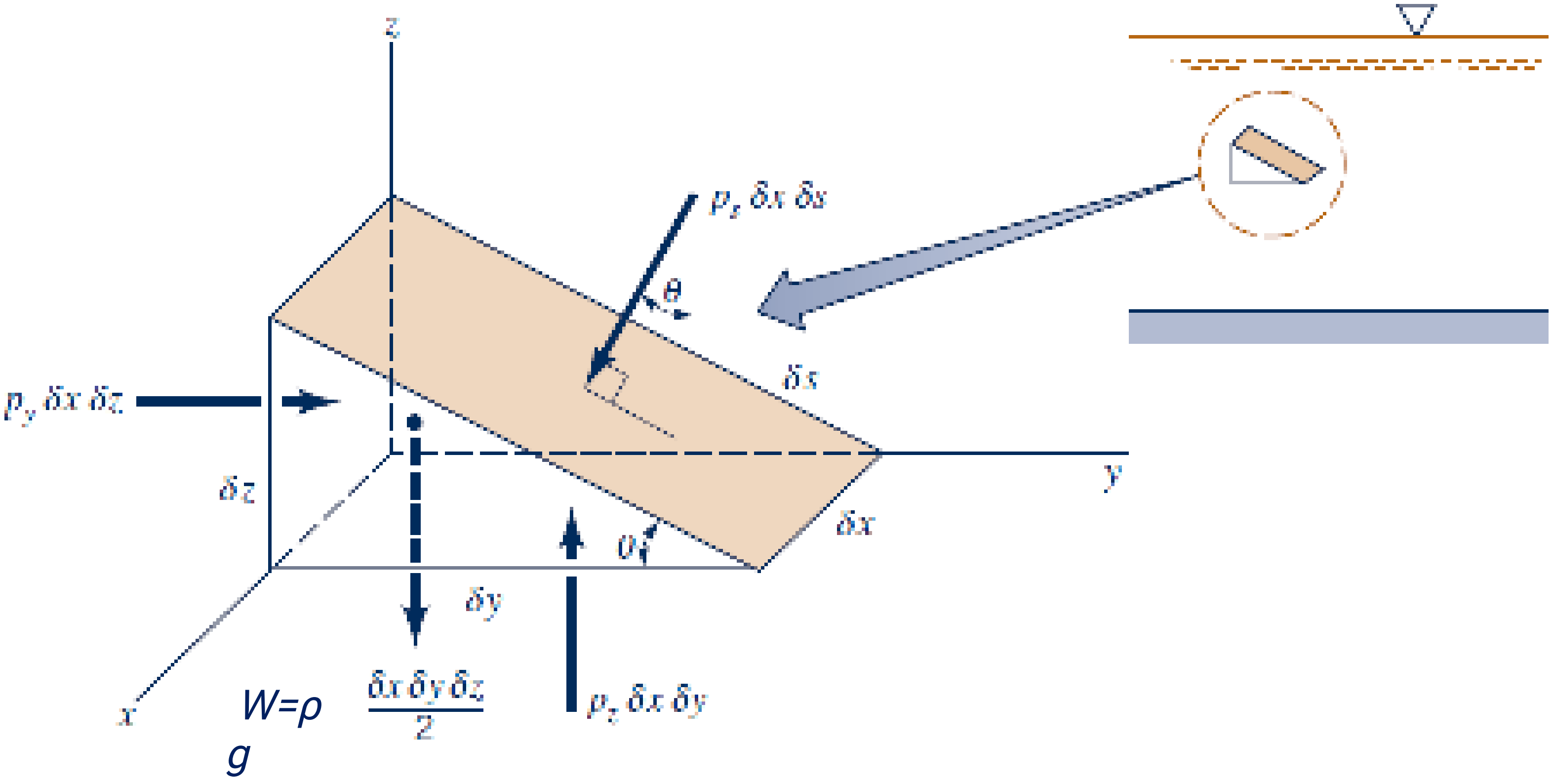
(1) pressure distribution in the atmosphere and the oceans,

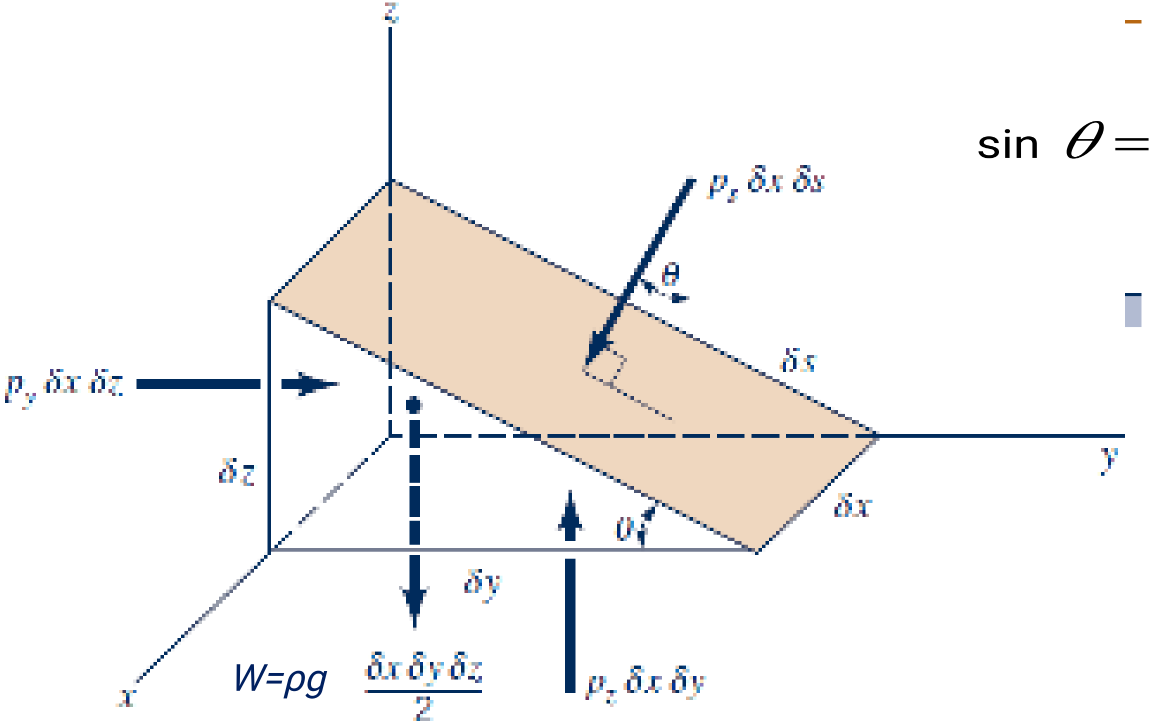
(2) The design of manometer, mechanical, and electronic pressure instruments,

(3) forces on submerged flat and curved surfaces,

Pressure at a point – The Pascal's law

Consider a fluid element. The forces acting on the fluid element are as shown below.



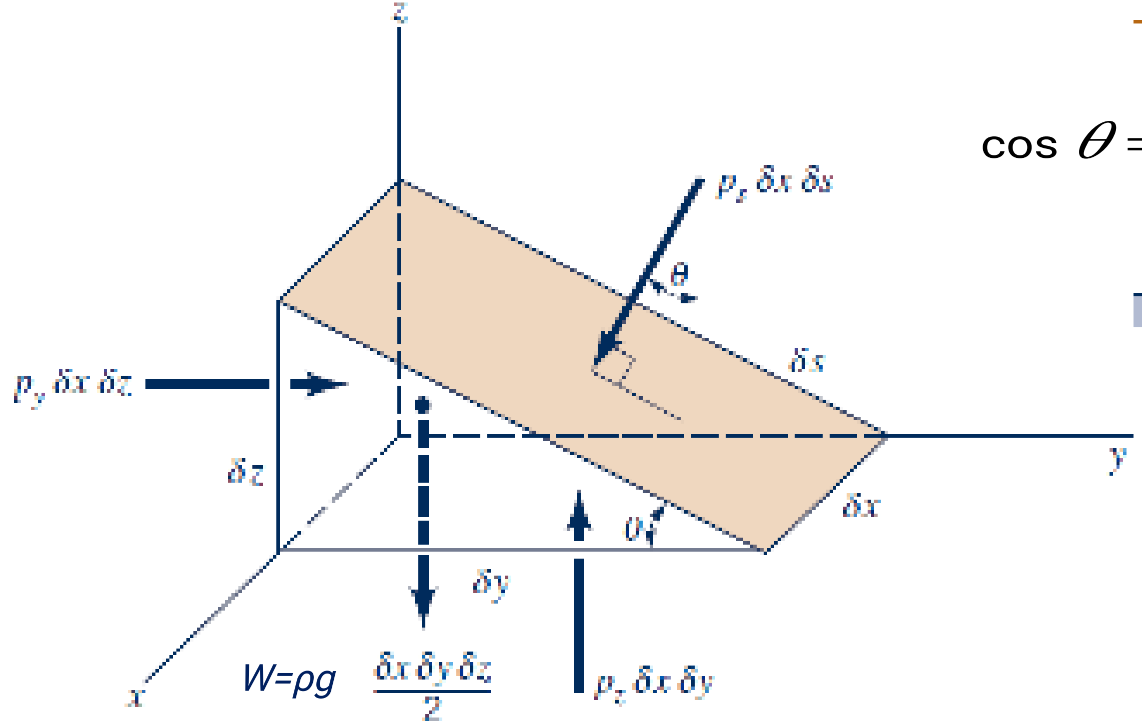


$$\sin\theta = \frac{\delta z}{\delta s}$$

$$\sum F_y = 0$$

$$p_y\,\delta x\,\delta z - p_s\,\delta x(\delta s\sin\theta) = 0$$

$$p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta z = 0 \quad \Rightarrow \quad p_y = p_s$$



$$\cos\theta = \frac{\delta y}{\delta s}$$

$$\sum F_z = 0$$

$$p_z\,\delta x\,\delta y - p_s\,\delta x(\delta s\cos\theta) - \rho g\,\frac{\delta x\,\delta y\,\delta z}{2} = 0$$

$$p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta y - \rho g\,\frac{\delta x\,\delta y\,\delta z}{2} = 0 \quad \Rightarrow p_z = p_s + \rho g\,\frac{\delta z}{2}$$

$$p_y = p_s \qquad\qquad p_z = p_s + \rho g \frac{\delta z}{2}$$

In the limit of $\delta x,\ \delta y,\ \delta z \rightarrow 0$, the fluid element tends to become a point

$$p_y = p_s \qquad\qquad p_z = p_s$$

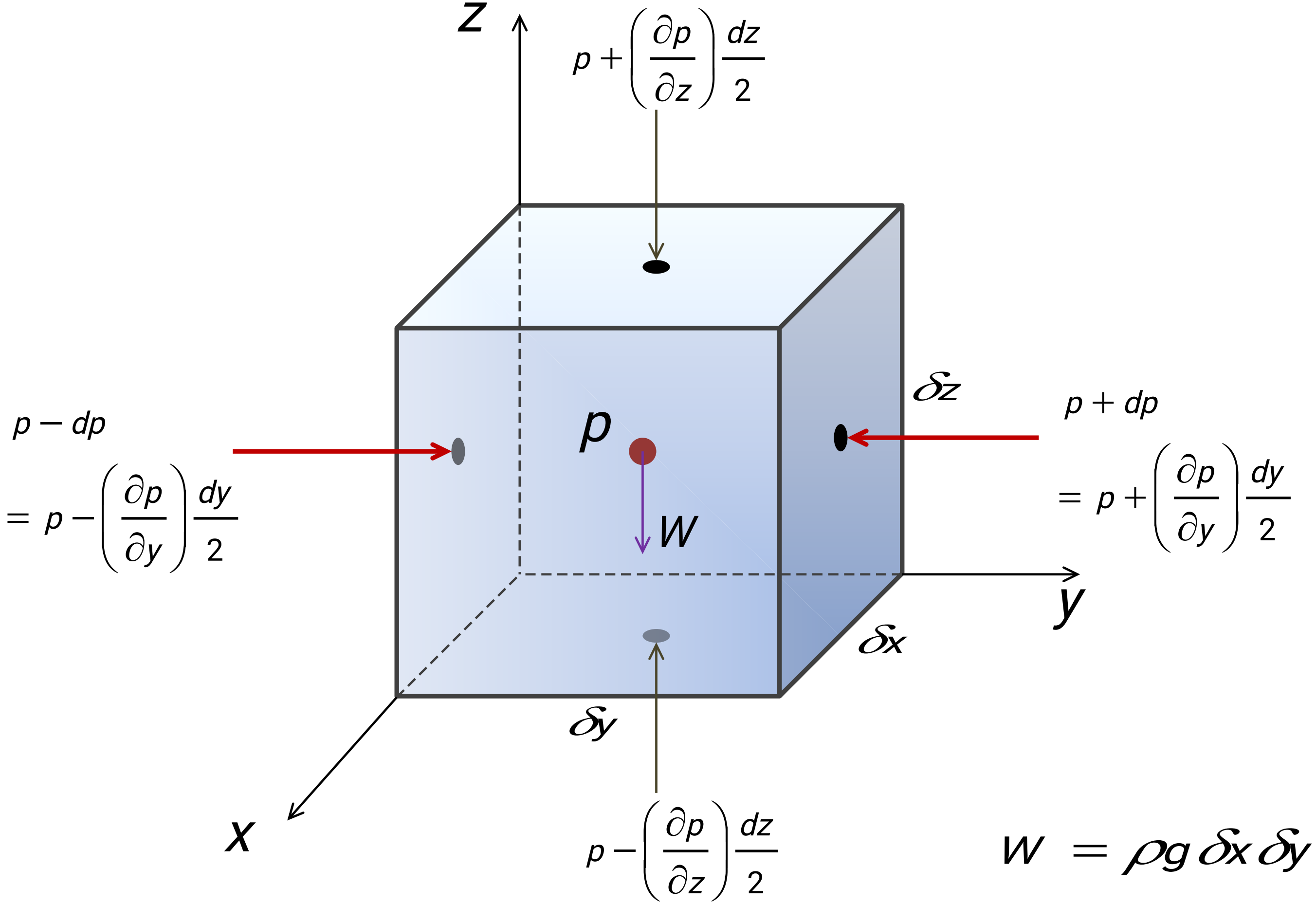
At a point, $\boxed{p_y = p_z = p_s}$

PASCAL'S LAW

**The pressure at a point in a fluid at rest is independent of direction

Pressure Variation in a Fluid at Rest – The Hydrostatic Law

Consider a fluid element as shown



$$W = \rho g\, \delta x\, \delta y\, \delta z$$

The fluid element is in equilibrium at rest

$$\sum F_y = 0$$

$$\left[p - \left(\frac{\partial p}{\partial y}\right)\frac{\delta y}{2}\right]\delta x\,\delta z - \left[p + \left(\frac{\partial p}{\partial y}\right)\frac{\delta y}{2}\right]\delta x\,\delta z = 0$$

$$\Rightarrow \frac{\partial p}{\partial y} = 0$$

Similarly, applying $\sum F_x = 0$

gives $\dfrac{\partial p}{\partial x} = 0$

$$\sum F_z = 0$$

$$\left[p - \left(\frac{\partial p}{\partial z}\right)\frac{\delta z}{2}\right]\delta x\,\delta y - \left[p + \left(\frac{\partial p}{\partial z}\right)\frac{\delta z}{2}\right]\delta x\,\delta y - \rho g\,\delta x\,\delta y\,\delta z = 0$$

$$\Rightarrow \frac{\partial p}{\partial z} = -\rho g$$

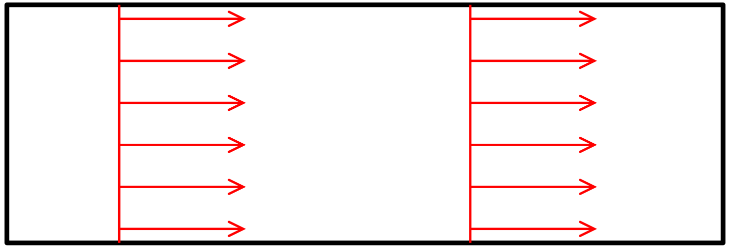
$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\rho g$$

**In a fluid at rest, pressure varies only in vertical direction. It does not vary horizontally.
**Pressure decreases with height

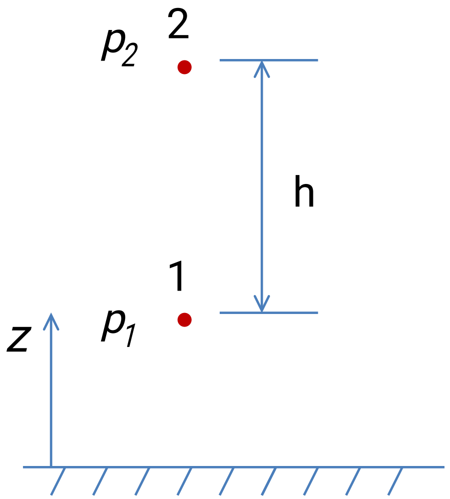
$$\boxed{\frac{dp}{dz} = -\rho g}$$ Hydrostatic Law

Note:

- The Pascal's law and Hydrostatic law haven been derived for fluid element at rest, i.e., $\Sigma F = 0$
- Since, from Newton's second law $\Sigma F = ma$, these laws are applicable for a flow with zero acceleration as well, i.e., flow with a constant velocity.



Relation for pressure variation with height



$$dp = -\rho g dz$$

$$\int_1^2 dp = -g \int_1^2 \rho dz$$

For an incompressible flow, density ρ is constant

$$\int_1^2 dp = -\rho g \int_1^2 dz$$

$$p_2 - p_1 = -\rho g (z_2 - z_1)$$

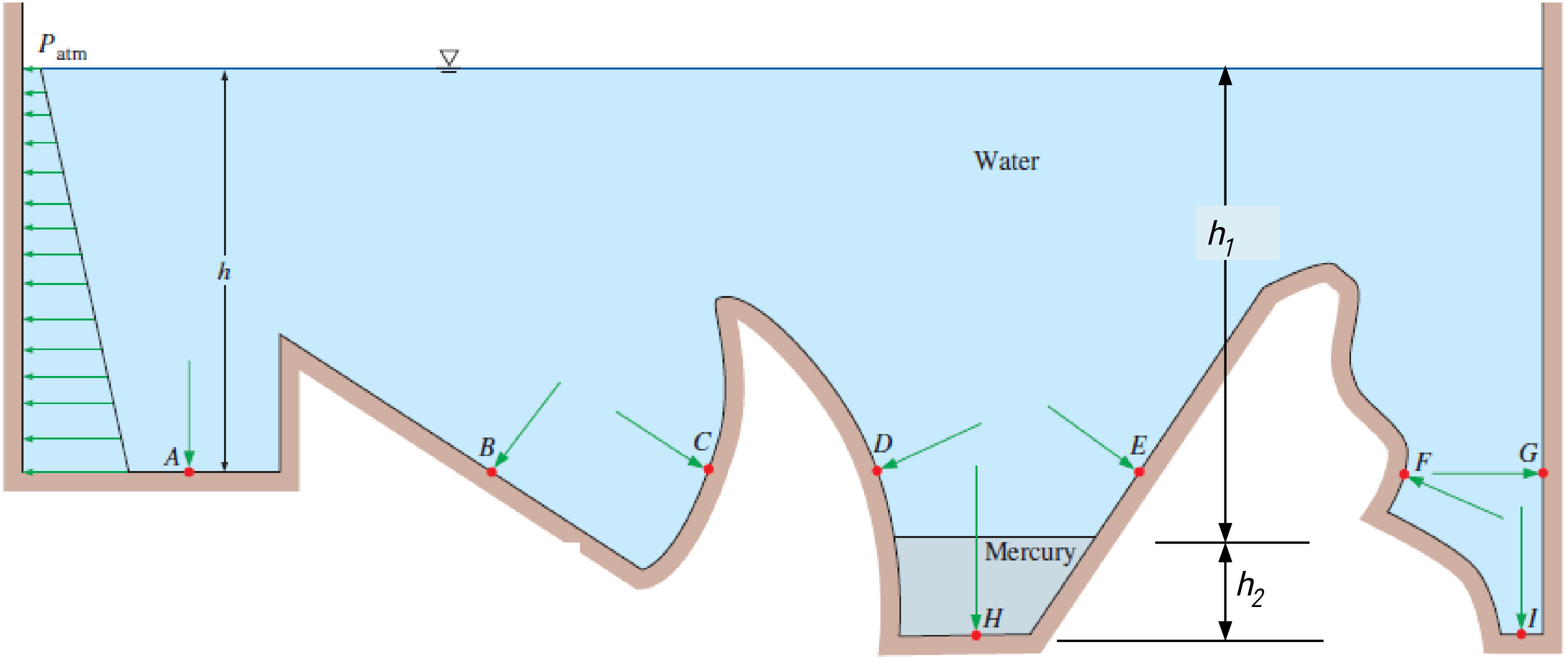
$$\Rightarrow \quad p_2 - p_1 = -\rho g (h)$$

$$\boxed{\begin{aligned} p_2 &= p_1 - \rho g h \\ \text{(or)} \quad p_1 &= p_2 + \rho g h \end{aligned}}$$

**In an incompressible fluid at rest, the pressure varies linearly along vertical direction

- **Pressure increases with depth by** $\rho g h$
- **Pressure decreases with height by** $\rho g h$

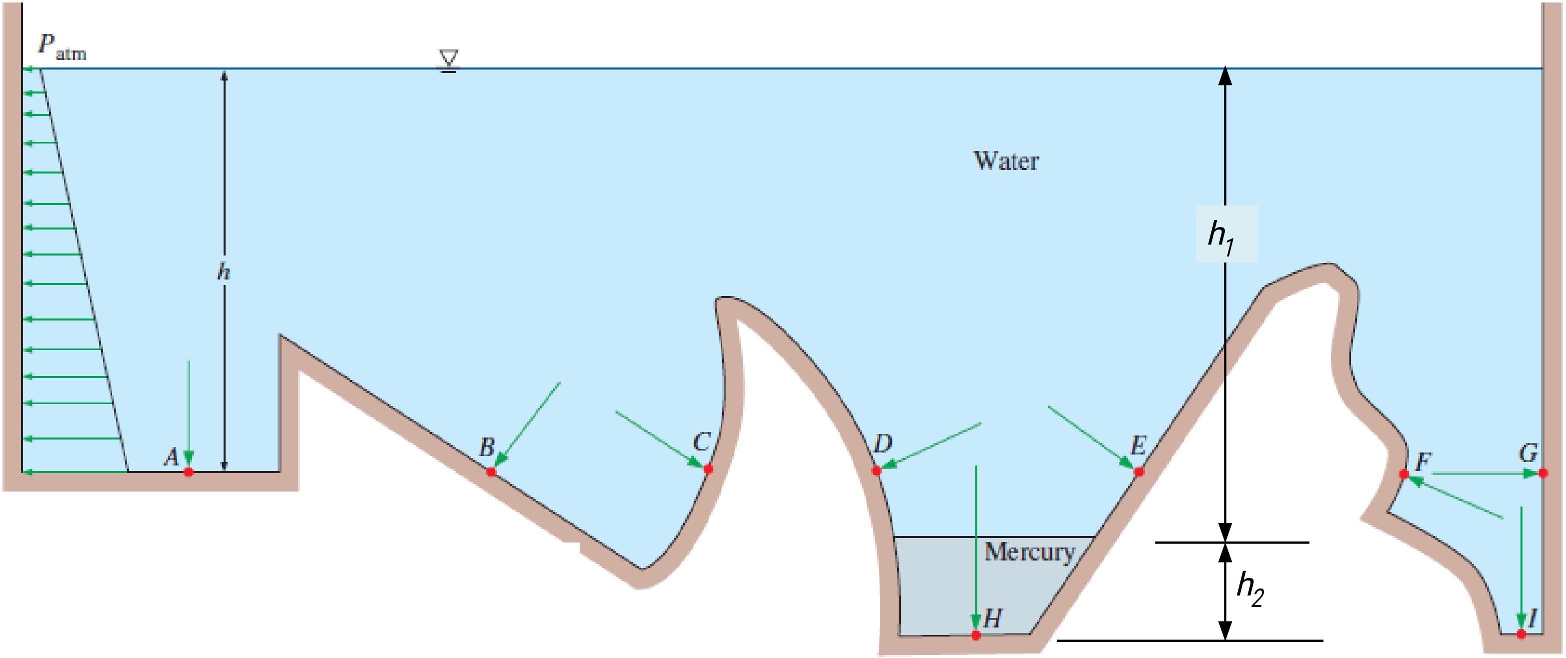
Example Comment on the pressures at points A, B, C, D, E, F, G, H and I



Points A-G are at the same depth (h)and in the same fluid (water)

$$p_A = p_B = p_C = p_D = p_E = p_F = p_G = p_{atm} + \rho_w gh$$

Example Comment on the pressures at points A, B, C, D, E, F, G, H and I



Are pressures at H and I the same ? NO

Because H and I are not in the same fluid. Density is different

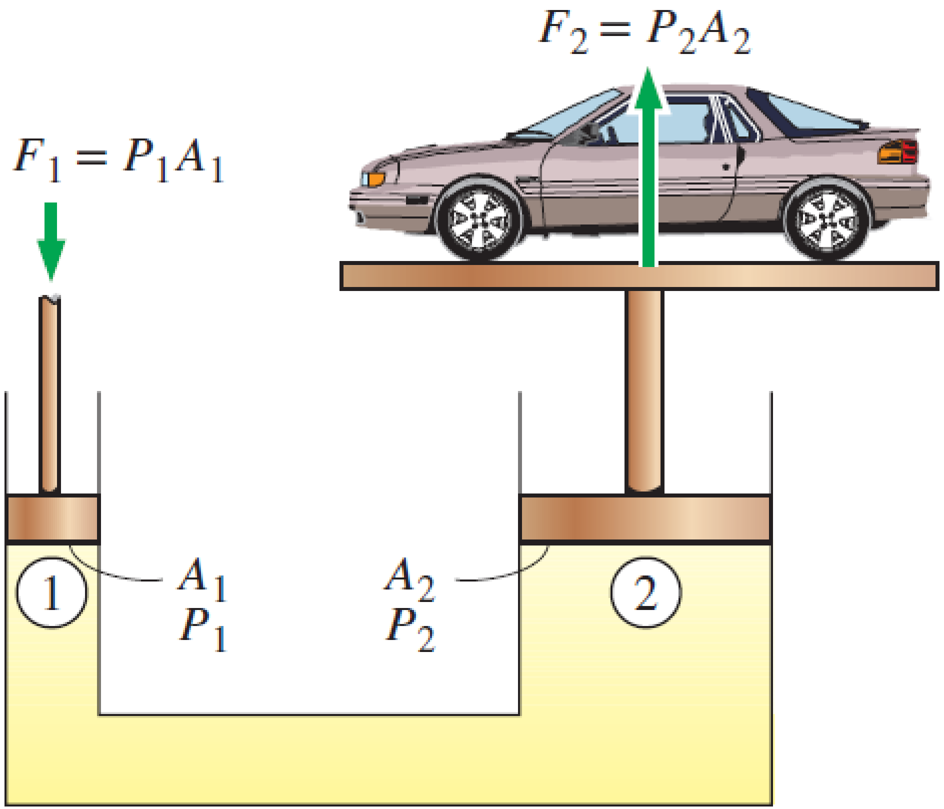
$$p_H \neq p_I$$

$$p_H = p_{atm} + \rho_w g h_1 + \rho_{Hg} g h_2$$

$$p_I = p_{atm} + \rho_w g (h_1 + h_2)$$

The transmission of pressure throughout a stationary fluid is the principle upon which many hydraulic devices are based.

Hydraulic Jack



$$P_1 = P_2$$

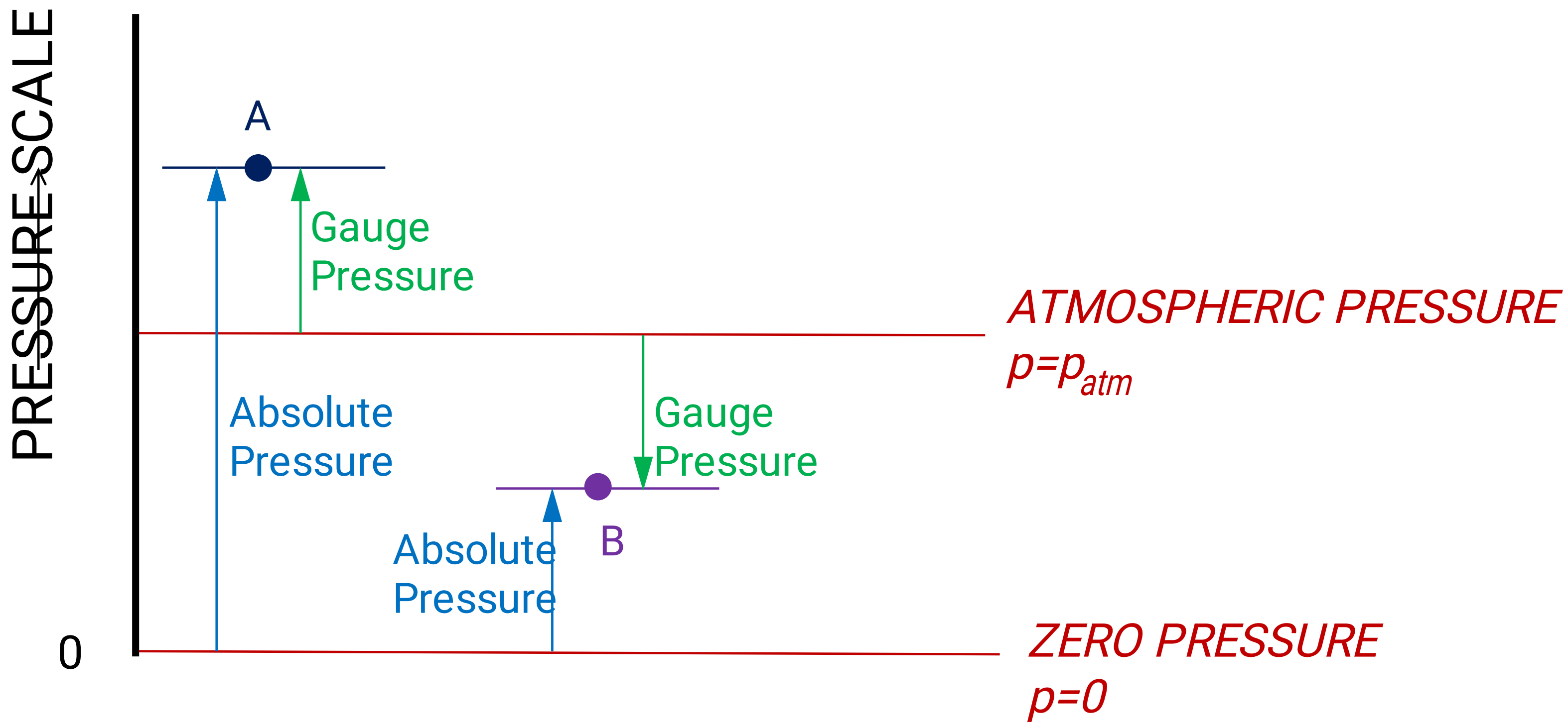
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

Measurement of Pressure

Pressure in a fluid measured in two different systems

1. With reference to the absolute zero (or) perfect vacuum - Absolute Pressure
2. With reference to the atmospheric pressure – Gauge Pressure



- Absolute pressure = Atmospheric pressure + Gauge pressure

$$p_{abs} = p_{atm} + p_{gage}$$

$$p_A = p_{atm} + p_{A,gage} \qquad\qquad p_B = p_{atm} - p_{B,gage}$$

- Negative Gauge pressure corresponds to Suction pressure or Vacuum (For example, -1000 Pa *gauge pressure* corresponds to 1000 Pa *Vacuum*)

- Pressure gauges reads gauge pressure

- If gauge pressure is zero, the pressure is atmospheric

Standard atmospheric pressure: p_{atm} = 1 atm = 1.01325 bar = 1.01325 x 10^5 Pa

If the absolute pressure is 3 atm, what is the gauge pressure? Answer: 2 atm

If the gage pressure is 20,000 Pa, what is the absolute pressure? Answer: 121325 Pa

If the Suction pressure/Vacuum pressure is 50000 Pa, what is the gauge pressure? Answer: -50000 Pa

If the Vacuum is 50000 Pa, what is the absolute pressure? Answer: 51325 Pa

If the gauge pressure is zero, what is the absolute pressure? Answer: Atmospheric pressure

If the absolute pressure is 100000 Pa, what is the gauge pressure? Answer: -1325 Pa