$Z[n^2] = \frac{z^2+z}{(z-1)^3}$

✡ $z[n] = \frac{z}{(z-1)^2}$

$z[\frac{1}{n!}] = e^{1/z}$

★ $Z[f(n)] = f(z)$ $\qquad$ $Z^{-1}[f(z)] = f(n)$

★ 1 $Z[a^n] = \frac{z}{z-a}$ $\qquad$ $Z^{-1}[\frac{z}{z-a}] = a^n$

If $a = 1, -1, -a$

2. $Z[a^{n-1}] = \frac{a}{z-a}$ $\qquad$ $Z^{-1}[\frac{a}{z-a}] = a^{n-1}$

3. $Z[n] = \frac{z}{(z-1)^2}$

★★ 4. $Z[na^{n-1}] = \frac{z}{(z-a)^2}$ $\qquad$ $Z^{-1}[\frac{z}{(z-a)^2}] = na^{n-1}$

5. $Z[na^n] = \frac{az}{(z-a)^2}$ $\qquad$ $Z^{-1}[\frac{az}{(z-a)^2}] = na^n$

6. $Z[n(n-1)] = \frac{2z}{(z-1)^3}$ $\qquad$ $Z^{-1}[\frac{2z}{(z-1)^3}] = n(n-1)$

★ 7. $Z[a^n \cos \frac{n\pi}{2}] = \frac{z^2}{z^2+a^2}$ $\qquad$ $Z^{-1}[\frac{z}{z^2+a^2}] = a^n \cos \frac{n\pi}{2}$

★ 8. $Z[a^n \sin \frac{n\pi}{2}] = \frac{az}{z^2+a^2}$ $\qquad$ $Z^{-1}[\frac{az}{z^2+a^2}] = a^n \sin \frac{n\pi}{2}$

9. $Z[(n-1)a^{n-2}] = \frac{1}{(z-a)^2}$ $\qquad$ $Z^{-1}[\frac{1}{(z-a)^2}] = (n-1)a^{n-2}$

10. $Z[(n-2)(n-1)a^{n-3}] = \frac{2}{(z-a)^3}$ $\qquad$ $Z^{-1}[\frac{2}{(z-a)^3}] = (n-2)(n-1)a^{n-3}$

11. $Z[(n+1)a^n] = \frac{z}{(z-a)^2}$ $\qquad$ $Z^{-1}[\frac{z}{(z-a)^2}] = (n+1)a^n$

★ Definition: $Z[x(n)] = \sum_{n=0}^{\infty} x(n) z^{-n} = X(z)$

x is a function of z

Properties

1. Linear: If $Z[x(n)] = X(z)$ & $Z[y(n)] = Y(z)$, then $Z[ax(n) + by(n)] = aX(z) + bY(z)$

2. scalar / change of scale: If $Z[x(n)] = X(z)$,

i) $Z[a^n x(n)] = X(z/a)$

ii) $Z[a^{-n} x(n)] = X(az)$

~~iii) $Z[a^n x(n)] = X(z/a)$~~

3. Left shifting thm: If $Z[x(n)] = X(z)$, $Z[x(n-k)] = z^{-k} X(z)$

4. First shifting property: If $Z[x(n)] = X(z)$, $Z[x(n+1)] = zX(z) - x(0)$

Q. 1 Find $z[r^n \cos n\theta]$ and $z[r^n \sin n\theta]$

$a = re^{i\theta}$, $e^{i\theta} = \cos\theta + i\sin\theta$

$a^n = (re^{i\theta})^n$

$$z[a^n] = \frac{z}{z-a}$$

$$z[(re^{i\theta})^n] = \frac{z}{z - re^{i\theta}}$$

$$z[r^n e^{i\theta n}] = \frac{z}{z - r(\cos\theta + i\sin\theta)}$$

$$z[r^n(\sin n\theta + \cos n\theta)] = \frac{z}{\underbrace{z - r\cos\theta}_{a} \underbrace{- ir\sin\theta}_{b}} \times \frac{(z - r\cos\theta + ir\sin\theta)}{(z - r\cos\theta + ir\sin\theta)}$$

$$= \frac{z(z - r\cos\theta + ir\sin\theta)}{z^2 + r^2(\cos^2\theta + \sin^2\theta) - 2zr\cos\theta}$$

$$= \frac{z((z - r\cos\theta) + i(r\sin\theta))}{z^2 + r^2 - 2z\cos\theta}$$

Real part: $z[r^n \cos n\theta] = \dfrac{z[z - r\cos\theta]}{z^2 - 2z\cos\theta + r^2}$

Imaginary part: $z[r^n \sin n\theta] = \dfrac{zr\sin\theta}{z^2 - 2zr\cos\theta + r^2}$

Q) Find $Z^{-1}\left[\frac{z^3}{(z-1)(z-2)}\right]$ via partial fraction

Step 1: $X(z) = \frac{z^3}{(z-1)(z-2)}$

Step 2:

$$\frac{X(z)}{z} = \frac{z^2}{(z-1)(z-2)}$$

Now, $\boxed{\frac{z^2}{(z-1)(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z-2)}}$

$$z^2 = A(z-2) + B(z-1)$$

$\boxed{z=2} \Rightarrow \boxed{4 = B}$

$\boxed{z=1} \Rightarrow \boxed{-1 = A}$

$$\Rightarrow \frac{z^2}{(z-1)(z-2)} = \frac{-1}{(z-1)} + \frac{4}{(z-2)} = \frac{X(z)}{z}$$

$$X(z) = \frac{-z}{(z-1)} + \frac{4z}{(z-2)}$$

$$Z^{-1}[X(z)] = -Z^{-1}\left[\frac{z}{z-1}\right] + 4Z^{-1}\left[\frac{z}{z-2}\right]$$

$$x(n) = -1^n + 4(2)^n$$

Q) Type 2: Find $z^{-1}\left[\frac{z^2}{(z+2)(z^2+4)}\right]$

Step 1: $X(z) = \frac{z^2}{(z+2)(z^2+4)}$

$$\frac{X(z)}{z} = \frac{z}{(z+2)(z^2+4)}$$

Now, $\frac{z}{(z+2)(z^2+4)} = \frac{A}{z+2} + \frac{Bz+C}{z^2+4}$

$z = A(z^2+4) + (Bz+C)(z+2)$

$z = Az^2 + 4A + Bz^2 + 2Bz + Cz + C(2)$ ①

Sub $z = -2$ in eq ①

$-2 = 8A + (C-2B)(0)$

$\boxed{A = -1/4}$

Take $z=0 \Rightarrow 0 = 4\left(\frac{-1}{4}\right) + 2C$

$2C = 1$

$\boxed{C = 1/2}$

~~$\Rightarrow z = 0 \Rightarrow 0 = \left(\frac{-1}{4}\right)(0) + 4\left(\frac{-1}{4}\right) + B$~~

~~$0 = \left(\frac{-1}{4}\right)(4) + C$~~

$z=1 \Rightarrow 1 = \left(\frac{-1}{4}\right)(5) + \left(B + \frac{1}{2}\right)(3)$

$= \frac{-5}{4} + \frac{(2B+1)3}{2}$

$= \frac{-5}{4} + \frac{(6B+3)}{2}$

$1 = \frac{-5}{4} + \frac{12B+6}{4}$

$4 = 12B + 1$

$12B = 3$

$\boxed{B = 1/4}$

comparing coe. of z^2

$z^2: 0 = A + B$

$0 = A + B$

$\boxed{B = 1/4}$

$\Rightarrow \frac{z}{(z+2)(z^2+4)} = \frac{-1/4}{(z+2)} + \frac{\frac{1}{4}(z) + \frac{1}{2}}{(z^2+4)}$

Step 3: $\frac{X(z)}{z} = \frac{-1}{4}\left[\frac{1}{z+2}\right] + \frac{1}{4}\left[\frac{z}{z^2+4}\right] + \frac{1}{2}\left[\frac{1}{z^2+4}\right]$

$X(z) = \frac{-1}{4}\left[\frac{z}{z+2}\right] + \frac{1}{4}\left[\frac{z^2}{z^2+4}\right] + \frac{1}{2}\left[\frac{z}{z^2+4}\right]$

$Z^{-1}[X(z)] = \frac{-1}{4} Z^{-1}\left[\frac{z}{z+2}\right] + \frac{1}{4} Z^{-1}\left[\frac{z^2}{z^2+4}\right] + \frac{1}{2} Z^{-1}\left[\frac{z}{z^2+4}\right]$

$x(n) = \frac{-1}{4}(-2)^n + \frac{1}{4} 2^n \cos\frac{n\pi}{2} + \frac{1}{2} 2^n \sin\frac{n\pi}{2}$

Type 3 Solve via partial fraction

15M

2. $z^{-1}\left[\dfrac{z^3}{(z-1)^2(z-2)}\right]$

Step 1:

$$X(z) = \frac{z^3}{(z-1)^2(z-2)}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z-1)^2(z-2)}$$

Now,

$$\frac{z^2}{(z-1)^2(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2} + \frac{C}{(z-2)}$$

$$z^2 = A(z-1)(z-2) + B(z-2) + C(z-1)^2$$

for $z=1$, $1 = -B \rightarrow \boxed{B=-1}$

for $z=2$, $4 = C \rightarrow \boxed{C=4}$

$\Rightarrow z^2 = A(z-1)(z-2) + (2-z) + 4(z-1)^2$

for $z=0$, $0 = A(2) + 2 + 4$

$A(2) + 6 = 0$

$\boxed{A=-3}$

$$\frac{z}{(z-1)^2(z-2)} = \frac{-3}{z-1} - \frac{1}{(z-1)^2} + \frac{4}{(z-2)}$$

$$\frac{X(z)}{z} = \frac{-3}{z-1} - \frac{1}{(z-1)^2} + \frac{4}{z-2}$$

$$X(z) = -3\left[\frac{z}{z-1}\right] - \frac{z}{(z-1)^2} + 4\left[\frac{z}{z-2}\right]$$

$$z^{-1}[X(z)] = -3z^{-1}\left[\frac{z}{z-1}\right] - z^{-1}\left[\frac{z}{(z-1)^2}\right] + 4z^{-1}\left[\frac{z}{z-2}\right]$$

$$= -3(1)^n - n + 4(2)^n$$

Convolution thm:

If $Z[x(n)] = X(z)$ & $Z[y(n)] = Y(z)$,
then
$$Z[x(n) * y(n)] = X(z)\,Y(z)$$

3. Find $Z^{-1}\left[\dfrac{z^2}{(z-a)(z-b)}\right]$ via convolution

Formula: $a + ar + ar^2 + \dots = \dfrac{a[1-r^{n+1}]}{1-r}$

$$Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right] = Z^{-1}\left[\frac{z}{(z-a)} \cdot \frac{z}{(z-b)}\right]$$

$$= Z^{-1}\left[\frac{z}{z-a}\right] * Z^{-1}\left[\frac{z}{z-b}\right]$$

$$= a^n * b^n$$

$$= \sum_{k=0}^{n} a^k b^{n-k}$$

$$= b^n \sum_{k=0}^{n} a^k b^{-k}$$

$$= b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$

$$= b^n \left[1 + \frac{a}{b} + \left(\frac{a}{b}\right)^2 + \dots\right]$$

$$= b^n \left[\frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\frac{a}{b}}\right]$$

$$= b^n \left[\frac{b^{n+1}-a^{n+1}}{b^{n+1}} \times \frac{b}{b-a}\right]$$

$$= \cancel{b^n}\left[\frac{(b^{n+1}-a^{n+1})\cancel{b}}{\cancel{b^n}\,\cancel{b}\,(b-a)}\right]$$

$$= \frac{b^{n+1}-a^{n+1}}{b-a}$$

5

Find $z^{-1}\left[\dfrac{8z^2}{(2z-1)(4z+1)}\right]$ by convolution thm

$$= z^{-1}\left[\frac{8z^2}{\cancel{2}(z-\frac{1}{2})\cancel{4}(z+\frac{1}{4})}\right]$$

$$= z^{-1}\left[\frac{z^2}{(z-\frac{1}{2})(z+\frac{1}{4})}\right]$$

$$= z^{-1}\left[\frac{z}{(z-\frac{1}{2})}\right] * z^{-1}\left[\frac{z}{(z+\frac{1}{4})}\right] = \left(\frac{1}{2}\right)^n * \left(\frac{-1}{4}\right)^n$$

$$= \sum_{k=0}^{n} \left(\frac{-1}{4}\right)^k \left(\frac{1}{2}\right)^{n-k}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{-1}{4}\times\frac{2}{1}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{-1}{2}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \left[1+\left(\frac{-1}{2}\right)+\left(\frac{-1}{2}\right)^2+\cdots\right]$$

$$= \frac{1}{2^n}\left[\frac{1-\left(\frac{-1}{2}\right)^{n+1}}{1-\left(-\frac{1}{2}\right)}\right]$$

$$= \frac{1}{2^n}\left[1-\left(\frac{-1}{2}\right)^{n+1}\right]\frac{2}{3}$$

$$= \frac{2}{3\cdot 2^n}\left[1-\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)^n\right]$$

$$= \frac{2^{1-n}}{3}\left[1+\frac{1}{2}\left(\frac{-1}{2}\right)^n\right]$$

$$= \frac{2^{1-n}}{3} + \frac{1}{3\cdot 2^n}\left(\frac{-1}{2}\right)^n$$

$$= \frac{2^{1-n}}{3} + \frac{(-1)^n}{3\cdot 2^{n+n}}$$

$$= \frac{2^{1-n}}{3} + \frac{1}{3}\left(\frac{-1}{4}\right)^n$$

$$z^{-1} = \frac{2}{3}(2)^n + \frac{1}{3}\left(\frac{-1}{4}\right)^n$$

$$= \frac{1}{2^n}\left(\frac{2}{3}\right)\left[1-\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)^n\right]$$

$$= \frac{2}{3}\left[\frac{1}{2^n}\right]\left[1+\frac{1}{2}\left(\frac{-1}{2}\right)^n\right]$$

$$= \frac{2}{3}\left[\frac{1}{2^n}\right] + \frac{\cancel{2}}{3}\left[\frac{1}{2^n}\right]\frac{1}{\cancel{2}}\left(\frac{-1}{2}\right)^n$$

$$= \frac{2}{3}\left(\frac{1}{2}\right)^n + \frac{1}{3}\left(\frac{-1}{4}\right)^n \;//$$

Q 4 Find $Z^{-1}\left[\dfrac{8z^2}{(2z-1)(4z-1)}\right]$ via convolution thm

$Z^{-1}\left[\dfrac{8z^2}{2(z-\frac{1}{2})4(z-\frac{1}{4})}\right] = Z^{-1}\left[\dfrac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})}\right]$

$Z^{-1}\left[\dfrac{z}{z-\frac{1}{2}}\right] Z^{-1}\left[\dfrac{z}{z-1/4}\right]$

$= \left(\frac{1}{2}\right)^n * \left(\frac{1}{4}\right)^n$

$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{4}\right)^{n-k}$

$= \frac{1}{4^n} \sum_{k=0}^{n} \left(\frac{1}{2}\times\frac{4}{1}\right)^k$

$= \frac{1}{4^n} \sum_{k=0}^{n} 2^k$

$= \frac{1}{4^n}\left[\dfrac{1-2^{n+1}}{1-2}\right]$

$= \frac{1}{4^n}\left[2^{n+1}-1\right]$ //

Q: 6M

Method of Residues

If $Z[f(n)] = F(z)$, $f(n) = \frac{1}{2\pi i}\int z^{n-1}F(z)\,dz$ ①

$\Rightarrow \int z^{n-1}F(z)\,dz = 2\pi i$ [sum of residues $z^{n-1}F(z)$ as its poles] ②

$\Rightarrow f(n) =$ sum of residues of poles

Simple pole

$[\text{Res } F(z)z^{n-1}]_{z=z_0} = \lim_{z\to z_0} (z-z_0)F(z)z^{n-1}$

Order 'm'

$$\left[\operatorname{Res} F(z)\, z^{n-1}\right]_{z=z_0} = \lim_{z \to z_0} \frac{1}{(m-1)!}\left(\frac{d}{dz}\right)^{m-1}\left[(z-z_0)^m F(z)\, z^{n-1}\right]$$

5. Find Inverse z transform of $\frac{z+3}{(z+1)(z-2)}$ via method of residues.

$$F(z) = \frac{z+3}{(z+1)(z-2)}$$

$$F(z)\, z^{n-1} = \frac{z^{n-1}(z+3)}{(z+1)(z-2)}$$

$z=-1,\ 2$ are both simple poles.

$$\left[\operatorname{Res} F(z)\, z^{n-1}\right]_{z=-1} = \lim_{z \to -1} \frac{\cancel{(z+1)}\, z^{n-1}(z+3)}{\cancel{(z+1)}(z-2)}$$

$$= \frac{(-1)^{n-1}(2)}{(-1-2)}$$

$$= \frac{\cancel{-}(-1)^n(2)}{\cancel{-}3}$$

$$R(-1) = \frac{2}{3}(-1)^n$$

$$\left[\operatorname{Res} F(z)\, z^{n-1}\right]_{z=2} = \lim_{z \to 2} \frac{\cancel{(z-2)}\, z^{n-1}(z+3)}{(z+1)\cancel{(z-2)}}$$

$$= \frac{2^{n-1}(5)}{3}$$

$$R(2) = \frac{5(2^n)}{6}$$

$f(n) = \{$Sum of residues of $F(z)\, z^{n-1}\}$

$$= R(-1) + R(2)$$

$$= \frac{2}{3}(-1)^n + \frac{5}{6}(2)^n$$

6. Find $z^{-1}\left[\frac{2z^2+4z}{(z-2)^3}\right]$ via method of residues

$$F(z) = \frac{2z^2+4z}{(z-2)^3}$$

$$F(z)\,z^{n-1} = z^{n-1}\left[\frac{2z^2+4z}{(z-2)^3}\right] \rightarrow \text{pole} = 2,\ \text{order } m=3$$

$$[\text{Res } F(z)\,z^{n-1}]_{z\to z_0} = \lim_{z\to z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} (z-z_0)^m F(z)\,z^{n-1}$$

$$R(2) = \lim_{z\to 2} \frac{1}{2} \frac{d^2}{dz^2} \frac{\cancel{(z-2)^3}\, z^{n-1}(2z^2+4z)}{\cancel{(z-2)^3}}$$

$$= \lim_{z\to 2} \frac{1}{2} \frac{d^2}{dz^2} [2z^{n+1}+4z^n]$$

$$= \lim_{z\to 2} \frac{1}{2} \frac{d}{dz} [2(n+1)z^n + 4n z^{n-1}]$$

$$= \lim_{z\to 2} \frac{1}{2} [2n(n+1)z^{n-1} + 4n(n-1)z^{n-2}]$$

$$= \lim_{z\to 2} \frac{1}{\cancel{2}} \times \cancel{2}n [(n+1)z^{n-1} + 2(n-1)z^{n-2}]$$

$$= n[(n+1)2^{n-1} + (n-1)2^{n-1}]$$

$$= n2^{n-1}[n+\cancel{1}+n-\cancel{1}]$$

$$= n2^{n-1}(2n)$$

$$= n^2 2^n$$

$f(n) =$ sum of residues $= 2^n n^2$

$$Z[y_{n+2}] = z^2 Y(z) - z^2 Y(0) - zY(1)$$

$$Z[y_{n+1}] = zY(z) + zY(0)$$

$$Z[y(n)] = Y(z)$$

(15M) 7. Solve $y_{n+2} + 6y_{n+1} + 9y_n = [redacted] 2^n$ given $y_0 = y_1 = 0$

CLA3 + UNI

$\rightarrow y(0) = y(1) = 0$

$$Z[y_{n+2}] + 6Z[y_{n+1}] + 9Z[y_n] = Z[2^n]$$

$$z^2Y(z) - \cancel{z^2y(0)}^{0} - \cancel{zy(1)}^{0} + 6[zY(z) + \cancel{zy(0)}^{0}] + 9[Y(z)] = \frac{z}{z-2}$$

$$z^2Y(z) + 6zY(z) + 9Y(z) = \frac{z}{z-2}$$

$$Y(z)[z^2 + 6z + 9] = \frac{z}{z-2}$$

$9 < \frac{3}{3}$
$6.$

Method 1: $$Y(z) = \frac{z}{(z-2)(z+3)^2}$$

$$Y(z)z^{n-1} = \frac{z^n}{(z+3)^2(z-2)}$$

$z = 2$ is a simple pole

$z = -3$ is a pole of order $m = 2$

$$[\text{Res}\, Y(z)z^{n-1}]_{z=z_0} = \lim_{z \to z_0} (z - z_0)F(z)z^{n-1}$$

$$R(2) = \lim_{z \to 2} \frac{\cancel{(z-2)}\, z^n}{(z+3)^2\cancel{(z-2)}}$$

$$= \frac{2^n}{(5)^2}$$

$$R(2) = \frac{2^n}{25}$$

$$[\text{Res}\, Y(z)z^{n-1}]_{z=z_0} = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(z-z_0)^m F(z) z^{n-1}]$$

$$R(-3) = \lim_{z \to -3} \frac{1}{1!} \frac{d^1}{dz^1} \left[\cancel{(z+3)^2} \frac{z^n}{\cancel{(z+3)^2}(z-2)} \right]$$

$$= \lim_{z \to -3} \frac{d}{dz} \frac{z^n}{z-2}$$

$$= \lim_{z \to -3} \frac{(z-2)(n)z^{n-1} - z^n(1)}{(z-2)^2}$$

$$= \frac{-5(n)(-3)^{n-1} - (-3)^n}{(-5)^2}$$

$$R(-3) = \frac{-5n(-3)^n - (-3)^n}{25}$$

$y(n) =$ sum of residues of $y(z)z^{n-1}$ of its poles

$$= R(2) + R(-3)$$

$$y(n) = \frac{2^n}{25} - \frac{5n(-3)^n - (-3)^n}{25} \quad //$$

8. Solve via z-transform;

$$y_{n+2} - 7y_{n+1} + 12y_n = 2^n, \; y(0) = y(1) = 0$$

$$Z[y_{n+2}] - 7Z[y_{n+1}] + 12\,Z[y_n] = Z[2^n]$$

$$z^2Y(z) - z^2\underbrace{y(0)}_{0} - z\underbrace{y(1)}_{0} - 7[zY(z) - z\underbrace{y(0)}_{0}] + 12\,Y(z) = \frac{z}{z-2}$$

$$z^2Y(z) - 7\,zY(z) + 12\,Y(z) = \frac{z}{z-2}$$

$$Y(z)[z^2 - 7z + 12] = \frac{z}{z-2}$$

$$12 \underset{-7}{\overset{3}{=}} 4$$

$$Y(z) = \frac{z}{(z-2)(z-3)(z-4)}$$

$$Y(z)z^{n-1} = \frac{z^n}{(z-2)(z-3)(z-4)}$$

$z = 2, 3, 4$ are all simple poles

$$[\mathrm{Res}\,Y(z)z^{n-1}]_{z \to z_0} = \lim_{z \to z_0} (z - z_0)Y(z)z^{n-1}$$

$$R(2) = \lim_{z \to 2} \frac{\cancel{(z-2)}\,z^n}{\cancel{(z-2)}(z-3)(z-4)} = \lim_{z \to 2} \frac{z^n}{(z-3)(z-4)}$$

$$= \frac{2^n}{(-1)(-2)} = 2^{n-1}$$

$$\left[\operatorname{Res} Y(z) z^{n-1}\right]_{z \to z_0} = \lim_{z \to z_0} (z - z_0) Y(z) z^{n-1}$$

$$R(3) = \lim_{z \to 3} \frac{\cancel{(z-3)}\, z^n}{(z-2)\cancel{(z-3)}(z-4)}$$

$$= \frac{3^n}{(1)(-1)}$$

$$R(3) = -3^n$$

$$R(4) = \lim_{z \to 4} \frac{\cancel{(z-4)}\, z^n}{(z-2)(z-3)\cancel{(z-4)}}$$

$$= \frac{4^n}{2(1)}$$

$$R(4) = 2^{2n-1}$$

$y(n) =$ sum of all residues of $Y(z) z^{n-1}$ of its poles

$$= R(2) + R(3) + R(4)$$

$$= 2^{n-1} - 3^n + 2^{2n-1}$$