

UNIT-5: SPACE APPLICATIONS

Types of Orbits

Circular Orbits ($e = 0$)

Circular orbits are the simplest type of orbit. In a circular orbit, the orbiting body maintains a constant distance from the central body. This means the **speed** of the orbiting body is also **constant**.

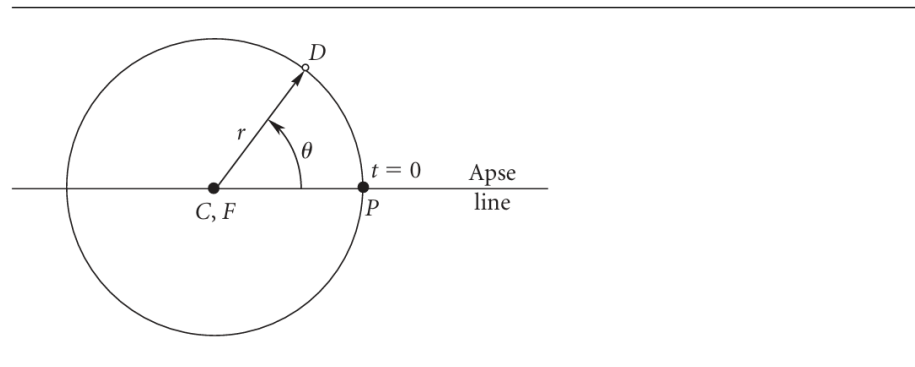
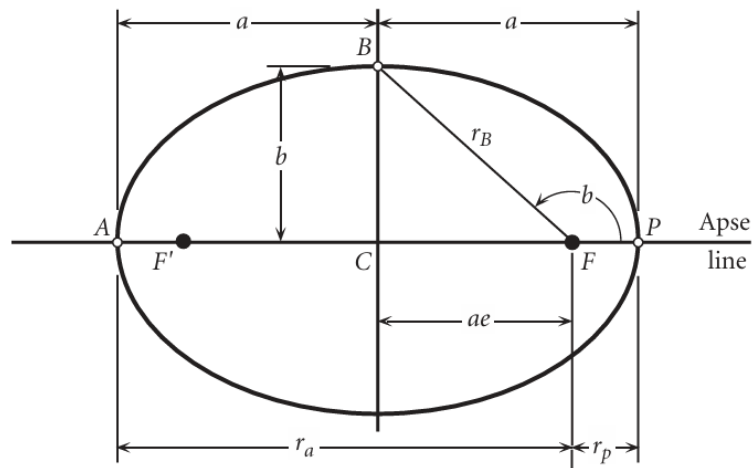


Figure 3.1 Time since periapsis is directly proportional to true anomaly in a circular orbit.

Elliptical Orbits ($0 < e < 1$)

In an elliptical orbit, the distance between the orbiting body and the central body changes throughout the orbit. The point of closest approach is called **periapsis** and the point of farthest distance is called **apoapsis**.

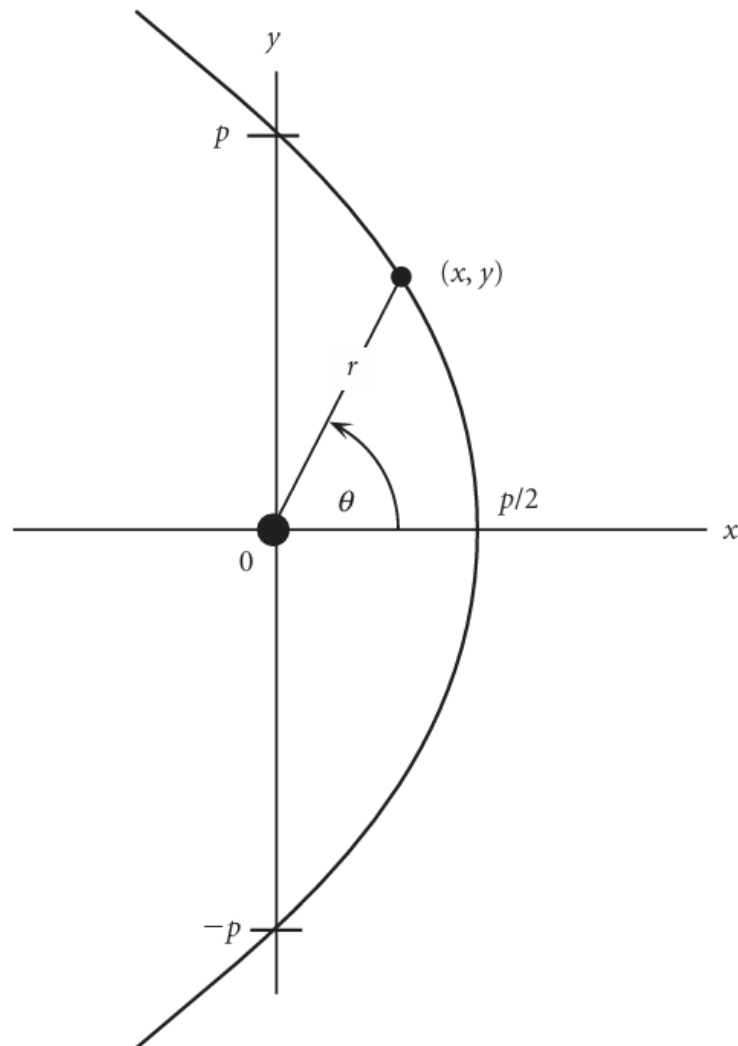
Elliptical orbits are characterised by their **eccentricity (e)**, a value between 0 and 1. An eccentricity of 0 is a circular orbit. As eccentricity increases towards 1, the ellipse becomes more elongated.



Parabolic Trajectories ($e = 1$)

Parabolic trajectories are the boundary between elliptical orbits and hyperbolic trajectories. A body on a parabolic trajectory has just enough energy to escape the gravitational pull of the central body.

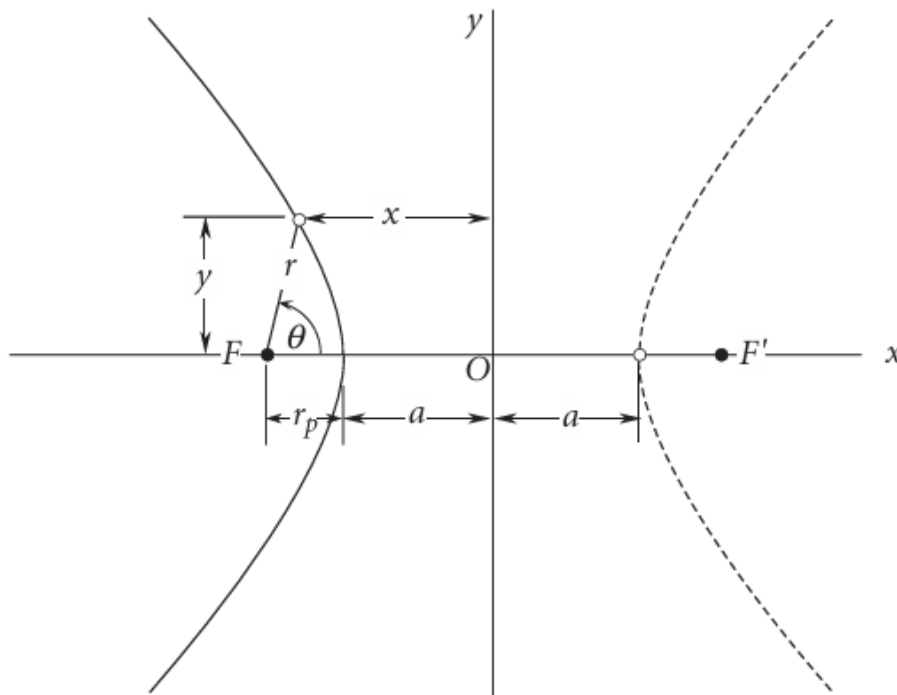
The velocity of a body on a parabolic trajectory **decreases** as it moves **away** from the central body, approaching zero at an **infinite** distance.



Hyperbolic Trajectories ($e > 1$)

A body on a hyperbolic trajectory has more than enough energy to escape the gravitational pull of the central body.¹ It will continue to move away from the central body at a **non-zero** velocity even at an **infinite** distance.

Hyperbolic trajectories are characterised by their **eccentricity (e)**, a value **greater** than 1. As eccentricity increases, the hyperbola becomes "flatter"



Classical Orbital Elements:

The six orbital elements are parameters that uniquely define the shape and orientation of an orbit in three-dimensional space, as well as the position of a body within that orbit. These elements are crucial for understanding and predicting the motion of celestial bodies, including satellites and spacecraft.

1. Specific Angular Momentum (h)

Specific angular momentum (h) is a measure of an object's orbital angular momentum per unit mass. It is a vector quantity that is **always perpendicular** to the orbital plane. The magnitude of h determines the **size and shape** of the orbit. A larger angular momentum corresponds to a larger and less eccentric orbit.

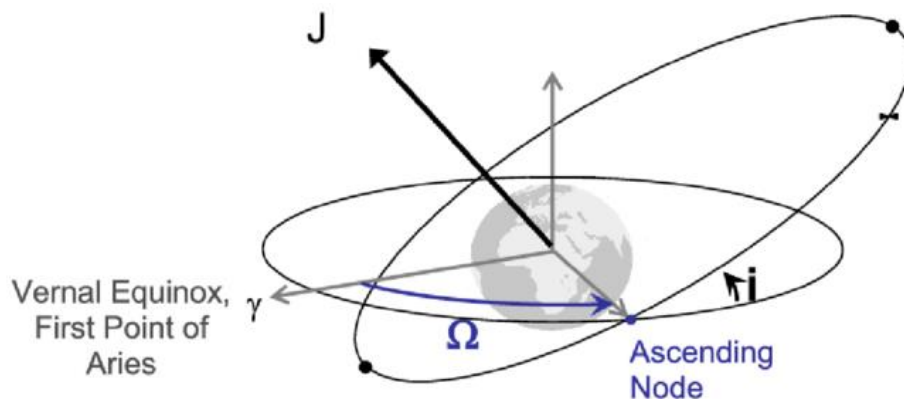
2. Inclination (i)

Inclination (i) is the angle between the orbital plane and a reference plane. For Earth-orbiting satellites, the reference plane is typically the Earth's equatorial plane. Inclination is a positive number between 0° and 180° .

- An inclination of 0° means the orbit lies **in the same plane** as the reference plane, like the equator.
- An inclination of 90° means the orbit passes **directly over the poles**.
- An inclination between 90° and 180° indicates a **retrograde orbit**, where the satellite travels in the opposite direction to the Earth's rotation.

3. Right Ascension of the Ascending Node (Ω)

The ascending node is the point where the orbit **crosses the reference plane from south to north**. Right ascension of the ascending node (Ω) is the angle between a reference direction in the reference plane and the ascending node. This angle is measured **eastward** in the reference plane. The reference direction is typically the **vernal equinox**, which is the direction of the sun from Earth at the start of spring.



4. Eccentricity (e)

Eccentricity (e) is a dimensionless parameter that describes the **shape** of the orbit. It is a measure of how elongated the orbit is, ranging from 0 to greater than 1.

- $e = 0$: Circular orbit (constant distance from the central body)
- $0 < e < 1$: Elliptical orbit (distance from the central body varies)
- $e = 1$: Parabolic trajectory (escape trajectory with zero velocity at infinity)
- $e > 1$: Hyperbolic trajectory (escape trajectory with non-zero velocity at infinity)

5. Argument of Perigee (ω)

The argument of perigee (ω) is the angle between the ascending node and the perigee, measured in the orbital plane. Perigee is the point in the orbit **closest** to the central body. The argument of perigee is a positive number between 0° and 360° .

6. True Anomaly (θ)

True anomaly (θ) is the angle between the perigee and the object's current position, measured in the orbital plane. It essentially pinpoints the location of the orbiting body along its orbital path at a specific time.

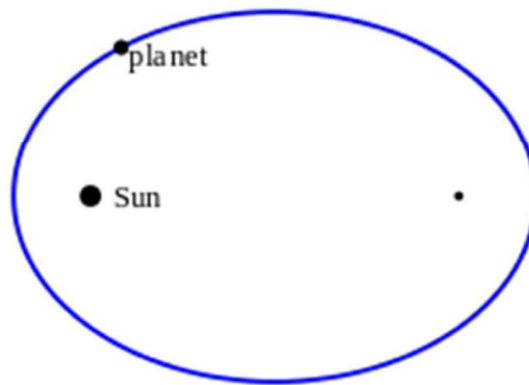
Kepler's law of Motions:

Johannes Kepler, a German astronomer, formulated three laws of planetary motion based on observations made by Tycho Brahe. These laws, published between 1609 and 1619, revolutionised our understanding of the solar system and laid the foundation for Newton's law of universal gravitation.

Kepler's First Law: Law of Ellipses

Kepler's First Law (Law of Ellipses) – 1609

The orbit of each planet is an ellipse with the sun at one focus



This law states that **planets move in elliptical orbits with the Sun at one of the two foci**. This replaced the long-held belief that planets moved in perfect circles.

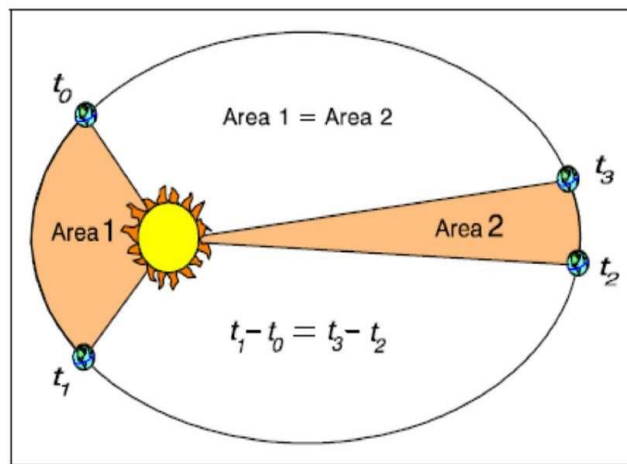
An ellipse is a closed curve where the sum of the distances from any point on the curve to the two foci is constant. The eccentricity (e) of an ellipse determines its shape.

- $e = 0$: The orbit is a perfect circle.
- $0 < e < 1$: The orbit is an ellipse, becoming more elongated as e approaches 1.

Kepler's Second Law: Law of Equal Areas

Kepler's Second Law (Law of Equal Areas) – 1609

The line joining any planet to the sun sweeps out equal areas in equal times.



Kepler's second law states that **a line that connects a planet to the Sun sweeps out equal areas in equal times**. This implies that a planet moves faster when it is closer to the Sun (perihelion) and slower when it is farther away (aphelion).

This law is a consequence of the conservation of angular momentum. As a planet moves closer to the Sun, its orbital speed increases to maintain a constant angular momentum.

Kepler's Third Law: Law of Harmonies

Kepler's Third Law (Law of Harmonies) – 1619

The square of the period of a planet is proportional to the cube of its mean distance from the sun.

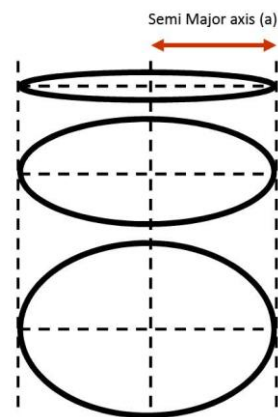
$$T^2 \propto a^3$$

The squares of the periods of two planet's orbits are proportional to each other as the cubes of their semi-major axes:

$$T_1^2/T_2^2 = a_1^3/a_2^3$$

In simple terms:

Orbits with the same semi-major axis will have the same period.



This law establishes a relationship between the orbital period of a planet and its average distance from the Sun. It states that **the square of the orbital period of a planet is proportional to the cube of the semimajor axis of its orbit.**

Mathematically, this can be expressed as:

$$T^2 \propto a^3$$

where:

- T is the orbital period of the planet.
- a is the semimajor axis of the planet's orbit (half of the longest diameter of the ellipse).

This law implies that planets farther from the Sun have longer orbital periods. For example, Earth's orbital period is one year, while Mars, being farther away, has an orbital period of about 1.88 Earth years.