Numericals on hydrostatics

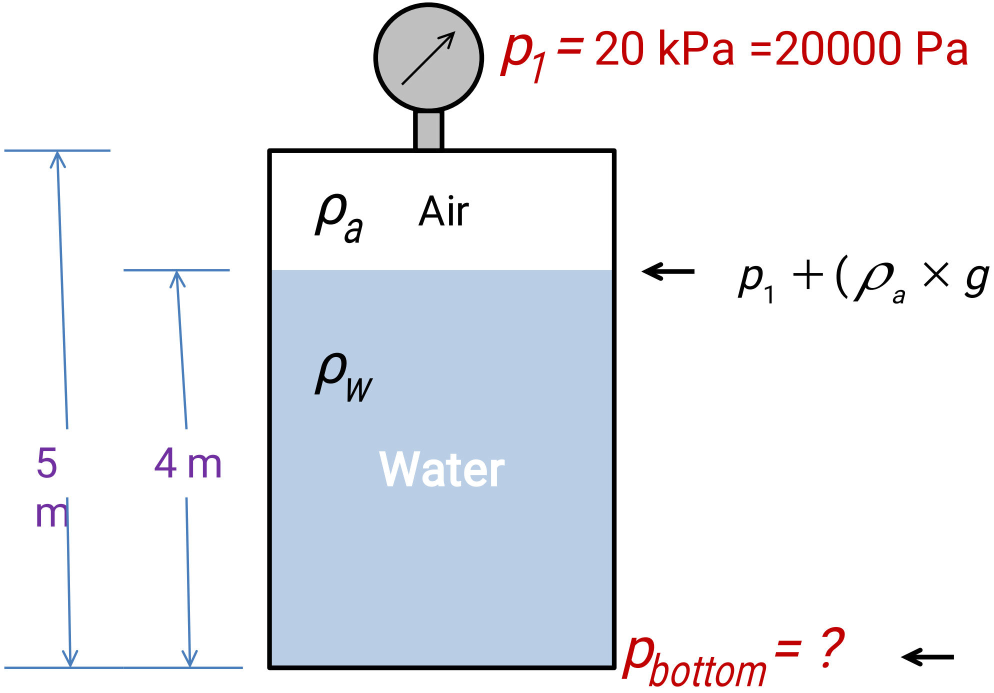
PROBLEM:

A closed, 5-m-tall tank is filled with water to a depth of 4 m. The top portion of the tank is filled with air which, as indicated by a pressure gage at the top of the tank, is at a pressure of 20 kPa. Determine the pressure that the water exerts on the bottom of the tank

Take

$\rho_a = 1.2 \text{ kg/m}^3$

$\rho_w = 1000 \text{ kg/m}^3$



$\leftarrow \; p_1 + (\rho_a \times g \times 1)$

From Hydrostatic law, Pressure increases with depth by $\rho g h$

$\leftarrow \; p_{bottom} = p_1 + (\rho_a \times g \times 1) + (\rho_w \times g \times 4)$

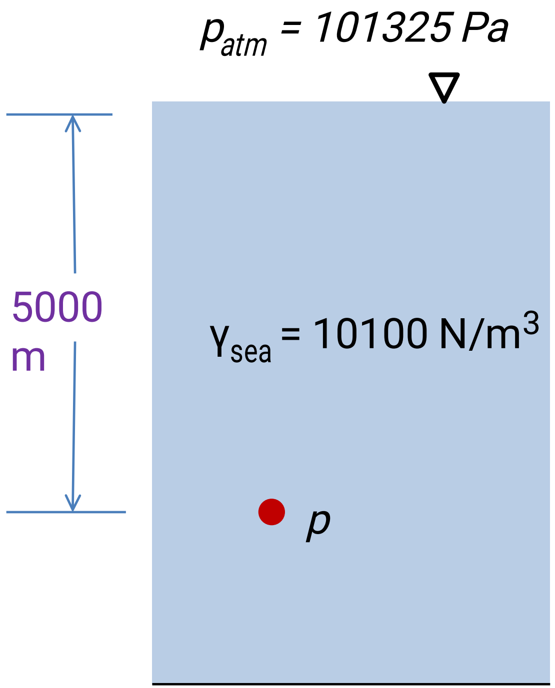
= 20000 + (1.2*9.8*1) + (1000*9.8*4)

= 59211.76 Pa

PROBLEM:

Bathyscaphes are capable of submerging to great depths in the ocean. What is the pressure at a depth of 5 km, assuming that seawater has a constant specific weight of 10.1 kN/m^3? Express your answer in absolute and gage pressures.

Given $\gamma_{sea} = \rho_{sea}\, g = 10100 \text{ N/m}^3$



Absolute pressure:

$$p = p_{atm} + (\rho_{sea} \times g \times 5000)$$

= 101325 + (10100 x 5000) Pa

= 50601325 Pa

= 50.6 MPa

Gage pressure:

$$p = (\rho_{sea} \times g \times 5000)$$

= (10100 x 5000) Pa

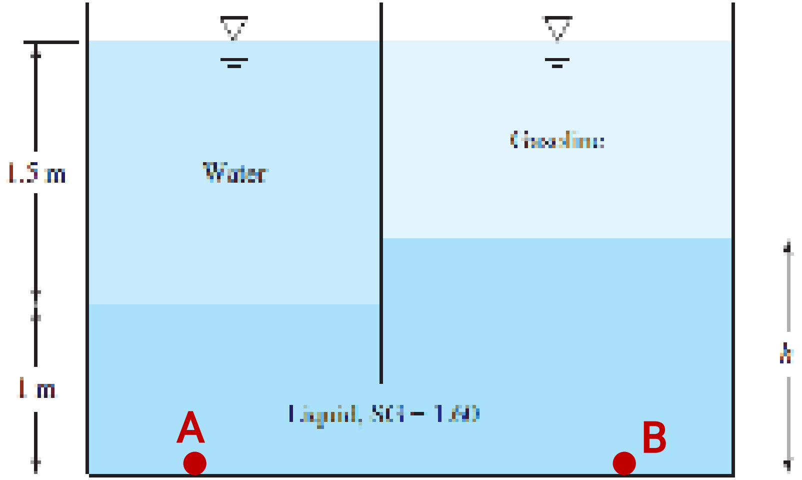
= 50500000 Pa

= 50.5 MPa

PROBLEM:

In the figure, the water and gasoline surfaces are open to the atmosphere and at the same elevation. What is the height h of the third liquid in the right leg? The density of water and gasoline are 1000 kg/m^3 and 711 kg/m^3 respectively.

p_{atm} = 101325 Pa



$$p_A = p_B$$

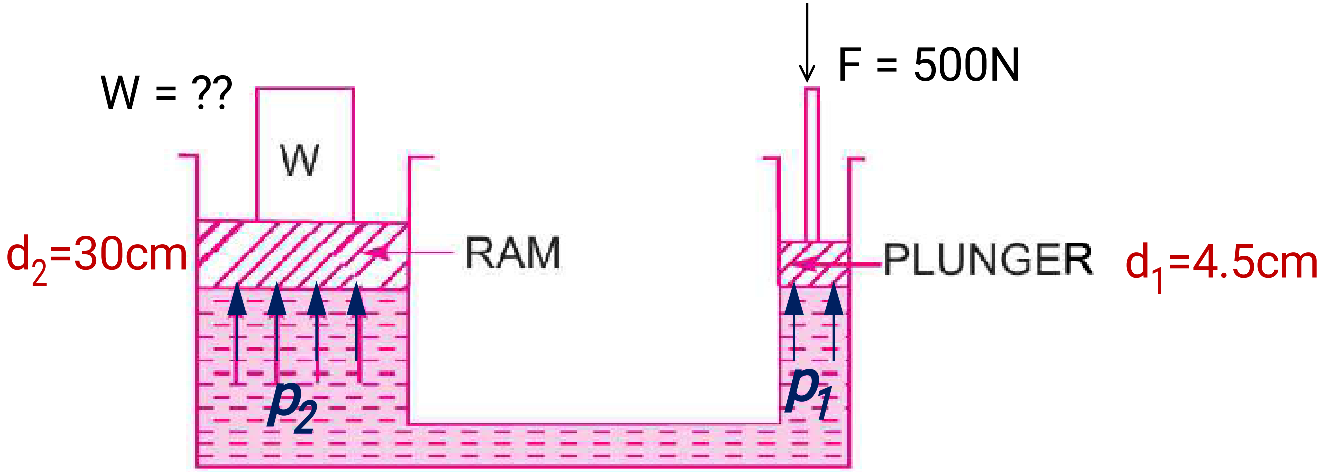
$$p_{atm} + (\rho_w \times g \times 1.5) + (\rho_l \times g \times 1) = p_{atm} + [\rho_g \times g \times (2.5 - h)] + (\rho_l \times g \times h)$$

$$(1000 \times 9.81 \times 1.5) + (1600 \times 9.81 \times 1) = [711 \times 9.81 \times (2.5 - h)] + (1600 \times 9.81 \times h)$$

$$h = 1.5\,m$$

PROBLEM:

A hydraulic press has a ram of 30cm diameter and a plunger of 4.5cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.



RAM and PLUNGER are at the same horizontal level $\Rightarrow$ $p_1 = p_2$

$$p_1 = p_2 \quad \Rightarrow \quad \frac{F}{A_1} = \frac{W}{A_2} \quad \Longrightarrow \quad \frac{500}{15.9 \times 10^{-4}} = \frac{W}{706.95 \times 10^{-4}}$$

Area of plunger: $A_1 = \dfrac{\pi d_1^2}{4} = \dfrac{\pi (4.5 \times 10^{-2})^2}{4} = 15.9 \times 10^{-4} \ \text{m}^2$

Area of ram: $A_2 = \dfrac{\pi d_2^2}{4} = \dfrac{\pi (30 \times 10^{-2})^2}{4} = 706.95 \times 10^{-4} \ \text{m}^2$

$\Rightarrow$ W = 22231 N
= 22.231 KN

PROBLEM:

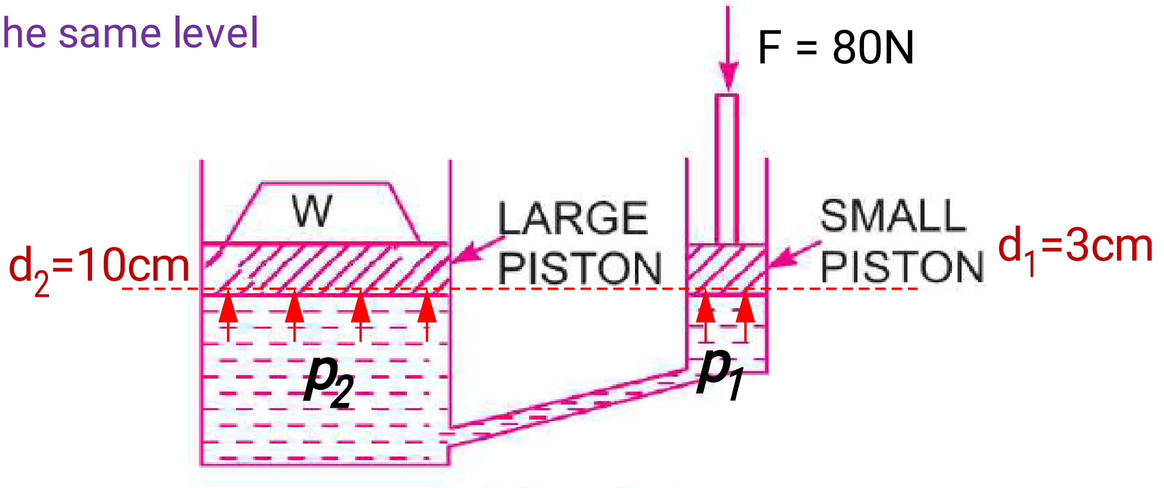
The diameters of a small piston and a large piston of a hydraulic jack are 3cm and 10cm respectively. A force of 80N is applied on the small piston. Find the load lifted by the large piston when:

(a) The pistons are at the same level

(b) Small piston is 40cm above the large piston

The density of liquid in the jack is 1000 kg/m3.

(a) The pistons are at the same level



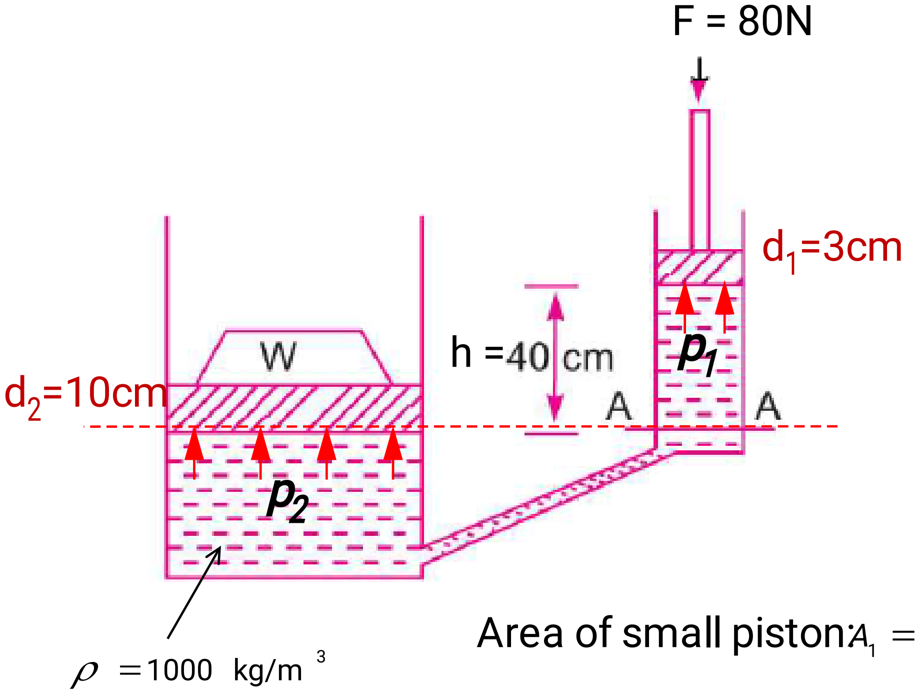
pistons at same level $\Rightarrow \quad p_1 = p_2 \quad \Rightarrow \quad \dfrac{F}{A_1} = \dfrac{W}{A_2}$

$$A_1 = \frac{\pi d_1^2}{4} =$$

$$A_2 = \frac{\pi d_2^2}{4} =$$

$$\Rightarrow \quad W = 888.9\ N$$

(b) Small piston is 40cm above the large piston



Pressure at section A-A $= p_2$

$$p_1 + \rho g h = p_2$$

$$\Rightarrow \frac{F}{A_1} + \rho g h = \frac{W}{A_2}$$

Area of small piston: $A_1 = \frac{\pi d_1^2}{4} = \frac{\pi (3 \times 10^{-2})^2}{4} = 7.07 \times 10^{-4}\ \text{m}^2$

Area of large piston: $A_2 = \frac{\pi d_2^2}{4} = \frac{\pi (10 \times 10^{-2})^2}{4} = 78.55 \times 10^{-4}\ \text{m}^2$

$$\frac{F}{A_1} + \rho g h = \frac{W}{A_2} \quad \Rightarrow \quad \frac{80}{7.07 \times 10^{-4}} + (1000 \times 9.81 \times 0.4) = \frac{W}{78.55 \times 10^{-4}}$$

$$\Rightarrow W = 919.65\ N$$

Manometry

Manometry

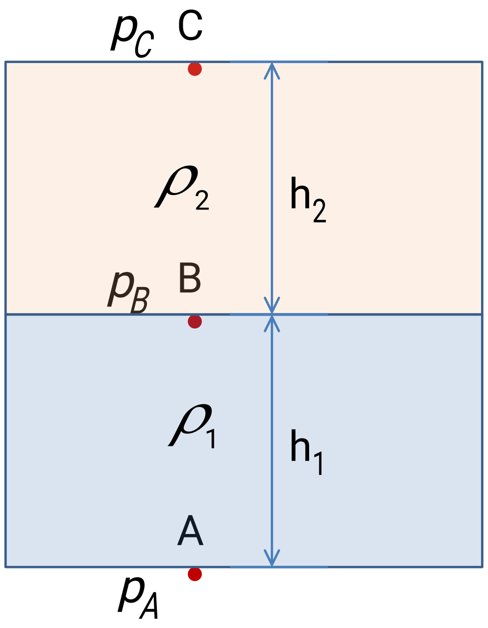
- A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes.

- Pressure-measuring devices based on this technique are called ***manometers.***

$$p_C = p_A - \rho_1 gh_1 - \rho_2 gh_2$$

$$p_B = p_A - \rho_1 gh_1$$

$$p_A$$



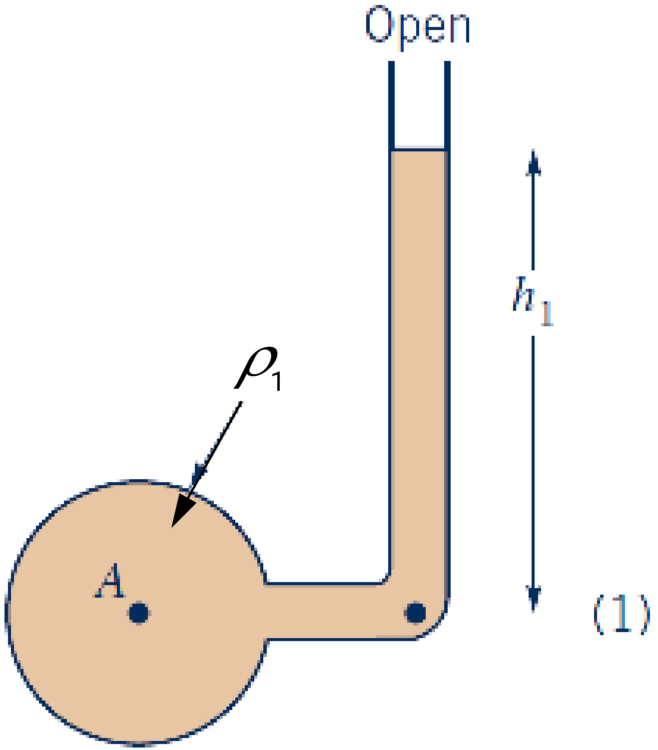
$$p_C$$

$$p_B = p_C + \rho_2 gh_2$$

$$p_A = p_C + \rho_2 gh_2 + \rho_1 gh_1$$

1. Piezometer Tube

- simplest type of manometer
- consists of a vertical tube, open at the top, and attached to the container in which the pressure is measured



$$p_{atm} + \rho_1 g h_1 = p_A$$

$$\Rightarrow \quad p_A - p_{atm} = \rho_1 g h_1$$

$$\Rightarrow \quad p_{A,\,gage} = \rho_1 g h_1$$

(since $p_A - p_{atm} = p_{A,\,gage}$)

Pressure head

We can rewrite the equation in last slide as $\frac{p_A}{\rho_1 g} - \frac{p_{atm}}{\rho_1 g} = h_1$

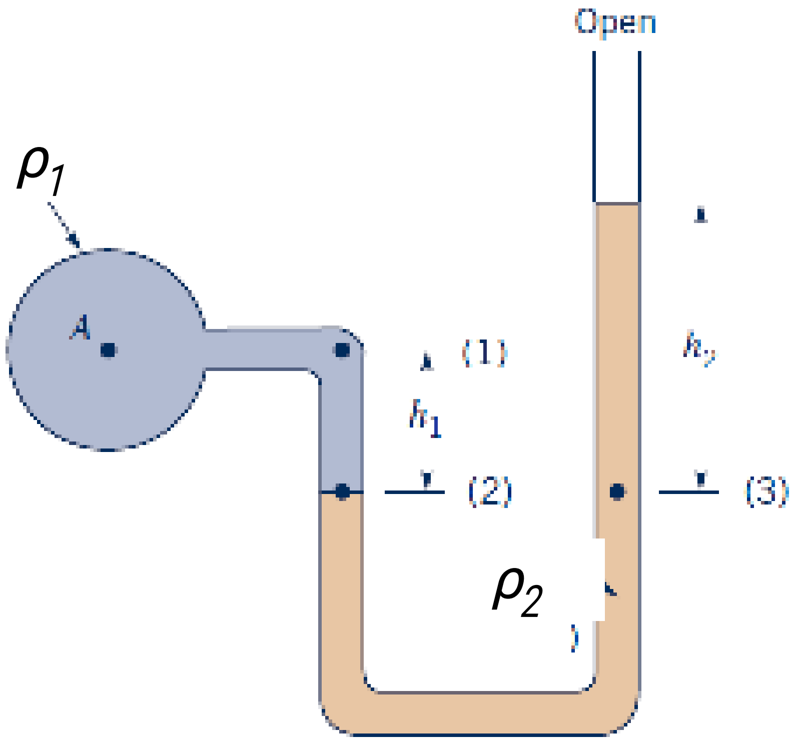
Pressure head is defined as $h = \frac{p}{\rho g}$

Pressure can be expressed in terms of the height h called pressure head

For example,

- atmospheric pressure p_{atm} = 1 atm or 101325 Pa
- Atmospheric pressure in *m of water* $h = \frac{101325}{1000 \times 9.81} = 10.33 \text{ m}$
- Atmospheric pressure in *m of mercury* $h = \frac{101325}{13600 \times 9.81} = 0.76 \text{ m} = 760 \text{ mm}$

2. Simple U-tube manometer



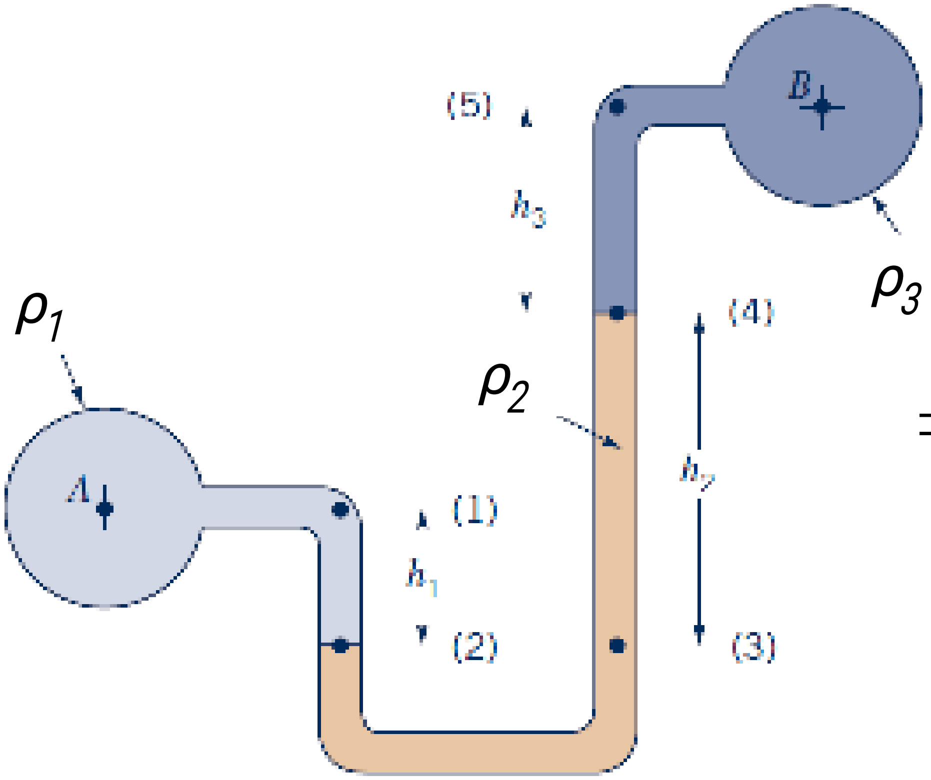
$$p_A + \rho_1 g h_1 - \rho_2 g h_2 = p_{atm}$$

$$\Rightarrow \quad p_{A,\,gage} + \rho_1 g h_1 - \rho_2 g h_2 = 0$$

(since $p_A - p_{atm} = p_{A,\,gage}$)

$$\Rightarrow \quad p_{A,\,gage} = (\rho_2 h_2 - \rho_1 h_1)\, g$$

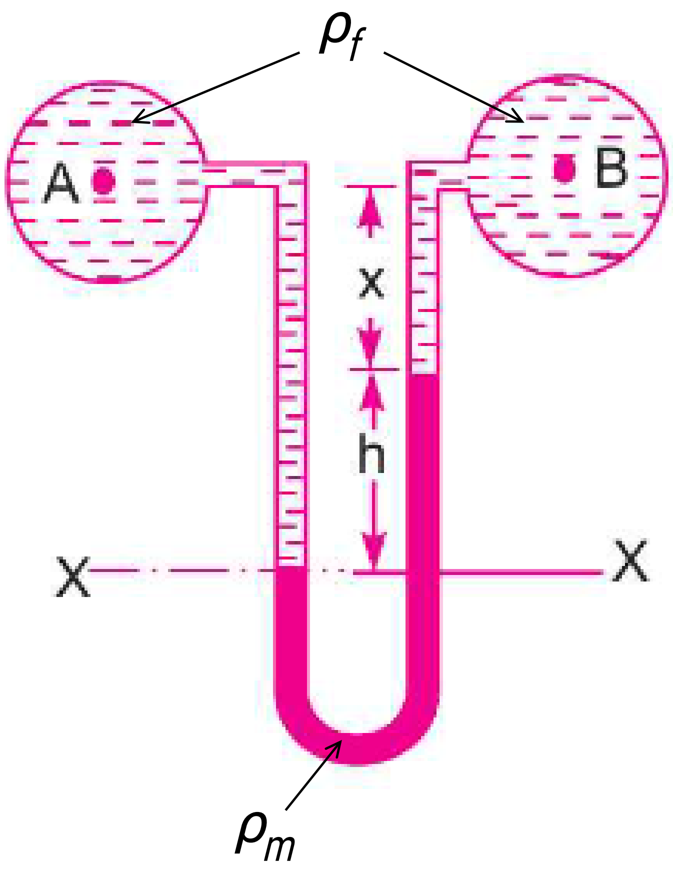
3. U-tube differential manometer



$$p_A + \rho_1 g h_1 - \rho_2 g h_2 - \rho_3 g h_3 = p_B$$

$$\Rightarrow \quad p_A - p_B = (\rho_3 h_3 + \rho_2 h_2 - \rho_1 h_1) g$$

In most of the cases, pipes A and B will be at same height and will have same fluid flowing in them (which is the case in our AFM lab experiments)



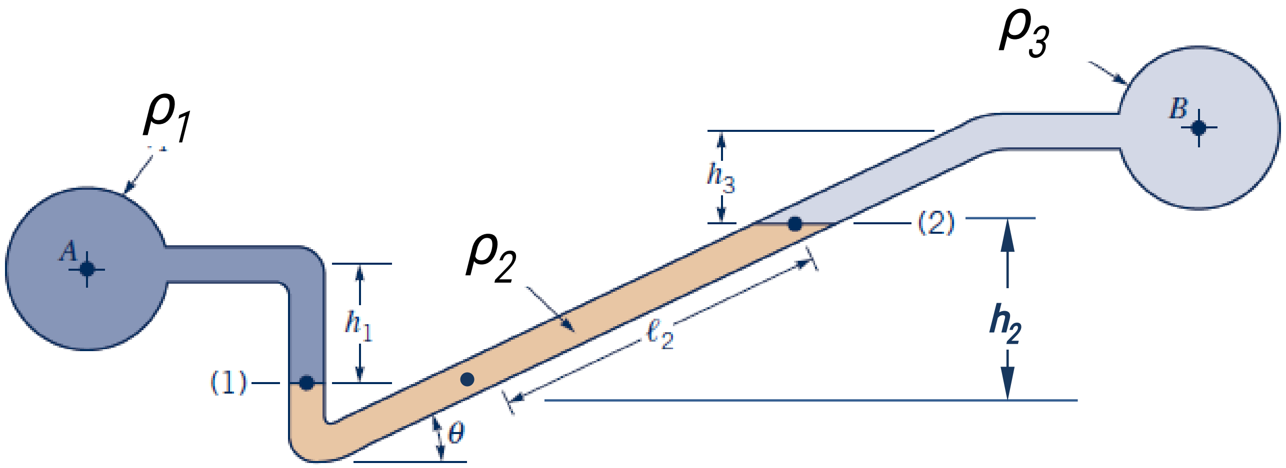
$$p_A + \rho_f g(h + x) - \rho_m gh - \rho_f gx = p_B$$

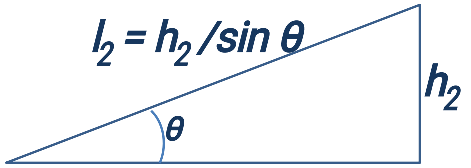
$$\Rightarrow \quad p_A - p_B = \rho_m gh - \rho_f gh$$

$$\Rightarrow \quad \frac{p_A - p_B}{\rho_f g} = \left(\frac{\rho_m}{\rho_f} - 1 \right) h$$

Differential head $H = \dfrac{p_A - p_B}{\rho_f g} = \left(\dfrac{\rho_m}{\rho_f} - 1 \right) h$

4. Inclined-tube manometer



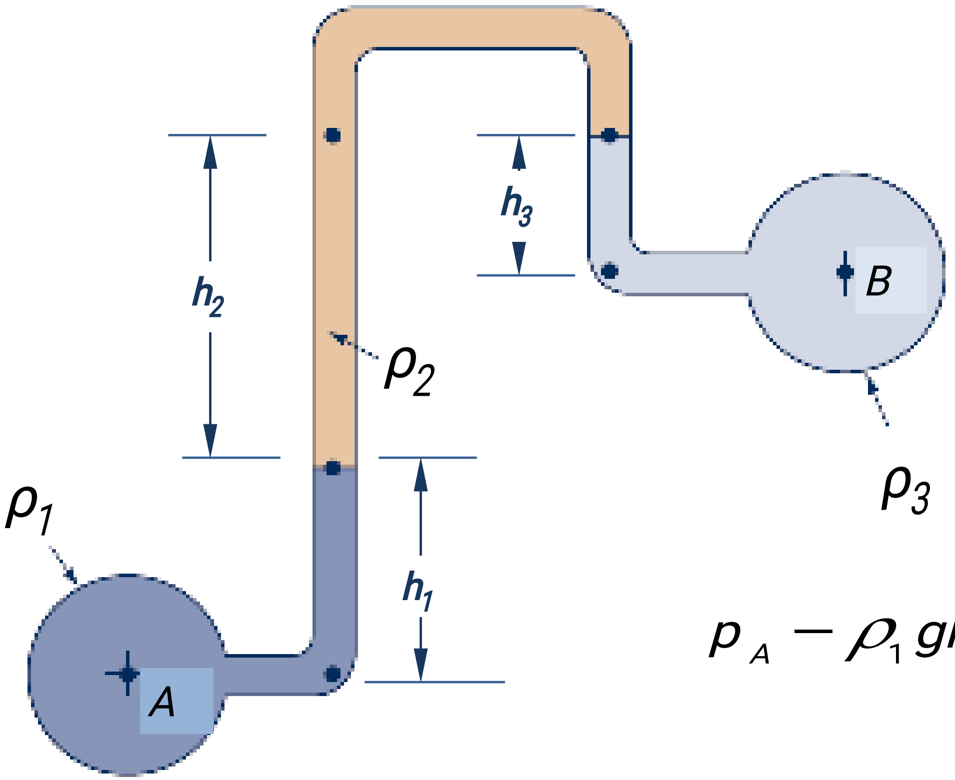


$$p_A + \rho_1 g h_1 - \rho_2 g h_2 - \rho_3 g h_3 = p_B$$

$$\Rightarrow \quad p_A + \rho_1 g h_1 - \rho_2 g l_2 \sin\theta - \rho_3 g h_3 = p_B$$

$$\Rightarrow \quad p_A - p_B = (\rho_3 h_3 + \rho_2 l_2 \sin\theta - \rho_1 h_1)\, g$$

5. Inverted U-tube differential manometer



$$p_A - \rho_1 g h_1 - \rho_2 g h_2 + \rho_3 g h_3 = p_B$$

$$\Rightarrow \quad p_A - p_B = \rho_1 g h_1 + \rho_2 g h_2 - \rho_3 g h_3$$