Basics of Heat Transfer

Modes of heat transfer

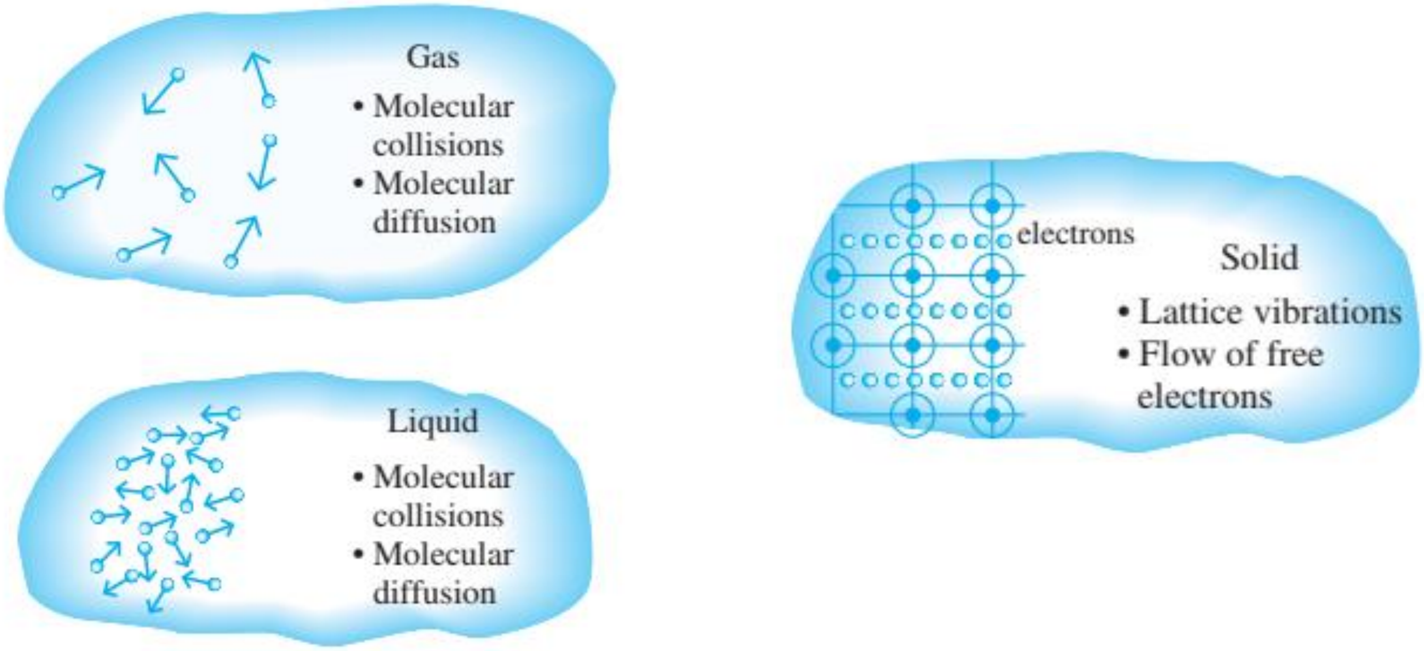
- we defined *heat* as the form of energy that can be transferred from one system to another as a result of temperature difference.
- Thermodynamics is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another.
- The science that deals with the determination of the *rates* of such energy transfers is the *heat transfer*.

Heat can be transferred in three different modes: *conduction*, *convection*, and *radiation*.

CONDUCTION

- **Conduction** is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

mechanisms of heat conduction in different phases of a substance :



Experiments have shown that :

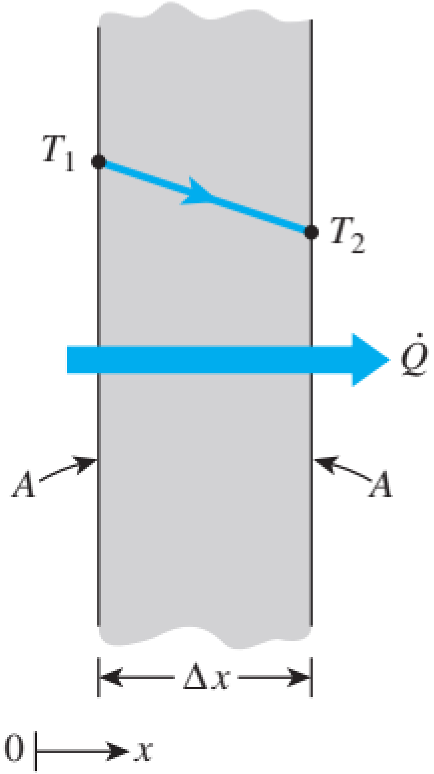


FIGURE 1–23
Heat conduction through a large plane wall of thickness Δx and area A.

$$\text{Rate of heat conduction} \propto \frac{\text{(Area)(Temperature difference)}}{\text{Thickness}}$$

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{\Delta x}$$

$$= -kA\frac{\Delta T}{\Delta x}$$

where the constant of proportionality k is the **thermal conductivity** of the material, which is a *measure of the ability of a material to conduct heat*

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx}$$

In the limiting case of $\Delta x \rightarrow 0$, the equation above reduces to the differential form:

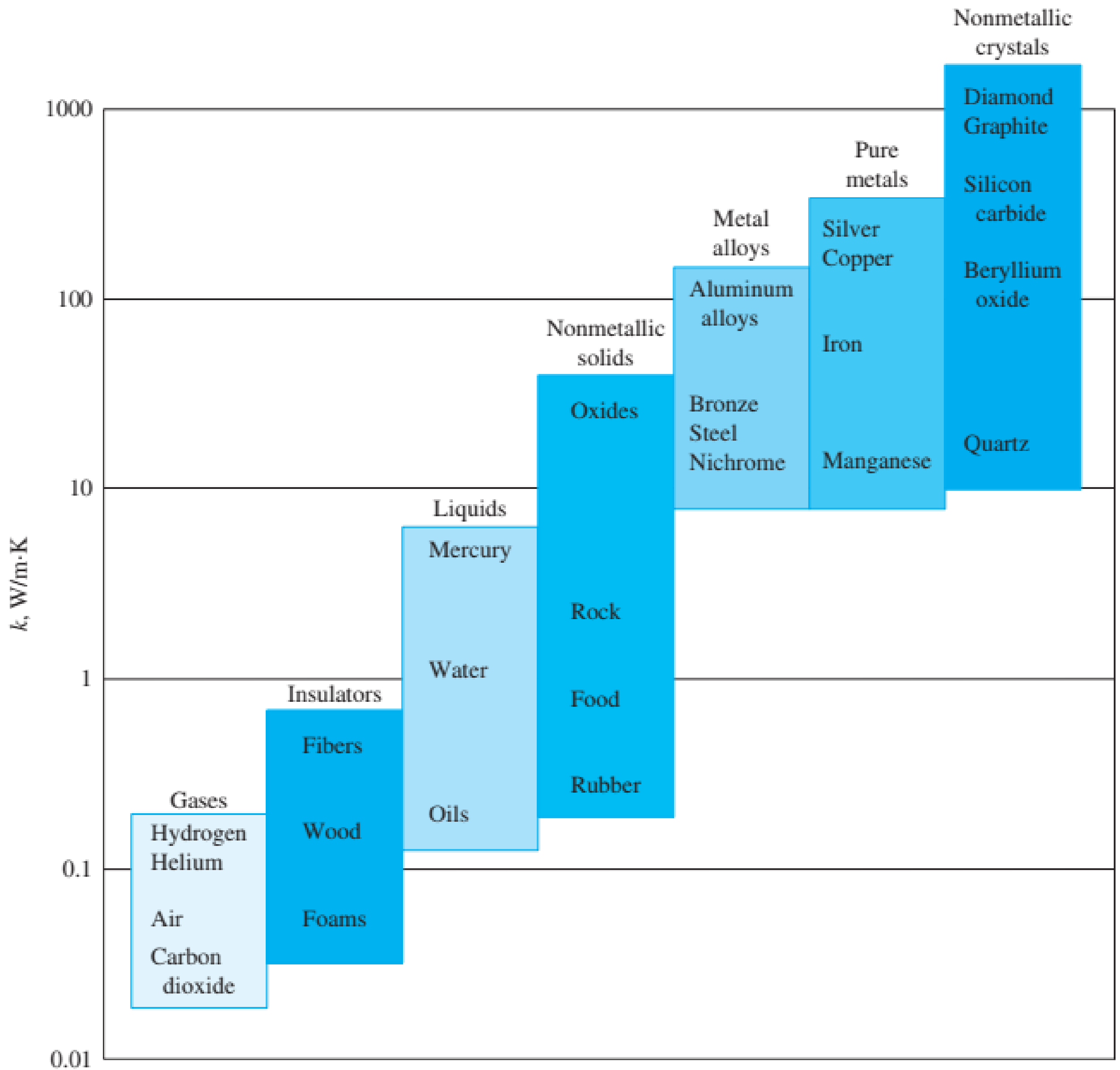
$$\dot{Q}_{\text{cond}} = -kA\,\frac{dT}{dx}$$

Fourier's law of heat conduction

Here dT/dx is the **temperature gradient**, which is the slope of the temperature curve on a *T-x* diagram

Thermal Conductivity *K*

- the **thermal conductivity** of a material can be defined as *the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.*
- The thermal conductivity of a material is a measure of the ability of the material to conduct heat.
- A high value of k indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator.*



Nonmetallic
crystals
Diamond
Graphite
Silicon
carbide
Beryllium
oxide
Quartz
Pure
metals
Silver
Copper
Iron
Manganese
Metal
alloys
Aluminum
alloys
Bronze
Steel
Nichrome
Nonmetallic
solids
Oxides
Rock
Food
Rubber
Liquids
Mercury
Water
Oils
Insulators
Fibers
Wood
Foams
Gases
Hydrogen
Helium
Air
Carbon
dioxide
1000
100
10
1
0.1
0.01
k, W/m·K

Thermal Diffusivity

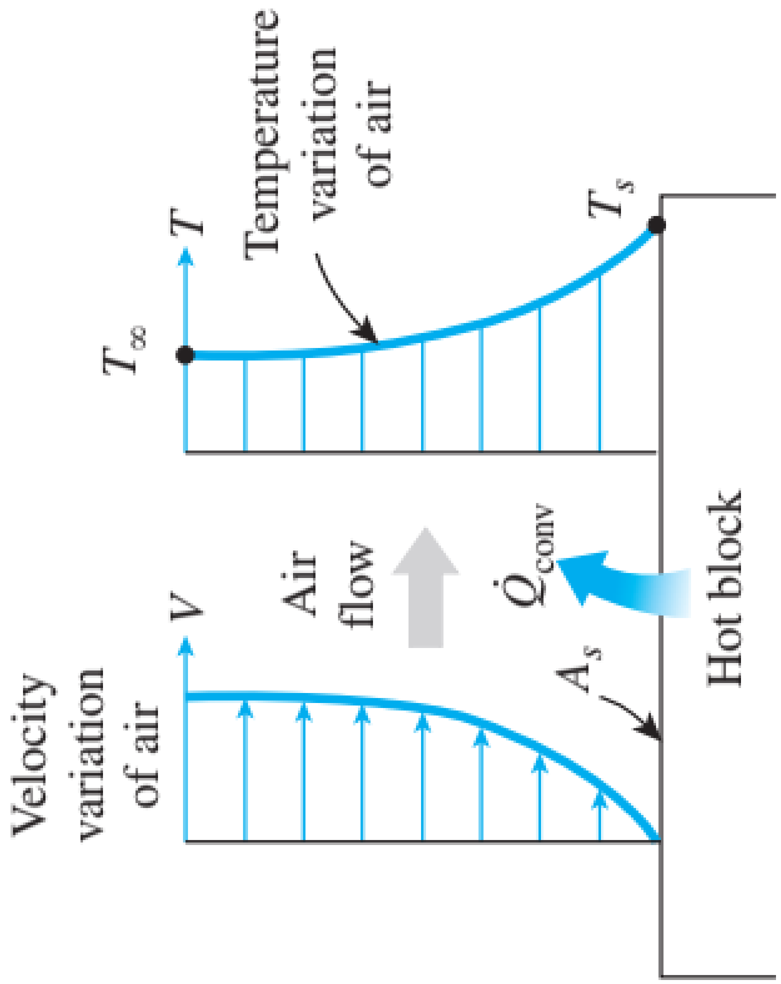
The material property that appears in the transient heat conduction analysis is the **thermal diffusivity**, which represents how fast heat diffuses through a material and is defined as

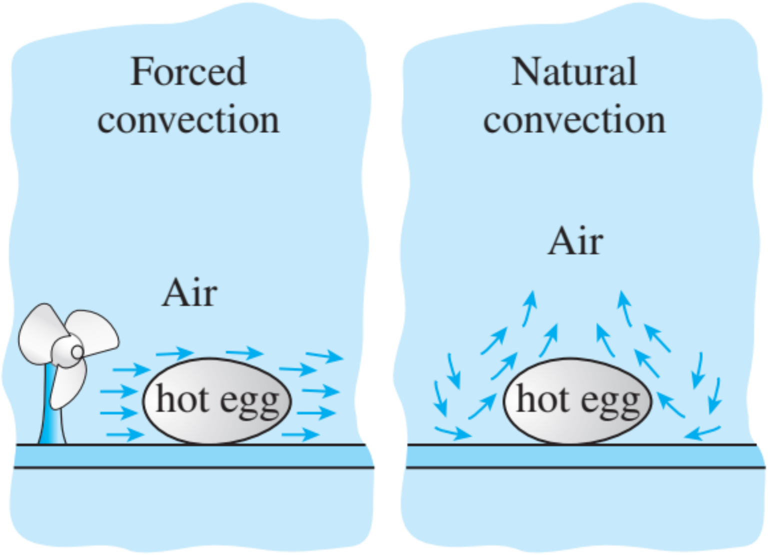
$$\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p}$$

The thermal conductivity k represents how well a material conducts heat, and the heat capacity ρc_p represents how much energy a material stores per unit volume.

CONVECTION

- **Convection** is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion,





- Convection is called **forced convection** if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.
- Convection is called **natural** (or **free**) **convection** if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.

The rate of *convection heat transfer* is

$$\dot{Q}_{\text{conv}} = hA_s\,(T_s - T_\infty)$$ **Newton's law of cooling**

where, h is the *convection heat transfer coefficient*

A_s is the surface area through which convection heat transfer takes place,
T_s is the surface temperature, and
T_∞ is the temperature of the fluid sufficiently far from the surface.

RADIATION

Radiation is the energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules

In heat transfer studies we are interested in *thermal radiation*, which is the form of radiation emitted by bodies because of their temperature. All bodies at a temperature above absolute zero emit thermal radiation

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature *Ts* (in K) is given by the **Stefan–Boltzmann law** as

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$$

where $\sigma = 5.670 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$ is the *Stefan–Boltzmann constant.*

The idealized surface that emits radiation at this maximum rate is called a **blackbody**, and the radiation emitted by a blackbody is called **blackbody radiation**

The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\dot{Q}_{\text{emit}} = \varepsilon\sigma A_s T_s^4$$

where ε is the **emissivity** of the surface. The property emissivity, whose value is in the range 0 to 1, is a measure of how closely a surface approximates a blackbody for which ε=1

When a surface of emissivity ε and surface area A_s at a *thermodynamic temperature* T_s is *completely enclosed* by a much larger (or black) surface at thermodynamic temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by (Fig. 1–40)

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s\,(T_s^4 - T_{\text{surr}}^4)$$

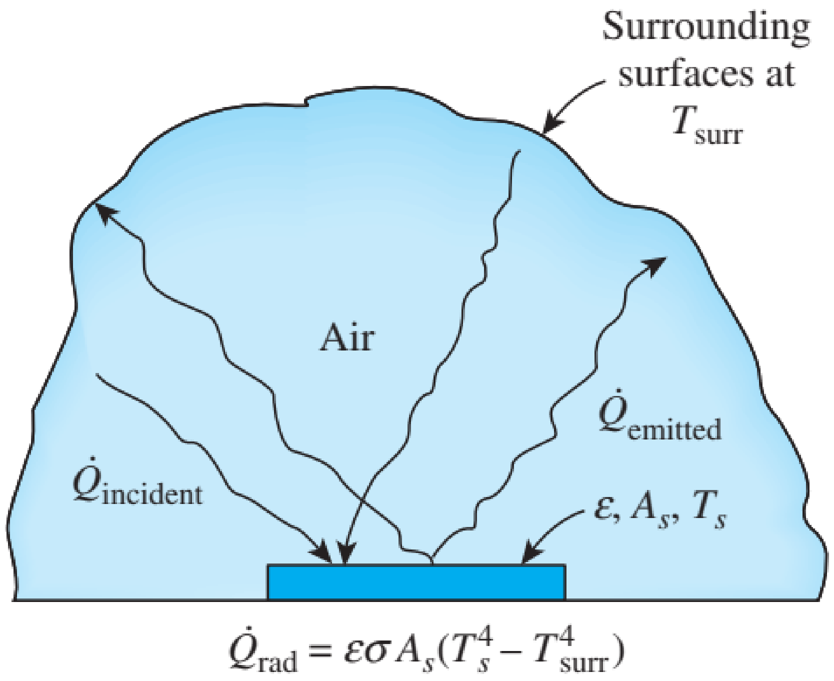
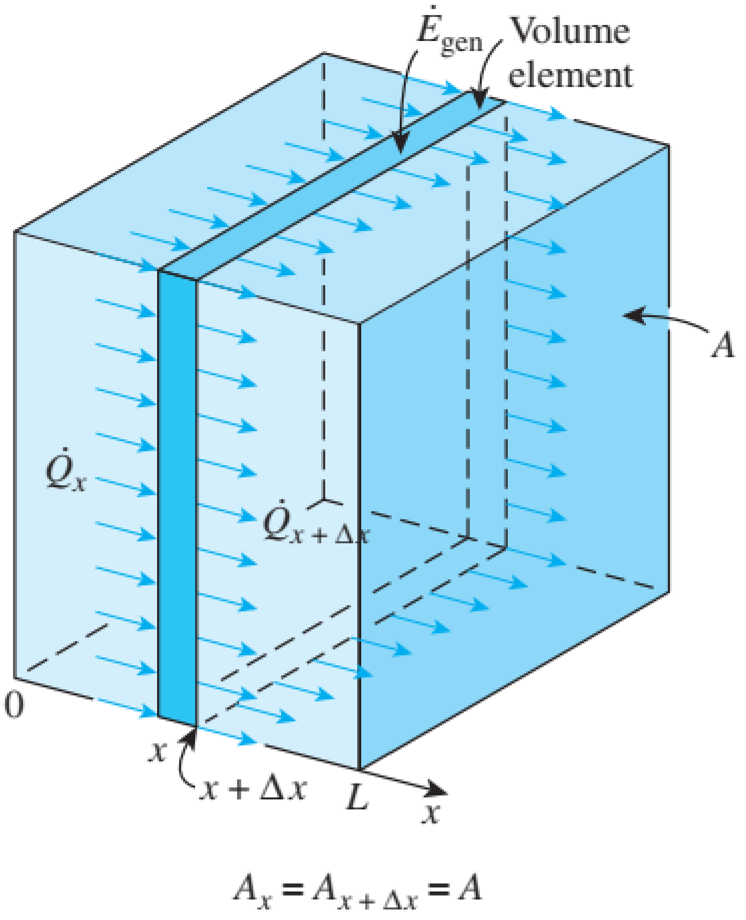


FIGURE 1–40

Radiation heat transfer between a surface and the surfaces surrounding it.

ONE-DIMENSIONAL HEAT CONDUCTION EQUATION



Heat Conduction Equation in a Large Plane Wall

Consider a thin element of thickness Δx in a large plane wall, as shown.

Assume the density of the wall is ρ, the specific heat is c, and the area of the wall normal to the direction of heat transfer is A.

An *energy balance* on this thin element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{E}_{\text{gen, element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

the change in the energy content of the element

$$\begin{aligned}\Delta E_{\text{element}} &= E_{t+\Delta t} - E_t \\ &= mc(T_{t+\Delta t} - T_t) \\ &= \rho c A \Delta x (T_{t+\Delta t} - T_t)\end{aligned}$$

the rate of heat generation within the element

$$\begin{aligned}\dot{E}_{\text{gen, element}} &= \dot{e}_{\text{gen}} V_{\text{element}} \\ &= \dot{e}_{\text{gen}} A \Delta x\end{aligned}$$

On substituting,

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{e}_{\text{gen}} A \Delta x = \rho c A \Delta x \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $A\Delta x$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x}-\dot{Q}_x}{\Delta x}+\dot{e}_{\text{gen}}=\rho c\,\frac{T_{t\,+\,\Delta t}-T_t}{\Delta t}$$

Taking the limit as $\Delta x \to 0$ and $\Delta t \to 0$,

$$\lim_{\Delta x \to 0}\frac{\dot{Q}_{x+\Delta x}-\dot{Q}_x}{\Delta x}=\frac{\partial \dot{Q}}{\partial x}$$

$$=\frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)$$ (From Fourier’s law of heat conduction)

On substituting,

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\,\frac{\partial T}{\partial x}\right)+\dot{e}_{\text{gen}}=\rho c\,\frac{\partial T}{\partial t}$$

The *one-dimensional transient* heat conduction equation in a plane wall becomes,

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \dot{e}_{\text{gen}} = \rho c\,\frac{\partial T}{\partial t}$$

For constant thermal conductivity k,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\,\frac{\partial T}{\partial t}$$

Further simplifications:

(1) *Steady-state:* $(\partial/\partial t = 0)$

$$\frac{d^2 T}{dx^2} + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

(2) *Transient, no heat generation:* $(\dot{e}_{\text{gen}} = 0)$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\,\frac{\partial T}{\partial t}$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0 \text{ and } \dot{e}_{\text{gen}} = 0)$

$$\frac{d^2 T}{dx^2} = 0$$

Consider a large plane wall of thickness $L = 0.2$ m, thermal conductivity $k = 1.2$ W/m·K, and surface area $A = 15$ m^2. The two sides of the wall are maintained at constant temperatures of $T_1 = 120°C$ and $T_2 = 50°C$, respectively. Determine (i) the variation of temperature within the wall and the value of temperature at $x = 0.1$m and (ii) the rate of heat conduction through the wall under steady conditions.

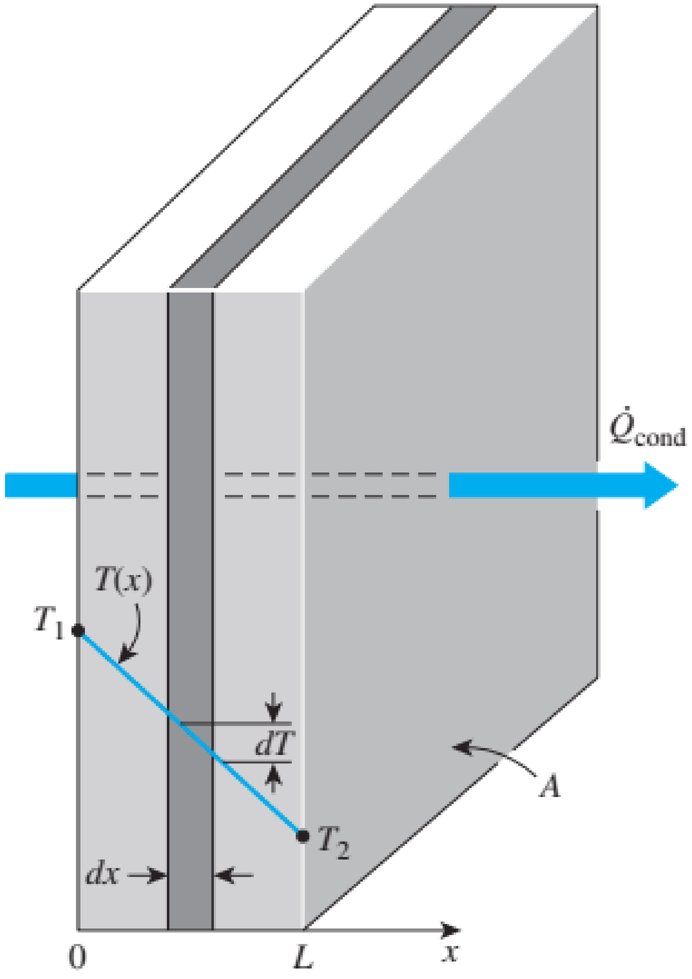
ONE DIMENSIONAL HEAT CONDUCTION EQUATION

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \dot{e}_{\text{gen}} = \rho c\,\frac{\partial T}{\partial t}$$

THREE DIMENSIONAL HEAT CONDUCTION EQUATION

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\,\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right) + \dot{e}_{\text{gen}} = \rho c\,\frac{\partial T}{\partial t}$$

STEADY HEAT CONDUCTION IN PLANE WALLS



$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \cancelto{0}{\frac{dE_{wall}}{dt}}$$

$$\dot{Q}_{cond,\ wall} = \text{constant}$$

Fourier's law of heat conduction for the wall can be expressed as

$$\dot{Q}_{cond,\ wall} = -kA\frac{dT}{dx}$$

On re-arranging the variables and integrating between 1 and 2,

$$\int_{x=0}^{L} \dot{Q}_{cond,\ wall}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L}$$

the rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness

Thermal Resistance Concept

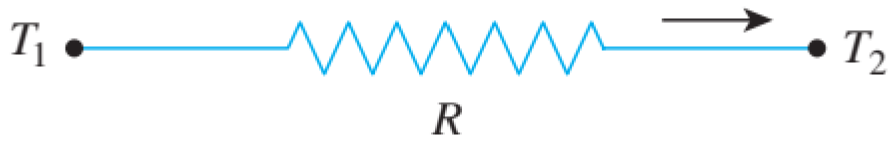
heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}}$$

where, $R_{\text{wall}} = \dfrac{L}{kA}$ is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall

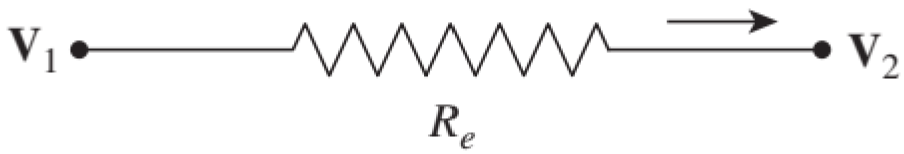
Analogy between thermal and electrical resistance concepts

$$\dot{Q} = \frac{T_1 - T_2}{R}$$



(*a*) Heat flow

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$



(*b*) Electric current flow

Thermal resistance R can be conduction resistance, convective resistance or radiation resistance

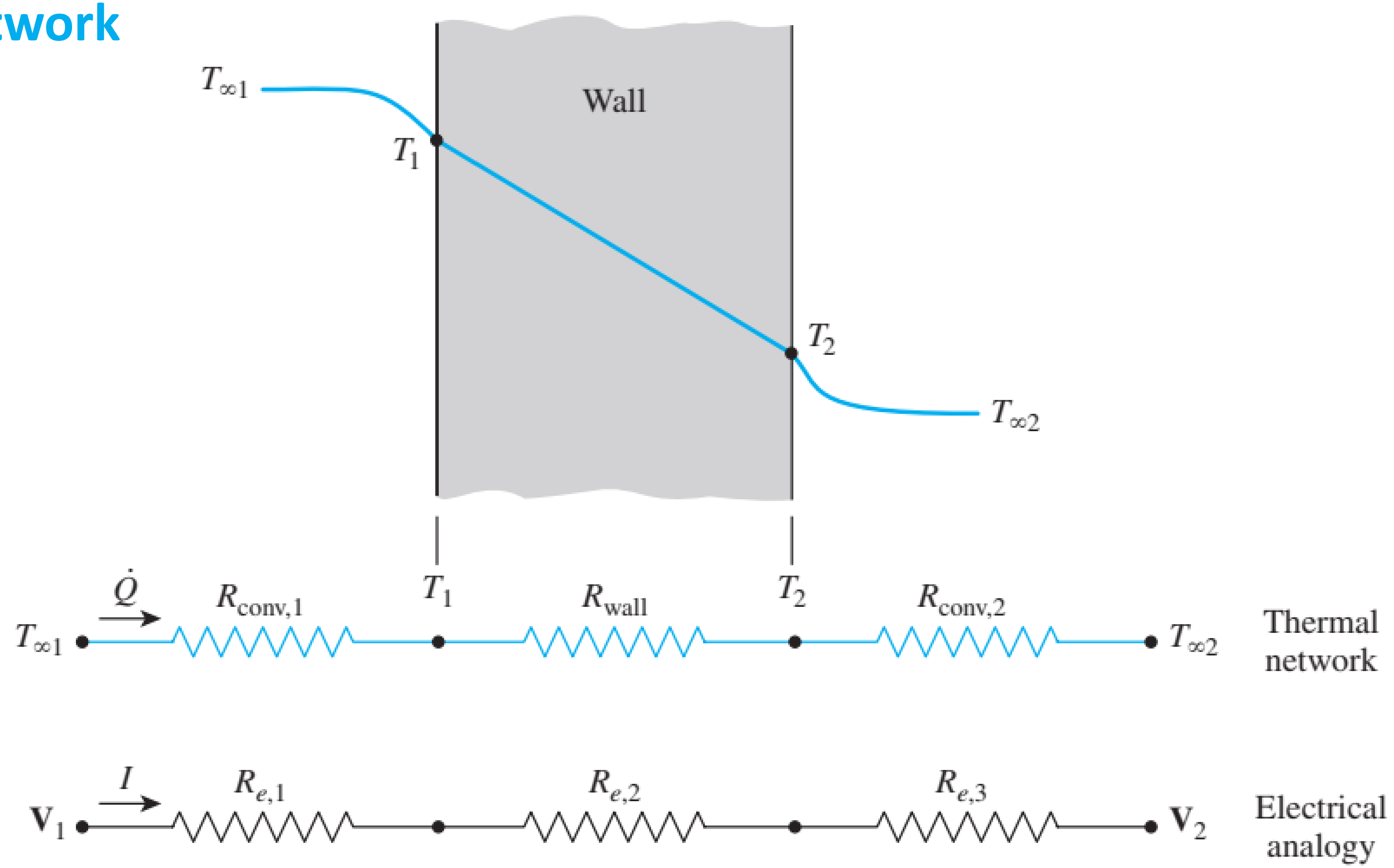
$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad R_{\text{wall}} = \frac{L}{kA}$$

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \qquad R_{\text{conv}} = \frac{1}{hA_s}$$

$$\dot{Q}_{\text{rad}} = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \qquad R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s}$$

Where, $h_{\text{rad}} = \dfrac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$

Thermal Resistance Network



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv},1} + R_{\text{wall}} + R_{\text{conv},2}}$$

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$ where, $$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

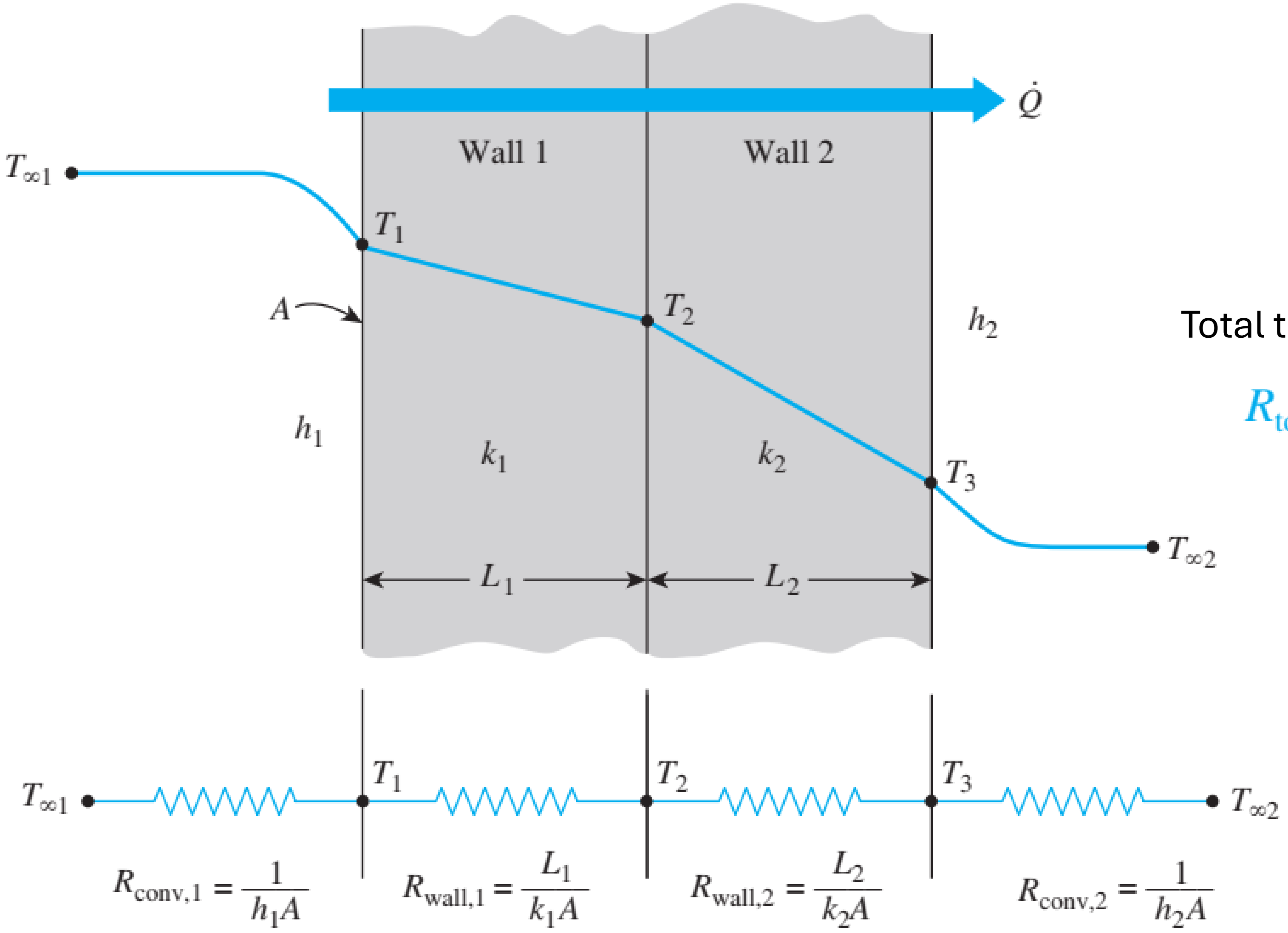
$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\,\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

which can be rearranged as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$

Once Q is known, this can be used to determine the intermediate temperatures T_1 or T_2.



Heat transfer rate

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

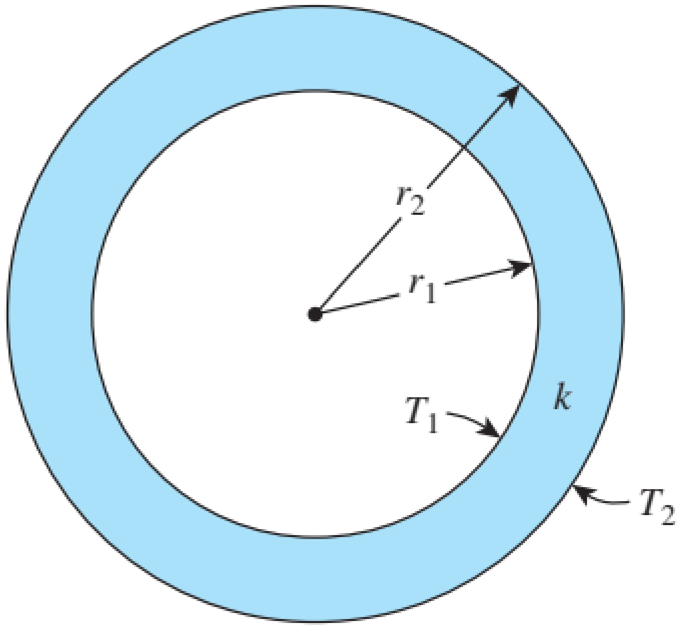
Total thermal resistance

$$R_{total} = R_{conv,\,1} + R_{wall,\,1} + R_{wall,\,2} + R_{conv,\,2}$$

$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m·K. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is 210°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2·K and $h_2 = 40$ W/m^2·K.

HEAT CONDUCTION IN CYLINDERS



Rate of heat conduction through a cylindrical pipe

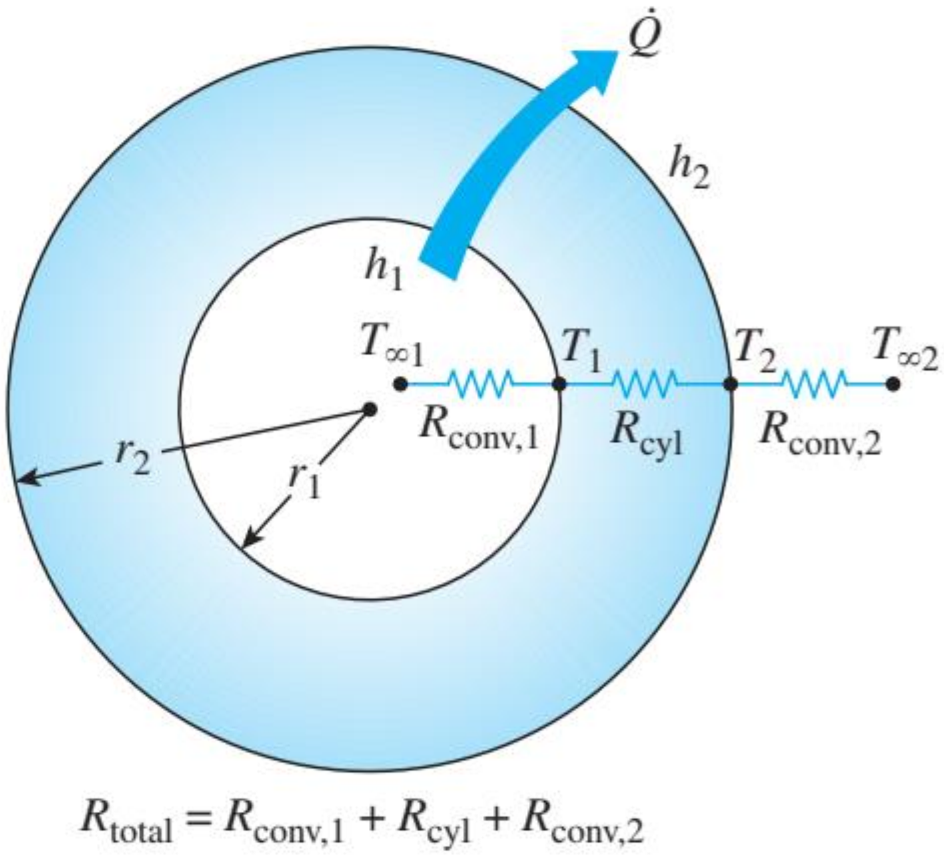
$$\dot{Q}_{\text{cond, cyl}} = -kA\,\frac{dT}{dr}$$

On rearranging and integrating between inner radius and outer radius

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT$$

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\,\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

Thermal Resistance Network in cylinder

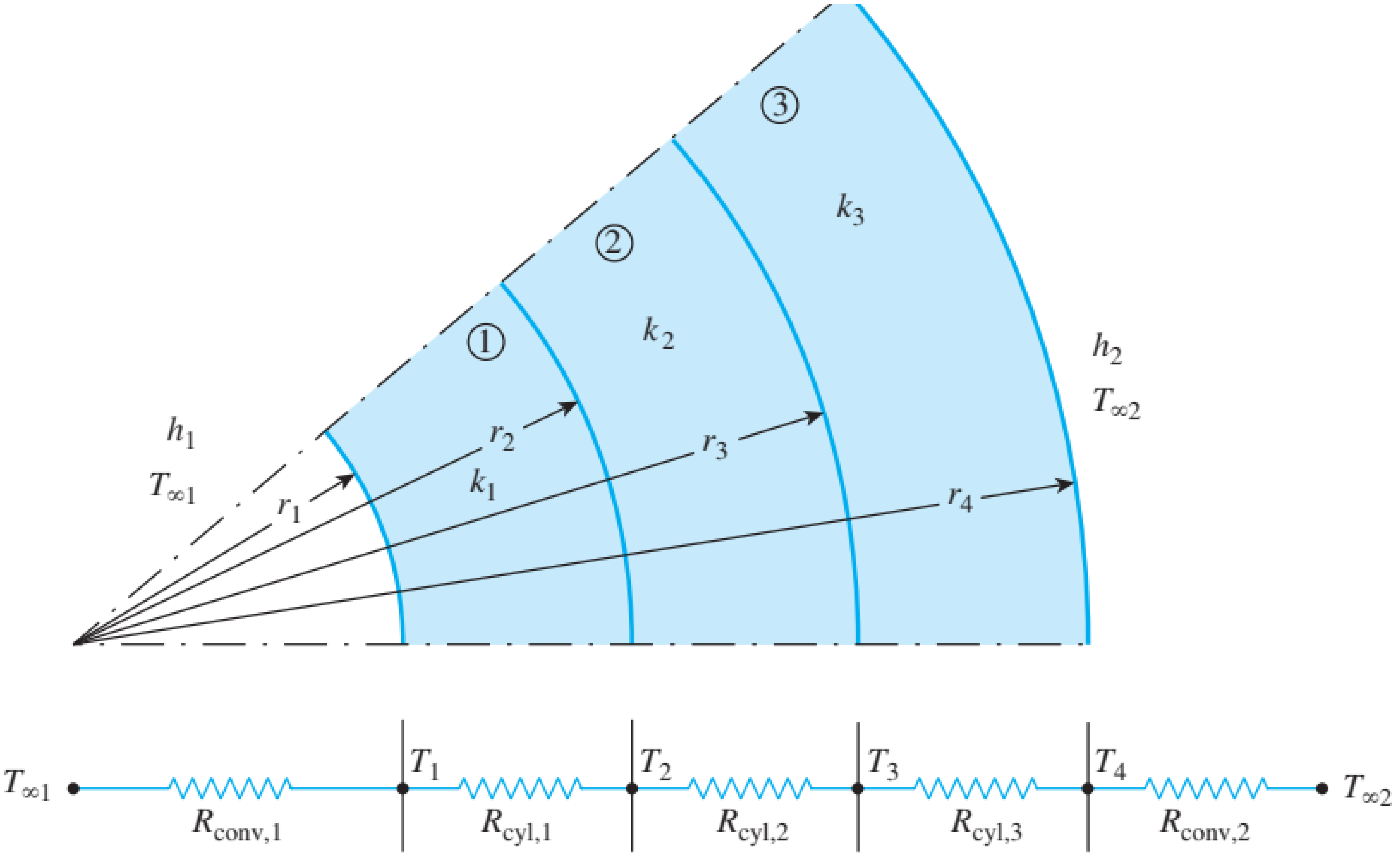


Heat transfer rate

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

Total thermal resistance is

$$R_{total} = R_{conv,\,1} + R_{cyl} + R_{conv,\,2}$$
$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv},\,1} + R_{\text{cyl},\,1} + R_{\text{cyl},\,2} + R_{\text{cyl},\,3} + R_{\text{conv},\,2} \\ &= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4} \end{aligned}$$

4 *An insulated steel pipe carrying a hot liquid. Inner diameter of the pipe is 25 cm, wall thickness is 2 cm, thickness of insulation is 5 cm, temperature of hot liquid is 100° C, temperature of surrounding is 20° C, inside heat transfer co-efficient is 730 W/m^2K and outside heat transfer co-efficient is 12W/m^2K. Calculate the heat loss per metre length of the pipe.*

Take k_{steel} = 55 W/mK, $k_{insulating\ material}$ = 0.22 W/mK