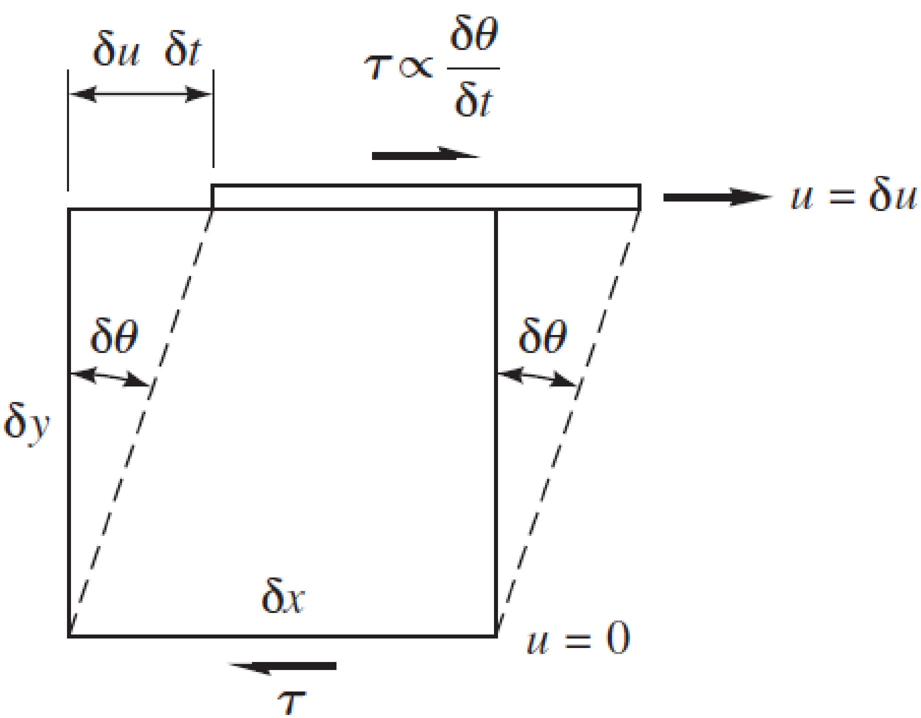
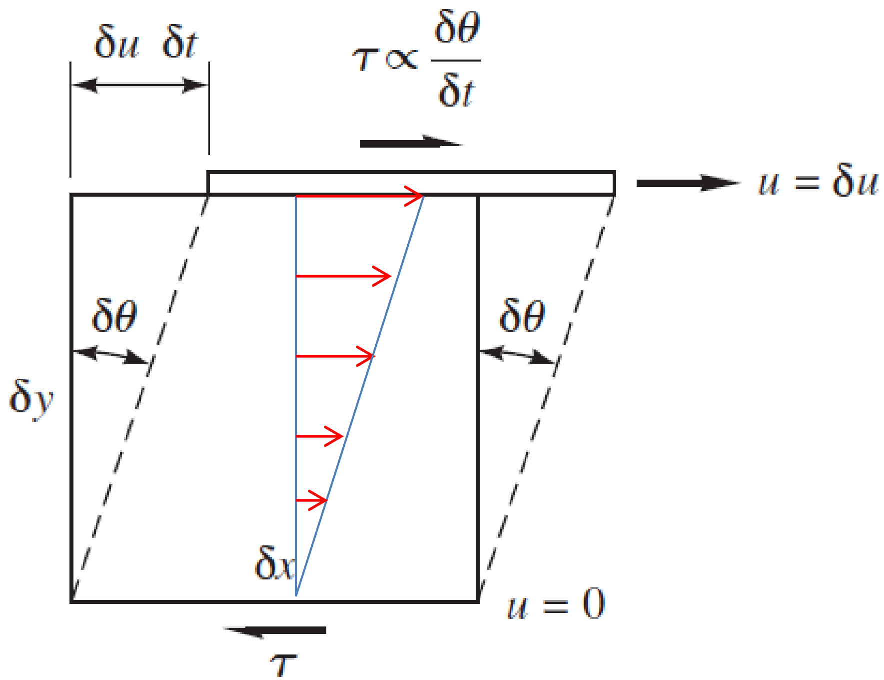
Viscosity

•Viscosity is a quantitative measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. The property which characterizes the fluid's resistance to shear force.
•Unlike solid's, the fluid's resistance to shear stress does not depend upon the deformation itself but on the rate of deformation.

shear stress $\propto$ shear strain rate



$$\tau \propto \frac{\delta\theta}{\delta t}$$



$$\tan \delta\theta = \frac{\delta u\ \delta t}{\delta y}$$

In the limit of infinitesimal changes

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

$$\tau \propto \frac{du}{dy} \Rightarrow \boxed{\tau = \mu \frac{du}{dy}}$$

Newton's Law of Viscosity

- **Applied Shear stress is proportional to the velocity gradient.**
- **The constant of proportionality is the *DYNAMIC VISCOSITY* (or) *VISCOSITY* (or) COEFFICIENT OF VISCOSITY (*μ*)**

Causes of viscosity

The causes of viscosity in a fluid are attributed to two factors:

1. intermolecular force of cohesion
2. molecular momentum exchange

Effect of temperature on viscosity

- For liquids, **viscosity decrease with increase in temperature.**

  Reason: In liquids, the forces of cohesion are more significant. molecular motion is negligible.

- For gases, **viscosity increase with increase in temperature.**

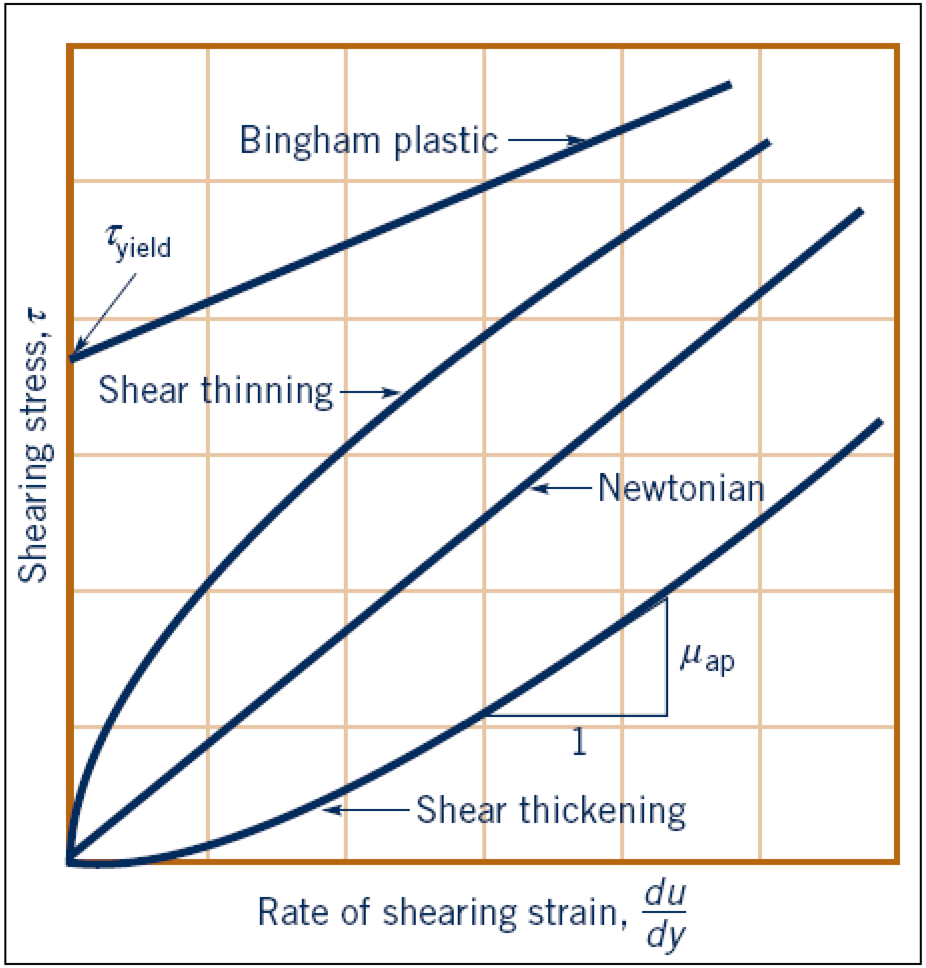
  Reason: In gases, molecular motion is more significant than the cohesive forces are negligible.

For air, the Sutherland's relation for dynamic viscosity is,

$$\boxed{\mu = 1.46 \times 10^{-6} \left(\frac{T^{3/2}}{T + 111} \right) \text{[kg/(m s)]}}$$

T in Kelvin, the equation is valid up to 3000 K.

Types of Fluids based on Viscosity



$$\tau \propto \frac{du}{dy}$$ --- Newtonian Fluid

Eg.: air, water, gases, crude oil, many common fluids

$$\tau \not\propto \frac{du}{dy}$$ --- Non-Newtonian Fluid

Eg.: blood, concrete slurry, paint, polymers etc

Shear thinning fluids - the apparent viscosity decreases with increasing shear rate—the harder the fluid is sheared, the less viscous it becomes. Eg. Polymers and colloidal suspensions.

Shear thickening fluids - the apparent viscosity increases with increasing shear rate—the harder the fluid is sheared, the more viscous it becomes. Eg. Water-sand mixture.

(***Apparent Viscosity** – the slope of the shearing stress versus rate of shearing strain graph)

Bingham plastic – neither a fluid nor a solid. Can withstand a finite shear stress without motion. But once the value exceed yield stress, it flows like a fluid. Eg. Thoothpaste

$\mu=0$ (or) $\tau=0$	--- Ideal Fluid
$\mu>0$	--- Real fluid

In this course, we only deal about Newtonian fluids $\tau = \mu \frac{du}{dy}$

Units of dynamic viscosity

SI unit of viscosity: N.s/m^2 (or) kg/(m.s) (or) Pa.s

CGS unit of viscosity:

- dyne.s/cm^2 (or) Poise ------ 1 N.s/m^2 = 10 Poise (P)
- centipoise ------- 1 Poise = 100 centiPoise (cP)

$$poise = \frac{dyne.s}{cm^2} = \frac{\frac{g\ cm}{s^2}.s}{cm^2} = \frac{\frac{0.001\text{kg}\ 0.01m}{s^2}.s}{(0.01m)^2} = 0.1\frac{kg}{ms}$$

$$= 0.1\frac{\frac{kg\ m}{s^2}\frac{s^2}{m}}{ms} = 0.1\frac{Ns}{m^2}$$

Kinematic viscosity

Kinematic viscosity(ν) is the ratio of Dynamic Viscosity to Density

$$\nu = \frac{\mu}{\rho}$$

Units of Kinematic viscosity

SI unit of kinematic viscosity: m^2/s

CGS unit of kinematic viscosity:

- cm^2/s (or) Stokes ---- 1 m^2/s = 10^{-4} Stokes (St)
- centiStokes ---- 1 Stokes = 100 centiStokes (cSt)

Problem 1:

If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which *u* is the velocity in metre per second at a distance *y* metre above the plate, determine the shear stress at *y*=0 and *y*=0.15m. Take dynamic viscosity of fluid as 8.63 poises.

Solution. Given : $u = \frac{2}{3}y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2.\ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2.\ Ans.}$$

Problem 2:

Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area $0.5 m^2$ between the two large plane surfaces at a speed of 0.6m/s, if:

(i) The thin plate is in the middle of the two plane surfaces and

(ii) The thin plate is at a distance of 0.8cm from one of the plane surfaces.

Take the dynamic viscosity of glycerine = 8.1×10^{-1} Ns/m^2

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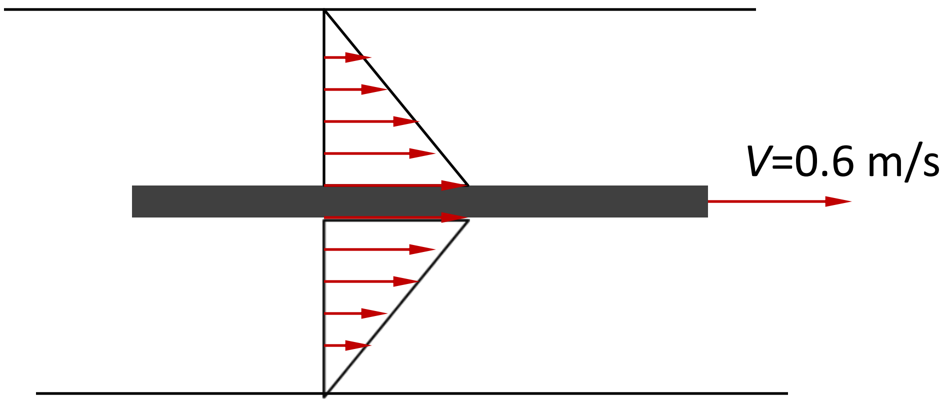

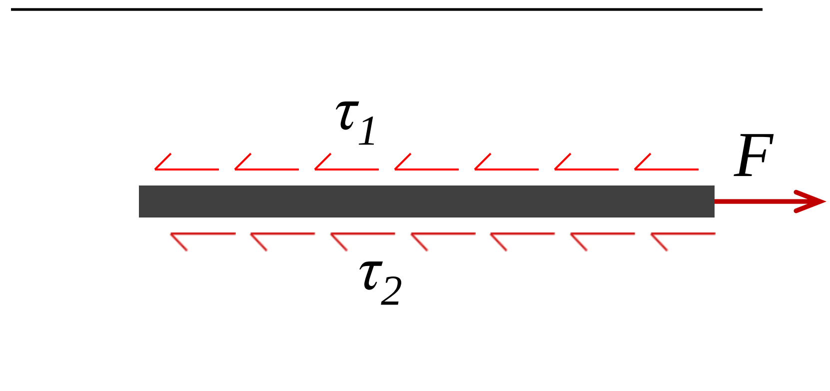

$$
\begin{aligned}
F &= F_1 + F_2 \\
&= \tau_1 A + \tau_2 A
\end{aligned}
$$

A – Surface area of plate

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Solution. Given :

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5 \text{ m}^2$

Velocity of thin plate, $u = 0.6$ m/s

Viscosity of glycerine, $\mu = 8.10 \times 10^{-1}$ N s/m^2

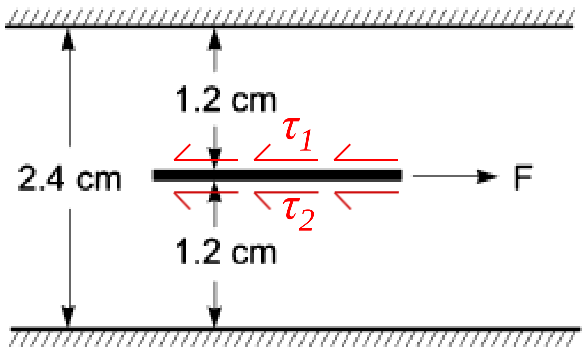


Fig. 1.7 (*a*)

Case I. When the thin plate is in the middle of the two plane surfaces [Refer to Fig. 1.7 (*a*)]

Let F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then $$F = F_1 + F_2$$

The shear stress (τ_1) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu \left(\frac{du}{dy}\right)_1$$

where du = Relative velocity between thin plate and upper large plane surface
= 0.6 m/sec

dy = Distance between thin plate and upper large plane surface
= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

$$\therefore \qquad \tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{.012}\right) = 40.5 \text{ N/m}^2$$

Now shear force, $\quad F_1$ = Shear stress × Area

$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu \left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

$\therefore$ Shear force, $\quad F_2 = \tau_2 \times \text{A} = 40.5 \times 0.5 = 20.25$ N

$\therefore$ Total force, $\quad F = F_1 + F_2 = 20.25 + 20.25 =$ **40.5 N. Ans.**

Case II. When the thin plate is at a distance of 0.8 cm from one of the plane surfaces [Refer to Fig. 1.7 (*b*)].

Let the thin plate is at a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface

$$= 2.4 - 0.8 = 1.6 \text{ cm} = .016 \text{ m}$$

(Neglecting thickness of the plate)

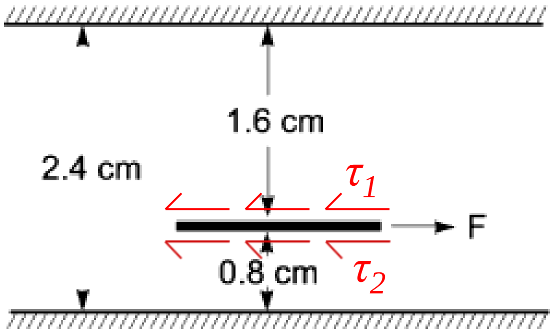


Fig. 1.7 (*b*)

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu \left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.18 \text{ N}$$

The shear force on the lower side of the thin plate,

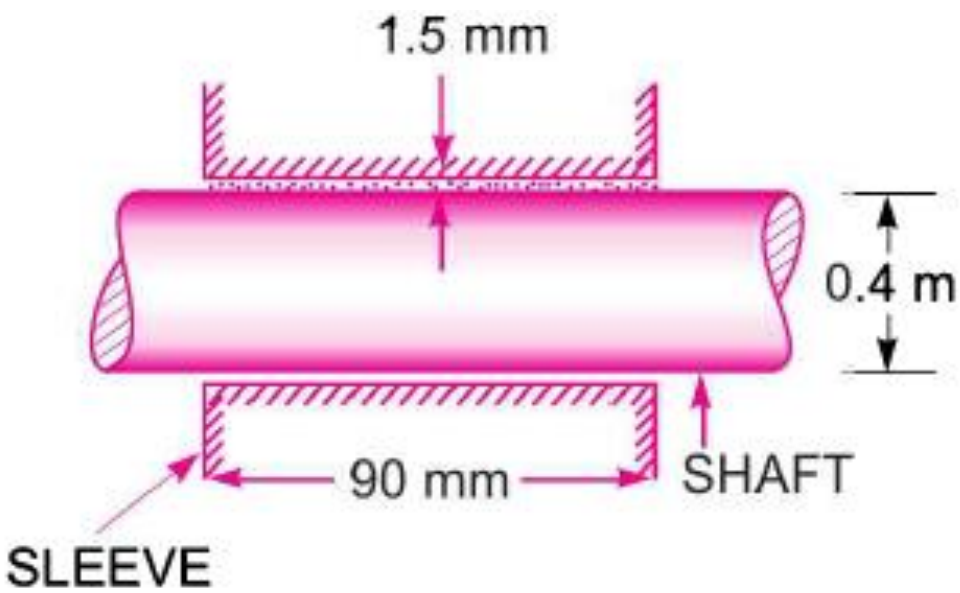
$$F_2 = \tau_2 \times A = \mu \left(\frac{du}{dy}\right)_2 \times A$$

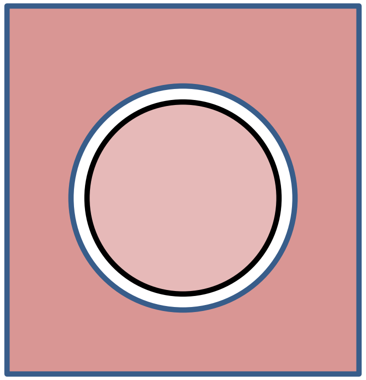
$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.36 \text{ N}$$

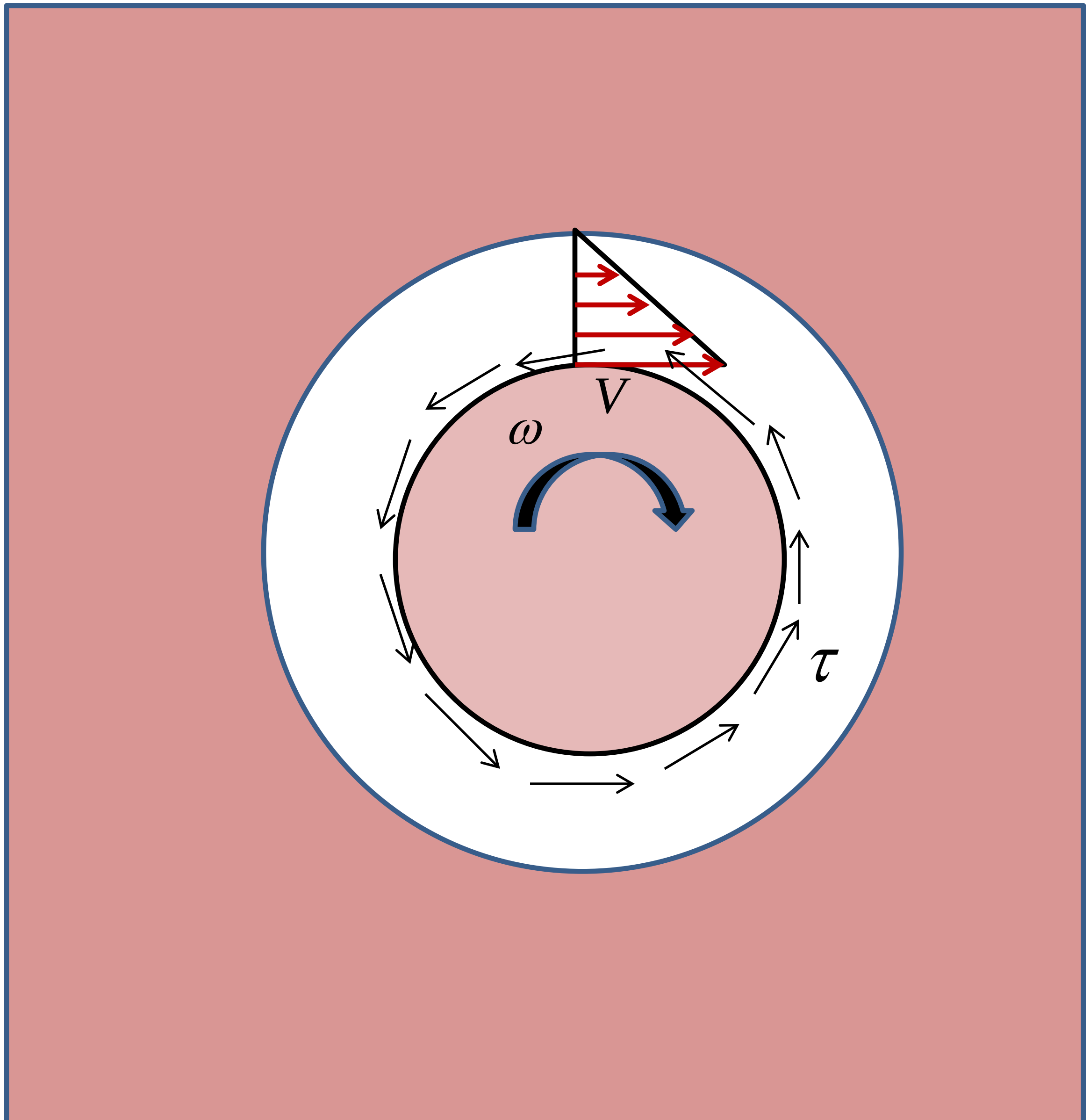
$\therefore$ Total force required $= F_1 + F_2 = 15.18 + 30.36 =$ **45.54 N. Ans.**

Problem 3:

The dynamic viscosity of an oil used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4m and rotates at 190rpm. Calculate the power lost in the bearing for a sleeve length of 90mm. The thickness of the oil film is 1.5mm.







$$V = \frac{D}{2}\omega = \frac{D}{2}\frac{2\pi N}{60}$$

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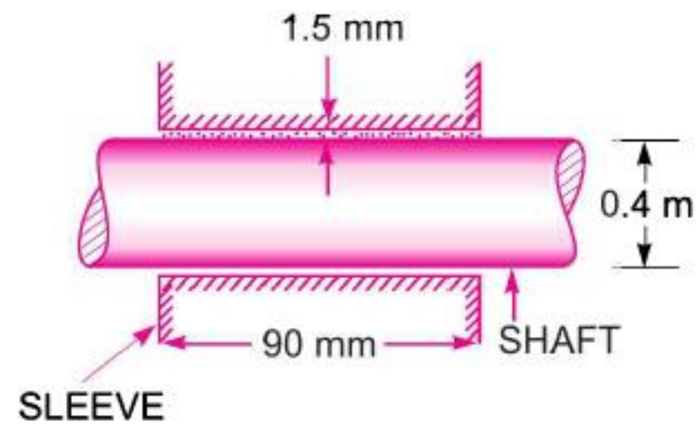


Solution. Given :

Viscosity $\mu = 6$ poise

$$= \frac{6}{10}\frac{\text{N s}}{\text{m}^2} = 0.6\ \frac{\text{N s}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

$\therefore$ Shear force on the shaft, F = Shear stress $\times$ Area

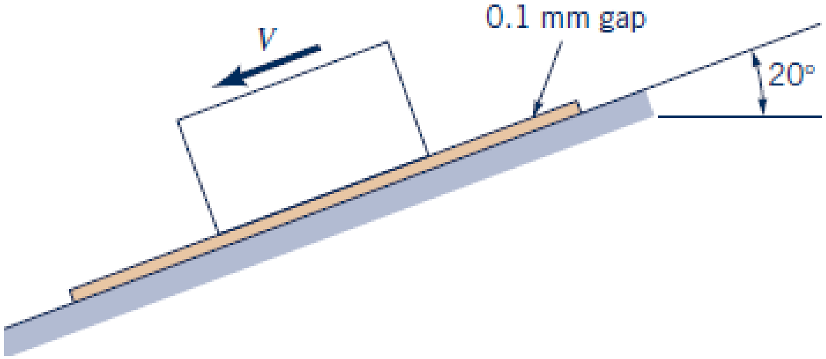
$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

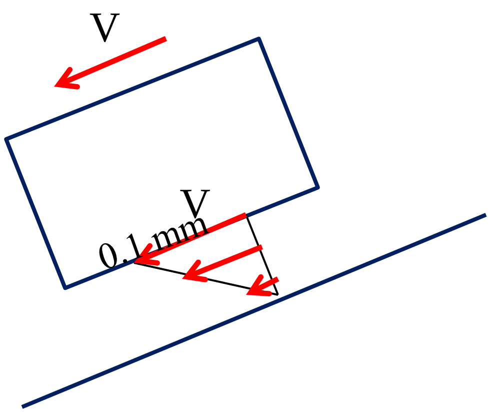
Torque on the shaft, $$T = \text{Force} \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01 \text{ Nm}$$

$\therefore$ *Power lost $$= \frac{2\pi NT}{60} = \frac{2\pi \times 190 \times 36.01}{60} = \mathbf{716.48\ W.\ Ans.}$$

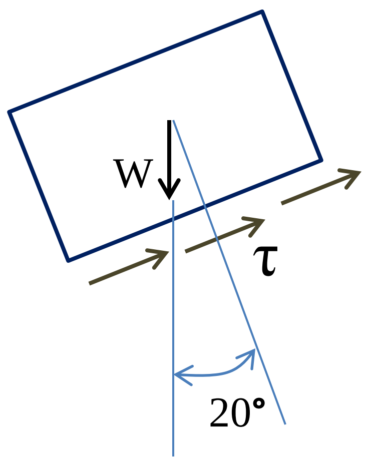
Problem 3:

A 10-kg block slides down a smooth inclined surface as shown in the figure. Determine the terminal velocity of the block if the 0.1mm gap between the block and the surface contains SAE oil of viscosity 0.38 Ns/m^2. Assume the velocity distribution in the gap is linear and the area of the block in contact with the oil is 0.1m^2.





$$W \sin 20 = \tau A$$



$$W = mg = 10 \times 9.81 = 98.1N$$

$$A = 0.1m^2$$

$$\tau = \mu \frac{du}{dy} = \mu \frac{V - 0}{(0.1 \times 10^{-3} - 0)} = 0.38 \frac{V}{10^{-4}}$$

$$V = 0.0883 \mathrm{m/s}$$