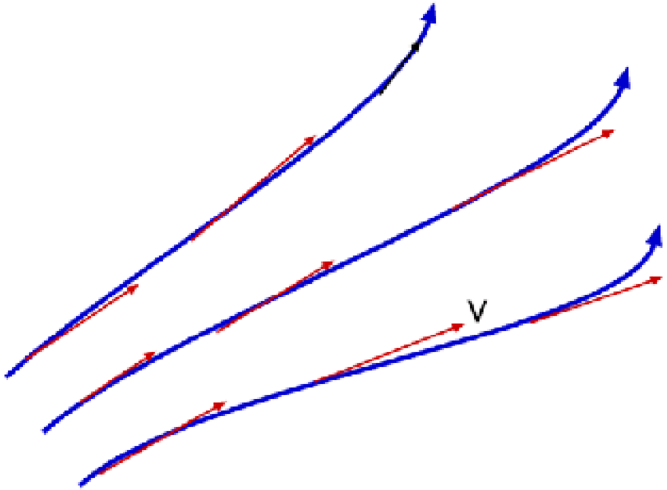
streamlines, path lines, and streak lines.

Geometrical representations of the of the velocity field in a flow

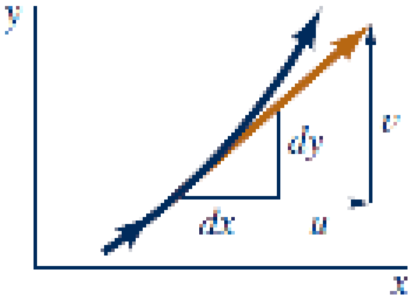
Streamline:

A streamline is a line that is everywhere tangent to the velocity field

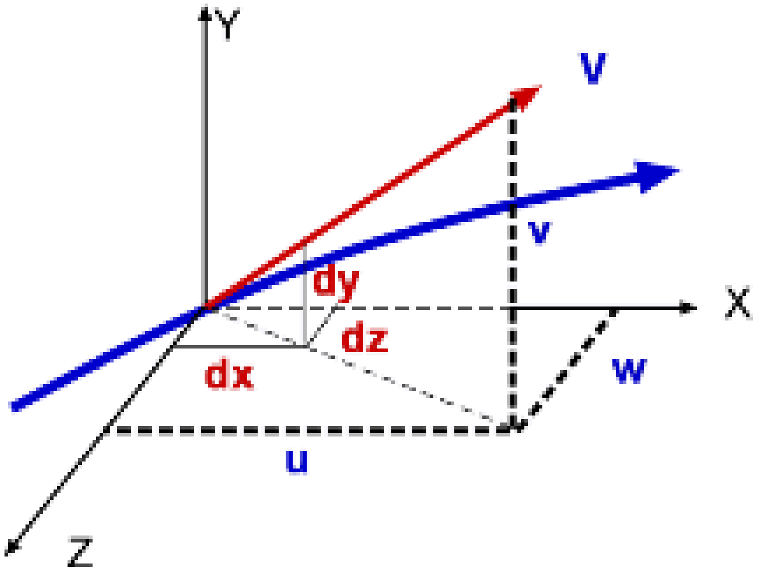


Example:
Streamline pattern around an airfoil

Equation of a streamline



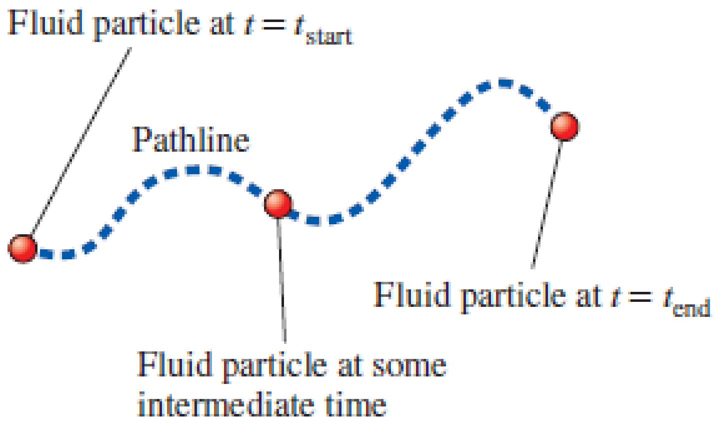
$$\frac{dy}{dx} = \frac{v}{u}$$



$$\frac{du}{u} = \frac{dv}{v} = \frac{dw}{w}$$

Pathline:

A pathline is the line traced out by a given particle as it flows from one point to another.



Streakline:

A streakline consists of all particles in a flow that have previously passed through a common point.

In a **steady flow, the path lines, streak lines and streamlines are identical.

Problem:

The x and y components of a velocity field are given by $u=x^2y$ and $v=-xy^2$. Determine the equation of the streamlines for this flow.

Streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = -\frac{xy^2}{x^2y} = -\frac{y}{x}$

or $\frac{dy}{y} = -\frac{dx}{x}$ which can be integrated as:

$\int \frac{dy}{y} = -\int \frac{dx}{x}$ Thus, $\ln y = -\ln y + \tilde{C}$, where $\tilde{C}$ is a constant.

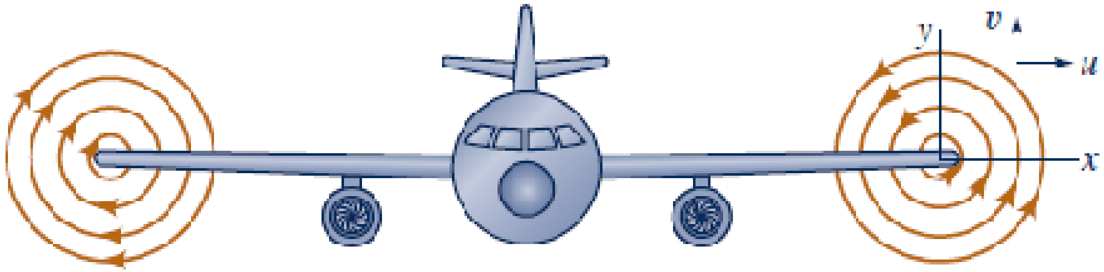
Thus, $\underline{\underline{xy = C}}$

Problem:

A flying airplane produces a swirling flow near the end of its wings as shown. This flow can be approximated by a velocity field $u=-Ky/(x^2+y^2)$ and $v=Kx/(x^2+y^2)$, where K is a constant and x y are measured from the center of the swirl.

(a) Show that the velocity is inversely proportional to the distance from the origin for this flow

(b) Show that the streamlines are circles.



(a) $V = \sqrt{u^2 + v^2} = \left[\frac{(-Ky)^2}{(x^2+y^2)^2} + \frac{(Kx)^2}{(x^2+y^2)^2} \right]^{1/2} = \frac{K}{\sqrt{x^2+y^2}}$

or

$\underline{\underline{V = \frac{K}{r}}}$, where $r = \sqrt{x^2+y^2}$

(b) Streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = \frac{\frac{Kx}{(x^2+y^2)}}{\frac{-Ky}{(x^2+y^2)}} = -\frac{x}{y}$

Thus,

$y\,dy = -x\,dx$ which when integrated gives

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$, where C_1 is a constant.

or

$\underline{\underline{x^2 + y^2 = \text{Constant}}}$