Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2023
Third Semester

21MAB201T – TRANFORMS AND BOUNDARY VALUE PROBLEMS
(For the candidates admitted from the academic year 2022-2023)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours — Max. Marks: 75

PART – A (20 × 1 = 20Marks)
Answer **ALL** Questions

Marks BL CO

1. The complete integral of p=q is — 1 1 1
   (A) $z=ax+by$ (B) $z=a(x+y)+c$
   (C) $z=ax+by+c$ (D) $z=ax-by+a$

2. The partial differential equation formed by eliminating arbitrary constants from z=(x+a)(y+b) — 1 2 1
   (A) z=p+q (B) z=p–q
   (C) Z=p/q (D) z=pq

3. The solution which has a number of arbitrary constants equal to the number of independent variables is — 1 1 1
   (A) Non integral (B) Complete integral
   (C) Particular integral (D) Singular integral

4. Find the particular integral of $\left(D^2+5DD'-6D'^2\right)z=e^{(2x+2y)}$ — 1 2 1
   (A) $\dfrac{xe^{(2x+2y)}}{14}$ (B) $\dfrac{e^{(2x+2y)}}{14}$
   (C) $\dfrac{xe^{(2x+2y)}}{24}$ (D) $\dfrac{e^{(2x+2y)}}{24}$

5. The constant a_0 of the Fourier series for the function $f(x)=k, 0\le x\le 2\pi$ is — 1 1 2
   (A) k (B) 2k
   (C) 0 (D) k/2

6. If f(x) is an even function in (–π, π) then the value of b_n in the Fourier series expansion of f(x) is — 1 1 2
   (A) $\frac{1}{\pi}\int_{-\pi}^{2\pi} f(x)\cos nx dx$ (B) $\frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx dx$
   (C) 0 (D) $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$

7. The RMS value of $f(x)$ in $a \le x \le b$ is 1 1 2

(A) 0

(B) $\sqrt{\dfrac{\int_a^b [f(x)]^2}{b-a}}$

(C) $\sqrt{\dfrac{\int_a^a [f(x)]^2}{b+a}}$

(D) $\sqrt{\dfrac{\int_a^a f(x)}{b-a}}$

8. The Half range sine series for $f(x)$ in a $(0,\pi)$ is 1 1 2

(A) $\sum_{n=1}^{\infty} a_n \cos nx$

(B) $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx$

(C) $\sum_{n=1}^{\infty} b_n \sin nx$

(D) $\frac{a_0}{2}-\sum_{n=1}^{\infty} a_n \cos nx$

9. The proper solution to the problems of vibration of the string is 1 1 3

(A) $y(x,t)=\left(Ae^{\lambda x}+Be^{-\lambda x}\right)\left(Ce^{\lambda at}+Be^{-\lambda at}\right)$

(B) $y(x,t)=\left(Ax+B\right)\left(Ct+D\right)$

(C) $y(x,t)=\left(A\cos^{\lambda x}+B\sin^{\lambda x}\right)(C\cos^{\lambda at}+D\sin^{\lambda at})$

(D) $y(x,t)=\left(Ax+B\right)$

10. The one dimensional heat equation is 1 1 3

(A) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

(B) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(C) $\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(D) $\frac{\partial u}{\partial x}=\alpha^2\frac{\partial^2 u}{\partial t^2}$

11. How many initial and boundary conditions are required to solve $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$ 1 2 3

(A) 2

(B) 3

(C) 5

(D) 4

12. The partial differential equation is elliptic if B^2-4AC 1 1 3

(A) >0

(B) ≥0

(C) ≤0

(D) <0

13. The Fourier transform of $f(x)=e^{\frac{-x^2}{2}}$ is 1 1 4

(A) e^{-s^2}

(B) $\frac{1}{e^{\frac{s^2}{2}}}$

(C) $\frac{1}{e^{\frac{-s^2}{2}}}$

(D) $\frac{1}{e^{x^2}}$

14. The Fourier cosine transform of e^{-ax} is 1 1 4

(A) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+x^2}$

(B) $\sqrt{\frac{1}{\pi}}\frac{s}{s^2+a^2}$

(C) $\sqrt{\frac{1}{\pi}}\frac{a}{s^2+a^2}$

(D) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$

15. If $F(s)=F(f(x))$, *then* $\int_{-\infty}^{\infty}|f(x)|^2$ is 1 1 4

(A) $\int_{-\infty}^{\infty}|f(x)|^2$

(B) $\int_{-\infty}^{\infty}|f(s)|^2$

(C) $\int_{0}^{\infty}|f(x)|^2$

(D) $\int_{0}^{\infty}|f(s)|^2$

16. $F\left[e^{iax}f(x)\right]=$ 1 1 4

(A) F(s+a) (B) F(s–a)
(C) F(sa) (D) F(s/a)

17. What is Z(5) 1 2 5

(A) $\dfrac{z}{z-1}$ (B) $\dfrac{5z}{z-1}$
(C) $\dfrac{1}{5}\dfrac{z}{z-1}$ (D) $\dfrac{z-1}{z}$

18. What is $Z^{-1}\left(\dfrac{z}{(z-a)^2}\right)$ w 1 1 5

(A) a^{n-1} (B) na^{n+1}
(C) na^{n-1} (D) a^{n+1}

19. Poles of $\phi(z)=\dfrac{z^n}{(z-1)(z-2)}$ are 1 2 5

(A) 1, 0 (B) 1, 2
(C) 0, 2 (D) 0, 3

20. Solution of $u_n=5u_{n-1}, n\ge 1, u_0=2$ is 1 2 5

(A) $u_n=5^n$ (B) $u_n=2(5^n)$
(C) $u_n=2^n$ (D) $u_n=5.(2^n)$

PART – B (5 × 8 = 40 Marks) Marks BL CO
Answer ALL Questions

21. a. Solve $x(y-z)p+y(z-x)q=z(x-y)$. 8 3 1

(OR)

b. Solve $\left(D^2-2DD'+D'^2\right)z=\cos(x-3y)$. 8 3 1

22. a. Find the Fourier series expansion of $f(x)=\pi^2-x^2\ in\ (-\pi,\pi)$. 8 3 2

(OR)

b. Find the Fourier series expansion of period 2π for the function *y=f(x)* which is defined in (0, 2π) by means of the table of values given below. Find the series upto the second harmonic. 8 3 2

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

23. a. A tightly stretched string with fixed end points x=0 and x=L is initially in a position given by $y(x,0)=y_0\sin^3\left(\dfrac{\pi x}{L}\right)$. If it is released from the rest from this position find the displacement y at any distance x from one end at any time t. 8 5 3

(OR)

b.	Solve $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ subject to the boundary conditions $u(0,t)=0$, $u(L,t)=0$, $u(x,0)=x$.	8	5	3
24. a.	Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & \lvert x\rvert\le 1\\ 0, & \lvert x\rvert>1\end{cases}$.	8	5	4
	(OR)			
b.	Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier transform.	8	3	5
25. a.	Find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$ using residue theorem.	8	3	5
	(OR)			
b.	Find the Z-transform of $\frac{1}{(n+1)(n+2)}, n>0$.	8	3	5

PART – C (1 × 15 = 15 Marks)
Answer **ANY ONE** Questions

		Marks	BL	CO
26.	Solve $y_{n+2}-3y_{n+1}+2y_n=2^n$, given $y_0=y_1=0$.	15	4	5
27.	A string is stretched and fastened to two points *x*=0 and *x*=L apart. Motion is started by displacing the string into the form $y=k(Lx-x^2)$, from which it is released at time t=0. Find the displacement of any point on the string at a distance of *x* from one end at time t.	15	4	3

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