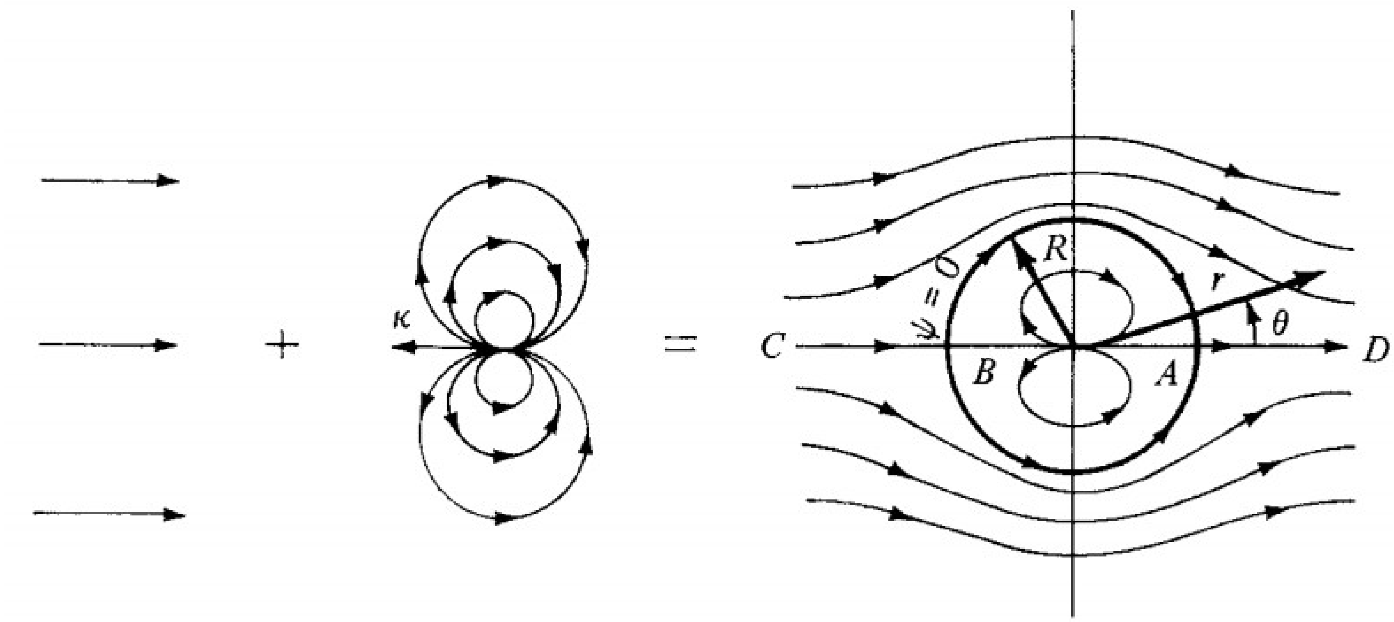
Type of flow	Velocity	ϕ	ψ
Uniform flow in x direction	$u = V_\infty$	$V_\infty x$	$V_\infty y$
Source	$V_r = \dfrac{\Lambda}{2\pi r}$	$\dfrac{\Lambda}{2\pi} \ln r$	$\dfrac{\Lambda}{2\pi}\theta$
Vortex	$V_\theta = -\dfrac{\Gamma}{2\pi r}$	$-\dfrac{\Gamma}{2\pi}\theta$	$\dfrac{\Gamma}{2\pi} \ln r$
Doublet	$V_r = -\dfrac{\kappa}{2\pi} \dfrac{\cos\theta}{r^2}$ $V_\theta = -\dfrac{\kappa}{2\pi} \dfrac{\sin\theta}{r^2}$	$\dfrac{\kappa}{2\pi} \dfrac{\cos\theta}{r}$	$-\dfrac{\kappa}{2\pi} \dfrac{\sin\theta}{r}$

NON-LIFTING FLOW OVER A CIRCULAR CYLINDER

NONLIFTING FLOW OVER A CIRCULAR CYLINDER

The combination of a ***uniform flow*** and a ***doublet*** produces the ***flow over a circular cylinder***



Uniform flow

$$\psi = V_\infty r \sin\theta$$

Doublet

$$\psi = \frac{-\kappa}{2\pi}\,\frac{\sin\theta}{r}$$

Flow over a cylinder

$$\psi = V_\infty r \sin\theta - \frac{\kappa}{2\pi}\,\frac{\sin\theta}{r}$$

$$\psi = V_\infty r \sin\theta - \frac{\kappa}{2\pi}\frac{\sin\theta}{r}$$

$$\psi = V_\infty r \sin\theta \left(1 - \underbrace{\frac{\kappa}{2\pi V_\infty r^2}}\right) \quad R^2$$

Let $R^2 \equiv \kappa/2\pi V_\infty$.

$$\boxed{\psi = (V_\infty r \sin\theta)\left(1 - \frac{R^2}{r^2}\right)}$$

It is the stream function for a uniform flow-doublet combination

When r=R $\Rightarrow$ $\psi = 0$

r=R is the equation of a circle

Therefore, $\psi = 0$ is the stream function for the flow over a circle of radius *R* as shown

Velocity field (Radial Velocity V_r and Tangential velocity V_θ)

Radial Velocity

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial \theta}\left[V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)\right]$$

$$= \frac{1}{r}(V_\infty r \cos\theta)\left(1 - \frac{R^2}{r^2}\right) = \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta$$

$$\boxed{V_r = \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta}$$

Tangential Velocity

$$V_\theta = -\frac{\partial \psi}{\partial r} = -\frac{\partial}{\partial r}\left[V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right)\right]$$

$$= -\left[(V_\infty r \sin\theta)\frac{2R^2}{r^3} + \left(1 - \frac{R^2}{r^2}\right)(V_\infty \sin\theta)\right]$$

$$= -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta$$

$$\boxed{V_\theta = -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta}$$

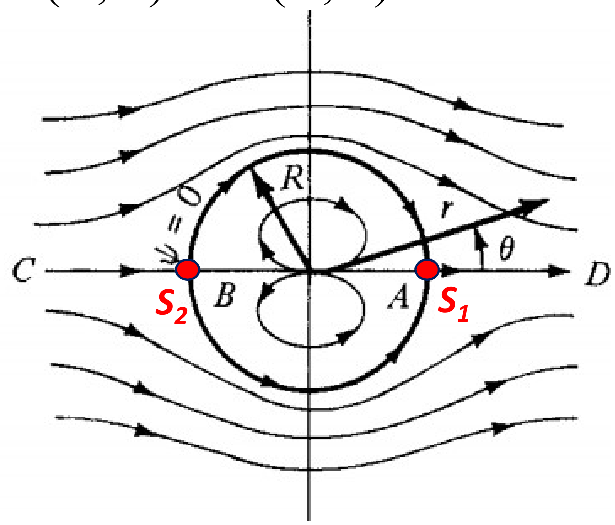
To locate the stagnation points

Stagnation points are the points in the flow where the *VELOCITY* is *ZERO*

$$V_r = 0 \quad \Rightarrow \quad \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta = 0$$

$$V_\theta = 0 \quad \Rightarrow \quad \left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta = 0$$

Simultaneously solving these two equations for r and θ, we find that there are two stagnation points, located at $(r, \theta) = (R, 0)$ and (R, π)



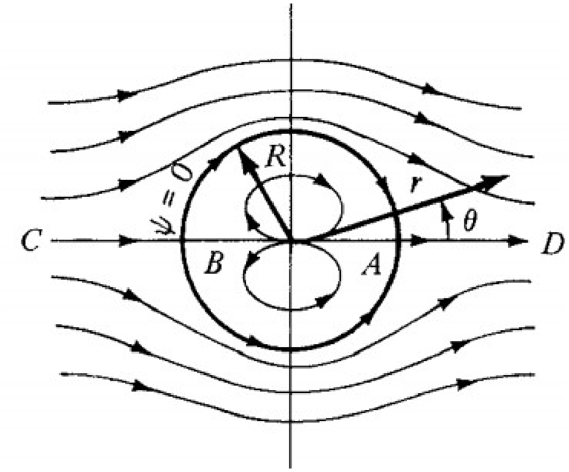
- Note that the $\psi = 0$ streamline, since it goes through the stagnation points, is the dividing streamline.

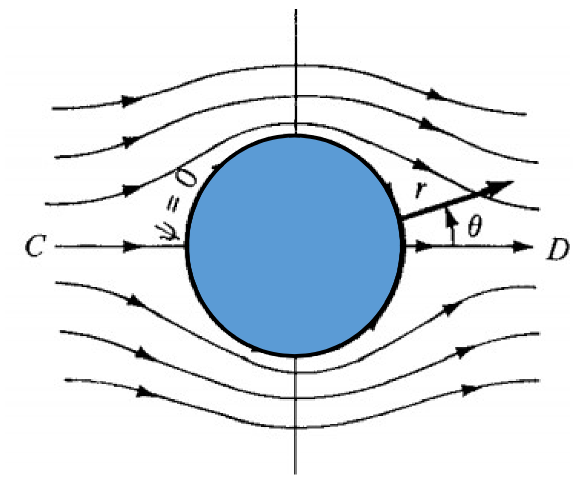
- That is, all the flow inside $\psi = 0$ (inside the circle) comes from the doublet, and all the flow outside $\psi = 0$ (outside the circle) comes from the uniform flow.

- Therefore, we can replace the flow inside the circle by a solid body, and the external flow will not know the difference.

- Consequently, the inviscid, irrotational, incompressible flow over a circular cylinder of radius R can be synthesized by adding a uniform flow with velocity V_∞ and a doublet of strength κ, where R is related to V_∞ and κ through

$$R = \sqrt{\frac{\kappa}{2\pi V_\infty}}$$





Velocity distribution over the cylinder

$$V_r = \left(1 - \frac{R^2}{r^2}\right) V_\infty \cos\theta$$

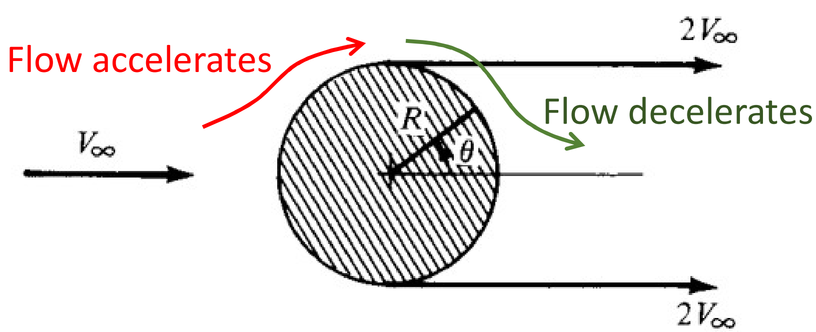
$$V_\theta = -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta$$

Substituting r=R

$$V_r = 0$$

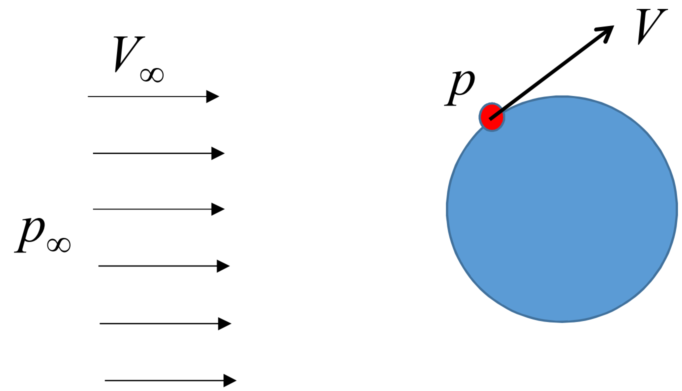
$$V_\theta = -2V_\infty \sin\theta$$

On the surface of the cylinder $V = V_\theta = -2V_\infty \sin\theta$



Pressure distribution over a cylinder

Coefficient of Pressure $C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}$



p_∞, V_∞ : Pressure and Velocity of free-stream

p, V : Pressure and Velocity at any point on the cylinder

Applying Bernoulli's equation b/w 1 and 2

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p + \frac{1}{2}\rho V^2$$

$$\Rightarrow \quad p - p_\infty = \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V^2 \quad \Rightarrow \quad \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

$$\Rightarrow \quad p - p_\infty = \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V^2$$

$$\Rightarrow \quad \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - \frac{\frac{1}{2}\rho V^2}{\frac{1}{2}\rho V_\infty^2} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

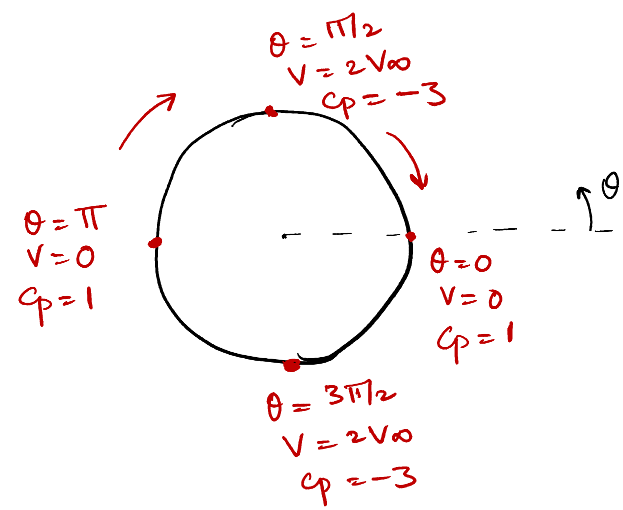
$$\Rightarrow \quad C_p = 1 - \left(\frac{V}{V_\infty}\right)^2$$

on the cylinder surface

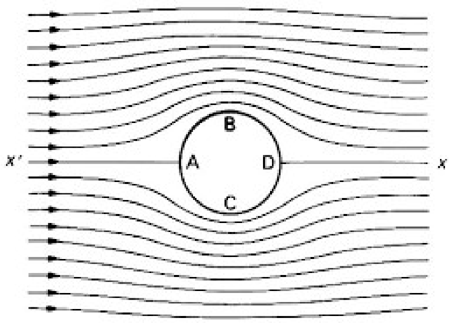
$$V = -2V_\infty \sin\theta$$

$$\Rightarrow \quad C_p = 1 - \left(\frac{-2V_\infty \sin\theta}{V_\infty}\right)^2$$

$$\Rightarrow \quad \boxed{C_p = 1 - 4\sin^2\theta}$$

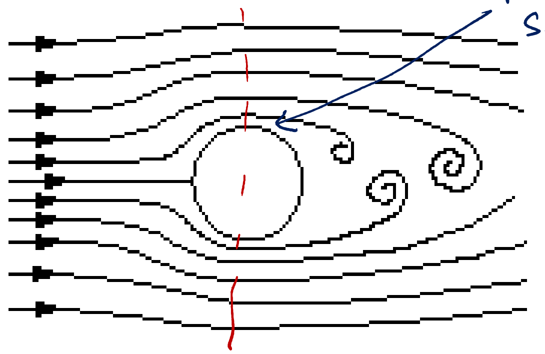


Potential flow (Theoretical)

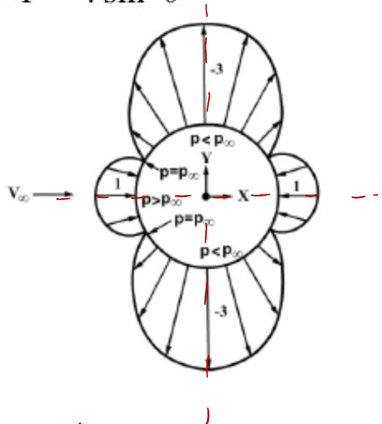


Real flow (Experimental)

Flow Separation

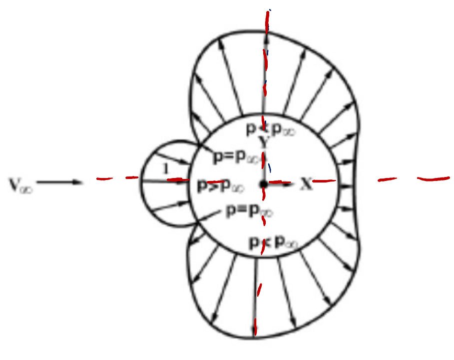


$C_p = 1 - 4\sin^2\theta$



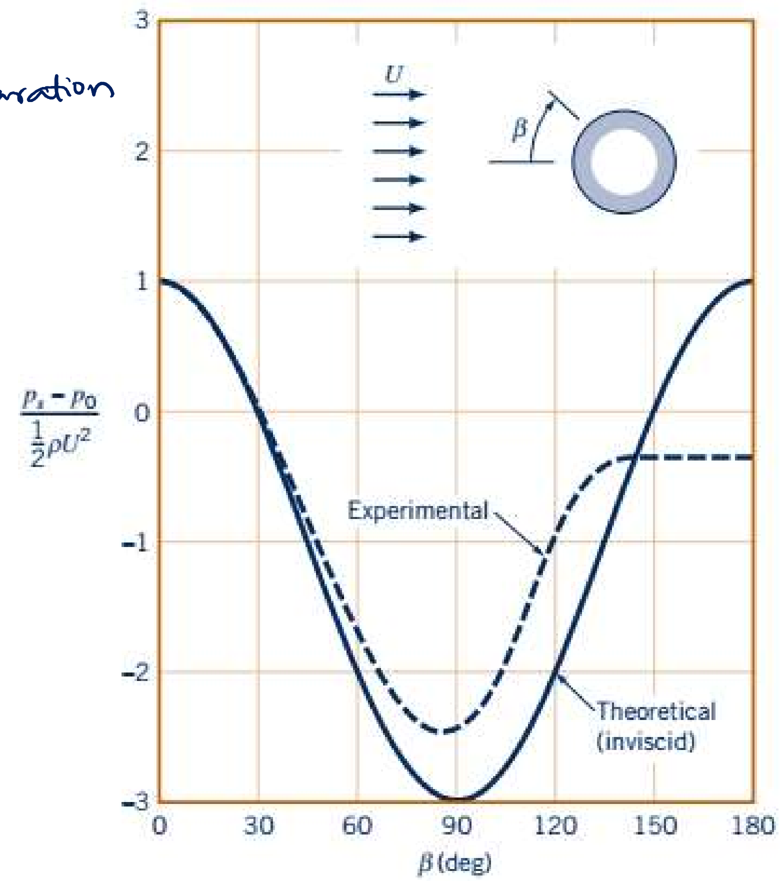
Lift = 0

Drag = 0



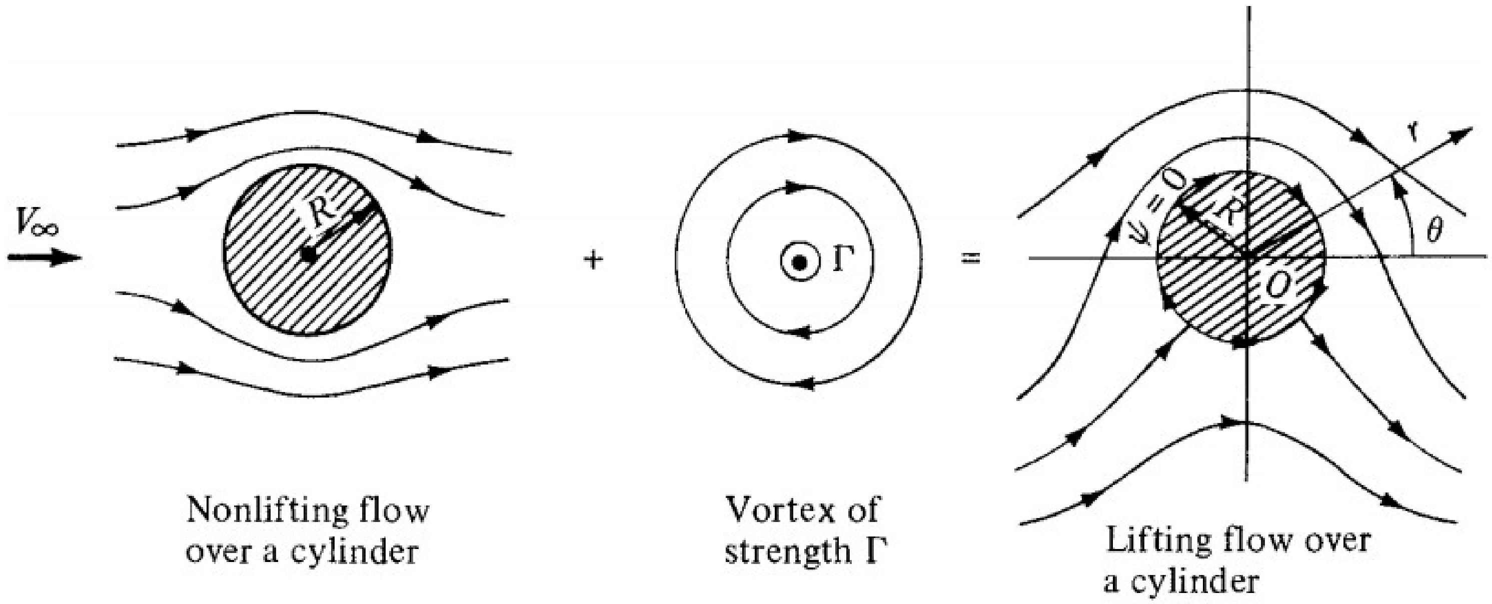
Lift = 0

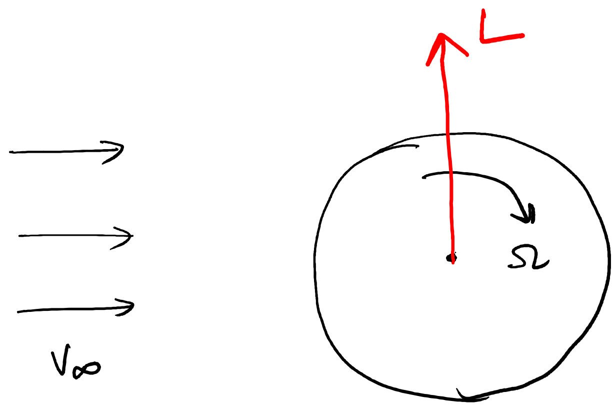
Drag ≠ 0

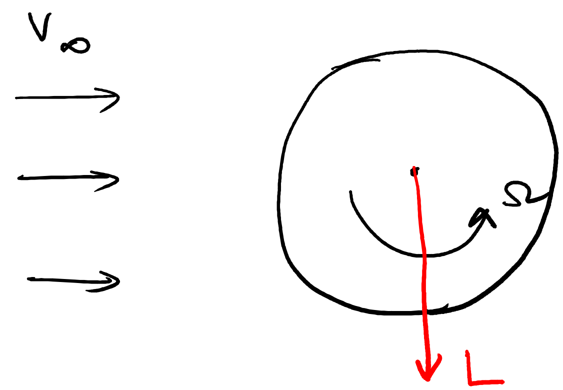


LIFTING FLOW OVER A CIRCULAR CYLINDER

LIFTING FLOW OVER A CIRCULAR CYLINDER – Flow over a rotating cylinder

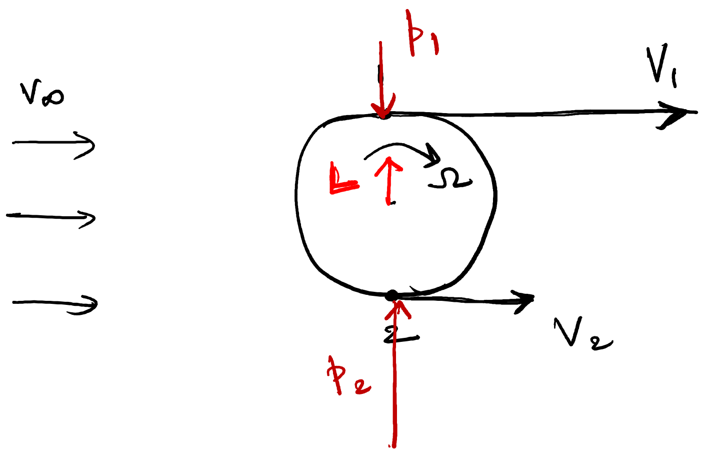






* A rotating cylinder or sphere moving in a fluid produces lift force → MAGNUS EFFECT

ROBIN - MAGNUS EFFECT



$V_1 > V_2$

$\Rightarrow p_1 < p_2$