ISENTROPIC PROCESSES

- A process during which the entropy remains constant is called an **isentropic process**.

- A *reversible* and *adiabatic* process is an isentropic process

$$\text{Isentropic process:} \qquad \Delta s = 0 \quad \text{or} \quad s_2 = s_1$$

- Many engineering systems or devices such as pumps, turbines, nozzles, and diffusers are adiabatic in their operation, and they perform best when the irreversibilities, such as the friction associated with the process, are minimized.

- Therefore, an isentropic process can serve as an appropriate simpler model for actual processes.

- Also, isentropic processes enable us to define efficiencies for processes to compare the actual performance of these devices to the performance under idealized conditions.
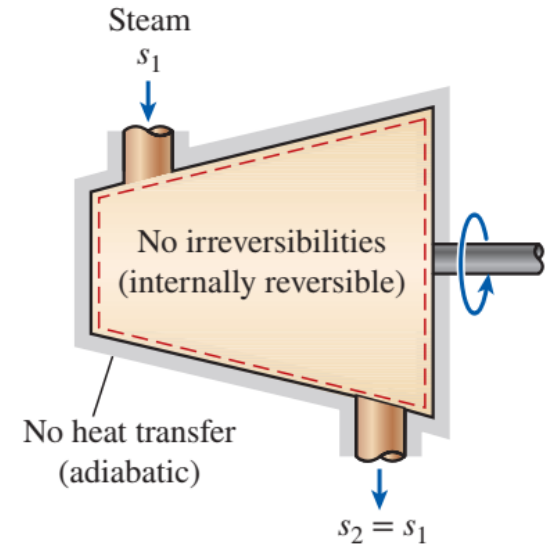
Steam
s_1

No irreversibilities
(internally reversible)

No heat transfer
(adiabatic)

$s_2 = s_1$

FIGURE 7–14
During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

PROPERTY DIAGRAMS INVOLVING ENTROPY

The two diagrams commonly used in the second-law analysis are the *temperature-entropy* and the *enthalpy-entropy* diagrams

The total heat transfer during an internally reversible process is

$$Q_{int\ rev} = \int_1^2 T dS$$

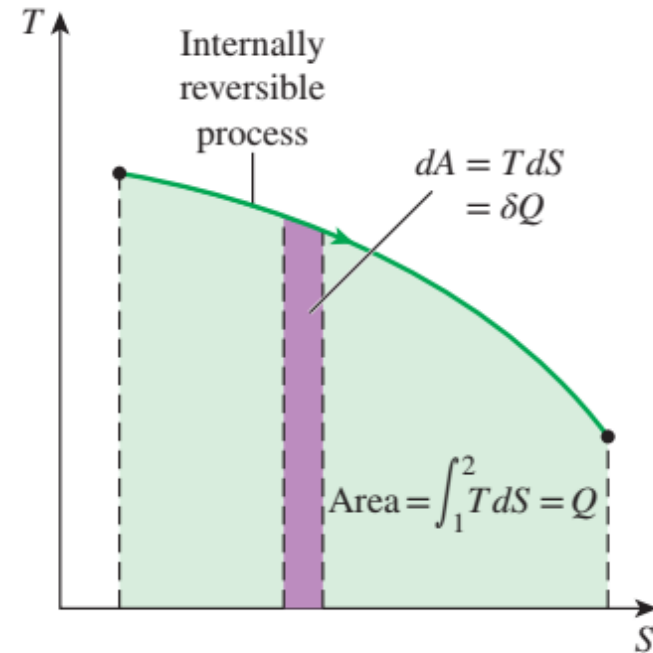which corresponds to the area under the process curve on a *T-S* diagram.

FIGURE 7–16
On a *T-S* diagram, the area under the process curve represents the heat transfer for internally reversible processes.

Another diagram commonly used in engineering is the enthalpy-entropy diagram, which is quite valuable in the analysis of steady-flow devices such as turbines, compressors, and nozzles.

The h-s diagram is also called a **Mollier diagram**

The coordinates of an h-s diagram represent two properties of major interest:
* enthalpy, which is a primary property in the first-law analysis of the steady-flow devices, and
* entropy, which is the property that accounts for irreversibilities during adiabatic processes.
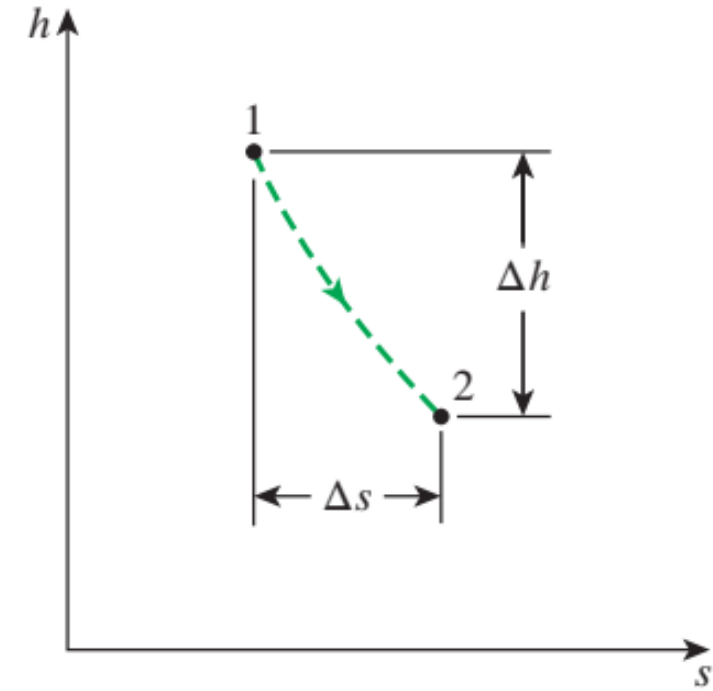

FIGURE 7–18

For adiabatic steady-flow devices, the vertical distance Δh on an h-s diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

T-ds relations - DIFFERENTIAL ENTROPY CHANGE RELATIONS

From the first law of thermodynamics,

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

We know that,

$$\delta Q_{\text{int rev}} = T\,dS$$

$$\delta W_{\text{int rev,out}} = P\,dV$$

On substituting these relations, we get,

$$T\,dS = dU + P\,dV$$

Taking the above eqn per unit mass,

$$T\,ds = du + P\,dv \qquad \text{------ (Eqn 1)}$$

Equation 1 is called first *T ds,* or *Gibbs equation*

From the definition of enthalpy,

$$h = u + Pv$$

$$\implies \quad dh = du + P\,dv + v\,dP$$

$$\implies \quad dh = T\,ds + v\,dp$$

$$\implies \quad T\,ds = dh - v\,dP \qquad \text{------ (Eqn 2)}$$

Equation 2 is called Second *T ds,* or *Gibbs equation*

They relate entropy changes of a system to the changes in other properties. They are property relations and therefore are independent of the type of the processes. valid for both reversible and irreversible processes

(Eqn 1) $\implies ds = \dfrac{du}{T} + \dfrac{P\,dv}{T}$ ------ (Eqn A)   (Eqn 2) $\implies ds = \dfrac{dh}{T} - \dfrac{v\,dP}{T}$ ------ (Eqn B)

The entropy change during a process can be determined by integrating either of these equations between the initial and the final states.

Entropy Change Of Liquids And Solids

liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process. Thus, $dv \cong 0$ for liquids and solids

$$ds = \frac{du}{T} = \frac{c\,dT}{T}$$

since $c_p = c_v = c$ and $du = c\,dT$ for incompressible substances

the entropy change during a process is determined by integration to be

$$\textit{Liquids, solids:} \qquad s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{avg}\ln\frac{T_2}{T_1}$$

A relation for isentropic processes of liquids and solids is obtained by setting the entropy change relation above equal to zero.

$$\textit{Isentropic:} \qquad s_2 - s_1 = c_{avg}\ln\frac{T_2}{T_1} = 0 \longrightarrow T_2 = T_1$$

the isentropic process of an incompressible substance is also isothermal

The Entropy Change Of Ideal Gases

For ideal gases,

$$PV = RT$$
$$du = c_v\, dT$$
$$dh = c_p\, dT$$

The entropy change of an ideal gas becomes

(Eqn A) $\implies$ $ds = c_v \dfrac{dT}{T} + R \dfrac{dV}{V}$

(Eqn B) $\implies$ $ds = c_p \dfrac{dT}{T} - R \dfrac{dP}{P}$

The entropy change for a process is obtained by integrating this relation between the end states:

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{V_2}{V_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

For constant specific heats,

$$s_2 - s_1 = c_{v,avg} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{p,avg} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

Isentropic Processes of Ideal Gases

Isentropic process → $s_2 - s_1 = 0$

$$\implies \ln\frac{T_2}{T_1} = -\frac{R}{c_v}\ln\frac{v_2}{v_1}$$

$$\implies \ln\frac{T_2}{T_1} = \frac{R}{c_p}\ln\frac{P_2}{P_1}$$

$$\implies \ln\frac{T_2}{T_1} = \ln\left(\frac{v_1}{v_2}\right)^{R/c_v}$$

$$\implies \ln\frac{T_2}{T_1} = \frac{R}{c_p}\ln\left(\frac{P_2}{P_1}\right)^{R/c_p}$$
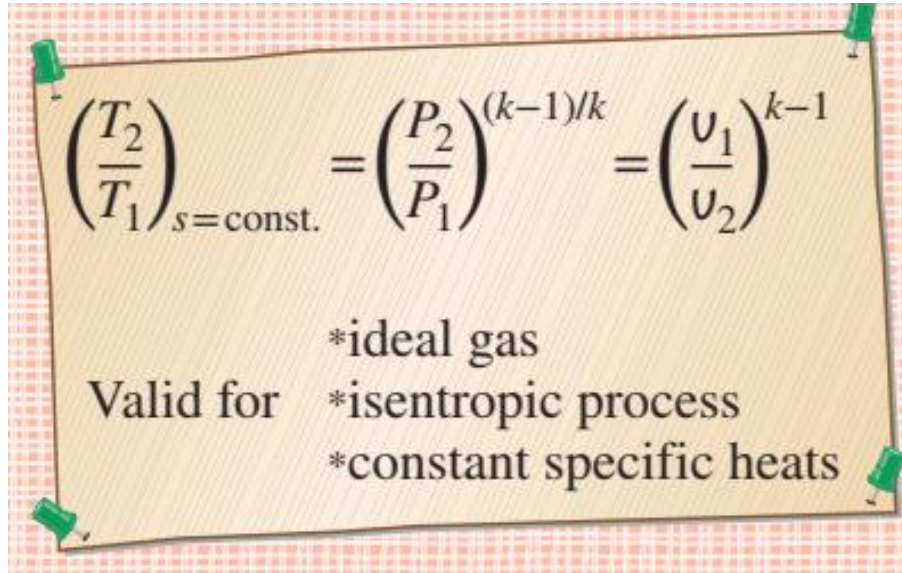
since $R = c_p - c_v$, $k = c_p/c_v$, and thus $R/c_v = k - 1$.

$$\&\quad R/c_p = (k-1)/k$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

Isentropic relations for ideal gas

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

*ideal gas

Valid for   *isentropic process

*constant specific heats

These equations can also be expressed in compact form as

$$Tv^{k-1} = \text{constant}$$

$$TP^{(1-k)/k} = \text{constant}$$

$$Pv^{k} = \text{constant}$$