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B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, MAY 2024

Third Semester

21MAB201T - TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted during the academic year 2022-2023 onwards)

Note:

i. **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.

ii. **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours **Max. Marks: 75**

PART - A (20 × 1 = 20 Marks)

Answer **all** Questions

	Question	Marks	BL	CO
1.	Form the PDE by eliminating the arbitrary constants from $z = ax^2 + by^2$. (A) $2z = qx + py$ (B) $2z = qx - py$ (C) $2z = px + qy$ (D) $2z = px - qy$	1	2	1
2.	Find the particular integral of $(D^2 + 5DD' - 6D'^2)z = e^{(2x+2y)}$ (A) $\frac{xe^{(2x+2y)}}{14}$ (B) $\frac{e^{(2x+2y)}}{14}$ (C) $\frac{xe^{(2x+2y)}}{24}$ (D) $\frac{e^{(2x+2y)}}{24}$	1	2	1
3.	What is the complementary function of $(D^2 - 5DD' + 6D'^2)z = x + y$? (A) $\varphi_1(y - 5x) + \varphi_2(y - x)$ (B) $\varphi_1(y + 2x) + \varphi_2(y + 3x)$ (C) $\varphi_1(y - 2x) + \varphi_2(y - 3x)$ (D) $\varphi_1(y + 5x) + \varphi_2(y + x)$	1	2	1
4.	Write the complete integral of $z = ax + by + \frac{a^2}{2} - \frac{b^2}{2}$? (A) $z = px + qy + \frac{p^2}{2} + \frac{q^2}{2}$ (B) $z = px + qy + 2p^2 - 2q^2$ (C) $z = px + qy + 2p^2 + 2q^2$ (D) $z = px + qy + \frac{p^2}{2} - \frac{q^2}{2}$	1	2	1
5.	Find the Fourier coefficient a_n of the function $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$ (A) 2 (B) 2π (C) 0 (D) $\frac{2}{\pi}$	1	2	2

6. The Fourier constant a_0 of the function $f(x) = |x|,\ -\pi < x < \pi$ is 1 2 2

(A) 2π

(B) $\frac{\pi}{2}$

(C) 4π

(D) π

7. The Fourier series of $f(x) = x^2,\ -\pi < x < \pi$ is given by $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} cosnx$ 1 2 2

Find $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

(A) $\frac{\pi^2}{12}$

(B) $\frac{\pi^2}{2}$

(C) $\frac{\pi^2}{4}$

(D) $\frac{\pi^2}{16}$

8. The Root Mean square value of the function $f(x) = 3x$, $(0, l)$ is 1 2 2

(A) $\sqrt{(3l)}$

(B) $\frac{l}{\sqrt{3}}$

(C) $l\sqrt{3}$

(D) $\frac{l}{3}$

9. The tension T caused by stretching the string before fixing it at the end points is ------------ at all times and at all points of the deflected string. 1 1 3

(A) increasing

(B) zero

(C) decreasing

(D) constant

10. If $u(x, t)$ is the temperature function, then the heat flow equation at steady state is given by 1 2 3

(A) $\frac{d^2u}{dx^2} = 0$

(B) $\frac{du}{dx} = 0$

(C) $\frac{d^2u}{dt^2} = 0$

(D) $\frac{du}{dt} = 0$

11. For the given PDE $xf_{xx} + yf_{yy} = 0,\ x < 0,\ y < 0$ find $B^2 - 4AC$ 1 2 3

(A) > 0

(B) $= 0$

(C) < 0

(D) ≤ 0

12. When the ends of a rod are non-zero for one-dimensional heat flow equation the temperature function $u(x,t)$ is modified as the sum of steady state and transient state temperatures. The transient part of solution is one which | 1 | 2 | 3

(A) increases with increase of time
(B) decreases with increase of time.
(C) decreases with decrease of time
(D) increases with decrease of time

13. Let $f(x)$ and $g(x)$ be two functions defined in the interval $(-\infty,\infty)$ Then the convolutions of the functions $f(x)$ and $g(x)$ is | 1 | 1 | 4

(A) $f*g=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)g(x-t)dt$
(B) $f*g=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}f(t)g(x-t)dt$
(C) $f*g=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}f(x)g(x-t)dt$
(D) $f*g=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(t)g(x-t)dt$

14. If $F(s)$ is the Fourier transform of $f(x)$ then the Fourier transform of $f(x-k)$ is | 1 | 2 | 4

(A) $e^{-kis}F(s)$
(B) $e^{kis}F(s)$
(C) $e^{ks}F(s)$
(D) $e^{-ks}F(s)$

15. Find $F(f(2x))$ if $F(s)$ is the Fourier transform of $f(x)$. | 1 | 2 | 4

(A) $\frac{1}{2}F\left(\frac{2}{s}\right)$
(B) $\frac{1}{s}F\left(\frac{2}{s}\right)$
(C) $\frac{1}{2}F\left(\frac{s}{2}\right)$
(D) $\frac{1}{s}F\left(\frac{s}{2}\right)$

16. If $F(s)$ is the Fourier transform of $f(x)$ *then* $F\left[\int_a^x f(x)dx\right]=$ | 1 | 2 | 4

(A) $\frac{F(s)}{-is}$
(B) $\frac{F(s)}{is}$
(C) $\frac{F(s)}{-s}$
(D) $\frac{F(s)}{s}$

17. $Z^{-1}\left(\frac{a^n}{n!}\right)=$ ---------. | 1 | 2 | 5

(A) $e^{a/z}$
(B) $e^{-a/z}$
(C) $e^{z/a}$
(D) $e^{-z/a}$

18. $(-z)^k \frac{d^k}{dz^k} F(z) = ------$ 1 1 5

(A) $Z(n^{-k} f(n))$
(B) $Z(n^{2k} f(n))$
(C) $Z(n^k f(n))$
(D) $Z(n^{-2k} f(n))$

19. The inverse Z transform of $\frac{z}{1-z} + \frac{2z}{(z-2)^2}$ is 1 2 5

(A) $1 + n.2^n$, $n = 0,1,2, \ldots$
(B) $-1 + n.2^n$, $n = 0,1,2, \ldots$
(C) $-1 + n2^{-n}$, $n = 0,1,2, \ldots$
(D) $1 + n2^{-n}$, $n = 0,1,2, \ldots$

20. Using Z transform find $Z(y_n)$ given $y_{n+1} + 5y_n = 0$ and $y_0 = -1$ 1 2 5

(A) $\frac{2z}{z-5}$
(B) $\frac{-z}{z+5}$
(C) $\frac{z}{z-5}$
(D) $\frac{z}{z+5}$

PART - B (5 × 8 = 40 Marks) Marks BL CO

Answer **all** Questions

21. (a) Solve $(D^2 - 6DD' + 5D'^2)z = e^{x+y} + xy$ 8 3 1

(OR)

(b) Find the general solution of $(4z - 5y)p + (5x - 3z)q = 3y - 4x$

22. (a) Find the Fourier series to represent $f(x) = x^2 - 2$ in $-2 < x < 2$. Hence find the sum of the series at $x = 0$. 8 3 2

(OR)

(b) Compute the first 2 harmonics of the Fourier series of $f(x)$ given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2

23. (a) If a string of length l is initially at rest in equilibrium position and each point of it is given the velocity $v_0 sin\left(\frac{\pi x}{l}\right)$, $0 < x < l$, determine the transverse displacement $y(x,t)$. 8 4 3

(OR)

(b) Find the solution to the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions

(i) u is finite as $t \to \infty$ (ii) $u(0,t) = 0,\ t > 0$ (iii) $u(l,t) = 0,\ t > 0$

(iv) $u(x,0) = \begin{cases} x, & 0 \le x \le l/2 \\ l - x, & l/2 \le x \le l \end{cases}$

24.	(a)	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \lvert x \rvert \le 1 \\ 0, & \lvert x \rvert > 1 \end{cases}$ Hence find $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\left(\frac{s}{2}\right) ds$	8	3	4
		(OR)			
	(b)	Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$			
25.	(a)	Find the inverse Z transform of $\frac{z}{z^2+2z+1}$ by long division method.	8	3	5
		(OR)			
	(b)	Find $Z^{-1}\left[\frac{z^3}{(z-1)^2\,(z-2)}\right]$ by residue method.			

PART - C (1 × 15 = 15 Marks)

Answer **any 1** Questions

		Marks	BL	CO
26.	A rod of length 20 cm has its ends A and B kept at 30^0C and 90^0C respectively until steady state conditions prevail. If the temperature at each end is then suddenly reduced to 0^0C and maintained so, find the temperature $u(x,t)$ at a distance x from A at time t.	15	4	3
27.	Using Z transform, solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0,\ y_1 = 1$.	15	3	5

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