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Probability and Mathematical Expectations

[25+ marks weightage
(20 to 25)]

* PROBABILITY :-

Ex: ① Red dice, blue dice

{(1,1) (1,2) ......
(2,1) (2,2) ......
(3,1) (3,2) ......
(4,1) (4,2) ......
(5,1) (5,2) ......
(6,1) (6,2) .... (6,5) (6,6)}

Total outcome = 36 $\left(\frac{1}{36}\right)$ each probability

Ex: ② 1 rupee coin, 2 rupee coin

(HH) (TT) (TH) (HT)

Total outcome = 4

$\left(\frac{1}{4}\right)$ each probability

* An experiment is called random experiment if it satisfies the following conditions:

(i) It has more than one possible outcome.

(ii) It is not possible to predict the outcome in advance

* Outcome and sample space :-

- A possible result of a random experiment is called outcome.

- The set of all possible outcomes of random experiment is called sample space

Example: 2 coins probability
sample space = (HH)(TT)(TH)(HT)

Example: A coin is tossed. If it shows head, drawn a ball of 3 blue and 4 white. Find sample space.

$$S = \{HB_1, HB_2, HB_3, HW_1, HW_2, HW_3, HW_4, T_1, T_2, T_3, T_4, T_5, T_6\}$$

1. no. of tails is exactly 2 — (TT)
2. no of tails atleast (min) 1 — (HT, TH, TT)
3. no of heads atmost (max) 1 — (HT, TH, TT)
4. second toss is not head — (HT, TT)
5. no. of tails is atmost 2 — (HH, HT, TH, TT)
6. no. of tails is more than 2 — ϕ null

Question — If two dice are tossed (~~read~~
(read and blue)

find ① sum of addition exactly 4

② addition of both dice more than 7
6 (1-6)

③ multiplication of both dice is less than 12 {(1-11)

④ subtraction of both dice less than 1 and more than 6 (5-2)

S.S. = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6),
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6),
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6),
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6),
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6),
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

(1) (1,3) (2,2) (3,1) $= \frac{3}{36} = \frac{1}{12}$ $\boxed{= 0.083}$

(2) (2,6) (3,5) (3,6) (4,4) (4,5) (4,6) (5,3) (5,4)
(5,5) (5,6) (6,2) (6,3) (6,4) (6,5) (6,6) $= \frac{15}{36}$ $\boxed{= 0.42}$

(3) (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4)
(2,5) (3,1) (3,2) (3,3) (4,1) (4,2) (5,1) (5,2) (6,1)
$= 19/36$ $\boxed{= 0.53}$

(4) (3,1) (4,1) (4,2) (5,1) (5,2) (5,3) (6,1) (6,2) (6,3) (6,4)
$= \frac{10}{36}$ $\boxed{= 0.28}$

* Types of Events :-

① Impossible event and Sure Event :-

The empty set ϕ and the S.S. describes events. In fact ϕ is called an impossible event and S, i.e the whole sample space is called sure event.

sure event $= S = \{1, 2, 3, 4, 5, 6\}$

Impossible event $= \phi$.

② Simple event :- If an event E has only one sample point of a sample space. (Elementary event)

$S = (HH, HT, TH, TT)$

$E_1 = (HH)$
$E_2 = (HT)$
$E_3 = (TH)$
$E_4 = (TT)$

③ Compound :- If an event has more than one sample point

E - exaclty one head
F - atleast one head
G - atmost one head

$E = \{HTT, THT, TTH\}$
$F = \{HTT, THT, TTH, HHT, HTH, THH, HHH\}$
$G = \{TTT, THT, HTT, TTH\}$

(4) Complementary Event :- Every event A, there corresponds another event A' called the complementary event to A. Event 'not A'.

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Let $A = \{HTH, HHT, THH\}$ be the event 'only 1 tail appear' outcome HTT, the event A has not occurred. But we may say that the event 'not A' has occurred. Thus, with every outcome which is not A, we say that 'not A' occurs.

Example 1: Find the probability of getting a numbered card, when a card is drawn from the pack of 52 cards. Find the prob. of getting numbered cards.

Sample space = $\frac{\cancel{36}^{\cancel{18}\ 9}}{\cancel{52}_{\cancel{26}\ 13}} = \frac{9}{13}$ $\cancel{\boxed{= 0.69}}$

Probability $\boxed{= \frac{9}{13}}$ Ans.

Example 2: There are 5 green and 7 red balls. 2 balls are selected one by one without replacement. Find the probability that first is green and second is red.

total outcome = 12

G_1, G_2, G_3, G_4, G_5 green ball = 5

$R_1, R_2, R_3, R_4, R_5, R_6, R_7$ red = 7

$^{12}C_2 = 66$

green balls = $^5C_1 = 5$

red balls = $^7C_1 = 7$

$^5C_1 \times {^7C_1} = 5 \times 7 = 35$

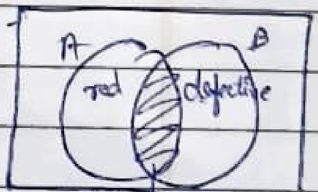
$\frac{5}{12} \times \frac{7}{11} \boxed{= \frac{35}{132}}$

Example 3 : One card is drawn at random from the pack of 52 cards.
1) find the probability that it is honored card. (A, J, Q, K (non-numbered))
2) It is a face card = 12

2) $\frac{\cancel{12}^{3}}{\cancel{52}_{\cancel{26}\,13}} = \frac{3}{13}$

1) $\frac{\cancel{16}^{\cancel{8}\,4}}{\cancel{52}_{\cancel{26}\,13}} = \frac{4}{13}$

Question 1: 10% of the bulbs produce in a factory are of red colour and 2% are red and defective. If one bulb is picked up determine the probability of its being
1) defective if it is red.



Intersection

A: 1 10% → red colour
2 2% → defective red colour bulb

1) $P(A) = 100\% = \frac{10}{100} = \frac{1}{10}$

2) $P(\overset{\text{defective}}{B}) = P(A \cap B) = 2\% = \frac{2}{100} = 0.02$

Pyeans

Probability of Ans:-

→ Let A and B be the events that the bulb is red and defective, respectively.

$P(A) = 10\%$

$= \frac{10}{100} = \boxed{\frac{1}{10}}$

$P(B) = P(A \cap B) = 2\% = \frac{2}{100} = \boxed{\frac{1}{50}}$

Baye's theorem :-

$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{1/50}{1/10}$

$= \frac{1 \times 10}{50 \times 1}$

$= \frac{1\cancel{0}}{5\cancel{0}}$

$\boxed{= 1/5}$

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Question 2 : Two dice are thrown together. Let A be the event 'getting six on the first dial' and B be the event 'getting 2 on the second dial'. Are the events A and B independent?

A: $A = (6,1)(6,2)(6,3)(6,4)(6,5)(6,6)$
$B = (1,2)(2,2)(3,2)(4,2)(5,2)(6,2)$

$P(A)$ = getting 6 on first

$P(B)$ = getting 2 on second

$$P(A) = \frac{6}{36} \boxed{= \frac{1}{6}}$$

$$P(B) = \frac{6}{36} \boxed{= \frac{1}{6}}$$

$$P(A \cap B) \boxed{= \frac{1}{36}}$$

~~Question 3:~~ Rules:-

→ Events A and B will be independent if
$P(A \cap B) = P(A) \cdot P(B)$

$$\frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6}$$

$$\boxed{\therefore \frac{1}{36} = \frac{1}{36}}$$

LHS = RHS

16/N

Let A denotes the events that atleast six girl will be choosen and B denotes exactly 2 girls will be choosen.

$P(B/A)$

$P(A) + P(A') = 1$

$P(A') = \frac{^8C_4}{^{12}C_4} = \frac{70}{495} = \boxed{\frac{14}{99} = P(A')}$

$^8C_4 = \frac{8!}{4(8-4)!}$

$= \frac{8\times7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1\,(4)!}$

$\therefore \boxed{= 70}$

$^{12}C_4$

$= \frac{12}{4(12-4)!}$

$= \frac{\cancel{12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}{\cancel{4\times3\times2\times1}\,(\cancel{8\times7\times6\times5}\times4\times3\times2\times1)}$

$= \frac{11880}{24}$

$\boxed{= 495}$

$P(A) = 1 - P(A')$

$= 1 - \frac{14}{99}$

$= \frac{99-14}{99}$

$\boxed{P(A) = \frac{85}{99}}$

$$\frac{8\times7\times6\times5\times4\times3\times2\times1}{2\times1\,(\ldots\times2\times1)}$$

$\frac{56}{2}$

$$P(A\cap B) = \frac{{}^{8}C_2 \times {}^{4}C_2}{{}^{12}C_4}$$

$$= \frac{6\times28}{495} = \frac{\cancel{168}^{56}}{\cancel{495}_{165}}$$

$$\boxed{= \frac{56}{165}}$$

$$P(B/A) = \frac{P(A\cap B)}{P(A)}$$

$$= \frac{56}{165} \div \frac{85}{99}$$

$$= \frac{56\times\cancel{99}^{\cancel{33}\,3}}{\cancel{165}_{\cancel{55}\,5}\times85}$$

$$= \frac{56\times3}{5\times85}$$

$$\boxed{= \frac{168}{425}}$$

* Three machines E_1, E_2, E_3 in a certain factory product 50%, 25%, 25% respectively of the total daily output of electric tube it is known that 4% of the tubes produce one each of machines E_1 and E_2 are defective and that 5% of those produce on E_3 are

E_1	E_2	E_3	
50%	25%	25%	→ 100%
↓	↓	↓	
4%	4%	5%	

Let D is event that pickup the tube is defective

Let D →

let A_1, A_2 and A_3 the event that the tube is produce on machines E_1, E_2, E_3 respectively

PROBABILITY DISTRIBUTION

* **Probability :-**

- Prob. of three types

1. Classical / Marginal
2. Subjective
3. Judgement

[1] Classical / Marginal = $\frac{\text{Favourable event}}{\text{Total no. of event}}$

Ex: coin, dice.

[2] Prob. which have no mathematically base is called subjective or judgement.

* Probability have two rule :-
① Addition
② Multiplication

[1] Addition rule : ~~P(A)~~

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

independent [mutually exclusive event]

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

[not mutually exclusive event]

[2] Multiplication rule :

$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$ → Independent event

(recession) $P\left(\frac{B}{A}\right) = P(B)$ → Independent event
→ conditional prob.

$P(A \cap B) = P\left(\frac{B}{A}\right) \times P(A)$ → dependent event

(or)

(conditional event)
$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$ → conditional event

* Consider the following events :-

Q: $P(I) = 0.4$, the prob. of the monetary authority increasing in the interest rate is 40%. (40%)
$P\left(\frac{R}{I}\right) = 0.7$, prob. of recession (R) given an increase in interest rate is 70%. What is prob. of $[P(R \cap I)]$ join prob. of recession and an increasing interest rate ?

A: $P(R \cap I) = P\left(\frac{R}{I}\right) \times P(I)$

$= 0.7 \times 0.4$

$= 0.28$

$\boxed{= 28\%}$

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addition.

(i) Using the informations in previous interest rate in recession example and the fact that the unconditional prob. of recession. Determine the prob. that either IRT or IRL.

$P(R) = 34\% = 0.34$

$P(R \text{ or } I) = P(R \cup I) = P(R) + P(I) - P(A \cap B)$

$= 0.34 + 0.4 - 0.28$

$= 0.74 - 0.28$

$\boxed{= 0.46}$ Ans

FORMULAS:-

$$\underset{(mean)}{\text{Expected value}} = \underset{(return)}{[Ex]} = \Sigma P_i x_i$$

$$\sigma_i^2 = \Sigma P_i (x - \Sigma R x)^2$$

$$\sigma_i = \sqrt{\sigma_i^2}$$

where

Σx = expected value (mean return)

σ_i^2 = variance of a security

(standard deviation) σ_i = risk of a security

$\Sigma R x$ = expected value from x

i = security

Question: Calculate the risk and return from stock A and stock B from the following data.

probability distribution of returns

Event	Prob.	R_A (%)	R_B (%)	$P.R_A$	$A - ER_A$	$(A-ER_A)^2$	$P_i(A - ER_A)^2$
boom	0.30	20	30	6.00	7	49	14.7
normal	0.50	12	10	6.00	-1	1	0.50
low	0.20	5	0	1.00	-8	64	12.8
	1.00			13.00			28

Expected value $(ER_A) = \Sigma P_i A_i$

$$\boxed{ER_A = 13\%} \text{ Ans}$$

$$\sigma_A^2 = \Sigma P_i (A - ER_A)^2$$
$$= 28\%$$

$$\sigma_A = \sqrt{28}$$
$$= 5.29\%$$