Unit: Work and Conservation of Energy

(Circular Motion at bottom!)

1. What is the difference between a dot product and a cross product? 2 "physics" examples.

Dot product is a multiplication of 2 vectors which results in a scalar. $a \cdot b = |a||b|\cos\theta$

Cross product is a multiplication of 2 vectors which results in a vector. $a \times b = |a||b|\sin\theta$ → Need to use right hand rule to establish direction.

Examples:

Dot Product -

1. work $= \vec{F} \cdot \vec{d} = |F||d|\cos\theta$ → dot product.

2. power $= \vec{F} \cdot \vec{v} = |F||v|\cos\theta$ → dot product.

Cross Product -

1. Torque $= \vec{F} \times \underbrace{\vec{R}}_{\text{Radius}} = |F||R|\sin\theta$ → cross product

2. $L = \underbrace{\vec{R}}_{\text{Radius}} \times \underbrace{\vec{P}}_{\text{Linear Momentum}} = |P||R|\sin\theta$ → cross product

2. What does it mean to do positive and negative work?

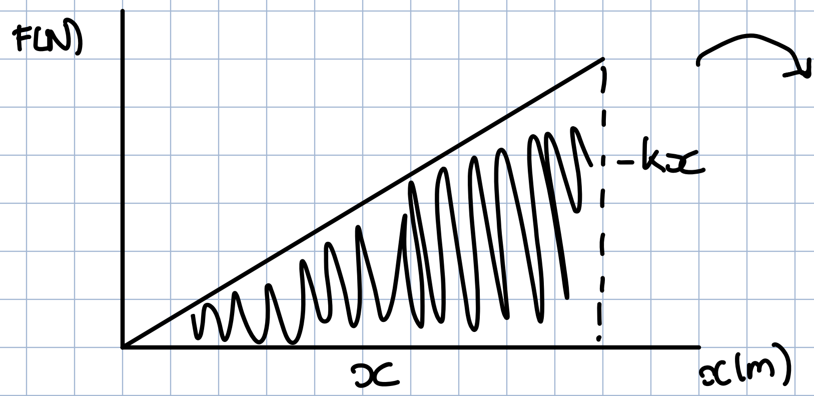
Positive work - $W > 0$ happens when force has a component in the same direction as displacement adding energy to the object. ex. pushing a shopping cart.

Negative work - $W < 0$ happens when force has a component that is opposite of the displacement thus removing energy from the object. ex. Gravity acting on a ball thrown up.

3. For a spring, $F_{Spring} = -kx$, so $W_{String} = PE_{String} = \frac{1}{2}kx^2$

Why is it 1/2 (explain by using the spectacular average or drawing a graph of Force vs Distance).

$F = -kx$



$\therefore A = \frac{1}{2}(-kx)(x)$

$= \frac{1}{2}kx^2$

This is where $\frac{1}{2}$ comes from as its a triangle

$W = |F||\Delta x|\cos\theta$

$F = \frac{F_f - F_i}{2} = \frac{-kx - 0}{2} = \frac{-kx}{2} \rightarrow W = \left|\frac{-kx}{2}\right||\Delta x|\cos(0) = \frac{1}{2}kx^2$

4. Calculus way for work $\rightarrow \int_{x_i}^{x_f} F(x)dx$ $\quad W = \int_{t_i}^{t_f} P(t)dt$

5. Calculus way for power $\rightarrow P = \frac{dw}{dt}$ and $P = \int_{t_i}^{t_f} F(t)dt$

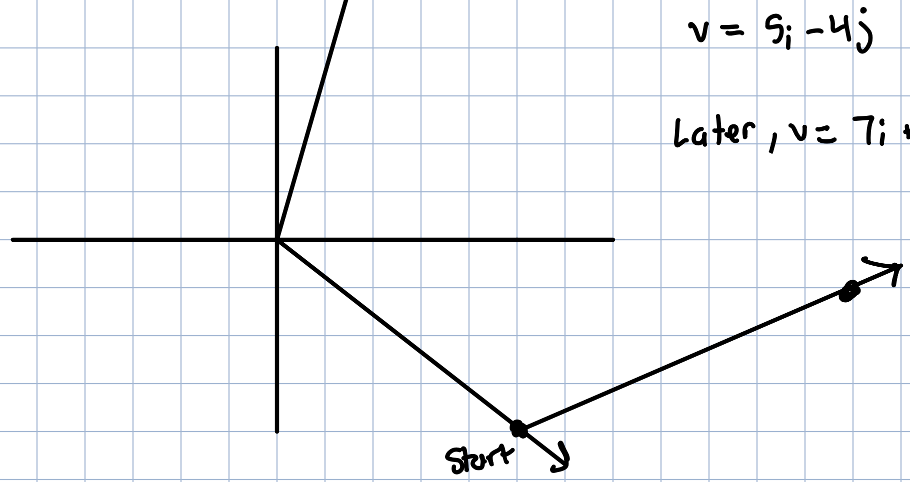
6. Calculus way to find potential energy from force? Relationship between force and potential energy?

$$PE(x) = -\int_{x_i}^{x_f} F(x)dx$$

Where $f(x)$ is a function for force (y axis) vs Δx (x axis) and x_i and x_f are bounds.

Problems:

1) 2kg particle



$v = 5i - 4j \quad \therefore \sqrt{5^2+4^2} = \sqrt{41}$ m/s

Later, $v = 7i + 3j \quad \therefore \sqrt{7^2+3^2} = \sqrt{58}$ m/s

$\therefore KE_i = \frac{1}{2}mv^2 = \frac{1}{2}(2kg)(\sqrt{41})^2$
$= 41J$

$KE_f = \frac{1}{2}mv^2 = \frac{1}{2}(2kg)(\sqrt{58})^2$
$= 58J$

Force changing the velocities direction is perpendicular to velocity $\therefore$ do not contribute work $\rightarrow \cos(90) = 0$

$\therefore$ work $= KE_f - KE_i$
$= 58J - 41J = 17J$

2)

$P = F \cdot v \cos\theta = 5 \times 2 \times \cos(0)$

$= 10$ watts

3) a) $F = -2x^3$

$m = 2.5kg$

Force is negative since the force acts in the opposite direction of displacement. since it is trying to restore to equilibrium

b) $PE = -\int F(x)dx$

$= -\int (-2kx^3)dx$

$= \int (2kx^3)dx = 2k\int x^3 dx = 2k\left(\frac{1}{4}x^4\right) = \frac{1}{2}kx^4$

c) $\sum TME_i = \sum TME_f$

$PE_i = KE_f$

$\therefore v_f = 2.4$ m/s

$$\frac{1800(0.3)^4}{2} = \frac{1}{2}(2.5)v_f^2$$

4)

a) $m = 5kg$

$\Delta y = 20m$

$\mu = 0.4$

$k = 10\,000$

$\Delta x = .2$

$ME_i = ME_f$

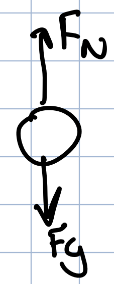
$mgh = \frac{1}{2}mv_f^2$

$gh = \frac{1}{2}v_f^2$

$9.8(20) = \frac{1}{2}v_f^2$

$v_f = 19.8 m/s$

b)



$\Sigma F_y = 0$

$F_N = F_g$

$F_N = 49N$

$w = Fd\cos\theta$

$$PE_i + KE_i + W_{nc} = PE_f + KE_f$$

$$\frac{1}{2}mv_i^2 + F_f d\cos\theta = \frac{1}{2}kx^2$$

$$\frac{1}{2}(5)(19.8)^2 - (49)(0.4)d = \frac{1}{2}(10000)(0.2)^2$$

$$d = 39.8m$$

c) all of the energy conserved was from A to B.

d) $ME_i = mgh = 5(9.8)(20) = 980J$

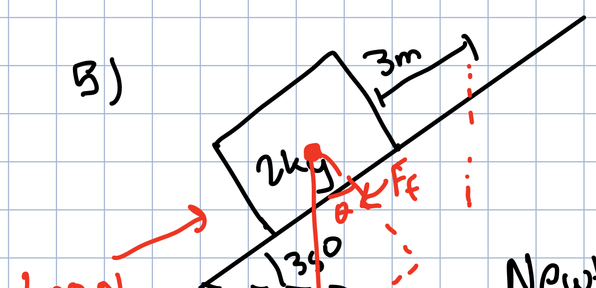
$ME_f = \frac{1}{2}kx^2 = \frac{1}{2}(10000)(0.2)^2 = 200J$

Energy difference $= 780J$

Only 200J was conserved

Change in energy was caused by the external force acting on the box, which is friction.

5)



$\mu = 0.2$

$F_f = 0.2(F_N) = 0.2(2)(9.8) = 3.92N$

Newtons Laws:

$F_g \sin\theta = 2(9.8)\sin(35) = 11.24\,N$

$\therefore \sum F_x = 200N - 11.24 - 3.92 = 184.84N = ma$

$184.84 = 2a$

$a = 92.42\,m/s^2$

$V_f^2 - V_i^2 = 2a\Delta x$

$V_f^2 - 0 = 2(92.42)(3)$

$V_f = 23.89\ m/s$

Energy:

$ME_i = ME_f$

$work_{nc} + W_c = KE_f$

$F\,d\cos\theta + F_f\,d\cos\theta = \frac{1}{2}mv_f^2$

$200(3) - 49(0.2)(3) = \frac{1}{2}(2)V_f^2$

$V_f = 23.89\ m/s$

Unit: Linear Momentum and Impulse

1. Is momentum a vector or scalar quantity?

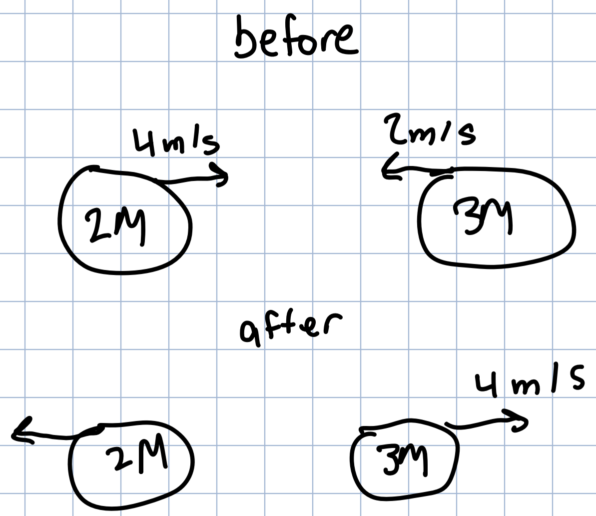
momentum is a vector quantity and moves in the same direction as velocity.

2. Is impulse a vector or scalar quantity?

Impulse is a vector quantity and moves in the same direction as force.

Problems:

1)



a) $P_i = P_f$

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$2M(4) - 3M(2) = 2M(v_{1f}) + 3M(4)$

$v_{1f} = 5M$ m/s left or $-5M$ m/s

b) This is an inelastic Collision

$KE_i = KE_f$

$KE_i = \frac{1}{2} m_1 (v_{1i})^2 + \frac{1}{2} m_2 (v_{2i})^2 = \frac{1}{2}(2M)(4)^2 + \frac{1}{2}(3M)(2)^2$

$= 22M$ J

$KE_f = \frac{1}{2} m_1 (v_{1f})^2 + \frac{1}{2} m_2 (v_{2f})^2 = \frac{1}{2}(2M)(-5)^2 + \frac{1}{2}(3M)(4)^2$

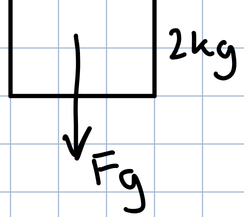
Inelastic since $22J \neq 49J$ $= 25M + 24M = 49M$ J

$\therefore$ Energy increased.

2)

Down = negative

Up = positive



$a = -9.8\ m/s^2$

Impulse $= P_f - P_i$

$= mv_f - mv_i$

$= mv_f - 0$

$= 2(v_f) = 2(-39.2)$

$= -78.4\ N \cdot s$

$v_f = v_i + at$

$v_f = (-9.8)(4\ \text{seconds})$

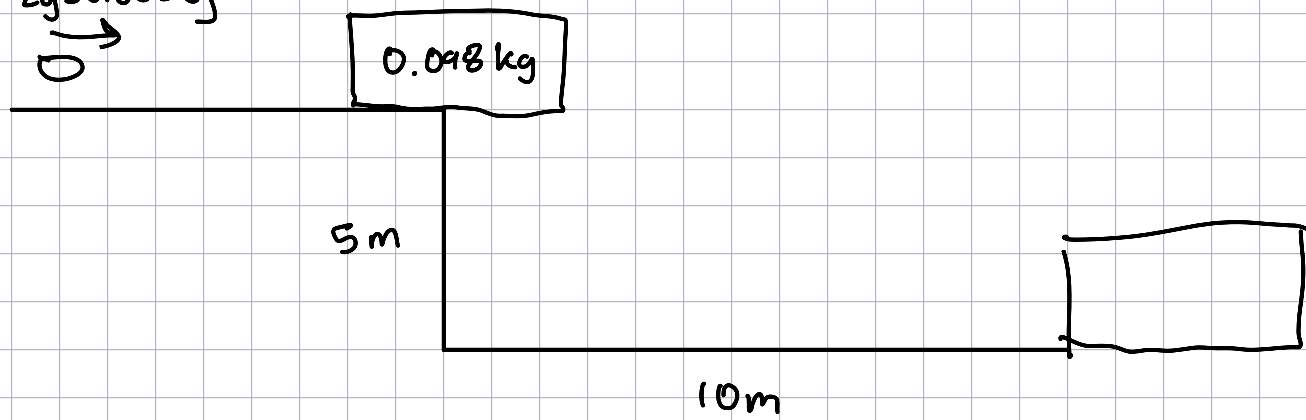
$v_f = -39.2\ m/s$

Momentum:

$P = mv = 2kg(-39.2)$

$= -78.4\ N \cdot s$

3) $2g = 0.002kg$



$P_i = P_f$

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$

$(0.002kg)(v_1) + (0.098kg)(0) = (0.002 + 0.098) v_f$

$\Delta y = v_i t + \frac{1}{2} a t^2$

$5 = \frac{1}{2}(9.8) t^2$

$t = 1.01s$

$\Delta x = v_f \cdot t$

$\therefore \frac{\Delta x}{t} = v_f$

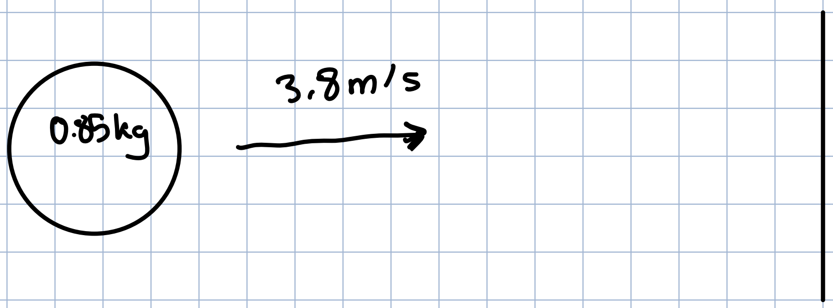
$\therefore \frac{10m}{1.01s} = v_f = 9.9\ m/s$

Could also use energy

$$(0.002kg)(V_1)+(0.098kg)(0)=(0.002+0.098)(9.9)$$

V_1 = speed of bullet = 495 m/s

4)



a) $P_i = P_f$

$$J=\Delta p = m(v_f - v_i)$$

$$J = 0.85\,(0-3.8)$$

$$J = -3.23 \text{ kg m/s}$$

Impulse experienced by the ball

By Newtons 3rd law, the wall experiences an equal and opposite impulse = +3.23 kg m/s

Since the ball comes to a stop, it is an inelastic collision. In a perfect inelastic collision, kinetic energy is not conserved since it is transformed into other energy forms.

The difference in energy is due to the conversion of kinetic energy into other forms, ex. heat

b) For the collision to be elastic, the ball would need to bounce back with a velocity of 3.8 m/s (opposite direction). $KE_i = KE_f$

$$\frac{1}{2}(0.85)(3.8)^2 = \frac{1}{2}(0.85)(V_f)^2$$

$V_f = -3.8$ m/s

Unit: Rotational Kinematics

1. What are the units for the angular quantities above?

v (m/s) $\rightarrow$ ω (rad/s

Δx (m) $\longrightarrow$ $\Delta\theta$ (rad)

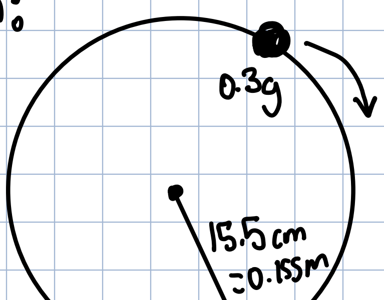
a (m/s^2) $\longrightarrow$ α (rad/s^2

2. When should you use the "R" relationship? Can the "R" relationship be used when there is slippage?

The "R" relationship does not apply when there is slippage because if the object is slipping forward or $v > R\omega$, the object moves faster than it roles and vice versa. ∴ The "R" relationship doesn't hold as friction plays a part in the velocity. Can only be used when an object is rotating with no slippage.

Problem:

1)



$\omega_f = 45\text{rpm} = 90\pi \text{ rad/min} = \frac{3}{2}\pi \text{ rad/s}$

0.92s to reach

a) $40\,rpm = 80\pi \frac{rad}{min} = \frac{4}{3}\pi \frac{rad}{s}$

$\frac{v}{r} = \omega$

$\frac{v}{.155m} = \frac{4}{3}\pi\ rad/s$

$v = 0.649\,m/s$

b) $\frac{3}{2}\pi\ rad/s = \omega_f = \omega_i + \alpha t$

$a_c = \frac{v^2}{r} = \frac{0.649^2}{0.155} = 2.7\,m/s^2$

$\frac{3}{2}\pi = 0 + \alpha(0.92s)$

$\alpha = 5.1\ rad/s^2$ at 40rpm

$a_t = \alpha R = 5.1(.155m) = 0.79\,m/s^2$

$a_{total} = \sqrt{a_t^2 + a_c^2} = \sqrt{2.7^2 + 0.79^2} = 2.8\,m/s^2$

c) $\sum F = ma = 2.8(0.0003) = 0.00085N$

Mini Unit: Center of Mass and Moment of Inertia

1. How do you calculate the center of mass?

$$\frac{\sum x_i m_i}{\sum m_i} \quad \text{or} \quad \frac{\int x\,dm}{\int dm}$$

2. How do you calculate the moment of inertia?

$$\sum m_i r_i^2 \quad \text{or} \quad \int r^2\,dm$$

3. $I \quad = I \quad + Md^2$

$I_{\text{any axis}} = I_{CM} + Md$

M = total mass of the system, d = distance from new I and I_{cm}

Problems:

1)

$\lambda = kx^2$

$k = (4M)/L^3$ and x is the distance from the rod.

$$\lambda = \frac{4M}{L^3}x^2 = \frac{dm}{dx}$$

$$\frac{4M}{L^3}x^2\,dx = dm$$

a) $$X_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\frac{4M}{L^3}\int_0^L x^3\,dx}{\frac{4M}{L^3}\int_0^L x^2\,dx}$$

$$= \frac{\frac{4M}{L^3}\cdot\left[\frac{1}{4}x^4\right]_0^L}{\frac{4M}{L^3}\cdot\left[\frac{1}{3}x^3\right]_0^L} = \frac{\frac{4M}{L^3}\cdot\frac{1}{4}L^4}{\frac{4M}{L^3}\cdot\frac{1}{3}L^3} = \frac{ML}{\frac{4M}{3}}$$

$= \frac{3}{4}L$ is the X_{cm}

b) $$I_{P_1} = \int r^2\,dm = \int r^2\cdot\frac{4M}{L^3}\cdot x^2\,dx$$

$$= \int_0^L x^2\cdot\frac{4M}{L^3}\cdot x^2\,dx = \frac{4M}{L^3}\int_0^L x^4\,dx$$

$$= \frac{4M}{L^3}\cdot\left[\frac{1}{5}x^5\right]_0^L = \frac{4M\cdot L^5}{5\cdot L^3} = \frac{4ML^2}{5}$$

c) $I_{P_1} = I_{cm} + Md^2$

$$\frac{4ML^2}{5} = I_{cm} + M\left(\frac{3}{4}L\right)^2$$

$$I_{cm} = \frac{4ML^2}{5} - \frac{9}{16}L^2 M = \frac{19ML^2}{80}$$

d) $I_{P_2} = I_{cm} + Md^2$

$$I_{P_2} = \frac{19ML^2}{80} + M\left(\frac{1}{4}L\right)^2 = \frac{19ML^2}{80} + \frac{ML^2}{16} = \frac{24ML^2}{80}$$

$$= \frac{3ML^2}{10}$$

2) $\lambda = 0.2\,kg/m^2 + 0.015x\,kg/m^2 + 0.003x^2\,kg/m^3$

a) $\lambda = \frac{M}{L}$

$\lambda dx = dm \rightarrow \text{Mass} = \int_0^{50} \lambda dx$

$$\therefore \text{Mass} = \int_0^{50} (0.2 + 0.015x + 0.003x^2)dx$$

$$= 153.75\,kg$$

b) $\lambda = \frac{M}{L}$

$\lambda = \frac{dm}{dx} \quad \therefore (0.2 + 0.015x + 0.003x^2)\,dx = dm$

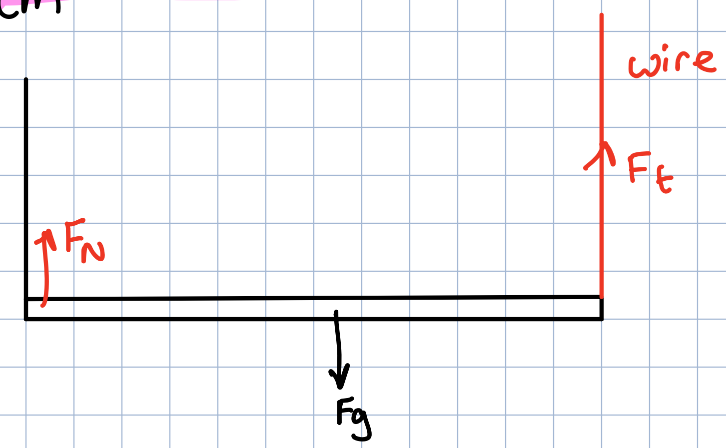
$$X_{cm} = \frac{\int x\,dm}{} = \int_0^{50} x(0.2 + 0.015x + 0.003x^2)\,dx$$

$$\int dm \quad \int_0^{50} (0.2 + 0.015x + 0.003x^2)\, dx$$

↓ Plug into calculator

$$\boxed{X_{cm} = 36.2\,m}$$

c)



$$\sum \tau = 0 = 50(F_t) - 36.2(F_g)$$

$$36.2 F_g = 50 F_t$$

$$36.2(153.75\,kg)(9.8) = 50 F_t$$

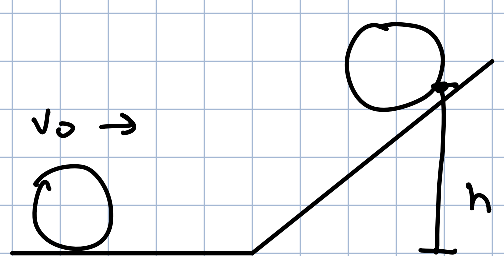
$$\boxed{F_t = 1090.887\,N}$$

d) $$\sum F_y = 0 = F_t + F_N - F_g$$

$$0 = 1090.887 + F_N - 36.2(153.75)$$

$\boxed{F_N = 417\,N \text{ acting upwords}}$.

3) Disk with moment of inertia $I = \frac{1}{2}MR^2$



a) $h = \frac{v_0^2}{2g}$

e) $h = \frac{v_0^2}{g}$

b) $h = \frac{2v_0^2}{3g}$

$3g$

c) $h = \frac{2v_0^2}{5g}$

$\sum TME_i = \sum TME_f$

d) $h = \frac{3v_0^2}{4g}$

$KE_i + KE_{rot\ i} = PE_f$

$\frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega^2 = Mgh$

$\frac{1}{2}Mv_0^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_0}{R}\right)^2 = Mgh$

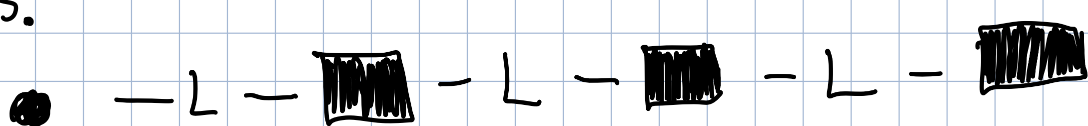
$\frac{3v_0^2}{4} = gh \quad \therefore \quad h = \frac{3v_0^2}{4g}$

Unit : Angular Dynamics

1. How do you figure out directionality of τ? Explain using Right hand Rule.

Start with your right hand at the distance from the axis of rotation and you curl your fingers to where force is, if you curl in a CW direction, then the direction is CW and vice versa. If your thumb is in the page, it is negative $\hat{k}$ and vice versa.

Problems:

1) 

a) $I = \sum m_i r_i^2 = 5(1.5)^2 + 5(3)^2 + 5(4.5)^2 = 157.5\ kg\ m^2$

$KE_{rot} = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}(157.5)(2)^2 = 315\ J$

b) $X_{cm} = \dfrac{\sum x_i m_i}{\sum m_i} = \dfrac{5(1.5)+5(3)+5(4.5)}{15} = 3\,m$

$I_p = I_{cm} + Md^2$

$157.5 = I_{cm} + 15(3)^2$

$I_{cm} = 22.5\ kgm^2$

$KE_{rot} = \frac{1}{2} I_{cm}\,\omega^2 = \frac{1}{2}(22.5)(2)^2 = 45J$

c) $I_{p_2} = I_{cm} + Md^2$

$I_{p_2} = 22.5 + 15(1.5)^2 = 56.3\,kg\,m^2$

$KE_{rot} = \frac{1}{2} I_{p_2}\,\omega^2 = \frac{1}{2}(56.3)(2)^2 = 112.5J$

d) $\omega_f = \omega_i + \alpha t$

$2 = 0 + \alpha(1.5)$

$\alpha = 1.33\ rad/s^2$

$\sum \tau = I\alpha = 157.5(1.33) = 209.5\,Nm$

i) $\sum \tau = I\alpha = r \cdot F$

$3F = 209.5$

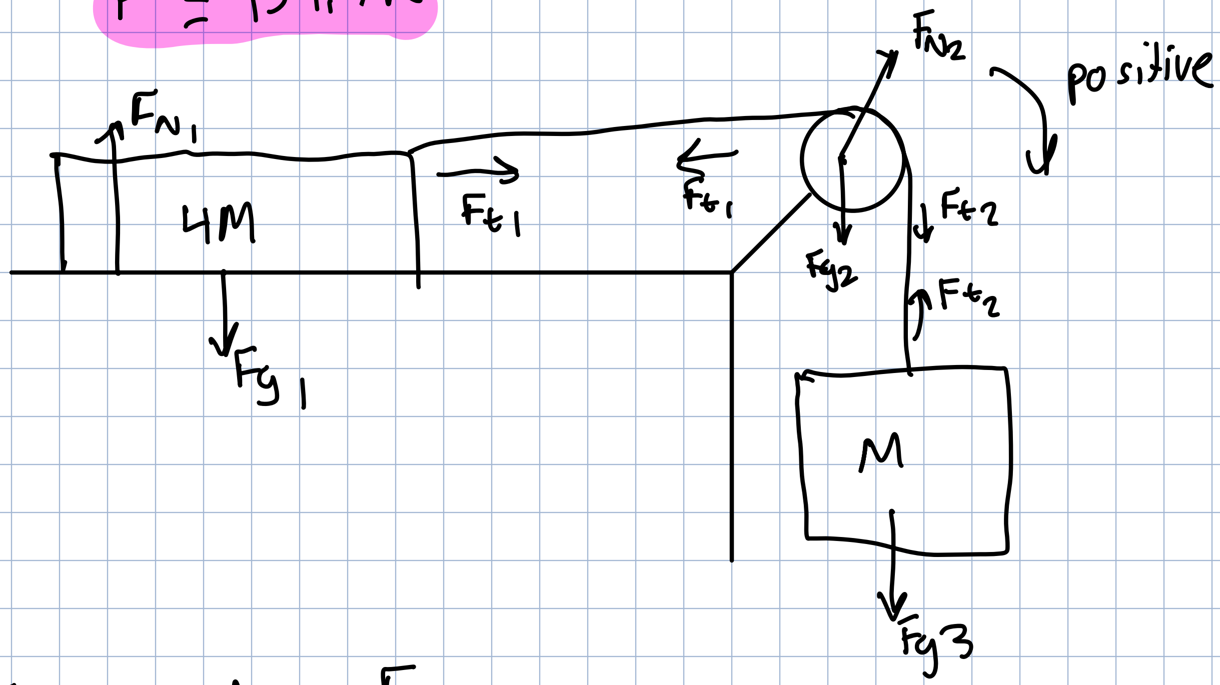
$F = 69.8N$

ii) $\sum \tau = I\alpha = r \cdot F$

$$1.5F = 209.5$$

$$F = 139.7\,N$$

2)



a) $$\sum F_x = 4Ma = F_{t_1}$$

$$\sum F_x = 8a = F_{t_1}$$

$$\sum F_y = Ma = F_{g3} - F_{t_2}$$

systems of equations

$$\sum F_y = 2a = 19.62 - F_{t_2}$$

$$\sum \tau = I\alpha = R(F_{t_2} - F_{t_1})$$

$$\sum \tau = \frac{1}{5}(0.5)(0.075)^2\left(\frac{a}{0.075}\right) = 0.075(F_{t_2} - F_{t_1})$$

$$F_{t_1} = 15.5\,N \quad \text{and} \quad a = 1.94\,m/s^2$$

$$F_{t_2} = 15.7\,N$$

b) ✓ Use same system of equations

$$a = 1.94\,m/s^2$$

c) $V_f^2 = V_i^2 + 2a\Delta y$

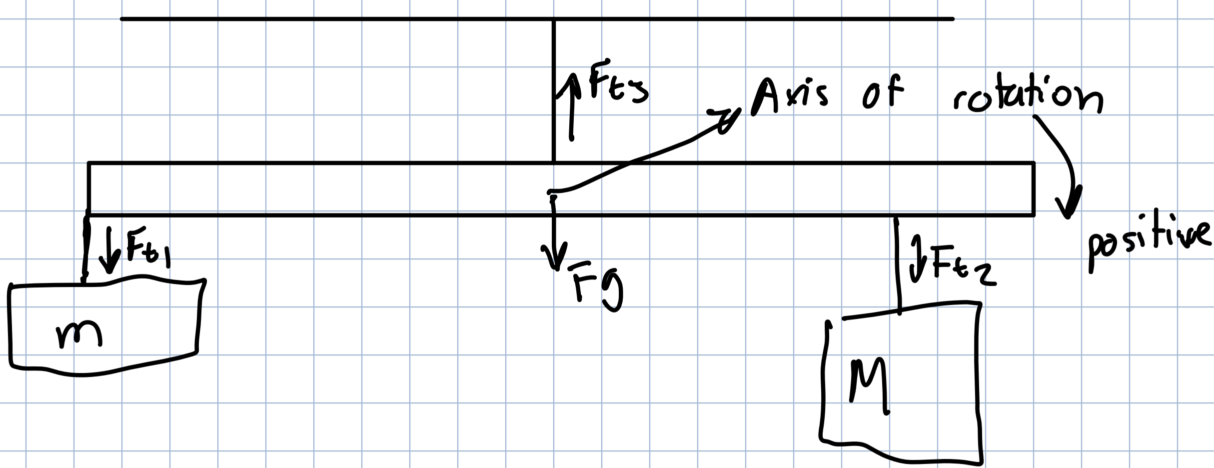
$V_f^2 = 0 + 2(1.94)(1.5)$

$V_f = 2.4 \text{ m/s}$

d) $L = I\omega = \frac{1}{5}MR^2\left(\frac{v}{R}\right) = \frac{1}{5}(0.5)(0.075)(2.4) = 0.018 \frac{\text{kgm}^2}{\text{s}}$

3)

a)



b) $\sum \tau = 0 = RF_{t_2} - RF_{t_1}$

$F_{t_2} = F_{t_1}$

$0.3M(9.81) = 0.5(3)(9.81)$

$M = 5\text{kg}$

c) $I = \sum r_i^2 m_i = (3)(0.5)^2 + 5(0.4)^2 + \frac{1}{2}(1)(1)^2$
$= 1.63 \text{kgm}^2$

$\sum \tau = I\alpha = RF_{t_2} - RF_{t_1}$

$$\sum \tau = 1.63\alpha = 0.4(5)(9.81) - 0.5(3)(9.81)$$

$$\alpha = 3.01 \text{ rad/s}^2 \quad CW$$

4)

a) $I = I_{rod} + I_{ball}$

$$I_{ball} = MR^2 = 0.2(2)^2 = 0.8 \text{kg m}^2$$

$$I_{rod} = \frac{1}{3}MR^2 = \frac{1}{3}(5)(2)^2 = 6.67 \text{kg m}^2$$

$$I = 6.67 + 0.8 = 7.47 \text{kg m}^2$$

b) $PE_{i\,rod} + PE_{i\,ball} = KE_{rot\,final}$

$$mgh_{rod} + mgh_{ball} = \frac{1}{2}I\omega^2$$

$$5(9.81)(1) + 0.2(9.81)(2) = \frac{1}{2}(7.47)\omega_f^2$$

$$\omega_f = 3.77 \text{ rad/s}$$

acceleration is 0 because the force acting on it is gravity which is perpendicular to the arm.

c) $v_{ball} = \omega_{ball} r = 3.77(2) = 7.53 \frac{m}{s}$

Unit: Uniform Circular Motion

1. Explain what uniform circular motion means

Uniform circular motion is changing the velocity, A constant centripetal force / acceleration, and with a constant speed. The constant speed makes it uniform.

2. Centripetal force is usualy the "net" force of the system unless there is a force actively pushing it away from the center of the circle.

3. Make up an example when centripetal force is not the net force of the system.

A cart at the bottom of the loop on a roller coaster. F_g actively pushes away from the center of the loop.

Problems:

1)
a) $33.3 rpm \cdot \frac{1 min}{60 sec} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot 0.155 m = 0.541 m/s$

b) $F_c = \frac{mv_r^2}{r} = \frac{0.0003 \cdot 0.541^2}{0.155} = 0.000565N$

$F_c = ma_c$

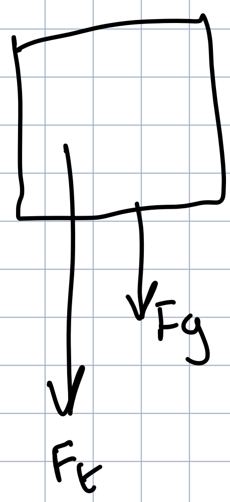
$0.000565N = 0.0003\, a_c$

$1.88\, m/s^2 = a_c$

c) $F_c = \frac{mv_r^2}{r} = \frac{0.0003 \cdot 0.541^2}{0.155} = 0.000565N$

2)

a)



$F_g = mg = 1.1(9.81) = 10.8N$

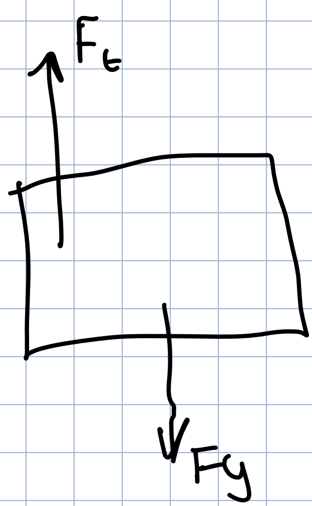
$F_c = \frac{mv_r^2}{r} = \frac{1.1 \cdot 4^2}{0.4} = 44N$

$F_c = F_g + F_t$

$44 = 10.8 + F_t$

$F_t = 33.2N$

b)



$F_g = mg = 1.1(9.81) = 10.8N$

$F_c = \frac{mv_r^2}{r} = \frac{1.1 \cdot 4^2}{0.4} = 44N$

$F_c = F_t - F_g$

$44 = F_t - 10.8$

$F_t = 54.8N$

[illegible]