Assignment - 1

Nisarg Parekh
SM21MTECH14002

PROBLEM

1.Show that the area of the triangle whose vertices are:

$$(a\tan\theta, b\cot\theta), (a\tan\phi, b\cot\phi), (a\tan\gamma, b\cot\gamma)$$

$$is \;\; \frac{4ab\ \sin(\theta-\phi)\sin(\phi-\gamma)\sin(\gamma-\phi)}{\sin 2\theta \sin 2\phi \sin 2\gamma}$$

SOLUTION

$$x_1 = a\tan\theta,\ x_2 = a\tan\phi,\ x_3 = a\tan\gamma,$$

$$y_1 = b\cot\theta,\ y_2 = b\cot\phi,\ y_3 = b\cot\gamma$$

area of triangle= $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$\frac{1}{2}[x1(y2-y3)+x2(y3-y1)+x3(y1-y2)]$$

area of triangle=

$$\frac{1}{2}[a\tan\theta(b\cot\phi - b\cot\gamma) + a\tan\phi(b\cot\gamma - b\cot\theta)$$
$$+a\tan\gamma(b\cot\theta - b\cot\phi)]$$

$$LHS = \frac{ab}{2}[(\tan\theta*\cot\phi - \tan\theta*\cot\gamma)+$$
$$(\tan\phi*\cot\gamma - \tan\phi*\cot\theta)+$$
$$(\tan\gamma*\cot\theta - \tan\gamma*\cot\phi)]$$

$\because$ taking “ab” as common

$$\therefore \frac{ab}{2}[\ (\ \frac{\sin\theta*\cos\phi}{\cos\theta*\sin\phi} - \frac{\sin\theta*\cos\gamma}{\cos\theta*\sin\gamma}\ )+$$
$$(\ \frac{\sin\phi*\cos\gamma}{\cos\phi*\sin\gamma} - \frac{\sin\phi*\cos\theta}{\cos\phi*\sin\theta}\ )+$$
$$(\ \frac{\sin\gamma*\cos\theta}{\cos\gamma*\sin\theta} - \frac{\sin\gamma*\cos\phi}{\cos\gamma*\sin\phi}\ )\ ]$$

$\because$ representing tan and cot in sin and cos form

$$\therefore \frac{ab}{2}[(\frac{\sin\theta*\cos\phi}{\cos\theta*\sin\phi} * \frac{4\sin\theta*\cos\phi*\sin 2\gamma}{4\sin\theta*\cos\phi*\sin 2\gamma} -$$
$$\frac{\sin\theta*\cos\gamma}{\cos\theta*\sin\gamma} * \frac{4\sin\theta*\cos\gamma*\sin 2\phi}{4\sin\theta*\cos\gamma*\sin 2\phi})+$$
$$(\frac{\sin\phi*\cos\gamma}{\cos\phi*\sin\gamma} * \frac{4\sin\phi*\cos\gamma*\sin 2\theta}{4\sin\phi*\cos\gamma*\sin 2\theta} -$$
$$\frac{\sin\phi*\cos\theta}{\cos\phi*\sin\theta} * \frac{4\sin\phi*\cos\theta*\sin 2\gamma}{4\sin\phi*\cos\theta*\sin 2\gamma})+$$
$$(\frac{\sin\gamma*\cos\theta}{\cos\gamma*\sin\theta} * \frac{4\sin\gamma*\cos\theta*\sin 2\phi}{4\sin\gamma*\cos\theta*\sin 2\phi} -$$
$$\frac{\sin\gamma*\cos\phi}{\cos\gamma*\sin\phi} * \frac{4\sin\gamma*\cos\phi*\sin 2\theta}{4\sin\gamma*\cos\phi*\sin 2\theta})]$$

$\because$ multiplying and dividing each term with 4$\sin X\cos X\sin 2X$

$$\therefore \frac{4ab}{2}[(\frac{\sin 2\gamma*(\sin\theta*\cos\phi)^2 - \sin 2\phi*(\sin\theta*\cos\gamma)^2}{\sin 2*\sin\theta*\sin\gamma})$$
$$+(\frac{\sin 2\theta*(\sin\phi*\cos\gamma)^2 - \sin 2\gamma*(\sin\phi*\cos\theta)^2}{\sin 2\theta*\sin\phi*\sin\gamma})$$
$$+(\frac{\sin 2\phi*(\sin\gamma*\cos\theta)^2 - \sin 2\theta*(\sin\gamma*\cos\phi)^2}{\sin 2\theta*\sin\phi*\sin\gamma})]$$

$\because$ simplifying above equation

$$\therefore \tfrac{4ab}{2}[\ \ (2\sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$
$$-2\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2 \tfrac{}{\sin 2\theta*\sin\phi*\sin\gamma)}\ +$$
$$(\tfrac{2\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2 - 2\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2}{\sin 2\theta*\sin\phi*\sin\gamma})+$$
$$(\tfrac{2\sin\phi\cos\phi*(\sin\theta*\cos\phi)^2 - 2\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2}{\sin 2\theta*\sin\phi*\sin\gamma})\ ]$$

$$\because \sin 2\theta = 2\sin\theta\cos\theta$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$
$$-\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2\ )$$
$$+(\ \sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2$$
$$-\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2\ )$$
$$+(\ \sin\phi\cos\phi*(\sin\gamma*\cos\theta)^2$$
$$-\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2\ )$$
$$+\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$$
$$-\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma]$$

$\because$ *adding and subtracting* $\sin\theta*\cos\theta$ $\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$$
$$-\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2-\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2$$
$$+\sin\phi\cos\phi*(\sin\gamma*\cos\theta)^2-\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2$$
$$+\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2+\sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$
$$-\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma\ ]$$

$\because$ *rearranging terms*

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\gamma\cos\theta(\ \sin\theta\cos\phi\sin\phi\cos\gamma-\cos\theta\cos\gamma\sin^2\phi-$$
$$\sin\theta\sin\gamma\cos^2\phi+\cos\theta\sin\theta\cos\phi\sin\gamma\ )$$
$$-\cos\gamma\sin\theta(\ \sin\theta\cos\phi\sin\phi\cos\gamma-\cos\theta\cos\gamma\sin^2\phi$$
$$-\sin\theta\sin\gamma\cos^2\phi+\cos\theta\sin\theta\cos\phi\sin\gamma\ )\ ]$$

$\because$ *taking common* $\sin\gamma\cos\theta$ *and* $\sin\theta\cos\gamma$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta-\sin\theta\cos\gamma)*(\ \sin\phi\cos\gamma\,(\sin\theta\cos\phi-\cos\theta\sin\phi)$$
$$-\cos\phi\sin\gamma\,(\sin\theta\cos\phi-\cos\theta\sin\phi)\ )\ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta-\sin\theta\cos\gamma)*(\sin\phi\cos\gamma-\sin\phi\cos\theta)$$
$$*(\sin\theta\cos\phi-\sin\phi\cos\theta)\ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin(\gamma-\theta))*(\sin(\phi-\gamma))*(\sin(\theta-\phi)\ )\ ]=R.H.S$$