Assignment - 1

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SM21MTECH14002

Problem

1.Show that the area of the triangle whose vertices are:

$(a\tan\theta, b\cot\theta), (a\tan\phi, b\cot\phi), (a\tan\gamma, b\cot\gamma)$

$$is \;\; \frac{4ab\ \sin(\theta-\phi)\sin(\phi-\gamma)\sin(\gamma-\phi)}{\sin 2\theta \sin 2\phi \sin 2\gamma}$$

Solution

$$\mathbf{x_1} = \left(a\tan\theta\right), \mathbf{x_2} = \left(a\tan\phi\right) \mathbf{x_3} = \left(a\tan\gamma\right),$$

$$\mathbf{y_1} = \left(b\cot\theta\right), \mathbf{y_2} = \left(b\cot\phi\right), \mathbf{y_3} = \left(b\cot\gamma\right)$$

area of triangle= $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

= $\frac{1}{2}\begin{vmatrix} a\tan\theta & b\cot\theta & 1 \\ a\tan\phi & b\cot\phi & 1 \\ a\tan\gamma & b\cot\gamma & 1 \end{vmatrix}$

= $\frac{ab}{2}\begin{vmatrix} \tan\theta & \cot\theta & 1 \\ \tan\phi & \cot\phi & 1 \\ \tan\gamma & \cot\gamma & 1 \end{vmatrix}$

$$\because taking\ a\ for\ C1\ and\ b\ from\ C2\ common$$

= $\frac{ab}{2}\begin{vmatrix} \tan\theta & \cot\theta & 1 \\ \tan\phi - \tan\theta & \cot\phi - \cot\theta & 0 \\ \tan\gamma - \tan\theta & \cot\gamma - \cot\theta & 0 \end{vmatrix}$

$$\because R2 = R2 - R1\ and\ R3 = R3 - R1$$

$$\therefore \frac{ab}{2}[(\tan\phi - \tan\theta) * (\cot\gamma - \cot\theta) - (\tan\gamma - \tan\theta) * (\cot\phi - \cot\theta)]$$

$$\because solving\ determinant$$

$$\therefore \frac{ab}{2}[\tan\theta(\cot\phi - \cot\gamma) + \tan\phi(\cot\gamma - \cot\theta) + \tan\gamma(\cot\theta - \cot\phi)]$$

$$\because solving\ above\ equation$$

$$\therefore \frac{ab}{2}[(\tan\theta * \cot\phi - \tan\theta * \cot\gamma) + (\tan\phi * \cot\gamma - \tan\phi * \cot\theta) + (\tan\gamma * \cot\theta - \tan\gamma * \cot\phi)]$$

$$\because solving\ above\ equation$$

$$\therefore \frac{ab}{2}[\ (\ \frac{\sin\theta * \cos\phi}{\cos\theta * \sin\phi} - \frac{\sin\theta * \cos\gamma}{\cos\theta * \sin\gamma}\ ) + (\ \frac{\sin\phi * \cos\gamma}{\cos\phi * \sin\gamma} - \frac{\sin\phi * \cos\theta}{\cos\phi * \sin\theta}\ ) + (\ \frac{\sin\gamma * \cos\theta}{\cos\gamma * \sin\theta} - \frac{\sin\gamma * \cos\phi}{\cos\gamma * \sin\phi}\ )\ ]$$

$\because$ representing tan and cot in sin and cos form

$$\therefore \frac{ab}{2}[(\frac{\sin\theta * \cos\phi}{\cos\theta * \sin\phi} * \frac{4\sin\theta * \cos\phi * \sin 2\gamma}{4\sin\theta * \cos\phi * \sin 2\gamma} - \frac{\sin\theta * \cos\gamma}{\cos\theta * \sin\gamma} * \frac{4\sin\theta * \cos\gamma * \sin 2\phi}{4\sin\theta * \cos\gamma * \sin 2\phi}) + (\frac{\sin\phi * \cos\gamma}{\cos\phi * \sin\gamma} * \frac{4\sin\phi * \cos\gamma * \sin 2\theta}{4\sin\phi * \cos\gamma * \sin 2\theta} - \frac{\sin\phi * \cos\theta}{\cos\phi * \sin\theta} * \frac{4\sin\phi * \cos\theta * \sin 2\gamma}{4\sin\phi * \cos\theta * \sin 2\gamma}) + (\frac{\sin\gamma * \cos\theta}{\cos\gamma * \sin\theta} * \frac{4\sin\gamma * \cos\theta * \sin 2\phi}{4\sin\gamma * \cos\theta * \sin 2\phi} - \frac{\sin\gamma * \cos\phi}{\cos\gamma * \sin\phi} * \frac{4\sin\gamma * \cos\phi * \sin 2\theta}{4\sin\gamma * \cos\phi * \sin 2\theta})]$$

$\because$ multiplying and dividing each term with $4\sin X \cos X \sin 2X$

$$\frac{4ab}{2}[\frac{\sin 2\gamma * (\sin\theta * \cos\phi)^2 - \sin 2\phi * (\sin\theta * \cos\gamma)^2}{\sin 2 * \sin\theta * \sin\gamma}$$
$$+(\frac{\sin 2\theta * (\sin\phi * \cos\gamma)^2 - \sin 2\gamma * (\sin\phi * \cos\theta)^2}{\sin 2\theta * \sin\phi * \sin\gamma})$$
$$+(\frac{\sin 2\phi * (\sin\gamma * \cos\theta)^2 - \sin 2\theta * (\sin\gamma * \cos\phi)^2}{\sin 2\theta * \sin\phi * \sin\gamma})]$$

$\because$ simplifying above equation

$$\therefore \frac{4ab}{2}[\ (\frac{2\sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2 - 2\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2}{\sin 2\theta*\sin\phi*\sin\gamma})$$
$$+\ (\frac{2\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2 - 2\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2}{\sin 2\theta*\sin\phi*\sin\gamma}) +$$
$$(\frac{2\sin\phi\cos\phi*(\sin\theta*\cos\phi)^2 - 2\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2}{\sin 2\theta*\sin\phi*\sin\gamma})]$$

$\because \sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin\gamma\cos\gamma * (\sin\theta * \cos\phi)^2$$
$$- \sin\phi\cos\phi * (\sin\theta * \cos\gamma)^2\ )$$
$$+ (\ \sin\theta\cos\theta * (\sin\phi * \cos\gamma)^2$$
$$- \sin\gamma\cos\gamma * (\sin\phi * \cos\theta)^2\ )$$
$$+ (\ \sin\phi\cos\phi * (\sin\gamma * \cos\theta)^2$$
$$- \sin\theta\cos\theta * (\sin\gamma * \cos\phi)^2\ )$$
$$+ \sin\theta * \cos\theta * \sin\phi * \cos\phi * \sin\gamma * \cos\gamma$$
$$- \sin\theta * \cos\theta * \sin\phi * \cos\phi * \sin\gamma * \cos\gamma]$$

$\because$ *adding and subtracting* $\sin\theta * \cos\theta * \sin\phi * \cos\phi * \sin\gamma * \cos\gamma$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\theta * \cos\theta * \sin\phi * \cos\phi * \sin\gamma * \cos\gamma$$
$$-\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2 - \sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2$$
$$+\sin\phi\cos\phi*(\sin\gamma*\cos\theta)^2 - \sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2$$
$$+\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2 + \sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$
$$- \sin\theta * \cos\theta * \sin\phi * \cos\phi * \sin\gamma * \cos\gamma\ ]$$

$\because$ *rearranging terms*

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\gamma\cos\theta(\ \sin\theta\cos\phi\sin\phi\cos\gamma$$
$$-\cos\theta\cos\gamma\sin^2\phi - \sin\theta\sin\gamma\cos^2\phi$$
$$+\cos\theta\sin\theta\cos\phi\sin\gamma\ ) - \cos\gamma\sin\theta$$
$$(\ \sin\theta\cos\phi\sin\phi\cos\gamma - \cos\theta\cos\gamma\sin^2\phi$$
$$- \sin\theta\sin\gamma\cos^2\phi + \cos\theta\sin\theta\cos\phi\sin\gamma\ )\ ]$$

$\because$ *taking common* $\sin\gamma\cos\theta$ *and* $\sin\theta\cos\gamma$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta - \sin\theta\cos\gamma) * (\ \sin\phi\cos\gamma$$
$$(\sin\theta\cos\phi - \cos\theta\sin\phi)$$
$$-\cos\phi\sin\gamma\,(\sin\theta\cos\phi - \cos\theta\sin\phi)\ \ )\ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta - \sin\theta\cos\gamma)*(\sin\phi\cos\gamma - \sin\phi\cos\theta)$$
$$*\,(\sin\theta\cos\phi - \sin\phi\cos\theta)\ \ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin(\gamma-\theta))*(\sin(\phi-\gamma))*(\sin(\theta-\phi)\ \ )\ ] = R.H.S$$