Assignment - 1

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Problem

1.Show that the area of the triangle whose vertices are:

$(a\tan\theta, b\cot\theta), (a\tan\phi, b\cot\phi), (a\tan\gamma, b\cot\gamma)$

$$is \ \ \frac{4ab\ \sin(\theta-\phi)\sin(\phi-\gamma)\sin(\gamma-\phi)}{\sin 2\theta\sin 2\phi\sin 2\gamma}$$

Solution

$$x_1 = a\tan\theta,\ x_2 = a\tan\phi,\ x_3 = a\tan\gamma,$$

$$y_1 = b\cot\theta,\ y_2 = b\cot\phi,\ y_3 = b\cot\gamma$$

area of triangle= $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} a\tan\theta & b\cot\theta & 1\\ a\tan\phi & b\cot\phi & 1\\ a\tan\gamma & b\cot\gamma & 1\end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a\tan\theta & b\cot\theta & 1\\ a\tan\phi - a\tan\theta & b\cot\phi - b\cot\theta & 0\\ a\tan\gamma - a\tan\theta & b\cot\gamma - b\cot\theta & 0\end{vmatrix}$$

$$\because R2 = R2 - R1\ and\ R3 = R3 - R1$$

$$\therefore \frac{1}{2}(a\tan\phi - a\tan\theta * b\cot\gamma - b\cot\theta)\ - (a\tan\gamma - a\tan\theta * b\cot\phi - b\cot\theta)$$

$$\because solving\ determinant$$

$$\therefore \frac{1}{2}[a\tan\theta(b\cot\phi - b\cot\gamma) + a\tan\phi(b\cot\gamma - b\cot\theta) + a\tan\gamma(b\cot\theta - b\cot\phi)]$$

$$\because solving\ above\ equation$$

$$\therefore \frac{ab}{2}[(\tan\theta * \cot\phi - \tan\theta * \cot\gamma) + (\tan\phi * \cot\gamma - \tan\phi * \cot\theta) + (\tan\gamma * \cot\theta - \tan\gamma * \cot\phi)]$$

$\because$ taking "ab" as common

$$\therefore \frac{ab}{2}[\ (\ \frac{\sin\theta * \cos\phi}{\cos\theta * \sin\phi} - \frac{\sin\theta * \cos\gamma}{\cos\theta * \sin\gamma}\ ) + (\ \frac{\sin\phi * \cos\gamma}{\cos\phi * \sin\gamma} - \frac{\sin\phi * \cos\theta}{\cos\phi * \sin\theta}\ ) + (\ \frac{\sin\gamma * \cos\theta}{\cos\gamma * \sin\theta} - \frac{\sin\gamma * \cos\phi}{\cos\gamma * \sin\phi}\ )\ ]$$

$\because$ representing tan and cot in sin and cos form

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$$\therefore \frac{ab}{2}[(\frac{\sin\theta * \cos\phi}{\cos\theta * \sin\phi} * \frac{4\sin\theta * \cos\phi * \sin 2\gamma}{4\sin\theta * \cos\phi * \sin 2\gamma} - \frac{\sin\theta * \cos\gamma}{\cos\theta * \sin\gamma} * \frac{4\sin\theta * \cos\gamma * \sin 2\phi}{4\sin\theta * \cos\gamma * \sin 2\phi}) + (\frac{\sin\phi * \cos\gamma}{\cos\phi * \sin\gamma} * \frac{4\sin\phi * \cos\gamma * \sin 2\theta}{4\sin\phi * \cos\gamma * \sin 2\theta} - \frac{\sin\phi * \cos\theta}{\cos\phi * \sin\theta} * \frac{4\sin\phi * \cos\theta * \sin 2\gamma}{4\sin\phi * \cos\theta * \sin 2\gamma}) + (\frac{\sin\gamma * \cos\theta}{\cos\gamma * \sin\theta} * \frac{4\sin\gamma * \cos\theta * \sin 2\phi}{4\sin\gamma * \cos\theta * \sin 2\phi} - \frac{\sin\gamma * \cos\phi}{\cos\gamma * \sin\phi} * \frac{4\sin\gamma * \cos\phi * \sin 2\theta}{4\sin\gamma * \cos\phi * \sin 2\theta})]$$

$\because$ multiplying and dividing each term with 4sin $X\cos X\sin 2X$

$$\therefore \frac{4ab}{2}[(\frac{\sin 2\gamma * (\sin\theta * \cos\phi)^2 - \sin 2\phi * (\sin\theta * \cos\gamma)^2}{\sin 2 * \sin\theta * \sin\gamma}) + (\frac{\sin 2\theta * (\sin\phi * \cos\gamma)^2 - \sin 2\gamma * (\sin\phi * \cos\theta)^2}{\sin 2\theta * \sin\phi * \sin\gamma}) + (\frac{\sin 2\phi * (\sin\gamma * \cos\theta)^2 - \sin 2\theta * (\sin\gamma * \cos\phi)^2}{\sin 2\theta * \sin\phi * \sin\gamma})]$$

$\because$ simplifying above equation

$$\therefore \frac{4ab}{2}[\ (\frac{2\sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2-2\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2}{\sin 2\theta*\sin\phi*\sin\gamma}) + (\frac{2\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2-2\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2}{\sin 2\theta*\sin\phi*\sin\gamma}) + (\frac{2\sin\phi\cos\phi*(\sin\theta*\cos\phi)^2-2\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2}{\sin 2\theta*\sin\phi*\sin\gamma})]$$

$$\because \sin 2\theta = 2\sin\theta\cos\theta$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2 - \sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2\ )$$
$$+(\ \sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2 - \sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2\ )$$
$$+(\ \sin\phi\cos\phi*(\sin\gamma*\cos\theta)^2 - \sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2\ )$$
$$+\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$$
$$-\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma]$$

$$\because\ \textit{adding and subtracting} \sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$$
$$-\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2-\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2$$
$$+\sin\phi\cos\phi*(\sin\gamma*\cos\theta)^2-\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2$$
$$+\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2+\sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$
$$-\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma\ ]$$

$$\because\ \textit{rearranging terms}$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\gamma\cos\theta(\ \sin\theta\cos\phi\sin\phi\cos\gamma-\cos\theta\cos\gamma\sin^2\phi- \sin\theta\sin\gamma\cos^2\phi+\cos\theta\sin\theta\cos\phi\sin\gamma\ )$$
$$-\cos\gamma\sin\theta(\ \sin\theta\cos\phi\sin\phi\cos\gamma-\cos\theta\cos\gamma\sin^2\phi -\sin\theta\sin\gamma\cos^2\phi+\cos\theta\sin\theta\cos\phi\sin\gamma\ )\ ]$$

$$\because \textit{taking common}\ \sin\gamma\cos\theta\ \textit{and}\ \sin\theta\cos\gamma$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta-\sin\theta\cos\gamma)*(\ \sin\phi\cos\gamma (\sin\theta\cos\phi-\cos\theta\sin\phi) -\cos\phi\sin\gamma(\sin\theta\cos\phi-\cos\theta\sin\phi)\ )\ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta-\sin\theta\cos\gamma)*(\sin\phi\cos\gamma-\sin\phi\cos\theta) *(\sin\theta\cos\phi-\sin\phi\cos\theta)\ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin(\gamma-\theta))*(\sin(\phi-\gamma))*(\sin(\theta-\phi)\ )\ ] = R.H.S$$