Assignment - 1

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PROBLEM

1.Show that the area of the triangle whose vertices are:

$$(a\tan\theta, b\cot\theta), (a\tan\phi, b\cot\phi), (a\tan\gamma, b\cot\gamma)$$

$$is \ \frac{4ab\ \sin(\theta-\phi)\sin(\phi-\gamma)\sin(\gamma-\phi)}{\sin 2\theta\sin 2\phi\sin 2\gamma}$$

SOLUTION

$$x_1 = a\tan\theta,\ x_2 = a\tan\phi,\ x_3 = a\tan\gamma,$$

$$y_1 = b\cot\theta,\ y_2 = b\cot\phi,\ y_3 = b\cot\gamma$$

area of triangle=

$$\frac{1}{2}[x1(y2-y3)+x2(y3-y1)+x3(y1-y2)]$$

area of triangle=

$$\frac{1}{2}[a\tan\theta(b\cot\phi - b\cot\gamma) + a\tan\phi(b\cot\gamma - b\cot\theta)$$

$$+a\tan\gamma(b\cot\theta - b\cot\phi)]$$

$$LHS = \frac{ab}{2}[(\tan\theta*\cot\phi - \tan\theta*\cot\gamma)+$$

$$(\tan\phi*\cot\gamma - \tan\phi*\cot\theta)+$$

$$(\tan\gamma*\cot\theta - \tan\gamma*\cot\phi)]$$

$\because$ taking "ab" as common

$$\therefore \frac{ab}{2}[\ (\ \frac{\sin\theta*\cos\phi}{\cos\theta*\sin\phi} - \frac{\sin\theta*\cos\gamma}{\cos\theta*\sin\gamma}\ )+$$

$$(\ \frac{\sin\phi*\cos\gamma}{\cos\phi*\sin\gamma} - \frac{\sin\phi*\cos\theta}{\cos\phi*\sin\theta}\ )+$$

$$(\ \frac{\sin\gamma*\cos\theta}{\cos\gamma*\sin\theta} - \frac{\sin\gamma*\cos\phi}{\cos\gamma*\sin\phi}\ )\ ]$$

$\because$ representing tan and cot in sin and cos form

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$$\therefore \frac{ab}{2}[(\frac{\sin\theta*\cos\phi}{\cos\theta*\sin\phi}*\frac{4\sin\theta*\cos\phi*\sin 2\gamma}{4\sin\theta*\cos\phi*\sin 2\gamma} -$$

$$\frac{\sin\theta*\cos\gamma}{\cos\theta*\sin\gamma}*\frac{4\sin\theta*\cos\gamma*\sin 2\phi}{4\sin\theta*\cos\gamma*\sin 2\phi})+$$

$$(\frac{\sin\phi*\cos\gamma}{\cos\phi*\sin\gamma}*\frac{4\sin\phi*\cos\gamma*\sin 2\theta}{4\sin\phi*\cos\gamma*\sin 2\theta} -$$

$$\frac{\sin\phi*\cos\theta}{\cos\phi*\sin\theta}*\frac{4\sin\phi*\cos\theta*\sin 2\gamma}{4\sin\phi*\cos\theta*\sin 2\gamma})+$$

$$(\frac{\sin\gamma*\cos\theta}{\cos\gamma*\sin\theta}*\frac{4\sin\gamma*\cos\theta*\sin 2\phi}{4\sin\gamma*\cos\theta*\sin 2\phi} -$$

$$\frac{\sin\gamma*\cos\phi}{\cos\gamma*\sin\phi}*\frac{4\sin\gamma*\cos\phi*\sin 2\theta}{4\sin\gamma*\cos\phi*\sin 2\theta})]$$

$\because$ multiplying and dividing each term with 4sin $X\cos X\sin 2X$

$$\therefore \frac{4ab}{2}[(\frac{\sin 2\gamma*(\sin\theta*\cos\phi)^2 - \sin 2\phi*(\sin\theta*\cos\gamma)^2}{\sin 2*\sin\theta*\sin\gamma})$$

$$+(\frac{\sin 2\theta*(\sin\phi*\cos\gamma)^2 - \sin 2\gamma*(\sin\phi*\cos\theta)^2}{\sin 2\theta*\sin\phi*\sin\gamma})$$

$$+(\frac{\sin 2\phi*(\sin\gamma*\cos\theta)^2 - \sin 2\theta*(\sin\gamma*\cos\phi)^2}{\sin 2\theta*\sin\phi*\sin\gamma})]$$

$\because$ simplifying above equation

$$\therefore \tfrac{4ab}{2}[\ (2\sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$

$$-2\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2\frac{}{\sin 2\theta*\sin\phi*\sin\gamma)}\ +$$

$$(\tfrac{2\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2 - 2\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2}{\sin 2\theta*\sin\phi*\sin\gamma})\ +$$

$$(\tfrac{2\sin\phi\cos\phi*(\sin\theta*\cos\phi)^2 - 2\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2}{\sin 2\theta*\sin\phi*\sin\gamma})\ ]$$

$\because \sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$
$$-\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2\ )$$
$$+(\ \sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2$$
$$-\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2\ )$$
$$+(\ \sin\phi\cos\phi*(\sin\gamma*\cos\theta)^2$$
$$-\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2\ )$$
$$+\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$$
$$-\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma]$$

$\because$ *adding and subtracting* $\sin\theta*\cos\theta$ $\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma$$
$$-\sin\gamma\cos\gamma*(\sin\phi*\cos\theta)^2-\sin\theta\cos\theta*(\sin\gamma*\cos\phi)^2$$
$$+\sin\phi\cos\phi*(\sin\gamma*\cos\theta)^2-\sin\phi\cos\phi*(\sin\theta*\cos\gamma)^2$$
$$+\sin\theta\cos\theta*(\sin\phi*\cos\gamma)^2+\sin\gamma\cos\gamma*(\sin\theta*\cos\phi)^2$$
$$-\sin\theta*\cos\theta*\sin\phi*\cos\phi*\sin\gamma*\cos\gamma\ ]$$

$\because$ *rearranging terms*

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$\sin\gamma\cos\theta(\ \sin\theta\cos\phi\sin\phi\cos\gamma-\cos\theta\cos\gamma\sin^2\phi-$$
$$\sin\theta\sin\gamma\cos^2\phi+\cos\theta\sin\theta\cos\phi\sin\gamma\ )$$
$$-\cos\gamma\sin\theta(\ \sin\theta\cos\phi\sin\phi\cos\gamma-\cos\theta\cos\gamma\sin^2\phi$$
$$-\sin\theta\sin\gamma\cos^2\phi+\cos\theta\sin\theta\cos\phi\sin\gamma\ )\ ]$$

$\because$ *taking common* $\sin\gamma\cos\theta$ *and* $\sin\theta\cos\gamma$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta-\sin\theta\cos\gamma)*(\ \sin\phi\cos\gamma\,(\sin\theta\cos\phi-\cos\theta\sin\phi)$$
$$-\cos\phi\sin\gamma\,(\sin\theta\cos\phi-\cos\theta\sin\phi)\ )\ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\sin\gamma\cos\theta-\sin\theta\cos\gamma)*(\sin\phi\cos\gamma-\sin\phi\cos\theta)$$
$$*\,(\sin\theta\cos\phi-\sin\phi\cos\theta)\ ]$$

$$\therefore \frac{4ab*2}{2*\sin 2\theta*\sin\phi*\sin\gamma}[$$
$$(\ \sin(\gamma-\theta))*(\sin(\phi-\gamma))*(\sin(\theta-\phi)\ )\ ]=R.H.S$$