Assignment - 1

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PROBLEM

1.Show that the area of the triangle whose vertices are:

$(a\tan\theta, b\cot\theta), (a\tan\phi, b\cot\phi), (a\tan\gamma, b\cot\gamma)$

$is \quad \dfrac{4ab\,\sin(\theta-\phi)\sin(\phi-\gamma)\sin(\gamma-\theta)}{\sin 2\theta\,\sin 2\phi\,\sin 2\gamma}$

SOLUTION

$\mathbf{x_1} = \left|a\tan\theta\right|, \mathbf{x_2} = \left(a\tan\phi\right)\,\mathbf{x_3} = \left(a\tan\gamma\right)$
$\mathbf{y_1} = \left(b\cot\theta\right), \mathbf{y_2} = \left(b\cot\phi\right), \mathbf{y_3} = \left(b\cot\gamma\right)$

$$area \quad of \quad triangle \quad = \quad \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad (1)$$

$$= \quad \frac{1}{2}\begin{vmatrix} a\tan\theta & b\cot\theta & 1 \\ a\tan\phi & b\cot\phi & 1 \\ a\tan\gamma & b\cot\gamma & 1 \end{vmatrix} \quad (2)$$

$$= \quad \frac{ab}{2}\begin{vmatrix} \tan\theta & \cot\theta & 1 \\ \tan\phi & \cot\phi & 1 \\ \tan\gamma & \cot\gamma & 1 \end{vmatrix} \quad (3)$$

$\because taking\ a\ for\ C1\ and\ b\ from\ C2\ common$

$$= \quad \frac{ab}{2}\begin{vmatrix} \tan\theta & \cot\theta & 1 \\ \tan\phi-\tan\theta & \cot\phi-\cot\theta & 0 \\ \tan\gamma-\tan\theta & \cot\gamma-\cot\theta & 0 \end{vmatrix} \quad (4)$$

$\because R2 = R2 - R1\ and\ R3 = R3 - R1$

$$= \quad \frac{ab}{2}\begin{vmatrix} \frac{\sin\theta}{\cos\theta} & \frac{\cos\theta}{\sin\theta} & 1 \\ \frac{\sin\phi}{\cos\phi}-\frac{\sin\theta}{\cos\theta} & \frac{\cos\phi}{\sin\phi}-\frac{\cos\theta}{\sin\theta} & 0 \\ \frac{\sin\gamma}{\cos\gamma}-\frac{\sin\theta}{\cos\theta} & \frac{\cos\gamma}{\sin\gamma}-\frac{\cos\theta}{\sin\theta} & 0 \end{vmatrix} \quad (5)$$

$\because \tan x = \frac{\sin x}{\cos x}\ and\ \cot x = \frac{\cos x}{\sin x}$

$$= \quad \frac{ab}{2}\begin{vmatrix} \frac{\sin\theta}{\cos\theta} & \frac{\cos\theta}{\sin\theta} & 1 \\ \frac{\sin\phi\cos\theta-\sin\theta\cos\phi}{\cos\phi\cos\theta} & \frac{\sin\theta\cos\phi-\sin\phi\cos\theta}{\sin\phi\sin\theta} & 0 \\ \frac{\sin\gamma\cos\theta-\sin\theta\cos\gamma}{\cos\gamma\cos\theta} & \frac{\sin\theta\cos\gamma-\sin\gamma\cos\theta}{\sin\gamma\sin\theta} & 0 \end{vmatrix} \quad (6)$$

$\because\ rearranging\ terms$

$$= \quad \frac{ab}{2}\begin{vmatrix} \frac{\sin\theta}{\cos\theta} & \frac{\cos\theta}{\sin\theta} & 1 \\ \frac{\sin(\phi-\theta)}{\cos\phi\cos\theta} & \frac{\sin(\theta-\phi)}{\sin\phi\sin\theta} & 0 \\ \frac{\sin(\gamma-\theta)}{\cos\gamma\cos\theta} & \frac{\sin(\theta-\gamma)}{\sin\gamma\sin\theta} & 0 \end{vmatrix} \quad (7)$$

$\because \sin A\cos B - \sin B\cos A = \sin(A-B)$

$$= \quad \frac{ab}{2\sin\theta\cos\theta}\begin{vmatrix} \sin\theta & \cos\theta & 1 \\ -\frac{\sin(\theta-\phi)}{\cos\phi} & \frac{\sin(\theta-\phi)}{\sin\phi} & 0 \\ \frac{\sin(\gamma-\theta)}{\cos\gamma} & -\frac{\sin(\gamma-\theta)}{\sin\gamma} & 0 \end{vmatrix} \quad (8)$$

$\because taking\ \frac{1}{\cos\theta}, \frac{1}{\sin\theta}\ common\ from\ column\ 1\ and\ 2$

$$= \quad \frac{ab\,(-\sin(\theta-\phi))\,(-\sin(\gamma-\theta))}{2\sin\theta\cos\theta}\begin{vmatrix} \sin\theta & \cos\theta & 1 \\ \frac{1}{\cos\phi} & \frac{1}{\sin\phi} & 0 \\ \frac{1}{\cos\gamma} & \frac{1}{\sin\gamma} & 0 \end{vmatrix} \quad (9)$$

$\because\ taking \sin(\theta-\phi)\ common\ from\ row\ 2$
$and\ \sin(\gamma-\theta)\ common\ from\ row\ 3$

$$= \quad \frac{ab\,\sin(\theta-\phi)\,\sin(\gamma-\theta)}{\sin 2\theta}\left[\frac{1}{\cos\phi\sin\gamma}-\frac{1}{\sin\phi\cos\gamma}\right] \quad (10)$$

$\because solving\ determinate$

$$= \quad \frac{ab\,\sin(\theta-\phi)\,\sin(\gamma-\theta)}{\sin 2\theta}\left[\frac{\sin\phi\cos\gamma-\cos\phi\sin\gamma}{\sin\phi\cos\phi\sin\gamma\cos\gamma}\right] \quad (11)$$

∵ solving above equation

$$= \frac{ab \; \sin(\theta - \phi) \; \sin(\gamma - \theta)}{\sin 2\theta} \left[\frac{4 \; \sin(\phi - \gamma)}{4 \; \sin \phi \cos \phi \sin \gamma \cos \gamma} \right]$$
(12)

∵ $\sin A \cos B \; - \; \sin B \cos A \; = \; \sin(A \; - \; B)$
multiply and divide by 4

$$= \frac{4ab \; \sin(\theta - \phi) \; \sin(\gamma - \theta) \; \sin(\phi - \gamma)}{\sin 2\theta \; 2 \sin \phi \cos \phi \; 2 \sin \gamma \cos \gamma}$$
(13)

$$= \frac{4ab \; \sin(\theta - \phi) \; \sin(\gamma - \theta) \; \sin(\phi - \gamma)}{\sin 2\theta \; \sin 2\phi \; \sin 2\gamma} = R.H.S$$

Putting Some Numerical Values

$$\frac{4 \; (1 * 1) \; \sin(60 - 45) \; \sin(30 - 60) \; \sin(45 - 30)}{\sin 2(60) \; \sin 2(45) \; \sin 2(30)}$$

putting value of $a = 1, \; b = 1 \; \theta = 60 \; \phi = 45$
$\gamma = 30$

$$= \frac{-0.133974}{0.75} = -0.178632$$