Assignment - 1

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SM21MTECH14002

Problem

1.Show that the area of triangle formed by lines

$$x\cos\alpha + y\sin\alpha = p, x\cos\beta + y\sin\beta = q,$$

$$x\cos\gamma + y\sin\gamma = r$$

$$is \ \frac{[p\sin(\beta-\gamma)+q\sin(\gamma-\alpha)+rsin(\alpha-\beta)]^2}{2\sin(\beta-\gamma)\sin(\gamma-\alpha)\sin(\alpha-\beta)}$$

Solution

$$Area\ formed\ by\ lines: x\cos\alpha + y\sin\alpha - p = 0,$$
$$x\cos\beta + y\sin\beta - q = 0\ x\cos\gamma + y\sin\gamma - r = 0 is$$

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$
$$a_3x + b_3y + c_3 = 0$$

$$= \frac{det\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}^2}{2\ C1\ C2\ C3}$$
$$where\ C1, C2, C3\ are\ cofactor\ of\ c1, c2, c3$$
$$respectively\ in\ the\ above\ matrix. \quad (1)$$

$$= \frac{det\begin{bmatrix} \cos\alpha & \sin\alpha & -p \\ \cos\beta & \sin\beta & -q \\ \cos\gamma & \sin\gamma & -r \end{bmatrix}^2}{2\ C1\ C2\ C3} \quad (2)$$

let's find Determinate first

$$Determinate = -\begin{vmatrix} \cos\alpha & \sin\alpha & p \\ \cos\beta & \sin\beta & q \\ \cos\gamma & \sin\gamma & r \end{vmatrix}$$
$$\because\ taking\ common\ C3 \xleftrightarrow{-} C3 \quad (3)$$

$$= -[\ \cos\alpha(r\sin\beta - q\sin\gamma) -$$
$$\sin\alpha(r\cos\beta - q\cos\gamma) + p(\cos\beta\sin\gamma - \sin\beta\cos\gamma)\ ]$$
$$\because\ expanding\ determinant \quad (4)$$

$$= -[\ (r\cos\alpha\sin\beta - q\sin\gamma\cos\alpha) -$$
$$(r\sin\alpha\cos\beta - q\cos\alpha) + p(\cos\beta\sin\gamma - \sin\beta\cos\gamma)\ ] \quad (5)$$

$$= -[\ r(\cos\alpha\sin\beta - \sin\alpha\cos\beta) -$$
$$q(\sin\gamma\cos\alpha + \cos\alpha) + p(\cos\beta\sin\gamma - \sin\beta\cos\gamma)\ ]$$
$$\because\ rearranging\ terms \quad (6)$$

$$= -[\ -r\sin(\alpha-\beta) - q\sin(\gamma-\alpha) - p\sin(\beta-\gamma)\ ]$$
$$\because \sin A\cos B - \sin B\cos A = \sin(A-B) \quad (7)$$

$$Determinate = [\ r\sin(\alpha-\beta) + q\sin(\gamma-\alpha) +$$
$$p\sin(\beta-\gamma)\ ] \quad (8)$$

Now we will find C1,C2,C3

$$C1 = \begin{vmatrix} \cos\beta & \sin\beta \\ \cos\gamma & \sin\gamma \end{vmatrix} = \cos\beta\sin\gamma - \cos\gamma\sin\beta$$
$$= -\sin(\beta-\gamma) \quad (9)$$

$$C2 = \begin{vmatrix} \cos\alpha & \sin\alpha \\ \cos\gamma & \sin\gamma \end{vmatrix} = \cos\alpha\sin\gamma - \cos\gamma\sin\alpha$$
$$= \sin(\gamma-\alpha) \quad (10)$$

$$C3 = \begin{vmatrix} \cos\alpha & \sin\alpha \\ \cos\beta & \sin\beta \end{vmatrix} = \cos\alpha\sin\beta - \cos\beta\sin\alpha$$

$$= -\sin(\alpha-\beta) \quad (11)$$

Putting equation number 8,9,10,11 in equation 2

$$= \frac{[r\sin(\alpha-\beta)+q\sin(\gamma-\alpha)+p\sin(\beta-\gamma)]^2}{2\ (-\sin(\beta-\gamma))\ (\sin(\gamma-\alpha))\ (-\sin(\alpha-\beta))} \quad (12)$$

$$\frac{[p\sin(\beta-\gamma)+q\sin(\gamma-\alpha)+rsin(\alpha-\beta)]}{2\sin(\beta-\gamma)\sin(\gamma-\alpha)\sin(\alpha-\beta)}$$

$$= R.H.S \quad (13)$$

putting value of p=-1, q=-1 ,r=-1 $\alpha = 60$ $\beta = 45$ $\gamma = 30$

$$= \frac{(-0.17638)^2}{-0.06698} = -0.004644198 \quad (14)$$