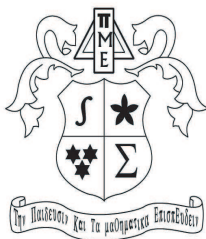




## MAA Student Chapters

The MAA Student Chapters program was launched in January 1989 to encourage students to continue study in the mathematical sciences, provide opportunities to meet with other students interested in mathematics at national meetings, and provide career information in the mathematical sciences. The primary criterion for membership in an MAA Student Chapter is “interest in the mathematical sciences.” Currently there are approximately 550 Student Chapters on college and university campuses nationwide.



## Pi Mu Epsilon

Pi Mu Epsilon is a national mathematics honor society with 397 chapters throughout the nation. Established in 1914, Pi Mu Epsilon is a non-secret organization whose purpose is the promotion of scholarly activity in mathematics among students in academic institutions. It seeks to do this by electing members on an honorary basis according to their proficiency in mathematics and by engaging in activities designed to provide for the mathematical and scholarly development of its members. Pi Mu Epsilon regularly engages students in scholarly activity through its *Journal* which has published student and faculty articles since 1949. Pi Mu Epsilon encourages scholarly activity in its chapters with grants to support mathematics contests and regional meetings established by the chapters and through its Lectureship program that funds Councillors to visit chapters. Since 1952, Pi Mu Epsilon has been holding its annual National Meeting with sessions for student papers in conjunction with the summer meetings of the Mathematical Association of America (MAA).

## **J. Sutherland Frame Lecture**

### **THE SINGULAR UNIFORMITY OF LARGE RANDOM SYSTEMS**

**Peter Winkler**

Dartmouth College

A random structure could be anything, yet somehow, when that structure is composed of many small parts, it often turns out to be shockingly predictable at least, in a probabilistic sense. A random graph on a million vertices, for example, has a long list of characteristics each with high probability.

In an attempt to understand this phenomenon, we'll take a little tour from zero-one laws to variational principles, contrasting graphs and permutations along the way.

*The J. Sutherland Frame Lecture is named in honor of the ninth President of Pi Mu Epsilon, who served from 1957 to 1966 and passed away on February 27, 1997. In 1952, Sud Frame initiated the student paper sessions at the annual Pi Mu Epsilon meeting, which is held at the Summer MathFests. He continually offered insight and inspiration to student mathematicians at these summer meetings.*

## **Jean B. Chan and Peter Stanek Lecture for Students**

### **FAIL: A MATHEMATICIAN'S APOLOGY**

**Laura Taalman**

James Madison University

The job of being a mathematician primarily consists of long periods of failure punctuated by short bursts of success which later seem to be somewhat obvious...but that's what we love about it! And, as it turns out, 3D printing kind of works the same way. In this talk we'll take a journey through many mathematical and 3D printing failures and try to laugh about it the best we can.

## **MAA Undergraduate Student Activity Session**

### **THE CASE OF THE MISSING VERTEX**

**Suzanne Dore**

Augsburg University

A vertex has gone missing in an un-labeled graph and taken all of its edges with it. Can we reconstruct the original graph, or at least some of its properties? What if we have the vertex-deleted graph for each of the vertices? Come join this mathematical investigation of the Graph Reconstruction Problem. This fanciful activity provides an introduction to Graph Theory and leads to an open question in the research. Bring a friend and writing utensil.

Plaza Courts 1

9:00A.M. – 10:55A.M.

9:00–9:15

**Hyperbolic Virtual Knots**

Or Eisenberg and Yi Wang

*Harvard and Columbia*

In the early 1980s, Thurston proved that many knots are hyperbolic, and therefore their hyperbolic volume could be used to distinguish between knots. We extend this notion to virtual knots.

9:20–9:35

**Links in Thickened Surfaces**

Natalia Pacheco-Tallaj and Zhen Liang

*Harvard University and University of Southern California*

Typically, mathematicians think about knots and links living in 3-dimensional space. But we will consider knots and links living in thickened surfaces. Can you determine properties of the knot or link? For instance, does the complement of the knot or link have negative curvature?

9:40–9:55

**Tilings and Multi-crossings**

Jonah Greenberg and Kabir Kapoor

*Williams College and Cornell University*

A multi-crossing is a crossing with more than two strands passing through it. We discuss how multi-crossings can be used to turn tilings of the Euclidean and hyperbolic planes into links in thickened surfaces, and how this allows a hyperbolic structure to be associated to a tiling.

10:00–10:15

**Modeling Anyonic Systems: Group Theoreticity of Integral Metaplectic Modular Categories**

Sydney Timmerman and Benjamin Warren

*Johns Hopkins University and Swarthmore College*

Topological quantum computation harnesses the swapping of non-Abelian anyons to perform computation that is topologically protected against decoherence. This swapping gives a representation of the braid group, where the strands are the anyon world lines. If the image of the braid group representation is infinite, quantum gates can usually be approximated to arbitrary accuracy by braiding anyons. Fusion categories provide one model for anyonic systems; in this model, each simple object corresponds to one anyon which can be fused with others to form a superposition. From each fusion category a braid group representation can be constructed. We specifically studied integral metaplectic modular categories modular fusion categories with integer dimension that have the same fusion rules as the quantum group category  $SO(N)_2$ . We were especially interested in proving that integral modular categories are group theoretical, which implies that the associated braid group representation has finite image and is necessarily non-universal. This would mean anyonic systems with these fusion rules cannot be used to create a universal topological quantum computer using only braiding.

10:20–10:35

**Gordian Adjacency for Positive Braid Knots**

Samuel Serra, Luke Seaton, David Luo, and Tolson Bell

*University of Colorado Boulder, Louisiana Tech University, Emory University,  
and Georgia Institute of Technology*

In 2014, Feller introduced the notion of Gordian adjacency. A knot  $K_1$  is said to be *Gordian adjacent* to a knot  $K_2$  if  $K_1$  is an intermediate knot on an unknotting sequence of  $K_2$ . In this paper, we study Gordian

adjacency between positive braids, in particular torus knots, because positive braids have computable unknotting numbers. By representing positive braid knots using braid words, we explore algorithmic methods to determine Gordian adjacency for positive braids.

10:40–10:55

**Equivalence of discrete Morse functions using persistent homology**

Yuqing Liu

*Ursinus College*

The past 15 years have witnessed an explosion of methods and applications in computational topology, including image processing, data analysis, and sensor networks. One method known as persistent homology relies on simple linear algebra techniques and takes as input a data set and outputs a bar code—a visual representation of how the topology of the data changes over time in an attempt to determine what features are important in the data and what features are simply “noise.” Closely related to persistence is discrete Morse theory, which has been used as a tool to improve certain homology computations. Several connections between persistence and discrete Morse theory have been made. In this talk, we will continue making connections between discrete Morse theory and persistent homology by introducing a new notion of equivalence of discrete Morse functions based on the persistence diagram induced by the sublevel complexes from a discrete Morse functions. We share several properties of this equivalence and compute some examples.

## Plaza Courts 2

9:00A.M. – 10:55A.M.

9:00–9:15

**Constructing a New Phylogenetic Treespace**

Rachel Klementowski and Molly Noel

*SUNY Geneseo and Ithaca College*

Phylogenetic tree topologies are an integral part of representing and understanding evolutionary relationships between taxa; however, a common problem faced when working with these topologies is the super-exponential increase in the number of possible topologies as the number of taxa,  $N$ , increases. We propose as a solution a function which maps these possible topologies into the more manageable Euclidean space,  $R^N$ , by computing the distance metric from each topology to  $N$  carefully selected benchmark trees. Ideally, the function will be one-to-one, so that given any two different topologies, each will map to a different point in  $R^N$ . Additionally, trees of similar topologies should map to relatively close points in  $R^N$  when compared with the mapping of more dissimilar topologies. We answer whether these requirements for our function hold for Robinson-Foulds distances and other metrics. Given that these requirements hold, we propose a method for moving from the  $R^N$  space back to the possible tree topologies, which will be useful for computing distance averages and recreating trees based on those averages.

9:20–9:35

**Computing Species Tree Branch Lengths**

Deirdre Coulter

*Providence College*

Phylogenetic trees represent the evolutionary relationship between taxa in Biology. The topology of the trees shows the order of evolution of the taxa, while the branch lengths represent the length of time between these evolutionary events at the vertices. Formulas for gene tree branches are well developed. We propose a method of generating branch lengths on species trees from a collection of DNA sequences. In our model, the species tree distance is given as the sum of the Jukes-Cantor gene tree distance and a new constant term, which depends on the mutation rate and effective population size. DNA sequence data from multiple individuals in the same species is needed to estimate this constant term. It is important to note that adding this constant to the gene tree distances does not change the topology of the tree, so the calculation of branch lengths can be computed without taking into account topological changes. We explore further use of this formula as a method for species delimitation.

9:40–9:55

**The Evolution of Cooperation in 2-Dimensional Mobile Populations**

Kyle Weishaar

*Regis University*

We investigate the effects of two-dimensional spatial structure and mobility on the evolution of cooperation, extending the existing one-dimensional model. We consider a finite, fixed size population of mobile cooperators and free-riders. A cooperator pays a cost to provide benefits to all individuals in its vicinity, whereas a free-rider offers no benefits and therefore pays no costs. Individuals are allowed to move around a 2-dimensional lattice to maximize their payoff. Our model allows simultaneous interactions among multiple individuals as all individuals in a neighborhood compete for the same limited resource. We create an exact stochastic simulation of this Markov process to investigate how various parameters of the model affect the evolution of cooperation.

10:00–10:15

**An EM algorithm to identify hidden ancestries in publicly available genetic summary data**

Kendra Koach, Tiffany Dinh, and Yinfei Wu

*University of Colorado Denver*

Most genetic association studies are performed with genetic information for each individual. Moderate effect sizes necessitate extremely large sample sizes. Extremely large public data exist (i.e., 10–100K) and are used in case-control association tests, to look up the frequency of potentially causal variants in rare disease studies, and for secondary data analysis. However, data is provided at the group rather than individual level leading to lost information. Specifically, ancestry information is broad (e.g., European, African, Asian) and does not include fine scale ancestry or admixture (e.g., African Americans with African and European ancestry). This results in confounding in association tests and incorrect assignment of the causal variant in rare diseases. In order to detect the proportions of finer scale ancestry in group data, we developed an EM algorithm that uses group genetic frequencies directly (rather than traditional methods that use data from individuals). In a wide variety of simulations, we show that our algorithm accurately and precisely detects ancestry proportions down to 1% of the summary data reducing inaccurate results due to hidden ancestries.

10:20–10:35

**Tracking Neural Activity: Automated Image Analysis**

BethAnna Jones and Amanda Stanley

*SUNY Geneseo and Grand Valley State University*

Modern optical methods such as high-definition two-photon imaging of calcium fluorescence allow us to view the activity of tens of thousands of neurons in the brain. However, due to the large number of images to sort through, manual methods of location, identification, and analysis of these individual neurons is tedious and time-consuming. Researchers need an automated system to complete in hours what may take an expert weeks or even years. Some automated techniques exist, but as shown by the 2017 SURIEM project, they find only roughly half the cells, and disagree on half of what they find. We aim to build on this work and improve automatic analysis techniques of neural images. Current leading methods analyze neural images using singular value decomposition and constrained non-negative matrix factorizations. We will compare the cells identified with these methods and improve them by i) setting better threshold values of criteria such as the roundness, size, and fluorescence statistics of the cells; ii) developing new statistical criteria for identifying cells; and iii) investigating correlation patterns across cells.

10:40–10:55

**Analyzing Behavior Quantitatively in Both Humans and Mice**

Katrina Gensterblum and Chanel Fraikin

*Michigan State University and University of California, San Diego*

It is well known that the brain controls behavioral choices. However, so far we have little idea about the relationship between brain activity and motion. In order to characterize how specific neural patterns relate to actions, we need a better taxonomy of the almost infinite variety of observed behaviors. First, we will work with recordings of mouse posture and motions. We will seek to reduce the dimensionality of the wide range of empirically recorded mouse postures to four to five variables, using local PCA for various combinations of length and height. We then will describe the dynamics of transitions between postures using a modified fiber bundle approach. Through this analysis we will identify regularities in the changing postures of the mice. We also work with human data, looking at interactions between pairs of people. Here we look closely at rhythmic motions to find periods of imitation and adaptation of one person towards the other. We do this by completing a wavelet analysis of the motion and by looking for significant correlation between the amplitudes of these motions. From this we expect to find some cases of behavioral modification due to the pairwise interaction.



**Plaza Courts 3****9:00A.M. – 10:55A.M.**

9:00–9:15

**Improving the Performance of Sublinear Compressive Sensing Algorithms**

Jaya Blanchard, Simon Miller, and Joshua Patana

*Bowdoin College, Oakland University, and Arizona State University*

This project analyzes compressive sensing as a way to learn functions of many variables. Compressive sensing provides a means of reconstructing signals using limited function evaluations. One such example of a function of many variables is predicting a user's network behavior based on their browser history, age, location, etc. With high-dimensional problems, standard algorithms fail because of high runtime and storage requirements. We will examine several sublinear-runtime compressive sensing algorithms designed for sparse data sets and high-dimensional problems. We hope to maximize efficiency by decreasing runtime, reducing error, and/or reducing the number of samples required. Our work is an extension of Bosu Choi, Mark Iwen, and Felix Krahmer's paper *Sparse harmonic functions: a new class of sublinear-time algorithms for approximating functions of many variables*.

9:20–9:35

**LLE Based Perceptual Image Hashing of Video**

Linda Beverly

*Cal State East Bay*

We investigated perceptual image hashing on video data by looking at video information that is visually similar in appearance. A perceptual image hash is a digital fingerprint obtained from an image or video data based on its visual appearance. This fingerprint may then be used to help identify other similar images or videos. We created a new method of perceptual image hashing that incorporates Locally Linear Embedding (LLE) and Discrete Cosine Transform (DCT) for the purpose of increasing accuracy and precision.

9:40–9:55

**Taste The Rainbow: Wavelength Selection in Hyperspectral Diffuse Optical Tomography**

Cara Lam, Thomas Mangan, and Riley Supple

*Goucher*

Diffuse optical tomography (DOT) is a medical imaging technology which utilises the near-infrared or visible light spectra to render images of soft-tissue through mapping the variations in light absorption and the scattering properties of tissues. The advantage of DOT over many imaging technologies is that it uses non-ionising radiation to generate images, making it safer for patients and more cost effective. Hyperspectral diffuse optical tomography (hyDOT) is a specific type of DOT which uses many different spectra across the range of visible light in the imaging process. There are diminishing returns to the accuracy of an image generated from more than a small number of wavelengths, leading us to hypothesize that certain combinations of spectra yield improvements in the optimization of our image. In this presentation, we investigate different methods of choosing the optimal wavelengths to form a basis for reconstructing an image in hyDOT.

10:00–10:15

**Optimal Investment Strategies and Portfolio Analysis of Leveraged Exchange-Traded Funds**

Jalen Harris, Jay Iyer, and Melanie Abel

*Fresno State University, Washington University in St. Louis, and University of Maryland*

In this research, we constructed a model for the expected return and risk of leveraged exchange traded funds (LETFs) by deriving a probability density function (PDF) in terms of the underlying ETF. Different distributions of the underlying ETF are tested in order to determine the most precise fit with historical data. Our model can show investors when it is optimal to invest in an LETF with positive or negative , and whether or not they should take a long or short position. We also created contour plots to help investors determine

their investment strategy when considering LETFs. Utilizing mean-variance optimization, we derived the efficient frontiers and calculated the sharpe ratios to understand the effect that LETFs have on a portfolio. Lastly, we completed some risk analysis with LETFs.

10:20–10:35

### **Performance of an Adaptive Portfolio**

Edward Nusinovich and Melanie Abel

*University of Maryland, College Park*

When looking at the financial markets, an investor is inclined to wonder if they can outperform conventional market strategies. In this investigation, we design seven metrics that demonstrate predictive power for market instability. After selecting a variety of assets that broadly represent the financial market sphere, we used these metrics to create a dynamic investment strategy over the 1985-2017 window. Using the metrics as indicators of good and bad market performance, we transitioned between an aggressive and conservative portfolio, based on the status of each year. In order to achieve this we used ideas from Markowitz portfolio theory, optimization and linear algebra. After examining the performance of the adaptive strategy and comparing it to three passive strategies, which are based on historical price data, the results suggest that a passive strategy that maintains fixed allocations remains the best option for most investors.

10:40–10:55

### **Sequence Pairs with Lowest Combined Autocorrelation and Crosscorrelation**

Eli Moore

*California State University, Northridge*

Signal processing deals with analysis and enhancement of signals. In mathematics, we can model these signals with binary sequences. When studying binary sequences, we are interested in their correlation properties. Specifically, we consider autocorrelation, which is a measure of how well a sequence  $f$  resembles shifted versions of itself, and crosscorrelation, which is a measure of the resemblance of two sequences  $f$  and  $g$  at various shifts. These correlation properties are significant for different reasons; for example, low autocorrelation is important for synchronization of signals, whereas low crosscorrelation reduces confusion of signals in communication networks. We want to design pairs  $(f, g)$  of sequences such that  $f$ 's autocorrelation,  $g$ 's autocorrelation, and their crosscorrelation are all low simultaneously. Pursley and Sarwate have previously proven that there is a lower bound on a combined measure of strength involving these three aspects of correlation. We have now shown that there exist infinitely many sequence pairs that meet this lower bound. In addition, we have shown Golay complementary pairs to be the only pairs that meet this bound.

Plaza Courts 4

9:00A.M. – 10:55A.M.

9:00–9:15

**Simple, Not Easy: A Study of Simple Groups**

Chris Dimmick

*Southern Oregon University*

A brief introduction to the study of finite simple groups from an undergraduate's perspective, with some introduction to their complete classification, and some applications of its results.

9:20–9:35

**Noetherian rings with unusual prime ideal structures**

Anya Michaelsen

*Williams College*

If  $R$  is a ring, then the set of prime ideals of  $R$ , called the spectrum of  $R$ , is a partially ordered set with respect to inclusion. Given a partially ordered set  $X$ , M. Hochster showed exactly when  $X$  can be realized as the spectrum of a commutative ring. It is unknown, however, when a partially ordered set can be realized as the spectrum of a **Noetherian** commutative ring. For example, if  $X$  is a 2-dimensional partially ordered set with two maximal elements,  $M$  and  $N$ , and one minimal element such that  $M$  contains countably many elements and  $N$  contains uncountably many elements, it is unknown whether or not  $X$  can be realized as the spectrum of a Noetherian commutative ring. In this talk, we will discuss progress on this problem.

9:40–9:55

**Combinatorial Neural Codes: A Study of k-Inductively Pierced Codes and Toric Ideals**

Alexandra Newlon, Matthew Hertel, Kira Laws, and Melissa Beer

*Colgate University, Michigan State University, Appalachian State University, and Franklin College*

Combinatorial neural codes are sets of 0/1 vectors that encode when a fixed set of neurons are firing. One area of interest is finding Euler diagrams that correspond to the firing patterns of place neurons represented by these codes. It is known that when a code is  $k$ -inductively pierced, there is an algorithm to construct these diagrams. Gross, Obatake, and Youngs have recently used algebraic techniques to determine when a code is  $k$ -inductively pierced. In this talk, we will present progress toward extending their work, including a study of codes for four or more neurons. The research presented was conducted at the 2018 SURIEM Program at Michigan State University.

10:00–10:15

**On the classification of seven-dimensional solvable Lie algebras**

Jacksyn Bakeberg and Kathryn Blaine

*McGill University and Bard College*

Low dimensional solvable Lie Algebras were completely classified up to dimension six. A general theorem asserts that if  $g$  is a solvable Lie Algebra of dimension  $n$ , then the dimension of its nilradical is at least  $\frac{n}{2}$ . For the seven-dimensional algebras, the nilradical's dimension could be 4, 5, 6 or 7. We give an update on this project and share our contribution. This research was conducted as part of the 2018 REU program at Grand Valley State University.

10:20–10:35

**Inverse Semigroups and Left Cancellative Categories**

Jennifer Gensler, Hannah King, and Ronen Wdowski

*California State University, Long Beach, Taylor University, and Rice University*

An inverse semigroup is a set  $S$  with an associative multiplication such that for each  $s$  in  $S$  there is a unique  $s^*$  in  $S$  such that  $s = ss^*s$  and  $s^* = s^*ss^*$ . Motivated by examples of  $C^*$ -algebras generated by

semigroups, we investigate constructions of inverse semigroups from left cancellative categories such as the inverse semigroup of the path category of a directed graph.

10:40–10:55

**Unique Product Groups and the Zero Divisor Conjecture**

William Craig

*Virginia Tech*

In this paper, we prove that any virtually solvable subgroup of a congruence subgroup is left orderable and therefore a unique product group. These results are important because being a unique product group is a sufficient condition to verify the zero divisor conjecture and trivial units conjecture for this group. We also construct infinitely many examples of finitely generated torsion free groups which are not unique product groups through a new generalization of the Hantzsche-Wendt group, one of the few known nontrivial examples of a torsion free nonunique product group. These generalized Hantzsche-Wendt groups are shown to have nonabelian free subgroups and are conjectured to have close relations to the generalizations of the Hantzsche-Wendt group which arise from topological and other group theoretical considerations.

## Plaza Courts 1

2:30P.M. – 4:25P.M.

2:30–2:45

**Solving a Simple Problem About Polynomials Using the Principles of Logic**

Steven Evans, Casey Christian, and Ian Ray

*Morehead State Univeristy and Morehead State University*

Every student of undergraduate mathematics has been asked to solve the following problem: given a polynomial  $p$  of any order (typically two or three) factorable into linear expressions, find the regions for which  $p$  satisfies a given inequality. When first given this problem, it is not uncommon for a student to be unaware of the principles governing the foundations of Mathematics, namely Logic. However, as one's mathematical maturity develops, the question of how to determine these regions from first principles is sure to arise. We describe a general framework for doing just this.

2:50–3:05

**Common Divergent Series: An Analysis of the Methods Used to Assign Values to Divergent Series**

Trevor Cross

*University of Wisconsin Washington County*

This study is an exploration of various methods of assigning values to divergent series and there meaning. More specifically, Grandis Series, the series of increasing integers but alternating signs, and the sum of all natural numbers. These series had an increasing influence on popular culture surrounding mathematics, and this study serves as a deeper, more structured view on the subject to understand what can be gained from these methods.

3:10–3:25

**Convergence of an infinite series of the form  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)\dots(n+k)}$ .**

Rabin Thapa

*Youngstown State University*

The infinite series of the form

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)\dots(n+k)} \quad \text{i.e.} \quad \sum_{n=1}^{\infty} \left[ \prod_{i=0}^k \frac{1}{(n+i)} \right]; \quad k \in \mathbb{N}$$

appears frequently in various problems related to series. This paper discusses an approach to find precisely what those series converge to by developing a general pattern to resolve them into their equivalent partial fractions followed by the telescoping forms of the series. Meanwhile, the connection of those series with the “Pascal’s Triangle” illustrates the underlying mathematical beauty and symmetry. In addition, the presentation contains applications of similar infinite series in various other scientific fields.

3:30–3:45

**Spectral and Stochastic Solutions to Boundary Value Problems on Magnetic Graphs**

Sawyer Robertson

*University of Oklahoma*

A magnetic graph is simply a graph  $G$  equipped with an orientation structure  $\sigma$  on its edges. The discrete magnetic Laplace operator  $\mathcal{L}_G^\sigma$ , a second-order difference operator for complex-valued functions on the vertices of  $G$ , has been an interesting and useful tool in discrete analysis for over twenty years. Its role in the study of quantum mechanics has been examined closely since its debut in a classic paper by Lieb and Loss. In this paper, we pose some boundary value problems associated to this operator, and adapt two classic techniques to the setting of magnetic graphs to solve them. The first technique uses the spectral properties

of the operator, and the second technique utilizes random walk techniques adjusted to this particular setting.

3:50–4:05

**Spectral Decimation for Families of Self-Similar Symmetric Laplacians on the Sierpinski Gasket**

Dylan King

Wake Forest University

The Sierpinski Gasket is a model structure for fractal analysis, and has a historically established standard Laplacian. We construct a one-parameter family of Laplacians on the Sierpinski Gasket that are symmetric and self-similar for the 9-map iterated function system obtained by iterating the standard 3-map iterated function system used to generate the Gasket. Spectral decimation is a classical technique used for the generation of eigenfunctions of linear operators on fractal structures. Our main result is the fact that all these Laplacians satisfy a version of spectral decimation that builds a precise catalog of eigenvalues and eigenfunctions for any choice of the parameter. We give a number of applications of this spectral decimation. We also prove analogous results for fractal Laplacians on the unit Interval, and this yields an analogue of the classical Sturm-Liouville theory for the eigenfunctions of these one-dimensional Laplacians.

4:10–4:25

**Feynman-Kac formulas under specific choices of semigroup generators**

Lorenzo Riva

Creighton University

The forthcoming paper *Combining continuous and discrete phenomena for Feynmans operational calculus in the presence of a  $(C_0)$  semigroup and Feynman-Kac formulas with Lebesgue-Stieltjes measures* (by L. Nielsen, to appear in *Integral Equations and Operator Theory*) contains, as its main result, an evolution equation which serves to describe how Feynmans operational calculus evolves with time in the presence of a  $(C_0)$  semigroup of linear operators. There are several examples in this paper which give rise to so-called, Feynman-Kac formulas with Lebesgue- Stieltjes measures (first investigated from a function space integral point of view by M. L. Lapidus in the late 1980s). However, due to the different approach, the Feynman-Kac formulas obtained in the paper by Nielsen have some significant differences from those obtained by Lapidus. An associated operator differential equation (essentially a nonhomogeneous Schrodingers equation) is also obtained in Nielsens paper. This talk will concentrate on the explanation of the newly-found Feynman-Kac formulas and some associated results.

Plaza Courts 2

2:30P.M. – 4:25P.M.

2:30–2:45

**No Talk**

2:50–3:05

**Recent Developments on the Stern's Diatomic Sequence and a Sister Function**

Aubrey Laskowski and Michael Schirle

*University of Illinois at Urbana-Champaign*

Stern's Diatomic sequence (OEIS A002487) is a curious mathematical object but has enjoyed little exposure in mainstream mathematics. The sequence has connections to combinatorial number theory via enumeration of the rationals and the Minkowski  $\phi$ -function. We will be presenting results on the maps  $\sigma_k(n) = \frac{s(n)}{s(n+k)}$  for fixed  $k$ , specifically on representing positive rationals in these forms. We also introduce a similarly generated sister function of the Stern sequence which we will call the Moritz function. This function provides exact formulas for previously unsimplifiable equations relating to the Stern sequence and exhibits interesting behavior by itself. Finally, we will discuss conjectures around regarding the Moritz function and the  $\sigma_k$  maps, such as solutions to  $\frac{s(n)}{s(n+k)} = c$ .

3:10–3:25

**Exploring patterns and properties of Fibonacci subsequences modulo m**

Jessica White

*University of New England*

The Fibonacci numbers have long interested mathematicians. Interest piqued in the Fibonacci numbers after DD Wall published his work around the numbers in 1960. Now, research in the Fibonacci numbers is prevalent, with patterns and characteristics of subsequences identified, and an entire journal dedicated to the field. The patterns and characteristics of subsequences of Fibonacci sequences modulo m, however, remain largely uncharacterized. Using Matlab software, the properties of Fibonacci subsequences modulo m are being examined.

3:30–3:45

**Pythagorean Triples la Mod**

Lindsey Hoeschen

*College of Saint Benedict*

People have studied Pythagorean triples for thousands of years, but no one has ever thought about Pythagorean triples in different mods. We know that all Pythagorean triples in the integers stay congruent in any mod. However, some families of triples, which we will call Neo-Pythagorean, cannot be traced back to a Pythagorean triple in the integers. In this talk, we will be exploring Neo-Pythagorean triples and other related issues.

3:50–4:05

**Patterns of Primes**

Justin Toyota

*University of Hawaii, West Oahu*

To investigate patterns of primes, we will test some formulas to determine whether they will generate primes using a computer program (CoCalc) and ideas from number theory. As it happens, some of these formulas are related to the lucky numbers of Euler. We also explore interesting properties of prime numbers such as Fermat's sum of two squares theorem and modular arithmetic.

4:10–4:25

**Lifting The Exponent of Integer Matrices**

Carl Schildkraut

*Lakeside School*

The canonical “Lifting the Exponent” lemma is a standard tool in Olympiad number theory which is used to find  $\nu_p(a^n - b^n)$  given various conditions, where  $\nu_p(N)$  is the degree of a prime  $p$  in the prime factorization of  $N$ . We extend this lemma to integer matrices by providing a closed form of the sequence  $x_n = \nu_p(\det(A^n - I))$  in terms of the factorization of the characteristic polynomial of  $A$  in  $\mathbb{F}_p$ . We also show that this quantity is asymptotic to the  $n$ th power of the Mahler measure of the characteristic polynomial of  $A$ , providing a possible application of this result.



Plaza Courts 3

2:30P.M. – 4:25P.M.

2:30–2:45

**No Talk**

2:50–3:05

**Assessing Critical Mass at UC-Berkeley: Creating Predictive Models for Affirmative Action Policies in Undergraduate Admissions in the United States**

Daniel Maes

*Williams College*

Affirmative action refers to hiring and recruiting practices designed to combat discrimination against members of certain demographic groups. These policies are used in settings ranging from federal contractors to local employment to public education, and can focus on race, ethnicity, or gender. Due to the broad applicability and direct social impact of affirmative action, substantial effort has gone into monitoring the necessity and effectiveness of these policies. We focus on the application of race-based affirmative action policies in public undergraduate college admissions in the U.S., specifically through a case study of admissions to the University of California, Berkeley. We make three primary contributions to policy assessment. First, we introduce a quantitative framework through which to interpret a key concept used in contemporary affirmative action litigation: "critical mass." Second, we construct a predictive model of college admissions demographics using Markov Chains. Third, we bring together the two previous contributions, using our quantified version of the critical mass criterion as a benchmark for assessing the outcomes of the predictive model.

3:10–3:25

**Geometry of Delegate Apportionment in the Kansas Republican Primary**

Dalton Nelson

*William Jewell College*

The Republican Party uses various apportionment methods for assigning amounts of pledged delegates in presidential primary elections. The Republican National Convention allows each state to choose which method of apportionment to use in that state's primary. Some states use self-created methods of assigning delegates that have never before been used - one of which is Kansas. In this talk, we study paradoxical behavior in the delegate apportionment method created by Kansas, focusing primarily on a paradox known as the *cutoff paradox*. We estimate the probability with which this paradox occurs, and we compare this to the probability with which the same paradox occurs in the most popular delegate apportionment method, Hamilton's method.

3:30–3:45

**Measuring Voting Power Using Observed Data**

Nick Schrock

*Goshen College*

The Shapley-Shubik and Banzhaf power indices have been used to measure the voting power of voters in a variety of political institutions, including the United Nations Security Council, the United States electoral college, and proportionate representation legislatures in a variety of countries. These power indices depend only upon the voting rules; hence, each assigns an equal power to each of the nine Justices on the United States Supreme Court, because each Justice has one vote and cases are determined by simple majority. Once actual people having particular ideological positions and behavioral patterns become members of a political institution, their true voting power can be measured more accurately by taking into account their past behavior and political alignment with respect to other voters; for example, the Justice seen as occupying the

central position in the liberal to conservative ordering of U.S. Supreme Court Justices is often considered more powerful than the other Justices. This paper examines power indices that take into account both the voting rules and the past actions of voters and applies these indices to particular political institutions.

3:50–4:05

**Graph theoretic models of interdependence in referendum elections**

Benjamin Becker and Lee Trent

*Hillsdale College and Rose Hulman Institute of Technology*

In referendum elections, voters are often required to cast simultaneous votes on multiple questions or proposals. The separability problem occurs when the proposals in the election are linked in voters minds, meaning that their preferences on the outcome of one or more proposals depend on the predicted outcomes of other proposals. In this talk, we will explore how graphs can be used to model the interdependence structures in voter preferences. We will also consider how these models can be used to study alternative voting methods and strategies. This research was conducted as part of the Grand Valley State University Summer Mathematics REU.

4:10–4:25

**Simulation of referendum elections via graph theoretic models of voter preferences**

Colby Brown and Kaleigh Roach

*University of Arizona and North Greenville University*

In referendum elections, voters are often required to cast simultaneous votes on multiple questions or proposals. The separability problem occurs when the proposals in the election are linked in voters minds, meaning that their preferences on the outcome of one or more proposals depend on the predicted outcomes of other proposals. In this talk, we will use graph theoretic models of voter preferences to generate diverse electorates and to study both the impact of separability on the outcomes of referendum elections and the potential for alternative voting methods to alleviate the separability problem. This research was conducted as part of the Grand Valley State University Summer Mathematics REU.

Plaza Courts 4

2:30P.M. – 4:25P.M.

2:30–2:45

**No Talk**

2:50–3:05

**Analyzing Games is Risky Business**

Joshua Jalowiec

*United States Air Force Academy*

The tools of mathematics are useful in analyzing games in order to find and evaluate strategies for play. Elements of such analysis might include formal game theory, the probability of events happening (ie - dice rolls, cards drawn), combinatorial analysis, and many other tools. Through the lens of graph theory, board games like Risk can be analyzed to determine aspects of the game that might go unfound or appear to run counter to common sense gameplay. We will specifically consider certain factors that a player should consider when formulating a strategy for the beginning of the game when choosing initial territories to occupy and deciding on an initial distribution of armies.

3:10–3:25

**Game R-chromatic and Game R-achromatic Numbers**

Emily Twardy

*College of Saint Benedict*

For any graph  $G$ , a collection  $R = \{R_1, R_2, \dots, R_t\}$  of subsets of the vertex set  $V(G)$  can be selected. Common choices for  $R$  are the collection of edges, the collection of closed neighborhoods, and the collection of open neighborhoods.  $R$ -chromatic numbers,  $\chi_R(G)$  are defined with respect to the collection  $R$  chosen. For example, the chromatic number  $\chi(G)$  is defined relative to the edge set so that  $\chi_E(G)(G) = \chi(G)$ . Using other choices for the collection  $R$  leads to other chromatic numbers, specifically  $\chi_N[v](G)$  and  $\chi_N(v)(G)$ . For game chromatic numbers, two players take turns coloring the vertices of a graph where one player has the objective of minimizing the number of necessary colors used to completely color the graph while the other player is trying to maximize the number of necessary colors. Applying the game idea to  $R$ -colorings, this talk defines these game  $R$ -colorings and presents results on certain families of graphs for both the minimum number of colors, the game  $R$ -chromatic number, denoted  $\chi(R, g)(G)$ , as well as the maximum number of colors for a minimal coloring, the game  $R$ -achromatic number, denoted  $\Psi(R, g)(G)$ .

3:30–3:45

**Coloring Graphs on the Earth and Moon**

Hannah Critchfield and Maia Wichman

*James Madison University and Grand Valley State University*

In Ringel's Earth-Moon problem, every country on Earth forms a single colony on the Moon. The problem is then to determine the number of colors needed to color the graph resulting from the map with the assumption that every country and its colony must receive the same color. In this talk, we discuss progress on this problem and some of its variations.

3:50–4:05

**Dynamics of Academic Affiliations: A Networked Approach**

Juan Mendoza

*Central Washington University*

Graph theory has been a useful mathematical framework for understanding structural properties of complex phenomenon, modeling as a network. In this work, we use graph theory to model interactions among aca-

demographic institutions with respect to changes in affiliation, that is, each academic institution is a node, and an edge exists between any two nodes at a given point in time, if a researcher from one institution to the other. This model allows us to investigate the patterns of movements among institutions and investigate the impact of affiliation dynamics on scientific research. We used a dataset from Association for Computing Machinery (ACM) Digital Library spanning over 30 years. The standardization and cleaning of the data required a substantial effort involving heuristics, data augmentation, natural language processing, and clustering techniques. Next, the network was created where the nodes of the network are the unique academic affiliations and the edges represent when a professor moves one affiliation. The network is a directed because there is movement from one affiliation to the other, weighted because there could be multiple movements to one affiliation.

4:10–4:25

**Vertex-Minimal Planar Graphs with Cyclic Symmetry of Even Order**

Carlie Triplitt and Colin Jones

*University of Science and Arts of Oklahoma and Eastern University*

In 1939, Frucht proved that for any finite group  $G$ , there exists a graph  $\Gamma$  such that the automorphism group of  $\Gamma$  is isomorphic to  $G$ . Naturally, this result gave rise to numerous extremal problems in graph theory. For instance, vertex-minimal graphs with a prescribed automorphism group are the subject of prior research by numerous authors. In this talk, we will discuss vertex-minimal *planar* graphs with cyclic automorphism groups of even orders.

## Plaza Courts 1

4:30P.M. – 6:05P.M.

4:30–4:45

**Matrix Models for the Circular Beta Ensemble**

Eli Cytrynbaum

*Williams College*

We construct families of random unitary matrices which have eigenvalues distributed according to the circular beta ensemble. We investigate properties of these matrices for beta approaching zero and infinity.

4:50–5:05

**Non-Hermitian Anderson Operators and their Spectral Properties**

Mira Gordin

*Brown University*

The Non-Hermitian Anderson operator has both real eigenvalues and complex eigenvalues located on smooth curves in the complex plane. We consider the distribution of the spacings between nearest eigenvalues and investigate the transition from Poisson to the picket fence distribution.

5:10–5:25

**The Rate of Convergence to the Circular Law for I.I.D. Matrix Ensembles**

Michael Curran

*Williams College*

We consider various families of matrices with independent identically distributed random coefficients and investigate the rate of convergence of their rescaled eigenvalue point process to the uniform measure on the unit disk.

5:30–5:45

**The Eigenvalue Distribution for the Symmetric Bernoulli Ensemble**

Ben Foster

*University of Pennsylvania*

We consider the symmetric matrices with random coefficients that are independent Bernoulli variables. We study the local and global eigenvalue distribution, as well as the distribution of eigenvalue spacings for these matrices.

5:50–6:05

**Finding a needle in  $\mathbb{C}$ : On Initial Estimates for the Polynomial Eigenvalue Problem**

Michael Robertson

*Davidson College*

We define a matrix polynomial as a polynomial in a complex variable with matrix coefficients. The polynomial eigenvalue problem consists of computing the eigenvalues of a given matrix polynomial, i.e., the values in  $\mathbb{C}$  for which the matrix polynomial evaluates to a singular matrix.

Recently, iterative methods intended to find the roots of a scalar polynomial have been adapted to solve for eigenvalues of a matrix polynomial. Although we often compare iterative methods by their local convergence, their efficiency also depends heavily upon the quality of the initial estimates.

In this talk, we use pictures to describe three inclusion sets of the eigenvalues: annuli derived from a generalized Pellet's Theorem, the Numerical Range, and the Gershgorin Set. For each of these sets, we calculate the average distance to a solution by taking the distance from each initial estimate to the closest eigenvalue, and then averaging those distances for many different polynomials. We use this average distance as a metric to compare the sets.

## Plaza Courts 2

4:30P.M. – 6:05P.M.

4:30–4:45

**Index Divisibility of Dynamically Generated Sequences from Polynomials with Rational Coefficients**

Angelina Schenck and Levi Lorenzo

*Mount Holyoke College and Hamilton College*

The index divisibility set of a sequence  $(s_n)$  is the set of positive integers  $n$  for which  $n \mid s_n$ . In recent years, there have been many papers investigating the index divisibility set of various sequences, most notably the Fibonacci and Lucas sequences. In another recent paper, Chen, Gassert and Stange characterize the index divisibility set for the sequence  $(\phi^n(0))$ , where  $\phi(x) = x^d + c$  is an integral polynomial of degree at least two. Motivated by these results, we consider polynomials with rational coefficients. In particular, if  $f(x)$  is a polynomial with rational coefficients and we let  $f^n(0) = \frac{a_n}{b_n}$ , we describe the index divisibility set for the sequence  $(a_n)$  and give a visualization of the set as a directed graph.

4:50–5:05

**Sums of  $k$ -th powers in Quaternion rings**

Sarah Wright and Luke Shuck

*McDaniel College*

Generalization of Waring's Problem – that for every natural number  $k$  there exists an integer  $g(k)$  such that every natural number can be written as the sum of at most  $g(k)$   $k$ -th powers – have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We extend current results on  $g(2)$  and  $g(4)$  for certain Quaternion rings with integer coefficients.

5:10–5:25

**Sums of  $k$ -th powers in ramified  $p$ -adic rings**

Tim Sherwin and Ashley Wright

*McDaniel College*

Generalization of Waring's Problem — that for every natural number  $k$  there exists an integer  $g(k)$  such that every natural number can be written as the sum of at most  $g(k)$   $k$ -th powers — have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We will examine values of  $g(k)$  for certain ramified extensions of  $\mathbb{Z}_p$ .

5:30–5:45

**Playing with Triangular Numbers**

Dipika Subramaniam

*Lake Forest College*

A triangular number is a number  $N$  that satisfies that  $N$  dots can be arranged in increasing order to form an equilateral triangle. This can be represented in the form :  $\frac{n(n+1)}{2}$ .

In his article titled Playing With Blocks, Professor Matthew McMullen noticed that sometimes, there are consecutive sequences of triangular numbers that add up to form another triangular number. Can we prove that for any  $k$  where  $k$  is a square, there are  $k$  consecutive triangular numbers, starting from the  $n$ th triangular number, that add up to form another triangular number, which we will call the  $m$ th triangular number? Moreover, how can we find the values of  $m$  and  $n$  for which this works?

5:50–6:05

**No Talk**

**Plaza Courts 3****4:30P.M. – 6:05P.M.**

4:30–4:45

**Bayesian Regression of Thermodynamic Models of Redox Active Materials**

Katherine Johnston

*California Institute of Technology*

Finding a suitable functional redox material is a critical challenge to achieving scalable, economically viable technologies for storing concentrated solar energy in the form of a defected oxide. Demonstrating the efficiency for thermal storage or solar fuel relies on a thermodynamic model derived from data obtained by experiments or DFT simulation. This projects purpose is to quantify the impact of the uncertainties in the data that feeds into the thermodynamic model on the predicted efficiency of the hydrogen conversion devices that rely on this model. The approach involves calibrating the model parameters using Bayesian inference, followed by model selection using Bayes factors to determine the best suited model form given the available data. Three data sets were considered: two materials for solar fuels by water splitting and one for thermal storage. The calibrated model fit the data well. Bayesian evidence values were calculated various ways and used to compare models with different numbers of parameters. All analysis was performed using the Sandia open source Uncertainty Quantification Toolkit and in collaboration with Bert Debusschere (Sandia Lab) and Ellen Stechel (ASU)

4:50–5:05

**Physical Applications of Fourier Analysis**

Liam Doherty

*Rowan College at Burlington County*

In this project we analyze physical phenomena which can be modeled with second order ODEs. Such equations often model oscillatory patterns such as waves in two dimensions. These waves may be purely sinusoidal or may have many components which can be summed in a series to produce a different type of wave such as sawtooth or square. To analyze a particular phenomenon, it may help to analyze them as signals in the frequency domain rather than time. We can transform time dependent equations into frequency dependent equations with the Fourier Transform. By doing so, we break down the system to get exact points of failure of structures such as mechanical systems, bridges, and high power electronic systems. Other applications include those in the defense industry such as RF protocols, neuroscience, and engineering. Our project shows the methods of analysis and provides physical examples of the possible failures that may occur, and how to correct them in the real world. Software used in research include National Instruments MultiSim, Wolfram Mathematica, and LaTeX. Using these three programs, results that illustrate the usefulness and applications of Fourier Analysis are obtained.

5:10–5:25

**The Geometry of Solar Panels based on its Efficiency**

Nikesh Prajapati and Rupak Kadel

*LaGuardia Community College*

The main purpose of this research is to examine the shape of a panel that accumulates more solar energy than traditional flat panels through mathematical flux. The calculations were performed through the use of python programming language. Until now, only traditional flat panels are used and are known to have high efficiency and flexibility. It is important to thoroughly understand the trajectory of sun during specific time and place to know the amount of sun rays that would expose in the solar panel. We chose New York as a designated destination to implant a ‘catenoid shaped solar panel’. Its efficiency is similar to the flux density. ‘Flux’ is used to measure the amount of sun rays passing through the catenoid. Since the trajectory of sun varies from place to place, we encountered more complex calculation while shifting our direction to New York. So, we coded in python since it has in-built library to calculate the triple integrals. As we considered

catenoid surface, we were bound to face some challenges like trimming some parts of the catenoid where sun rays were not exposed. However, the good news is we were able to find out some information about its efficiency.

5:30–5:45

**Mathematical Model and Computer Simulation of a Multilevel System of Rigid Spheres**

Stefano Monzon, Omar Khan, and Isabelle Gehrs

*University of San Diego*

This interdisciplinary project presents a mathematical model and computer visualization of a hierarchical system of many rigid spheres (particles) that simulates elastic collisions between sub-systems on each level of hierarchy and their resulting motion. We model each level of the system as a collection of spheres confined by a surrounding spherical enclosure. Based on the math of collision detection an innovative collision sequencing algorithm is used. An interactive computer program displays the system's motion using computer graphics in real time. One of the main purposes of our simulation is to allow the study of the dynamic variables that describe the system, such as energy distributions and rates of energy transfer between the sub-systems, as it transitions between states.

5:50–6:05

**Thermoviscoelastic Rod and Nonlinear Timoshenko Beam System with Dynamic Contact**

Zach Rail

*Arkansas State University*

In this work, we consider mathematical and numerical approaches to a rod-beam system. The rod-beam system is motivated by microelectromechanical systems (MEMS). One end of the beam is clamped the other end is jointed to a thin, vertical, thermoviscoelastic rod. When the top of the rod touches a rigid obstacle, Signorinis contact conditions and Barbers heat exchange condition are applied. The beam model combines a Kirchhoff type equation with the Timoshenko beam theory. The motion of the jointed rod and beam is described by four partial differential equations and several boundary conditions and complementarity conditions. We employ time-discretizations on a time interval and finite element methods over the spatial domain to propose fully discrete numerical schemes. In addition, we use the Newton-Raphson method to compute each time step numerical approximation satisfying a nonlinear system in the discrete case.



**Plaza Courts 4****4:30P.M. – 6:05P.M.**

4:30–4:45

**Comparing Tree and Divisorial Gonality**

Ivan Aidun

*Oberlin College*

Given a graph with lengths assigned to its edges, there are multiple ways to define an important invariant of the graph called its gonality. In this talk, we present on two different versions, tree gonality and divisorial gonality, and present families of graphs where these invariant agree, and other families where they differ.

4:50–5:05

**Computing Graph Gonality with Brambles**

Frances Dean

*Williams College*

The gonality of a graph is an important invariant that is difficult to compute for general families of graphs. One strategy is to find upper and lower bounds on the gonality. In this talk we use a tool called brambles to find lower bounds on the gonality of families of graphs.

5:10–5:25

**Computing Graph Gonality with Strong Separators**

Sammy Rosofsky

*Williams College*

The gonality of a graph is an important invariant that is difficult to compute for general families of graphs. One strategy is to find upper and lower bounds on the gonality. In this talk we use a tool called strong separators to find upper bounds on the gonality of families of graphs.

5:30–5:45

**Graph Gonality and Newton Polygons**

Teresa Yu

*Williams College*

A tropical plane curve is defined by a tropical polynomial, and is dual to a subdivision of the polynomial's Newton polygon. Such a tropical plane curve contains a metric graph which has an associated gonality. In this talk we show how to relate this gonality to the Newton polygon, through a property of the polygon called lattice width.

5:50–6:05

**Graphs of Gonality Three**

Julie Yuan

*University of Minnesota*

One of the most important invariants of a metric graph is its gonality. Graphs of gonality one are trees, and graphs of gonality two are the hyperelliptic graphs, which have been completely classified. In this talk, we present a classification of graphs of gonality three.

## Plaza Courts 1

9:30A.M. – 12:05P.M.

9:30–9:45

**Triangle Inscribed-Triangle Picking**

Arman Maesumi

*The University of Texas at San Antonio*

Given a triangle  $ABC$ , we derive the probability distribution function and the moments of the area of an inscribed triangle  $RST$  whose vertices are uniformly distributed on  $AB$ ,  $BC$  and  $CA$ . The theoretical results are confirmed by a Monte Carlo simulation.

9:50–10:05

**A Missing Entry in Sullivan's Dictionary?**

Colby Kelln

*University of Michigan*

Sullivan's dictionary highlights some similarities between 1-dimensional complex dynamics and 3-dimensional hyperbolic geometry. Many mathematicians attempted to use this correspondence to translate proofs from one field to the other. Some succeeded, some failed, but nonetheless this dictionary has driven enormous expansion in both fields since the 1980s. More recently, some mathematicians conjectured that there might be some missing entries. One of these possible missing entries is the degeneracy of Julia sets and degeneracy of limit sets of quasi-Fuchsian groups. The goal of this project is to use computer programs to generate images of degenerate Julia sets and limit sets that share similar behavior.

10:10–10:25

**The Nine Point Circle via Complex Numbers**

Maverick Maynard and Justin Merritt

*South Dakota State University*

We will be proving the nine point circle exists via complex numbers. The nine point circle can be constructed for any given triangle by using the midpoint of each side of the triangle, the foot of each altitude, and the midpoint of the line segment from each vertex of the triangle to the orthocenter. Some people may know of the nine point circle, but we will be establishing it by utilizing geometric properties of the complex plane. By relating the orthocenter of the triangle and the points on the nine points circle to triangle vertices, we will be able to verify our topic.

10:30–10:45

**Kähler-Einstein Metrics on Compact Fano Manifolds via Effective Approximations**

Pilar Orellana

*UC Riverside*

Kähler-Einstein metrics emerge when a complex, topological manifold, under additional conditions, admits a metric that is both Einstein and Kähler. They are beautiful objects which arise naturally in many facets of mathematics—and moreover, are of great importance in the study of string theory. We want to determine under what conditions a compact Fano manifold of Type I cohomogeneity one admits Kähler-Einstein metrics; for which is done by verifying the classes of the manifolds being Fano manifolds and their stability; however, by using the standard methods currently available to us, this proves to be quite a cumbersome task which yields very limited results. In order to overcome this obstacle, we have developed new specialized methods which are effective at retrieving large-scale information of classes of these compact Fano manifolds and their corresponding Kähler-Einstein properties.

10:50–11:05

**The Curious Pivot Points of Best-Fit Lines**

Sable Levy

*St. Edward's University*

In this paper we explore the pivot points of regression lines for discrete data sets. Previous literature has shown that the best-fit line of a data set pivots about a fixed pivot point when we repeat one of the data points. This paper explores the underlying geometric framework of pivot points, as well as what happens to them when we repeat multiple points in the data set.

11:10–11:25

**Circle Packing on a Square Flat Torus with Two Classes of Disks**

Susanna Manning and Bridget Parker

*University Alabama Huntsville and Central Michigan University*

The study of maximally dense packings of disjoint circles is a problem in Discrete Geometry. The goal is to find the optimal density and corresponding arrangements of circles in various containers. The optimal densities are known for packings of small numbers of equal circles into hard boundary containers, including squares, equilateral triangles and circles. In this presentation, we will explore packings of small numbers of circles onto a boundaryless container called a square flat torus and instead of using equal circles, we use two classes of circles each with a different common radius. Using numerous pictures, we will introduce all the basic concepts (including the notion of a square flat torus, an optimal packing, and the graph of a packing), illustrate some maximally dense arrangements, and outline the proofs of their optimality. This research was conducted as part of the 2018 REU program at Grand Valley State University.

11:30–11:45

**Euclidean Theorems in Spherical and Hyperbolic Geometries**

Geneva Collins and Lucas Perryman-Deskins

*North Carolina State University and Willamette University*

During the Edo period (1603-1867 CE) Japan was almost completely closed off from the rest of the world and developed its own mathematical tradition called *wasan*. Part of this tradition was to hang tablets, known as *sangaku*, in the eaves of a shire or temple as an act of devotion. The *sangaku* tablets were made of solid wood and each contained a colorful illustration of one or more results in Euclidean geometry. The tablets were created by people from many different social classes including farmers and samurai. In this presentation we will explore the generalization of some of these ancient Japanese theorems (or other more modern ones) in Euclidean geometry to both spherical and hyperbolic geometry. The basics of spherical and hyperbolic geometry will be explained. This research was conducted as part of the 2018 REU program at Grand Valley State University.

11:50–12:05

**No Talk**

Plaza Courts 2

9:30A.M. – 12:05P.M.

9:30–9:45

**Graph Theoretical Design Strategies for Modeling Self-Assembling DNA**

Chandler Stimpert and Hector Dondiego

*Lewis University*

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods developed with concepts from undergraduate level graph theory have resulted in significantly increased efficiency. In this talk, we present the results of a summer undergraduate research project applying graph theoretical and linear algebra techniques to constructing complexes like general bi-partite graphs and lattice-based graphs which can be created from self-assembling DNA. In particular, we explore various design strategies given three different laboratory constraints.

9:50–10:05

**A Wheely Tough Problem: Self-Assembling DNA Wheel Graphs**

Gabriel Lopez

*California State University, San Bernardino*

Emerging laboratory techniques have been developed using the Watson-Crick complementarity properties of DNA strands to achieve self-assembly of graphical complexes. One recent focus in DNA nanotechnology is the formation of nanotubes, which we model with a two-dimensional lattice that wraps around to form a tube. The vertices of the lattice graph represent  $k$ -armed branched junction molecules, called tiles. Using concepts from graph theory, we seek to determine the minimum number of tile and bond-edge types necessary to create the desired self-assembled complex. Although results are known for a few infinite classes of graphs, triangle lattice graphs are an open problem. Motivated by the study of triangle lattice graphs, we present results for a related problem: finding the minimum number of tile and bond-edge types for the wheel graph. While some laboratory settings allow for the possibility of the formation of smaller complexes using the same set of tiles, we examine the problem under the restriction that no smaller complete complex may be formed.

10:10–10:25

**Star Ideals of Graphs**

Morgan Gauvin, Carlos Munoz, and Hugh Mckenny

*Bethany Lutheran College, San Jose State University, and Hobart and William Smith Colleges*

We will be investigating the relationship between commutative algebra and graph theory through an exploration of square-free monomial ideals. Our primary examination will be a generalization of edge ideals, in which the generators of our ideal are constructed from the complete bipartite subgraphs  $K_{1,t}$  of a finite simple graph  $G$ . Our paper attempts to answer the following question: Given a square-free monomial ideal  $I$  in  $R = k[x_1, \dots, x_n]$ , is there a finite simple graph  $G$  whose star ideal is  $I$ ? We identify and classify families of ideals whose generators do not produce graphs and make some general claims about star ideals.

10:30–10:45

**Exploring 3-Path Domination**

Rebecca Jackson and Rayan Ibrahim

*Charleston Southern University and CUNY-College of Staten Island*

The dominating set of a graph  $G$  is a set of vertices  $S$  such that for every  $v \in V(G)$  it must be the case that  $v \in S$  or  $v$  is adjacent to a vertex in  $S$ . The domination number, denoted  $\gamma(G)$ , represents the minimum number of vertices needed to construct a dominating set. In 1998, Haynes and Slater introduced

paired-domination. Paired-domination considers a dominating set whose induced subgraph contains a perfect matching. The paired-domination number, denoted  $\gamma_p(G)$ , represents the minimum number of vertices needed to construct a paired-dominating set. Building upon paired-domination, we introduce 3-path domination. We define a 3-path dominating set of  $G$  to be  $S = \{P_1, P_2, \dots, P_k \mid P_i \text{ is a 3-path}\}$  such that the vertex set  $V(S) = V(P_1) \cup V(P_2) \cup \dots \cup V(P_k)$  is a dominating set. The 3-path domination number, denoted by  $\gamma_3(G)$ , represents the minimum number of 3-paths needed to dominate  $G$ . We show that the 3-path domination problem is NP-complete, as well as additional results regarding bounds on  $\gamma_3(G)$  and closed formulas for families of graphs.

10:50–11:05

### On Perfect Domination Ratios of Archimedean Lattices

Yunfan Zhao

*Johns Hopkins University*

A set of vertices  $S$  is said to dominate a graph  $G = (V, E)$  if every vertex in  $V$  is either in set  $S$  or is adjacent to a vertex in set  $S$ . A dominating set is a perfect dominating set if every vertex not in the dominating set is dominated exactly once. For a finite graph  $G$ ,  $D$  is a minimal perfect dominating set if it is a perfect dominating set of  $G$  and has cardinality less than or equal to every other perfect dominating set of  $G$ . The perfect domination ratio of  $G = \frac{|D|}{|V|}$ . We extend the notion of perfect domination ratio to infinite periodic graphs. Archimedean lattices are vertex-transitive infinite graphs formed by regular polygons. We have determined exact perfect domination ratios for most Archimedean lattices. We also study nonisomorphic perfect dominating sets on some Archimedean lattices.

11:10–11:25

### Limit characterizations through spanning trees in multigraphs: an exploration

Joshua Steier

*Seton Hall University*

Nikolopolous et. al., using Cayley's Theorem and Kirchhoff's Matrix Theorem, established limiting results for the number of spanning trees of certain families of graphs. Focusing as they did on edge deletions, we applied similar techniques to similar families of multigraphs. We examined existing results involving threshold graphs and split graphs. Utilizing various matrix properties and a general formula for the number of spanning trees on complete multigraphs, we conjecture limit characterization for underlying complete multigraphs with fixed multiplicity.

11:30–11:45

### Investigating bounds for the Bond and Site Percolation Thresholds of a Two-Uniform Lattice

Gabrielle Moss

*Johns Hopkins University*

A percolation model is an infinite graph, from which edges are deleted independently with probability  $p$ . The percolation threshold of an infinite graph is the critical probability  $p_c$  above which there exists a connected, infinite component. Most research has focused on calculating the percolation threshold of the Archimedean lattices. Instead, this research focuses on the two-uniform lattices, which are lattices where each vertex has one of two sequences of faces surrounding it. Each vertex of this graph has one of two face sequences: triangle-square-triangle-dodecagon or triangle-dodecagon-dodecagon. Due to its structure, very accurate bounds for its bond and site percolation threshold may be computed. The bounds are obtained using the substitution method, which is based on the equivalence of stochastic ordering and coupling in probability theory, with computational efficiencies involving partition lattices, non-crossing partitions, symmetry reductions, and network flow algorithms. In this talk, we will discuss this method, the final calculated bounds for

the  $(3,4,3,12;3,122)$  lattice, and the new intuition developed from these results.

11:50–12:05

**Inversions in Directed Acyclic Graphs**

Meraiah Martinez, Otto Osterman, and Xuming Liang

*Benedictine College, The University of Texas at Dallas, and Harvey Mudd*

An inversion of a labeled graph is an ordered pair of vertices  $(x,y)$  such that  $x \not\leq y$  and there exists a path from  $x$  to  $y$ . We investigate the number of labeled directed acyclic graphs with a given number of inversions.

**Plaza Courts 3**

9:30–9:45

**9:30A.M. – 12:05P.M.****No Talk**

9:50–10:05

**Youngstown Temperature Forecast**

David Gessler

*Youngstown State University*

We predicted temperatures in Youngstown over a 30-year period (1987–2017). We used several regression models to fit the data and determined the error between the values predicted and the actual values to ultimately choose the best model. The results indicated that mean temperature sine model is the best model, among the four chosen models, for predicting weather.

10:10–10:25

**Evaluating of an Ice Hockey Goalie: A new application of Sabermetrics**

Helen Cooney

*Neumann University*

Sabermetrics is the field of study dealing with the statistical analysis of baseball data (Costa, Huber, and Saccoma, pg.1). In recent years, similar statistical techniques have been applied to other sports. The sport of ice hockey appears to be lacking in the depth of statistical analysis used. This paper discusses the use of sabermetrics in ice hockey. In particular, we analyze the relationship between goalies allowing "weak" goals and their team's likelihood of winning games. This analysis ultimately provides us with a new statistic that helps measure the value of a goalie to his or her team. This paper begins with some background about the history of sabermetrics, beginning with the technique's popularization in baseball during the 1980s (Society for American Baseball, A guide to Sabermetric research). We then examine how such techniques are used in other sports, and in other fields all together. Next, we discuss some of the currently used statistics in ice hockey and, in particular, what defines a weak goal. Finally, data were gathered and used to find a new statistic for use in ice hockey analysis. The value of this new statistic is discussed and validated.

10:30–10:45

**Analysis of AP College Football Polls, II**

Samuel Guo

*University of Illinois at Urbana-Champaign*

We analyzed individual ballots of voters in AP College Football Polls using several well-known ranking metrics and statistical methods. Our data set consisted of around 4000 individual ballots spanning four seasons of college football. We report on the results of this analysis.

10:50–11:05

**Comparing University Rankings**

Weiru Chen and Qianqian Li

*University of Illinois at Urbana-Champaign*

We compare 4 major university rankings - US News and World Report Global University Rankings, Shanghai Rankings (ARWU), QS Top University Rankings, and Times Higher Education Rankings - in selected subject areas. Our methods include well-known ranking metrics such as Kendall tau, Spearman's Footrule, and the Hamming distance, as well as statistical methods such as correlation methods. We report on the results of this analysis.

11:10–11:25

**Introducing a Bayesian Framework for the Symmetric Rendezvous Problem on the Line**

Jeff Braun

*Johns Hopkins University*

A famous open problem in the field of Rendezvous Search is to ascertain the rendezvous value of the symmetric rendezvous problem on the line wherein both agents begin 2 units apart. We provide a new, Bayesian framework to both create new strategies for the agents to follow and to provide a new interpretation of previously posited strategies. Additionally, we have developed a method that modifies any strategy, even those with potentially infinite expected meeting time, into a new strategy that is guaranteed to have a finite expected meeting time. This process, combined with using our Bayesian framework to create new strategies, yields an upper bound that is within 1 bound for the symmetric rendezvous value.

11:30–11:45

**Rendezvous Search on the Edges of Platonic Solids**

Elanor West and Xiao (Annie) Xie

*Johns Hopkins University*

A classic "rendezvous search" problem is the "astronaut problem," in which two agents are placed on a sphere and move around until they meet. Research focuses on finding an optimal strategy for both agents to use. We consider a model that utilizes discrete units of time, with movement along the edges of platonic solids. The search ends when the two agents can see each other. We compare the mean times to meet on all five solids under an unbiased random walk strategy, and we alter assumptions and strategies in various versions of the search to see how certain changes affect the mean time to end. One version involves the possibility of waiting on any given turn under both biased and unbiased random strategies. We also examine multi-step strategies, which involve a random step and a predetermined sequence of directions. The calculations of expected meeting times all involve first-step Markov chain decompositions.

11:50–12:05

**No Talk**



**Plaza Courts 4****9:30A.M. – 12:05P.M.**

9:30–9:45

**Random Walks and the Heat Equation**

Tait Weicht and Alejandro Becerra

*Seattle Pacific University and University of Houston*

Random walks are processes useful for understanding a variety of stochastic behaviors from the movement of molecules to the flight path of moths. Through a series of various transformations, such as Laplace and Fourier transforms, we can find the probability of trapping a random walker at the origin for a discrete walk and in a disk for a continuous one. In particular, we will use properties of Brownian motion and the heat equation to model this Markov process. Moreover, we are studying both simple random walks, those with an equal probability of turning, and direct random walks, those which have a tendency to continue in the same direction. Our goal is to relate the trap size and diffusion coefficient for a heat equation to the behavior of a directed random walker.

9:50–10:05

**A Cost Benefit Analysis of Cyber Defense Improvements**

Tung Thai

*Wentworth Institute of Technology*

In the past few years, several major cybersecurity attacks on supervisory control and data acquisition (SCADA) devices have been reported. Such attacks can result in damages to the economy and have an impact on society. In 2010, Ten et al presenting a attack tree mode of impact analysis. We have implemented the attack tree structure developed by Ten, in concert with typical financial loss data to implement Monte Carlo techniques to generate a new cost benefit analysis of various security improvement scenarios. Time to attack is modeled as an exponentially distributed random variable obtained via maximum likelihood analysis; financial losses are modeled using regression to generalized logistic functions via gradient descent. Under these conditions, hypothetical future losses are simulated for a variety of intrusion scenarios and improvement schemes; we incorporate budgetary constraints in an effort to advise the prioritization of system improvements.

10:10–10:25

**Data Reduction on High-Dimensional Data Sets Using Principal Component Analysis**

Jessica Sandcork

*Creighton University*

Given a data set with a considerably large amount of variables, traditional statistical testing procedures will lead to inflated Type I error rates without proper adjustment. Multiple testing procedures must be introduced to accommodate such a large quantity of variables. However, a data set with a large number of variables may also exhibit exceptionally high levels of correlation between variables, which weakens the statistical power of multiple testing procedures. In this presentation, we explore the usage of data reduction on a highly correlated, large variable data set with the ultimate goal of breaking the high rates of correlation between many variables to determine what is truly significant. We propose using principal component analysis (PCA) to reduce the number of variables by searching for linear combinations of highly correlated variables that explain a large proportion of variability in the overall data set. By reducing the dimensionality of the data set, we hope to be able to apply multiple testing procedures without the additional challenge of highly correlated variables.

10:30–10:45

**Come Converge! Lets Talk About Clustering: Part One**

Alanis Chew and Madeline Cope

*Youngstown State University*

Data clustering is the unsupervised classification of patterns into groups or clusters that is based on recognizing the similarities or dissimilarities between the data. It can be used in a wide variety of disciplines, ranging from astrophysics to zoology. The two main branches of data clustering are hierarchical and partitional. This talk will focus on two techniques, Mutual Nearest Neighbor, a hierarchical clustering technique, and Spectral Clustering, a partitional clustering technique. Mutual Nearest Neighbor is an algorithm that merges the least dissimilar mutual pair, and Spectral Clustering is a graph theoretic partitioning that uses eigenvalues of the graph for dimensionality reduction. These data clustering techniques are two of many that can be used to analyze a wide range of data sets.

10:50–11:05

**Come Converge! Lets Talk About Clustering: Part Two**

Julie Phillis

*Youngstown State University*

Data clustering is the unsupervised classification of patterns into groups or clusters that is based on recognizing the similarities or dissimilarities between the data. In this talk, the use of the Spectral Clustering technique and the Mutual Nearest Neighbors technique will be discussed in studying two very different data sets. Two different methods of measuring cluster validity are then applied to evaluate the resulting clusters and what they say about the data. These techniques and methods will aid in answering thoughtful questions posed by the researchers.

11:10–11:25

**Spherical Visual Cryptography**

Lindsey Wise

*Appalachian State University*

This paper introduces the concept of performing visual cryptography on spherical models. This method utilizes a variation of the scheme introduced by Naor and Shamir for 2 shares using triangular subpixels. The construction the spherical shares is based on the theory of spherical models given by Wenninger.

11:30–11:45

**On Error Free Transformations and Applications to Polynomial Equations**

Aidan O'Neill and Thomas Cameron

*Davidson College*

We begin our presentation with a discussion of floating-point techniques. Knuth's algorithm for summing two floating point numbers allows us to sum two floating point numbers with accuracy comparable to twice the working precision; the Veltkamp-Dekker algorithm for splitting floating point numbers and then evaluating their product similarly allows us to multiply two floating point numbers in twice the working precision. Following the work of Stef Graillat et al., we apply these floating point techniques to develop a compensated Ruffini-Horner method which evaluates a polynomial in twice the working precision. We then apply this compensated Ruffini-Horner method to develop root solvers that can similarly compute simple zeros in twice the working precision. Finally, we provide numerical experiments to verify the accuracy of the compensated Horner's method and the corresponding root solvers.

11:50–12:05

**No Talk**

**Plaza Courts 1****2:30P.M. – 4:25P.M.**

2:30–2:45

**Sudoku Pair Latin Squares**

David Diaz

*Southern Connecticut State University*

Named for a paper by Euler, Latin Squares are an arrangement of  $n$  symbols in an  $n \times n$  array so that each symbol appears once in each column and row. Sudoku Puzzles are Latin Squares of order 9 with  $3 \times 3$  Latin subregions. Can Sudoku-like Latin Squares be constructed when  $n$  is neither prime nor a prime power? We define Sudoku Pair Latin Squares [(a,b)-SPLS] of order  $n = a \times b$  as Latin Squares tiled naturally with  $a \times b$  and  $b \times a$  Latin subregions. It is conjectured that there exist (a,b)-SPLS for all natural numbers a,b. We will present a new construction for the infinite class of  $(3,3k+1)$ -SPLS.

2:50–3:05

**Rainbows, Integers, and Binary, oh my!**

Hunter Rehm

*University of Wisconsin-La Crosse*

Problems in anti-Ramsey theory typically ask questions of the form: If a system is structured and we partition the system into smaller pieces can we guarantee that the smaller pieces break the structure? The open question that we focused on followed the same format. We wanted to find out exactly how much structure we could give a system before we could guarantee that a smaller piece of our system must break the structure. The systems we are concerned with are subsets of the integers and our structure breaking pieces are called rainbow sums. To our surprise, the binary representation played a pivotal role while answering our question. We are now investigating the question for cyclic groups and more!

3:10–3:25

**A Few Ripe Red-Blue Cherries**

Scott Herman

*Choate Rosemary Hall*

This talk shows how to play and evaluate the partizan game Red-Blue Cherries. You will learn how to use the simplicity theorem together with theorems on paths and trees to answer the unsolved problems about this game in Games of No Chance 4. You will see examples of Red-Blue Cherries games that evaluate to star, switch, all half integers,  $\frac{1}{4}$ , and  $\frac{3}{4}$ .

3:30–3:45

**The Combinatorics of RNA**

Katrina Teunis

*Grand Valley State University*

RNA, much like DNA, is made up of four building blocks called nucleotides, Adenine, Guanine, Cytosine, and Uracil. These nucleotides form a string that likes to fold in on itself and bond together using what are called bonding pairs, Adenine with Uracil and Guanine with Cytosine. The order and number of nucleotides present will determine how many times the strand of RNA can fold. Using these guidelines, we moved into the theoretical and consider what happens when we have  $n$  bonding pairs. For each  $n$  and  $m$ , we sought to find the smallest number  $k$  such that there does not exist a string with  $n$  bonding pairs of length  $2m$ , that folds in exactly  $k$  ways.

3:50–4:05

**On the Subsets of Spreads**

William Stowe

*Augustana College*

A spread is a discrete set of points such that no three are in a straight line. In this talk, we will define a function that generates the centers of subsets of spreads. We will see a pattern involving Pascals Triangle that emerges when counting necessarily similar subsets in the domain and range of this function.

4:10–4:25

**No Talk**

**Plaza Courts 2****2:30A.M. – 4:25P.M.**

2:30–2:45

**Modeling Public Opinion**

Arden Baxter

*Rollins College*

The population dynamics of public opinion have many similarities to those of epidemics. For example, models of epidemics and public opinion share characteristics like contact rates, incubation times, and recruitment rates. Generally, epidemic dynamics have been presented through epidemiological models. In this paper we adapt an epidemiological model to demonstrate the population dynamics of public opinion given two opposing viewpoints. We find equilibrium solutions for various cases of the system and examine the local stability. Overall, our system provides sociological insight on the spread and transition of a public opinion.

2:50–3:05

**Modeling Dengue Outbreaks**

Jisun Otterson

*Dixie State University*

Dengue is the most important vector-borne viral disease of humans caused by bites from mosquitoes carrying the virus. The cycle of dengue transmission can be broken by using the bacterium Wolbachia, which reduces the level of dengue virus in the mosquito and shortens the host mosquito's lifespan. Infecting mosquitoes with Wolbachia and releasing them can spread this bacterium to the local mosquito population. We developed a mathematical model using a system of nonlinear first-order differential equations to investigate how using Wolbachia could reduce the spread of the Dengue virus in human populations. By incorporating local temperature data and using MATLAB computational software to simulate the spread of Dengue fever in different regions, we determined three results: 1) the range of temperature of the region that Wolbachia-carrying mosquitoes need to be released in human populations, 2) the average number of releases in a year, 3) the number of the Wolbachia-carrying mosquitoes that need to be released each time. Our numerical simulations suggest that the Dengue fever outbreak can be diminished by releasing a small number of Wolbachia-carrying mosquitoes at the right time.

3:10–3:25

**Vanishing Viscosity Limits for a generalized MHD equation**

Sierra Brown

*Creighton University*

In general, fluid motion is governed by partial differential equations. For example, viscous fluids subject to magnetic fields are governed by the Magneto-hydrodynamic (MHD) equation. The equation has parameters governing dispersion terms: one governing viscosity and one governing magnetic diffusion. Setting those parameters equal to zero gives rise to a diffusion-free version of the equations, and a vanishing viscosity problem determines if the solutions to the original equations will converge to a solution of the diffusion-free version of the equation as the parameters are sent to zero. In this project, we will be studying this problem on a generalized version of the MHD equation called the MHD-alpha equation. To accomplish this, we will adapt analogous results for the Navier-Stokes equation (which governs non-magnetic viscous fluids) for flow in a pipe.

3:30–3:45

**Adaptation of Conventional Integrating Factor Method  
in Bernoulli's and Euler's Differential Equations**

Huy "Simon" Vuong

*Pasadena City College*

With a few improvised modification, the idea of using Integrating Factor could work well with more complex forms of Ordinary Differential Equations (ODEs) such as Bernoulli's and Euler's DEs. Using this new technique, the written solutions to more kinds of ODEs are shorter yet more intuitive and easier to be reviewed, which the author believes to be beneficial for Mathematics Education in the near future.

3:50–4:05

**Signatures of Chaos in the Brillouin Zone of a Soft Sinai Lattice**

Ariel Barr

*University of Texas at Austin*

When the classical dynamics of a particle in a finite 2-D billiard undergoes a transition to chaos, the quantum dynamics of the particle also shows manifestations of chaos in the form of scarring of wave functions and changes in energy level spacing distributions. If we tile an infinite plane with such billiards, we find that the Bloch states on the lattice undergo avoided crossings, energy level spacing statistics change from Poisson-like to Wigner-like, and energy sheets of the Brillouin zone begin to mix as the classical dynamics of the billiard changes from regular to chaotic behavior. We study the effect of broken spatial and dynamical symmetries on the band structure of two lattices with unit cells that are soft versions of the classic Sinai billiard. Broken dynamical symmetries and the presence of chaos can diminish the feasibility of changing and controlling band structure in a wide variety of 2-D lattice-based devices, including 2-D solids, optical lattices, and photonic crystals. This work was made possible by generous support from the Welch Foundation. These results were published in the journals *Physical Review E* and *Chaos*.

4:10–4:25

**Come Heat and High Water**

Dylan Keaweehu

*University of Hawaii West Oahu*

What is the effect of rising greenhouse gas to the sea level, to the sea temperature, and ultimately, to Hawaii? We study the Budyko energy balance model of the zonally averaged annual temperature. Adapting the model to incorporate observations allowing for extrapolation into future climate. Simulations produced by open source computer programs will be presented.

## Plaza Courts 3

2:30A.M. – 4:25P.M.

2:30–2:45

**Predictive Modeling and Analysis of Golf Using the Massey Method**

Kevin Gannon

*Lewis University*

The use of predictive modeling in the analysis of sports data is an exciting, but challenging task. Golf is particularly difficult to predict because it incorporates individual scores into a total team score. There are many mathematically inspired sports ranking systems, but the Massey Method is among the most elegant and simple. This method involves setting up and solving a system of equations using least squares. We can possibly improve this method by incorporating weights into the system. In this talk, we will discuss the results of a summer undergraduate research project which tested the predictive power of using a weighted Massey Method to predict golf results from the NCAA Division II Great Lakes Valley Conference. In particular, we incorporated and tested different features and weights to our model in order to build off of the results from past research teams.

2:50–3:05

**Will Facebook be #1 Forever? A Competition Analysis**

Alyssa O'Brien

*Benedictine University*

In the early 2000's, online social networking was dominated by two sites: Facebook and MySpace. At that time, MySpace was the #1 social network in terms of site visits and users, and Facebook was an up-and-comer. However, by 2007, their roles reversed, and Facebook not only took the #1 spot, which it has not relinquished since, but MySpace began a rapid decline in usage, eventually becoming obsolete in 2012 (though people argue it was obsolete well before then). But, there are many more online social networking sites now, such as Twitter, Snapchat, LinkedIn, and Google+, all of which are in direct competition with Facebook for users. So, the big question for fans of Facebook is: Can another network come and knock Facebook off of its #1 spot, just as they did with MySpace? Or, is Facebook here to stay? To answer this question, we will present a variation of a standard competition model that allows users to potentially be on multiple networks at the same time. The model analysis will incorporate both theoretical and computational (using MATLAB) work to see what needs to happen in order for Facebook to become obsolete, or if that's even possible!

3:10–3:25

**Probabilistic Counting-Out Game on a Line**

Michelle Shu and Tingting Ou

*The Johns Hopkins University*

Our research focuses on a novel problem posted on a question-and-answer website. There are  $n$  people in a line at positions  $1, 2, \dots, n$ . For each round, we randomly select a person at position  $k$ , where  $k$  is odd, to leave the line, and shift the people at position  $i$  such that  $i > k$  to position  $i - 1$ . We continue to select people until there is only one person left, who then becomes the winner. We are interested in two questions: which initial position has the greatest chance to win and which has the longest expected time to stay in the line. We have derived recursions to solve for exact values of the winning probabilities and expected time, the exact formula for the winning probabilities, and the asymptotic behaviors of the expected survival time. We have also considered a variation of the problem, where people are grouped into triples, quadruples, etc., and the first person in each group is at the risk of being selected. We will also present various sequences we have discovered while solving the variations, including the hexanacci numbers, as well as other possible extensions and related findings concerning this problem.

3:30–3:45

**The Mathematics of Poker: Extending the Nash-Shapley Model**

Kevin Grosman and Ajay Dugar

*University of Illinois at Urbana-Champaign*

In 1950, John Nash and Lloyd Shapley proposed a three-player poker model and derived optimal probabilistic strategies for each of the players. Their model remains one of the only mathematical poker models involving more than two players. The Nash-Shapley model assumes there are only two kinds of cards, high and low. At the beginning of the game, each player is dealt a card, chosen at random from the two kinds. The game then proceeds for up to five rounds of betting or passing actions. Nash and Shapley derived optimal betting probabilities for each player and each round of this game. We implemented the Nash-Shapley model in Mathematica in order to have the ability to extend the model in new directions. In particular, we explored different player profiles (Random, Optimal, Semi-Optimal, Naive, Tight/Loose, Contrarian) in an effort to uncover the effects of different strategies on player profits. We did so by running simulations, as well as working out the relevant probabilities and expected values theoretically.

3:50–4:05

**Conservative Strategy Analysis in Poker**

Uttam Bhetuwal

*Troy University*

In a no limits Texas Hold'em poker tournament, players pay to play in the form of blinds and antes. This means that if a player were to play conservatively by folding and only checking when necessary, they would eventually run out of money. However, players may decide to utilize this strategy near the end of day 1 of a tournament if they know they will have decent amount left in their stack at the end. We will examine the effectiveness of this strategy and formulate a function which will determine how many chips we will have left, if beginning with "x" many chips, and playing "n" hours in a No Limit Hold'em tournament.

4:10–4:25

**No Talk**



## Plaza Courts 4

2:30A.M. – 4:25P.M.

2:30–2:45

**Mathematical modeling and experimental calibration of a simple natural dynamic walking**

Ryan Pakledinaz and Bhavika Gummadi

*Lawrence Technological University and Northville High School*

Our research seeks to model and analyze the mechanisms of the human gait in both the stable state and under unstable conditions. At the current stage, we are focusing on a dynamical system in 2-dimensional space simulating human legs as constrained movable double pendulum. We test the model with human gait data collected at the Biomedical Engineering laboratory with wearable sensors. In the future we intend to expand the model into in three dimensional space via mathematical calculation and affirmation using the lab data.

2:50–3:05

**Gauze and Effect Within Wound Healing**

Lindsey Chludzinski and Olivia Hall

*Youngstown State University and Youngstown State University*

Wound progression through the stages of healing is an important focus for many medical professionals and mathematicians. Chronic wounds, such as diabetic wounds, fail to properly advance through the healing process, which is detrimental to a patients health. To address this problem, professionals must understand the differences between an acute and a chronic wound. Understanding how a wound heals under different circumstances enables us to develop and improve treatment options. Traditionally, research of the phases of wound healing must be performed in the lab with live cultures. Such studies are both costly and time consuming. The purpose of our study is to develop a visual representation and a mathematical model of the movement of macrophages and similar cells in a wound throughout the different stages of healing based on experimental measurements. Using MATLAB, we are developing an adaptable computer simulation that predicts the movement of specific cells prior to performing long term studies by modeling the wounds. Ultimately, this model will permit researchers to maximize the impact of their time in the lab and hasten the process of discovery in chronic wound treatment.

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3:10–3:25

**Modeling African Trypanosomiasis: Performing bifurcation analysis and finding  $R_0$  for gHAT**

Noah Brady

*Creighton University*

How can we reduce the transmission of a lethal parasite between humans and Tsetse flies? We use a combined SIS-SEI model to answer that question, calculating  $R_0$  and performing a sensitivity analysis, to determine small changes that make a big difference in the progression of an epidemic. The goal of this study is to advise policy change that would reduce death and unnecessary suffering caused by African sleeping sickness in the Democratic Republic of Congo.

3:30–3:45

**Modeling the Treatment of HIV in Children**

Kelley France

*University of Central Oklahoma*

We build a differential equations model to study how the length of HIV treatment in young children affects the treatment success. The recent news of remission in some children who had been treated briefly for the virus early in life, shows that understanding the dynamics of the treatment at this stage of life is crucial to uncovering the potential for remission. Taking into account the uninfected T cell population, drug sensitive

and drug resistant infected productive T cell populations, drug sensitive and drug resistant infected T cells in the eclipse phase, and drug sensitive and drug resistant free virus, we build a differential equations model of HIV treatment reflective of treatment in young children. We present our model and the results, identifying conditions under which the child would enter remission. We conclude by discussing how this information can be used to minimize the risks to other children/patients.

3:50–4:05

**The Dynamics of an Epidemiological Model for HPV  
with Partial Vaccination in a Heterogeneous Population**

Stefano Chiaradonna

*Benedictine University*

The Human papillomavirus (HPV) is one of the most prevalent sexually transmitted diseases in the United States. HPV-16 and HPV-18 are the primary agents of cervical cancer, and HPV-6 and HPV-11 are responsible for most genital warts and juvenile-onset recurrent respiratory papillomatosis. Highly efficacious vaccines have been developed to prevent these high-risk types of HPV, which are typically administered in three doses. However, younger adolescents need only two-doses of the full three-dose vaccine regimen [33]. We propose and analyze a mathematical model that investigates the implications of the population not completing the vaccine regimen as well as the scenario of younger adolescents receiving two-dosages. By having differing age groups, the model can target a specific age group for vaccination to optimize the control of HPV spread, which could lead to the eradication of the disease.

4:10–4:25

**No Talk**

Plaza Courts 5

9:10A.M. – 10:55A.M.

9:00–9:15

**No Talk**

9:20–9:35

**No Talk**

9:40–9:55

**Easier Said than Done: The Collatz Conjecture**

Daniel Welchons

*Concordia University Irvine*

The development of the mathematical discipline has long rested on mathematicians finding solutions to previously unsolved problems. Difficult problems provide mathematics with goals toward which to work and milestones for measuring success. Some of these problems are, naturally, quite complex, but others require little or no specialized knowledge to understand. The Collatz Conjecture is one such problem in number theory. Starting with a simple function and statement, it appears to lack nuance, yet it continues to resist all mathematical attempts to prove or disprove it. This paper examines the utility of unsolved problems such as the Collatz Conjecture and then moves to a more direct investigation of the mechanics of the problem, while presenting some original results regarding patterns which arise while exploring Collatz and its connection to binary bit strings. Finally, it investigates the specific benefits of studying the Collatz Conjecture.

10:00–10:15

**An Investigation of Recurring Decimals**

Sarah Dickey

*Elmhurst College*

Each rational number has a decimal representation which eventually becomes periodic. If the representation has the periodic portion non-zero, then the representation is referred to as a recurring (or repeating) decimal. For example,  $1/7$  can be written  $0.\overline{142857}_{10}$  where the bar over the numbers denotes the repeating periodic digits and the subscript 10 denotes the base. A fraction that has a zero-periodic portion in base ten frequently becomes a recurring decimal when represented in another base. For example,  $5.1\overline{0}_{10}$  is represented as  $101.000\overline{11}_2$  in base 2. By manipulating and understanding the properties of recurring decimals, patterns emerged that make it easier to classify and comprehend their behavior. A goal of this project was to investigate the cyclic properties of these repeating decimals, estimate the length of the repetition, determine prime factorization of recurring decimals and its patterns, and consider creating a terminating decimal in a different base.

10:20–10:35

**Higher Order Primes and Functions**

Andrew Giddings

*University of Wisconsin-Platteville*

The Sieve of Eratosthenes is a method of extracting the set of prime numbers from the set positive integers. It is well known that the prime numbers are intimately connected to the Riemann zeta function, and that many interesting results follow from the study of the Riemann zeta function. It is lesser known that applying the Sieve of Eratosthenes to the index of the prime numbers will result in the so-called set of prime-indexed primes 3, 5, 11, 17, 31, . . . More generally, the application of the Sieve of Eratosthenes  $n$ -times will yield the set of  $n$ th order prime-indexed primes. In this talk, we discuss the corresponding Beurling zeta function and

some of our recent results on a few arithmetic functions attached to these zeta functions.

10:40–10:55

**The Prime Number Theorem: A Historical Look at How Mathematicians Proved It**

Anthony Dickson

*Youngstown State University*

Mathematics is interesting not only for the results that come forth from it, but also the ways in which said results are found. The process of proving a difficult result which may take years, decades, centuries even requires more than just plain ingenuity and intelligence, but also cooperation among fellow mathematicians and a study of past scholars works. We will take a historical look at the Prime Number Theorem (which states how one can predict the distribution of primes) and see how mathematicians worked on it for over a century, utilizing various techniques, to finally state the conjecture and, eventually, prove it. This will be a historical presentation with select proofs from various people sprinkled throughout to give an overall impression of how the theorem was finally proven.

**Plaza Courts 6****9:10A.M. – 10:55A.M.**

9:00–9:15

**No Talk**

9:20–9:35

**Predictive Modeling and Analysis of Golf Using the Massey Method and Artificial Intelligence Part 1**

Brandon Joutras

*Lewis University*

Ranking sports teams and predicting future results from past games can be challenging. Golf is particularly difficult to predict because it incorporates individual scores into a total team score. One linear algebra based system which can be used to help predict sport outcomes is the Massey Method. This method is relatively simple and basically involves setting up and solving a system of equations using least squares. We can possibly improve this method by incorporating weights into the system through intelligent models. In this first part of a two-part talk, we will discuss the results of the predictive power of our weighted Massey Method and the challenges of choosing our intelligent models.

9:40–9:55

**Predictive Modeling and Analysis of Golf Using the Massey Method and Artificial Intelligence Part 2**

Adrian Siwy

*Lewis University*

Ranking sports teams and predicting future results from past games can be challenging. Golf is particularly difficult to predict because it incorporates individual scores into a total team score. One linear algebra based system which can be used to help predict sport outcomes is the Massey Method. This method is relatively simple and basically involves setting up and solving a system of equations using least squares. We can possibly improve this method by incorporating weights into the system through intelligent models. This second part of a two-part talk, discusses the implementation and results of a project which used an intelligence system, NEAT (NeuroEvolution of Augmenting Topologies), to improve the predictive power of the Massey Method.

10:00–10:15

**Predicting and Comparing Medal Counts in the Summer and Winter Olympics**

Emmy Yarmy

*Lynchburg College*

This presentation uses variables unrelated to athletic ability to predict medal counts in the Summer and Winter Olympics. Furthermore, the two seasons of games are compared to see if certain factors are more important when winning a medal in one season than another. The variables used are GDP per capita in US dollars, a dummy variable measuring whether or not the country has a socialist history, and the latitude of the countries capital city. The response variable is medals. Since there are so many countries with zero medals during the games, a normal regression was not possible to run. After testing other models, we determined that a zero-inflated negative binomial regression would be the best model to use. This output was found using R. I found that wealthier countries tend to compete in the winter games than the summer, but with both groups more GDP increased the number of medals on average holding other variables constant. It is also noteworthy that countries located further north tend to do better in both games and that countries with a socialist history tend to not be in the zero-groups.

10:20–10:35

**A Statistical Approach to the Effect of Suspensions in the NFL**

Preston Biro  
*Texas A&M University*

The number of NFL players suspended has increased dramatically in the last decade. While teams would certainly prefer their players to be available for every game, it is unclear what level of effect a suspended player has on the rest of the team. Does the suspension cause a distraction that weighs down the rest of the team, or can teams effectively block out these issues? Do players tend to return from a suspension with a vigor to prove their worth on the field, or do they return from their leave of absence unprepared? From the management level, is it advantageous to cut players who are going to miss multiple games a year, or could the benefits of their abilities outweigh the games missed? Do some teams do better harboring players with suspect track records, or does the same fate effect all teams? This talk will take a statistical approach at examining the true effects of teams employing players with suspension problems.

10:40–10:55

**Analysis of AP College Football Polls, I**

Qingqing Yang  
*University of Illinois at Urbana-Champaign*

We analyzed individual ballots of voters in AP College Football Polls using several well-known ranking metrics and statistical methods. Our data set consisted of around 4000 individual ballots spanning four seasons of college football. We report on the results of this analysis.

Plaza Courts 7

9:10A.M. – 10:55A.M.

9:00–9:15

**No Talk**

9:20–9:35

**Exploring Finite-Time Blow-up of Separable Differential Equations**

Jacob Hines

*Hendrix College*

As has been previously proven (Jared Williams, Hendrix '03), one can determine whether an autonomous differential equation will blow up in finite time (i.e. have a vertical asymptote) without actually solving the equation. However, the autonomous case is extremely narrow and only covers relatively few cases. Using this result along with a straightforward proposition, I have extended Williams' result into the separable nonautonomous case, which requires an additional condition in some cases. We also apply our criterion to quantify some properties of certain differential equations. Specifically, we are able to describe a condition under which a given differential equation will have a separatrix. Then, constructing such a differential equation, we can apply our results in order to compute an equation for the separatrix.

9:40–9:55

**Bessel Equation and Calculus of Variations**

Niyousha Davachi

*University of Texas at Arlington*

Bessel's equation arises when finding separable solutions to Laplace's equation and the Helmholtz equation in cylindrical or spherical coordinates. Wave propagation, heat conduction in a cylindrical object and modes of vibration of a drum are some of its few applications to count. The Lagrange formalism is established for this ODE, whose solutions are given by the Bessel Equation itself, by using standard and non-standard Lagrangians. This new class of non-standard Lagrangian requires auxiliary conditions, which is a novel phenomenon in the calculus of variations.

10:00–10:15

**The Hare-Lynx Paradox**

Lydia Pane

*Gonzaga University*

The Hudson Bay Company's pelt-trading data has been used as a proxy for population densities of the Showshoe hare and Canadian lynx in the Boreal forest. Both populations exhibit cyclic behavior; however, peak population density of the lynx precedes that of the hare leading to the paradoxical conclusion that hares eat lynx. We present a novel mathematical model with four trophic levels to explain the paradox.

10:20–10:35

**Understanding and Quantifying the Importance of Habitats for Migratory Species**

Summer Stafford

*University of Redlands*

Understanding the influence of specific habitats on the survival of a migratory species is an essential part of making successful management decisions. Migration is a complicated process, and mathematical models offer a way to understand the importance of different parts of the migratory annual cycle. The National Institute for Mathematical and Biological Symbiosis Habitat for Migratory Species working group has worked to develop generalized approaches to this problem. First, a generalized network model allows for perturbation analysis in order to understand habitat value in the network, which is typical in population ecology, but also

very complicated when considering new species. Thus, we use two other time dependent graph approaches to approach the problem without the need for computationally intense simulations: per capita contribution and criticality. to approach the problem without the need for computationally intense simulations. While per capita contribution measures the expected contribution of an individual to the network after one annual cycle, criticality ranks nodes in the network on how relatively critical they are to the overall population growth. These frameworks are intended to be general, not species specific, so that they can be applied in a standard way to a wide range of species. Our work is focused on testing the utility of the metrics, and on developing consistent, mathematically supported approaches to calculating each metric.

10:40–10:55

**The Effect of Prey Dispersal on a Two-Patch Predator-Prey System**

Victoria Kelley

*James Madison University*

We consider the effect of prey dispersal in a two-patch predator-prey model in which the two patches are qualitatively different. In particular, we assume patch two has a significantly smaller carrying capacity and a correspondingly higher predation rate. Scaling the model under these assumptions introduces a parameter of arbitrarily small order, allowing for an asymptotic analysis. We show that the predator and prey will always coexist for biologically reasonable parameter values. Furthermore, we prove the existence and uniqueness of a coexistence equilibrium and determine the stability regions in the parameter space. Using numerical simulations, we illustrate the varying effects of prey dispersal on the stability of the coexistence equilibrium and find parameter values for which a Hopf bifurcation occurs.



## Plaza Courts 5

2:30P.M. – 4:25P.M.

2:30–2:45

**The Probability of Generating a Semidirect Product with two Generators**

Jonathan Coffin

*Saint John's University*

When looking at cyclic groups, the probability of picking one random element which generates the group can be determined rather quickly by applying Eulers phi function. The question then arises, what about non-cyclic groups? In the past other researchers have found formulas for this probability for abelian groups with two and three generators. We will look at the probability of generating a particular type of non-abelian group: Semidirect products of two cyclic groups.

2:50–3:05

**Examples of Graphs that Admit No Normal Nonabelian Sylow  $p$ -Subgroup**

Mark Nichols

*St. Norbert College*

When studying a given graph, we frequently wish to determine whether or not the graph occurs as the prime character degree graph, denoted  $\Delta(G)$ , for some finite solvable group  $G$ . En route to showing a graph does not occur as  $\Delta(G)$ , one known strategy is to show there are no normal nonabelian Sylow  $p$ -subgroups. One way to ensure this result is to show  $\Delta(G)$  satisfies specific technical conditions, of which the graphs presented in this talk exhibit.

3:10–3:25

**On the Analysis of Cycles in the Symmetric Group**

Michael English

*Clark Atlanta University*

In the realm of Abstract Algebra, the symmetric group, denoted by  $S_n$ , defined over a finite set on  $n$  symbols is the group whose elements are all permutation operations that can be performed on  $n$  distinct symbols. The generalized formula  $\frac{|S_n|}{\prod_{i=1}^n (i^{a_i}) * (a_i!)}$  determines the number of permutations in  $S_n$  with a given cycle structure. In the proposed research, we extract information from the generalized formula and produce a simplified formula  $\frac{n!}{(n-m)m!}$  to calculate the order of elements of the symmetric group that have a cycle configuration. Moreover we calculate successive cycle sizes. That is, cycles of size  $n-1, n-2, \dots, n-m+1$ , where  $m$  is the number of fixed points in a permutation configuration or the number of one element orbits in equivalent cyclic notation. With this simplification, calculating the order of cycles in  $S_n$  becomes efficient, and allows for the illumination of various fresh properties and new ways to describe old properties. For future work, understanding the behavior of cycles in the symmetric groups, in particular the group  $S_3$ , can help provide insight into a connection with the triangular periodic functions. The triangular periodic functions are developed by inscribing a triangle inside of a circle. Unpublished work by Lewis and Mickens give an explicit representation of the triangular periodic functions. This research is supported by NSF LSAMP Award 1305041 and NSF Award 1700408.

3:30–3:45

**Problem 21: An Exploration of Dial Rings**

Vladimir Sworski

*Cleveland State University*

Imagine a game with a series of dials arranged in a ring. When one of the dials is turned, so too are the adjacent dials. Some of the dials will start straight up, others on their sides. Mathematically, how can we solve for a specific position? What is the minimal set of moves? We answer questions like these and more in

an exploration involving linear algebra, a Multivariable Chinese Remainder Theorem, and module theory.

3:50–4:05

**Parameters of Locally Recoverable Codes with Multiple Recovery Sets**

Sam Kottler

*Colorado College*

A code is a set of vectors, called codewords. Usually we look at codes that actually form vector spaces. Codes can be used for redundancy and error correction, when storing or transferring data. One way to do this is with locally recoverable codes (LRCs) in which any position of a codeword can be recovered from a fixed subset of other positions, called a recovery set. An interesting problem is called the availability problem, which addresses constructing LRCs with multiple disjoint recovery sets for each position. This talk will discuss minimum distance and other parameters of families of such codes constructed from curves over finite fields.

4:10–4:25

**A New Approach to Optimal Solution of Rubiks Cubes with Half Turns**

Kellie Halladay

*SUNY Fredonia*

How many moves are required to solve a Rubiks cube in the quickest way possible? There is a method for finding the maximum number of moves to solve a Rubiks cube in the minimum number of steps. This number is known as Gods number. We will consider finding Gods number in the case where we only allow 180 degree turns on the cube. Is the size of the cube a factor in determining the optimal number of moves needed? Take a 2x2x2 Rubiks cube, for example. It seems that this cube would require less moves than a 3x3x3 to solve. How do these 2 cubes compare? Are there Rubiks puzzles that are larger than a 2x2x2 but smaller than a 3x3x3? Yes, and these intermediate steps in between the 2x2x2 and the 3x3x3 are vital to the analysis of the 3x3x3.

**Plaza Courts 6**

2:30–2:45

**2:30P.M. – 4:25P.M.****Why So Stressed? Examining the Relationship Between  
Inquiry-Based Learning and Freshman Mathematics Anxiety**

Abby Large

*Concordia University Irvine*

The objective of this research was twofold: to investigate the method of Inquiry-based Learning (IBL) as it is applied in a mathematics classroom and to evaluate the level of mathematics anxiety of female students relative to male students. Literature from previous similar research projects was referenced to aid in this study. Surveys were run in Core Math Classrooms at Concordia University Irvine to evaluate the anxiety levels of male and female students. These surveys were based on the Revised Mathematics Anxiety Rating Scale (RMARS), which was customized to be used specifically in Core Math classrooms. The updated RMARS scale used in this research was found to have a 95% reliability score when compared to the original RMARS. Four classrooms were evaluated, giving a database of slightly under a hundred students. The research showed that there is no significant difference in the anxiety levels of male and female students.

2:50–3:05

**MYMathApps Tutorials**

Matthew Weihing

*Texas A&M University*

For the past year, I have been working under Dr. Yasskin at Texas A&M University to develop online calculus tutorials that run in a browser to be incorporated into his online textbooks. Before my project, he had programs called Maplets which generate random practice problems for given calculus concepts to help students learn these concepts step by step. However, Maplets require the student to have Maple installed on his/her device and they require Java for display. Maple does not run natively in a browser and Java will not run on most tablets. So they can't easily be incorporated into Dr. Yasskin textbooks. In response, I have created new, browser based tutorials that teach the same calculus concepts as the Maplets and can be incorporated into any page of the textbooks. These are powered by the Sage computer algebra system. Sage can perform all of the necessary math functions like integrals and derivatives that Maple can do, except it's free and open source. It can't, however, make interactive graphics. For this, I use three.js which is a WebGL based JavaScript library which is far more powerful than Maple's graphics. These, in conjunction with MathLex, a parser; MathJax, to render LaTeX, and some of my own JavaScript code, I can replicate all of the Maplets into these new tutorials. We will demonstrate several of the tutorials.

3:10–3:25

**Does Mastery-based Testing Help with Test Anxiety:  
A Preliminary Analysis of the Impact of MBT on Student Anxiety Levels**

Lauren Klamerus

*Lewis University*

Many students suffer from math anxiety and test anxiety which can hamper their abilities in mathematics courses. In this first of a two part talk, we present the results of a year-long research project analyzing the impact of an alternate assessment method called, "Mastery-based Testing" (MBT). Using this assessment method, students are given problems in which they can only receive full credit for the problem after they demonstrate mastery of the concept being tested. Each test includes similar questions over the same concepts from previous tests which allows students who have not mastered an idea to retest and reevaluate old concepts. In order to help determine the effectiveness of Mastery-based Testing, we used statistical methods to analyze survey data collected from MBT and traditionally assessed classes. This talk focuses on the research regarding test anxiety levels throughout the semester and whether there was a difference between

the anxiety levels of MBT students and traditionally assessed students.

3:30–3:45

**Does Mastery-based Testing Encourage a Growth Mindset:**

**A Preliminary Analysis of the Impact of MBT on the Growth Mindset and Attitudes of Students**

Christy Carlson

*Lewis University*

In this second half of a two part talk, we present the results of a year-long research project analyzing the impact of an alternate assessment method called, Mastery-based Testing (MBT). Using this assessment method, students are given problems in which they can only receive full credit for the problem after they demonstrate mastery of the concept being tested. Each test includes similar questions over the same concepts from previous tests which allows students who have not mastered an idea to retest and reevaluate old concepts. In order to help determine the effectiveness of Mastery-based Testing, we used statistical methods to analyze survey data collected from MBT and traditionally assessed classes. This talk focuses on the research regarding student attitudes and perceptions about mathematics and growth mindset. In particular, we explored whether there was a difference between the growth-mindset of MBT students and traditionally assessed students and whether there was a change in attitudes throughout the semester.

3:50–4:05

**On Developing an Early Warning System: A Follow-Up Longitudinal, Cohort Study  
of College Student Development and Learning at a Four-Year Public University (N=8,500)**

Brian Darrow, Jr.

*Southern Connecticut State University*

Research on students learning and development has contributed to the creation of a comprehensive psychoeducational survey instrument, which is administered to all new full-time, first-year students at a four-year public university. The survey measures students learning and development, as well as such competencies as future orientation, academic habits of mind, and self-regulation. Statistical analyses were conducted on longitudinal, cohort data collected from over 8,000 students over the course of ten years including students responses to precollege and college surveys, collegiate credits/grades, and retention/graduation statistics to identify patterns and anomalies of students persistence, retention, and graduation from the university. Preliminary results of the study indicate that institutions should focus on the amenable characteristics of a students educational profile rather than their unchangeable past academic experiences and demographic information. The results of the completed study have the potential to influence policy, inform interventions for vulnerable student populations, and contribute to holistic understanding of student success in college.

4:10–4:25

**Turn Down the Lights: Basis Reduction Techniques in Hyperspectral Diffuse Optical Tomography**

Riley Supple

*Goucher College*

Diffuse optical tomography (DOT) is a process in medical imaging which uses visible to near-infrared light to get two-dimensional images of soft tissue in order to reconstruct an image. DOT revolutionizes medical imaging in that it is non-ionizing, is relatively low cost, and produces high contrast images, however it cannot penetrate tissue deeply and the images are of low resolution. Hyperspectral diffuse optical tomography (hyDOT) is an advancement in DOT technology which utilizes multiple wavelengths, in comparison to DOT which only utilizes one. Using this new approach, hyDOT presents more information resulting in a more accurate and informative image. In order to make hyDOT as efficient as possible, we will present a basis reduction method for determining which wavelengths are the most useful and which are providing information that is not necessary to reconstruct the image. Here we will discuss the proper orthogonal decomposition

method (POD) which uses vector analysis to reduce the dimensionality of the output space.

**Plaza Courts 7****2:30P.M. – 4:25P.M.**

2:30–2:45

**A Statistical Approach to Analyze Temporal Changes in the Spatial Distribution of Cells**

Nicholas McDonald

*Clarkson University*

Cells are highly dynamic structures that continuously interact with neighboring cells in multicellular organisms. Cell culture models are often used to understand some of the basic components of such interaction. Using a human cervical cancer cell line, we investigated how the cell distribution evolves on a petri dish, focusing primarily on the first day after plating. Time-lapse images were captured on a sample space of 3530x2647 square micron and statistical analysis was performed on the spatial location of the cells in a few frames spanned over 27 hours. We have used several methods to accurately assess the change in cellular spatial distribution, which include comparison of the standard distances, tests for complete spatial randomness using quadrat-counting and Ripley's K Function, and application of average nearest neighbor technique to compute a scalar value that can be compared among frames. We find that cell distribution gets more randomly distributed over the first 12 hours after which they tend to cluster. Our results suggest that cell movements are random at the early stage of culture but influenced by cellular interaction at later time-points, leading to cell clustering.

2:50–3:05

**Modeling African Trypanosomiasis: Performing bifurcation analysis and finding  $R_0$  for gHAT**

Noah Brady

*Creighton University*

How can we reduce the transmission of a lethal parasite between humans and Tsetse flies? We use a combined SIS-SEI model to answer that question, calculating  $R_0$  and performing a sensitivity analysis, to determine small changes that make a big difference in the progression of an epidemic. The goal of this study is to advise policy change that would reduce death and unnecessary suffering caused by African sleeping sickness in the Democratic Republic of Congo.

3:10–3:25

**Stochastic vs. SIR Models: A comparison using influenza spread data**

Jarrin Peters

*Salisbury State University*

The use of models can provide valuable insight to naturally occurring phenomena, which in turn grants us powerful advantages when it comes to combatting many of the issues we face as humans. In particular, various mathematical models are used to understand the spread of infectious diseases. In this segment, I compare and contrast the application of two such models to influenza spread data from the British Medical Journal (March 4, 1978, p. 587), namely, an SIR (Susceptible, Infected, Recovered) model and a stochastic model. SIR models are based on differential equations while stochastic models are probabilistic in nature. I will describe each model and discuss their differences, advantages, and disadvantages when used to model this specific outbreak of influenza.

3:30–3:45

**Columnaris Disease and the Population Dynamics of Infected Fish**

Allison Gerk

*St. Norbert College*

*Flavobacterium columnare* is a bacterial pathogen that forms biofilms on the surface of freshwater fish. As the biofilm grows, the resulting infection causes a fatal disease to fish known as columnaris. *F. columnare* grows not only on the surface of healthy fish but also on deceased fish. During this biofilm growth, *F.*

*columnare* sheds from the host fish and enters the aquatic ecosystem where it resides, remaining viable, until it infects another fish. Consequently, shedding is a threat to the healthy fish population. In this talk, we present a model for the population dynamics of freshwater fish after *F. columnare* is introduced into the healthy population. We explore the interconnected relationship between healthy fish, infected fish, and deceased fish as a result of the shedding of *F. columnare* due to biofilm growth.

3:50–4:05

**New Songs in the Deep: A Passive Acoustic Analysis  
of the Temporal and Spatial Distribution of Omuras Whales (*Balaenoptera omurai*)**

Bridget Mueller-Brennan

*University of Illinois at Urbana-Champaign*

We consider the acoustic behavior of the Omuras whale, *Balaenoptera omurai*, a species first described in 2003. The first live sighting and in-situ study of a population of these whales began off the coast of Madagascar in 2013. *B. omurai* song is a stereotyped broadband pattern with an average duration of 9 sec and repetition rate of 3 min. This consistency allows the use of passive acoustic monitoring. The goal of this study is to use acoustic data to determine the temporal and spatial distribution of *B. omurai* in the Nosy Be region of Madagascar. Passive acoustic monitoring devices were placed at 4 sites based on previous acoustic and visual data to record from Dec 2015 to Nov 2016. These data were analyzed using a spectrographic display and an automated detector was developed and run over the data from all 4 sites. The detectors performance was verified manually. Our results suggest that *B. omurai* demonstrate distinct variation in habitat utilization but are present and singing in the area all year, indicating that they have no distinct seasonality and are non-migratory. This population of *B. omurai* appears to be resident to Nosy Be, so protecting this region is vital to the conservation of the only studied population of this newly discovered whale. Whales are an important part of the ecosystem, so understanding their ecology and protecting them is key to keeping our oceans healthy.

4:10–4:25

**Modeling the Rate of Recombination in Bacterial Species**

Kara Schatz

*Xavier University*

When bacteria reproduce, they typically create near-exact copies of their genome to pass to the next generation. There are two main sources of genetic variation: mutations during DNA replication and homologous recombination, which occurs when a bacterium uptakes foreign DNA and incorporates it into its own genome. Thus, recombination is one of the leading mechanisms of bacterial evolution and adaptation. However, recombination events are difficult to detect, so we do not have a full understanding of the extent of their presence or the extent of their impacts. Various methods have been developed to estimate the rate of recombination, but the conclusions have been inconsistent. Therefore, we developed a model to detect recombination events and estimate the relative impact of recombination in comparison to mutation on divergence in bacterial populations. Because homologous recombination often involves the transfer of key traits that improve bacterial fitness, understanding its prevalence gives us better insight into bacterial adaptation and persistence as well as their levels of clonality or sexuality.

## Plaza Courts 5

4:30P.M. – 6:25P.M.

4:30–4:45

**Patterns in Variations of the Fibonacci Sequence**

Daniel Gotshall

*Saint Peters University*

Zeckendorf explored the decomposition of integers into sums of nonconsecutive Fibonacci numbers. Thinking of each Fibonacci number in the sequence as being inside a “bin”, Zeckendorf’s rule for “legal” decompositions can be reworded as sums of numbers in bins separated by 1 or more bins. Zeckendorf never explored sequences that result when the bins are enlarged to include more than one number or when “legal” decompositions are redefined to require that summands used in the decomposition are separated by at least  $n$  bins, where  $n$  is the number of different bin sizes. We do both these things and call the resulting sequences bin sequences. In this talk we will describe how to generate bin sequences from a given list of bin sizes using multiple recurrence relations. Next, we will explain why it is necessary to use these to start the bin sequences before a single recurrence relation can be used, which is both similar to and more complex than the well-known Fibonacci recurrence relation ( $f_n = f_{n-1} + f_{n-2}$ ). Finally, we will show why two different lists of bin sizes produce similar sequences.

4:50–5:05

**On a Generalization of the Fibonacci Sequence**

Victoria Robinson

*University of Mississippi*

Fibonacci numbers have fascinated mathematicians for centuries, and they continue to charm us with their beauty and their habit of occurring in surprising and unrelated places. One defines the Fibonacci sequence by the recurrence relation:  $F_n = F_{n-1} + F_{n-2}$  for  $n = 3, 4, 5, \dots$  and the initial conditions  $F_1 = 1$  and  $F_2 = 1$ . So one can easily verify that the Fibonacci sequence is  $1, 1, 2, 3, 5, \dots$ . The goal of the present research is to study properties of a generalization of the Fibonacci sequence. Motivated by Marcia Edson and Omer Yanyenie’s research on a generalization of the Fibonacci sequence in 2009, a decision to study a generalization given by

$$f_n = f_{n-1} + (-1)^n f_{n-2}, \quad n = 2, 3, 4, \dots$$

and the initial conditions  $f_1 = 1, f_2 = 1$ . We proceed to study the properties of this generalization through computer software to compare the classical Fibonacci sequence and the generalization, form conjectures based on observations and previous findings in the classical Fibonacci sequence, and prove some relations using the Principle of Mathematical Induction to form theorems.

5:10–5:25

**A probabilistic analysis of sign pattern matrices**

Nicholas Bragman

*Hofstra University*

Sign pattern matrices are matrices where the only possible entries are  $+$ ,  $-$  and  $0$ . For an  $n \times n$  square matrix, assuming no element is exactly zero, we see that there are  $2^{n^2}$  possible sign patterns. We know that the absolute value of a determinant is preserved under row transposition, row negation, and transposition, and we use these properties to construct equivalence classes for sign pattern matrices of a given size. We say that two sign pattern matrices are equivalent if they can be obtained from one another solely through the three aforementioned operations. We focus on the probability of nonnegative determinant for each equivalence class, sign patterns that yield quick classification, and some characteristics of a few special case sign patterns.



5:30–5:45

**Perspective Drawing: How to Find the Immersion Point**

Robert Lehr

*Southwestern University*

We take a fresh perspective on an old idea and create an alternate way to answer the question: where should we stand in front of an image in two-point perspective to view it correctly? We review known geometric and algebraic techniques, then use the cross ratio to derive a simple algebraic formula and a technique that makes use of slopes on a perspective grid.

5:50–6:05

**Isoperimetric Problem Solution in 2 Dimensional Taxicab Metric**

Rory O'Dwyer

*Texas A&M*

Euclidean space demands that the shape which minimizes surface area for a given volume is a sphere, and that the shape of two fixed volumes that minimizes surface area is two spheres intersecting along a circle and separated from each-other along said border by 120 degrees. In continuation of this vein, the solution of the isoperimetric problem in the 2 dimensional taxicab metric is quite simple: its just a square. The problem becomes interesting, however, when two 2-dimensional bodies are involved, and the minimizing shape fundamentally changes depending on the relative sizes of the two bubbles.

6:10–6:25

**No Talk**

**Plaza Courts 6****4:30P.M. – 6:25P.M.**

4:30–4:45

**B-Splines vs. Bézier Curves in Computer Graphics**

Amanda Matson

*Kent State University*

A spline refers to a class of functions that are used in operations involving data interpolation and/or smoothing. Splines have several applications in curve fitting, 3D modeling, animation, and are widely used in industry. Two commonly used splines are the B-spline and Bézier. These splines have user-defined control points. The purpose of my study is to observe which spline is better for computer graphics by comparing these two spline functions mathematically and computationally. I analyze the smoothness of the functions and evaluate their order of continuity, and I show the B-splines most advantageous property is local control in which changing a control point only affects a limited part of the spline.

4:50–5:05

**Special Relativity and the Hyperbola**

Franquiz Caraballo Alba

*Plattsburgh State University*

Relativity is well-known for its counter-intuitive phenomena such as time dilation and length contraction. These phenomena occur when we shift the frame of reference from one observer to another. These shifts are called Lorentz transformations, and their usual formulae do not give a window into the reasoning behind them. However, intuitive ways of representing these transformations have been developed, if rarely discussed in Physics courses. In this talk, we will first discuss Minkowski spacetime, the four-dimensional vector space where events and observers exist according to the theory of special relativity. Then we will introduce the Lorentz group, the group of Lorentz transformations in this space and finally, we will simplify the elements of this group into a compact and more intuitive form using hyperbolic trigonometric functions.

5:10–5:25

**Mapping Sound Waves in Octave**

Caroline Howell

*Troy University*

The purpose of this research project is to map sound waves through a computer program, octave. In this presentation, we will explore the true nature of sound waves, the different paths that sound waves travel, and the importance of using octave in mapping sound waves. Using Octave, it makes calculating and graphing sound waves more efficient. The data for the experiment is collected by setting a speaker in a normal room and gathering information about the intensity and loudness of the sound waves from a detector set at certain distances from the speaker. Through this, we may calculate how different interference paths affect the sound received by the detector. The true nature of a sound wave is not linear, but logarithmic. Therefore, in Octave, we may graph this to see if this nature of sound waves is reflected through the experiment. The importance of using octave is to go further in mapping sound waves, rather than be limited by the real world circumstances.

5:30–5:45

**Improvements in Sliced Inverse Regression (SIR)**

Chris Kwan

*Georgia Tech*

Slice Inverse Regression (SIR) is a tool in which we reduce the dimension of a given input variable  $x$  without going through the process of model-fitting. There has been changes of the algorithm such as various ways of computing the principal component directions. We will explore ways to improve SIR since it does not work that well in monotonic functions. This improvement will help in improving SIR and how it performs

in monotonic functions.

5:50–6:05

**Modeling Dependence among Stock Prices Using Copulas**

Xiao Yuan

*Indiana University Purdue University*

A copula is a function that connect the joint distribution of random variables to their respective marginal distributions. Recently, using copulas to model the joint distribution of random variables has become more attractive due to their many appealing features. For instance, one great advantage is that using a copula permits the modeling of dependence structure of variables independent of their marginal distributions. Applications of copula modeling can be seen almost everywhere, including finance, risk management, engineering, and climate forecasting. In this study, the dependence among the performance of several stocks is investigated using copula models; specifically, elliptical copula models, including Gaussian copula and student t-copula. Time series methods are also used to prepare the data for analysis. After the study, we found that copula models have captured these dependence very well. By understanding these dependence among the stocks, the investors can take better measures to control the investment risks in the stock market.

6:10–6:25

**No Talk**

## Plaza Courts 7

4:30P.M. – 6:25P.M.

4:30–4:45

**Powered by Coffee: A Markov Chain Analysis of Campus Coffee Consumption**

Rebecca Cooper

*University of Mount Union*

In this project, I took a hands-on approach to studying the connections between mathematics and marketing. The goal of this research was to predict brand loyalty and future market shares of coffee, using the University of Mount Union's students, staff, and faculty as the target consumer demographic, and Alliance coffee shops as the suppliers. This presentation brings mathematics out of theory and into practical application. Markov Chains are mathematical modelling tools that determine the probability of achieving a future state based on our current situation. Through the use of Markov Chains, we are able to answer questions such as: Can small coffee shops around our campus compete with established international brands? How many coffee shops can the Mount Union community sustain? What will the market share of these shops be in 5 years? 10 years? 20?

4:50–5:05

**Optimizing Congressional Voting Districts using a Genetic Algorithm**

Kathleen Buch

*Xavier University*

We examine the existence of bias in the congressional redistricting process within Ohio by creating an overall nonpartisan score for each district. The score is the sum of a weighted function with the parameters, based on the Ohio Legislatures redistricting competition in 2009; compactness, population equality, fairness, and competitiveness. Although measuring compactness and population equality may be straightforward, defining fairness and competitiveness can become convoluted and subjective. Our fairness component is calculated using the efficiency gap method originally developed by Nicholas Stephanopoulos of the University of Chicago and Eric McGhee of the Public Policy Institute of California. We score competitiveness using an equation based on the percentage of votes won by a single political party. The weighted nature of our function is subject to change given the user and what parameters they wish to give more importance. We then utilize a genetic algorithm to optimize the score of districts. The algorithm provides a method by which to improve upon the score of each district by altering district lines and recalculating. This process allows for users to see how districts could be altered with a preference on certain district characteristics.

5:10–5:25

**Using Game Theory and Probability to Analyze International Conflict Deterrence**

Nicolas Beike

*Youngstown State University*

International conflicts seem to constantly be in the news today. You may ask yourself how these world leaders decide to respond during these conflicts. These leaders have many options available to them when a threat is detected. Most of the time they could ignore this threat or retaliate by increasing economic sanctions to even threatening invasion or even nuclear warfare. Some leaders could even decide to initiate the threat before the opponent can. In the talk we will discuss how using game theory and probability can help us analyze the decisions of these leaders and the results that occur from their decisions. We will be focusing the discussion on the deterrence of conflict and how this equilibrium can be achieved.

5:30–5:45

**A Not-So-Fair Guessing Game and the Math Behind It**

Samuel Delatore

*Youngstown State University*

This paper introduces a variation of a common game of chance, where two people attempt to guess a randomly chosen integer from 1 to  $N$ , with the added condition that either party automatically loses if their guess is above the correct number. Then, the optimal strategies for both a specific case and the general case are derived.

5:50–6:05

**No Talk**

6:10–6:25

**No Talk**

## Plaza Courts 5

9:30A.M. – 12:25P.M.

9:30–9:45

**The Pursuit of  $\rho^{\wedge}(\mathbb{Z}_p^r, m, h)$ : On Minimum Sumset Sizes**

Jason Heath

*Gettysburg College*

We are interested in finding the minimum size of an  $h$ -fold restricted sumset of an  $m$ -element subset of  $\mathbb{Z}_p^r$  for some prime number  $p$ , given by the notation  $\rho^{\wedge}(\mathbb{Z}_p^r, m, h)$ . Here, we present upper bounds for  $\rho^{\wedge}(\mathbb{Z}_p^r, m, h)$  for all  $h, m, p$ , and  $r$  and compare them to  $\rho^{\wedge}(\mathbb{Z}_{p^r}, m, h)$ . Further, we compare our bounds to the (already known) values of the minimum size of an  $h$ -fold *unrestricted* sumset of an  $m$ -element subset of  $\mathbb{Z}_p^r$ , denoted  $\rho(\mathbb{Z}_p^r, m, h)$ .

9:50–10:05

**Building low rank matroids**

Bao Van

*St. Norbert College*

We explore low rank matroids constructed by an iterative process of adding elements to circuits of other low rank matroids. Throughout this process we can ask, for example, how likely a resulting matroid is to be graphic, representable, or connected.

10:10–10:25

**Hamiltonicity of Unbalanced Tripartite Graphs**

Elizabeth Wrightsman

*Texas State University*

A graph with a path that travels through every vertex exactly once and returns to the starting vertex is a Hamiltonian graph. In 1952, Dirac proved that for a graph to be Hamiltonian, it is sufficient that every vertex neighbors more than half of the remaining vertices. This result paved the way for the study of hamiltonicity in graphs. Graphs whose vertices can be split into sets so that no edge joins two vertices in the same set are multipartite graphs. A tripartite graph is a multipartite graph in which the vertices are partitioned into three independent sets. A multipartite graph is balanced if all sets have an equal number of vertices, and otherwise it is unbalanced. Improvements of Dirac's result are known in balanced multipartite graphs, but there are no results known for unbalanced multipartite graphs. We focus our attention on unbalanced tripartite graphs. In this work, we extend and improve previously known results and present several sufficient conditions for determining hamiltonicity of unbalanced tripartite graphs. Hamiltonicity of tripartite graphs has many applications such as optimizing neural networks, enhancing municipal services, configuring data systems and computer network topology.

10:30–10:45

**Upper Chromatic Number of n-Dimensional Cubes**

Saurabh Bhandari

*Troy University*

I will talk about my research to find an algebraic function that calculates the upper-chromatic number of any given  $n$ -dimensional cube. I will also cover fundamental topics and definitions related to Hypergraph and Mixed-Hypergraph coloring.

10:50–11:05

**Harmonious Graphs**

Nischal Subedi

*Troy University*

Let  $G = (V, E)$  be a simple graph with vertex set  $V$  and edge set  $E$ , where  $|E| = m$ . A one-to-one function  $f : V \rightarrow \{0, 1, \dots, m-1\}$  is a harmonious labeling of  $G$  provided that if  $u, v$  are the endpoints of an edge, then the induced edge labels defined by  $f(u) + f(v) \pmod{m}$  are distinct. If there exists a harmonious labeling for  $G$ , then the graph  $G$  is said to be harmonious. When  $G$  is a tree, as  $|V| = |E| + 1$ , it is allowed for there to be exactly one repeated vertex label (but not a repeated edge label) for the tree to be harmonious. We will provide examples to demonstrate harmonious labelings for various classes of trees and other graphs.

11:10–11:25

### Quiver Hall-Littlewood Functions and Kostka-Shoji Polynomials

William Craig

*Virginia Tech*

For a quiver  $Q$  and the algebra  $\Lambda$  of symmetric functions, we define the quiver Hall-Littlewood symmetric function  $H_{\mu(\bullet)}^{i,a} \in \Lambda^Q$  as a generalization of the Hall-Littlewood polynomials. When written in the Schur basis, the coefficients  $K_{\lambda(\bullet), \mu(\bullet)}$  of each Schur polynomial  $s_{\lambda(\bullet)}$  form a natural generalization of the Kostka-Foulkes polynomials, which appear in the study of coordinate rings of closures of conjugacy classes of nilpotent matrices. Previous work by Shoji and by Finkelberg & Ionov treat the case where  $Q$  is a cyclic quiver, and the results have important applications to the geometry of affine flag varieties. In the presentation, an explicit formula for  $K_{\lambda(\bullet), \mu(\bullet)}$  when  $Q$  is a directed line on a two-vertex quiver is given, and conjectures about even more general cases are proposed.

11:30–11:45

### Fast Multiplication with the Schnhage-Strassen algorithm

Harrison Welch

*Austin Peay State University*

The ever-expanding cyberspace of data gives rise to the need for stronger more efficient algorithms to handle extremely large numbers. In this talk I will introduce the SchnhageStrassen (1971) algorithm for large number multiplication. Inside the algorithm, there are many different mathematical concepts including the Fast Fourier transform, Convolution theory, and Ring theory among other concepts and theories. The historical evolution provides great context from the classical long multiplication algorithm learned in grade school to the SchnhageStrassen algorithm to the modern, sometimes slightly faster Frer's algorithm.

11:50–12:05

### The Rational-Float Data Type

Katherine Mantych

*Elmhurst College*

Computer programmers have a variety of data types to select from when storing numeric values. When storing an integer, a programmer could use an int, a fixed-point data type. In other cases, the programmer may be working with non-integer rational numbers, in which case she would need to store the value as a floating-point data type. The fraction of a floating-point data type contains the sequence of bits to the right of the decimal point. For rational values, the fraction will contain a repeating sequence. Floating-point data types allocate a specified number of bits to represent a number. Intuitively, the more bits allocated, the more accurate the representation of the stored value, however rational numbers can reduce the bits needed to represent them by only storing one iteration of the repeating sequence. This research focused on the development of the rational-float, a new data type that only stores one iteration of repeating bits. The algorithms used to compute operations on the rational-float as well the method for memory allocation will be discussed.

12:10–12:25

### Bitcoin, Blockchain Technology and the Future of Commerce

Daniel Plummer  
*Howard University*

In 2007, Satoshi Nakamoto published a paper titled, Bitcoin: A Peer-to-Peer Electronic Cash System. The concept sought to create electronic cash to allow individuals to complete transactions directly by using digital signatures as proof-of-work that removes the need for a trusted third party to prevent double spending. We define an electronic coin as a chain of digital signatures. Each owner transfers the coin to the next by digitally signing a hash of the previous transaction and the public key of the next owner and adding these to the end of the coin. Satoshi Nakamoto 2007 Bitcoin and Blockchains are linked lists that record transactions to the general ledger through Hash functions. UBitShop seeks to act as a trusted third-party for processing secure, private retail transactions where shoppers choose both a secure public and private key and are routed to the Amazon.com website where items are listed in a major, established cryptocurrency, International merchandisers can sell their goods by using Ethereum smart contracts. Some of the problems that need to be resolved are: mobile phone hacking and viruses, establishing certified commodities warehouses, transaction speed. Specifically, security of cold storage wallets with commodities clearinghouses have proved vulnerable to theft of cryptocurrencies.



**Plaza Courts 6****9:30A.M. – 12:25P.M.**

9:30–9:45

**Image Recognition Using Polynomial Regression and Artificial Neural Networks**

Jacob Kirsch

*Saint John's University*

My presentation will summarize the basics of the mathematics behind an artificial neural network (one of the major pillars of artificial intelligence and machine learning), as well as look at a way to combine polynomial regression techniques with the techniques of artificial neural networks to estimate non-linear functions in a clear way. I will then show examples of how this regression analysis matches up against a common artificial neural network using a standard image recognition dataset.

9:50–10:05

**Applications of Sampling-based Motion Planning to Robotics**

Everett Yang

*Texas A&M University*

Consider the classical problem of moving a robot from point A to point B in a known environment. Though this problem seems simple, finding an exact solution is computationally intractable in nearly all cases. Many techniques have been developed for solving this problem, with sampling-based planning showing the most promise. That is, given the space of all possible robot configurations, we randomly sample and connect configurations to create a graph that contains representative feasible trajectories. There are many distinct sampling distributions, each being effective only for certain environments, so choosing the appropriate distribution becomes essential. This talk will explore the mathematics behind sampling-based motion planning and how constrained problems may be solved efficiently through the reparameterization of constraints.

10:10–10:25

**Structural Properties of Twisted Hermitian Codes and Applications to Cryptography**

Keller Blackwell

*University of South Florida, Tampa*

The advent of quantum computing has renewed interest in code-based cryptography. The McEliece public-key cryptosystem is a code-based scheme that is thought to achieve post-quantum security through the NP-hardness of decoding a random linear code; however, its reliance on binary Goppa codes forces massive key sizes and relatively low data rates. Implementation of the McEliece PKC with more structured codes can improve feasibility, but care must be taken to avoid incurring vulnerabilities under cryptanalysis. For instance, while Reed-Solomon codes are highly structured, they have a small dimensional Schur square that makes them not suitable for McEliece PKC. Recently, twisted Reed-Solomon codes were introduced to increase the dimension of the Schur square. In this talk, we introduce a new variant of Hermitian codes, called twisted Hermitian codes, and consider their parameters. Furthermore, we identify a subfamily of these new codes that achieve large Schur square dimension and show their resistance to distinguisher attacks.

10:30–10:45

**A Convention for Drawing Knots and Links on the Real Projective Plane**

Henry Potts-Rubin

*College of Wooster*

We explore placing knots and links on certain surfaces — namely, the torus, the Klein bottle, and the real projective plane ( $\mathbb{R}P^2$ ). Torus knots and links have been studied fairly comprehensively, as have Klein knots and links. Real projective plane knots and links, however, have not been examined. We begin by introducing torus and Klein knots and links, with results from previous research. We follow this up by defining a specific convention for placing knots and links on the real projective plane, based on its planar representation. We

also investigate braids as they relate to torus, Klein, and  $\mathbb{R}P^2$  knots and links, investigating a predictable braid word for  $\mathbb{R}P^2$  knots and links obtained from our convention.

10:50–11:05

### **Knots and Links on the Real Projective Plane**

Isaac Weiss

*College of Wooster*

In this talk, we will discuss the history of research regarding knot theory and knots on surfaces. More specifically, we look at the history of torus and Klein knots. We will then continue to discuss our own research into knots and links on the real projective plane ( $\mathbb{R}P^2$ ). We discuss a convention of drawing knots on rectangular diagrams and how it affects the outcome of the resulting link or knot.

11:10–11:25

### **Virtual Knots**

Kate O'Connor

*The College of New Jersey*

Invented by Lou Kauffman in 1999, virtual knots extend the usual knots (now called classical knots) by allowing so-called virtual crossings in addition to the classical crossings. We discuss how a virtual knot can be associated to a classical knot living in a thickened surface, and how the genus of the surface provides an invariant of virtual knots.

11:30–11:45

### **Topology of the Mandelbrot Set**

Amelia Pompilio

*University of Dayton*

A brief exploration of the topological properties of the Mandelbrot set, based on honors thesis research progress.

11:50–12:05

### **The Jones Polynomial of the Generalized Half-Twist**

Rebecca Sorsen

*University of Nebraska-Lincoln*

The Jones Polynomial is a specific knot invariant that can yield extremely useful information; however, the Jones Polynomial is computationally complex. While computers can calculate the Jones Polynomial up to a certain complexity, we are interested in much larger, complex examples. Specifically, we will focus on a special class of knots with many twisted, parallel strands, called the generalized half twist. We will discuss the properties of the Kauffman bracket expansion of the generalized half twist and apply our findings to the Jones Polynomial.

12:10–12:25

### **Exploring Homology through the Particle Model**

Rachel Chaiser

*University of Puget Sound*

For a given space we can define a set of topological invariants called homology groups. These groups provide information about the holes in a given space, and as such, can be used to distinguish spaces that we want to think of as different. There is a collection of spaces called Eilenberg-MacLane spaces that can be used as building blocks for other spaces. They are simple in that an Eilenberg-MacLane space has only one nontrivial homotopy group, but they are also often infinite dimensional. Cartan showed that the homology

groups of certain Eilenberg-MacLane spaces have a nice addition and multiplication on them. We will be looking at some of these spaces, specifically ones with fundamental group  $\mathbb{Z}/p\mathbb{Z}$ , through the particle model (essentially a configuration space), to visualize the generators and relations of these homology groups.

## J. Sutherland Frame Lectures

2018	Peter Winkler	<i>The Singular Uniformity of Large Random Systems</i>
2017	Ingrid Daubechies	<i>Bones and Teeth: Analyzing Shapes for Evolutionary Biology</i>
2016	Robin Wilson	<i>Combinatorics - The Mathematics That Counts</i>
2015	Noam Elkies	<i>G-Sharp, A-Flat, and the Euclidean Algorithm</i>
2014	Keith Devlin	<i>Fibonacci and the First Personal Computing Revolution</i>
2013	Gilbert Strang	<i>Matrices I Admire</i>
2012	Melanie Matchett Wood	<i>The Chemistry of Primes</i>
2011	Margaret H. Wright	<i>You Can't Top This: Making Things Better with Mathematics</i>
2010	Nathaniel Dean	<i>Incomprehensibility</i>
2009	Persi Diaconis	<i>The Mathematics of Perfect Shuffles</i>
2008	John H. Conway	<i>The Symmetries of Things</i>
2007	Donald E. Knuth	<i>Negafibonacci Numbers and the Hyperbolic Plane</i>
2006	Donald Saari	<i>Ellipses and Circles? To Understand Voting Problems??!</i>
2005	Arthur T. Benjamin	<i>Proofs that Really Count: The Art of Combinatorial Proof</i>
2004	Joan P. Hutchinson	<i>When Five Colors Suffice</i>
2003	Robert L. Devaney	<i>Chaos Games and Fractal Images</i>
2002	Frank Morgan	<i>Soap Bubbles: Open Problems</i>
2001	Thomas F. Banchoff	<i>Twice as Old, Again, and Other Found Problems</i>
2000	John H. Ewing	<i>The Mathematics of Computers</i>
1999	V. Frederick Rickey	<i>The Creation of the Calculus: Who, What, When, Where, Why</i>
1998	Joseph A. Gallian	<i>Breaking Drivers' License Codes</i>
1997	Philip D. Straffin, Jr.	<i>Excursions in the Geometry of Voting</i>
1996	J. Kevin Colligan	<i>Webs, Sieves and Money</i>
1995	Marjorie Senechal	<i>Tilings as Differential Gratings</i>
1994	Colin Adams	<i>Cheating Your Way to the Knot Merit Badge</i>
1993	George Andrews	<i>Ramanujan for Students</i>
1992	Underwood Dudley	<i>Angle Trisectors</i>
1991	Henry Pollack	<i>Some Mathematics of Baseball</i>
1990	Ronald L. Graham	<i>Combinatorics and Computers</i>
1989	Jean Cronin Scanlon	<i>Entrainment of Frequency</i>
1988	Doris Schattschneider	<i>You Too Can Tile the Conway Way</i>
1987	Clayton W. Dodge	<i>Reflections of a Problems Editor</i>
1986	Paul Halmos	<i>Problems I Cannot Solve</i>
1985	Ernst Snapper	<i>The Philosophy of Mathematics</i>
1984	John L. Kelley	<i>The Concept of Plane Area</i>
1983	Henry Alder	<i>How to Discover and Prove Theorems</i>
1982	Israel Halperin	<i>The Changing Face of Mathematics</i>
1981	E. P. Miles, Jr.	<i>The Beauties of Mathematics</i>
1980	Richard P. Askey	<i>Ramanujan and Some Extensions of the Gamma and Beta Functions</i>
1979	H. Jerome Keisler	<i>Infinitesimals: Where They Come From and What They Can Do</i>
1978	Herbert E. Robbins	<i>The Statistics of Incidents and Accidents</i>
1977	Ivan Niven	<i>Techniques of Solving Extremal Problems</i>
1976	H. S. M. Coxeter	<i>The Pappus Configuration and Its Groups</i>
1975	J. Sutherland Frame	<i>Matrix Functions: A Powerful Tool</i>

## MAA Lectures for Students

2018	Laura Taalman	<i>FAIL: A Mathematician's Apology</i>
2017	David Richeson	<i>Four Tales of Impossibility</i>
2016	Colin Adams	<i>Zombies &amp; Calculus: A Survival Guide</i>
2015	Joseph Gallian	<i>Seventy-Five Years of MAA Mathematics Competitions</i>
2014	Jack Graver	<i>The Founding of Pi Mu Epsilon 100 Years Ago</i>
2013	Frank Morgan	<i>Optimal Pentagonal Tilings</i>
2012	Ivars Peterson	<i>Geometreks</i>
2011	Roger Nelson	<i>Math Icons</i>
2010	Sommer Gentry	<i>Faster, Safer, Healthier with Operations Research</i>
2009	Colm Mulcahy	<i>Mathemagic with a Deck of Cards on the Interval Between 5.700439718 and 806581751709438785716606368564037 669752895054408832778240000000000000</i>
2008	Laura Taalman	<i>Sudoku: Questions, Variations and Research</i>
2007	Francis Edward Su	<i>Splitting the Rent: Fairness Problems, Fixed Points, and Fragmented Polytopes</i>
2006	Richard Tapia	<i>Math at Top Speed: Exploring and Breaking Myths in Drag Racing Folklore</i>
2005	Annalisa Crannell & Marc Frantz	<i>Lights, Camera, Freeze!</i>
2004	Mario Martelli	<i>The Secret of Brunelleschi's Cupola</i>
2004	Mark Meerschaert	<i>Fractional Calculus with Applications</i>
2003	Arthur T. Benjamin	<i>The Art of Mental Calculation</i>
2003	Donna L. Beers	<i>What Drives Mathematics and Where is Mathematics Driving Innovation?</i>
2002	Colin Adams	<i>"Blown Away: What Knot to do When Sailing" by Sir Randolph "Skipper" Bacon III</i>
2002	M. Elisabeth Pate-Cornell	<i>Finding and Fixing Systems' Weaknesses: The Art and Science of Engineering Risk Analysis</i>
2001	Rhonda Hatcher	<i>Ranking College Football Teams</i>
2001	Ralph Keeney	<i>Building and Using Mathematical Models to Guide Decision Making</i>
2000	Michael O'Fallon	<i>Attributable Risk Estimation: A Tale of Mathematical/Statistical Modeling</i>
2000	Thomas Banchoff	<i>Interactive Geometry on the Internet</i>
1999	Edward G. Dunne	<i>Pianos and Continued Fractions</i>
1999	Dan Kalman	<i>A Square Pie for the Simpsons and Other Mathematical Diversions</i>
1998	Ross Honsberger	<i>Some Mathematical Morsels</i>
1998	Roger Howe	<i>Some New and Old Results in Euclidean Geometry</i>
1997	Aparna Higgins	<i>Demonic Graphs and Undergraduate Research</i>
1997	Edward Schaefer	<i>When is an Integer the Product of Two and Three Consecutive Integers?</i>
1996	Kenneth Ross	<i>The Mathematics of Card Shuffling</i>
1996	Richard Tapia	<i>Mathematics Education and National Concerns</i>
1995	David Bressoud	<i>Cauchy, Abel, Dirichlet and the Birth of Real Analysis</i>
1995	William Dunham	<i>Newton's (Original) Method, or, Though This Be Method, Yet There is Madness</i>
1994	Gail Nelson	<i>What is Really in the Cantor Set?</i>
1994	Brent Morris	<i>Magic Tricks, Card Shuffling and Dynamic Computer Memories</i>
1993	Richard Guy	<i>The Unity of Combinatorics</i>
1993	Joseph Gallian	<i>Touring a Torus</i>
1992	Peter Hilton	<i>Another Look at Fibonacci and Lucas Numbers</i>
1992	Caroline Mahoney	<i>Contemporary Problems in Graph Theory</i>
1991	Lester Lange	<i>Desirable Scientific Habits of Mind Learned from George Polya</i>

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## CONGRATULATIONS TO THE 2016 ANDREE AWARD WINNER!

Bradley Dougherty  
*Northwest Missouri State University*

### **On the average of a function and the mean value theorem for integrals**

The Richard V. Andree Awards are given annually to the authors of the papers, written by undergraduate students, that have been judged by the officers and councilors of Pi Mu Epsilon to be the best that have appeared in the Pi Mu Epsilon Journal in the past year.

## THE PME JOURNAL—CALL FOR STUDENT PAPERS

Mathfest is all about communication—so is the PME Journal. At the conference, listening is as important as talking. For a journal, readers are as important as authors. The Pi Mu Epsilon Journal readership and authorship is YOU. Send in your mathematical findings under a cover letter and feel the excitement of waiting for the referees' comments. The referees help you become better authors, but they are not, by any means, proof readers or error checkers, so submit only a polished, carefully crafted manuscript. But even if you find improvement unimaginable, most of the time rewriting is required before publication. The process is time consuming but the reward is great. Your paper in print in the PME Journal, now covered by JSTOR, will be accessible forever.

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Pi Mu Epsilon would like to express its sincere appreciation to the following individuals and organizations for their generous donations:

- The American Mathematical Society, the American Statistical Association, the Council on Undergraduate Research, Budapest Semesters in Mathematics, and BioSIGMAA for the sponsorship of the Awards for Outstanding Presentations.
- Dover Publications, Cambridge University Press, and Princeton University Press for donating book prizes.
- The Youngstown State University Print Shop for printing this Abstract Booklet at cost.
- Nancy Blachman, founder of the Julia Robinson Mathematics Festival ([jrmf.org](http://jrmf.org)), for contributions toward subsistence grants for Pi Mu Epsilon speakers.