Retrieval Experiment: Dense vs Hybrid+Rerank

Overview

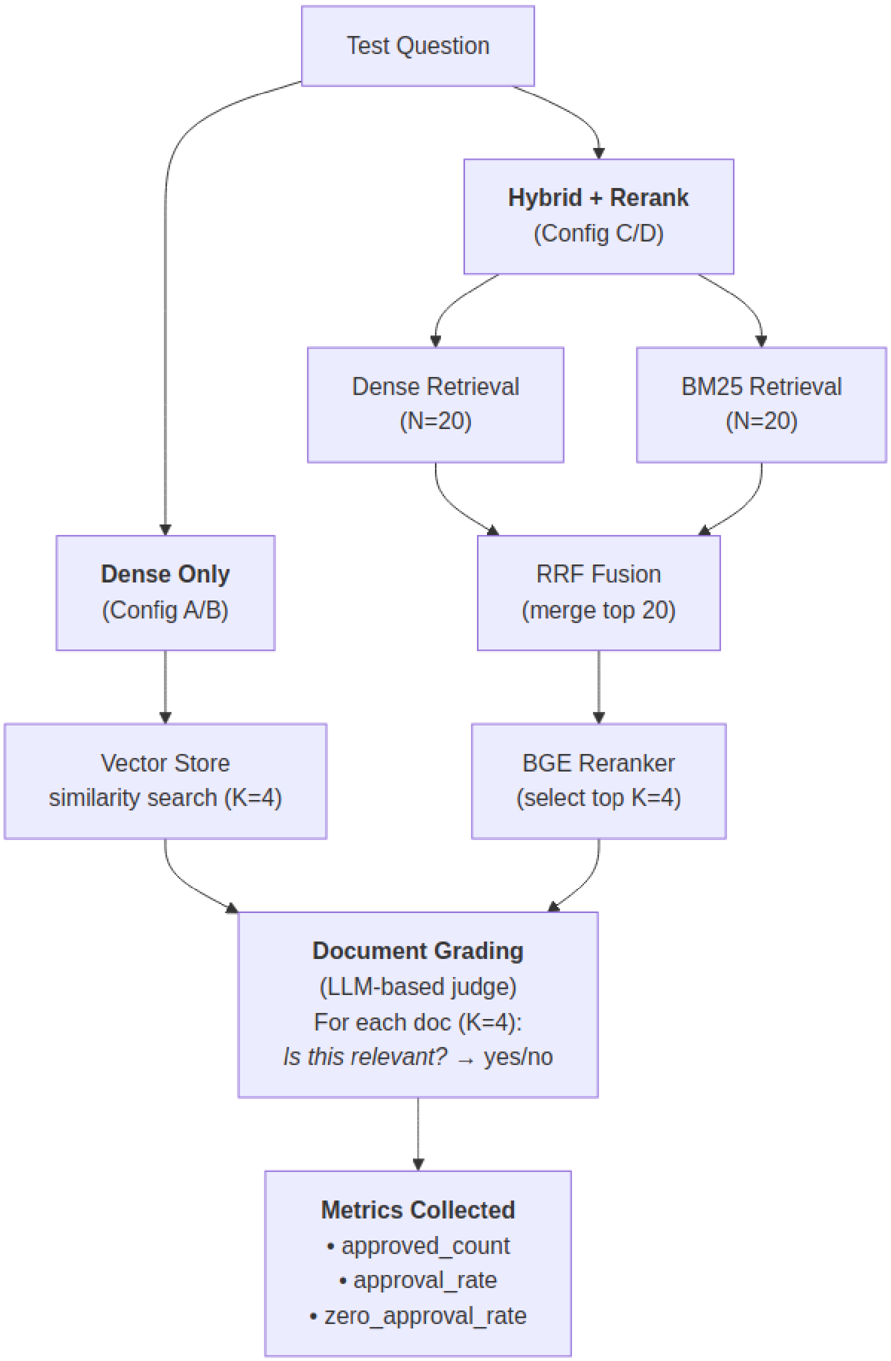
This experiment compares dense-only retrieval against a hybrid pipeline (Dense + BM25 + RRF + Rerank) for Case-Based Reasoning on mathematical problem solving. Retrieval quality is evaluated using an LLM-based document grader that judges whether retrieved examples contain transferable problem-solving techniques.

Experimental Setup

Configurations

Config	Dense Embeddings	BM25	RRF Fusion	Reranker
A. Dense (Nomic)	Nomic-embed-v1.5	-	-	-
B. Dense (Qwen3)	Qwen3-Embedding-8B	-	-	-
C. Hybrid+Rerank (Nomic)	Nomic-embed-v1.5	Yes	Yes	BGE-reranker-v2-m3
D. Hybrid+Rerank (Qwen3)	Qwen3-Embedding-8B	Yes	Yes	BGE-reranker-v2-m3

Pipeline


Test Question
Hybrid + Rerank
(Config C/D)
Dense Retrieval
(N=20)
BM25 Retrieval
(N=20)
Dense Only
(Config A/B)
RRF Fusion
(merge top 20)
Vector Store
similarity search (K=4)
BGE Reranker
(select top K=4)
Document Grading
(LLM-based judge)
For each doc (K=4):
Is this relevant? → yes/no
Metrics Collected
• approved_count
• approval_rate
• zero_approval_rate

The hybrid pipeline performs parallel dense and BM25 retrieval (N=20 each), fuses results via Reciprocal Rank Fusion (RRF, k=60), then reranks with a cross-encoder to select the top K=4 documents.

Datasets

Dataset	Test Queries	Retrieval Corpus (Training Set)
GSM8K	1,319	7,473
MATH	5,000	7,500

Technical Components

Component	Implementation
Dense Embeddings	Nomic-embed-v1.5 (137M params) or Qwen3-Embedding-8B
Vector Store	ChromaDB
BM25	rank_bm25.BM25Okapi
Reranker	BAAI/bge-reranker-v2-m3 (CrossEncoder)
Grader LLM	Qwen2.5-32B-Instruct via vLLM

Documents are graded in a single batched vLLM call. The grader outputs a binary "yes"/"no" judgment on whether each document provides adaptable problem-solving techniques for the query.

Hyperparameters

Parameter	Value	Description
N_INITIAL	20	Documents retrieved per method
N_RERANK	20	Documents passed to reranker after RRF
TOP_K	4	Final documents graded
RRF_K	60	RRF smoothing constant

Results

GSM8K (1,319 queries)

Rank	Config	Avg Approved	Approval Rate	Zero Rate	Coverage
1	**B. Dense (Qwen3)**	**3.40**	**85.0%**	**1.4%**	**98.6%**
2	D. Hybrid+Rerank (Qwen3)	3.16	79.0%	3.0%	97.0%
3	A. Dense (Nomic)	3.14	78.5%	2.4%	97.6%
4	C. Hybrid+Rerank (Nomic)	3.11	77.7%	3.0%	97.0%

Approval Distribution (GSM8K)

Config	0 docs	1 doc	2 docs	3 docs	4 docs
Dense (Qwen3)	18	58	120	304	819
Hybrid+Rerank (Qwen3)	39	92	163	349	676
Dense (Nomic)	32	117	152	351	667
Hybrid+Rerank (Nomic)	40	105	173	355	646

MATH (5,000 queries)

Rank	Config	Avg Approved	Approval Rate	Zero Rate	Coverage
1	**B. Dense (Qwen3)**	**3.48**	**86.9%**	**1.5%**	**98.5%**
2	A. Dense (Nomic)	2.86	71.4%	6.1%	93.9%
3	D. Hybrid+Rerank (Qwen3)	2.79	69.8%	5.7%	94.3%
4	C. Hybrid+Rerank (Nomic)	2.63	65.8%	7.7%	92.3%

Coverage (queries with at least 1 relevant example)

Config	GSM8K	MATH
Dense (Qwen3)	**98.6%** (1301/1319)	**98.5%** (4927/5000)
Dense (Nomic)	97.6% (1287/1319)	93.9% (4697/5000)
Hybrid+Rerank (Qwen3)	97.0% (1280/1319)	94.3% (4716/5000)
Hybrid+Rerank (Nomic)	97.0% (1279/1319)	92.3% (4614/5000)

Analysis

BM25 Impact

Dataset	Dense Qwen3	Hybrid Qwen3	Penalty
GSM8K	85.0%	79.0%	**-6.0pp**
MATH	86.9%	69.8%	**-17.1pp**

Dataset	Dense Nomic	Hybrid Nomic	Penalty
GSM8K	78.5%	77.7%	-0.8pp
MATH	71.4%	65.8%	**-5.6pp**

The BM25 penalty increases with dataset complexity. On the more diverse MATH dataset, adding BM25 to Dense Qwen3 reduces approval rate by 17.1 percentage points.

Hyperparameter Sweep (GSM8K, Hybrid Qwen3)

N_INITIAL	N_RERANK	TOP_K	Approval	Avg Approved	Zero Rate
10	**10**	4	**79.9%**	**3.20**	**2.4%**
50	20	4	79.7%	3.19	2.7%
20 (baseline)	20	4	79.0%	3.16	3.0%
100	50	4	78.4%	3.13	2.7%
50	50	4	78.2%	3.13	2.5%

Smaller retrieval pools outperform larger ones. Expanding the candidate pool introduces more noise from BM25 that the reranker fails to filter.

Zero-Approval Query Analysis (Dense Nomic, GSM8K)

32 queries (2.4%) received zero approved documents. Breakdown by problem type: - 31% ratio/proportion problems - 22% rate/time problems - 22% multi-step problems

Root cause: embeddings match on topic similarity rather than mathematical reasoning structure. A “bakery” problem about unit conversion retrieves other “bakery” problems about quantity counting, not structurally similar unit conversion problems from other domains.

Key Findings

1. **Dense Qwen3 dominates** across both datasets (85-87% approval, 98.5%+ coverage).
2. **Dense outperforms Hybrid** for both embedding models on both datasets.
3. **Embedding model quality is the dominant factor.** Upgrading from Nomic (137M) to Qwen3 (8B) yields +6.5pp on GSM8K and +15.5pp on MATH — far exceeding any pipeline-level improvement.
4. **BM25 degrades retrieval quality for math.** Keyword matching surfaces topically similar but structurally different problems. The penalty grows with dataset complexity (-6pp GSM8K, -17pp MATH).
5. **The BGE reranker cannot compensate** for BM25 noise injection, as it lacks math reasoning awareness.
6. **These results contradict Anthropic’s Contextual Retrieval findings** (49-67% failure rate reduction from hybrid retrieval), suggesting that domain-specific retrieval tasks — particularly mathematical reasoning — do not benefit from generic hybrid pipelines.

Recommendation

Use **Dense Qwen3** for the CBR retrieval pipeline. It is the simplest configuration and outperforms all alternatives. The hybrid approach adds complexity while degrading quality.