CS105 (DIC on Discrete Structures)
Problem set 6

- Attempt *all* questions.
- Apart from things proved in lecture, you cannot assume anything as "obvious". Either quote previously proved results or provide clear justification for each statement.

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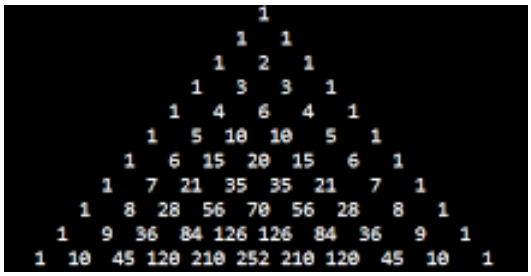
1. How many functions are there from a set of size 15 to a set of size 12?
2. Prove the following identities by counting the size of a suitably designed set in two different ways.
   (a) $\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k}\binom{n}{k}$
   (b) $\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$
3. Find the number of (i) all words (ii) four letter words that can be formed using the letters from the word "MISSISSIPPI".
4. Find the number of integral solutions for $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ with each $x_i \geq 2$. Next, generalize this to find the number of solutions for $\sum_{i=1}^{k} x_i = n$ with each $x_i \geq t$ and express it in a closed form involving n, t, and k.
5. The drama club has m members and the dance club has n members. For the upcoming paf, a committee of k people needs to be formed with at least one member from each club. If the clubs have exactly r members in common, what is the number of ways the committee may be chosen? Justify your answer.
6. Two hundred students participated in a math contest. The had six problems to solve. Each problem was correctly solved by at least 120 participants. Prove, using double counting, that there must be two participants such that every problem was solved by at least one of these students.

Programming assignment (Out of Syllabus)

These questions are not for the exam, but just for your own interest. Solve them at your own risk!

1. Write a program to draw the Pascal's triangle of any size. That is, the program should take as input an integer n and write down the numbers of the first n rows of the triangle (as a triangle). For example, for input $n = 10$, the output should look like:



2. Now, modify the program to replace every odd number in the triangle by the symbol # and every even number by an empty blank space. Print the output of this new program for $n = 17$ and $n = 33$. Can you see something special about this picture? Can you explain this phenomenon mathematically?