CS105 (DIC on Discrete Structures)
Problem set 7

- Attempt *all* questions.
- Apart from things proved in lecture, you cannot assume anything as "obvious". Either quote previously proved results or provide clear justification for each statement.

Basic

1. Let $B(n)$ be the no. of bit strings of length n that contain the string 01.
   (a) Write a recurrence relation for $B(n)$.
   (b) Determine the initial conditions.
   (c) How many strings are there of length 7 are there that contain 01?
   (d) Solve the recurrence to obtain an expression for $B(n)$ in terms of n.

2. Consider the standard deck of 52 playing cards. A balanced hand is a subset of 13 cards containing four cards of one suit and three cards of each of the remaining three suits. Find N , the number of balanced hands. Find the number of ways of dealing the cards to four (distinguishable) players so that each player gets a balanced hand. Is this number equal to $N(N-1)(N-2)(N-3)$?

3. Find the coefficients of x^{10} in
   (a) $(1+x)^{12}$
   (b) the power series of $x^4/(1-3x)^3$

4. Consider a grid from $(0,0)$ to (n,n). Starting from the point $(0,0)$, we wish to take units steps ONLY in the direction of the positive X and Y axes (i.e., right and up), and reach the point (n,n). Find a recurrence relation for the number of ways of doing so, if we are not allowed to go above the line joining $(0,0)$ and (n,n), i.e, the diagonal.

Advanced

5. Write a recurrence for the number of derrangements. That is, no. of ways to arrange n letters into n addressed envelopes such that no letter goes to the correct envelope.

6. Using generating functions, prove Pascal's identity: $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ where $r < n \in \mathbb{Z}^+$.

7. Using generating functions, find the number of ways of selecting k objects from n different kinds of objects if repetitions are allowed, and we must select at least 2 objects of each kind?

8. Use generating functions to determine the number of different ways to give 15 (identical) chocolates to 6 children so that each child receives at least one chocolate but not more than three chocolates.

9. Solve the following recurrences:

   (a) $T(n) = 5T(n-1) - 6T(n-2)$ with $T(0) = 6, T(1) = 30$.

   (b) $T(n) = n(T(n/2))^2$ with $T(1) = 6$.

   (c) $T(n) = \sqrt{n}T(\sqrt{n}) + n$ with $T(2) = 2$.