CS105 (DIC on Discrete Structures)
Problem set 9

- Attempt *all* questions.

- Apart from things proved in lecture, you cannot assume anything as "obvious". Either quote previously proved results or provide clear justification for each statement.

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1. In any group of 7 people, show that there must be a group of 3 people who are mutual friends (that is, each pair of them are friends), or a group of 3 people who are mutually unknown to each other. You may assume that between any two people in the group, they are either friends or don't know each other.

2. Prove or disprove: any finite simple graph with at least two vertices contains at least two vertices of the same degree.

3. Anushka and her husband Virat gave a party at which there are 4 other married couples. Some pairs of people shake hands when they meet, but naturally no couple shake hands with each other. At the end of the party Virat asks everyone else how many people they have shaken hands with, and he receives nine different answers. How many people shook hands with Anushka? Model the problem as a graph and solve it.

4. Consider any five points $p1, \ldots, p5$ in the interior of a square S of side length 1. Prove that there are two of these five points which are separated by a distance of at most $\sqrt{2}/2$.

5. Write the formal proof showing that $R(3, 4) = 9$. Note that we already saw the proof for $R(3, 4) \leq 9$ in class, but your exercise is to formally rewrite it in your own words. And in addition to show that this number is indeed optimal (which we did not do in class).

6. In any group of people, we are told that any two either don't know each other or, if they know each other, they like or dislike each other. In a group of 17 people, show that the following property P is true: there must exist 3 who know and mutually like each other or 3 who know but mutually dislike each other or 3 who don't know each other.