

CS105 2023 Discrete Structures: Basics, Strong Induction

Exercise Problem Set 2

Part 1

1. For each of the statements below, write its negation.
 - (a) All computer science students like coffee.
 - (b) $(\forall x(x^2 > x)) \wedge (\exists x(x^2 = 2))$
2. Consider the proposition: for all natural numbers $n \geq 5, n^2 < 2^n$.
 - (a) Give a proof by induction. Do you need Strong Induction for it? Can you use Strong Induction?
 - (b) Also give a proof by contradiction, using Well-Ordering-Principle.

Part 2

3. Given $n \in \mathbb{N}$, consider a sequence of numbers where the n^{th} term of the sequence is defined by a “recurrence”. That is,
$$u_0 = 1, u_1 = 3, u_n = 2u_{n-1} - u_{n-2}, \text{ for } n \geq 2$$
 - . Prove using Strong Induction that for all $n \geq 0, u_n = 2n + 1$. Also highlight why you need the Strong Induction in your proof.
4. Consider the following game:
 - There are two piles of matches.
 - Two players take turns removing any positive (i.e., non-zero) number of matches they want from one of the two piles.
 - The player who removes the last match wins.

Show (by Strong induction!) that, if the two piles contain the same number of matches initially, then the second player can always win the game.