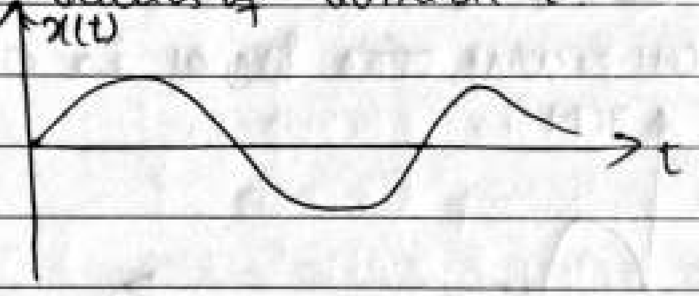
Signal:-

Signal can be defined as any function, in general, of time that carry information.

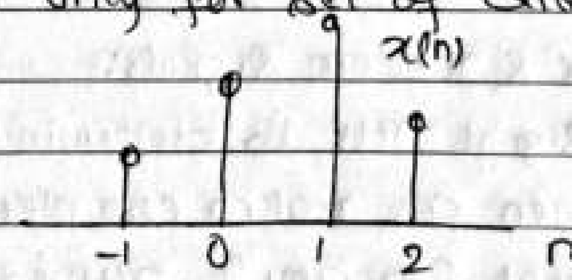
Types of Signal:-

1) continuous & discrete-time signals:-

A signal $x(t)$ is said to be continuous type if it can be defined for all possible real number values of variable 't'.



The discrete time signal $x(n)$ can be defined only for set of discrete values of 'n'.



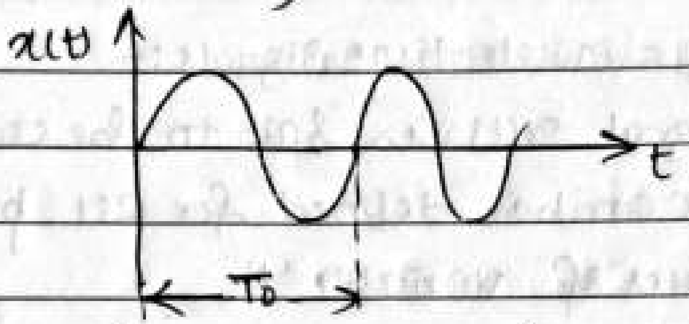
2) Real & complex signals:-

A real $x_r(t)$ signal take its values in the set of real numbers i.e. $x_r(t) \in R$ where as the complex signal $x_c(t)$ take its values in the set of complex numbers i.e. $x_c(t) \in C$. In mathematical analysis, a signal is generally expressed in exponential form consisting of real & imaginary parts

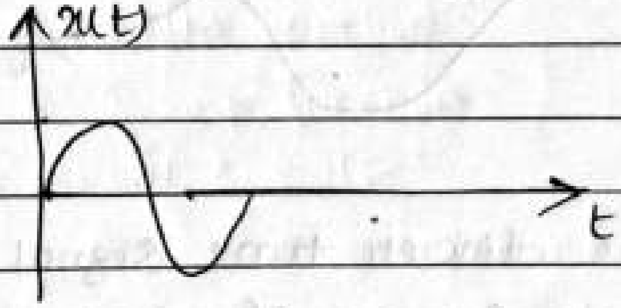
$x(t) = A \exp j(2\pi f_0 t + \theta_0)$, $A \geq 0$

3) <u>Periodic & non-periodic signals:</u>

A signal $x(t)$ is said to be periodic if $x(t) = x(t+T_0)$ where T_0 is time period.



Otherwise the signal is aperiodic.



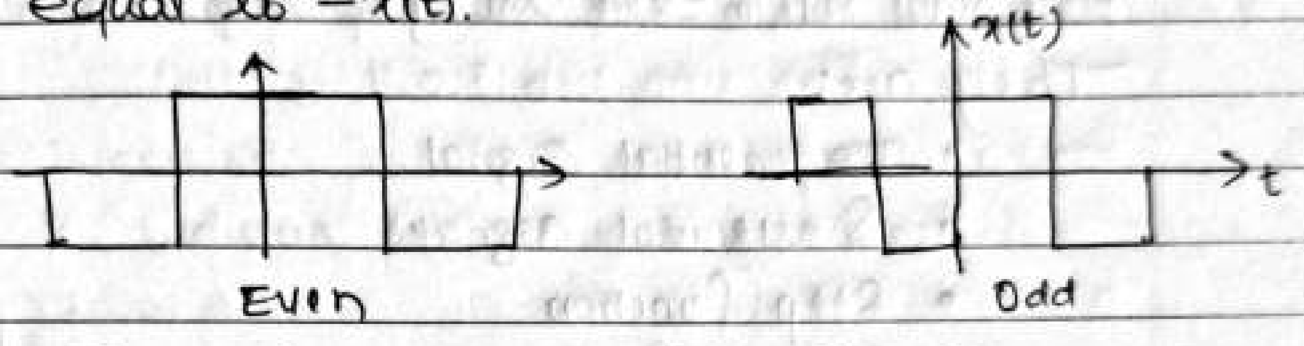
4) <u>Deterministic & random signals:-</u>

A signal $x(t)$ is deterministic if for any 't', $x(t)$ is given as real or complex number. For random signal for any 't', $x(t)$ is random variable i.e. it can be defined only by probability density function. In real world, all signals are of random nature but simplicity in modeling & processing we consider most of signals to be deterministic.

5) <u>Even & odd signals:-</u>

These types of signals are identified by their symmetry with respect to point of origin

If $x(-t) = x(t)$, then the signal is even.
For a signal to be odd symmetry, $x(-t)$ should be equal to $-x(t)$.



6) Energy type & power type signals:-

A signal is called energy type signal if its energy E_x is well defined & finite.

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt < \infty$$

Therefore for a signal $x(t)$ to be of energy type, the value of E_x should be

$$0 < E_x < \infty$$

A signal is called power type if its power is finite.

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt < \infty$$

And well defined

$$0 < P_x < \infty$$

Elementary Signals:-

There are several elementary signals which play vital role in the study of signals & systems. They are:-

- Exponential signal
- Sinusoidal signal
- Step function
- Signum function
- Sinc function
- Rectangular pulse
- Delta function
- Ramp function

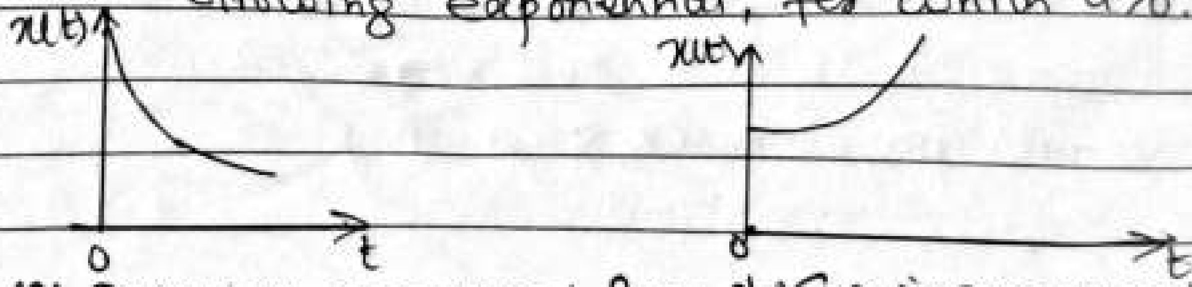
Exponential Signal:-

A real exponential signal, in its most general form, is written as,

$$x(t) = Be^{at}$$

Where both B & a are real parameters. The parameter B is the amplitude of the exponential signal measured at time t=0. Depending on whether the other parameter 'a' is positive or negative, we may identify two special cases:

- Decaying exponential, for which $a<0$.
- Growing exponential, for which $a>0$.



(a) Decaying exponential form (b) Growing exponential form

Date :
Page:

Sinusoidal Signal:-

In continuous time, the general expression for a sinusoidal signal may be written as

$$x(t) = A\cos(\omega t + \phi)$$

where A is the amplitude, ω is the freq^n in radians per second & ϕ is phase angle in radians.

A sinusoidal signal is an example of periodic signal, the period of which is

$$T = \frac{2\pi}{\omega}$$

Now,

$$\begin{aligned} x(t+T) &= A\cos[\omega(t+T)+\phi] \\ &= A\cos(\omega t + \omega T + \phi) \\ &= A\cos(\omega t + 2\pi + \phi) \\ &= A\cos(\omega t + \phi) \\ &= x(t) \end{aligned}$$

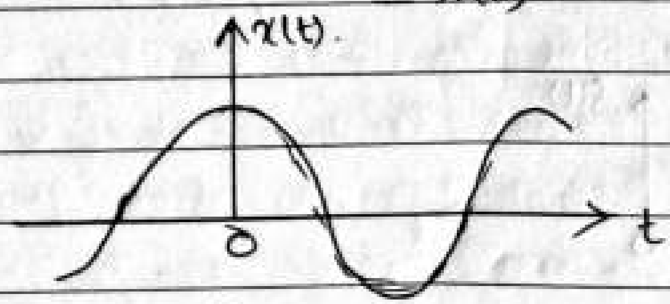


fig: Sinusoidal Signal

Step function:-

The continuous-time version of unit-step function is defined by

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Date:
Page:

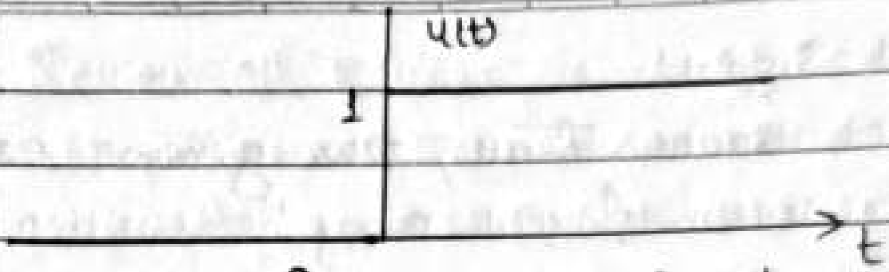


fig: Unit Step Signal

Rectangular pulse:-

Rectangular pulse $x(t)$ may be written in mathematical form as

$$x(t) = \begin{cases} A & 0 \le |t| < \tau \\ 0 & |t| > \tau \end{cases}$$

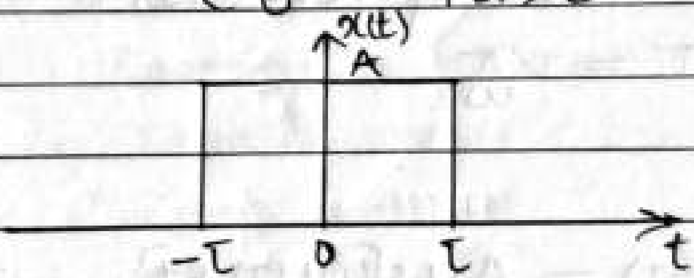


fig: Rectangular pulse

Impulse function (delta function):-

It is defined as

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \;, \; \delta(t) = 0 \;, \; t \neq 0$$

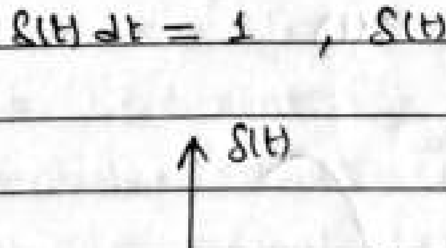


fig: Delta function

Sinc function:-

A Sinc signal has its maximum value equal to 1 at $t=0$ & gradually tends to zero for t tending to infinite.

Date:

Page:

$$\mathrm{Sinc}(t) = \begin{cases} \frac{\sin(\pi t)}{\pi t}, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

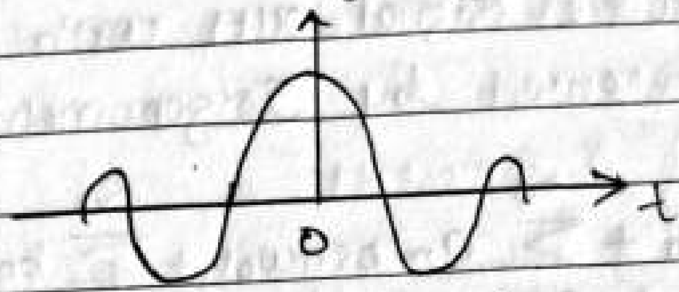


fig: Sinc function

Signum function:-

It is used to define the sign of a signal.

$$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{cases}$$

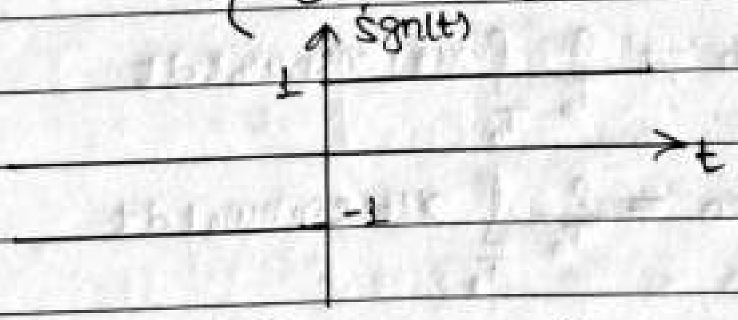


fig: Signum function

Unit ramp function:-

It is defined as

$$r(t) = \begin{cases} t, & t > 0 \\ 0, & t < 0 \end{cases}$$

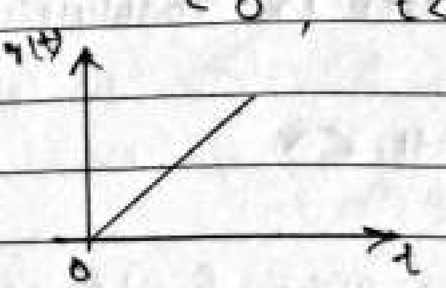


fig: Ramp function

Date :
Page :

Review of Fourier Series & Fourier transform :-

Trigonometric Fourier series :-

A periodic signal $x(t)$ with a period of T_0 may be represented by Trigonometric Fourier series as,

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

Where a_0, a_n & b_n are known as Fourier coefficients.

$$\omega_0 = \frac{2\pi}{T_0}$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t\, dt$$

Dirichlet conditions for the existence of Fourier series

Fourier series will exist if & only if the periodic signal $x(t)$ satisfies the following conditions. These are known as Dirichlet conditions.

1) The function $x(t)$ is absolutely integrable, that is,

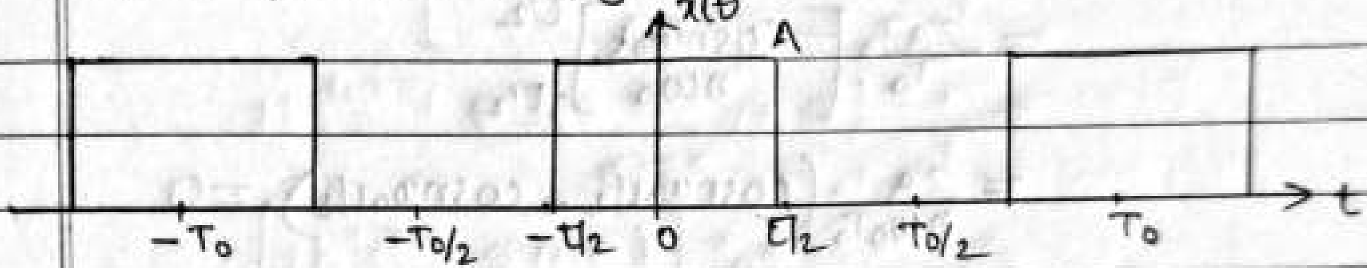
$$\int_{T_0} |x(t)|\, dt < \infty$$

2) $x(t)$ should have only finite number of maxim

& minima in the given interval of time.

3) $x(t)$ must have only finite number of discontinuities in the given interval of time.

Obtain Fourier series for the rectangular pulse train shown in fig.



Soln:-

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt$$

$$= \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A\,dt = \frac{A\tau}{T_0}$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\int_{-\tau/2}^{\tau/2} A\cos n\omega_0 t\,dt$$

$$= \frac{2A}{T_0}\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{2A}{T_0 n\omega_0}\left[\sin n\omega_0 \tau/2 - \sin(-n\omega_0 \tau/2)\right]$$

Date:
Page:

$$= \frac{4A}{n.2\pi}\sin\left(\frac{n\pi\tau}{T_0}\right) = \frac{2A}{n\pi}\sin\left(\frac{\pi n\tau}{T_0}\right)$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\sin n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\int_{-\tau/2}^{\tau/2} A\sin n\omega_0 t\,dt$$

$$= \frac{2A}{T_0}\left[\frac{-\cos n\omega_0 t}{n\omega_0}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{2A}{n\omega_0 T_0}\left(\cos n\omega_0\tau/2 - \cos n\omega_0\tau/2\right) = 0$$

Hence,

$$x(t) = \frac{A\tau}{T_0} + \sum_{n=1}^{\infty}\frac{2A}{n\pi}\sin\left(\frac{n\pi\tau}{T_0}\right)\cos\left(\frac{2\pi n t}{T_0}\right)$$

Exponential Fourier Series:-

$$x(t) = \sum_{n=\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\,dt$$

Fourier transform:-

Fourier transform may be expressed as

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt$$

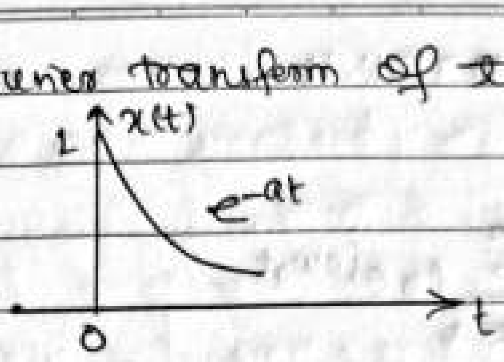
Inverse Fourier transform is given by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\,d\omega$$

Date:

Page:

Find Fourier transform of the signal shown in fig.



Solⁿ:-

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_{0}^{\infty} = \frac{1}{a+j\omega}$$

Parseval's power theorem:-

This theorem relates the average power P of a periodic signal to its Fourier series coefficients. The Parseval's power theorem states that the ~~total power~~ average power of a periodic signal x(t) is equal to the sum of the average power of individual Fourier coefficients i.e. Cn.

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2$$

Proof:-

$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_{T_0} x(t)\,x^*(t)\,dt$$

We've,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$P = \frac{1}{T_0}\int_{T_0} x(t)\left[\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}\right]^* dt$$

$$= \frac{1}{T_0}\int_{T_0} x(t)\sum_{n=-\infty}^{\infty} C_n^* e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{T_0}\sum_{n=-\infty}^{\infty} C_n^*\left[\int_{T_0} x(t)e^{-jn\omega_0 t}\,dt\right]$$

$$= \frac{1}{T_0}\sum_{n=-\infty}^{\infty} C_n^* \cdot C_n$$

$$\boxed{P = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} |C_n|^2}$$

Parseval's Energy theorem :

It states that the total energy of the signal x(t) is equal to the sum of energies of individual spectral components in the frequency domain.

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

Proof:-

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Date:
Page:

$$= \int_{-\infty}^{\infty} x(t)\, x^*(t)\, dt$$

Where,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\, e^{-j\omega t}\, d\omega$$

$$E = \int_{-\infty}^{\infty} x(t) \cdot \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\, e^{-j\omega t}\, d\omega\, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\left[\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt\right] d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\, X(\omega)\, d\omega$$

$$\boxed{E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega}$$

Types of System :-

1) Continuous-time System
2) Discrete-time System

A continuous time system is a physical device which performs required operation on a continuous-time signal. It is represented as

$$\underset{\text{Input}}{x(t)} \longrightarrow \boxed{\text{Continuous-time System}} \longrightarrow \underset{\text{Output}}{y(t)}$$

Date:
Page:

A discrete time system is a physical device which performs required operation on discrete-time signal.

$x(n)$ Input $\longrightarrow$ $\boxed{\text{Discrete-time System}}$ $\longrightarrow$ $y(n)$ Output

Properties of Systems:-

1) Linearity:-

A system is linear if it follows superposition principle. Let $x_1(t)$ produces output $y_1(t)$ & input $x_2(t)$ produces output $y_2(t)$.

Then, for linear system,

$$a_1x_1(t) + a_2x_2(t) \rightarrow a_1y_1(t) + a_2y_2(t) \quad \text{--- (1)}$$

where a_1 & a_2 are arbitary constants.

For discrete-time systems,

$$a_1x_1(n) + a_2x_2(n) \longrightarrow a_1y_1(n) + a_2y_2(n) \quad \text{--- (2)}$$

If (1) or (2) is not satisfied, then the system is non-linear.

\# Determine whether the following systems are linear or not:

i) $y(t) = t\,x(t)$

ii) $y(n) = 2x(n) + 6$

i) Soln:-

$$y_1(t) = t\,x_1(t)$$

$$y_2(t) = t\,x_2(t)$$

Date :
Page:

Applying superposition in input,

$$x(t) = a_1 x_1(t) + a_2 x_2(t)$$

Then, o/p will be

$$y_3(t) = t\left[a_1 x_1(t) + a_2 x_2(t)\right] \quad \text{--- (i)}$$

Applying superposition in output,

$$y_4(t) = a_1 y_1(t) + a_2 y_2(t)$$
$$= a_1 t x_1(t) + a_2 t x_2(t) \quad \text{--- (ii)}$$

$\therefore y_3(t) = y_4(t)$, the system is linear.

ii) Soln:-

$$y_1(n) = 2x_1(n) + 6$$
$$y_2(n) = 2x_2(n) + 6$$

Applying superposition in input,

$$x(n) = a_1 x_1(n) + a_2 x_2(n)$$

Then, o/p will be

$$y_3(n) = 2\left[a_1 x_1(n) + a_2 x_2(n)\right] + 6 \quad \text{--- (i)}$$

Applying superposition in output,

$$y_4(n) = a_1 y_1(n) + a_2 y_2(n)$$
$$= a_1\left[2x_1(n) + 6\right] + a_2\left[2x_2(n) + 6\right] \quad \text{--- (ii)}$$

$\because y_3(n) = y_4(n)$, the system is non-linear.

2) Time Invariant:-

A system is time invariant if its input-output characteristics do not change with time. It means that its input-output characteristics are not changing with time shifting.

If

$$x(t) \rightarrow y(t)$$

Date :
Page :

Then,

$x(t-t_0) \rightarrow y(t-t_0)$

where t_0 is time delay.

Similarly for discrete-time system,

If

$x(n) \rightarrow y(n)$

Then

$x(n-n_0) \rightarrow y(n-n_0)$

where n_0 is time delay

Determine whether the following systems are time invariant or not:

i) $y(t) = x^2(t)$

ii) $y(n) = nx(n)$

i) Soln:-

If we delay input by t_0, then o/p will be

$y_1(t) = x^2(t-t_0)$ --- (i)

If we delay output by t_0, then

$y(t-t_0) = x^2(t-t_0)$ --- (ii)

$\therefore y_1(t) = y(t-t_0)$, the system is time-invariant

ii) $y(n) = nx(n)$

If we delay input by k unit, then O/p will be,

$y_1(n) = nx(n-k)$ --- (i)

If we delay output by k unit, then

$y(n-k) = (n-k)\, x(n-k)$ --- (ii)

$\therefore y_1(n) \neq y(n-k)$, the system is time-variant.

Date: ______
Page: ______

3) Causality:-

A system is said to be causal if output at any instant of time depends only on present & past inputs. But the output doesn't depend on future inputs.

E.g. the system described by the eqn

$y(n) = x(n) + x(n-1)$ is causal &

$y(t) = x(t) + x(t+1)$ is non-causal.

4) Stability:-

An initially relaxed system is BIBO stable if & only if every bounded input produces bounded output.

If input is bounded i.e.

$|x(n)| < M_x < \infty$

then for BIBO stable system, o/p will be

$|y(n)| < M_y < \infty$ for all n.

Determine whether the following systems are stable or not:

i) $y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$

ii) $y(n) = r^n x(n), \quad |r| > 1$

Soln:-

i) $y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$

Assume

$|x(n)| < M_x < \infty,$

Then,

$$|y(n)| = \left|\frac{1}{3}\left[x(n)+x(n-1)+x(n-2)\right]\right|$$

$$\leq \frac{1}{3}\left[|x(n)|+|x(n-1)|+|x(n-2)|\right]$$

$$\leq \frac{1}{3}(M_x+M_x+M_x)$$

$$\leq M_x < \infty$$

Hence the system is BIBO stable.

ii) $y(n) = r^n x(n)$, $|r|>1$

& Assume,

$$|x(n)| < M_x < \infty$$

Then,

$$|y(n)| = |r^n x(n)|$$

$$= |r^n||x(n)|$$

As $n \to \infty$, $|r^n| \to \infty$.

$$|y(n)| \to \infty$$

The system is BIBO unstable.

5) Static or dynamic:-

A system is static if output at any instant of time depends on input at the same time.

E.g. $y(n) = x^2(n) + 5x(n) + 10$

A system is dynamic if output at any instant of time depends on input at the same time as well as at other times.

Date:
Page:

E.g. $y(n) = x(n) + 5x(n-1)$

$y(n) = 3x(n+2) + x(n)$

Block diagram of general communication system:-

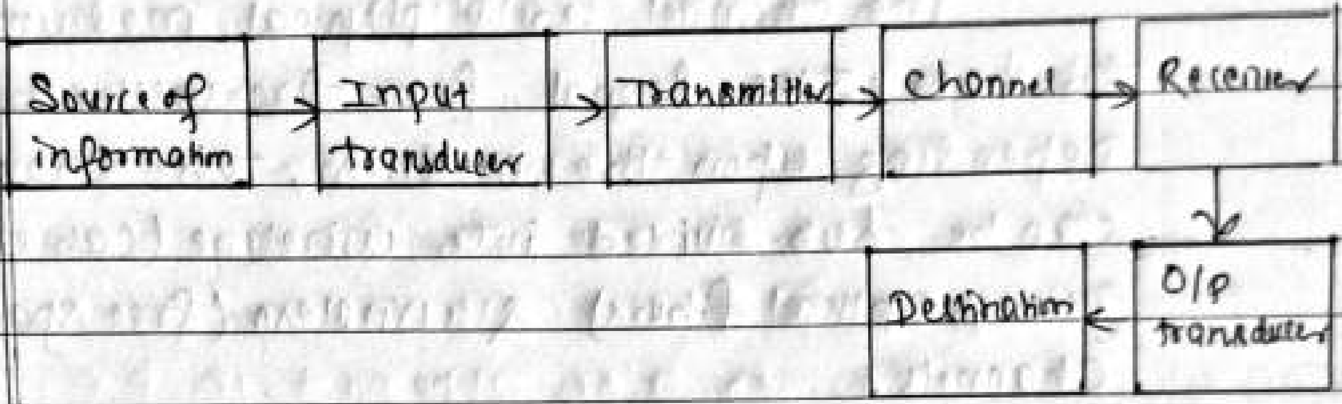


fig: Block diagram of analog communication system

communication system is designed to send information from a source generating that information to one (point to point communication) or more (broadcasting) receivers of that information.

In very simplified form, the analog communication system comprises of the source of information, input transducer, transmitter, channel, receiver, output transducer & the recipient of the information.

Source of information could be any device or person or event whose output could be voice, picture, text, sequence of symbols etc.

Input transducer converts the input information into electrical signal suitable for further processing.

Transmitter shapes the signal to match

it with the characteristics of the channel.

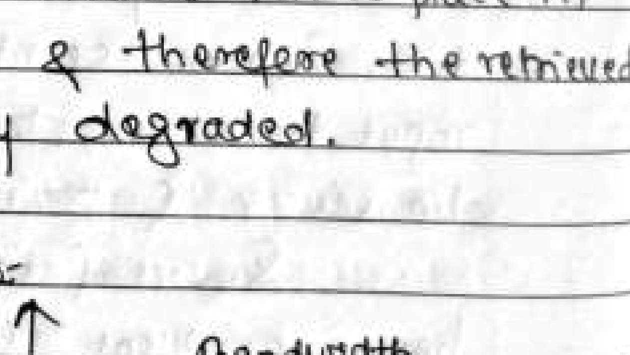
The signal processing could be amplification and / or modulation & radiation, in case of wireless communication.

The channel is a physical medium used to pass the signal from transmitter to receiver. Depending upon the medium, the channels can be sub-divided into wireline (cables, wave-guide, optical fibers) & wireless (free space) channels.

The receiver unit of a communication system recover the required information bearing signal from the mix of various signal at its input. Basic processes involved in the receiver are filtering, demodulation & amplification. The signal processing in the receiver takes place in the presence of noise & therefore the retrieved information is usually degraded.

Concept of bandwidth:-

Bandwidth may be defined as the portion of the electromagnetic spectrum occupied by a signal.



We may also define the bandwidth as

Date:
Page:

the frequency range over which an information signal is transmitted.

Bandwidth is the difference between upper & lower frequency limits of the signal.

The bandwidth of voice signal for telephony.

$$BW = (3400 - 300)Hz = 3100 Hz$$

Noise & its effect on communication system:-

Noise is unwanted / man made or natural random signal that adds to the received signal. It degrades the performance of communication system.

Types of noise:-

i) Thermal noise
ii) Shot noise
iii) Partition noise
iv) Flicker noise (low frequency noise)
v) Transit time noise (High frequency noise)
vi) Generation & recombination noise

Thermal noise:-

It is due to random movement of free electrons in the conductor. Average thermal noise can be estimated as,

$$P_n = KTB \text{ Watts}$$

where, K - Boltzman's constant $= 1.38 \times 10^{-23}$ J/K

Date:
Page:

T = Temperature of conductor
B = Bandwidth of noise (Hz)
P_n = Average noise power
PSD of thermal noise spectrum will be
$S_n = KT$ (Watt/Hz)

Shot noise:-
It is also called anode current noise. It is due to random fluctuation of electron emission from the cathode.
It is also appeared in p-n junction diode while crossing the potential barrier.

Partition noise:-
It is due to random fluctuations in division when current is divided into two or more paths. Partition noise is produced when emitter current is divided into base & collector current.

Flicker noise / LF noise:-
Below a frequency of few KHz, a noise appear in the device is called flicker noise.

Transit time / HF noise:-
It is produced when signal period is very low or the frequency of signal is very high.

Date :

Page:

Generation-recombination noise:-

It is due to random ionization of impurities in semiconductor. It is due to doping of impurities into semiconductor.

Need of modulation:-

1) To separate signal from different transmitters
2) Reduces size of antenna.
3) Transmit information to long distance without interference
4) Increases range of communication
5) Wireless communication

Continuous-wave linear Modulation

Date:

Page:

Modulation:-

It is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave.

The message signal is referred to as the modulating wave & the result of the modulation process is referred to as the modulated wave.

At the receiving end of the communication system, we usually require the message signal to be recovered. This is accomplished by using a process known as demodulation or detection, which is the inverse of the modulation process.

Amplitude Modulation:-

Consider a sinusoidal carrier wave c(t) defined by

$$c(t) = A_c \cos(2\pi f_c t) \cdots (1)$$

Where the peak value A_c is called carrier amplitude & f_c is called carrier frequency.

Let m(t) denote the baseband signal that carries specification of the message, we refer to m(t) as the message signal.

Amplitude modulation is defined as the process in which the amplitude of the carrier wave c(t) is varied linearly with the

Date:
Page:

message signal m(t).

Time domain description of DSB-AM:-

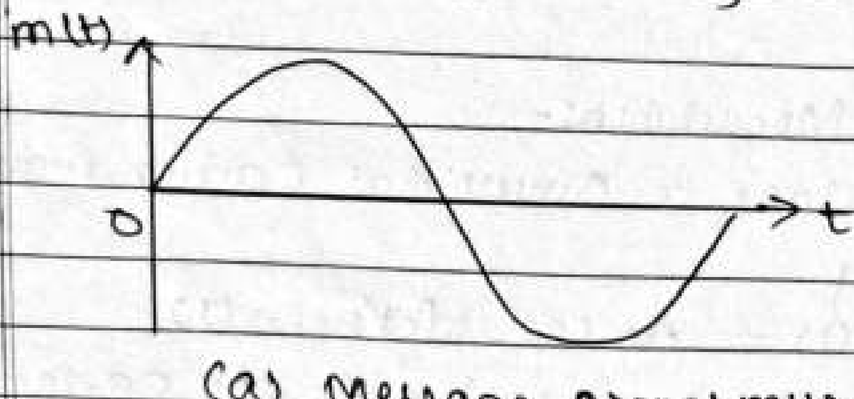
The standard form of AM wave is defined by

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t) \quad \text{---(2)}$$

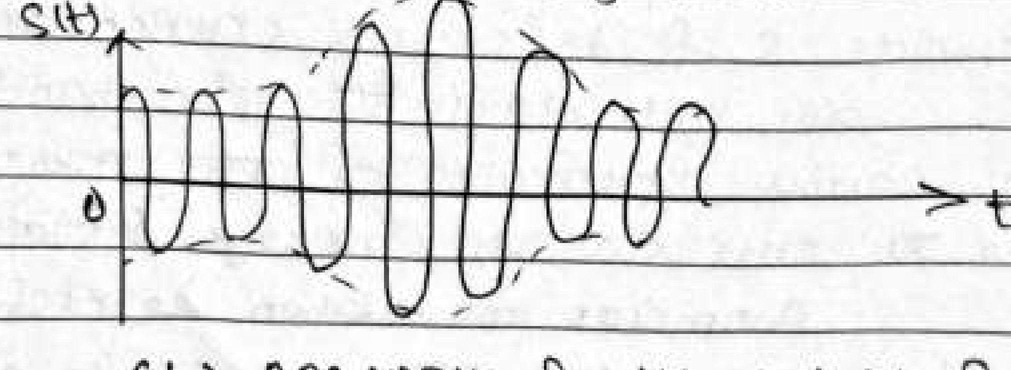
where k_a is a constant called amplitude sensitivity of the modulator.

The amplitude of the time function multiplying $\cos(2\pi f_c t)$ in eqn(2) is called envelope of AM wave s(t). Using a(t) to denote this envelope, we may thus write,

$$a(t) = A_c[1 + k_a m(t)] \quad \text{---(3)}$$

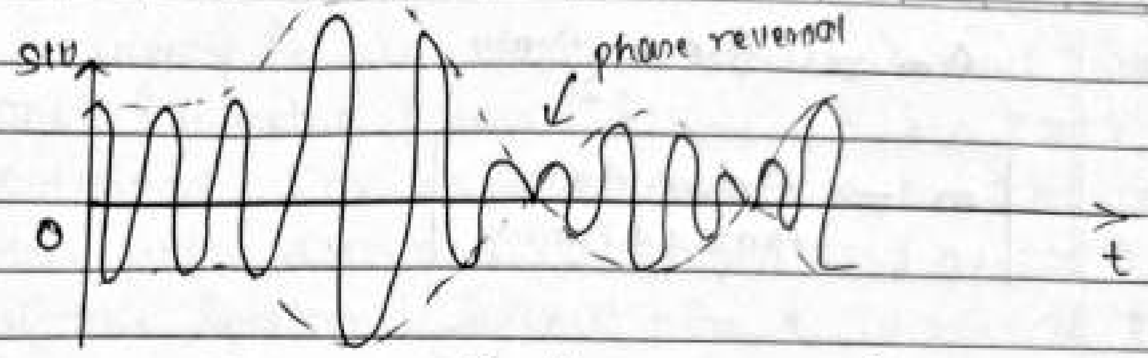


(a) Message signal m(t)



(b) AM wave for $|k_a m(t)| < 1$ for all t

Date :
Page:



(C) AM wave for $|k_a m(t)| > 1$ for some t.

The maximum absolute value of $k_a m(t)$ multiplied by 100 is referred to as percentage modulation. Accordingly, fig(b) corresponds to percentage modulation less than or equal to 100% whereas fig(c) corresponds to percentage modulation in excess of 100%.

Modulation index or modulation factor :-

It is defined as the ratio of amplitude of modulating & carrier waves.

$$m = \frac{A_m}{A_c}$$

It has values between 0 & 1.

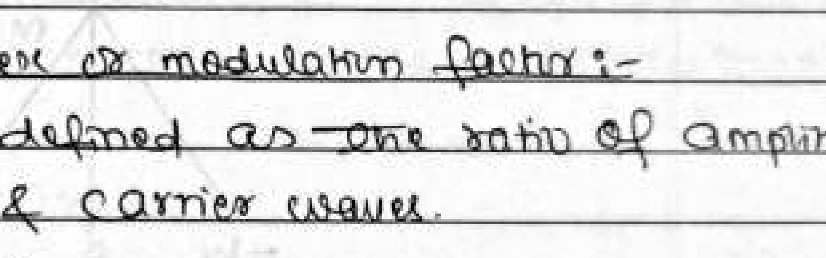
$$\text{Percent modulation} = \frac{A_m}{A_c} \times 100\%$$

It can be calculated graphically as

$A_{max} = A_c + A_m$

$A_{min} = A_c - A_m$

$A_c = \frac{A_{max} + A_{min}}{2}$



$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$\boxed{m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

Spectral representation of DSB-AM :-

To develop frequency description of AM wave, we take Fourier transform of both sides of eqn(i).

Let $S(f)$ denote Fourier transform of $s(t)$ & $M(f)$ denote Fourier transform of $m(t)$.

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{K_a A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

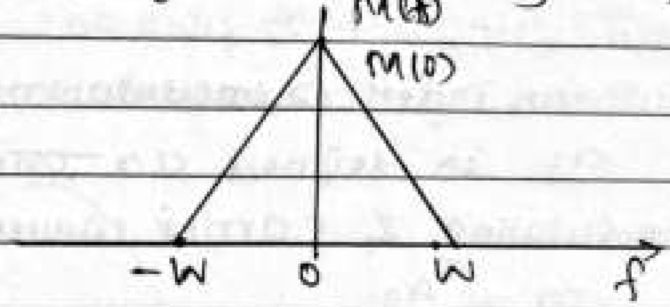


fig: Spectrum of message signal

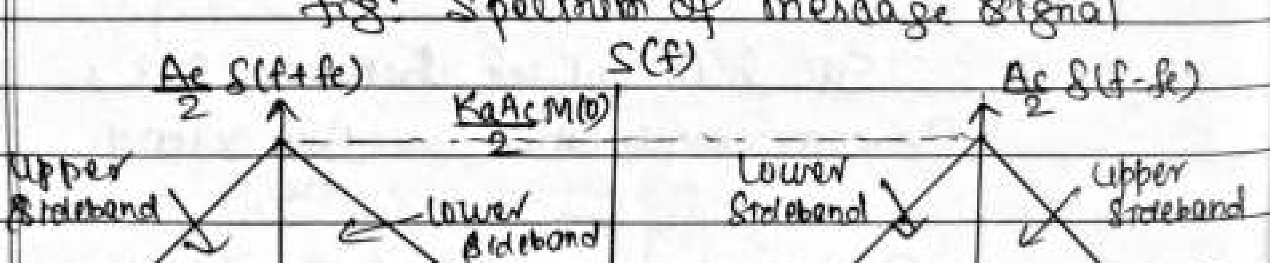


fig: Spectrum of AM wave

Spectrum may be described as follows:-

1) For positive frequencies, the portion of the spectrum of the modulated wave lying above the carrier

Date:
Page:

frequency f_c is called upper sideband, where as the symmetric portion below f_c is called lower sideband. For negative frequencies, the image of the upper sideband is represented by the portion of the spectrum below $-f_c$ & image of lower sideband by the portion above $-f_c$. The condition $f_c > W$ ensures that the sidebands don't overlap.

2) For positive frequencies, the highest frequency components of AM wave is $f_c + W$ & lower frequency component is $f_c - W$. The difference between these two frequencies defines the transmission bandwidth, B for AM wave which is exactly twice the message bandwidth W, that is,

$$B = 2W$$

Single tone AM:-

Let, $m(t) = A_m \cos(2\pi f_m t)$

AM wave,

$$s(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

$$= A_c[1 + m\cos(2\pi f_m t)] \cos(2\pi f_c t) \quad [\because m = k_a A_m]$$

$$= A_c \cos(2\pi f_c t) + \frac{mA_c}{2} \cos(2\pi(f_c + f_m)t) + \frac{mA_c}{2} \cos(2\pi(f_c - f_m)t))$$

Power in AM wave,

Date:
Page:

$$P_t = P_c + P_{LSB} + P_{USB}$$

Carrier power $= P_c = \frac{(A_c/\sqrt{2})^2}{R} = \frac{A_c^2}{2R}$

Lower sideband power $= P_{LSB} = \left(\frac{mA_c/2}{\sqrt{2}}\right)^2 \cdot \frac{1}{R}$

$$= \frac{m^2 A_c^2}{8R}$$

Upper sideband power $= P_{USB} = \left(\frac{mA_c/2}{\sqrt{2}}\right)^2 \cdot \frac{1}{R}$

$$= \frac{m^2 A_c^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{m^2 A_c^2}{4R} = \frac{A_c^2}{2R}\left(1 + \frac{m^2}{2}\right)$$

$$\boxed{P_t = P_c\left(1 + \frac{m^2}{2}\right)}$$

likewise,

$$P_t = I_t^2 R \qquad P_c = I_c^2 R$$

$$I_t^2 R = I_c^2 R\left(1 + \frac{m^2}{2}\right)$$

$$\boxed{I_t = I_c\sqrt{1 + \frac{m^2}{2}}}$$

Efficiency of AM:

$$\eta = \frac{\text{Useful power at output}}{\text{Total power consumed}} = \frac{P_{LSB} + P_{USB}}{P_t} \times 100\%$$

$$= \frac{m^2}{2 + m^2} \times 100\%$$

Date:
Page:

Maximum efficiency = 33.33% when $m=1$.

A sinusoidal carrier has amplitude of 10V & frequency 30kHz. It is amplitude modulated by a sinusoidal voltage of amplitude 3V & frequency 1kHz. Modulated voltage is developed across 50Ω ~~resistor~~ resistance.

a) Determine modulation index.

b) Write the equation for modulated wave.

c) Plot the modulated wave showing maxima & minima of waveform.

d) Draw the spectrum of modulated wave.

Soln:-

$A_c = 10V$, $f_c = 30kHz$, $A_m = 3V$, $f_m = 1kHz$

$R_L = 50\Omega$

a) Modulation index,

$$m = \frac{A_m}{A_c} = \frac{3}{10} = 0.3$$

b) Eqn of AM wave,

$$s(t) = A_c[1 + m\cos 2\pi f_m t]\cos 2\pi f_c t$$
$$= 10[1 + 0.3\cos(2\pi \times 10^3 t)]\cos(6\pi \times 10^4 t)$$

c) $A_{max} = A_c + A_m = 13V$

$A_{min} = A_c - A_m = 7V$

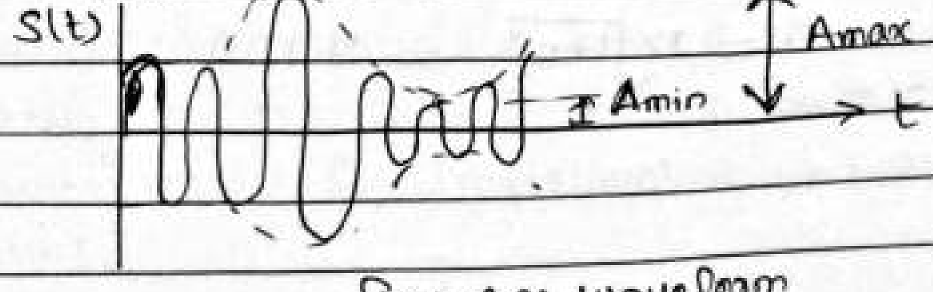


fig: AM waveform

Date :

Page:

2) Upper sideband frequency $= f_c + f_m = 31\,\text{KHz}$

lower " " $= f_c - f_m = 29\,\text{KHz}$

Amplitude of each sideband $= \frac{mA_c}{2} = 1.5V$

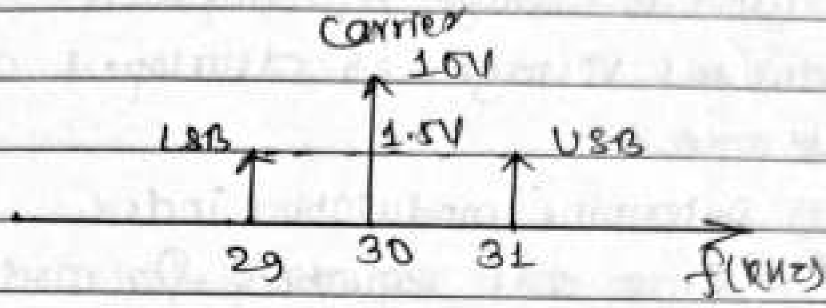


fig: Spectrum of AM wave

\# The antenna current of AM transmitter is 8A if only the carrier is sent, but it increases to 8.93A if the carrier is modulated by a single sinusoidal wave. Determine the percentage modulation. Also find the antenna current if the percent of modulation changes to 0.8.

Soln:-

$I_c = 8A$, $I_t = 8.93A$

$$I_t = I_c\sqrt{1+\frac{m^2}{2}}$$

$m = 0.701 = 70.1\%$

Again,

$m = 0.8$ $I_t = ?$

$$I_t = I_c\sqrt{1+\frac{m^2}{2}} = 9.19A$$

Date:
Page:

An AM transmitter radiates 9KW of power when the carrier is unmodulated & 10.125KW when the carrier is sinusoidally modulated. Find the modulation index, percentage of modulation. Now if another sine wave corresponding to 40% modulation is transmitted simultaneously, then calculate the total radiated power.

Soln:-

$P_c = 9KW$, $P_t = 10.125 KW$

$P_t = P_c(1+m^2/2)$

$m = 0.5 = 50\%$

Again,

$m_1 = 0.5$, $m_2 = 0.4$, $P_t = ?$

$m_t = \sqrt{m_1^2+m_2^2} = 0.64$

$P_t = P_c\left(1+\frac{m_t^2}{2}\right) = 10.84 KW$

Generation of DSB-AM:-

i) Square-law modulator:-

A square-law modulator requires three features: a means of summing the carrier & modulating waves, a nonlinear element & a bandpass filter for extracting the desired modulation products. Semiconductor diodes & transistors are the most common non-linear devices used for implementing square law modulators.

The transfer characteristic of non-linear device can be represented as

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \quad \text{---} (1)$$

where a_1 & a_2 are constants. The input voltage $v_1(t)$ consists of the carrier wave plus the modulating wave, that is,

$$v_1(t) = A_c \cos(2\pi f_c t) + m(t) \quad \text{---} (2)$$

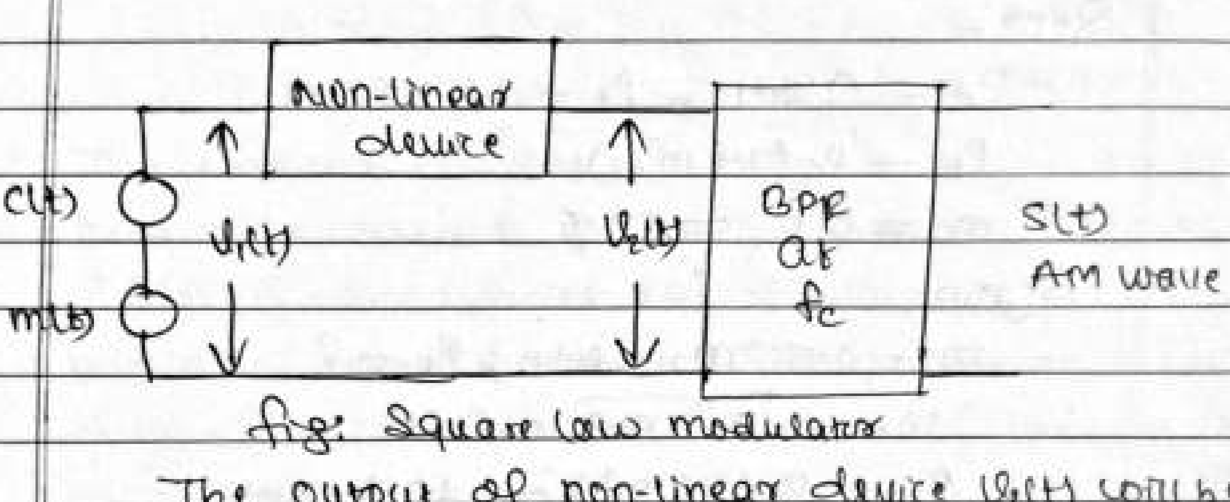


fig: Square law modulator

The output of non-linear device $v_2(t)$ will be

$$v_2(t) = a_1\{A_c\cos(2\pi f_c t) + m(t)\} + a_2\{A_c\cos(2\pi f_c t) + m(t)\}^2$$

$$= a_1 A_c \cos 2\pi f_c t\left[1 + \frac{2a_2}{a_1} m(t)\right] + a_1 m(t) + a_2 m^2(t)$$

$$+ \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2}\cos(4\pi f_c t) \quad \text{---} (3)$$

BPF centered at f_c will filter out the frequency component centered at $2f_c$, message signal $m(t)$, dc component & therefore the output of BPF will be.

$$s(t) = a_1 A_c[1 + k_a m(t)]\cos(2\pi f_c t) \quad \text{---} (4)$$

Date: ____________
Page: ____________

Where $K_a = \frac{2a_2}{a_1}$

(ii) Switching modulator :-

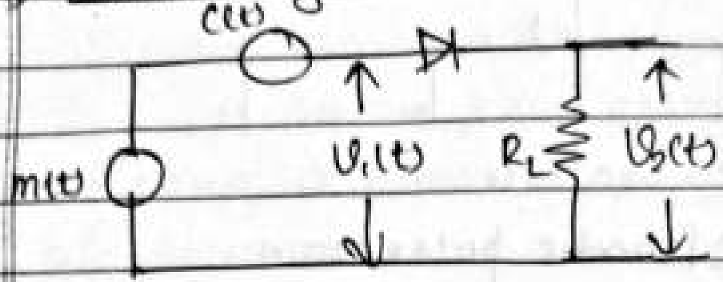


fig: Switching modulator

It is assumed that the carrier wave c(t) applied to the diode is large in amplitude. We also assume that the diode acts as ideal switch, i.e. it presents zero impedence when it is forward biased [corresponding to c(t)>0] & infinite impedence when it is reverse-biased [corresponding to c(t)<0].

Input voltage $V_1(t)$ is given by

$$V_1(t) = A_c\cos(2\pi f_c t) + m(t) \quad \text{---(1)}$$

Where $|m(t)| << A_c$, the resulting load voltage $V_2(t)$ is

$$V_2(t) = \begin{cases} V_1(t) & , c(t)>0 \\ 0 & , c(t)<0 \end{cases} \quad \text{---(2)}$$

That is, the load voltage $V_2(t)$ varies periodically between the values $V_1(t)$ & zero at a rate equal to carrier frequency f_c.

We may express eqn(2) as,

$$V_2(t) = [A_c\cos(2\pi f_c t) + m(t)]\, g_p(t) \quad \text{--(3)}$$

Where $g_p(t)$ is a periodic pulse train of duty cycle

Date: ______
Page: ______

equal to one half & period $T_0 = 1/f_c$ as shown in fig.

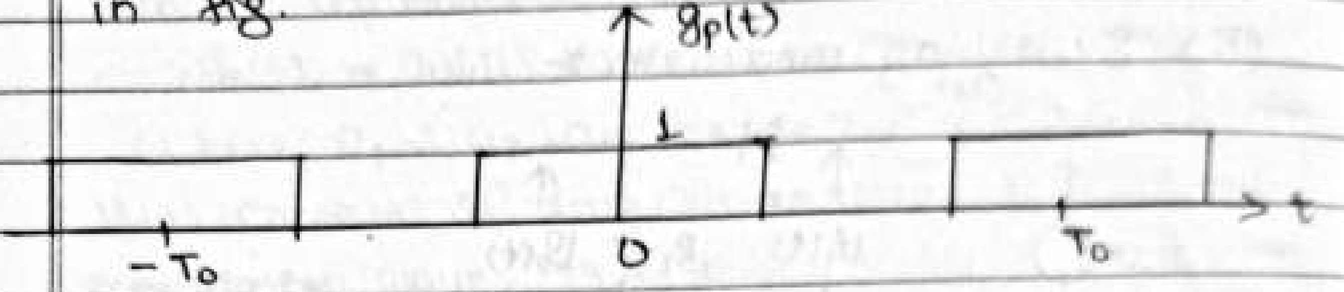


fig: Periodic pulse train

Representing $g_p(t)$ by its Fourier Series,

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)] \quad \text{--(1)}$$

$$= \frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) + \text{odd harmonic components} \quad \text{--(4)}$$

Substituting eq^n(4) in eq^n(3), $V_2(t)$ is expressed as

$$V_2(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos(2\pi f_c t) + \text{unwanted terms} \quad \text{--(5)}$$

The first term of eq^n(5) is the desired AM wave with amplitude sensitivity $K_a = \frac{4}{\pi A_c}$. The unwanted terms are removed from the load voltage $V_2(t)$ by means of a band pass filter.

Detection of AM waves:-

The process of detection or demodulation provides means of recovering the message signal from an incoming modulated wave. In effect, detection is

The inverse of modulation. We describe two devices for the detection of AM waves, namely, the square-law detector & the envelope detector.

i) Square-law detector:-

A square law detector is essentially obtained by using a square-law modulator for the purpose of detection.

The transfer characteristics of non-linear device is

$$V_2(t) = a_1 V_1(t) + a_2 V_1^2(t) \quad \text{--- (1)}$$

Where $V_1(t)$ & $V_2(t)$ are the input & output voltages respectively & a_1 & a_2 are constants.

When such a device is used for the demodulation of AM wave, we've for the input,

$$V_1(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t) \quad \text{- (2)}$$

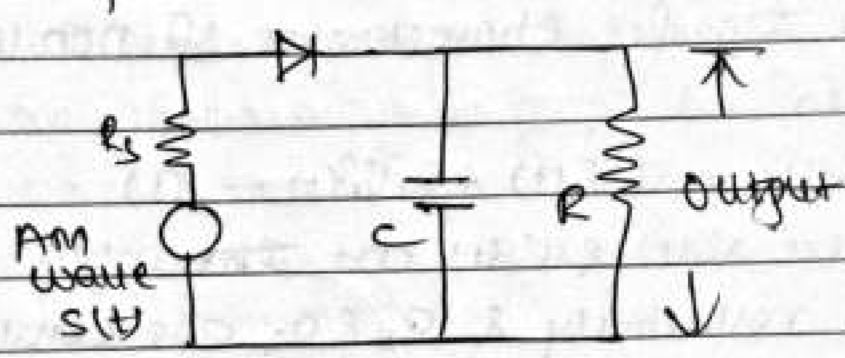
Therefore, substituting eqn (2) in eqn (1), we get,

$$V_2(t) = a_1 A_c [1 + K_a m(t)] \cos(2\pi f_c t) + \tfrac{1}{2} a_2 A_c^2 [1 + 2K_a m(t) + K_a^2 m^2(t)] [1 + \cos(4\pi f_c t)] \quad \text{- (3)}$$

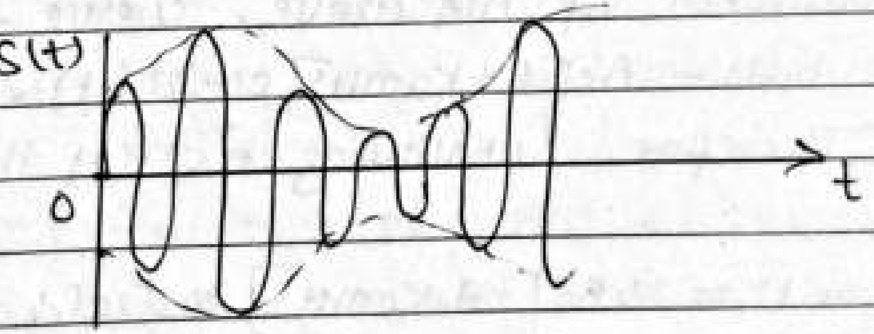
The desired signal $a_2 A_c^2 K_a m(t)$ can be extracted by means of low-pass filter.

ii) Envelope detector:-

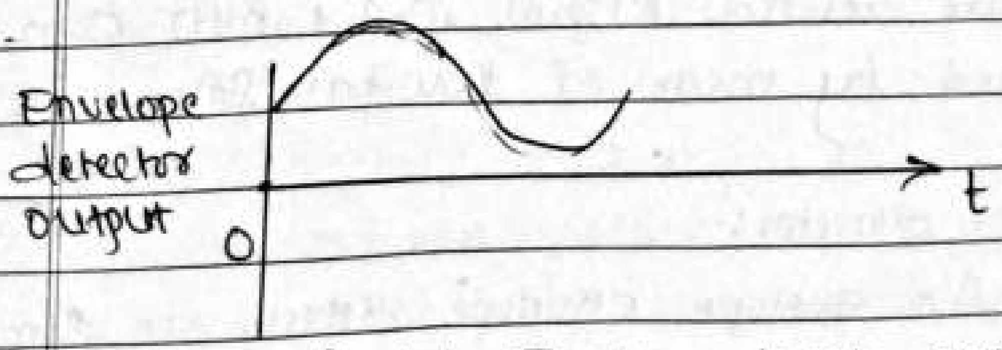
An ~~envelope~~ envelope detector is simple & yet highly effective device that is well-suited for the demodulation of a narrow-band AM wave, for which the percentage modulation is less than 100%. Ideally, an envelope detector produces an output signal that follows the envelope of the input signal waveform exactly, hence the name. Fig shows the circuit diagram of an envelope detector that consists of a diode & resistor-capacitor filter.



fig(a): circuit diagram



fig(b): AM wave input



fig(c): Envelope detector output

The operation of this envelope detector is as follows : on the positive half-cycle of the input signal, the diode is forward biased & the capacitor C charges up rapidly to the peak value of the input signal. When the input signal falls below this value, the diode becomes reversed-biased & the capacitor C discharges slowly through the load resistor R_L. The discharging process continues until the next positive half cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again & the process is repeated.

In order to minimize the distortion & maximize the filtration of high frequency ripples, the time constant of RC filter is selected in the following manner :

$$W << \frac{1}{RC} << f_c$$

W is message bandwidth

Double Sideband Suppressed Carrier (DSB-SC) modulation :-

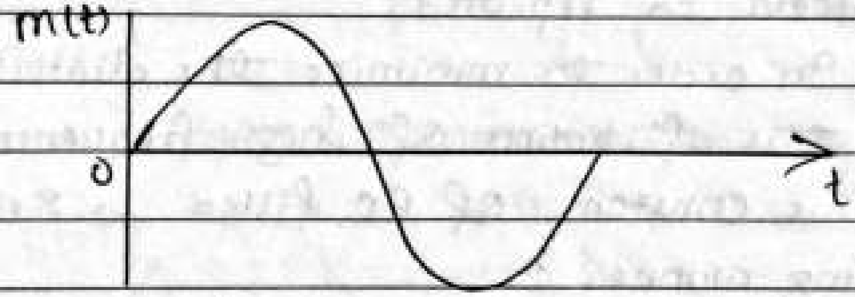
In the standard form of AM, the carrier wave is completely independent of the message signal which means that the transmission of the carrier wave represents a waste of power. We may suppress the carrier component from the modulated wave, resulting in double-sideband suppressed carrier modulation.

Time domain description of DSB-SC wave:-

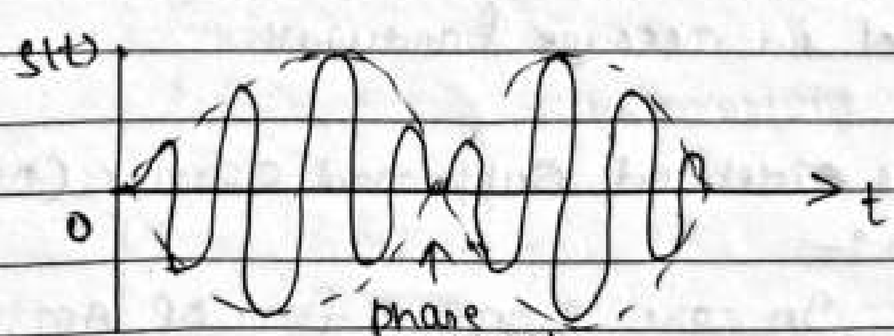
To describe DSB-SC modulated wave as a function of time, we write,

$$s(t) = c(t)m(t)$$
$$= A_c\cos(2\pi f_c t)\, m(t) \quad \text{---(1)}$$

This modulated wave undergoes a phase reversal whenever the message signal m(t) crosses zero as shown in fig.



fig(a) : Message signal



fig(b) : DSB-SC modulated wave

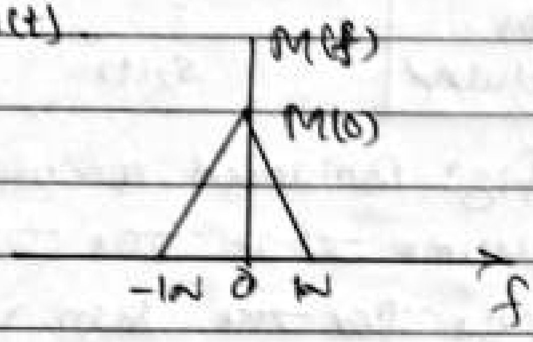
Date:
Page:

Frequency domain description of DSB-SC wave:-

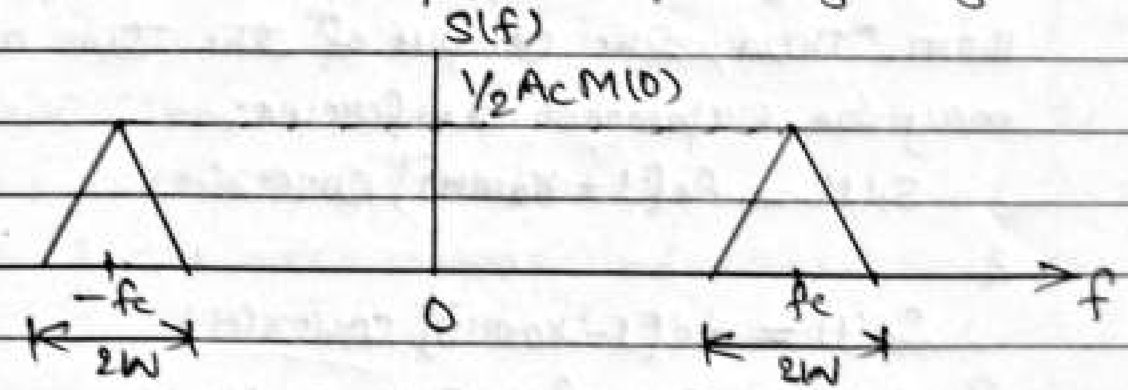
By taking the Fourier transform of both sides of eqn(1), we get,

$$S(f) = \frac{1}{2} A_c [M(f-f_c) + M(f+f_c)] \quad \ldots (2)$$

Where $S(f)$ is Fourier transform of modulated wave $s(t)$ & $M(f)$ is Fourier transform of message signal $m(t)$.



(a) Spectrum of message signal



(b) Spectrum of DSB-SC modulated wave

Generation of DSB-SC waves:-

1) Balanced Modulator:-

A balanced modulator consists of two standard amplitude modulators arranged in a balanced configuration so as to suppress the carrier wave,

Date:
Page:

as shown in the block diagram.

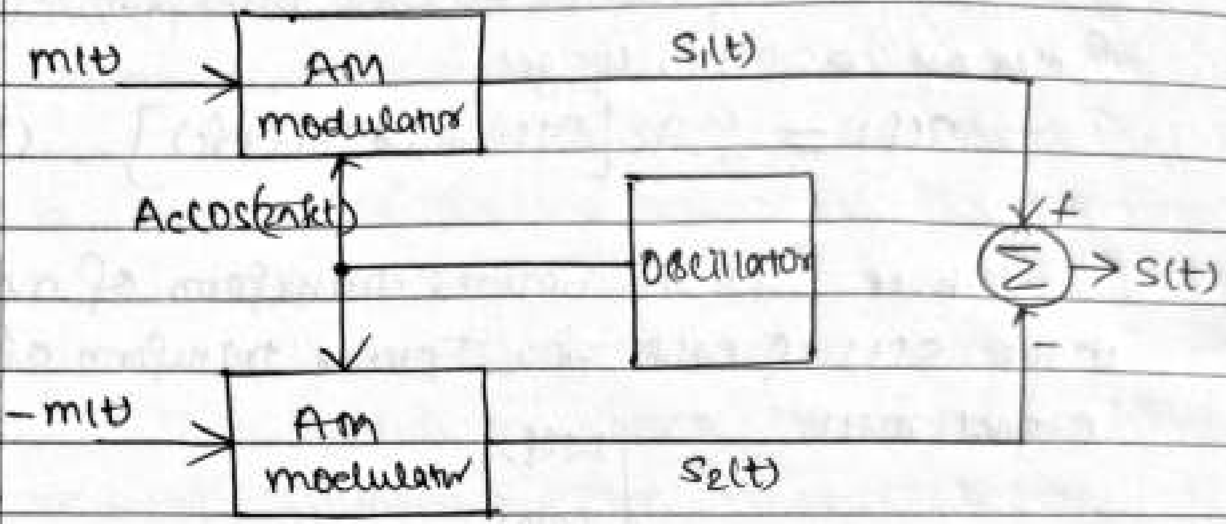


fig: Balanced modulator

We assume that the two modulators are identical except for the sign reversal of the modulating wave applied to the input of one of them. Thus, the output of the two modulators may be expressed as follows:-

$$S_1(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t)$$

&

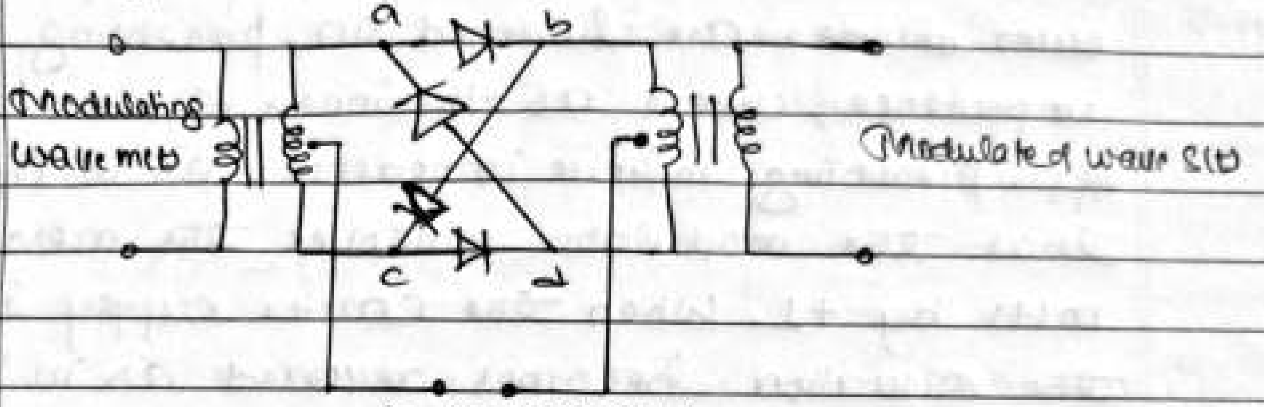
$$S_2(t) = A_c[1 - K_a m(t)] \cos(2\pi f_c t)$$

Subtracting $S_2(t)$ from $S_1(t)$, we obtain,

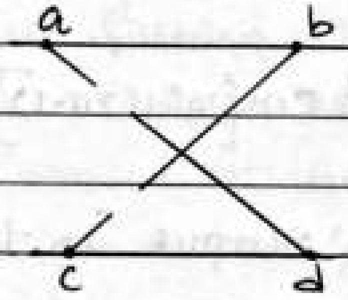
$$S(t) = S_1(t) - S_2(t)$$
$$= 2K_a A_c \cos(2\pi f_c t)\, m(t)$$

Hence, except for the scaling factor $2K_a$, the balanced modulator output is equal to the product of the modulating wave & the carrier, as required.

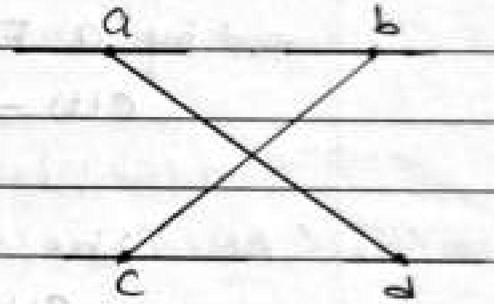
2) Ring Modulator :-



fig(a) : Ring Modulator



fig(b): The condition when the outer diodes are switched on & the inner diodes are switched off.



fig(c): The condition when the outer diodes are switched off & inner diodes are switched on.

It is also known as lattice or double-balanced modulator. The four diodes in fig(a) form a ring in which they all point in the same way. The diodes are controlled by a square-wave carrier c(t) of frequency fc, which is applied by means of two center-tapped transformers. We assume that the diodes are ideal & the transformers are perfectly

Date:
Page:

balanced. When the carrier supply is positive, the outer diodes are switched on, presenting zero impedence, where as the inner diodes are switched off, presenting infinite impedence, as in fig(b), so that the modulator multiplies the message signal m(t) by +1. When the carrier supply is negative, the situation becomes reversed as in fig(c) & the modulator multiplies the message signal by -1.

The square wave carrier c(t) can be represented by a Fourier series,

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]$$

The ring modulator output is therefore

$$s(t) = c(t)m(t)$$

$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)} \cos[2\pi f_c t(2n-1)]\, m(t)$$

$$= \frac{4}{\pi} m(t) \cos(2\pi f_c t) + \ldots$$

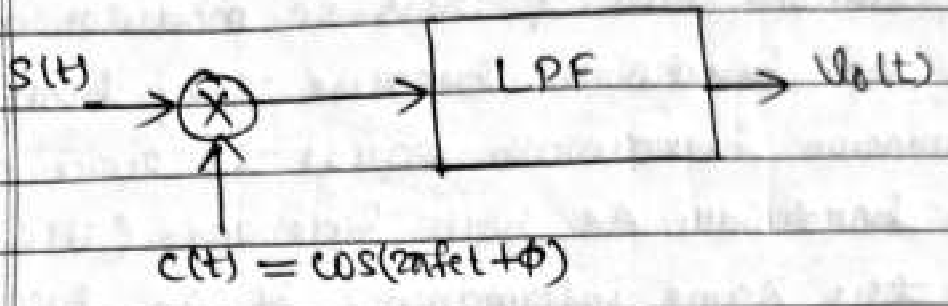
After passing through BPF centered at fc,

$$\boxed{s(t) = \frac{4}{\pi} m(t) \cos(2\pi f_c t)}$$

Date: ____________

Page: ____________

Synchronous demodulation of DSB-SC wave:-



The message signal m(t) is recovered from a DSB SC wave s(t) by first multiplying s(t) with a locally generated sinusoidal wave & then low pass filtering the product as shown in fig.

$$V(t) = \cos(2\pi f_c t + \phi)\, s(t)$$

$$= A_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi)\, m(t)$$

$$= \frac{1}{2} A_c \cos\phi\, m(t) + \frac{1}{2} A_c \cos(4\pi f_c t + \phi)\, m(t)$$

Now,

$$V_0(t) = \frac{1}{2} A_c \cos\phi\, m(t)$$

The demodulated signal $V_0(t)$ is therefore proportional to m(t) when the phase error ϕ is a constant. The amplitude of this demodulated signal is maximum when $\phi = 0$ & is minimum (zero) when $\phi = \pm\pi/2$. The zero demodulated signal, which occurs for $\phi = \pm\pi/2$, represents the quadrature null effect of the synchronous detector.

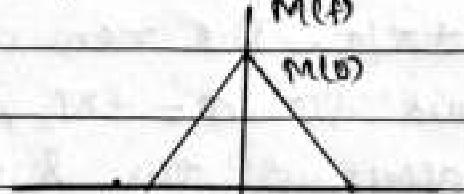
Date: ______
Page: ______

Single Sideband (SSB) modulation:-

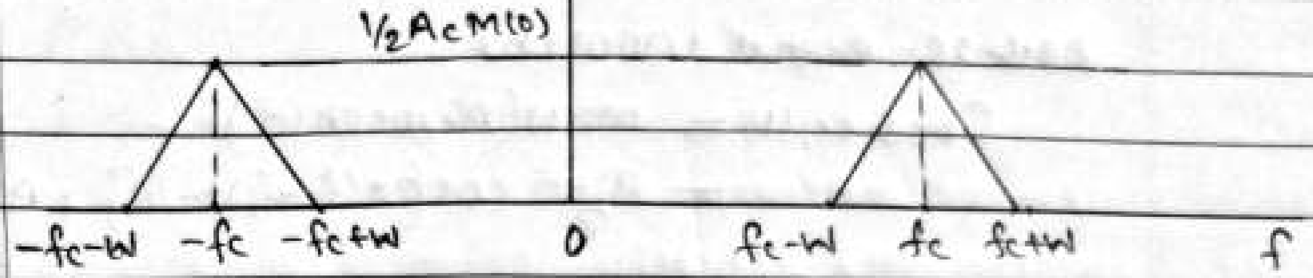
Standard AM & DSB-SC modulation are wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth. As both sidebands (USB & LSB) carry the same information, it is sufficient to transmit only one sideband to recover the information at the receiving end. In this way, we will conserve the power as well as the bandwidth. This type of modulation is called single sideband modulation.

Frequency domain description of SSB wave:-

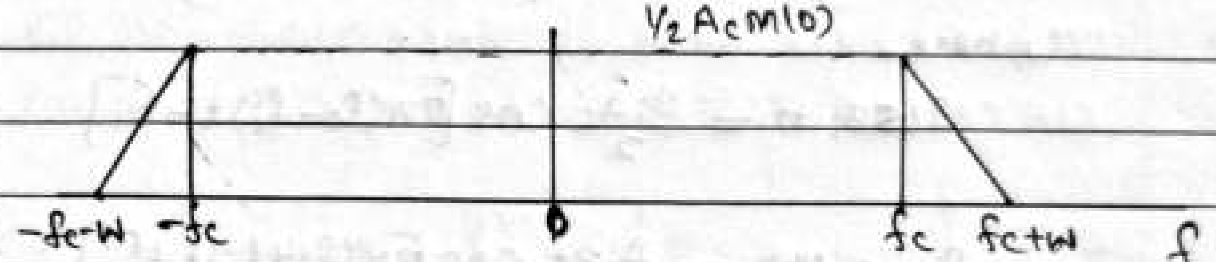
Consider a message signal m(t) with a spectrum M(f) limited to the band $-W \le f \le W$ as shown in fig(1). The spectrum of DSB-SC modulated wave, obtained by multiplying m(t) by the carrier wave $A_c \cos(2\pi f_c t)$, is shown in fig(2). When only uppersideband is transmitted, the resulting SSB modulated wave has the spectrum shown in fig(3) & when only the lower sideband is transmitted, the spectrum is shown in fig(4).



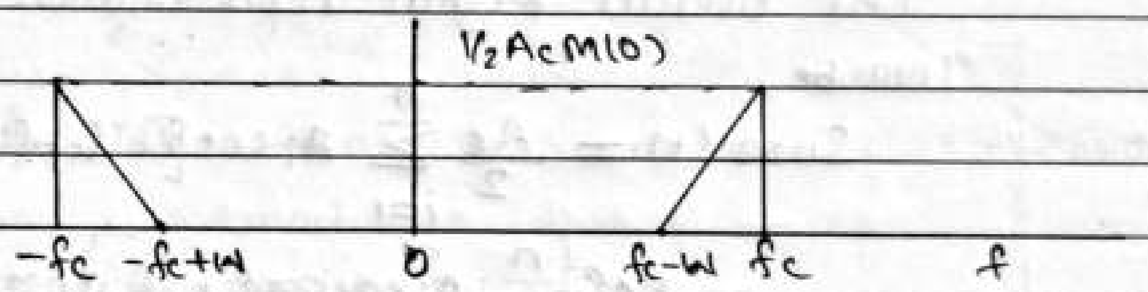
fig(1): Message Spectrum



fig(2): DSB-SC Spectrum



fig(3): SSB Spectrum with upper sideband transmitted



fig(4): SSB spectrum with lower sideband transmitted

Time domain description of SSB wave:-

In real case, the modulating signal is not the sinusoidal but a complex signal m(t) which can be represented as sum of large number of sinusoidal signals of various amplitude, frequency & phase i.e.

$$m(t) = \sum_{i=1}^{n} A_i \cos(2\pi f_i t + \theta_i) \quad \text{---} (1)$$

For i-th component,

$$m_i(t) = A_i \cos(2\pi f_i t + \theta_i) \quad \text{---} (2)$$

Date:

Page:

DSB-SC signal would be,

$$S_{DSB\text{-}SC_i}(t) = m_i(t)(A_c \cos 2\pi f_c t)$$

$$= \frac{A_i A_c}{2} \cos[2\pi(f_c - f_i)t - \theta_i] + \frac{A_i A_c}{2} \cos[2\pi(f_c + f_i)t + \theta_i] \quad \text{--(3)}$$

Where,

$$S_{LSB_i}(t) = \frac{A_i A_c}{2} \cos[2\pi(f_c - f_i)t - \theta_i] \quad \text{--(4)}$$

$$S_{USB_i}(t) = \frac{A_i A_c}{2} \cos[2\pi(f_c + f_i)t + \theta_i] \quad \text{--(5)}$$

The overall signal representation for USB will be,

$$S_{USB}(t) = \frac{A_c}{2} \sum_{i=1}^{n} A_i \cos[2\pi(f_c + f_i)t + \theta_i]$$

$$= \frac{A_c}{2} \left\{ \left[\sum_{i=1}^{n} A_i \cos(2\pi f_i t + \theta_i) \right] \cos 2\pi f_c t - \left[\sum_{i=1}^{n} A_i \sin(2\pi f_i t + \theta_i) \right] \sin 2\pi f_c t \right\}$$

$$= \frac{A_c}{2} \left[m(t) \cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t \right] \quad \text{--(6)}$$

Where,

$\hat{m}(t)$ = Hilbert transform of original message signal

Similarly,

$$S_{LSB}(t) = \frac{A_c}{2} \left[m(t) \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t \right] \quad \text{--(7)}$$

Generation of SSB wave :-

1) Frequency discrimination method :-

SSB modulator based on frequency discrimination consists basically of a product modulator & a filter designed to pass the desired sideband of DSB-SC modulated wave at the product modulator output & reject the other sideband.

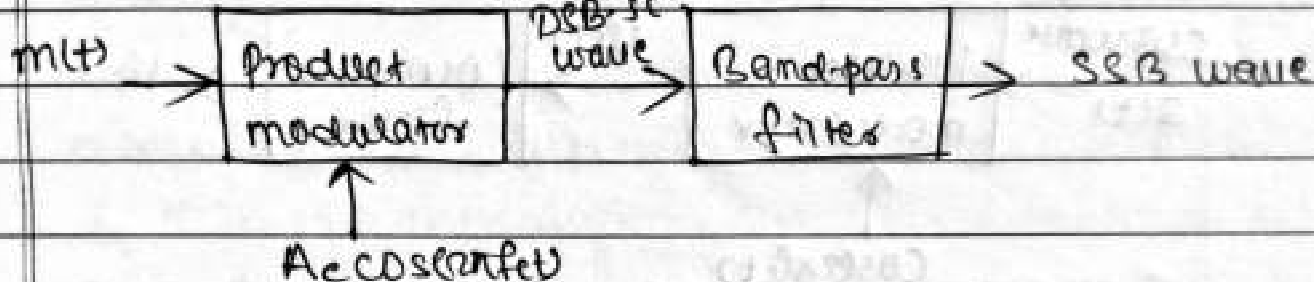
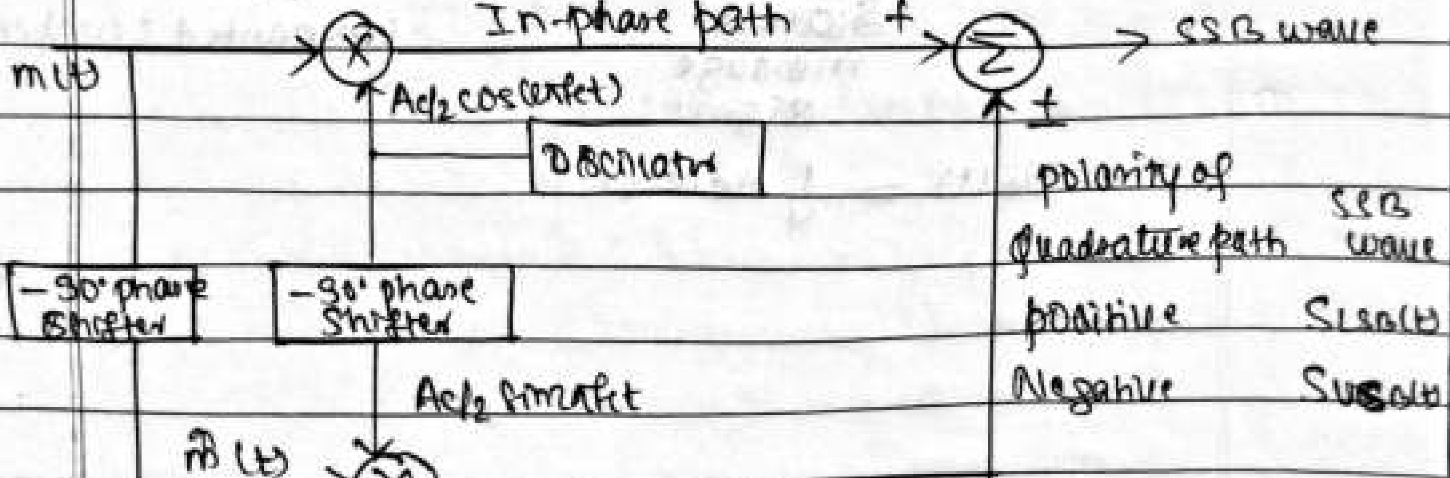


fig: Block diagram of frequency discrimination method

2) Phase discrimination method:-

SSB wave expression can be implemented directly by using DSB-SC modulators, Hilbert transformer & adder as shown below:



Demodulation of SSB wave:-

Modulating signal $m(t)$ can be recovered by using coherent detection, which involves applying SSB wave, together with a locally generated carrier $\cos(2\pi f_c t)$, to a product modulator & then low-pass filtering the modulator output as shown in fig.

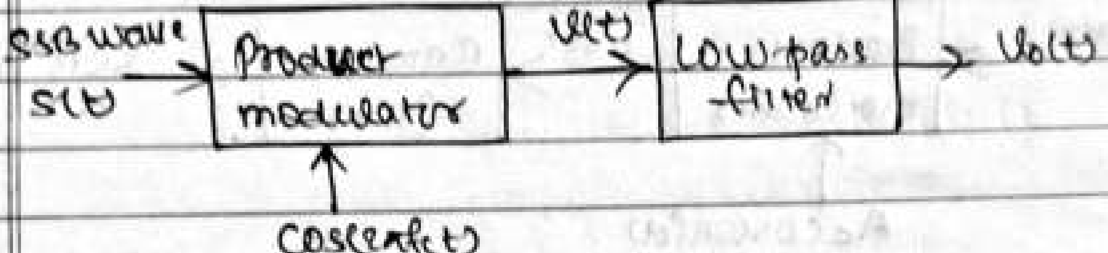


fig: Coherent detection of SSB wave

$$v(t) = \cos(2\pi f_c t)\, s(t)$$

$$= \frac{1}{2} A_c \cos(2\pi f_c t)\left[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin(2\pi f_c t)\right]$$

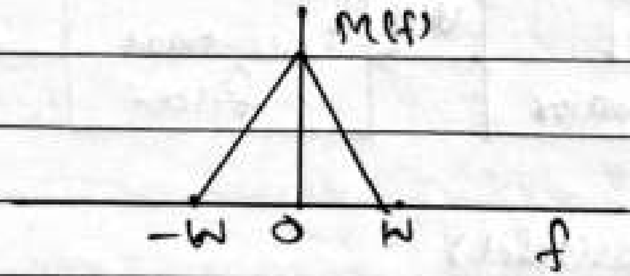
$$= \underbrace{\frac{1}{4} A_c m(t)}_{\text{Scaled message signal}} + \underbrace{\frac{1}{4} A_c\left[m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)\right]}_{\text{Unwanted component}}$$

$$V_o(t) = \frac{1}{4} A_c m(t)$$

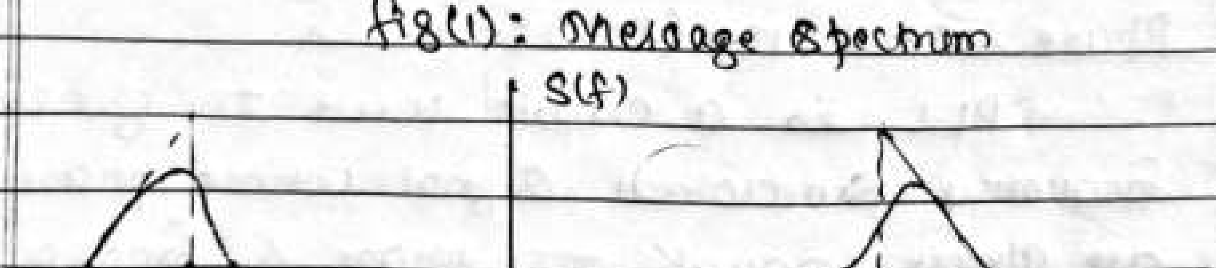
Date: ____________
Page: ____________

Vestigial Sideband (VSB) modulation :-

When the message signal contains significant components at extremely low frequencies, the upper & lower sidebands meet at the carrier frequency. This means that the use of SSB modulation is inappropriate for the transmission of such message signals owing to the difficulty of isolating one sideband. This difficulty suggests another scheme known as vestigial sideband (VSB) modulation, which is compromise between SSB & DSB-SC modulation. In this modulation scheme, one sideband is passed almost completely whereas just a trace, or vestige of the other sideband is retained. The spectrum of VSB wave is shown below :-



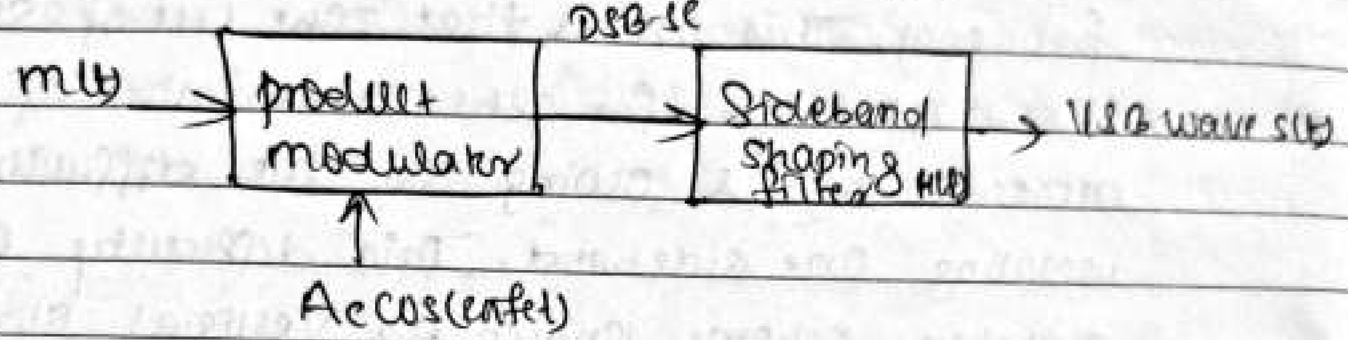
fig(1): Message spectrum



fig(2): Spectrum of VSB wave

Generation of VSB wave:-

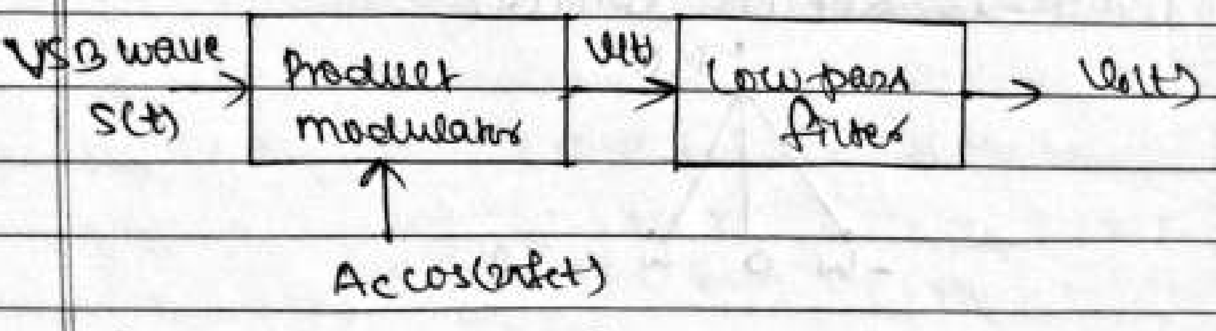
To generate VSB modulated wave, we pass DSB-SC modulated wave through a sideband shaping filter as shown in fig.



$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)$$

VSB demodulator:-

Message signal m(t) can be recovered by using coherent demodulator, as shown in fig.



Phase locked loop (PLL):-

PLL is a circuit used to generate high frequency sinusoidal signal whose phase & frequency are almost equal to phase & frequency of reference signal (incoming signal). It consist of a multiplier, Voltage controlled oscillator (VCO) & low pass filter.

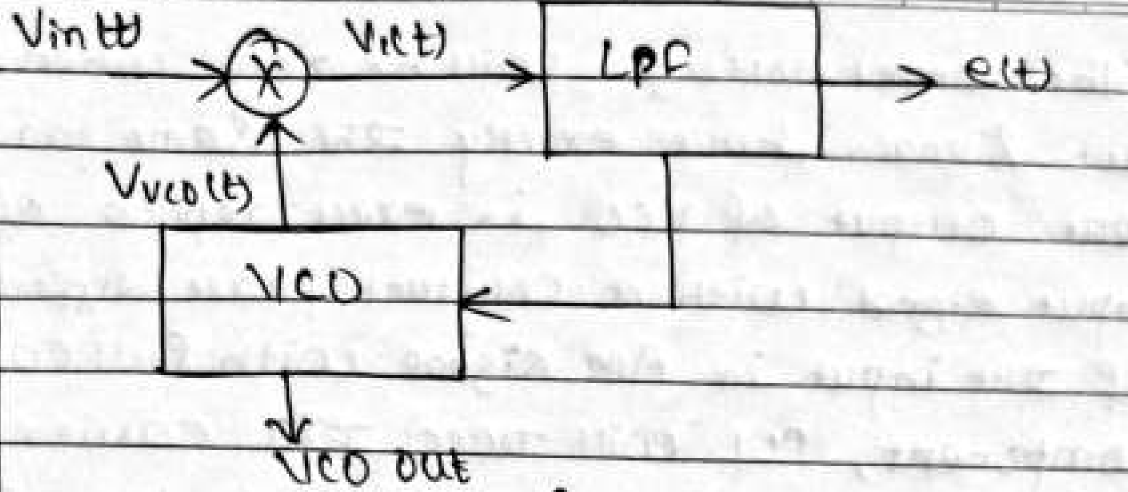


fig: PLL

The phase (& hence the frequency) of VCO is controlled by the error signal obtained at the output of the LPF.

Let us consider,

$V_{in}(t) = A_c \cos(2\pi f_c t + \psi_c)$ & $V_{vco}(t) = A \sin(2\pi f_c t + \theta)$

Here it is assumed that the frequency of two signals is same but differ in phase only.

The output of multiplier,

$$V_1(t) = V_{in}(t)\, V_{vco}(t)$$

$$= \frac{A_c A}{2} \sin(\theta - \psi_c) + \frac{A_c A}{2} \sin[4\pi f_c t + (\psi_c + \theta)]$$

The second term of which is filtered out & the output of LPF,

$$e(t) = \frac{A_c A}{2} \sin(\theta - \psi_c)$$

Assuming that the phase difference is small

$$e(t) \simeq \frac{A_c A}{2} (\theta - \psi_c)$$

The control voltage will be zero when the two signals have exactly the same phase. Hence the output of VCO is exact replica of the input signal with a constant phase difference of 90°. If the input is AM signal with full carrier component, PLL will track the carrier component & VCO output will be a single tone carrier signal.

Superheterodyne receiver for standard AM radio:-

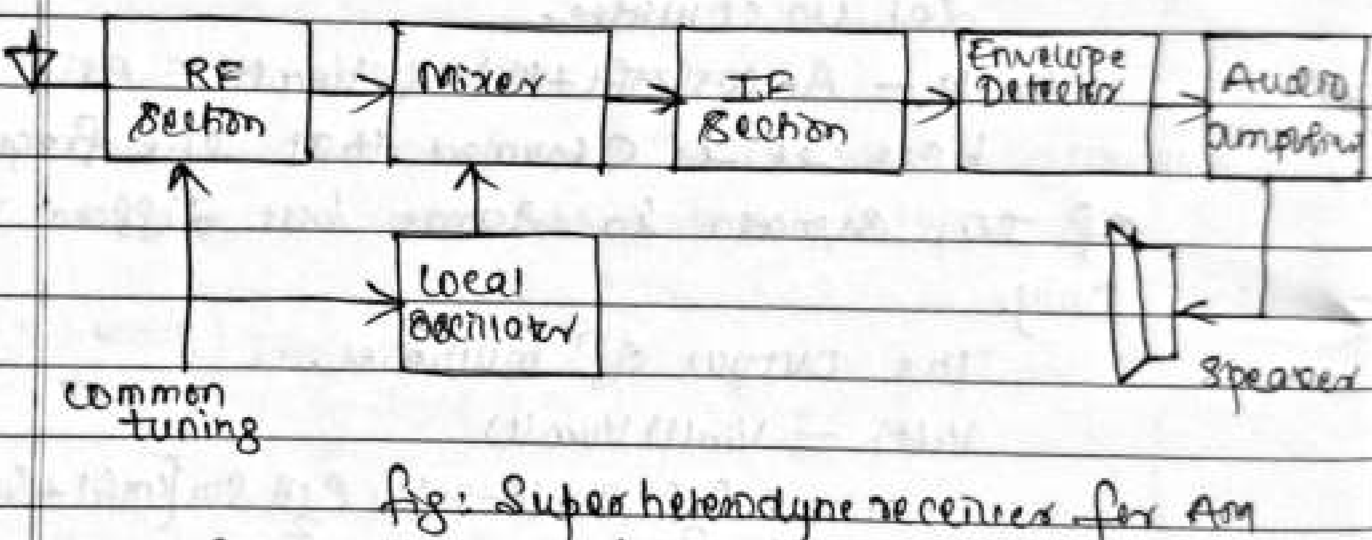


fig: Superheterodyne receiver for AM

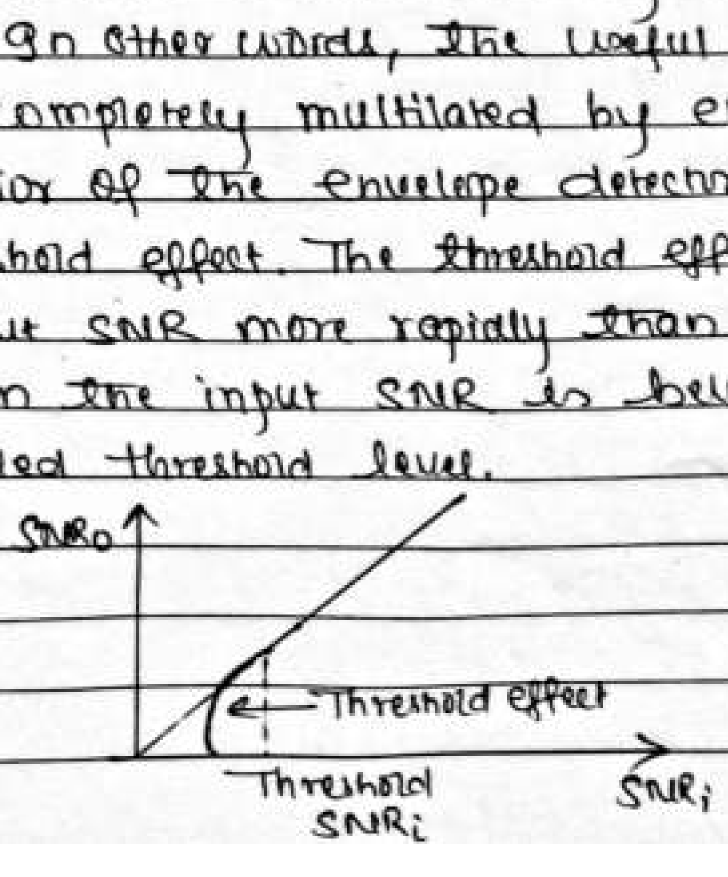
Commercial AM broadcast receivers employ heterodyning (mixing) of frequencies during the process of receiving. The circuits can be tuned to the desired station of given frequency band. The bandwidth of these tank circuits are usually 10KHz. The amplified RF signal is fed to a mixer circuit. The mixer along with the local oscillator circuit converts all the selected carrier frequen-

Date:
Page:

cies into a standard intermediate frequency (IF). This is done to ensure required amplification & & selectivity by designing an amplifier with fixed center frequency & bandwidth. The output of mixer has a constant center frequency of 455 KHz. The amplified IF signal is then fed to the envelope detector. The recovered message signal is then fed to the audio amplifier & finally to the speaker.

Threshold effects in AM:-

When the noise level is greater than the signal level, the envelope of AM signal contains no independent message signal m(t). The message signal now is modulated by random noise component. In other words, the useful signal component is completely multilated by envelope detector. Such behavior of the envelope detector present is called threshold effect. The threshold effect deteriorate the output SNR more rapidly than the input SNR when the input SNR is below certain level, called threshold level.



Angle modulation (non-linear modulation):-

In angle modulation, the angle of the carrier signal is varied according to the modulating signal. In general case, the angle modulated signal can be expressed as,

$s(t) = A_c \cos[\theta(t)]$, where $\theta(t)$ is proportional to the modulating signal $m(t)$.

Instantaneous phase:-

The instantaneous phase of an angle modulated signal is

$\theta_i(t) = 2\pi f_c t + \phi(t)$, where $\phi(t)$ = phase deviation

Instantaneous frequency:-

The instantaneous frequency is the derivative of instantaneous phase.

$$W_i(t) = \frac{d\theta_i(t)}{dt} = 2\pi f_c + \frac{d\phi(t)}{dt}$$

where $\frac{d\phi(t)}{dt}$ is called frequency deviation.

Phase modulation (PM):-

In PM, the phase deviation of the carrier signal $\phi(t)$ is directly proportional to the modulating signal $m(t)$.

$$\phi(t) = K_p m(t)$$

where K_p = phase sensitivity or phase deviation constant

The expression for PM can be written as

$$S_{pm}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

Frequency modulation (FM):-

In FM, the frequency deviation in the carrier signal is directly proportional to the modulating signal m(t).

$$\frac{d\phi(t)}{dt} = K_a m(t) = 2\pi K_f m(t)$$

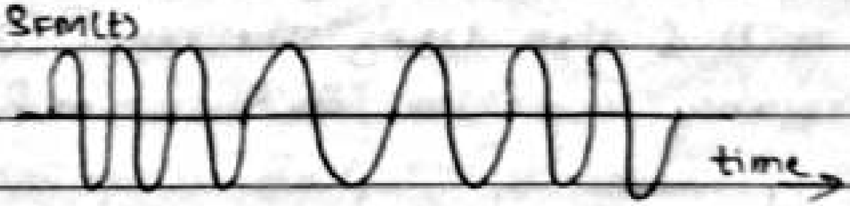
Where $K_a = 2\pi K_f$ = frequency sensitivity or frequency deviation constant.

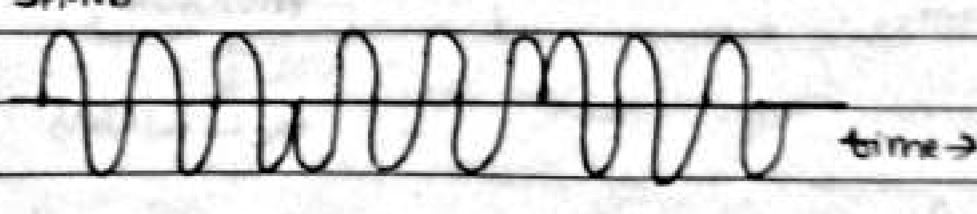
$$\frac{d\theta_i(t)}{dt} = 2\pi f_c + 2\pi K_f m(t)$$

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int m(t)\,dt$$

The expression for FM can be written as.

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\,dt\right]$$





Date:
Page:

Relationship between frequency modulation & phase modulation:-

FM wave may be regarded as PM wave in which the modulating wave is $\int m(t)dt$ in place of $m(t)$. This means that FM can be generated by first integrating $m(t)$ & then using the result as the input to a phase modulator as shown in fig.

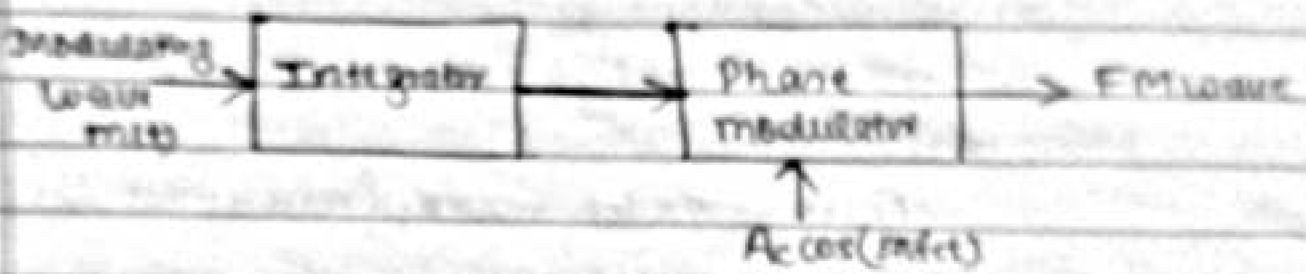


fig: Scheme for generating FM wave by using phase modulator

PM wave can be generated by first differentiating $m(t)$ & then using the result as the input to a frequency modulator as shown in fig.

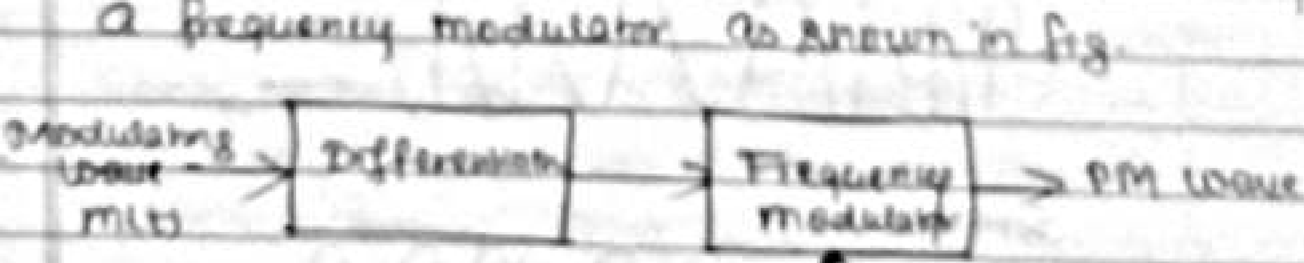


fig: Scheme for generating PM wave by using frequency modulator

Single tone PM:-

Let the modulating signal m(t) be,

$$m(t) = A_m \cos(2\pi f_m t)$$

Then, the eqn for PM will be,

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p A_m \cos 2\pi f_m t]$$

$$= A_c \cos[2\pi f_c t + \beta_{PM} \cos 2\pi f_m t]$$

where,

$\beta_{PM} = K_p A_m$ = modulation index or peak phase deviation of PM

Single tone FM:-

Let the modulating signal m(t) be,

$$m(t) = A_m \cos(2\pi f_m t)$$

The eqn for FM can be expressed as

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \frac{A_m \sin(2\pi f_m t)}{2\pi f_m}\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)\right]$$

where $\Delta f = K_f A_m$ = peak freqn deviation

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta_{FM} \sin(2\pi f_m t)]$$

where $\beta_{FM} = \frac{\Delta f}{f_m}$ = modulation index of FM

Date:
Page:

Spectral representation of FM:-

Let the carrier signal be

$$c(t) = A_c \cos 2\pi f_c t$$

& the modulating signal be

$$m(t) = A_m \cos 2\pi f_m t$$

Then the FM signal is expressed as

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

In exponential form, above expression can be written as,

$$S_{FM}(t) = Re\left[A_c \exp j(2\pi f_c t + \beta \sin 2\pi f_m t)\right]$$

$$= Re\left[A_c \exp(j2\pi f_c t) \exp(j\beta \sin 2\pi f_m t)\right]$$

As $\beta \sin 2\pi f_m t$ is a periodic function with period $T_m = 1/f_m$, it is evident that the function $\exp(j\beta \sin 2\pi f_m t)$ is a periodic signal with the same period. Therefore, it can be expanded in Fourier series as,

$$\exp(j\beta \sin 2\pi f_m t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$$

Where $C_n = \frac{1}{T_m} \int_{-T_m/2}^{T_m/2} \exp(j\beta \sin 2\pi f_m t) \exp(-j2\pi n f_m t) dt$

Let, $x = 2\pi f_m t$

$dx = 2\pi f_m dt$

When $t = -T_m/2$, $x = -\pi$

$t = T_m/2$, $x = \pi$

Then,

$$C_n = \frac{1}{T_m} \int_{-\pi}^{\pi} \exp(j(\beta \sin x - nx)) \frac{dx}{2\pi f_m}$$

Date: ________
Page: ________

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp j(\beta \sin x - nx)\,dx = J_n(\beta) = \text{Bessel function}$$

Now,

$$S_{FM}(t) = \text{Re}\left[A_c \exp(j2\pi f_c t) \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t}\right]$$

$$\boxed{S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]}$$

Taking Fourier transform,

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty} J_n(\beta)\left[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m))\right]$$

The above eq^n represents infinite sum of harmonic signal with frequencies $f_c + nf_m$. Therefore the spectrum of FM signal has infinite number of sidebands whose magnitude depend upon the Bessel coefficients.

Narrow band & wideband FM:-

For small values of modulation index β compared to one radian, the FM wave assumes a narrow band form consisting essentially of a carrier, an upper side frequency component & lower side frequency component.

For small values of β, we've,

$$J_0(\beta) \cong 1$$

$$J_1(\beta) \cong \beta/2 \quad J_{-1}(\beta) = -\beta/2$$

$$J_n(\beta) \cong 0, \quad n > 1$$

With these substitution, the expression of FM becomes,

$$s(t) \simeq A_c\cos(2\pi f_c t) + \frac{\beta A_c}{2}\cos[2\pi(f_c+f_m)t]$$

$$\frac{\beta A_c}{2}\cos[2\pi(f_c-f_m)t]$$

An FM wave so characterized is said to be narrow band.

For large values of modulation index β compared to one radian, the FM wave contains a carrier & infinite number of side-frequency components located symmetrically around the carrier. An FM wave so defined is said to be wideband.

Transmission bandwidth of FM:-

The spectrum of FM consists of carrier component & infinite number of sidebands. Therefore, the bandwidth of FM is theoretically infinite. The number of sidebands are limited in such a way that the radiated power is at least 98% of the total power.

Hence,

Bandwidth of FM, $B = 2nf_m$

It is established that for 98% of the total power to transmit, the number n &

Date:
Page:

modulation index β are related by the eq,

$$n = \beta + 1$$

$$\boxed{B = 2(\beta+1)f_m}$$ (Carson's rule)

where $\beta = \frac{\Delta f}{f_m}$

In North America, the maximum value of frequency deviation is fixed at 75KHz for commercial FM broadcasting by radio. Find the bandwidth of FM wave of modulation frequency is 15KHz.

Soln:-

Frequency deviation $(\Delta f) = 75$ KHz

Modulation frequency $(f_m) = 15$ KHz

Modulation index,

$$\beta = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$$

Bandwidth of FM wave,

$$B = 2(\beta+1)f_m$$

$$= 2(5+1)\times 15 = 180 \text{ KHz}$$

A single tone FM signal is given by

$$S_{FM}(t) = 10\sin(10^8 t + 3\sin 10^4 t)$$

calculate the carrier & modulating frequencies, modulation index, frequency deviation & power dissipated in 100 Ω resistor.

Soln:-

$$S_{FM}(t) = 10\sin(10^8 t + 3\sin 10^4 t)$$

Date:

Page:

Comparing this eq^n with standard eq^n,

$S_{FM}(t) = A_c \sin[2\pi f_c t + \beta \sin(2\pi f_m t)]$

$A_c = 10$, $2\pi f_c = 10^8$ $f_c = 15.91$ MHz

$2\pi f_m = 10^4$, $f_m = 1591.5$ Hz

$\beta = 3$

$\beta = \frac{\Delta f}{f_m}$

$\Delta f = \beta \times f_m = 3 \times 1591.5 = 4.774$ kHz

$P = \frac{(A_c/\sqrt{2})^2}{R} = \frac{(10/\sqrt{2})^2}{100} = 0.5$ W

20 MHz carrier is modulated by 400Hz modulating signal. The carrier voltage is 5V & maximum deviation is 10 kHz. Write down mathematical expression for FM. If modulating frequency is increased to 2kHz keeping everything else constant, write down expression for FM wave.

Sol^n:-

$\omega_c = 2\pi f_c = 2\pi \times 20 \times 10^6 = 1.25 \times 10^8$ rad/sec

$\omega_m = 2\pi f_m = 2\pi \times 400 = 2513$ rad/sec

$\beta = \frac{\Delta f}{f_m} = \frac{10 \times 10^3}{400} = 25$, $A_c = 5V$

$S_{FM}(t) = A_c \sin[2\pi f_c t + \beta \sin 2\pi f_m t]$

$= 5 \sin[1.25 \times 10^8 t + 25 \sin 2513 t]$

When $f_m = 2$ kHz, $\omega_m = 2\pi f_m = 12566.3$ rad/sec

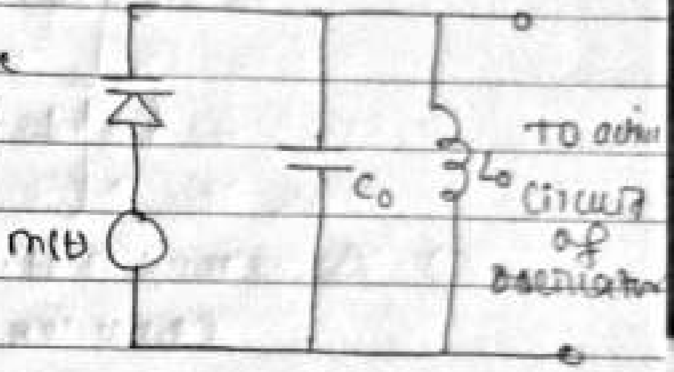
$\beta = \frac{\Delta f}{f_m} = \frac{10}{2} = 5$

Date: ______
Page: ______

$$S_{FM}(t) = 5\sin(1.25\times10^8 t + 5\sin 12566.3t)$$

Generation of FM :-

1) Direct method :-



In direct method, the frequency of oscillation of an oscillator is varied according to the modulating signal. The simplest form of this type of modulator is an oscillator circuit having a varactor diode in its frequency determining section. The applied m(t) will change the capacitance of the varactor diode & subsequently the frequency of the oscillation.

The total capacitance of the LC tank circuit will be,

$$C(t) = C_0 + K_0 m(t)$$

For $m(t) = 0$, $C(t) = C_0$, the frequency of oscillation is the carrier frequency,

$$f_c = \frac{1}{2\pi\sqrt{L_0 C_0}}$$

And for non-zero m(t), the instantaneous frequency of oscillation will be,

$$f_i = \frac{1}{2\pi\sqrt{L_0(C_0 + K_0 m(t))}}$$

$$= \frac{1}{2\pi\sqrt{L_0 C_0}\sqrt{1+\frac{K_0 m(t)}{C_0}}}$$

$$= f_c\left[1+\frac{K_0 m(t)}{C_0}\right]^{-1/2}$$

$$= f_c\left[1-\frac{K_0}{2C_0}m(t)\right]$$ which is the basic relation of FM

2) Armstrong method:-

Consider first the generation of a narrow band wave. To do this, we begin with the expression for FM wave,

$$s(t) = A_c\cos[2\pi f_c t+\phi(t)]$$

Where $\phi(t) = 2\pi k_f \int m(t)\,dt$

$\phi(t)$ is small compared to one radian, we may use the following approximations,

$$\cos[\phi(t)] \simeq 1$$

$$\sin[\phi(t)] \simeq \phi(t)$$

$$s(t) \simeq A_c\cos(2\pi f_c t) - A_c\sin(2\pi f_c t)\phi(t)$$

$$= A_c\cos(2\pi f_c t) - 2\pi k_f A_c\sin(2\pi f_c t)\int m(t)\,dt$$

It defines narrow-band FM wave.

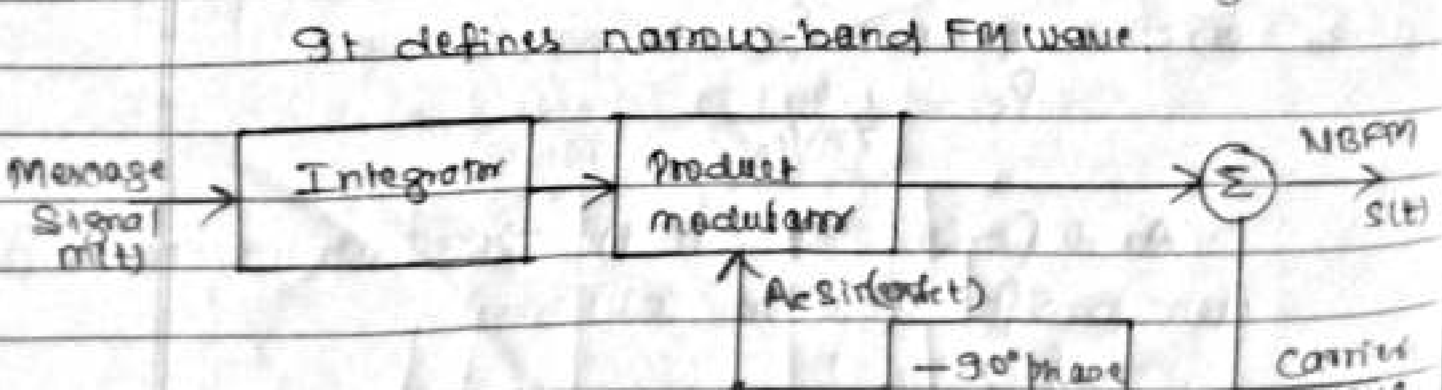


fig: NBFM

Date:
Page:

The next step in the indirect FM method is that of frequency multiplication. Basically, a frequency multiplier consists of a non-linear device followed by bandpass filter.

NBFM wave → [Memoryless non-linear device] → [Band-pass filter] → WBFM wave

Demodulation of FM:-

1) Limiter-discriminator method:-

The FM signal for $m(t)$ is expressed as.

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

where $\phi(t) = 2\pi k_f \int m(t)\, dt$

Extraction of $m(t)$ from the above equation involves the following three steps:

- Amplitude limit
- Discrimination (Differentiation)
- Envelope detection

During propagation of the FM signal from transmitter to receiver, the amplitude of the FM may undergo changes due to fading & noise. Therefore before further processing, the amplitude of FM signal is limited to reduce the effect of fading & noise.

The next stage is extraction of $m(t)$ from within the cosine function. This can be accomplished by differentiating FM signal,

$$\frac{ds(t)}{dt} = A_c \frac{d\cos(2\pi f_c t+\phi(t))}{d(2\pi f_c t+\phi(t))} \cdot \frac{d(2\pi f_c t + 2\pi k_f \int m(t)dt)}{dt}$$

$$= A_c[2\pi f_c + 2\pi k_f m(t)] \sin[2\pi f_c t + \phi(t)]$$

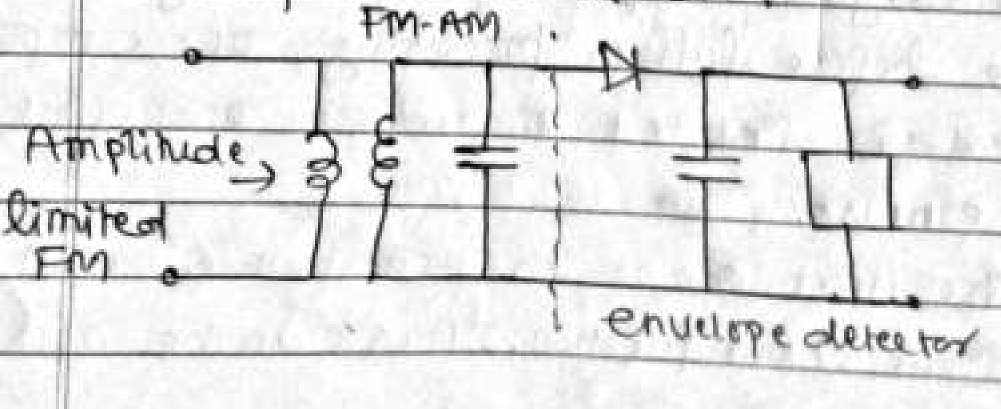
This is equivalent as a standard AM signal with carrier component & sidebands containing m(t). The process of differentiation, in this case, is also known as FM to AM conversion.

The last stage is envelope detection. As the output of the differentiator is now AM signal with m(t) within the amplitude of a carrier signal, simple envelope detection will result in a signal,

$$S_{det}(t) = A_c[2\pi f_c + 2\pi k_f m(t)]$$

The first component is dc component & second component is the message signal multiplied by some constant coefficient.

The implementation of FM discriminator based on limiter-discriminator principle is simply dc tuned LC tank circuit followed by envelope detector & LPF.



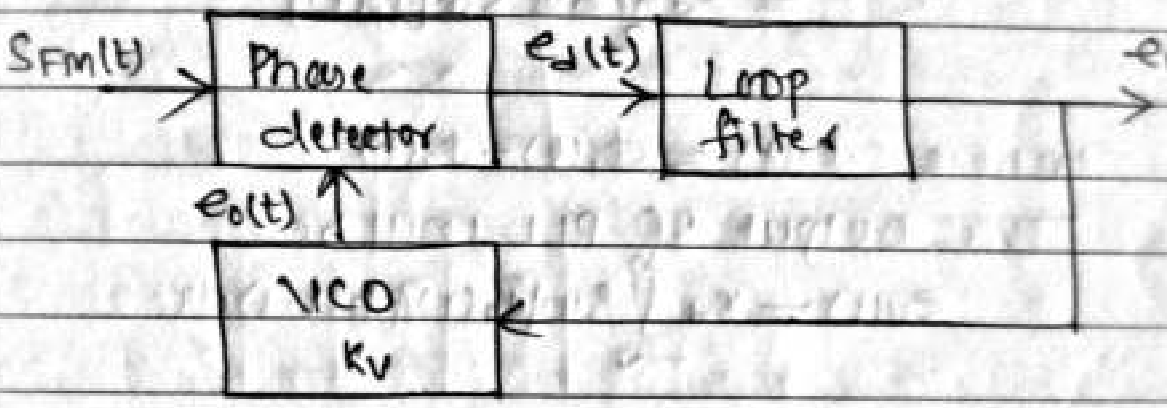
2) PLL as FM detector:-

Phase locked loop (PLL) consist of:

i) Phase detector (multiplier followed by LPF) that produce voltage proportional to the phase difference between incoming carrier signal & the signal produced by the local oscillator.

ii) Loop filter with impulse response h(t)

iii) Voltage controlled oscillator (VCO)



At beginning, let $e_v(t)=0$. This can happen when the input FM signal is zero. In this case, $e_d(t)=0$. At this stage, VCO is calibrated in such a way that its free running frequency of oscillator is equal to the carrier frequency of incoming FM signal. A constant phase shift of 90° is also added to the signal of VCO.

$$e_o(t) = A_v\cos(\omega_c t - \pi/2) = A_v \sin\omega_c t$$

Now let us assume that $e_v(t) \neq 0$. In this case, the output of VCO is

$$e_o(t) = A_v \sin(\omega_c t + \phi_o(t))$$

where, $\phi_o(t) = 2\pi K_v \int e_v(t)\,dt$ (K_v = VCO sensitivity)

Date:
Page:

FM signal is expressed as

$$S_{FM}(t) = A_c \cos[\omega_c t + \phi_i(t)]$$

where,

$$\phi_i(t) = 2\pi k_f \int m(t)\,dt$$

Now the error voltage at the output of PD (after LPF) would be,

$$e_d(t) = \frac{K_d A_c A_v}{2} \sin[\phi_i(t) - \phi_o(t)]$$

$$= \frac{K_d A_c A_v}{2} \sin[\phi_e(t)]$$

where $\phi_e(t) = \phi_i(t) - \phi_o(t)$

The output of PLL will be

$$e_v(t) = K_d \int_{-\infty}^{\infty} \sin[\phi_e(\tau)]\, h(t-\tau)\, d\tau$$

When PLL enters into lock mode (i.e. when two frequency & phase match), the error voltage will be nearly equal to zero.

$$\phi_e(t) = \phi_i(t) - \phi_o(t) \simeq 0$$

$$\phi_i(t) \simeq \phi_o(t)$$

$$2\pi k_f \int m(t)\,dt \simeq 2\pi k_v \int e_v(t)\,dt$$

$$\boxed{e_v(t) \simeq \frac{k_f}{k_v}\, m(t)}$$

That is, in locked mode, the output voltage of the PLL is nearly equal to the message signal.

Date: ____________
Page: ____________

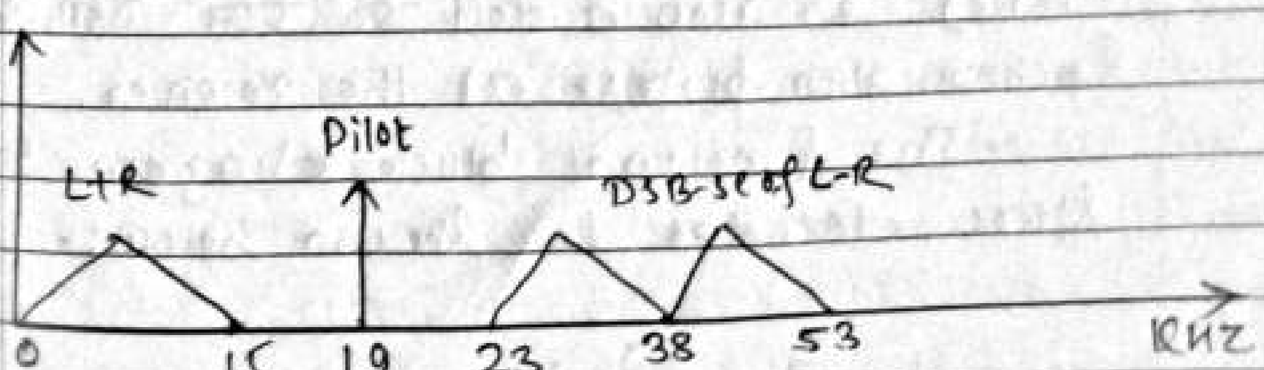
Introduction to Stereo FM transmission & reception:-

Stereo basically means two or double & in broadcasting stereo means transmission of two independent channels of the same source for better effect. The group of signals are categorized as left & right channels. The system capable of transmitting both left & right channel signals & reproducing these two signals as separate signals at the receiving end is called stereo system.

In stereo FM broadcast system, two separate channels left (L) & Right (R) are combined in the following manner to constitute the baseband FM signal.

- Sum of L & R channels in their original baseband form.
- Pilot tone at 19KHz as synchronizing signal & as indicator of stereo transmission.
- DSB-SC of difference of L & R.

The base-band spectrum of FM stereo signal is shown below:-



Date: ___
Page: ___

For stereo FM broadcasting, channels in VHF band from 88 to 108 MHz with spacing between channels 200 KHz are allocated. The allowable peak deviation per channel is 75KHz.

The functional block diagram of the stereo FM transmitter is shown below:

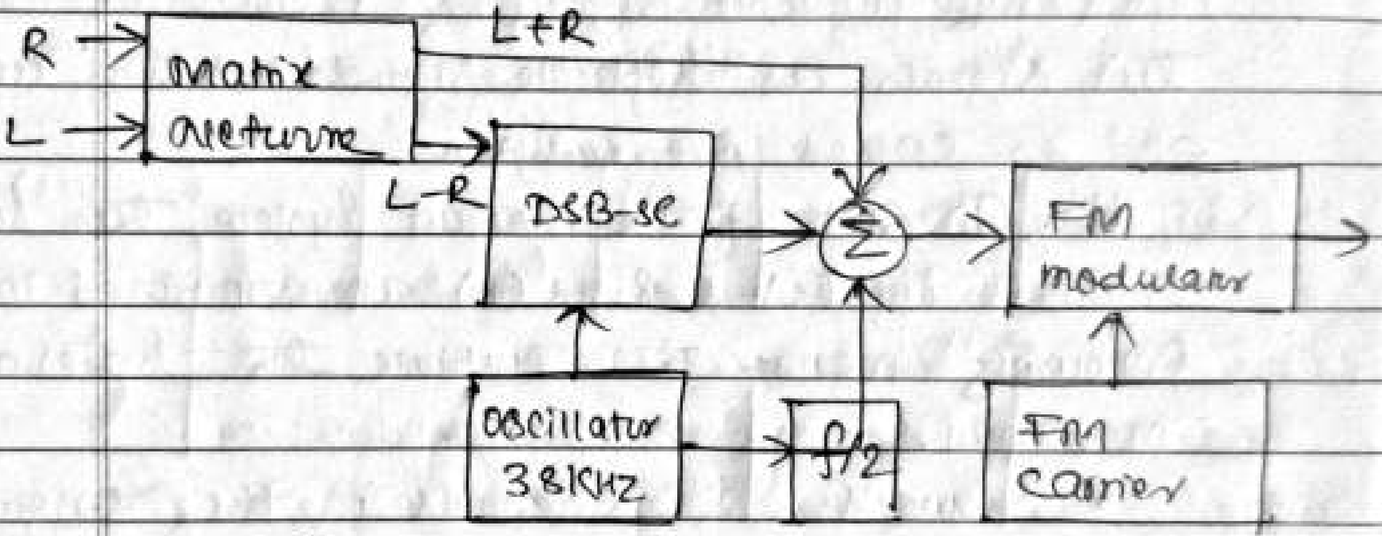


fig: Stereo FM transmission

From the matrix of resistors, L+R & L-R signals are generated from L & R signals. Since L+R signal is directly passed to the modulator, only L-R signal undergo DSB-SC at 38 KHz. The half of the carrier frequency (19KHz) is also added to the DSB-SC for synchronization purpose at the receiver.

The functional block diagram of the stereo decoder is shown below:-

Date: ______
Page: ______

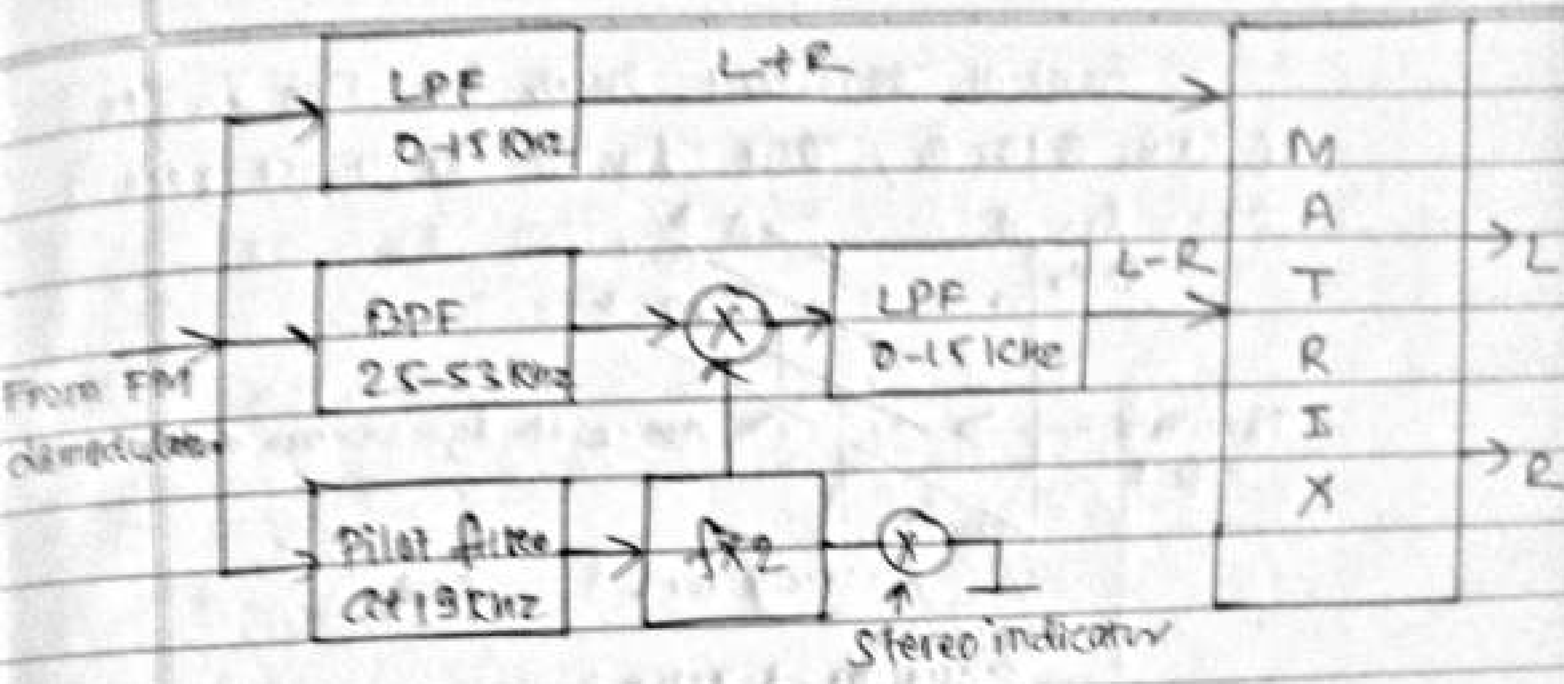


fig: Stereo FM receiver

Non-stereo FM receiver can receive stereo signal because there is baseband L+R signal in the stereo base-band spectrum.

Threshold effect in FM:-

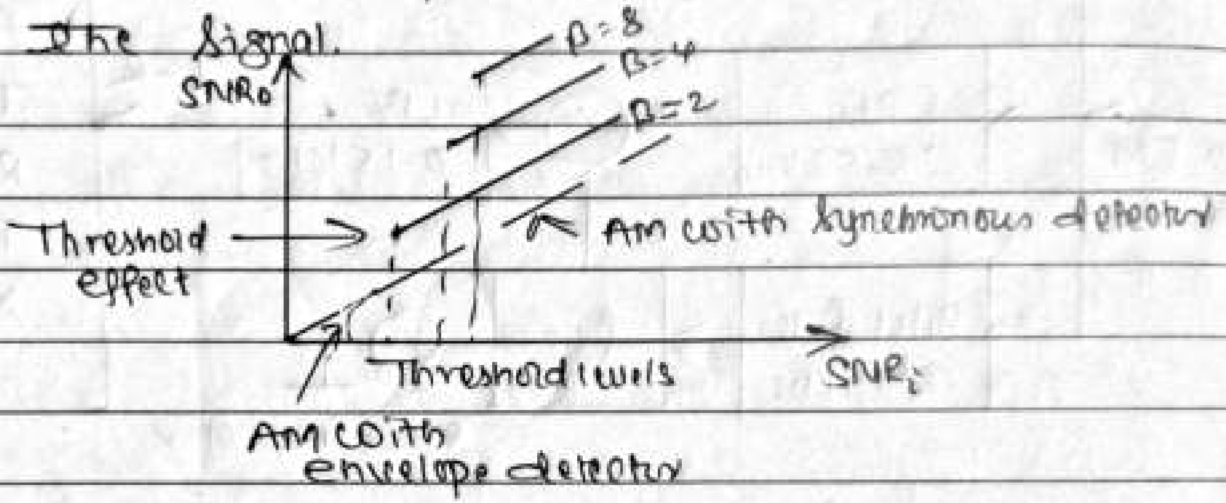
As the detection gain γ in the FM is proportional to β^2, for given SNR_i rise in β will increase γ or output SNR_o. But with increase in β, the system bandwidth also increases as Carson's rule says,

$$B = 2(\beta+1)f_m$$

Increase in bandwidth will subsequently increase the input noise power & decrease SNR_i.

There exists the threshold effect that for SNR_i less than the threshold level, FM system will no more detect the message signal.

General threshold level for FM is 10dB.
if $SNR_i \leq 10dB$, the system will not receive the signal.



Before reaching the threshold level, the FM system produces noise clicks.

The threshold effect in the FM can be considerably reduced by use of PLL as demodulator & by pre-emphasis, de-emphasis networks.

Introduction to Digital Communication System

Block diagram of digital communication system:-

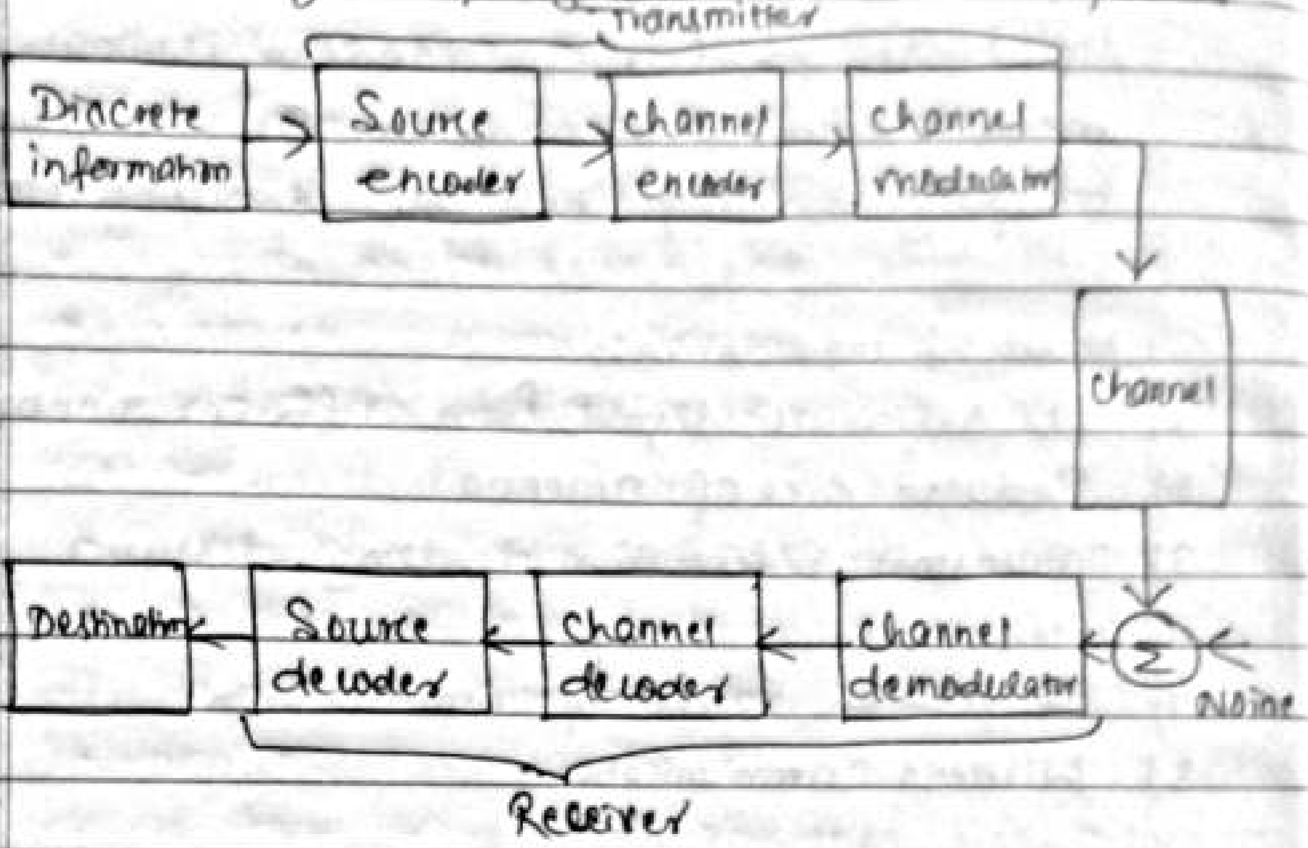


fig: Block diagram of digital communication system

Discrete information:-

- Sequence of symbols A, Q, 1, 5 etc

Source encoder:-

- converts into a binary sequence of 1's & 0's.

Source decoder:-

- converts the binary output of the channel decoder into sequence of symbols with minimum error & maximum efficiency.

Channel encoder:-

- enhances reliability & efficiency of high speed digital signal transmission.
- may use start bit, stop bit & parity bit.

high / low	0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	1
	start bit								parity bit	stop bit

- use error control bits.
- These bits doesn't carry any information.
- Each block of k information bearing bits, r control bits are added.

Channel decoder:-

It recovers information bearing bits streams from coded bit streams with min^m error & max^m efficiency.

Channel modulator/demodulator:-

Channel modulator converts bit streams to electrical signal suitable for transmission over communication channel.

Channel demodulator converts electrical signals to bit streams.

Channel:-

- Physical medium
- two types of communication channel.

Guided propagation:-

- coaxial cable, telephone wire, optical fiber, twisted pair cable.

Free propagation:-

- wireless broadcast
- Channel has finite frequency bandwidth.

Advantages of Digital Communication System over analog communication systems:-

- Relatively inexpensive digital circuits may be used.
- Privacy is preserved by using data encryption
- Greater dynamic range (the difference between the largest & smallest values) is possible.
- Data from voice, video & data sources may be merged & transmitted over a common digital transmission system.
- In long-distance systems, noise doesn't accumulate from repeater to repeater.
- Errors in detected data may be small, even when there is a large amount of noise on the received signal.
- Errors may often be corrected by the use of coding.

Disadvantages of digital communication systems:-

- Generally, more bandwidth is required than that for analog system.
- Synchronization is required.

Date:

Page:

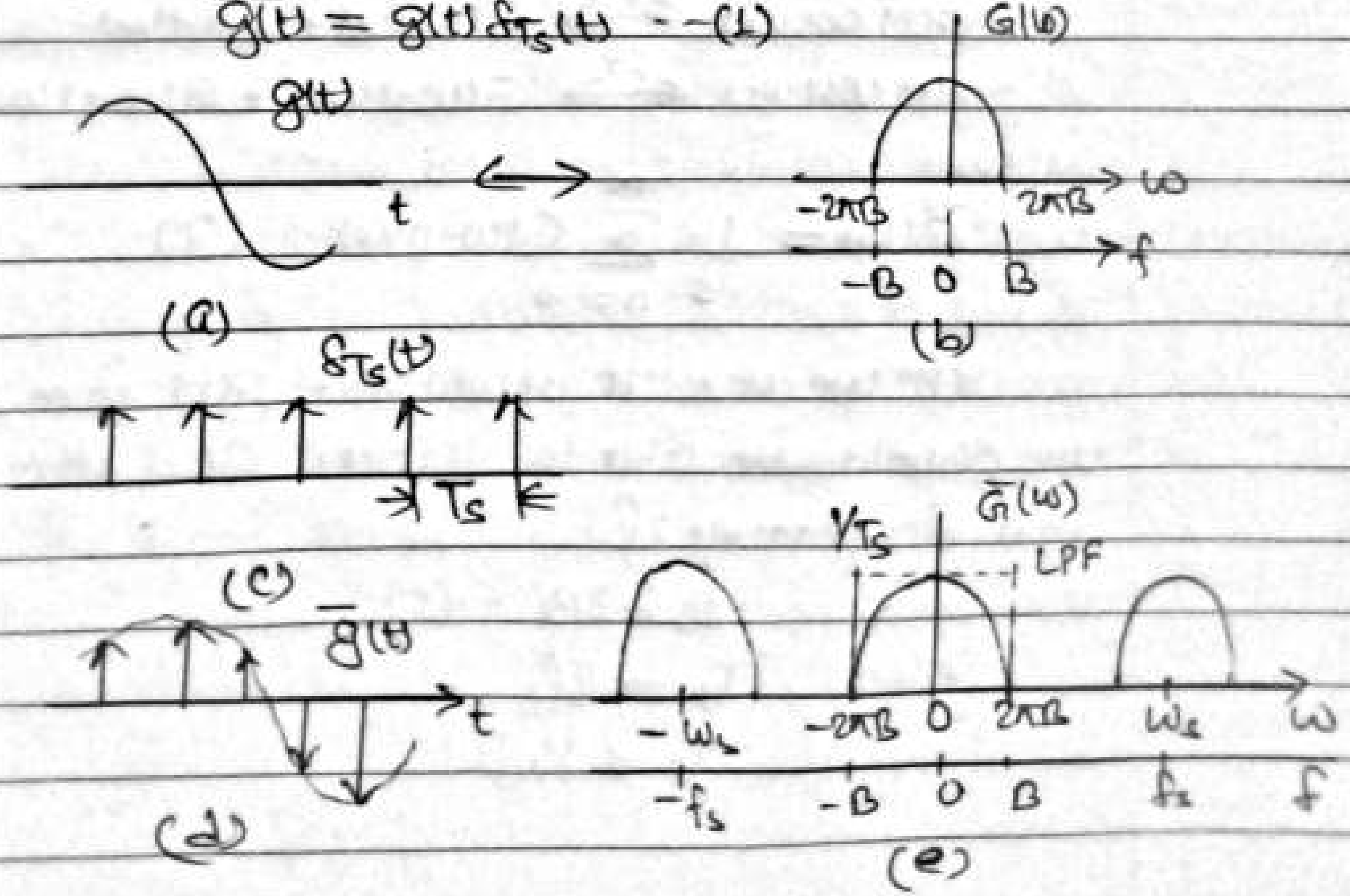
Nyquist Sampling theorem:-

A signal whose spectrum is band limited to B Hz can be reconstructed exactly from its samples taken uniformly at a rate $R > 2B$ Hz (samples per second). In other words, the minm sampling frequency is $f_s = 2B$ Hz.

Proof:-

Consider a signal $g(t)$ whose spectrum is band-limited to B Hz. Sampling $g(t)$ at a rate of f_s Hz can be accomplished by multiplying $g(t)$ by an impulse train $\delta_{T_s}(t)$, consisting of unit impulses repeating periodically every T_s seconds, where $T_s = 1/f_s$. This results in the sampled signal $\bar{g}(t)$ shown in fig. Thus,

$$\bar{g}(t) = g(t)\,\delta_{T_s}(t) \quad \text{---(1)}$$



Because the impulse train $\delta_{T_s}(t)$ is a periodic signal of period T_s, it can be expressed in Fourier ~~trans~~ series,

The trigonometric Fourier series is

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + \cdots\right] \quad \text{-(2)}$$

where, $\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$

Therefore,

$$\bar{g}(t) = g(t)\delta_{T_s}(t)$$

$$= \frac{1}{T_s}\left[g(t) + 2g(t)\cos\omega_s t + 2g(t)\cos 2\omega_s t + \cdots\right] \quad \text{--(3)}$$

Now, ~~to~~ find $\bar{G}(\omega)$, take Fourier transform of (3),

$$2g(t)\cos\omega_s t \xleftrightarrow{FT} G(\omega-\omega_s) + G(\omega+\omega_s)$$

$$2g(t)\cos 2\omega_s t \xleftrightarrow{FT} G(\omega-2\omega_s) + G(\omega+2\omega_s)$$

Now,

$$\bar{G}(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(\omega - n\omega_s) \quad \text{--(4)}$$

If we are to reconstruct $g(t)$ from $\bar{g}(t)$, we should be able to recover $G(\omega)$ from $\bar{G}(\omega)$. This is possible if

$$f_s > 2B \quad \text{--(5)}$$

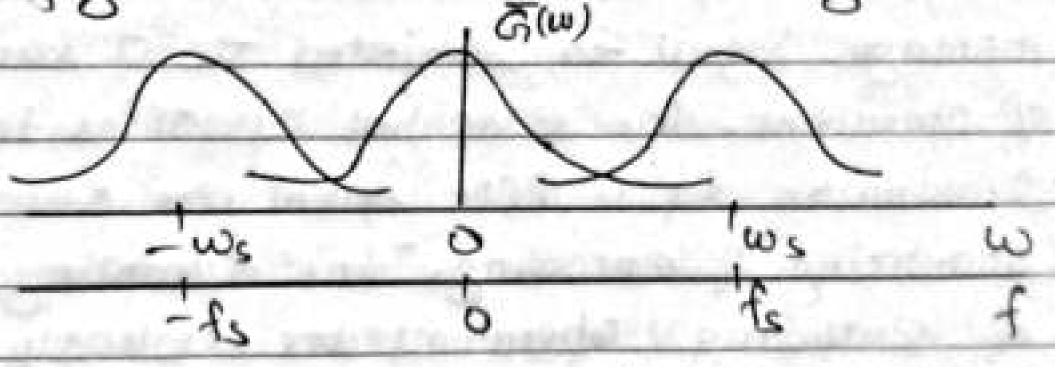
Also, $T_s = 1/f_s$

$$T_s < 1/2B$$

A signal g(t) which is bandlimited to B Hz can be reconstructed exactly from its samples by passing sampled signal through ideal low pass filter.

Aliasing effect:-

All practical signals which are time limited, are non-bandlimited, they have infinite bandwidth & spectrum $\bar{G}(\omega)$ consists of overlapping cycles of $G(\omega)$ repeating every f_s Hz as shown in fig. This is called aliasing.



Pulse amplitude modulation (PAM):-

In PAM, the amplitude of carrier consisting of a periodic train of rectangular pulses is varied in proportion to sample value of message signal. In this type of modulation, pulse duration is held constant.

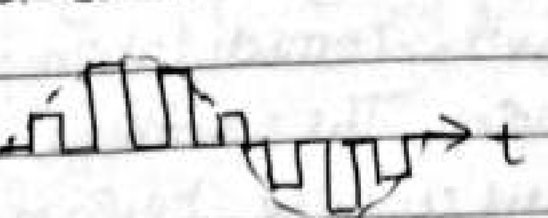


fig: PAM

Reconstruction:-

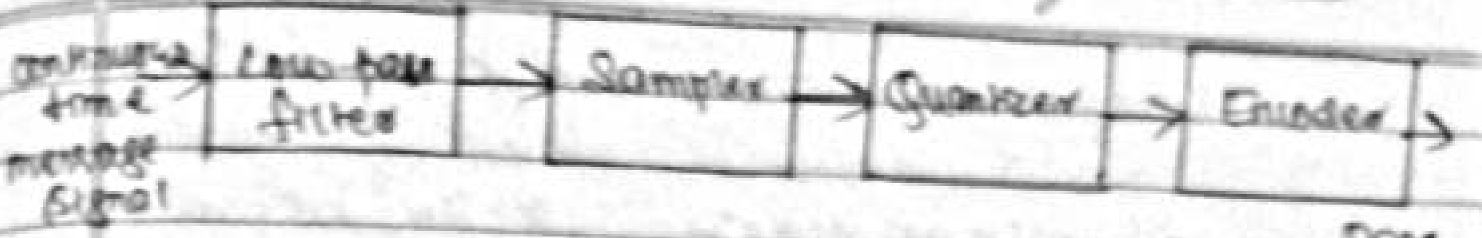
Sampled message signal → low pass interpolation filter → Equalizer → Reconstructed message signal

- low pass interporation filter has bandwidth equal to message bandwidth.
- Equalizer corrects distortion.

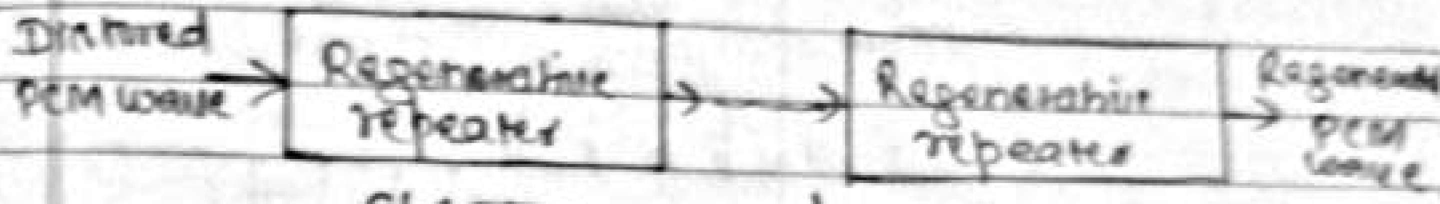
Pulse code modulation (PCM):-

PCM Systems are complex in that the message signal is subjected to a large number of operations. The essential operations in the transmitter of a PCM System are sampling, quantizing & encoding. The sampling, quantizing & encoding operations are usually performed in the same circuit, which is called analog to digital converter. Regeneration of impaired signals occurs at intermediate points along the transmission path (channel) as shown in fig. At the receiver, the essential operations consist of one last stage of regeneration followed by decoding, then demodulation of train of quantized samples. The operations of decoding & reconstruction are usually performed in the same circuit, called digital to analog converter.

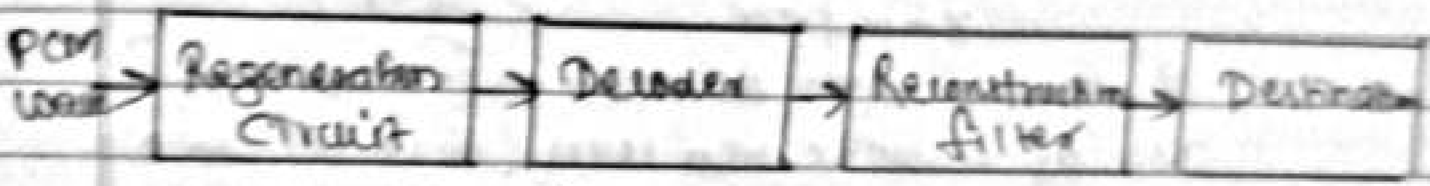
Date: ______
Page: ______



(a) Transmitter



(b) Transmission path

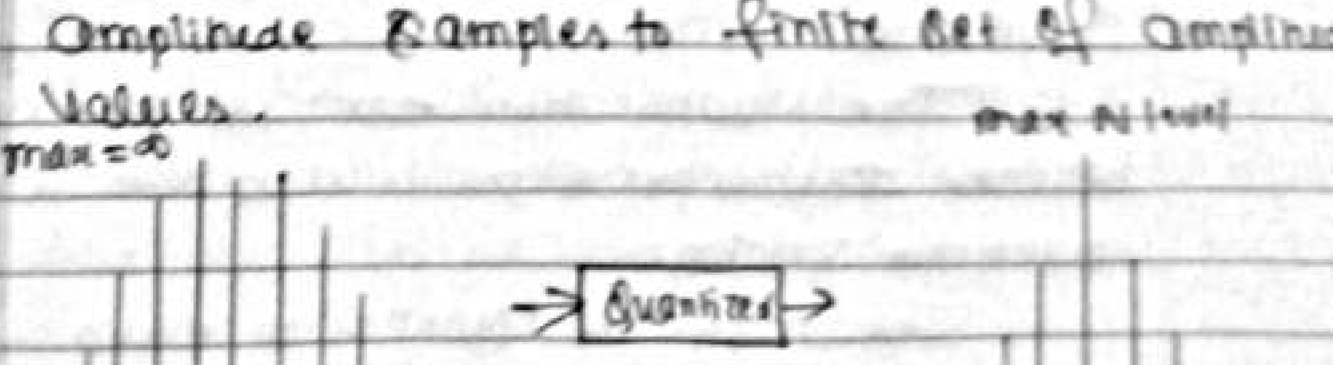


(c) Receiver

fig: Basic elements of PCM System

Sampling process converts continuous time signal to discrete time signal with amplitude that can take any values from zero to maximum level.

But quantization process converts continuous amplitude samples to finite set of amplitude values.

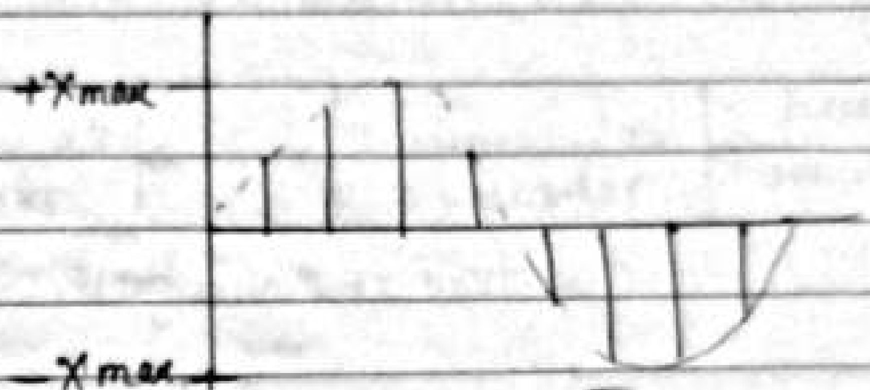


Encoding process translate the discrete set of

Date:
Page:

sample values to a more appropriate form of a sig

Uniform quantization:-



Range of input sample is [-Xmax, Xmax].
No. of quantization level (Q-level) is N.
Where,
$N = 2^n$, where n is no. of bits per sample

Step size Δ or the length of Q-level,

$$\Delta = \frac{2X_{max}}{N} = \frac{2X_{max}}{2^n} = \frac{X_{max}}{2^{n-1}}$$

Step size is also called quantum.

Quantization Error:-

The quantization error is the difference between the input signal level & the level of quantized version.

The maximum quantization error is Δ/2

In uniform quantization, the step size Δ is constant for entire dynamic range of the input discrete signal level.

Q-error & Q-noise is produced during the process of quantization because of rounding of the sampled values of a continuous message signal of the nearest representation level.

actual level

$q_e = Q\text{-error}$

representation level

It is evident that q_e lies in between $-\Delta/2$ to $\Delta/2$.

The average power of Q-noise is therefore equals to

$$P_q = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q_e^2 \, dq_e$$

$$= \frac{1}{\Delta}\left[\frac{q_e^3}{3}\right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta}\left(\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right)$$

$$P_q = \frac{\Delta^2}{12}$$

From above eq^n, it is seen that Q-noise in uniform quantization is dependent only upon the step size. Reducing the step size, P_q can be reduced.

Date: ______
Page: ______

Signal to quantization noise ratio (SQNR) :-

The average quantization noise power can be expressed in terms of signal level & no of bits per representation level as,

$$P_q = \frac{\Delta^2}{12} = \frac{4x_{max}^2}{12\times4^n} = \frac{x_{max}^2}{3\times4^n}$$

Assume average signal power is $\overline{x}^2$,

$$SQNR = \frac{\overline{x}^2}{x_{max}^2}\times3\times4^n$$

If $\tilde{x}^2 = \frac{\overline{x}^2}{x_{max}^2}$ is normalized signal power, then

$$SQNR = 3\times4^n\times\tilde{x}^2$$

Since x_{max} is the max^m level of input signal, the normalized signal power is always less or equal to unity.

Therefore, the upper limit of SQNR,

$$= 3\times4^n$$

In terms of dB, SQNR will be,

$$SQNR(dB) = P_x(dB) + 10\log3 + 10n\log4$$
$$= (P_x + 4.8 + 6n)dB$$

It means that for each extra bit (n) used for representing each quantization level coding, SQNR increased by 6 dB.

Date:
Page:

Transmission bandwidth of PCM:-

No. of quantization levels (N) may be represented by n bits as,

$$N = 2^n$$

Sampling rate,

$$f_s \geq 2f_m$$

~~Signaling~~

Signaling rate,

$$R = nf_s$$

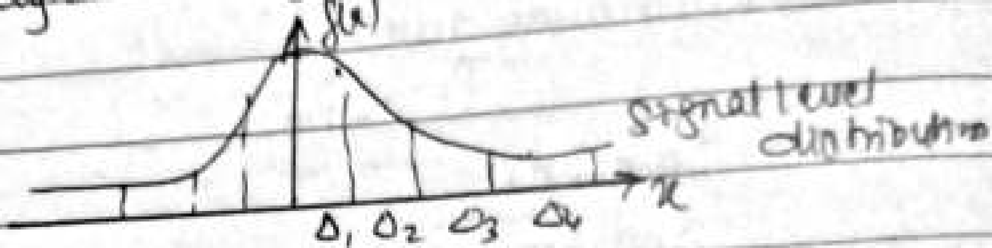
Bandwidth needed for PCM transmission is given by half of the signaling rate,

$$B/W \geq \frac{1}{2}R \geq nf_m$$

Non-uniform quantization - companding

Uniform quantizer produces highest or optimum SQNR provided the input signal has a uniform power density function. The average quantization noise power P_q is fixed at $\Delta^2/12$ regardless of the value of the sample being quantized. In such case, if at most of the time, the signal level remains small, the apparent SQNR will be much lower than the design value. This situation is prominent in voice signal where medium voice level prevails at most of the time where as loud voice levels occur rarely. For such signals, uniform ~~quantization~~ quantization may not yield optimum result.

Non-uniform quantizer employ variable step size quantization. In this case, the average SQNR can be maintained high & constant over entire dynamic range of the input signal.



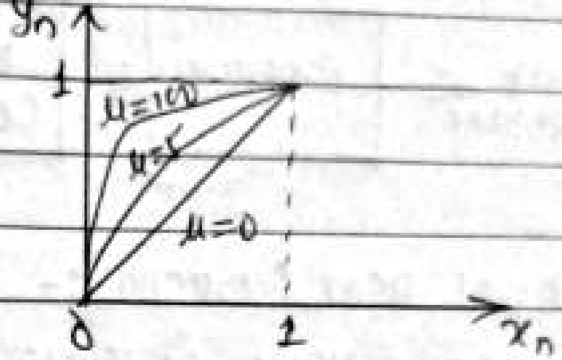
For signal values with highest probability, the step size is made smaller. As the probability decreases, step size is increased correspondingly so that

$$\Delta_1 < \Delta_2 < \Delta_3 < \Delta_4$$

Direct realization of variable step size quantization is very complicated. The indirect method of ~~a~~ non-uniform quantization involves compressing of baseband signal & then applying compressed signal to uniform quantizer. At the receiving end, the decoded message is expanded in the reverse manner to recover original baseband signal. The process of compressing & expanding is called companding.

There are two compression laws, called μ & A laws that are commonly used in practice. In μ-law, the normalized output signal y_n is related to the normalized input signal x_n in following manner,

$$|y_n| = \frac{\log(1+\mu|x_n|)}{\log(1+\mu)} \quad ---(1)$$

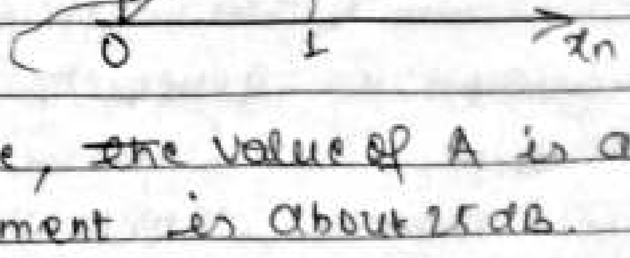


$\mu=0$ is the case of uniform quantization. Practical values of μ is within 100 to 300. Standard PCM employ $\mu=125$ compandor with 7-bits (128 quantization levels) quantization yielding system improvement (SQNR) of about 24dB.

The A-law compander utilizes the following relationship between input & output signal levels.

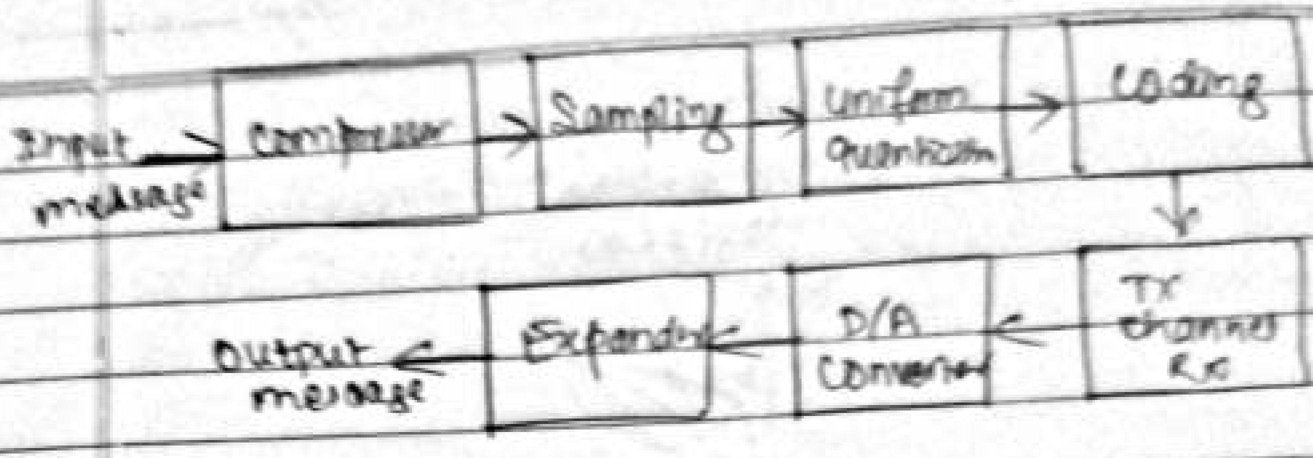
$$|y_n| = \frac{A|x_n|}{1+\log A} \quad \text{for } 0 \le |x_n| \le V_A$$

$$= \frac{1+\log(A|x_n|)}{1+\log A} \quad \text{for } V_A \le |x_n| \le A$$



For practical purpose, the value of A is around 100 & the improvement is about 25dB.

Page ______



Differential PCM (DPCM):-

When a voice or video signal is sampled at a rate slightly higher than the Nyquist rate, the resulting sampled signal is found to exhibit a high correlation between adjacent samples. The meaning of this high correlation is that the signal doesn't change rapidly from one sample to the next. When these highly correlated samples are encoded in a standard PCM system, the resulting encoded signal contains redundant information.

Let m(k) is the kth sample. Instead of transmitting m(k), we transmit the difference d(k) = m(k) − m(k−1). At the receiving end, knowing d(k) & the previous sample value, we can reconstruct m(k) by adding d(k) & m(k−1).

Since the difference between successive sample is generally smaller than the sample values, the peak value of transmitted samples are reduced considerably. Because the quantis interval,

$$\Delta = \frac{2X_{max}}{N}$$

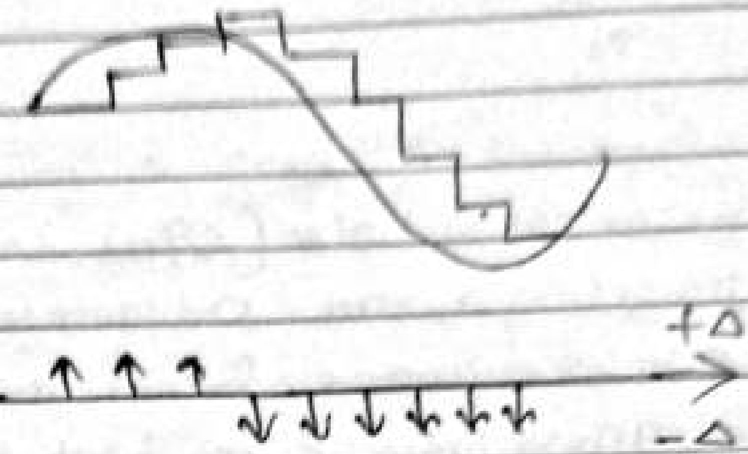
X_{max} is reduced & hence Δ is reduced. As a result, quantization noise ($\Delta^2/12$) is also reduced.

The straight forward advantage of DPCM is that 8-bit encoded signal may be transmitted using 4-bits only & in both case quantization noise power remaining same.

Delta modulation (DM):-

Delta modulation is the simplified version of DPCM. In DM, the difference between original sample & its approximation is quantized in one of two possible levels $+\Delta$ or $-\Delta$ & each level is converted into 1-bit codeword (e.g. $+\Delta \to 1$, $-\Delta \to 0$). Thus, DM uses only one bit to represent each sampled level.

Staircase approximation with step size Δ is used in DM. This technique is feasible only when sample to sample correlation is very high. To make the samples highly correlated, the sampling rate is chosen much higher than the Nyquist rate. But since the coding is 1-bit coding, the bandwidth requirement as well as the signalling rate for given signal with DM becomes considerably low.



SQNR of PCM is greater than that of DM system. PCM system can be realized by using two A/D & D/A chips where as DM system can be implemented by single chip CODEC (coder-decoder).

Adaptive delta modulation (ADM):-

The performance of delta modulator can be improved significantly by making the step size of the modulator assume a time-varying form. In particular, during a steep segment of the input signal, the step size is increased. Conversely, when the input signal is time varying slowly, the step size is reduced. In this way, the step size is adapted to input signal. The resulting method is called adaptive delta modulation (ADM).

Introduction To Information Theory:-

A message is a sequence of symbols intended to reduce uncertainty of the receiver. If the sequence of symbols doesn't change the uncertainty level of the receiver, then the message doesn't contain any information.

Consider the following three hypothetical headlines in a morning paper:

1. Tomorrow the sun will rise in the east.
2. United states invades Cuba.
3. Cuba invades the United states.

The reader will hardly notice the first headline. He or she will be very, very interested in the second. The third will attract much more attention than the previous two headlines. From viewpoint of common sense, the first headline convey hardly any information, second convey a large amount of information & third conveys yet a larger amount of information. If we look at the probabilities of occurance of these three events, we find the probability of occurance of first event is unity, second is very low & third is practically zero. If P is probability of occurance of a message & I is the information gained from the message, it is evident from previous discussion that when $P \to 1$, $I \to 0$ & when $P \to 0$, $I \to \infty$ & in general smaller P gives a larger I. Hence this suggests,

$$I = \log(1/p)$$

The unit of I depends upon the base assigned to log.

a) base is 'e' - unit is 'nat'.

b) base is 10 - unit is Hartley or decit.

c) base is 2 - unit is bit.

Average Information per Message: Entropy of a source

Consider a memoryless source m emitting messages $m_1, m_2, \ldots, m_n$ with probabilities $P_1, P_2, \ldots P_n$ respectively ($P_1 + P_2 + \ldots + P_n = 1$). A memoryless source implies that each message emitted is independent of the previous message. The information content of message m_i is I_i, given by.

$$I_i = \log_2(1/P_i) \text{ bits}$$

The probability of occurrance of m_i is P_i. Hence the average information per message emitted by the source is given by $\sum_{i=1}^{n} P_i I_i$ bits. The average information per message of a source m is called its entropy, denoted by H(m). Hence,

$$H(m) = \sum_{i=1}^{n} P_i I_i \text{ bits}$$

$$= \sum_{i=1}^{n} P_i \log(1/P_i) \text{ bits}$$

$$= -\sum_{i=1}^{n} P_i \log P_i \text{ bits}$$

\# A discrete source emits one of five symbols once every millisecond with probabilities 1/2, 1/4, 1/8, 1/16 & 1/16 respectively. Determine the source entropy & information rate.

Soln:-

Source entropy,

$$H = \sum_{i=1}^{M} P_i \log_2(1/P_i)$$

$$= 1/2 \log_2(2) + 1/4 \log_2(4) + 1/8 \log_2(8) + 1/16 \log_2(16) + 1/16 \log_2(16)$$

$$= \frac{15}{8} \text{ bits/symbol}$$

Symbol rate, $R_s = \frac{1}{T} = \frac{1}{10^{-3}} = 1000$ symbol/sec

Information rate,

$R_{inf} = R_s \times H$

$= 1000 \times 15/8$

$= 1875$ bits/sec

\# An analog signal bandlimited to 10KHz is quantized in 8 levels of a PCM system with probabilities of 1/4, 1/5, 1/5, 1/10, 1/10, 1/20, 1/20 & 1/20 respectively. Find the entropy & rate of information.

<u>Soln:-</u>

Sampling frequency(f_s) = 2×10 KHz = 20,000 messages/sec

Entropy,

$$H = \sum_{i=1}^{M} P_i \log_2(1/P_i)$$

$$= \frac{1}{4}\log_2 4 + \frac{1}{5}\log_2 5 + \frac{1}{5}\log_2 5 + \frac{1}{10}\log_2 10 + \frac{1}{10}\log_2 10$$

$$+ \frac{1}{20}\log_2 20 + \frac{1}{20}\log_2 20 + \frac{1}{20}\log_2 20$$

$= 2.74$ bits/message

Rate of information,

$R_{inf} = R_s \times H$

$= 20,000 \times 2.74$

$= 54,800$ bits/sec

Shannon's channel capacity theory:-

It states that

$$C = B \log_2(1+SNR) \text{ bits/sec}$$

where,

C = channel capacity

B = channel Bandwidth

SNR = Signal to noise ratio

Implication of theorem:-

1) Indicates the upper limit of data transmission for reliable communication. A designer thus can estimate C for required SNR & B for reliable communication.

2) Trade off between B & SNR for given C.

The channel capacity is limited by various extra- neous factors that the designer can not play with. For example, the maximum frequency that can be transmitted over a pair of cable is limited by its construction. Bandwidth of the signal can be compressed & SNR can be increased by increasing signal power or by introducing low noise devices.

Example:-

A signal with data rate $R = 10{,}000$ bits/sec is required to transmit over a channel with bandwidth of $B = 3000$ Hz. Minimum channel capacity,

$$C_{min} = R$$

$$SNR = 2^{(C/B)} - 1 = 2^{(10{,}000/3000)} - 1 \simeq 9.$$

It means signal power must be 9 times higher than the noise power.

But if $B = 10{,}000$ Hz, $SNR = 1$

Bandwidth compression from 10,000 to 3000 Hz is possible but at the cost of increasing signal power by 9 times.

3) Bandwidth compression:-

Shannon channel capacity theorem indicates that it is possible to transmit signal with upper frequency f_{max} through a channel having bandwidth less than f_{max}.

Example:-

Let a signal $x(t)$ has upper frequency limit of f_{max}. Let us sample $x(t)$ at $f_s = 3f_{max}$ (1.5 times greater than Nyquist rate)

Then, data rate will be

$$R = 3nf_{max}$$

where, n = no. of bits/sample

If $B = \frac{f_{max}}{2}$

Then, for $C_{min} = R$, $n = 6$, Required SNR will be

$$SNR = 2^{(C/B)} - 1 = 2^{3 \times 6 \times f_{max}/(f_{max}/2)} - 1 \approx 6.8 \times 10^{10}$$

If we increase signal power by 7×10^{10} times in comparison to noise power, we can transmit a signal through a channel having bandwidth equal to half of the signal bandwidth.

Limitations of Shannon's channel theorem :-

1) As the noise in the channel tends to zero, the value of SNR will tend to infinity. Subsequently the channel capacity C will tend to infinity. It means that the noiseless channel has an infinite capacity. This type of channel is referred to as ideal channel.

2) As the bandwidth of channel B tends to infinity, the channel capacity reaches an upper limit Cmax. This is because noise power is proportional to the bandwidth & as bandwidth is increased, the noise power also increases correspondingly.

$$C = B\log_2\left(1 + \frac{S}{\eta B}\right)$$

Where, S = Signal power

η = psd of white noise

$$C = \left(\frac{S}{\eta}\right) \log_2\left(1 + \frac{S}{\eta B}\right)^{\eta B/S}$$

Replacing $\frac{S}{\eta B}$ by x & considering the limit,

$$\lim_{x \to 0} (1+x)^{1/x} = e$$

We get,

$$\lim_{B \to \infty} C = C_{max} = \frac{S}{\eta} \log_2 e = 1.44 \frac{S}{\eta}$$

Base band (BB) digital communication:

For the baseband transmission of digital data, the use of discrete pulse amplitude modulation (PAM) provides the most efficient form of discrete pulse modulation in terms of power & bandwidth use. In discrete PAM, the amplitude of the transmitted pulses is varied in a discrete manner in accordance with the given digital data.

The basic elements of baseband binary PAM system are shown in fig.

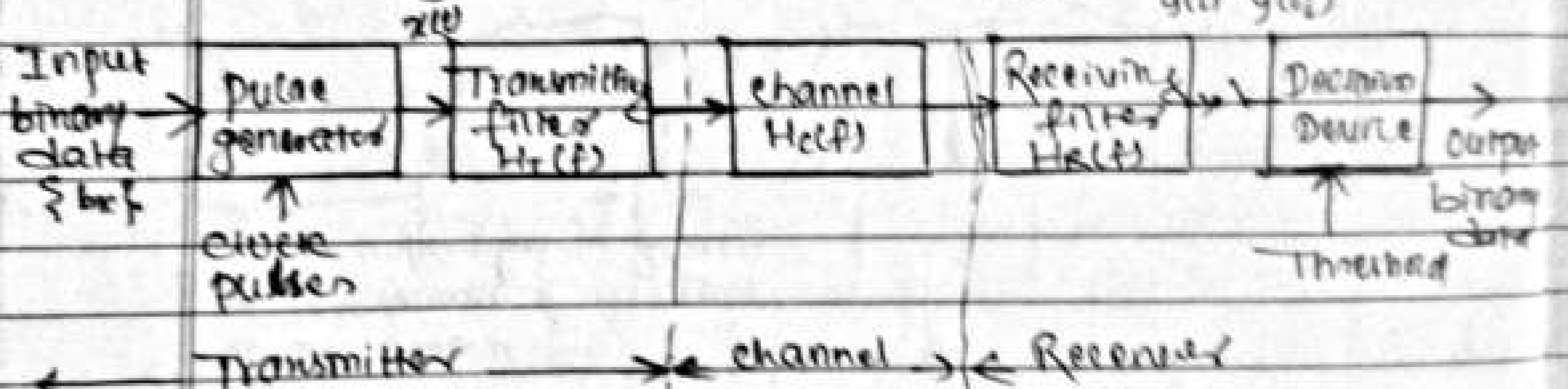


fig: Baseband binary data transmission system

The signal applied to the input of the system consists of a binary data sequence $\{b_k\}$ with a bit duration of T_b seconds, b_k is in the form of 1 or 0. This signal is applied to a pulse generator, producing the pulse waveform

$$x(t) = \sum_{K=-\infty}^{\infty} A_k g(t-KT_b) \quad \dots (1)$$

where $g(t)$ denotes a shaping pulse with its value at time $t=0$ defined by

$g(0) = 1$

The amplitude A_k depends on the identity of the input bit b_k. Specifically, we assume that,

$$A_k = \begin{cases} +a, & \text{if the input bit } b_k \text{ is symbol } 1 \\ -a, & \text{if the input bit } b_k \text{ is symbol } 0 \end{cases} \quad \text{--- (2)}$$

The signal $x(t)$ from the output of the pulse generator is passed through transmitting filter, the channel & the receiving filter having transfer function $H_T(f)$, $H_C(f)$ & $H_R(f)$. Furthermore the additive noise is added to the signal in the channel. The output of receiving filter is thus

$$y(t) = \mu \sum_{k=-\infty}^{\infty} A_k\, g(t - kT_b - t_d) + n_o(t)$$

where, μ = scaling factor

t_d = delay introduced by the system

$n_o(t)$ = noise

$y(t)$ is passed through a decision-making device with decision making instance of timing clock. If $y(t)$ is above threshold, then the output is 1 & if below threshold, then output is 0.

Signature

Baseband representation of digital data (line coding):-

There are different formats for representation of binary data sequence.

Unipolar format:-

In unipolar format (on-off signaling), symbol 1 is represented by transmitting a pulse, whereas symbol 0 is represented by switching off the pulse. When a pulse occupies the full duration of

Signature

a symbol, the unipolar format is said to be of the non return to zero (NRZ) type. When it occupies a fraction (usually one-half) of the symbol duration, it is said to be of return to zero (RZ) type.

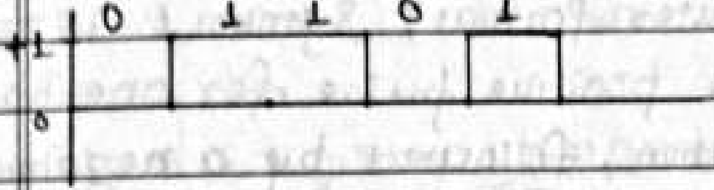


fig: NRZ unipolar format

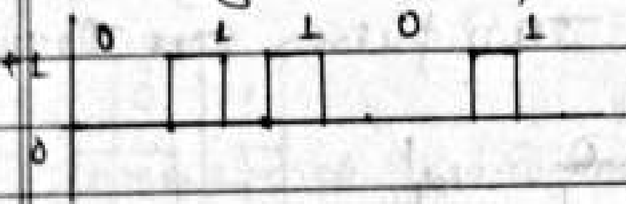


fig: RZ unipolar format

Polar format:-

In polar format, a positive pulse is transmitted for symbol 1 & negative pulse for symbol 0. It can be of NRZ or RZ.

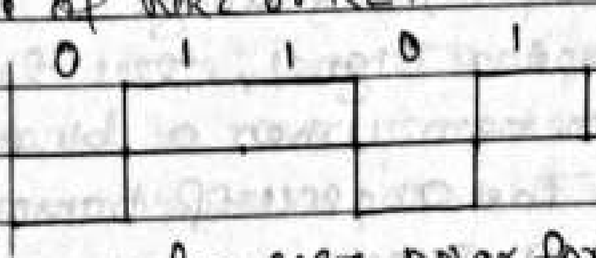


fig: NRZ polar format

Bipolar format:-

In bipolar format, positive & negative pulses are used alternatively for the transmission of 1s (with alteration taking place at every occurrence of 1) & no pulses for the transmission of 0s. It can be of NRZ or RZ type.

Signature

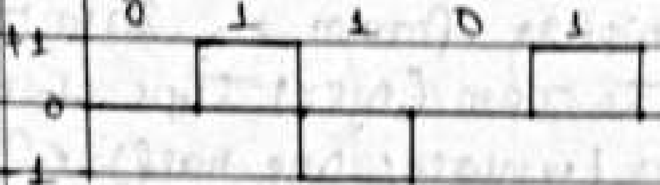


fig: NRZ bipolar format

Manchester format (biphase baseband signalling):

In Manchester format, symbol 1 is represented by transmitting a positive pulse for one-half of the symbol duration, followed by a negative pulse for the remaining half of the symbol duration. For symbol 0, these two pulses are transmitted in reverse order.

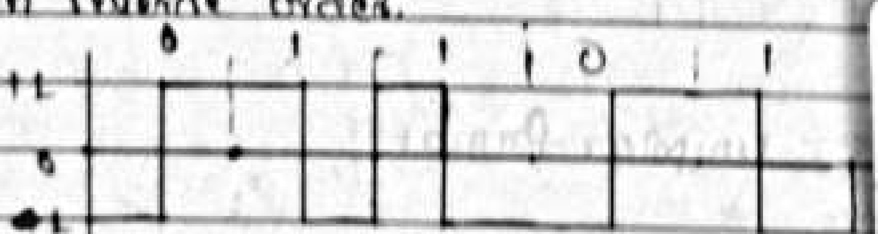


fig: Manchester format

Inter-symbol interference (ISI) in BB digital communication :-

The receiving filter output $y(t)$ is compared with threshold at $t = mT_b$,

$$y(t=mT_b) = \mu A_m + \mu \sum_{k=-\infty (k \neq m)}^{\infty} A_k\, g(mT_b - kT_b)$$

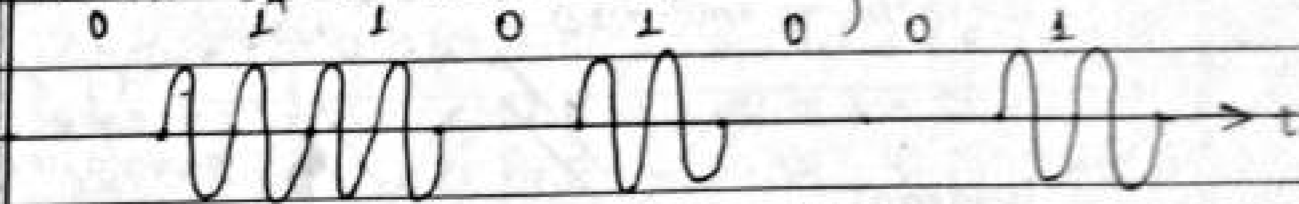
The first term of the above expression μA_m is the m^{th} decoded bit & the second term represents the residual effect of all other transmitted bits on the m^{th} bit being decoded. The residual effect is called intersymbol interference (ISI). ISI arises due to dispersion of pulse shape by the filters & channel.

Signature

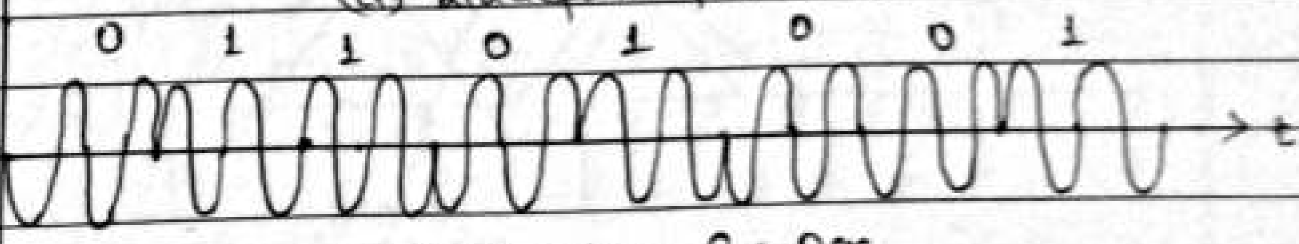
Digital Modulation formats:-

Modulation is defined as the process by which some characteristic of a carrier is varied in accordance with a modulating wave. In digital communications the modulating wave consists of binary data or an M-ary encoded version of it. For the carrier, it is customary to use a sinusoidal wave. With a sinusoidal carrier, the feature that is used by the modulator to distinguish one signal from another is a step change in the amplitude, frequency or phase of the carrier. The result of this modulation process is amplitude-shift keying (ASK), frequency-shift keying (FSK) or phase-shift keying (PSK) respectively as illustrated in fig for a special case of a source of binary data.

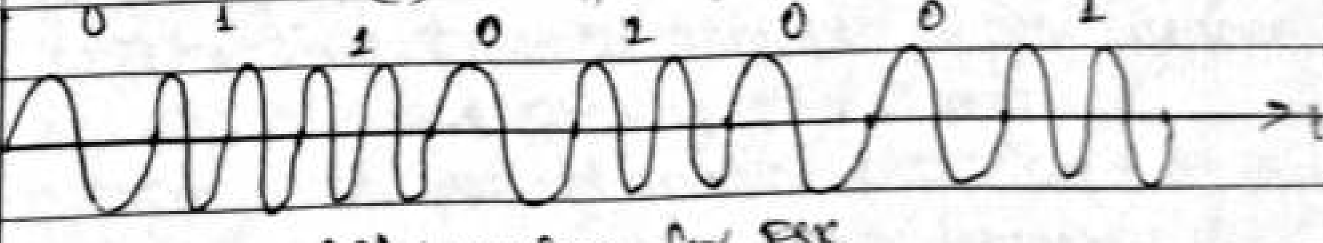
Binary data



(a) Waveform for ASK



(b) Waveform for PSK



(c) Waveform for FSK

To perform demodulation at the receiver, we've the choice of coherent or non-coherent detection. In the ideal form of coherent detection, exact replicas of the possible arriving signals are available at the receiver. In non-coherent detection, on the other hand, knowledge of the carrier wave's phase is not required. The complexity of the receiver is thereby reduced but at the expense of an inferior error performance, compared to a coherent system.

Binary Amplitude shift keying (BASK):-

In amplitude shift keying or on-off keying (OOK), symbol 1 is represented by transmitting a sinusoidal carrier of amplitude $\sqrt{2E_b/T_b}$ & symbol 0 is represented by switching off the carrier. In other words, the amplitude of the carrier signal is keyed according to the modulating digital signal. Mathematically, the output of ASK is

$$s(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{for } m(t) = 1 \\ 0 & \text{for } m(t) = 0 \end{cases} \quad \text{---(1)}$$

where, E_b = Bit energy

T_b = Bit duration

Signal space diagram of ASK:-

The ASK waveform of eqn(1) for symbol '1' can be represented as

$$s(t) = \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

$$= \sqrt{E_b} \cdot \phi_1(t) \quad \text{---- (2)}$$

This means that there is only one carrier function $\phi_1(t)$. The signal space diagram will have two points on $\phi_1(t)$. One will be at zero & other will be at $\sqrt{E_b}$.

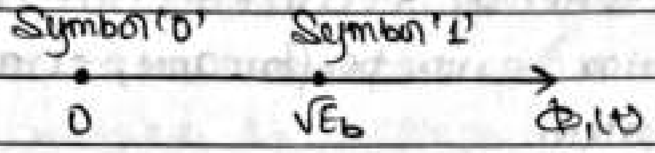


fig: Signal space diagram of ASK

Generation of ASK Signal:-

ASK signal can be generated by simply applying the incoming binary data (represented in unipolar form) & the sinusoidal carrier to the two inputs of a product modulator (i.e. balanced modulator). The resulting output will be the ASK waveform.

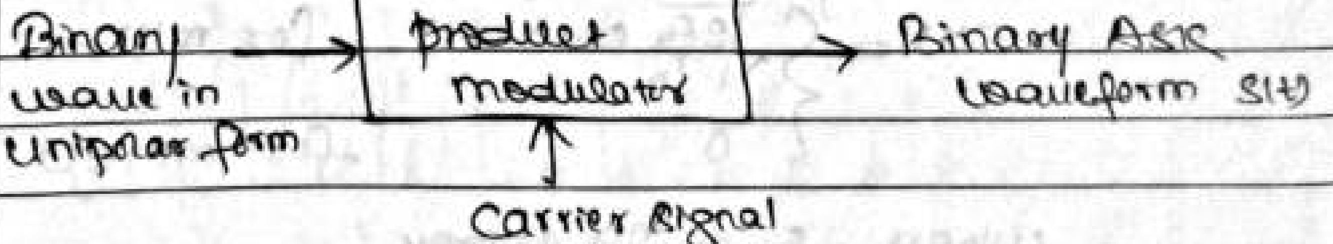


fig: Generation of binary ASK waveform

Signature

Coherent demodulation of binary ASK :-

Fig shown the coherent detection of binary ASK.

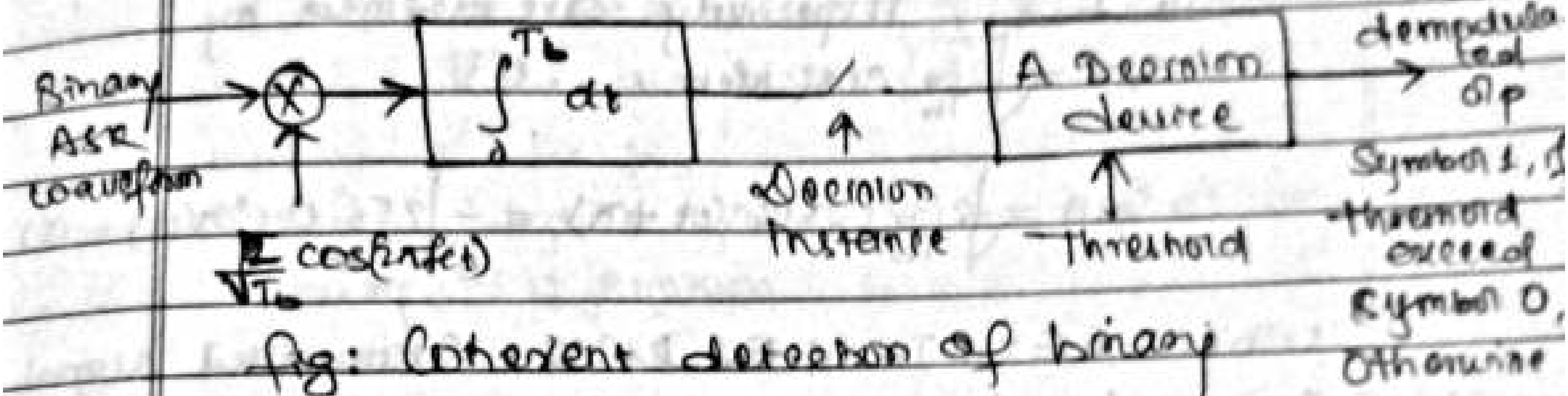


fig: Coherent detection of binary ASK signals

It consists of a product modulator which is followed by an integrator & a decision device. The incoming ASK signal is applied to one input of the product modulator. The other input of the product modulator is supplied with a sinusoidal carrier which is generated with the help of a local oscillator. The integrator operates on the output of the multiplier for successive bit intervals & essentially performs a low-pass filtering action. Now the decision making device compares the output of the integrator with a preset threshold. It makes a decision in favour of symbol 1 when threshold is exceeded & in favour of symbol 0, otherwise.

Signature

Binary phase shift keying (BPSK):-

In a coherent binary PSK system, the pair of signals, $S_1(t)$ & $S_2(t)$, used to represent binary symbols 1 & 0, respectively are defined by

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \text{---(1)}$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \text{---(2)}$$

where $0 \le t \le T_b$ & E_b is the transmitted signal energy per bit.

A pair of sinusoidal waves that differ only in a relative phase-shift of 180°, as defined above, are referred to as antipodal signals.

From (1) & (2), it is clear that there is only one basis function of unit energy, namely,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b \quad \text{--(3)}$$

Then, the transmitted signals $S_1(t)$ & $S_2(t)$ in terms of $\phi_1(t)$ will be

$$S_1(t) = \sqrt{E_b}\,\phi_1(t) \quad 0 \le t \le T_b \quad \text{--(4)}$$

& $$S_2(t) = -\sqrt{E_b}\,\phi_1(t) \quad 0 \le t \le T_b \quad \text{--(5)}$$

A coherent binary PSK system is therefore characterized by having a signal space that is one-dimensional (i.e. $N=1$) & with two message points (i.e. $M=2$) as shown in fig.

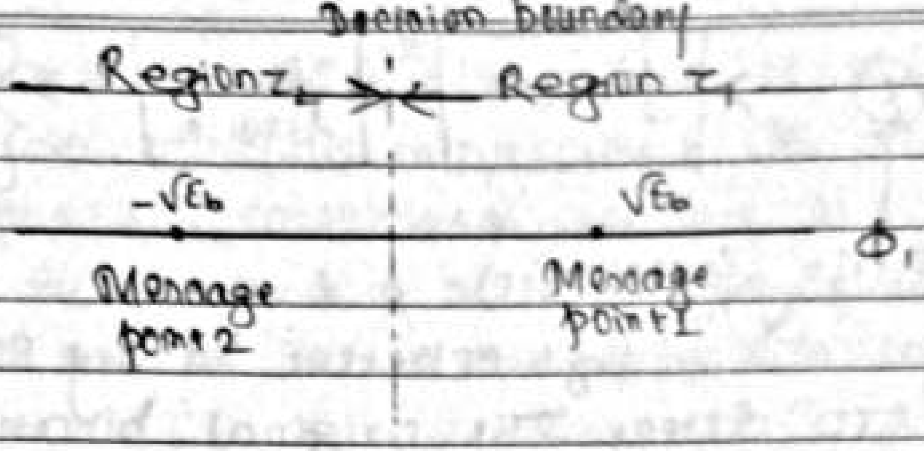


fig: Signal space diagram for coherent binary PSK system

Generation of PSK signal:-

To generate a binary PSK wave, we see from eqn(3) to (5) that we have to represent the input binary sequence in polar form with symbols 1 & 0 represented by constant amplitude levels of $+\sqrt{E_b}$ & $-\sqrt{E_b}$ respectively. This binary wave & a sinusoidal carrier wave $\phi_1(t)$ are applied to a product modulator as shown in fig below:

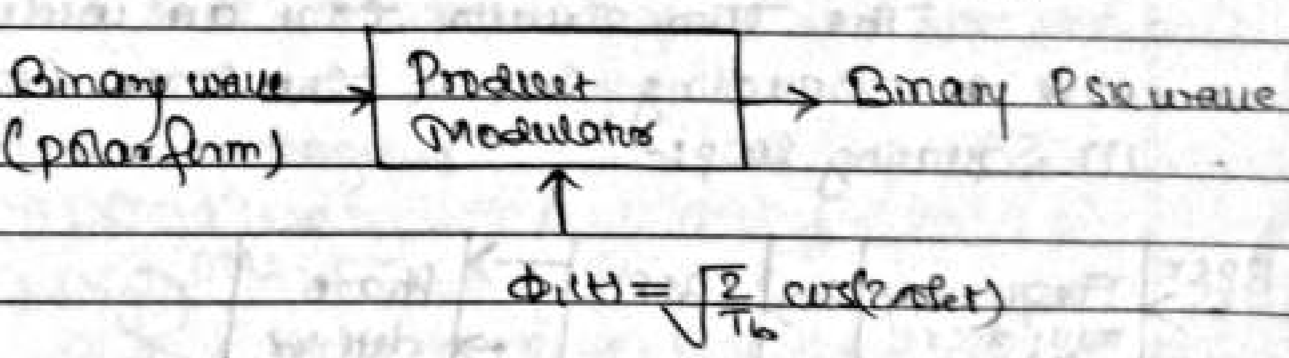


fig: Binary PSK transmitter

Coherent demodulation of PSK signal:-

Fig shown the coherent detection of binary PSK wave.

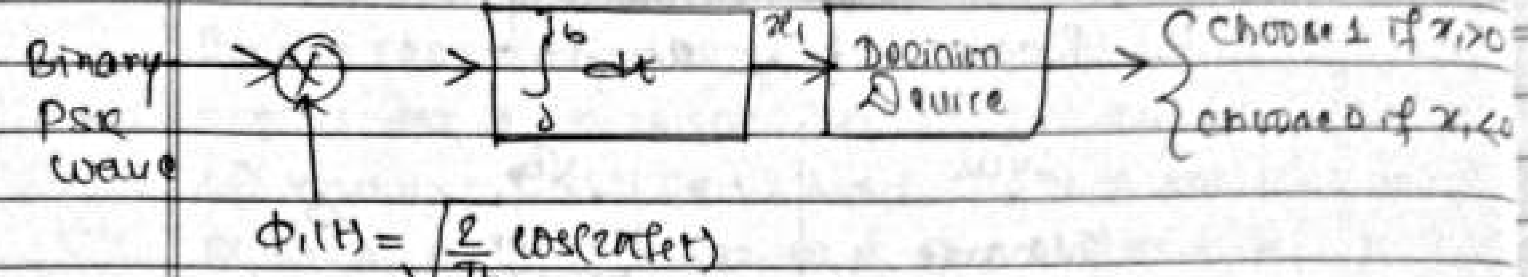


fig: coherent binary PSK receiver

To detect the original binary sequence of 1s & 0s, we apply noisy PSK wave $x(t)$ (at the channel output) to a correlator, which is also supplied with a locally generated coherent reference signal $\phi_1(t)$. The correlator output x_1 is compared with a threshold of zero volts. If $x_1 > 0$, the receiver decides in favor of symbol 1. On the other hand, if $x_1 < 0$, it decides in favor of symbol 0.

Binary Frequency Shift Keying (BFSK):

In a binary FSK system, symbols 1 & 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. A typical pair of sinusoidal waves is described by

$$S_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases} \quad \text{--- (1)}$$

where $i = 1, 2$ & E_b is the transmitted signal energy per bit & the transmitted freq^n equals

Signature

$f_i = \frac{n_c + i}{T_b}$ for some fixed integer n_c & $i = 1, 2$ ...(2)

Thus, symbol 1 is represented by $s_1(t)$ & symbol 0 by $s_2(t)$.

From eqn(1), the basis functions is

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases} \quad ...(3)$$

where $i = 1, 2$. Correspondingly, the coefficient s_{ij} for $i = 1, 2$ & $j = 1, 2$ is defined by

$$s_{ij} = \int_0^{T_b} s_i(t) \phi_j(t)\, dt$$

$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\, dt$$

$$= \begin{cases} \sqrt{E_b}, & i = j \\ 0, & i \ne j \end{cases} \quad ...(4)$$

Thus a coherent binary FSK system is characterized by having a signal space that is two-dimensional (i.e. $N = 2$) with two message points (i.e. $M = 2$), as shown in fig.

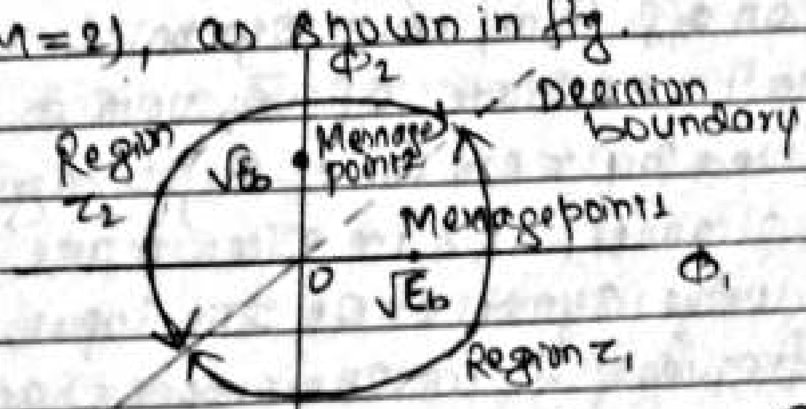


fig: Signal space diagram for coherent binary FSK system

Fig shown BFSK transmitter & receiver.

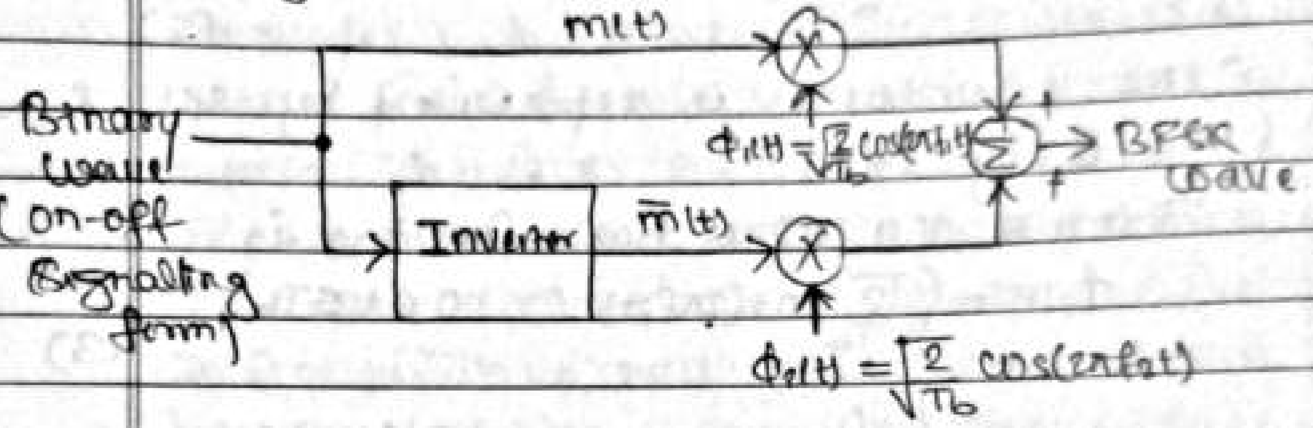


fig: BFSK transmitter

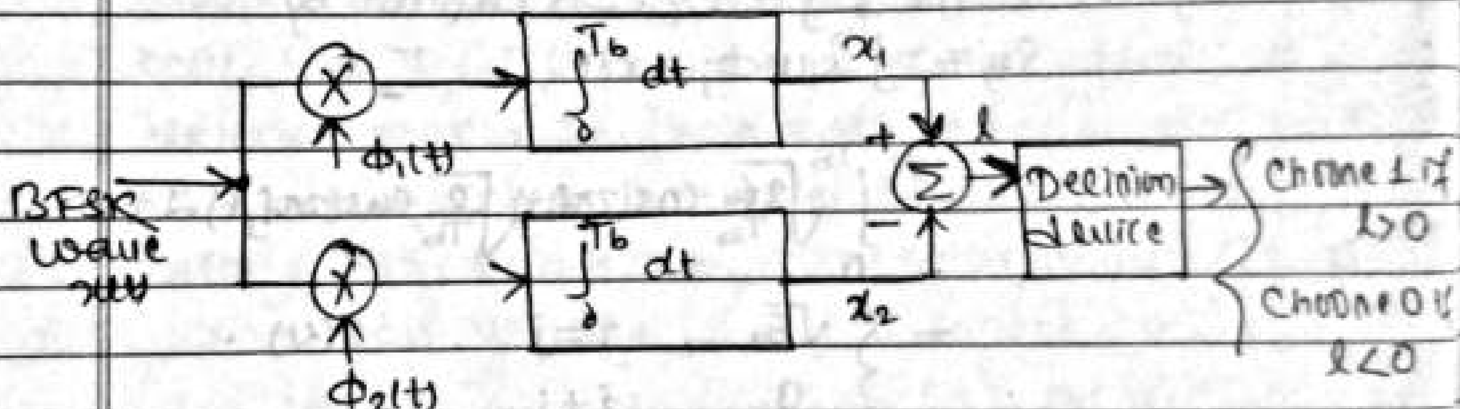


fig: BFSK receiver

BFSK transmitter :-

The input binary sequence is represented by in its on-off form, with symbol 1 represented by a constant amplitude of $\sqrt{E_b}$ volts & symbol 0 represented by zero volts. By using inverter in the lower channel, we in effect make sure that when we've symbol 1 at the input, the oscillator with frequency f_1 in the upper channel is switched on while the oscillator with frequency f_2 in the lower channel is switched off with the result that frequency f_1 is transmitted. Conversely when we have symbol 0 at the input, the oscillator in the upper channel is switched off & the oscillator in the lower channel is switched on, with the result that frequency f_2 is transmitted. The two frequencies f_1 & f_2 are chosen to equal integer multiples of bit rate $1/T_b$.

BFSK receiver :-

In order to detect the original binary sequence given the noisy received wave $x(t)$, we may use the receiver shown in fig 9.1 consists of two correlators with a common input, which are supplied with locally generated coherent reference signals $\phi_1(t)$ & $\phi_2(t)$. The correlator outputs are then subtracted one from the other, & the resulting difference, l compared with the threshold of zero volts. If $l>0$, the receiver decides in favor of 1, on the other hand, if $l<0$, it decides in favor of 0.

M-ary data communication systems :-

In an M-ary signalling scheme, we may send one of M possible signals, $s_1(t), s_2(t), \ldots s_M(t)$, during each signalling interval of duration T. For almost all the applications, the number of possible signals $M = 2^n$, where n is the integer. The symbol duration $T = nT_b$, where T_b is the bit duration. These

Signature

signals are generated by changing the amplitude, phase or freq^n of a carrier in M discrete steps. Thus, we've M-ary ASK, M-ary PSK & M-ary FSK digital modulation schemes. The QPSK system is an example of M-ary PSK with M=4.

Another way of generating M-ary signals is to combine different methods of modulation into a hybrid form. For example, we may combine discrete changes in both the amplitude & phase of a carrier to produce M-ary amplitude ~~and~~ phase keying (APK). A special form of hybrid modulation, called M-ary QAM, has some attractive properties.

M-ary signalling schemes are preferred over binary signalling schemes for transmitting digital information over band-pass channels when the requirement is to conserve bandwidth at the expense of increased power. In practice, we rarely find a communication channel that has the exact bandwidth required for transmitting the output of an information source by means of binary signalling schemes. Thus, when the band-width of the channel is less than the required value, we may use M-ary signalling schemes so as to utilize the channel efficiently.

To illustrate the bandwidth-conservation

Signature

capability of M-ary signalling schemes, consider the transmission of information consisting of a binary sequence with bit duration T_b. If we were to transmit this information by means of binary PSK, for example, we require a bandwidth inversely proportional to T_b. However, if we take blocks of n bits & use an M-ary PSK scheme with $M = 2^n$ & symbol duration $T = nT_b$, the bandwidth required is inversely proportional to $1/nT_b$. This shows that the use of M-ary PSK enables a reduction in transmission bandwidth by the factor $n = \log_2 M$ over binary PSK.

Quadrature Amplitude Modulation (QAM):-

In this modulation scheme, the carrier experiences amplitude as well as phase modulation. The general form of M-ary QAM is defined by the transmitted signal,

$$s_i(t) = \sqrt{\frac{2E_0}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}}\, b_i \sin(2\pi f_c t) \quad 0<t<T \quad \text{---(1)}$$

where, E_0 = energy of signal with lowest amplitude

a_i & b_i = Pair of independent integers

Signature

The signal $s_i(t)$ consists of two phase-quadrature carriers, each of which is modulated by a set of discrete amplitudes, hence the name "quadrature amplitude modulation".

The signal $s_i(t)$ can be expanded in terms of a pair of basis functions:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \le t \le T \quad \text{---(2)}$$

$$\& \quad \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \le t \le T \quad \text{---(3)}$$

Fig shows the block diagram of an M-ary QAM transmitter & receiver.

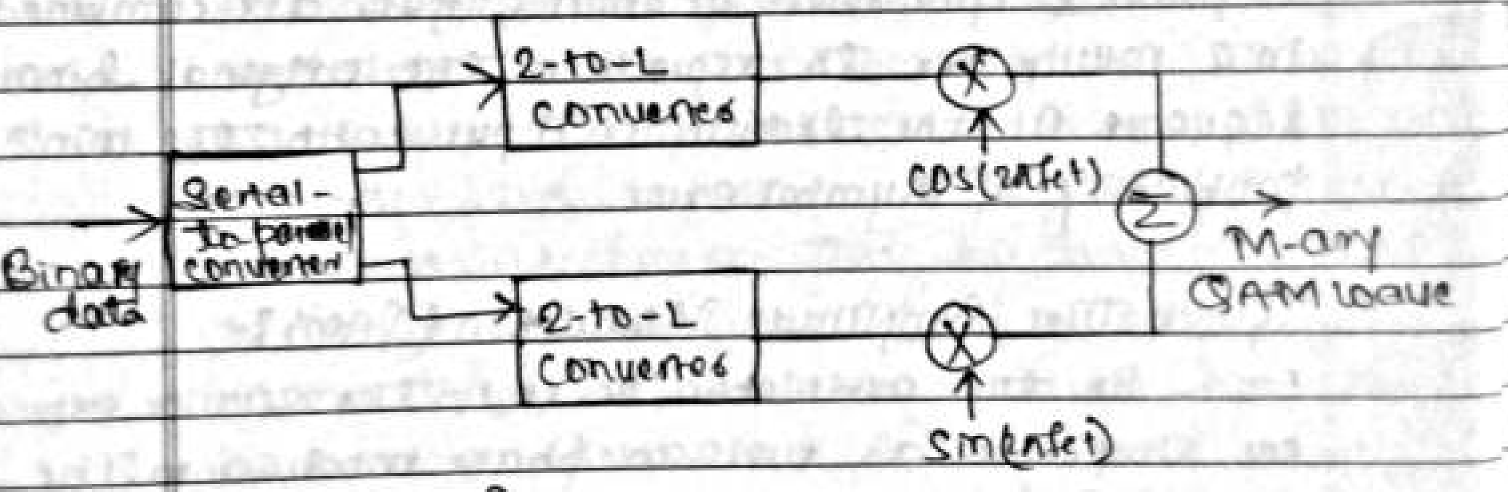


fig: Transmitter

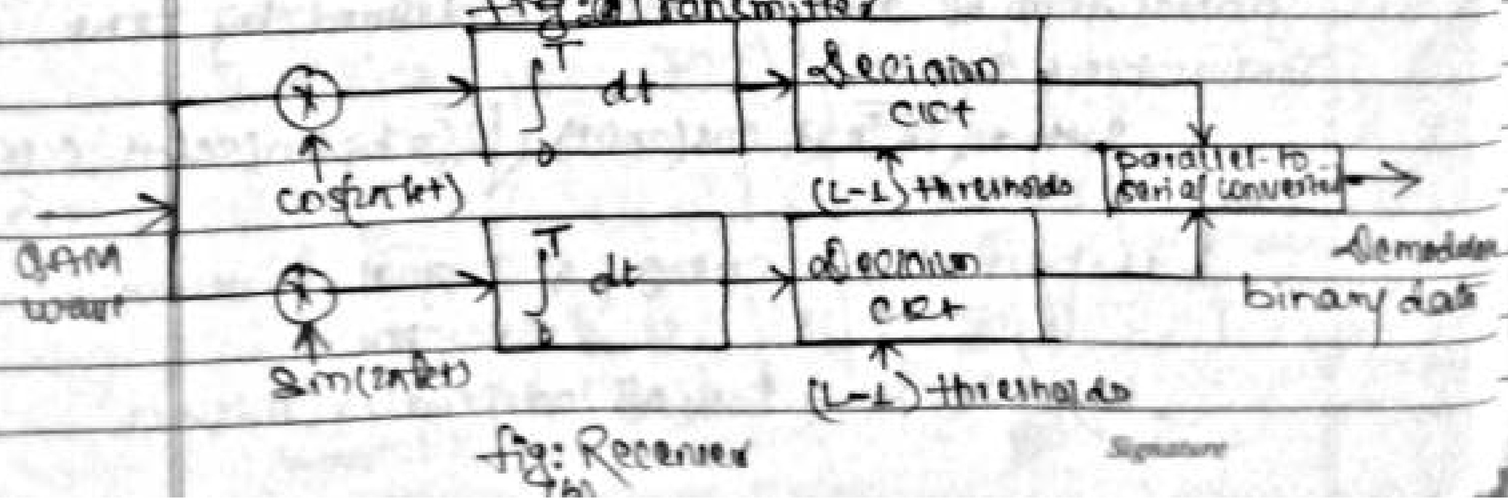


fig: Receiver

The serial-to-parallel converter accepts a binary sequence at a bit rate $R_b = 1/T_b$ & produces two parallel binary sequences whose bit rates are $R_b/2$ each. The 2-to-L level converters, where $L = \sqrt{M}$, generate polar L-level signals in response to the respective in-phase & quadrature channel inputs. Quadrature-carrier multiplexing of the two polar L-level signals so generated produces the desired M-ary QAM signal.

Fig(b) shows the block diagram of the corresponding receiver. Decoding of each baseband channel is accomplished at the output of the decision circuit, which is designed to compare the L-level signals against (L-1) decision-thresholds. The two binary sequences so detected are then combined in the parallel-to-serial converter to reproduce the original binary sequence.

Introduction to multiplexing:-

It is the process of combining two or more signal into a single wave from which the signals can be individually recovered. In our case, the signals are voice channels & we can literally combine more than 1000 such channels for transmission over a medium. The medium has to be able to accomodate the required bandwidth.

Types of multiplexing
- FDM (Frequency Division Multiplexing)
- TDM (Time Division Multiplexing)

Frequency Division Multiplexing:-

It is the method of allocating a unique band of frequencies in a comparatively wideband frequency spectrum of the transmission medium to each communication channel on a continuous time basis. To accomodate large number of voice channels, telephone companies established 4kHz as the standard bandwidth of voice channel including guard band in SSB modulation.

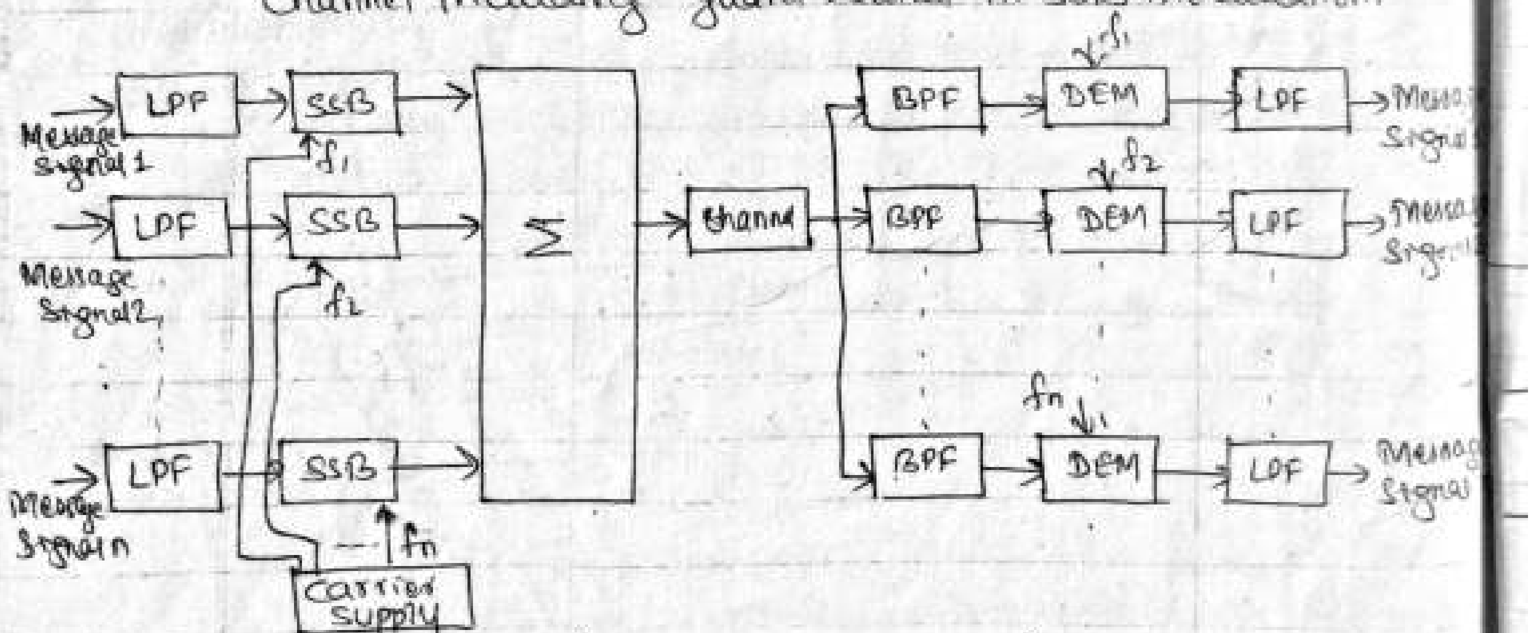
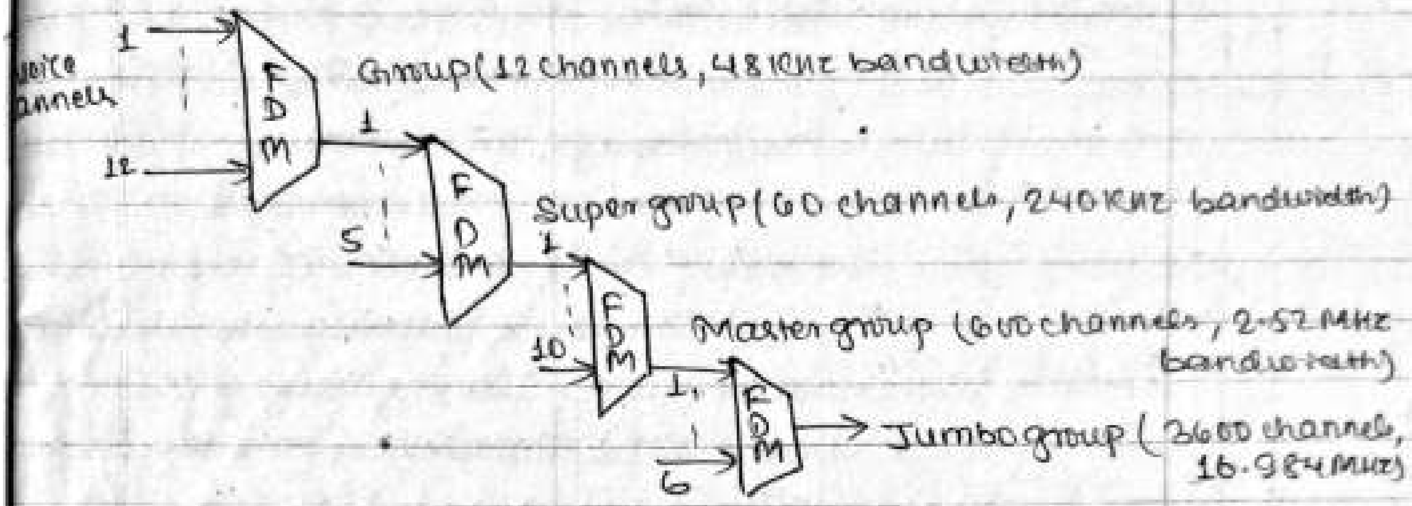


fig: Block diagram of FDM system

LPF - low pass filter SSB - single sideband modulator
BPF - band pass filter DEM - Demodulator

FDM in telephony:-



Time Division Multiplexing:-

It involves the sharing of a transmission media by establishing a sequence of time slots during which each individual source can transmit signal. Thus the entire bandwidth of the facility is periodically available to each user for a restricted time interval. Thus over the same communication channel, samples of no of message signals can be transmitted serially & then recovered & separated at the receiving end. In other words, the time interval between two adjacent samples of one message can be used to transmit samples of other messages & such technique is known as time division multiplexing.

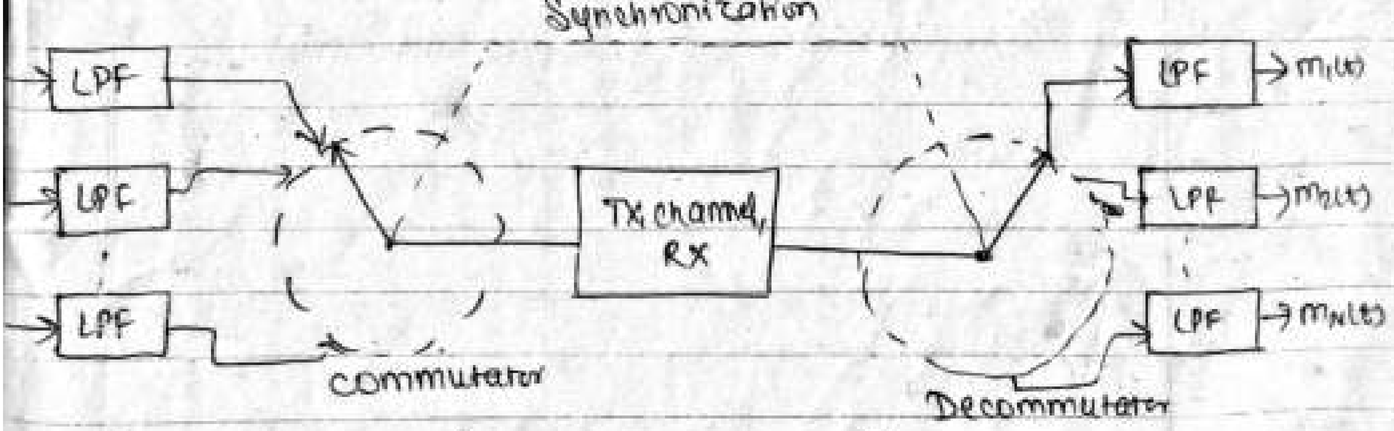


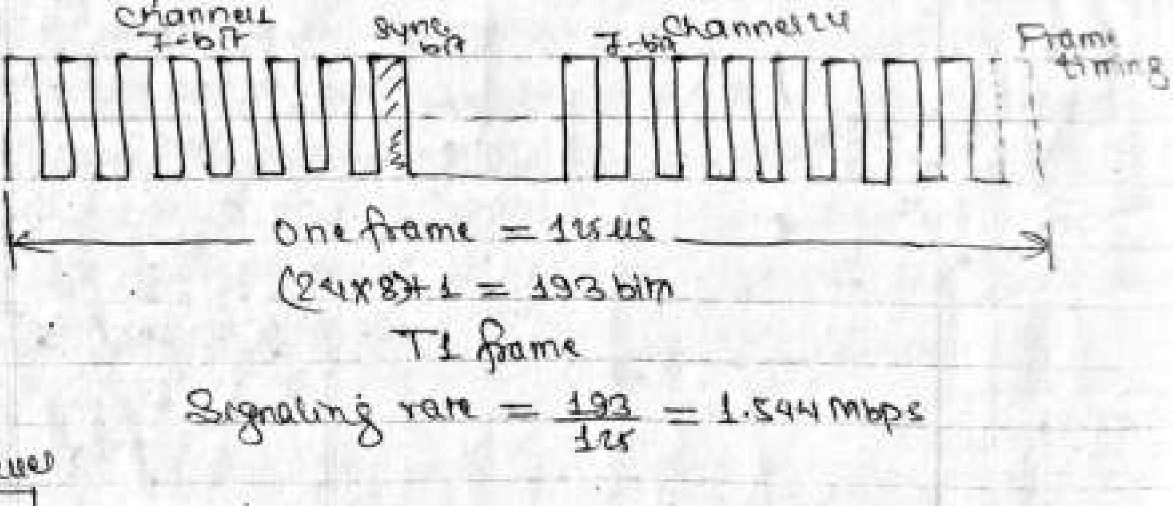
fig: Block diagram of TDM System

TDM consist of commutator at the transmission end & decommutator at receiving end.

The signals to be multiplexed are first individually band limited by LPF. The commutator takes samples of each signal sequentially at fixed interval of time. These samples are then transmitted through the common channel using digital transmission technique. The decommutator at the receiving end separates the sequentially transmitted signal into individual signals. Commutator & decommutator are synchronized using timing signal. LPF at the receiving end convert the samples of message signal into original continuous signal.

T1 hierarchy in digital telephony:-

In T1 system, 24 voice channels are sampled at $f_s = 8\text{kHz}$ ($T_s = 125\,\mu s$). Each sample is quantized & converted into 7-bit PCM codeword. 8th additional bit is added for synchronization purpose. Thus, total no. of bits per voice channel is 8.



$(24\times 8)+1 = 193$ bits

T1 frame

$$\text{Signaling rate} = \frac{193}{125} = 1.544\ \text{Mbps}$$

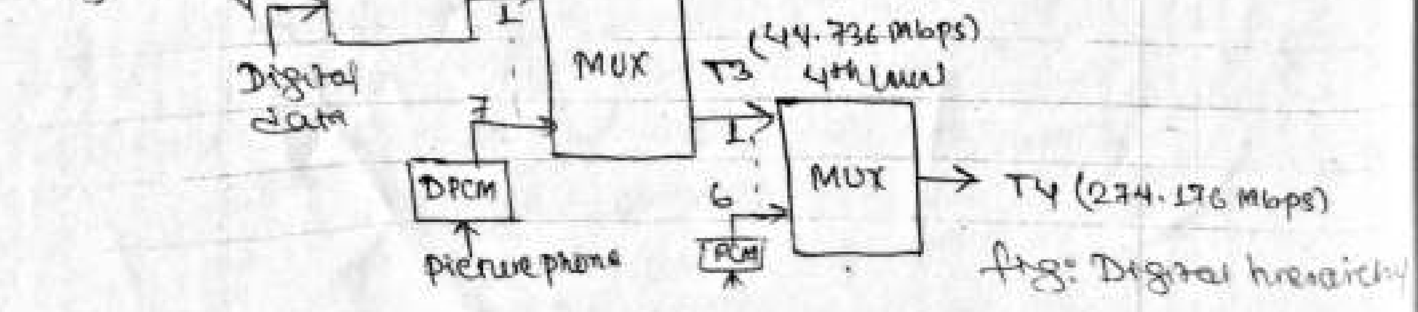


fig: Digital hierarchy

E1 hierarchy:-

Channel 0	Channel 1		Channel 15	Channel 16	Channel 17		Channel 31

Synchronizing the MUX at each end of the link | voice or data | CCS or CAS | voice or data

CCS - common channel Signaling
CAS - channel associated signaling

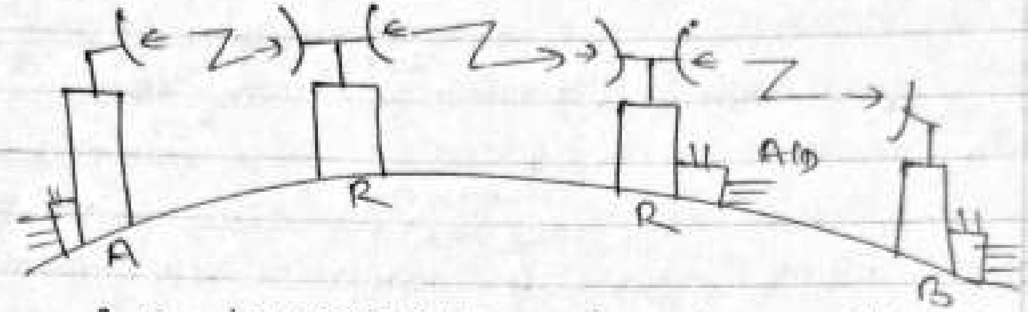
Channel 0 is for timing used to synchronize the multiplexers at each end of the link. Channels 1 to 15 & 17 to 31 are for voice or data while channel 16 is used for CCS or CAS. Every 3.91 us, 8-bits from one channel is sent down the line followed by 8-bits from the next channel during next 3.91 us & so on in a round robin fashion throughout all the channels, thus 32 channels are used once every 125 us.

$$\text{Signaling rate} = \frac{32 \times 8}{125} = 2.048 \text{ Mbps}$$

E2, 4×E1 - 8.448 Mbps
E3, 4×E2 - 34.368 Mbps
E4, 4×E3 - 139.264 Mbps
E5, 4×E4 - 565.148 Mbps

Introduction to multiple access techniques - FDMA, TDMA, CDMA:-

Terrestrial microwave link:-



A, B = terminal sites , R = repeater, A|D = add & drop lines
fig: Microwave link

A microwave link is made up of two terminal sites & usually one or more repeater sites as shown in fig. At the transmitting terminal site, voice channels are multiplexed (FDM) which is then modulated into RF carrier. At the receiving site, the RF carrier is demodulated & demultiplexed into individual voice channels. A repeater site is characterized by two antennas for the two directions. It receives, amplifies & retransmits the RF signals to the next site in sequence. Some repeaters may add & drop a few lines serving some local connections.

Microwave system may be either long-haul type or short-haul type. A long-haul system has only a small number of add & drop points but has many simple repeaters & covers a long distance end-to-end. A short haul system consists of a relatively small number of repeaters with frequent add & drop points.

Of the three commonly used microwave carrier bands, 4, 6 & 11 GHz, 4 GHz is used for long haul transmission. The frequencies around 11 GHz carrier experience severe attenuation during rainfall & hence are not used in long haul transmission. ~~Gt~~ 6 GHz band is useful for both long & short haul. Very high gain antennas are required for microwave communications.

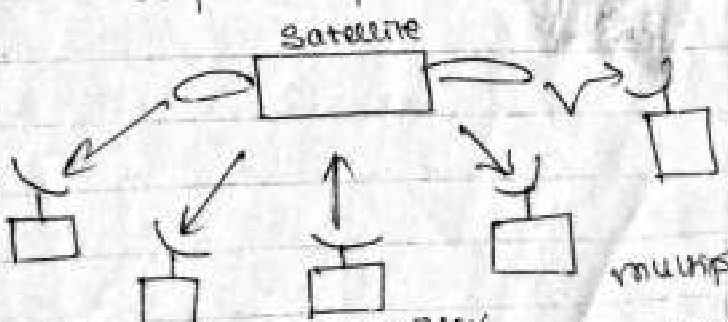
Application of terrestrial microwave system

- primary use of terrestrial microwave system is in long-haul ~~transmission~~ telecommunication service, as an alternative to coaxial cable or optical fiber.
- requires far fewer amplifiers or repeaters than coaxial cable over the same distance, but requires line of sight transmission.
- Microwave is commonly used for both voice & television transmission.
- another common use of microwave is for short point to point links between buildings, or as data link between local area network.

Satellite communication System :-

A communication satellite is, in effect, a microwave relay station. It is used to link two or more ground based microwave transmitter / receiver, known as earth stations or ground stations. The satellite receives transmission on one frequency band (uplink), amplifies the signal & transmit it on another frequency (downlink). A single orbiting satellite will operate on a number of frequency bands, called transponder channels. Fig shown two common configurations for satellite communication,

(a) point to point link via satellite microwave



(b) Broadcast link via satellite microwave

Application :-

- Television distribution
- Long distance telephone transmission
- private business networks

Optical fiber communication :-