Numericals: Remaining. P.S.: Marks are flexible

copy-1 (Theory only).

Chapter 1: 3–16
Chapter 2: 17–35
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Chapter 4: 68–96
Chapter 5: 97–118
Chapter 6: 119–140
Chapter 7: (141–167) (copy 2: 170–177)
Chapter 8: (175–187)

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IOE-ERC.

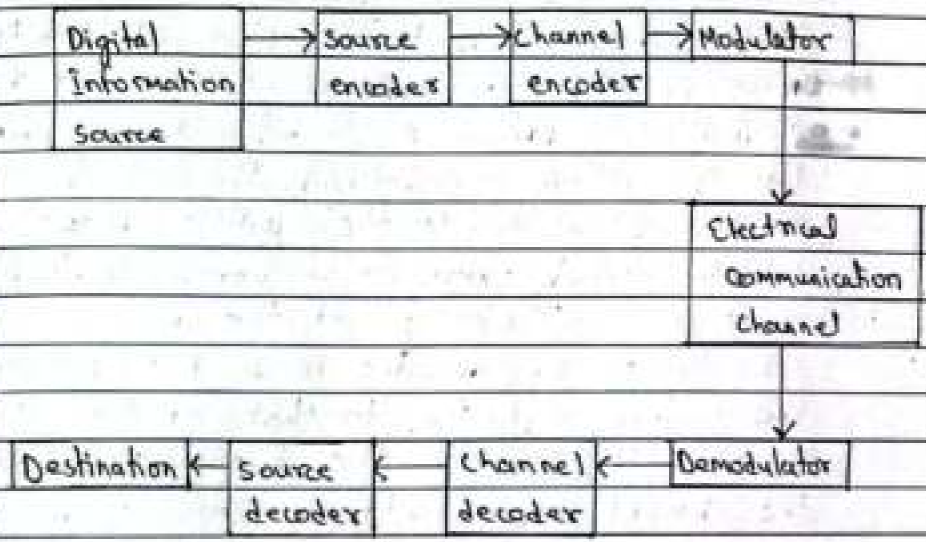
References: Communication systems, Dr. Sanjay Sharma
Handwritten note of communication system-I
Old question solution.

Chapter - 1

1. Introduction (3Hours / 5 marks)

Block diagram of digital communication system: 73Sh[5]

The following figure shows the model of a digital communication system. The overall purpose of the system is to transmit the message or sequences of symbols coming out of a source to a destination point at as high a rate and accuracy as possible.



(i) Digital information source: In case of DIS, the information source produces a message signal which is not continuously varying with time. Rather the message signal is discontinued with time. The output of discrete information sources consists of a sequence of discrete symbols or letters. An analog information source may be transformed into discrete information source through the process of sampling & Quantizing.

(ii) Source Encoder and Decoder: The symbols produced by the information source are given to the source encoder. These symbols cannot be transmitted directly. They are first converted into digital form (0's & 1's) by source encoder. Each binary '1' and '0' is known as a bit. The group of bits is called a codeword. The source encoder assigns codewords to the symbols. For each distinct symbol, there is a unique codeword.

Similarly, at the receiver end, some sort of decoder is used to perform the reverse operation to that of the source encoder. It converts the binary output of the channel decoder into a symbol sequence. Some decoders also use memory to store codewords. The decoders and encoders ~~are~~ can be synchronous or asynchronous.

(iii) Channel encoder and decoder: After signal conversion via source encoder, the signal is transmitted through the channel. The communication channel adds noise and interference to the signal being transmitted. Hence errors are introduced in the binary sequence received at the receiver end. Thus channel coding is done to avoid these kind of errors. Channel encoder adds some redundant binary bits to the input sequence. These redundant bits are always added with some properly defined logic.

Similarly channel decoder at receiving end decodes these redundant bits and checks for any errors thus reconstructing error free accurate bit sequence reducing the effects of channel noise and distortion.

(iv) Digital Modulators and Demodulators: Modulation is the process of superimposing low frequency message signal on high frequency carrier signal for efficient transmission of signal. Similarly Demodulation at receiver gets the original message signal back from modulated signal.

Here in digital communication system, digital modulation techniques are used. The carrier signal used by digital modulators is always continuous sinusoidal wave of high frequency. Amplitude shift keying (ASK), Phase shift keying (PSK), frequency shift keying (FSK), differential phase shift keying (DPSK) and minimum shift keying (MSK) are the examples of various digital modulators. However, since these modulators use a continuous carrier wave, therefore they are known as digital CW modulators. At the receiving end, the digital demodulator converts the input modulated signal into the sequence of binary bits.

6. Communication channel: The connection between transmitter and receiver is established through a communication channel. The communi-cation can take place through wireline, wireless or fiber optic channels. The other media such as optical disks, magnetic tapes & disks, etc may also be called as a communica-tion channel since they can also carry data through them.

Advantages of Digital communication system: G9ch[2]

(i) The digital communication systems are simpler and cheaper compared to analog communication systems because of advances made in the IC technology.

(ii) In digital communication, the speech, video and other data may be merged and transmitted over a common channel using multiplexing.

(iii) Using data encryption, only permitted receivers may be allowed to detect the transmitted data.

(iv) Since the transmitted signal is digital in nature, therefore a large amount of noise interference may be tolerated.

(v) Since in digital communication, channel coding is used, therefore errors may be detected and corrected in receivers.

(vi) Digital communication is adaptive to other advanced branches of data processing such as digital signal processing, image processing & data compression, etc.

76ch Q) Define any three types of noises: 76ch [3]

= Noise in a communication system is basically undesired or unwanted signals that get randomly added to the actual information carrying signal. They are subcategorized into:

(i) External noise: Natural & Man-made noise
(ii) Internal noise: Generated by electronic equipment involved in the system itself.

Three types of noise:

(i) Thermal noise: Electrical signal is transmitted when electrons move randomly. The random motion of the electrons is the reason for the thermal energy received by the conductor. These free electrons are non-uniformly distributed within conductor. Due to this a possibility exists that at one end the number of free electrons will be comparatively higher than at the other end. This non-uniform distribution of electrons provides the average voltage to be zero, however the average power is not zero in this case. So, this non-zero power is nothing but noise. And as it is the action of thermal action. Hence also known as thermal noise power. It is also known as Johnson or white noise.

(ii) Shot noise: It is the result of random variation in the appearance of electrons and holes at the output side of the device. These random movements are the result of discontinuities in the device which is used by the system. The shot noise generates sound like several lead shots are striking over a metal plate or tube. Non-linearity or discontinuity in the system generates shot noise.

(iii) Partition Noise: Here the name itself is indicating the cause for generation of this type of noise. As it gets generated when the system is composed of multiple paths, & during the flow the current gets divided in those paths. These are nothing but the result of random variation in the divisions. Due to this reason some devices offer low partition noise while some offer high.

Source coding: 70 Asd [1]

A conversion of the output of a discrete memoryless source (DMS) into a sequence of binary symbols (ie, binary code word) is called source coding. The device that performs this conversion is called the source encoder. Figure below shows a source encoder.

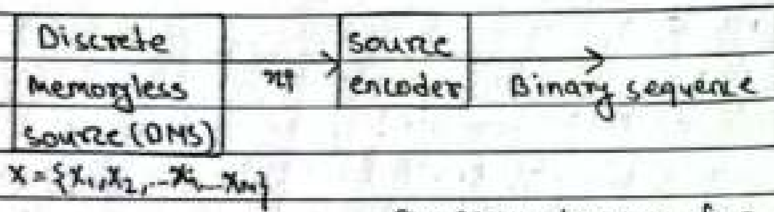


Fig: Block diagram for source coding.

Importance of source coding in Digital communication system: 73Ch, 71Ch, 70Ch, 69Ch [6,2]

An objective of source coding is to minimize the average bit rate required for representation of the source by reducing the redundancy of the information source. By reducing the redundancy of the encoding, we can make the probability of errors approach zero.

Significance of variable length coding with example: 72Ku

A variable length coding technique in which codeword length is not fixed. To obtain this, short codes are assigned to frequently occuring events and long codes are assigned to infre-quent events. The advantage / significance of VLC is that it does not degrade the signal quality in any way. The reconstructed signal will exactly match the input signal so that if the signal is adequately described by a series of events, using VLCs to communicate them to the decoder will not change the events. A example is shown in table below

x_i	code 1	code 2	code 3	code 4
x_1	0	0	0	1
x_2	1	10	01	01
x_3	00	110	011	001
x_4	11	111	0111	0001

69Ch [5] Importance of channel coding: They are used to protect the digital information from noise & interference & reduce the number of bit errors by introducing redundant bits

Source coding vs channel coding: 74 As [3]

Source coding	Channel coding
• It is defined as the conversion of output of discrete memoryless source into a sequence of binary symbols.	It is defined as adding extra bits so that we can control error at the receiver side.
• It's main purpose is to reduce redundancy	It's main purpose is to reduce noise & error correction.
• This causes reduce in Bandwidth consumption	This causes extra Bandwidth consumption as extra bits are added.
• It's methods are - shanon-fano coding - Huffman coding	It's methods are - Block code - cyclic code

Q) Explain Shannon-Fano coding. 73 Sh [3]

= The steps for shannon-Fano algorithm are given below:

1) List the source symbols in the order of decreasing probability.

2) Partition the set into two sets that are as close to equiprobables as possible, & assign 0 to upper set and 1 to lower set

3) Continue this process, each time partitioning the sets with as nearly equal probabilities as possible until further partitioning is not possible.

An example is given below: In shannon fano coding the ambiguity may arise in the choice of approximately equiprobable sets.

x_i	$P(x_i)$	step 1	step 2	step 3	step 4	code
x_1	0.30	0	0			00
x_2	0.25	0	1			01
x_3	0.20	1	0			10
x_4	0.12	1	1	0		110
x_5	0.08	1	1	1	0	1110
x_6	0.05	1	1	1	1	1111

Entropy $H(x) = -\sum_{i=1}^{6} P_i \log_2(P_i)$

$= \sum_{i=1}^{6} P_i \log_2(1/P_i)$

$= 0.3 \times \log_2\left(\frac{1}{0.3}\right) + 0.25 \times \log_2\left(\frac{1}{0.25}\right) + 0.20 \times \log_2\left(\frac{1}{0.20}\right)$
$+ 0.12 \log_2\left(\frac{1}{0.12}\right) + 0.08 \log_2\left(\frac{1}{0.08}\right) + 0.05 \log_2\left(\frac{1}{0.05}\right)$

$= 2.36$ bits/symbol.

Average length $(L) = \sum_{i=1}^{6} P(x_i) n_i$

$= 2(0.3) + 2(0.25) + 2(0.2) + 3(0.12) + 4(0.08) + 4(0.05)$

$= 2.38$ bits/symbol.

efficiency, $\eta = \frac{H(x)}{L} \times 100\% = 99\%$.

Huffman coding: 72ch[4], 70ch[3]

Huffman encoding results in optimum code. Thus it is the code that has the highest efficiency. The huffman encoding procedure is as follows:

1) list the source symbols in order of decreasing probability.

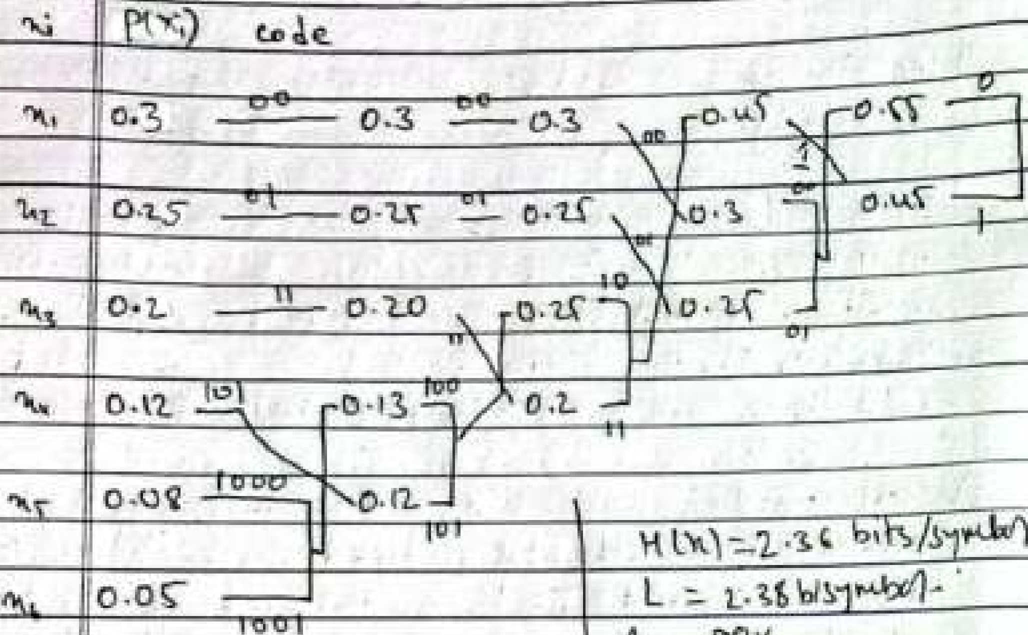
2) Combine the probabilities of the two symbols having the lowest probabilities, & reorder the resultant probabilities, this step is called reduction 1. The same procedure is repeated until there are two ordered probabilities remaining.

3) Start encoding with the last reduction, which consist of exactly two ordered probabilities. Assign 0 as the first digit in the codewords for all the source symbols associated with the first probability; assign 1 to the second probability.

4) Now go back & assign 0 and 1 to the second digit for the two probabilities that were combined in the previous reduction step. Retaining all assignments made in step 3.

5) keep regressing this way until the first column is reached.

Lets take the same example taken above.



$H(x) = 2.36$ bits/symbol
$L = 2.38$ b/symbol.
$\eta = 97\%$.

∴

0.3	00
0.25	01
0.2	11
0.12	101
0.08	1000
0.05	1001

Chapter - 2

Sampling theory: (4 Hours / 8 Marks)

* Sampling theorem: Nyquist theorem and its proof.
76Ch[3] 73sh[5], 78Bh[2], 72Ch[2], 71Ch[2], 74Ch[2]

Sampling is the fundamental operation in signal processing. It converts continuous time signal to discrete time signal.
The statement of sampling theorem can be given in two parts:

(i) A band-limited signal of finite energy, which has no frequency-component higher than f_m Hz, is completely described by its sample values at uniform intervals less than or equal to $1/2f_m$ second apart.

(ii) A band-limited signal of finite energy, which has no frequency components higher than f_m Hz, may be completely recovered from the knowledge of its samples taken at rate of $2f_m$ samples per second.

The first part represents the representation of signal in its samples & minimum sampling rate required to repeat a continuous-time signal into its samples.

The second part of the theorem represents reconstruction of the original signal from its samples. It gives sampling rate required for satisfactory reconstruction of signal from its samples.

Combining two parts, the sampling theorem may be stated as under:

"A continuous-time signal may be completely represented in its samples & recovered back if the sampling frequency is $f_s \geq 2f_m$. Here f_s is the sampling frequency & f_m is the maximum frequency present in the signal."

Proof: 73sh[5],

To prove the sampling theorem, we shall show that a signal whose spectrum is band-limited to f_m Hz, can be reconstructed exactly without any error from its samples taken uniformly at a rate $f_s > 2f_m$ Hz

Let us consider continuous time signal $x(t)$ whose spectrum is band-limited to f_m Hz. This means

that the signal $x(t)$ has no frequency components beyond f_m Hz. Therefore, $X(\omega)$ is zero for $|\omega| > \omega_m$, i.e.,

$$X(\omega) = 0 \quad \text{for } |\omega| > \omega_m$$

$$\text{where, } \omega_m = 2\pi f_m.$$

figure (a) shows CT signal $x(t)$. Let $X(\omega)$ be its FT or frequency spectrum as shown in figure (b). sampling of $x(t)$ at a rate of f_s Hz (f_s samples per second) may be achieved by multiplying $x(t)$ by an impulse train $\delta_{T_s}(t)$. The impulse train consists of unit impulses repeating periodically every T_s seconds, where $T_s = 1/f_s$.

This multiplication results in sampled signal $g(t)$. This sampled signal consists of impulses spaced every T_s seconds (the sampling interval).

The resulting or sampled signal may be written as

$$g(t) = x(t)\,\delta_{T_s}(t) \quad \text{—(i)}$$

Since, $\delta_{T_s}(t)$ is a periodic signal it can be represented as fourier series:

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \dots\right] \quad \text{—(ii)}$$

Here, $\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$.

Putting the value of $\delta_{Ts}(t)$ from (ii) in eq^n (i) the sampled signal is

$$g(t) = \frac{1}{T_s}\left[x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + 2x(t)\cos 3\omega_s t + \dots\right] \quad \text{--- (iii)}$$

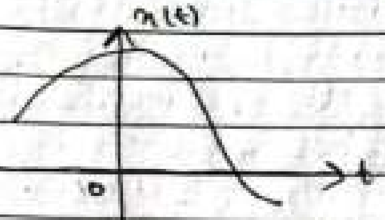


Fig a: ct signal $x(t)$

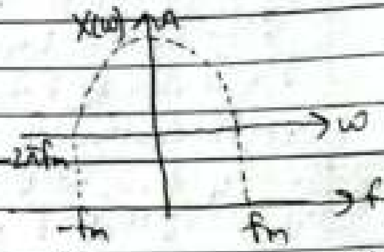


Fig b: Frequency spectrum of $x(t)$

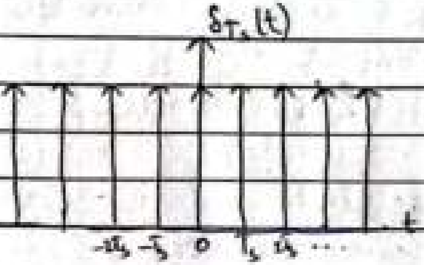


Fig c: impulse train as sampling function

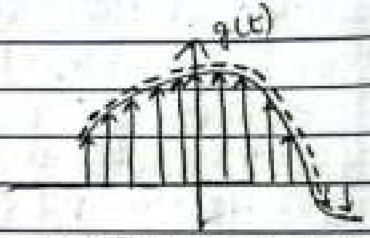


Fig d: Sampled signal

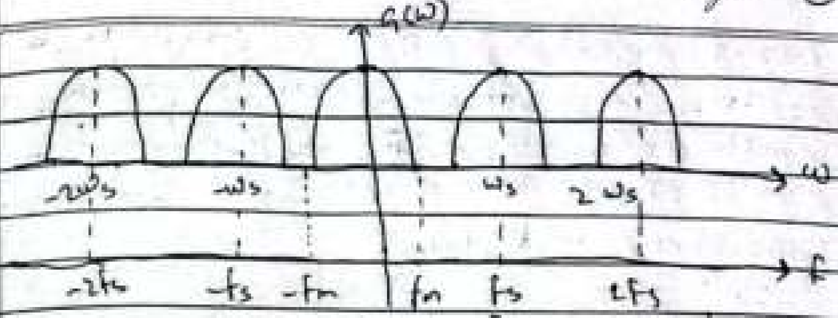


Fig e: spectrum of sampled signal.

Now to obtain $G(\omega)$ we have to take fourier transform of right hand side.

FT of $x(t) \longrightarrow X(\omega)$

FT of $2x(t)\cos\omega_s t \longrightarrow [X(\omega-\omega_s) + X(\omega+\omega_s)]$

FT of $2x(t)\cos 2\omega_s t \longrightarrow [X(\omega-2\omega_s) + X(\omega+2\omega_s)]$

and so on.

Therefore on taking FT, eq^n (3) becomes,

$$G(\omega) = \frac{1}{T_s}\left[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s) + X(\omega-3\omega_s) + X(\omega+3\omega_s) + \dots\right] \quad \text{--- (iv)}$$

or, $$G(\omega) = \frac{1}{T_s}\sum_{-\infty}^{\infty} X(\omega - n\omega_s) \quad \text{--- (v)}$$

Thus from eqn (iv) & (v) it is clear that the spectrum $G(\omega)$ consists of $X(\omega)$ repeating periodically with period $\omega_s = \frac{2\pi}{T_s}$ rad/sec. or $f_s = \frac{1}{T_s}$ Hz

as shown in fig (e).

Now, if we have to reconstruct $x(t)$ from $g(t)$, we must be able to recover $X(\omega)$ from $G(\omega)$. This is possible if there is no overlap between successive cycles of $G(\omega)$. fig (e) shows that this requires

$$f_s > 2f_m \quad \text{—— (vi)}$$

But the sampling interval $T_s = 1/f_s$

$$\text{Hence, } T_s < 1/2f_m \quad \text{—— (vii)}$$

Therefore, as long as the sampling frequency f_s is greater than twice the maximum signal frequency f_m, $G(\omega)$ will consist of non-overlapping repetitions of $X(\omega)$. If this is true, fig (e) shows that $x(t)$ can be recovered from its samples $g(t)$ by passing the sampled signal $g(t)$ through an ideal low pass filter of bandwidth f_m Hz. This provides the sampling theorem.

Factors to consider while sampling:

(i) The sampled signal waveform consists of finite amplitude & duration pulses, rather than ideal pulses

(ii) Reconstruction filters are not ideal

(iii) Input waveforms are time limited rather than band limited.

Q) If two bandlimited signals $x_1[t]$ and $x_2[t]$ have bandwidths of ω_1 & ω_2 Hertz respectively, estimate the maximum sampling interval required for the signal given by $y[t] = x_1[t]x_2[t]$.

= sol^n

$$y[t] = x_1[t]\,x_2[t] \rightarrow y[j\omega] = \frac{1}{2\pi}\{x_1(j\omega) * x_2(j\omega)\}$$

we know that,

$x_1(j\omega) \rightarrow 0$ for $|\omega| > \omega_1$

$x_2(j\omega) \rightarrow 0$ for $|\omega| > \omega_2$

convolution of two signal will result in a signal that is non-zero with atleast one of the signal being non-zero.

Therefore, $y(j\omega) = 0$ for $|\omega| > (\omega_1 + \omega_2)$

Nyquist rate = $2W_m$

$= 2(\omega_1 + \omega_2)$

Max^n sampling interval $= \dfrac{2\pi}{\text{min sampling frequency}}$

$= \dfrac{2\pi}{2(\omega_1+\omega_2)}$

$= \dfrac{\pi}{\omega_1+\omega_2}$ //

Q) With mathematical derivations show that original bandlimited signal can be reconstructed from its samples taken at Nyquist rate. 70 Asd [5]

= The low pass filter that is used to recover original signal from its samples is known as interpolation filter. Since an ideal low pass filter is practically impossible we use a practical low-pass filter as shown in figure below.

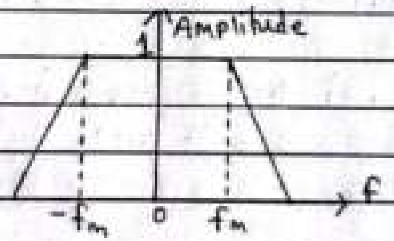


Fig: Practical low pass filter.

The process of reconstructing a continuous time signal $x(t)$ from its samples is called as interpolation. As discussed earlier, a signal $x(t)$ band limited to f_m Hz can be reconstructed (interpolated) completely from its samples by passing through an ideal low passed filter of cut off frequency f_m Hz.

The expression for sampled signal is written as

$$g(t) = x(t) \cdot \delta_{Ts}(t) \quad \text{--- (i)}$$

$$= \frac{1}{T_s}\left[x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + \cdots\right] \quad \text{--- (ii)}$$

From above eq^n it can be seen that sampled signal contains a component $\frac{1}{T_s} \times x(t)$.

To recover $x(t)$, sampled signal must be passed through LPF of BW f_m Hz and gain T_s. Therefore, the reconstruction or interpolating filter Transform function is expressed as;

$$H(\omega) = T_s \times \text{rect}\left(\frac{\omega}{4\pi f_m}\right) \quad \text{--- (iii)}$$

The impulse response $h(t)$ of this filter is the inverse Fourier transform of $H(\omega)$, i.e.,

$$h(t) = F^{-1}\left[H(\omega)\right] = F^{-1}\left[T_s \, \mathrm{rect}\left(\frac{\omega}{4\pi f_s}\right)\right]$$

$$= 2 f_m T_s \, \mathrm{sinc}(2\pi f_m t) \quad \text{--- (iv)}$$

Assuming sampling is done at Nyquist rate, then

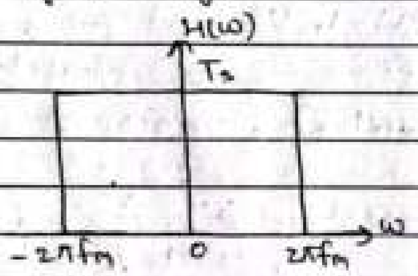
$$T_s = \frac{1}{2f_m}$$

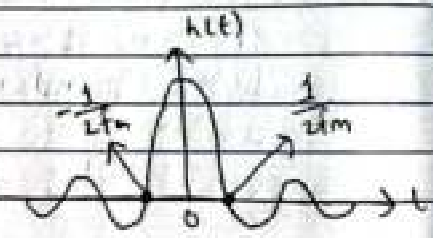
$$2 f_m T_s = 1 \quad \text{--- (v)}$$

Putting the value of $2f_mT_s$ in eq^n (iv)

$$h(t) = 1 \cdot \mathrm{sinc}(2\pi f_m t) = \mathrm{sinc}(2\pi f_m t) \quad \text{--- (vi)}$$

graphically;





From figure above, it can be seen that $h(t) = 0$ at all Nyquist sampling instants $t = \pm n/2f_m$ except at $t=0$.

Now when the sampled signal $g(t)$ is applied to this filter, the o/p will be $x(t)$. Each sample in $g(t)$ produces a sinc pulse of height equal to strength of sample. Addition of the sinc pulses produced by all the samples results in $x(t)$.

For instant, the k^{th} sample of the input $g(t)$ is the impulse $x(kT_s)\delta(t-kT_s)$. The filter output of this impulse will be $x(kT_s)\, h(t-kT_s)$. Therefore the filter o/p to $g(t)$, which is $x(t)$, may be expressed as a sum,

$$x(t) = \sum_k x(kT_s)\, h(t-kT_s) \quad \text{--- (vii)}$$

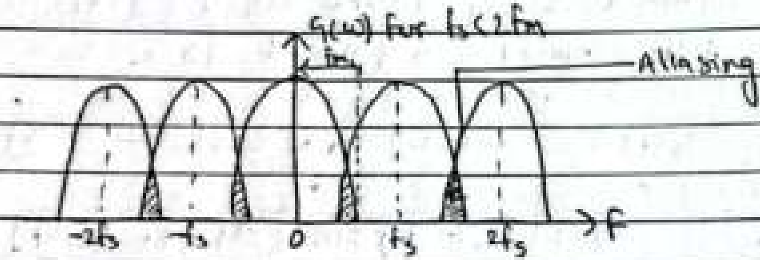
$$= \sum_k x(kT_s)\, \mathrm{sinc}[2\pi f_m(t-kT_s)] \quad \text{--- (viii)}$$

$$\therefore x(t) = \sum_k x(kT_s)\, \mathrm{sinc}(2\pi f_m - k\pi) \quad \text{--- (ix)} \quad \left\{ T_s = \frac{1}{2f_m} \right.$$

eq^n (ix) is the interpolation formula which provides values of $x(t)$ between samples as a weighted sum of all the sample values.

Aliasing effect: 74Ats, 70Asd, 73sh, 71sh 73Ch.

when a continuous-time band-limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then successive cycles of spectrum $G(\omega)$ of the sampled signal $g(t)$ overlap with each other as shown in figure below. Here, the signal is said to be undersampled in this case and some amount of aliasing is produced in this process. Aliasing is the phenomenon in which a high frequency component in the frequency-spectrum takes identity of a lower-frequency component in the spectrum of the sampled signals.



In this case shown in figure above, it is not possible to recover original signal $x(t)$ from sampled signal without distortion. To avoid aliasing we must:

(i) use pre alias filter to limit band of frequencies of the signal to f_m Hz.
(ii) Sampling frequency 'f_s' must be selected such that, $f_s > 2f_m$

Sub-sampling theory: 74ch[2], 78 Kar[2]

Previously, we studied sampling theorem for low-pass signals. However when the given signal is bandpass signal, then a different criteria must be used to sample the signal. The sub-sampling theory is related with sampling of bandpass signal, which is expressed as under.

'The bandpass signal $x(t)$ whose maximum bandwidth is $2f_m$ can be completely represen-ted into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth. Here, f_m is maximum frequency component present in the signal'

Hence if the bandwidth is $2f_m$, then minimum sampling rate for bandpass signal must be $4f_m$ samples per second.

$$\therefore f_s = 4f_m$$

$$\therefore T_s = \frac{1}{f_s} = \frac{1}{4f_m}.$$

Natural sampling: 76Ch[6], 78Bh[5], 71sh[6]

Since in instantaneous sampling, τ approaches 0 which is not possible in real life. Thus this method is not suitable to use in real life. Thus we use Natural sampling which is a practical method and has a finite width equal to τ.

Let us consider an analog continuous-time signal $x(t)$ to be sampled at the rate of f_s Hz.
where, $f_s >$ Nyquist rate.

Let $c(t) \rightarrow$ sampling f^n which is train of periodic of width τ & frequency equal to f_s Hz.

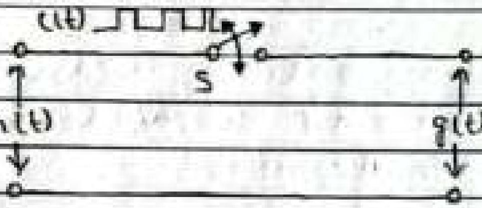


Fig: functional diagram of Natural sampler.

With the help of natural sampler shown above, a sampled signal $g(t)$ is obtained by multiplication of sampling function $c(t)$ & i/p signal $x(t)$

from figure, when $c(t)$ = high, switch 'S' = Closed
Therefore,

$g(t) = x(t)$ when $c(t) = A$ (Amplitude of $c(t)$)
$g(t) = 0$ when $c(t) = 0$ — (ii)

Mathematically, $g(t) = c(t) \cdot x(t)$ — (iii)

we know that exponential Fourier series for any periodic waveform is expressed as

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n t/T_0} \quad \text{— (iv)}$$

for $c(t)$,
$T_0 = T_s = 1/f_s$ = Period of $c(t)$

$f_0 = f_s = \frac{1}{T_0} = \frac{1}{T_s}$ = frequency of $c(t)$

$\therefore$ from (iv), we have

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_s n t} \quad \text{— (v)} \quad \left\{ \text{from } \frac{1}{T_0} = f_s \right.$$

Since $c(t)$ = rectangular pulse train:
$$C_n = \frac{TA}{T_0} \operatorname{sinc}(f_n \cdot T) \quad \text{— (vi)}$$

here, T = pulse width = τ
& f_n = harmonic frequency = nf_s
or, $f_s = \frac{n}{T_0} = nf_0$.

Hence, $C_n = \frac{\tau A}{T_0} \text{sinc}(f_n \tau)$ — (vii)

∴ using eq^{n}s (v) and (vi),

$$c(t) = \sum_{n=-\infty}^{\infty} \frac{\tau A}{T_s} \text{sinc}(f_n \tau)\, e^{j2\pi f_s n t} \quad \text{— (viii)}$$

Now, from (iii), we have,

$$g(t) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \text{sinc}(f_n \tau)\, e^{j2\pi f_s n t} \cdot x(t) \quad \text{— (ix)}$$

This is the required time-domain representation for naturally sampled signal $g(t)$.

Now to get the frequency-domain representation of the naturally sampled signal $g(t)$, lets take its fourier transform:

$$G(f) = FT[g(t)]$$

$$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(f_n \cdot \tau)\, FT[e^{j2\pi f_s n t} \cdot x(t)] \quad \text{— (x)}$$

From the frequency shifting property of FT,

$$e^{j2\pi f_s n t} \cdot x(t) \longleftrightarrow X(f - f_s n) \quad \text{— (xi)}$$

$$\therefore G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(f_n \cdot \tau)\, X(f - f_s n) \quad \text{— (xii)}$$

since $f_n = n f_s$ = harmonic frequency

∴ eq^n (xii) becomes,

$$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(n f_s \cdot \tau)\, X(f - f_s n) \quad \text{— (xiii)}$$

Hence, we write
spectrum of Naturally sampled signal:

$$G(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(n f_s \cdot \tau)\; X(f - f_s n) \quad \text{— (xiv)}$$

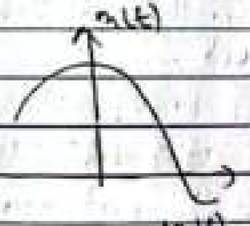


Fig: Ct signal x(t)

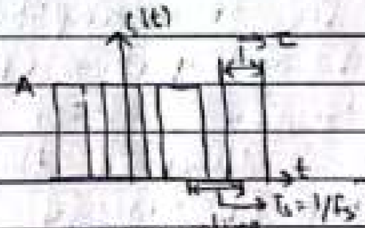


Fig: Sampling function

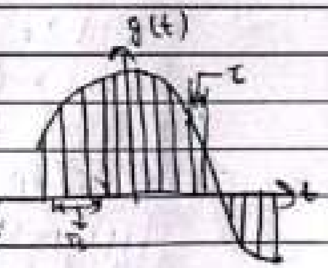


Fig: naturally sampled signal waveform g(t)

Aperture effect: 74ch[2], 73sh[7], 73sh[3], 74ch[6]
71sh[2] 69ch[5]

Let us take one pulse of rectangular pulse train and its spectrum as shown in figure below:

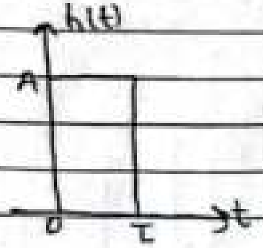


Fig: one pulse of rectangular pulse train

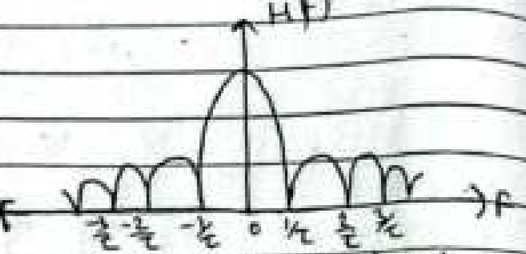


Fig: spectrum of pulse shown in previous figure.

During flat top sampling, to convert varying amplitudes of pulse to flat top pulses we use a sinc function. Because of this, there would be decrease in amplitude. This distortion is named as aperture effect.

- The high frequency roll-off of H(f) acts like a low-pass filter and thus attenuates the upper portion of message signal spectrum. These high frequencies of x(t) are affected causing aperture effect. As T increases, aperture effect is

more prominent

Remedy for aperture effect:

(i) Using the sampling pulse(T) as narrow as possible
(ii) By using equalizers during reconstruction, having transfer function of $H_{eq}(f) = \frac{1}{P(f)}$
(iii) By using good filter design by following sampling theorem properly.

Chapter: 3

Pulse Modulation systems: (8 Hours / 14 marks)

70Ch [1.5] or 5 — Pulse Amplitude modulation may be defined as that type of modulation in which the amplitudes of regularly spaced rectangular pulses vary accordingly to instantaneous value of the modulating or message signal.

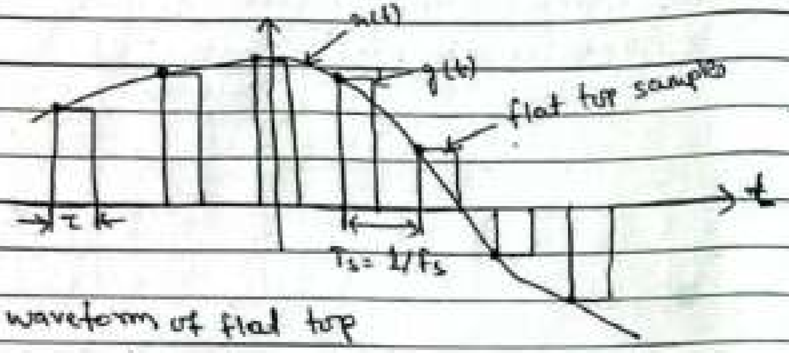
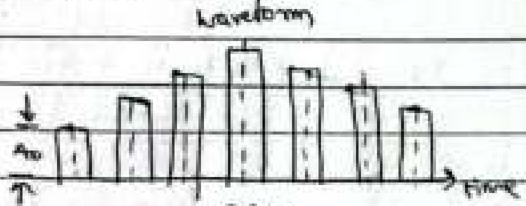
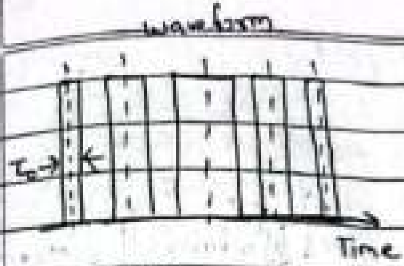


Fig: waveform of flat top sampled PAM.

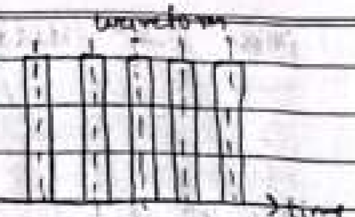


PAM

70Ch [1.5] or 5 — Pulse width modulation may be defined as the type of modulation in which the width of modulated pulses varies in proportion with the amplitude of modulating signal.



PWM



PPM

70Ch [1.5] or — In Pulse position modulation, the amplitude & width of the pulses are kept constant but the position of each pulse is varied in accordance with the amplitudes of the sampled values of the modulating signal. The position of the pulses is changed with respect to the position of reference pulses.

Pulse code Modulation: In PCM, the message signal is sampled and amplitude of each sample is approximated (rounded off) to the nearest one of a finite set of discrete levels. This will enable us to represent both time and amplitude in discrete form.

Figure below shows the basic elements of a PCM system. It consists of three main parts i.e transmitter, transmission path and receiver.

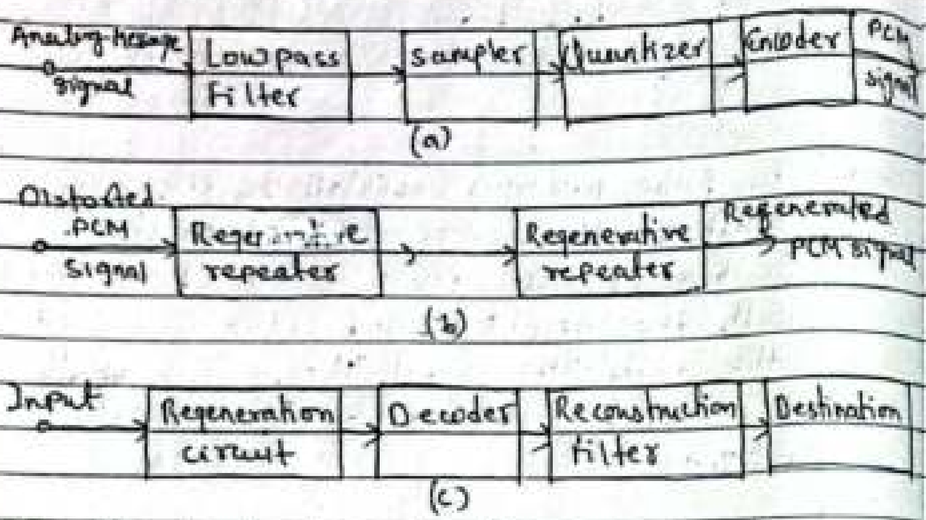


Fig: The basic elements of a PCM signal
(a) Transmitter
(b) Transmission path
(c) Receiver.

The essential operations in the transmitter of a PCM system are sampling, quantizing & encoding as shown in figure above. Sampling is an operation in which analog signal is sampled according to the sampling theorem resulting in a discrete-time signal. The quantizing & encoding operation is performed

in same circuit which is known as analog-to-digital converter (ADC).
Also, essential operations in the receiver are regeneration of impaired signals, decoding & demodulation of the train of quantized samples. These operations are usually performed in the same circuit which is known as a digital-to-analog converter (DAC).

Further, at intermediate points along the transmission route from the transmitter to the receiver, regenerative repeaters are used to reconstruct (i.e. regenerate) the transmitted sequence of coded pulses in order to combat the accumulated effects of signal distortion & noise.

Quantization refers to the use of a finite set of amplitude levels and the selection of a level nearest to a particular sample value of the message signal as the representation for it. In fact, this operation combined with sampling, permits the use of coded pulses for representing the message signal. Thus, it is the combined use of quantization and coding that distinguishes pulse code modulation from analog modulation techniques.

78AS[8]
94AS[7]
71Ch[4]

Signal to Quantization noise Ratio for linear/uniform quantization:

We know that in a pulse code Modulation system for linear quantization the signal quantization noise ratio is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

Normalized noise power is given as $\frac{\Delta^2}{12}$

$\therefore$ Therefore $\frac{S}{N} = \frac{\text{Normalized signal power}}{(\Delta^2/12)}$ —— (i)

we know that number of bits 'v' & quantization levels are related as,

$$q = 2^v \quad \text{—— (ii)}$$

Let us assume that input $x(nT_s)$ to a linear quantizer has continuous amplitude in the range $-x_{max}$ to $+x_{max}$. Therefore total amplitude range becomes: $x_{max} - (-x_{max}) = 2x_{max}$.

Now, stepsize will be

$$\Delta = \frac{2x_{max}}{q} \quad \text{—— (iii)}$$

$$\Delta = \frac{2x_{max}}{2^v} \quad \{\text{from (ii)}\}$$

Now substituting this value in (i)

$$\frac{S}{N} = \frac{\text{normalized signal power}}{\left(\frac{2x_{max}}{2^v}\right)^2 \times \frac{1}{12}}$$

Let normalized signal power be denoted as 'P'.

Then, $\frac{S}{N} = \frac{P}{\frac{4x^2_{max}}{2^{2v}} \times \frac{1}{12}} = \frac{3P \times 2^{2v}}{x^2_{max}}$

This is the required relation for signal to quantization noise ratio for linear quantization in a PCM system.

Hence, $\boxed{SQNR: \frac{S}{N} = \frac{3P}{x^2_{max}} \cdot 2^{2v}}$ —— (iv)

This exp^n shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per sample.

If $x(t) \rightarrow$ normalized.
$x_{max} \rightarrow 1$

$\therefore$ SQNR $= 3 \times 2^{2v} \times P$ —— (v)

If $P \rightarrow$ normalized
$P \leq 1$

$\therefore$ SQNR $\leq 3 \times 2^{2v}$ —— (vi)

Now as both of them are normalized we can express SQNR in decibels;

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) dB \leq 10 \log_{10}[3 \times 2^{2v}] \leq (4.8 + 6v) \text{ dB}$$

$$\therefore \boxed{\left(\frac{S}{N}\right) dB \leq (4.8 + 6v) \text{ dB}} \quad \text{—— (vii)}$$

Non-Uniform quantization:

Basic process? 72Kar[3]
Need? (VVI) [2]
uniform vs non-uniform quantization 73ch[4]

A uniform quantizer is that type of quantizer in which the 'step size' remains same throughout the input range. It is of two types: Mid thread and Mid rise. Similarly, a non-uniform quantizer is that type of quantizer in which the 'step size' varies according to the input values.

In a non-uniform quantizer, the quantizer characterstics is nonlinear and step size is also not constant instead it is variable. dependent on the amplitude of the input signal. Here, the step size is reduced with the reduction in signal level. For weak signals ($P \ll 1$), the step size is small, therefore the quantization noise reduces, to improve SQNR for weak signals. The step size is thus varied a/c to the signal level to keep the SQNR adequately high. This is non-uniform quantization. It is achieved through companding.

VVI 2 Marks

Need of Non-Uniform Quantization:

(i) Robust Quantization: In uniform quantization, the quantizer has linear characteristic. Thus stepsize remains same throughout the range of quantizer. Because of this, over the complete range of inputs, the maximum quantization error also remains the same. It is very prominent as sometimes quantization error can reach 30 to 50% of original voltage causing significant error. Thus in such cases, we use non-uniform quantization in which SQNR remains essentially constant over a wide range of i/p power levels. A quantizer that satisfies all these requirements is known as Robust quantizer which can be achieved by using a nonuniform quantization.

(ii) For speech and music signals: speech and music signals are characterized by large crest factor. This means that for such signals the ratio of peak to rms value is quite high. For large crest factor of speech and music signal, P should be very very less than 1 (P<<1) because of which SQNR is very less. Thus this type of signal should use non-uniform quantization to tackle the problem. In SQNR as P decreases,

noise increases. Noise is dependent on Δ. Thus by varying stepsize (Δ), we can keep SQNR at required value. This is the another need of non-uniform quantization.

UM

uniform quantization	non uniform quantization
• It is the type of quantization in which the quantization levels are uniformly spaced.	It is the type of quantization in which quantization levels are unequal.
• The step size (Δ) is constant throughout i/p power levels.	The step size (Δ) keeps on varying according to i/p power levels.
• It is particularly divided into two categories: Mid-thread and Mid-rise quantization	It is not categorized further. However companding can be done by μ-law & A-law methods.
• It has significant amount of quantization error	It reduces quantization error.
• SQNR is variable/changing continuously.	SQNR is essentially constant.

Companding:

- Explain [1.5, 2, 3]
- necessity
- Types: [4] {Block diagram & wave forms}

Non-uniform quantization is very difficult to achieve via regular methods. It can be practically achieved via a method called companding. The reason why Non-Uniform Quantization is difficult to implement is because we do not know about the changes in the signal level beforehand. Thus we use companding.

Companding is a term derived from two expression; Companding = compressing + Expanding.

Compression is the process of amplifying weak signals and attenuating strong signals before applying them to a uniform quantizer. The block that provides compression is called as compressor.

Similarly at receiver exactly opposite is followed which is called expansion. The circuit used for providing expansion is called as an expander.

The compression of signal at the transmitter and expansion at receiver is combined to be called companding. The compression and expansion is obtained by passing the signal through the amplifier having non-linear transfer characteristic. The process of companding is illustrated in block diagram below:

Input → [Compressor] → [Uniform Quantizer] → [Expander] → output

Fig: A companding model.

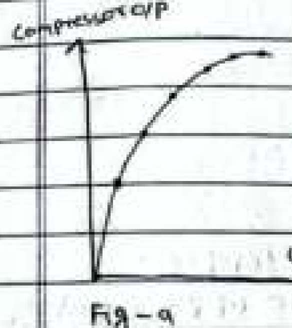


Fig-a

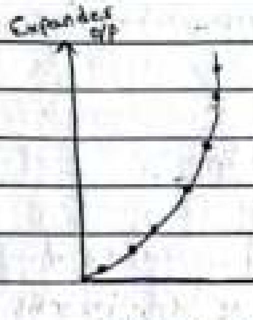


Fig-b

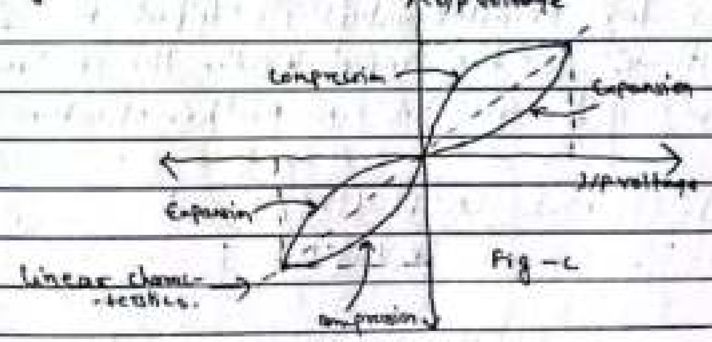


Fig-c

Fig a: compressor characteristic

Fig b: expander characteristic

Fig c: compander characteristic

Necessity of companding: In uniform quantization, Δ is fixed. Thus quantization noise is fixed. But, signal power is not fixed which might cause poor SNR. This might affect the quality of signal. Thus we use companding to avoid this.

Types of companding: There are two types of companding techniques. They have different compressor characteristics compared to each other. They are listed below:

(i) μ-law companding
(ii) A-law companding.

* μ-law companding:

In the μ-law companding, the compressor characteristic is continuous. It is approximately linear for smaller values of input levels and logarithmic for high input levels. The μ-law compressor characteristics is mathematically expressed as

$$z(x) = (\text{sgn } x)\,\frac{\ln(1+\mu|x|/x_{max})}{\ln(1+\mu)} \quad \text{—(1)}$$

where, $0 \leq |x|/x_{max} \leq 1$

Here,
$z(x) \rightarrow$ output of compressor
$x \rightarrow$ input of compressor
$|x|/x_{max} \rightarrow$ normalized i/p wrt maximum value x_{max}
$\text{sgn } x \rightarrow \pm 1$ (+ve & −ve values of i/p & o/p)

The μ-law compressor characteristic is shown below for different values of μ. Practical value of μ is 255.
$\mu \rightarrow 0$ means uniform quantization.

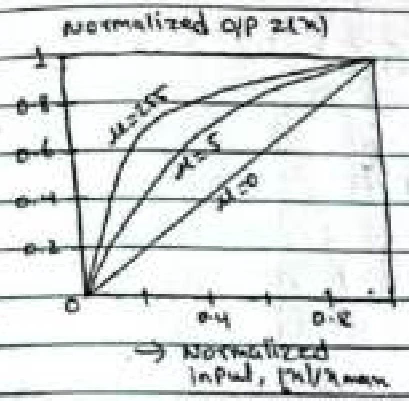


Fig: compressor characteristics of a μ-law compressor

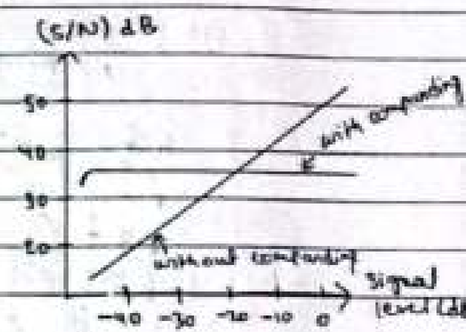


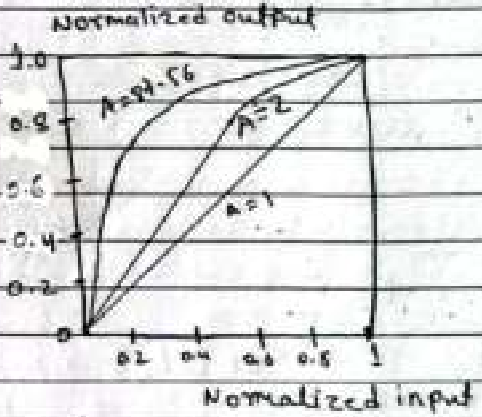
Fig: PCM performance with μ-law companding.
SNR is almost constant while using companding

uses:
(i) It is used for speech & music signals.
(ii) It is used for PCM telephone systems in US, Canada & Japan.

* A law companding:

In the A law companding the compressor characteristic is piecewise, made up of a linear segments for low level inputs and a logarithmic segments for high level inputs.
Figure below shows A-law compressor characteristics for different values of A.

$A \to 1 \to$ linear characteristic / uniform quantization
$A \to 87.56 \to$ Practically used value.



Mathematically,

$$\frac{z(x)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\log_e A} & \text{for } 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\ \dfrac{1+\log_e[A|x|/x_{max}]}{1+\log_e A} & \text{for } \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases} \quad \text{— (ii)}$$

Uses:
- PCM telephone systems in Europe.

SQNR for DM: 76 Ch [5], 73 Ch [6], 70 Ch [6]

We know that the condition to avoid the slope overload distortion is expressed as

$$A < \frac{\Delta}{\omega_m T_s} = \frac{\Delta}{2\pi} \frac{f_s}{f_m} \quad \text{— (i)}$$

Therefore, the maximum value of the output signal power is expressed as,

$$P_{max} = \left(\frac{A}{\sqrt{2}}\right)^2 \quad \left\{ \text{Since } P \text{ is proportional to square of RMS value} \right\}$$

Hence, we have $P_{max} = \dfrac{A^2}{2} = \dfrac{\Delta^2 f_s^2}{8\pi^2 f_m^2}$ — (ii)

Now, we require to obtain the expression for quantization noise power.
The quantization error in DM lies in range $(-\Delta, \Delta)$

$\therefore$ maximum quantization error, $\epsilon_{max} = \pm\Delta$

The error shown above is uniformly distributed as shown in figure. Thus, the PDF is an uniform distribution function which is defined as under:

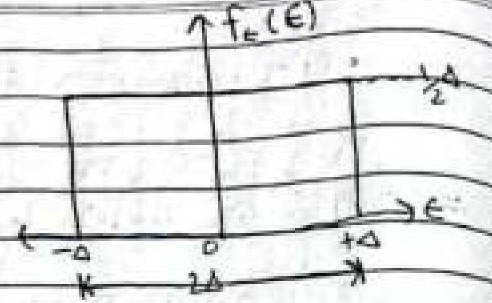


Fig: PDF of quantization

$$f_\epsilon(\epsilon) = \begin{cases} \frac{1}{2\Delta} & \text{for } -\Delta \le f_\epsilon(\epsilon) \le +\Delta \\ 0 & \text{Otherwise} \end{cases}$$

Mean square value or the variance of the quantization noise is given by,

$$\overline{\epsilon^2} = \int_{-\Delta}^{\Delta} \epsilon^2 f_\epsilon(\epsilon)\, d\epsilon$$

$$= \int_{-\Delta}^{\Delta} \epsilon^2 \cdot \frac{1}{2\Delta}\, d\epsilon = \frac{1}{2\Delta}\left[\frac{\epsilon^3}{3}\right]_{-\Delta}^{\Delta} = \frac{1}{2\Delta}\left[\frac{\Delta^3}{3} + \frac{\Delta^3}{3}\right]$$

$$\overline{\epsilon^2} = \frac{1}{2\Delta} \times \frac{2\Delta^3}{3}$$

$$\therefore \overline{\epsilon^2} = \frac{\Delta^2}{3} \quad \text{--- (iii)}$$

Normalized quantization noise power $N_q = \frac{\overline{\epsilon^2}}{1} = \frac{\Delta^2}{3}$ --- (iv)

The delta modulated signal is passed through a reconstruction low pass filter (LPF) at the output of a DM receiver. The BW of this LPF is f_M such that

$$f_M \ge f_m \text{ and } f_M \ll f_s$$

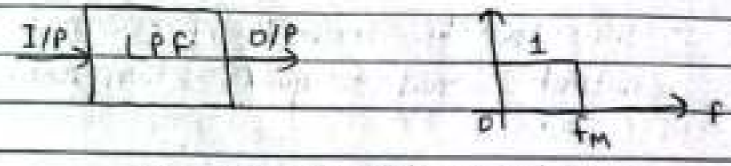


Fig: LPF at delta modulation.

Now assuming N_q is distributed uniformly over frequency band upto f_s, the output quantization noise power within the bandwidth f_M is given by,

Normalized noise power at filter O/P, $N_q' = \frac{\Delta^2}{3} \times \frac{f_M}{f_s}$ --- (v)

Substituting (ii) & (v) in one place, we get,

$$\left[\frac{S}{N_q}\right]_o = \frac{P_{max}}{N_q'} = \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2} \times \frac{3 f_s}{\Delta^2 f_M}$$

$$\left[\frac{S}{N_q}\right]_o = \frac{3 f_s^3}{8\pi^2 f_m^2 f_M} \quad \text{— (vi)}$$

since $f_s = \frac{1}{T_s}$

$$\therefore \boxed{\left[\frac{S}{N_q}\right]_o = \frac{3}{8\pi^2 f_m^2 f_M T_s^3}} \quad \text{— (vii)}$$

This is the desired expression for the output signal to quantization noise ratio.

II Adaptive delta Modulation: short notes: 72kar[4]

To overcome quantization errors due to slope overload and granular noise, the stepsize (Δ) is made adaptive to variations in the input signal $x(t)$. Particularly in the steep segment of the signal $x(t)$, the step size is increased. Also, if the input is varying slowly, the step size is reduced. Then, this method is known as Adaptive Delta Modulation (ADM). The adaptive delta modulators can take continuous changes in step size or discrete changes in step size.

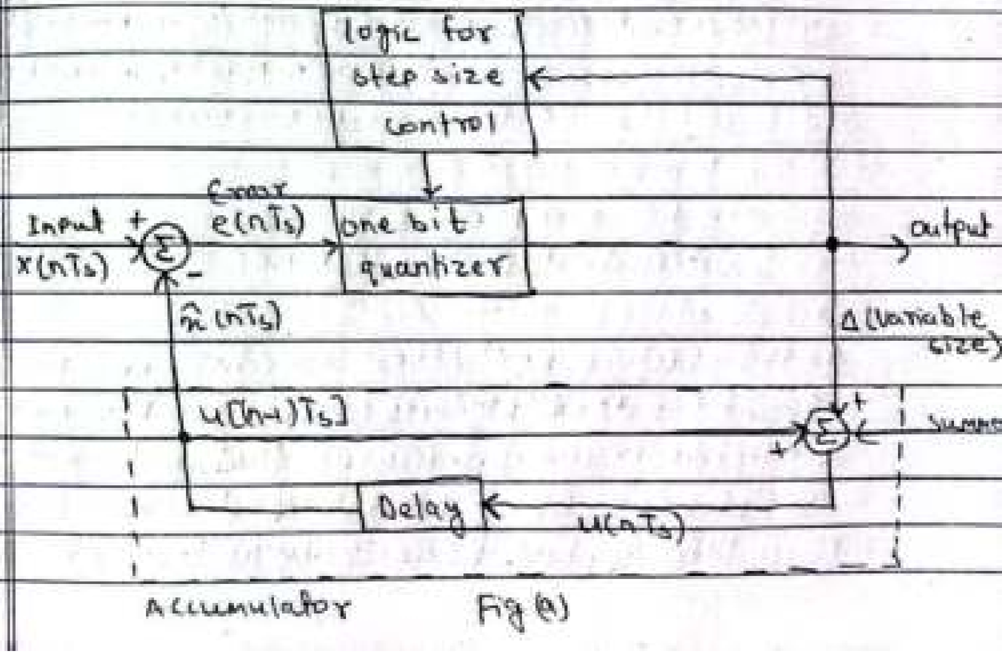


Fig (a)

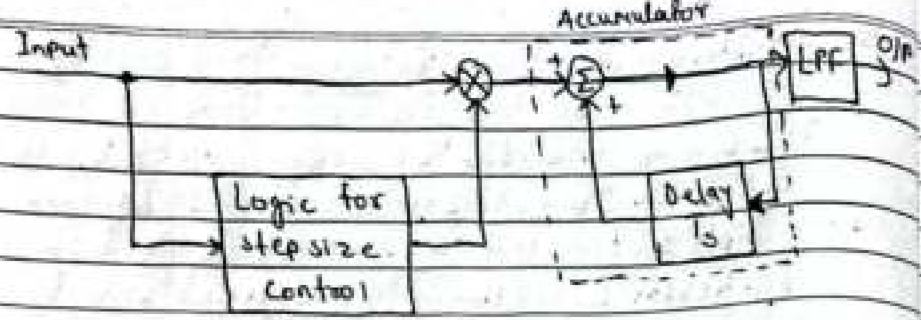


Fig (b)

Fig : a : Transmitter of ADM
b : Receiver of ADM.

The transmitter of ADM is shown in figure (a). The logic for step size control is added in the diagram. It depends on o/p of one bit quantizer. For eg: one bit quantizer o/p → high
step size may be doubled for next sample.

one bit quantizer → low
step size → Reduce by one step.

In the receiver part, there are two portions. The first portion produces step size for incoming bit following same process as that in transmitter. The previous i/p & present i/p decide step size. It is then applied to an accumulator which builds up staircase waveform. The lowpass filter then smoothens out the staircase waveform to reconstruct the original signal.

* Advantages:

(i) DM use only one bit per sample thus transmitter & receiver's design are simple.
(ii) ADM's SNR becomes better reducing slope overload distortion & idle noise.
(iii) Because of variable stepsize, dynamic range of ADM is wider than DM.
(iv) Utilization of BW is better than delta modulation.

Differential Pulse Code Modulation (DPCM) : 70Ch[4]

- Why DPCM is better than PCM? 71Ch[2]
- Working principle with figure and equations : 71Ch[5]

We may see that the samples of signal are highly correlated with each other because any signal does not change fast. This means its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with a little difference. When these samples are encoded by a standard PCM system, the resulting encoded signal contains some redundant infor-mation.

If this redundancy is reduced, then overall bit rate will decrease & number of bits required to transmit one sample will also be reduced. This results in saving bandwidth as well. This type of digital pulse modulation scheme is known as Differential Pulse Code Modulation (DPCM) and because of above mentioned reasons, it is superior over PCM.

* Working Principle

DPCM works on the principle of prediction. The value of present samples is predicted from past samples. The prediction may not be exact but it is very close to the actual sample value.

Sampled signal $\longrightarrow x(nT_s)$

Predicted signal $\longrightarrow \hat{x}(nT_s)$ {Produced by Prediction Filter}

Comparator finds difference between these two

signals which is known as prediction error $e(nT_s)$

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{—(i)}$$

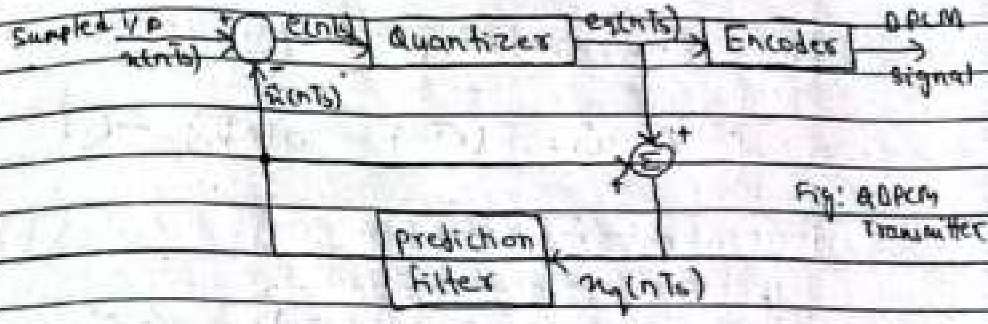


Fig: DPCM Transmitter

The prediction filter produces predicted value by taking inputs as the sum of quantizer O/P & previous prediction. This makes prediction close to the actual sampled signal.

We can observe that $e_q(nT_s)$ is very small & can be encoded using small number of bits. Thus number of bits per sample are reduced in DPCM.

The quantizer O/P is given as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \quad \text{—(ii)}$$

where, $q(nT_s)$ = quantization error

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \quad \text{—(iii)}$$

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \quad \text{--- (iv)}$$

from eqⁿ (i);

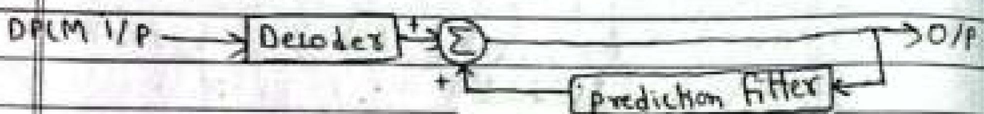
$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$\therefore e(nT_s) + \hat{x}(nT_s) = x(nT_s) \quad \text{--- (v)}$$

in eqⁿ (iv), putting (v).

$$\therefore \boxed{x_q(nT_s) = x(nT_s) + q(nT_s)} \quad \text{--- (vi)}$$

Similarly at receiver, the decoder first reconstructs the quantized error signal from incoming binary signal. DPCM receiver is shown below:



The prediction filter o/p & quantized error signals are summed up to give quantized version of the original signals. Thus the signal at receiver differs from actual signal by quantization error $q(nT_s)$ which is introduced permanently in the reconstructed signal.

Linear Predictive coding (LPC): Short notes 70th [6]

LPC synthesizes the analog signal first. Then the parameters of the waveform synthesizer are encoded and transmitted instead of the actual signal. LPC is well suited for speech synthesis and transmission.

Figure below shows the block diagram of LPC Transmitter:

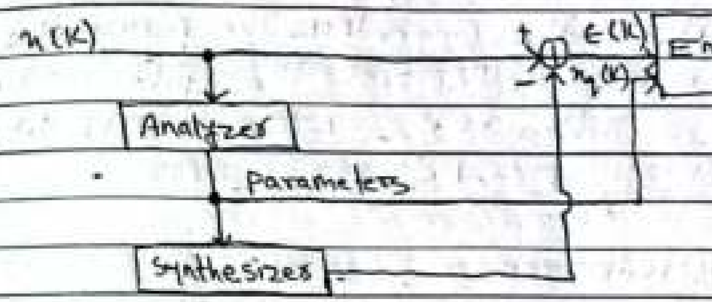
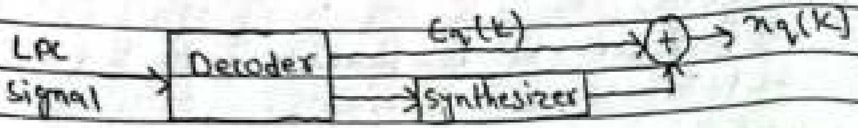


Fig: Transmitter

The voice input signal is first sampled to obtain signal $x(k)$ at the input of LPC transmitter. These sampled values are applied to an analyzer which analyzes them to determine the parameters for synthesizer. The synthesizer reconstructs the approximated speech signal $x_q(k)$. The reconstructed speech signal & sampled signal $x(k)$ are compared to produce error $E(k)$. The error signal & parameter values are encoded by the encoder to form a digital signal which is known as LPC signal

Figure below shows the LPC Receiver:



From the received LPC signal, the decoder or demultiplexer separates out the error signal 'Eq(k)' & the parameters. The parameters are applied to a synthesizer. The synthesizer output is then added to the error signal to produce speech signal $x_q(k)$.

1 LPC code word → 80 bits
- → 1 bit: for voiced/unvoiced switch
- → 6 bits: Pitch frequency of the voice
- → few bits: to represent error.

LPC has low bit rate of about 3K to 8 Kbits/sec.

---

* → These topics are not asked in exams but do learn about them.

*# T1 carrier system: T1-digital system is the basic Time division multiplexing scheme utilized to transmit large number of PCM signals over a common channel.
- This system can accomodate 24 voice channels, S_1 to S_{24}.
- Each signal is band limited to 3.3 KHz.
- sampling rate = 8 KHz.

* Basic block diagram:

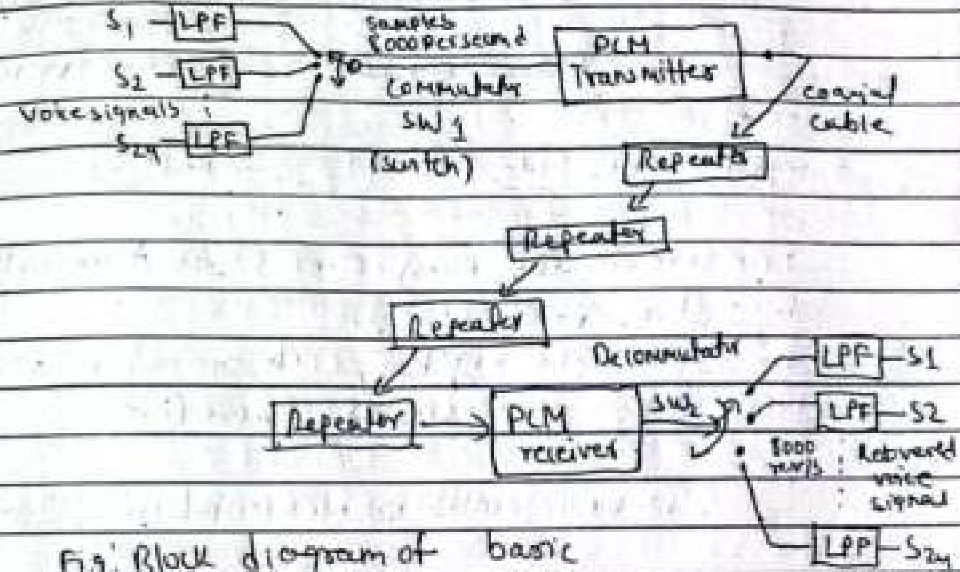


Fig: Block diagram of basic PCM-TDM system or T1 carrier system.

* Frame diagram of T1: 70 Asd [3]

sampling Rate: 8000 samples/sec = 8000 TPS.

1 frame = 1 revolution = 24 channels = 24x8 bits
= 192 bits/frame

S_1	S_2	S_3	S_4		S_{23}	S_{24}
8 bits	8 bits	8 bits	8 bits		8 bits	8 bits

|← 24x8 = 192 bits/frame →|

Fig: one frame & bits per frame

* signalling (bit) rate: 70 Ch [3], 70 Asd [3]

Bit rate means number of bits transmitted by system per second.
In T1 carrier sigstem, each signal is sampled 8000 times per second, therefore,

1 frame (1 revolution of commutator) = 1/8000 = 125 μs

But 1 frame contains 193 bits. Hence 193 bits are transmitted in 125 μs.

∴ number of bits in 1 sec = $\frac{193}{125 \times 10^{-6}} = 1.544 \times 10^6$

so, bit rate of T1 carrier system = 1.544 M bits/sec

* Bandwidth requirement: 70 Ch [3]

minimum BW = $\frac{1}{2}$ (bit rate) = $\frac{1}{2} \times (1.544 \times 10^6)$
= 772 KHz.

∴ $\boxed{\text{Bandwidth of T1 system} = 772\ \text{kHz}}$

* Duration of each bit:

193 bits = 125 μs
∴ 1 bit = (125/193) μs = 0.6476 μs.

short notes: 76 ch[5]

71ch[4]
69ch[4

E1 - digital hierarchy: Explain: 74 As[3], 72 ch[4], 74 ch[4]

E-1 is the european version of T1. Both T1 & E-1 are conceptually identical but their capacities and number of voice channels which they can carry will be different.

E1 consists of:

- 32 channels: 2 time slots are reserved for signalling and synchronization while remaining 30 are for voicecalls or data communication.
- 1 channel = 8 bits
  ∴ Total bits = $8 \times 32 = 256$ bits
- Sampling frequency = 8 KHz.
- Each slot of E1 has bitrate of 64 kbps.
  Thus total bitrate = 2048 Kbps = 2.048 Mbps.

Frame diagram:

1 frame = 32 channels = $32 \times 8 = 256$ bits / frame.

0	1	2	3	...	30	31
8-bits	8 bits	8 bits	8 bits		8 bits	8 bits

← $32 \times 8 = 256$ bits / frame →

Signalling (bit) rate: 70 ch[3] 74 As[3]

Sampling rate = 8000 samples per second.

Time taken to send 1 frame = $\frac{1}{8000} = 125 \mu s$.

1 frame has 256 bits.

Thus, 256 bits are transmitted in 125 µs.

∴ number of bits in 1 sec = $\frac{256}{125 \times 10^{-6}} = 2.048 \times 10^6$

So, bit rate of E1 carrier system = 2.048 Mbps

Bandwidth requirement: 70 ch[3]

Min. BW = $\frac{1}{2}$ (bit rate) = $\frac{1}{2} \times 2.048$ Mbps

= 1.024 MHz

∴ BW of E1 system = 1.024 MHz

Chapter 4

Baseband Data communication System (7hours/ 12Marks)

70Asd[2] **Information:**

The amount of information received from the knowledge of occurance of a event may be related to the likelihood or probability of occurance of that event. The message assbciated with the least likelihood event thus consists of maximum information. The amount of information in a message depends only upon the uncertainty of the underlying event rather than its actual content.

Entropy: 70Asd[2]

In practice, we transmit long sequences of symbols from an information source. Thus, we are more interested in average information that a source produces than the information content of a single symbol. However, flow of information can fluctuate because of randomness in selection of symbols. Thus, average information content is important in context of long message.

Entropy is the average information content of a

sequence of symbols. Mathematically, it is given by,

$$H(x) = -\sum_{i=1}^{m} P(x_i) \cdot \log_2 P(x_i) \text{ bits/symbol.} \quad \text{—(i)}$$

Information Rate: 76Ch[4]

If the time rate at which source X emits symbols is r; the information rate R of the source is given by,

$$R = r H(X) \text{ b/s} \quad \text{—(ii)}$$

Here, R → information rate
H(X) → Entropy/Average information
r → Rate at which symbols are generated.

Information rate R is represented in average number of bits of information per second. It is calculated as under:

$$R = \left[r \text{ in } \frac{\text{symbols}}{\text{Second}} \right] \times \left[H(X) \text{ in } \frac{\text{information bits}}{\text{Symbol}} \right] \quad \text{—(iii)}$$

or, R = Information bits / second, —(iv)

Shannon's channel capacity theorem: 71ch[1], 72ch[2]

According to this theorem, "The capacity of a channel 'C' to transmit information without error is limited by its Bandwidth and noise in the channel."
Mathematically,

$$C = B\log_2(1+SNR) \text{ bits/second} \quad \text{--- (1)}$$

In this expression,

B = Channel Bandwidth in Hz
$SNR = \frac{S}{N}$ → S = Signal power
N = Noise power
C = Channel Bandwidth.

Here, $N=0$ for noiseless channel
and C = infinite however this case is impossible in real life.

similarly B & S can be exchanged for one anoth
when $B\uparrow \rightarrow S\downarrow$ and $S\uparrow \rightarrow B\downarrow$

We will talk about Noiseless channel & infinite Bandwidth in next section.

Theoretical limits of Shannon channel capacity Theorem

71ch[3], 72ch[3]

1) As the noise in the channel tends to zero, the value of SNR will tend to infinity. Subsequently the channel capacity C will tend to infinity. It means that the noiseless channel has an infinite capacity. This type of channel is referred to as ideal channel. This is not possible practically as Noise is always finite in real life.

2) As bandwidth of channel B tend to infinity, the channel capacity reaches to an upper limit C_{max} (Instead of $C \rightarrow \infty$) because as the channel BW increases noise power increases correspondingly. Noise power is proportional to channel BW.

76ch [4]
76ch [6]

$$C = B\log_2(1+SNR)$$
$$C = B\log_2\left(1+\frac{S}{N}\right) \quad \text{--- (1)}$$

we have, $N = \eta B$
where, $\eta \rightarrow$ psd of white noise.

Equation (1) can be rewritten as,

$$C = \frac{S}{\eta} \cdot \frac{2B}{S} \log_2\left(1+\frac{S}{2B}\right)$$

or, $$C = \frac{S}{\eta} \log_2\left(1+\frac{S}{2B}\right)^{\frac{2B}{S}}$$

or, $$C = \frac{S}{\eta} \log_2\left(1+\frac{S}{2B}\right)^{1/\frac{S}{2B}} \quad \text{—(ii)}$$

let $x = \frac{S}{2B}$.
as $B \to \infty$, $x \to 0$

we know that,
$$\lim_{x\to 0} \log_2(1+x)^{1/x} = \log_2 e$$

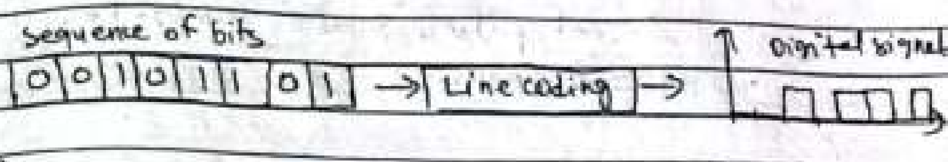
$$\therefore \lim_{B\to\infty} C = \frac{S}{\eta} \log_2 e$$

$$\therefore \lim_{B\to\infty} C = 1.44 \frac{S}{\eta} \quad \text{—(iii)}$$

$$\therefore \boxed{\lim_{B\to\infty} C = C_{max} = 1.44 \frac{S}{\eta}} \quad \text{—(iv)}$$

Short Notes: Line codes [4] 72kar

Line coding is the process of converting binary data, a sequence of bits to a digital signal.



There are various line codes. They are listed below:

(i) Unipolar RZ & NRZ: In this format, waveform has single polarity. Waveform can have +5V or +12V when high. The waveform is simple ON-OFF.

(ii) Polar RZ & NRZ: In this format, '1' is represented by +ve voltage polarity while '0' is represented by -ve voltage polarity.

(iii) Bipolar NRZ: In this format, successive 1 are represented by pulses with alternate polarity & '0' are represented by no pulses.

(iv) split phase Manchester Format: In this format,

When '1' is transmitted, then a +ve half interval is followed by -ve half interval pulse
When '0' is transmitted, then -ve half interval pulse is followed by +ve half interval pulse. Symbol takes +ve as well as -ve value.

Intersymbol Interference (ISI): VVVI

In communication system, when the data is being transmitted in the form of pulses (ie, bits) the output produced at the receiver due to other bits or symbols interferes with the output produced by the desired bit. This is known as intersymbol interference (ISI). The isi will introduce errors in the detected signal.

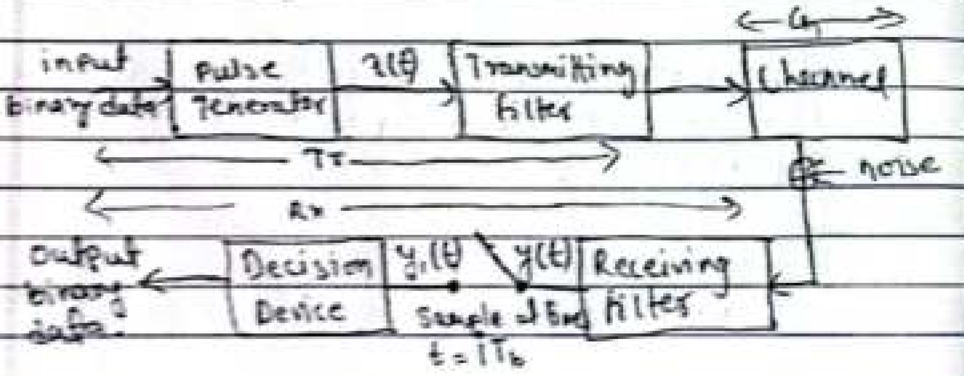


Figure above shows elements of binary PAM system.

i/p → binary data $\{b_k\}$ with bit duration T_b seconds.

This input is applied to Pulse generator to produce discrete PAM signal which is given by,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k V(t - kT_b) \quad \text{--- (i)}$$

where, $V(t) \rightarrow$ basic pulse, normalized such that $V(0) = 1$

Thus this $x(t)$ is transmitted via a channel which introduces noise to the signal.

The receiving filter o/p is written as;

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k P(t - kT_b) + n(t) \quad \text{--- (ii)}$$

where, $\mu \rightarrow$ scaling factor.
$n(t) \rightarrow$ noise
$P(t - kT_b) \rightarrow$ combined impulse response of receiving filter

The receiving filter o/p $y(t)$ is sampled at time $t_i = iT_b$ with i taking on integer values to provide following expression:

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(iT_b - kT_b) + n(t_i) \quad \text{--- (iii)}$$

when $k = i$

$$y(t_i) = \mu\, a_i\, p(0) + n(t_i)$$
$$k = i \quad = \mu a_i$$

$$\therefore y(t_i) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k\, p(iT_b - kT_b) + n(t_i) \quad \text{--- (iv)}$$

This is the receiver o/p $y(t)$ at instant $t = t_i$

From eqn (iv);

$\mu a_i \rightarrow$ produced by i^{th} transmission bit & theoretically only this should exist. but because of residual effect of all transmitted bits second term is also present. This residual effect is intersymbol interference.

Methods to reduce ISI:

(i) The function which produces a zero ISI is a sinc function. Hence, instead of a rectangular pulse if we transmit a sinc pulse then the ISI can be reduced to zero. This is known as Nyquist pulse shaping. {Fig (a)}

(ii) The fourier transform of a sinc pulse is a rectangular function. Here to preserve all the frequency components, the frequency response of the filter must be exactly flat in the passband & zero in the attenuation band {fig b}

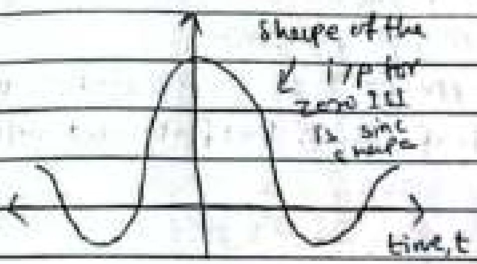


Fig: Ideal pulse shape for zero ISI

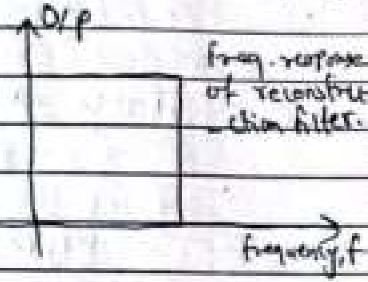


Fig: Frequency response of the filter.

Ideal solution:

The Nyquist criterion for distortionless baseband transmission in absence of noise is given by

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = \frac{1}{R_b} = T_b \quad \text{—— (i)}$$

The LHS of eqⁿ (i) represents a series of shifted spectrums.

for $n=0$

LHS = $P(f)$; which represents a frequency function with the narrowest band.

Range of frequencies for $P(f) = (-B_0, B_0)$
where, $B_0 \rightarrow$ half the bit rate.

Hence, $B_0 = \frac{R_b}{2}$ —— (ii)

∴ $P(f)$ can be specified in following term,

$$P(f) = \frac{1}{2B_0} \, \text{rect}\left(\frac{f}{2B_0}\right) \quad \text{—— (iii)}$$

This is shown graphically in fig (a) below.

This is the spectrum of signal which produces zero ISI. Hence the signal that produces zero ISI can be obtained by taking the IFT of $P(f)$.

$$\therefore \; p(t) = F^{-1}[P(f)]$$

$$p(t) = F^{-1}\left[\frac{1}{2B_0} \, \text{rect}\left(\frac{f}{2B_0}\right)\right]$$

$$\therefore \; p(t) = \text{sinc}(2B_0 t) \quad \text{—— (iv)}$$

Fig (b) shows plot of this function.

f^n $p(t) \rightarrow$ impulse response of an ideal LPF with Bandwidth B_0.

Thus the shape of a pulse should be sinc rather than being a rectangular one to eliminate ISI.

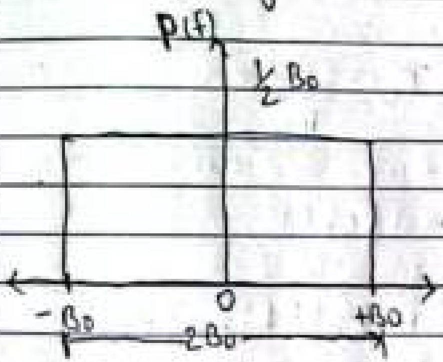


Fig (a): Graphical representation of $P(f)$

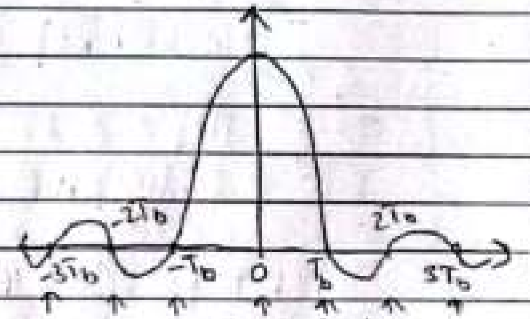


Fig: Time-domain representation

★ Difficulties of ideal solution:

(i) The abrupt transition at $\pm B_0$ from flat to zero is not physically realizable.

(ii) Due to discontinuity of $P(f)$ at $\pm B_0$, there is practically no margin of error in sampling times at receiver end.

Raised Cosine spectrum: [3 marks – 6 Marks]

Because of two difficulties by ideal Nyquist channel we cannot realize them practically. Instead we can overcome them by increasing the bandwidth from its minimum value $B_0 = R_b/2$ to an adjustable value between B_0 and $2B_0$.

A condition is put on overall frequency response $P(f)$ to satisfy the given condition.

From (78.1)

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b = \frac{1}{R_b}$$

Expanding LHS we get;

$$\ldots + P(f+R_b) + P(f) + P(f-R_b) + P(f-2R_b) + \ldots = T_b \quad \text{—(1)}$$

But, $B_0 = \frac{R_b}{2}$ $\therefore R_b = 2B_0$.

putting $R_b = 2B_0$, taking only the values having $n = -1, 0, 1$ & restricting frequency band to $[-B_0, B_0]$, we get

$$P(f+2B_0) + P(f) + P(f-2B_0) = \frac{1}{2B_0} \quad \{-B_0 \le f \le B_0\}$$

It is possible to devise several bandlimited functions that satisfy above eq^n, one of them is raised cosine spectrum which consists of a flat portion and a roll off portion. The raised cosine spectrum is mathematically expressed as;

$$P(f) = \begin{cases} 1/2B_0 \quad \text{(flat portion)} & \ldots 0 \le |f| < f_1 \\ \frac{1}{4B_0}\left\{1 - \sin\left[\frac{\pi(|f| - B_0)}{2B_0 - 2f_1}\right]\right\} & \ldots f_1 \le |f| < 2B_0 - f_1 \\ 0 & \ldots |f| \ge 2B_0 - f_1 \end{cases}$$

frequency parameter f_1 & BW B_0 are related as:

$a = 1 - \frac{f_1}{f_m}$; where, a = rolloff factor indicates excess BW over ideal sol^n B_0.

Responses for different roll-off factors, a.

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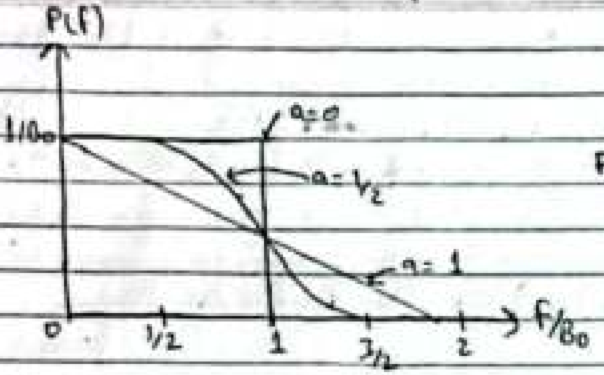

Fig: Frequency response.

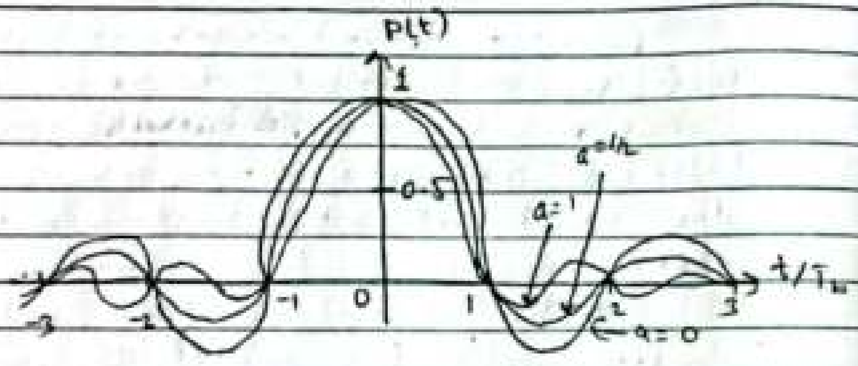

Fig: Time response.

observations:

(i) for a=0.5 & 1, P(f) changes gradually wrt frequency. Hence it is Practically realizable.

(ii) Time response has sinc shape & all sinc fⁿ pass through zero at $t = \pm T_b, \pm 2T_b, \ldots$

(iii) Amplitude of sidelobes ↑ when a ↓.

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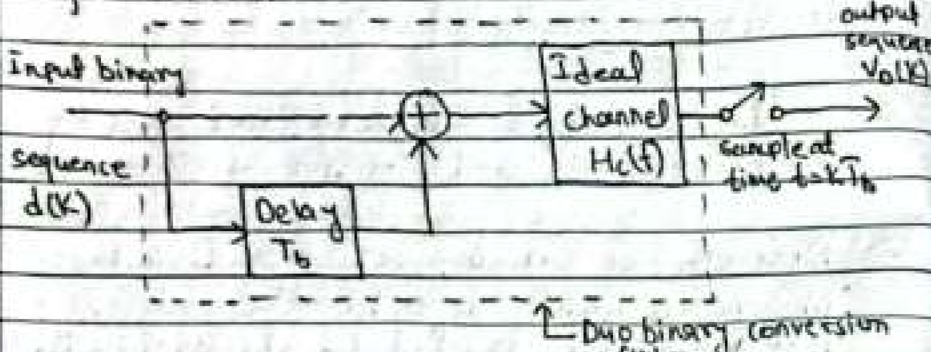
(iv) with a=0, BW requirement is max^m equal to $2B_o$

Correlative coding:

a) Explain one type of correlative coding with its impulse response and transfer function. 73Ch[6]

→ Normally we consider ISI as an undesirable phenomenon which degrades the system performance. But by adding the ISI to the transmitted signal in a controlled manner it is possible to achieve a bit rate of 2B bits per second in a channel of bandwidth B Hz. Such schemes are called as correlative coding or partial response signalling schemes. These schemes are based on assumption that ISI introduced to transmitted signal is known, therefore its effect at the receiver can be compensated. Thus using correlative coding it is possible to achieve the theoretical maximum signalling rate of 2B bits per second for a channel bandwidth of B Hz. It was predicted by Nyquist and can be achieved by using realizable & perturbation tolerant filter.

One type of correlative coding is duo-binary signalling. It's basic scheme is shown in figure below:



The binary signal d(K) is first passed through a simple filter which consists of a single delay element. The present signal d(K) & its delayed version d(K-1) are added to get the duobinary signal at the output of the coder. This is mathematically expressed as:

$$V_0(K) = d(K) + d(K-1) \quad \text{—— (i)}$$

Here, d(K) sequence of the uncorrelated binary digits at the input is converted into a sequence $V_0(K)$ of correlated digits. This correlation betⁿ adjacent pulses is equivalent to introducing ISI which is in control of designer.

Let, delay element is ideal, thus its Transfer Function is given by.

$$H_0(f) = e^{-j2\pi f T_b} \quad \text{—— (ii)}$$

TF of filter containing delay & adder as shown in figure will be,

$$1 + H_0(f) = 1 + e^{-j2\pi f T_b} \quad \text{—— (iii)}$$

∴ Overall TF of this basic filter cascaded with an ideal channel is given by,

$$H(f) = [1 + H_0(f)]\,[H_c(f)]$$
$$H(f) = H_c(f)\,[1 + e^{-j2\pi f T_b}] \quad \text{—— (iv)}$$

Substituting $1 = e^{+j\pi f T_b} \cdot e^{-j\pi f T_b}$ and $e^{-j2\pi f T_b} = e^{-j\pi f T_b} \cdot e^{-j\pi f T_b}$

$$H(f) = H_c(f)\left[e^{+j\pi f T_b} \cdot e^{-j\pi f T_b} + e^{-j\pi f T_b} \cdot e^{-j\pi f T_b}\right]$$

$$H(f) = 2H_c(f)\, e^{-j\pi f T_b}\left[\frac{e^{+j\pi f T_b} + e^{-j\pi f T_b}}{2}\right]$$

$$\therefore \boxed{H(f) = 2H_c(f)\, e^{-j\pi f T_b} \cos(\pi f T_b)} \quad \text{—— (v)}$$

is the reqd TF of duobinary system.

R → bitrate | B = ½ R
B → BW

For impulse response, we need to take IFT of H(f).

$\therefore h(t) = IFT[H(f)]$

$h(t) = IFT[2 \; e^{-j\pi f T_b} \cos(\pi f T_b)] \quad |f| \le R/2$

we can imagine cosine function is being multiplied by rectangular function $(-R/2, R/2)$

$\therefore h(t) = IFT[2 e^{-j\pi f T_b} \cos(\pi f T_b) . \text{rect}(f/R)]$

The corresponding impulse response is given by,

$$h(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t-T_b)/T_b]}{\pi(t-T_b)/T_b}$$

$$\text{or, } h(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b}$$

$$h(t) = \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)} \quad \text{— (vi)}$$

Hence impulse response is a sinc function as depicted in figure below:

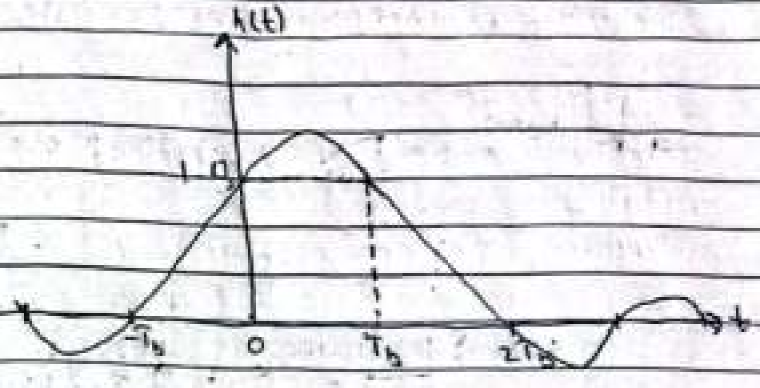


Fig: Impulse response of duobinary filter.

Duobinary Encoding: 72ch[4], 71sh[6]

BW of a communication system depends on the max^m frequency (f_m) in the spectrum of the modulating baseband signal. The BW will reduce with decrease in f_m. The duobinary encoding is a mode of encoding a binary bit stream in which the max^m frequency f_m will decrease as compared to that in the unencoded data. Thus if a carrier is modulated by a duobinary encoded waveform, then the bw of the modulated signal will be smaller than if the unencoded data were used.

Duobinary encoder is shown in figure below

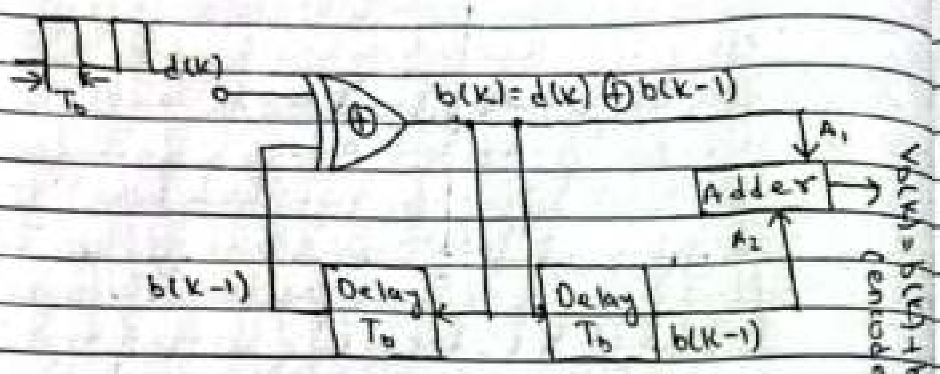


Fig: Duobinary encoder

The input signal d(k) is the data bit stream with a bit duration equal to T_b. It changes its values between 0 & 1 levels.

lets assume '0' → −1V
'1' → +1V

b(k) → O/P of Ex-OR gate given by

$$b(k) = d(k) \oplus b(k-1) \quad \text{—(i)}$$

This is further delayed by one bit & b(k) & b(k−1) are applied to Adder to produce encoded signal $V_D(k)$.

$$V_D(k) = b(k) + b(k-1)$$
$$= \boxed{d(k) \oplus b(k-1)} + \boxed{d(k-1) \oplus b(k-2)} \quad \text{—(ii)}$$

d(k) → changes bet^n +1V & −1V.
∴ b(k) → changes bet^n +1V & −1V.

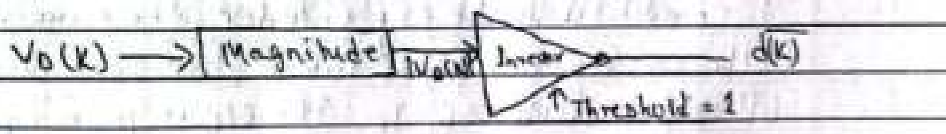
Hence, b(k−1) will change bet^n ±1V.

∴ $V_D(k)$ will change as under:

$V_D(k) = +2V$ {b(k) = +1V, b(k−1) = −1V}
= 0 {b(k) = b(k−1)}
= −2V {b(k) = −1V, b(k−1) = +1V}

Since $V_D(k)$ depends on b(k) & b(k−1) there is correlation bet^n values of $V_D(k)$. The name of this coding is appropriate because in each bit interval, the encoded voltage $V_D(k)$ results from combination of two bits.

Figure below shows a duobinary decoder:



O/P 1 when magnitude is +1V or greater
O/P 0 when magnitude is 0V.

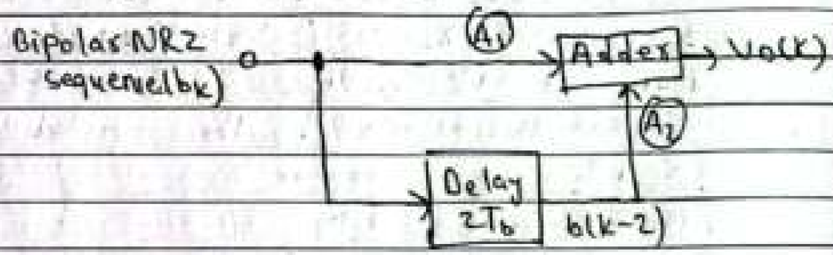
* Problems in duobinary encoder: 71ch[6], 73ch[2]

1) If a carrier is modulated by a duo binary encoded waveform, then the bandwidth of the modulated signal will be smaller than if the unencoded data were used.

2) If there is an error in the received signal then it will propagate in the system.

To solve above difficulties:

To eliminate first issue, we should modify the duo binary encoder as follows.



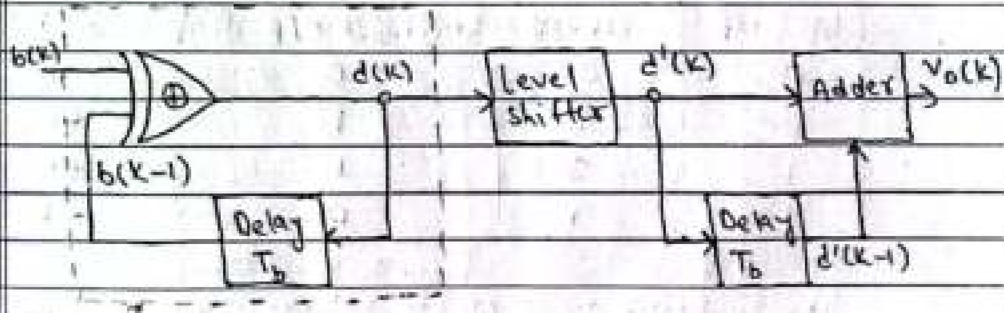
In modified duobinary encoder the correlation between binary digits takes place over duration of $2T_b$ instead of T_b. It's O/P is given by

$$V_o(K) = b(K) - b(K-2)$$

However even in this method the error is not eliminated. The error present in the received signal may be propagated through the whole system.
To eliminate the error, it must be precoded. Hence a modified duobinary encoder with precoder can solve the above two major difficulties. It will be discussed in detail in next section.

Duobinary-Encoder with Precoder (Differential Encoder)
74ch[6], 73sh[6]

The modified duobinary system discussed above has an important limitation, which is that if there is an error in the received signal then it will propagate in the other values of b(k).
The duo-binary system with precoder is free from such a problem. It's block diagram is shown below



Precoder or differential encoder ←— Modified duobinary Encoder —→

The precoder is added to the modified duobinary encoder to obtain the duobinary encoder with precoder as shown in figure above. The precoder consists of an Ex-OR gate along with a delay unit that introduces a delay of one bit duration T_b.

The precoder output d(k) is given by,

$$d(K) = b(K) \oplus d(K-1) \quad \text{--- (i)}$$

where, b(K) = unipolar stream of binary digits.

Eqn shows that precoder o/p is equal to zero, if both its i/p are alike (both 0 or 1) and it is 1 if any one of the input is 1.

operation of precoder:

b(K) i/p	d(K-1)	Precoder O/P d(K)	d'(K)
0	0	0	-1
0	1	1	+1
1	0	1	+1
1	1	0	-1

The precoder O/P is then applied to

a level shifter which converts it into a bipolar signal having values +1 or -1 as follows:

$$d'(K) = \begin{cases} +1 & \text{if } d(K) = 1 \\ -1 & \text{if } d(K) = 0 \end{cases} \quad \text{--- (ii)}$$

d'(K) is applied to basic duobinary encoder. Its O/P i.e, $V_0(K)$ is given by,

$$V_0(K) = d'(K) + d'(K-1) \quad \text{--- (iii)}$$

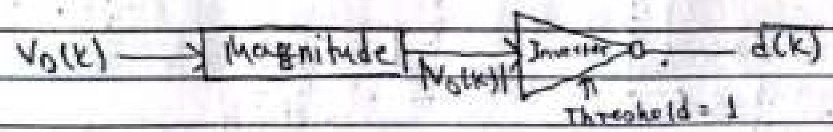
As d'(K-1) is delayed version of d'(K) it will also be a bipolar signal having value ±1

$$\therefore V_0(K) = \begin{cases} \pm 2 & \text{if } d'(K) = d'(K-1) \\ 0 & \text{if } d'(K) \neq d'(K-1) \end{cases} \quad \text{--- (iv)}$$

As this is duobinary signal,

$$V_0(K) = \pm 2 \quad \text{when } d(K) = 0$$
$$V_0(K) = 0 \quad \text{when } d(K) = 1 \quad \text{--- (v)}$$

Decoder Block diagram:



Eye Diagram: 78Bh[5], SN: 72ch[5], 76ch[5], 69ch[4]

Eye pattern is a pattern displayed on the screen of a cathode ray oscilloscope (CRO). The shape of this pattern resembles with the shape of human eye. Hence it is known as eye pattern. Eye pattern is a practical way to study the intersymbol interference (ISI) & its effects on a PCM or data communication system.

The eye pattern is obtained on the CRO inter--face by applying the received signal to vertical deflection plates (Y-plates) of the CRO and a sawtooth wave at the transmission symbol rate i.e. $(1/T_b)$ to the horizontal deflection plates (X-plates) as shown in figure(a) below. The resulting oscilloscope display is shown in figure (b) is called as the eye pattern which resembles human eye.

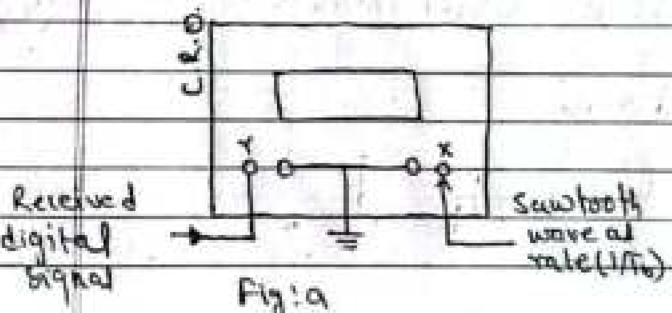


Fig: a

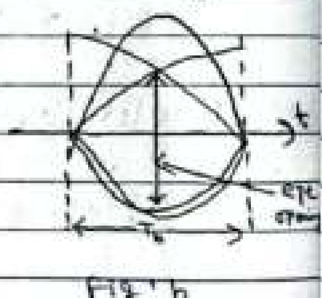


Fig: b

The interior region of the eye pattern is known as the eye opening. The eye pattern provides great deal of information about the performance of the system.

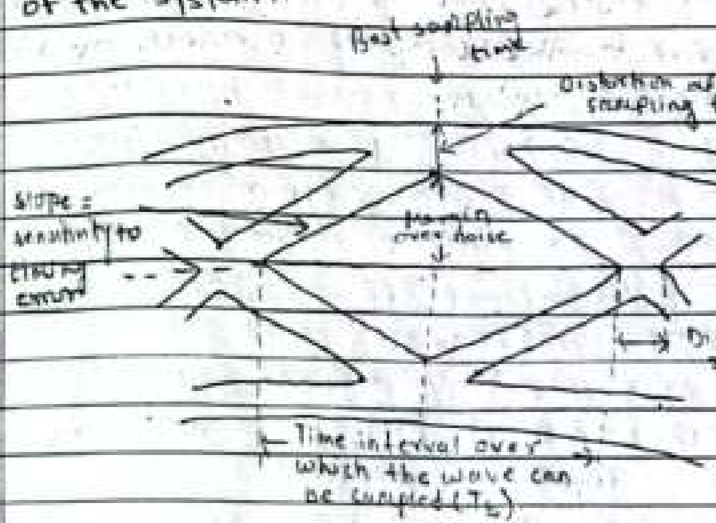


Fig: Interpretation of eye diagram

Information obtained from eye pattern:

(i) The width of the eye opening defines the time-interval over which the received wave can be sampled, without an error due to ISI. The best time for sampling is when eye is open widest.

(ii) The sensitivity of the system to the timing error is determined by the rate of closure of the eye as the sampling rate is varied. The height of eye opening at a specified sampling time defines the margin over noise.

(iii) When the effect of ISI is severe, the eye is completely closed & it is impossible to avoid errors due to the combined presence of ISI & noise in the system.

Chapter-5

Bandpass (Modulated) data communication systems (4H/8

No chapter 5 in examquestions → 74ch, 74As, 73Sh, 71Ch

Q) What are the design goals of digital Modulation techniques? 73Ch[2]

= To send baseband signals over radio links or satellites is not possible because of impractically large antennas that have to be used. Hence the spectrum of the message signal has to be shifted to higher frequencies. This is achieved by using the baseband digital signal to modulate a sinusoidal carrier. This is called as digital carrier Modulation. There are various design goals of digital modulation techniques. They are:

(i) Binary & M-ary schemes: In binary we send any one of two possible signals during each signalling interval of duration T_b. Similarly in Mary we can send any one of M possible signals during each signal interval of duration T_b. M-ary scheme needs less BW but error performance is poor compared to binary schemes.

(ii) Probability of Error (Pe): The value of Pe of a system indicates its performance in presence of AWGN (additive white Gaussian Noise). Pe should be as small as possible.

(iii) Power spectra: It is a graph of power spectral density plotted on Y axis vs frequency plotted on X axis. It gives information about the bandwidth requirement & co-channel interference.

(iv) BW efficiency: It is defined as the ratio of the data rate (bits/sec) to the effectively utilized channel BW. It is denoted by ρ.

$$\rho = \frac{R_b}{B} \text{ bits/Hz}$$

It depends on multichannel encoding & spectral shaping.

(v) Passband Transmission Model: The passband transmission model consists of 3 parts: Transmitter, communication channel & receiver.

Coherent Modulation: This technique employs coherent detection. Here, receiver carrier is phase locked with the transmitter carrier. It's design is complex but provides low error probability.

Non-coherent Modulation: Here, receiver carrier is not phase locked with transmitter carrier. It's design is simple but error probability increases.

Binary phase shift keying:

- Modulation technique
- Demodulation technique
- Signal space diagram

} 73Ch[8]

It falls under the coherent modulation technique. It is the most efficient of the 3 digital modulation (ASK, FSK & PSK). Hence, BPSK is used for high bit rates. In BPSK, phase of the sinusoidal carrier is changed according to the data bit to be transmitted. Also, a bipolar NRZ signal is used to represent the digital data coming from the digital source.

In a binary phase shift keying (BPSK), the binary symbols '1' and '0' modulate the phase of the carrier.

BPSK signal can be defined as;

$$s(t) = b(t)\sqrt{2P}\ \cos(2\pi f_c t) \longrightarrow (1)$$

where, $b(t) = +1$ when binary '1' is transmitted
$= -1$ when binary '0' is transmitted

BPSK signal may be generated by applying carrier signal to a balanced modulator. The binary data signal (0s & 1s) is converted to NRZ bipolar signal by an NRZ encoder. Here, the bipolar signal b(t) is applied as a modulating signal to the balanced modulator.

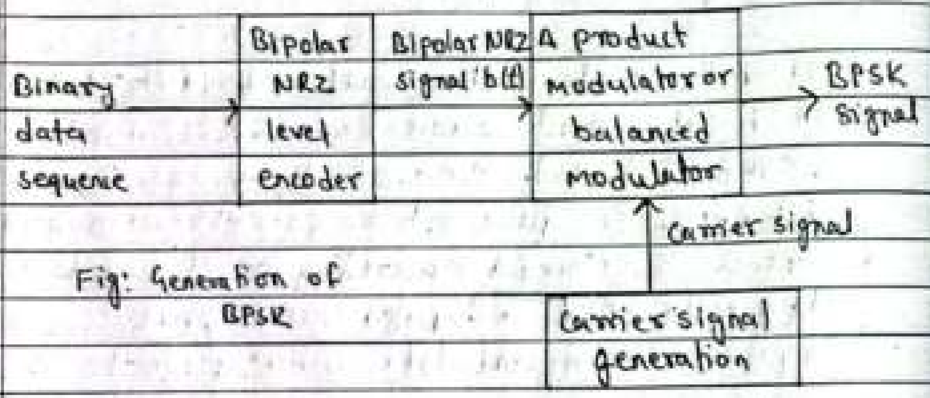


Fig: Generation of BPSK

A NRZ level encoder converts the binary data sequence into bipolar NRZ signal. The table below shows i/p digital & corresponding bipolar NRZ signal

S.N.	i/p digital signal	Bipolar NRZ signal b(t)	BPSK O/P signal
1.	Binary '0'	$b(t) = -1$	$-\sqrt{2P}\cos\omega_c t$
2.	Binary '1'	$b(t) = +1$	$+\sqrt{2P}\cos\omega_c t$

Here,

$P = \frac{E_b}{T_b}$; $E_b \rightarrow$ Signal energy
$T_b \rightarrow$ bit duration

$\omega_c \rightarrow 2\pi f_c$

Figure below shows the block diagram of the scheme to recover baseband signal from BPSK signal. The Transmitted BPSK signal is given as:

$$s(t) = b(t)\sqrt{2P}\ \cos(2\pi f_c t)$$

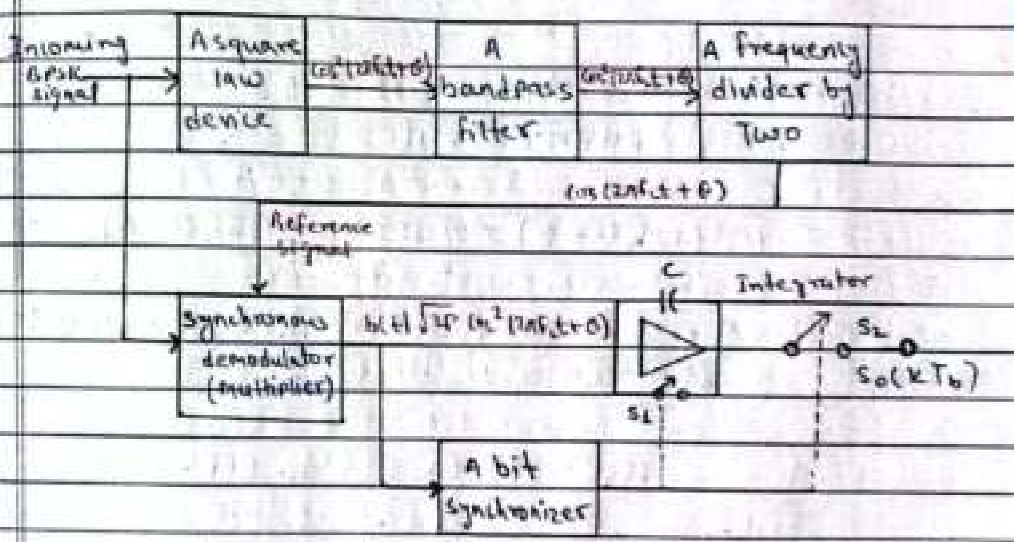


Fig: Reception of baseband signal in BPSK signal

The transmitted BPSK signal undergoes phase change depending upon timedelay from transmitter end to receiver end. This phase change is phase shift in transmitted signal, θ. Thus, the signal at input of receiver can be written as;

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t + \theta) \quad \text{--- (i)}$$

Since it is a coherent detection, the carrier is separated.

The received signal is passed through square law device. Thus we get

$\cos^2(2\pi f_c t + \theta)$ { we have neglected amplitude as we are interested in carrier

we know that,

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\therefore \cos^2(2\pi f_c t + \theta) = \frac{1}{2} + \frac{1}{2}\cos 2(2\pi f_c t + \theta)$$

This signal is passed through Bandpass filter thus we get the o/p

$\cos 2(2\pi f_c t + \theta)$ from BPF.

This signal has frequency equal to $2f_c$ thus it is

passed through frequency divider by two whose o/p will be $\cos(2\pi f_c t + \theta)$

The coherent demodulator multiplies the i/p signal & the recovered carrier signal. Hence at o/p of multiplier, we get:

$$b(t)\sqrt{2P}\cos(2\pi f_c t + \theta) \times \cos(2\pi f_c t + \theta) = b(t)\sqrt{2P}\cos^2(2\pi f_c t + \theta)$$

$$\Rightarrow b(t)\sqrt{\frac{2P}{4}}\,[1 + \cos 2(2\pi f_c t + \theta)]$$

$$\Rightarrow b(t)\sqrt{\frac{P}{2}}\,[1 + \cos 2(2\pi f_c t + \theta)] \quad \text{--- (ii)}$$

This signal is applied to bit synchronizer and integrator. The integrator integrates the signal over one bit period. The bit synchronizer takes care of starting & ending times of bit. By closing and opening s_1 & s_2 (switches) integrator performs integration causing phase changes and sends the values generated to output of Receiver.

The output $s_0(kT_b)$ generated in receiver depends upon the value $b(kT_b)$ given on the input side.

The following figure shows the signal space diagram of BPSK signal

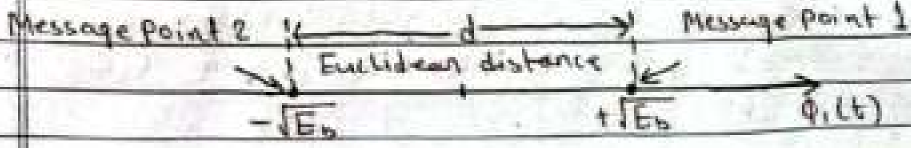


Fig: SS diagram of BPSK signal.

where,

Signal energy, $E_b = P T_b$ | P = Power
T_b = bit duration

Drawback: In the detection of BPSK, we square the $b(t)\sqrt{P}\cos(\omega t+\theta)$ to regenerate the carrier. However if the received signal is

$$-b(t)\sqrt{2P}\cos(2\pi f_c t+\theta)$$

the squared signal remains same as before. Here, the recovered carrier is unchanged even if the i/p signal has changed its sign. Therefore it is not possible to determine whether the received signal is equal to b(t) or −b(t), which results in ambiguity. Its remedy is we can solve this problem by using DPSK.

Coherent Binary Frequency shift keying [BFSK]

- Modulation — 70ch[8]
- Demodulation — 72ch[6]
- Signal space diagram — 69ch[8]
- Non-coherent detection

In binary frequency shift keying (BFSK), the frequency of a sinusoidal carrier is shifted according to the binary symbol. In other words, the frequency of a sinusoidal carrier is shifted between two discrete values. However the phase of the carrier is unaffected. This means we have two different frequency signals according to binary symbols. Let there be frequency shift by Ω then we can write following equations.

If $b(t) = '1'$ then $S_H(t) = \sqrt{2P_s}\cos(2\pi f_c+\Omega)t$ — (i)

If $b(t) = '0'$ then $S_L(t) = \sqrt{2P_s}\cos(2\pi f_c-\Omega)t$ — (ii)

Here there is increase or decrease in frequency by Ω.
Let us use the following conversion table to combine above eqs

b(t) Input	d(t)	$P_H(t)$	$P_L(t)$
1	+1V	+1V	0V
0	−1V	0V	+1V

The equation (i) & (ii) can combinely be written as!

$$s(t) = \sqrt{2P_s}\cos[(2\pi f_c + d(t)\Omega)t] \quad \text{— (iii)}$$

when 1 is transmitted.

$$\text{carrier frequency, } f_H = f_c + \frac{\Omega}{2\pi} \quad \text{— (iv)}$$

when '0' is transmitted

$$\text{carrier frequency, } f_L = f_c - \frac{\Omega}{2\pi} \quad \text{— (v)}$$

It may be observed from table above that $P_H(t)$ is same as $b(t)$ but $P_L(t)$ is inverted of $b(t)$. The block diagram of BFSK generation is shown below!

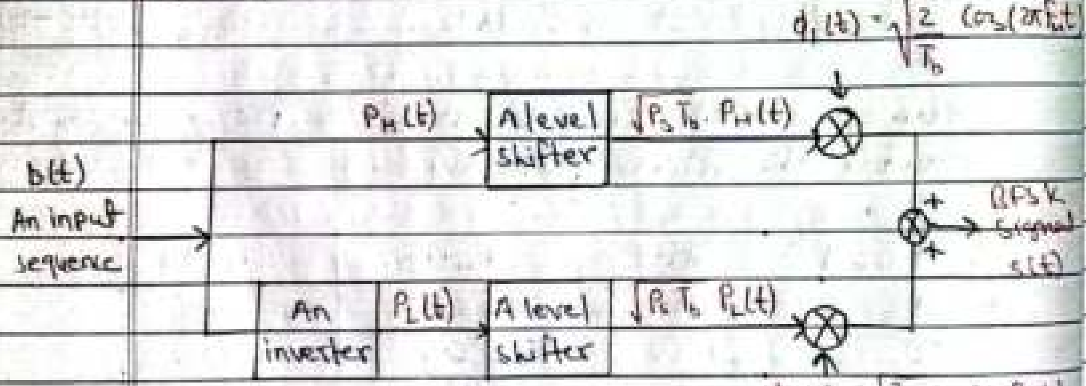


Fig: BD of BFSK generation

We know that i/p sequence $b(t)$ is same as $P_H(t)$. An inverter is added after $b(t)$ to get $P_L(t)$. The level shifter converts +1 level to $\sqrt{P_s T_b}$ while zero is unaffected. The transmitted signal will have a frequency of either f_H or f_L.
Further, there are product modulators after level shifter. The two carrier signals $\phi_1(t)$ & $\phi_2(t)$ are used. $\phi_1(t)$ & $\phi_2(t)$ are orthogonal to each other.

output from both modulators are not possible at a time since $P_H(t)$ & $P_L(t)$ are complementary to each other!

Similarly figure below shows the block diagram of demodulation of BFSK using coherent detection technique.
The detector consists of two correlators that are individually tuned to two different carrier frequencies to represent symbols '1' and '0'. A correlator consists of a multiplier followed by an integrator. Then, the received binary FSK signal is applied to the multipliers of both the correlators. To the other input of the multipliers, carriers with frequency f_{c1} and f_{c2} are applied

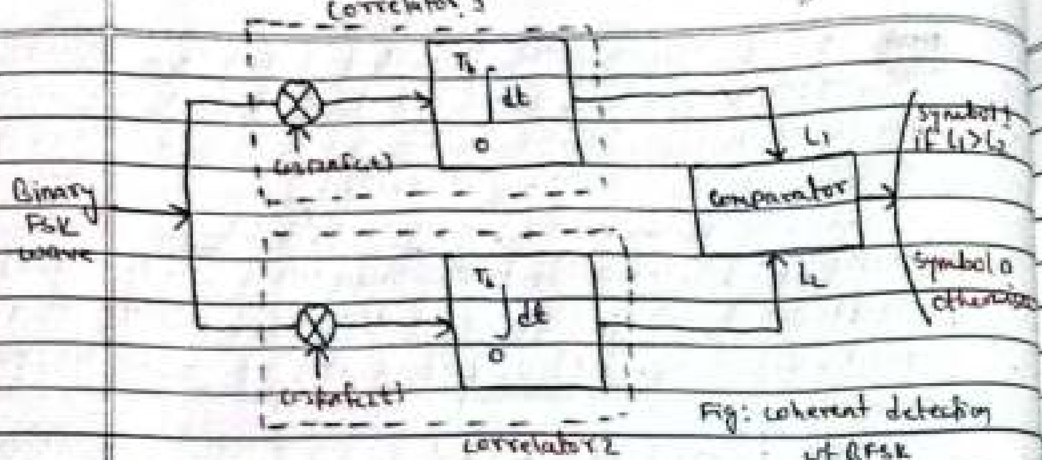


Fig: coherent detection of BFSK

The multiplied output of each multiplier is subsequently passed through integrators generating output L_1 & L_2 in the two paths. The output of the two integrators are fed to the decision making device.

If $L_1 > L_2 \longrightarrow$ output is symbol 1
If $L_1 < L_2 \longrightarrow$ output is symbol 0

signal space diagram:

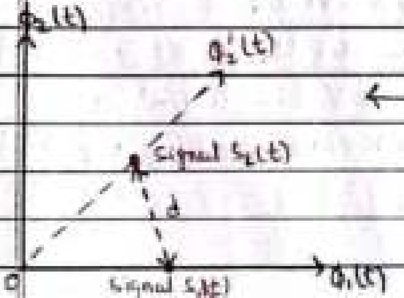


Fig: ssd of non orthogonal BFSK

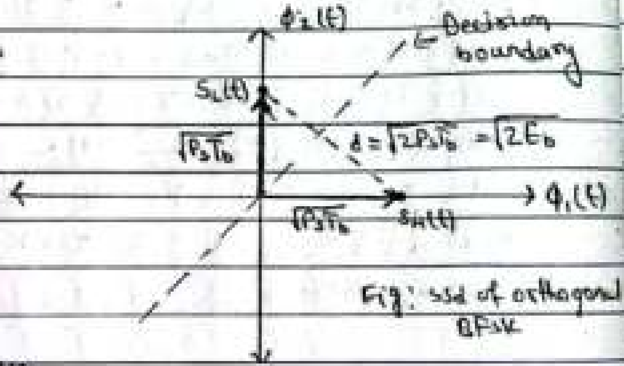


Fig: ssd of orthogonal BFSK

Binary FSK waves may be demodulated non-coherently using envelope detector. The received FSK signal is applied to bank of two bandpass filters, one tuned to frequency f_{c1} and the other tuned to f_{c2}. Each filter is followed by an envelope detector. The resulting outputs of the two envelope detectors are sampled and then compared to each other.

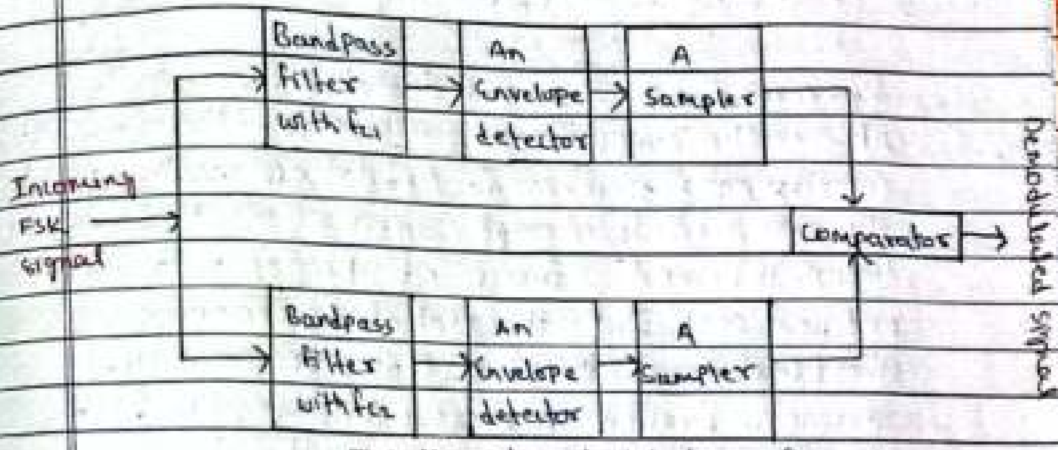


Fig: Non-coherent detection of FSK binary signal

A decision is made in favour of symbol '1' if the envelope detector output derived from the filter tuned to frequency f_{c1} is larger than that derived from the second filter. Otherwise, a decision is made in favour of the symbol '0'.

Differential phase shift keying (DPSK):

- what do you understand by differential coding? 71sh[5]
- Modulation: 75Bh[6], 72Kar[2], 71sh[5]
- demodulation: 75Bh[6], 72Kar[2], 71sh[5]
- Signal space diagram: 72Kar[1]
- need: 75Bh[1]
- why DPSK is preferred over PSK. 72Kar[1]

DPSK is the non-coherent version of the PSK. It is differentially coherent modulation method. It doesnot need a coherent carrier at the demodulator. The input sequence of binary bits is modified such that the next bit depends upon the previous bit. Therefore, in receiver, the previous received bits are used to detect the present bits.

In DPSK, the input binary sequence is first differentially encoded and then modulated using a product modulator as shown in figure below. The differentially encoded sequence {d(k)} is generated from the input binary sequence {b(k)} by complementing the modulo2sum of b(k) and d(k-1). The effect is to leave the symbol

d(k) unchanged from the previous symbol if the incoming binary signal b(k) is 1 & to toggle d(k) if b(k) is 0.

$$\therefore\ d(k) = b(k) \oplus d(k-1)$$

The block diagram of DPSK transmitter is shown below. It consists of one bit delay element and encoder circuit interconnected so as to generate differentially encoded sequence from the input binary sequence. The output is passed through product Modulator to obtain DPSK signal.

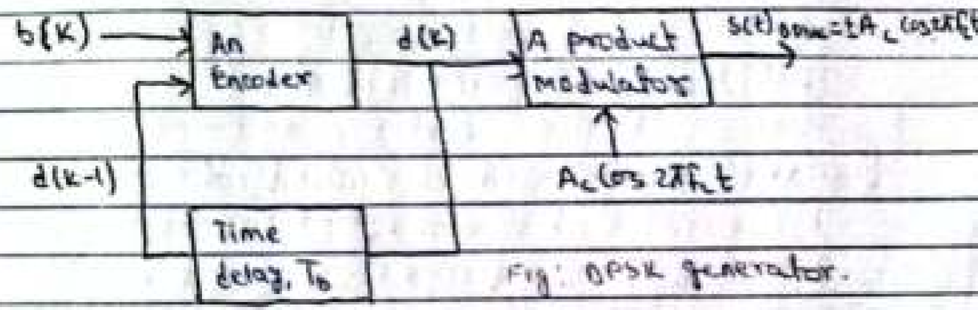


Fig: DPSK generator.

At receiver, the original sequence is recovered from the demodulated differentially encoded signal through a complementary process as shown in figure below. The received DPSK signal and its time delayed version is applied to each one of the input of the multiplier.

The output of the multiplier is then given to the integrator which produces output value.

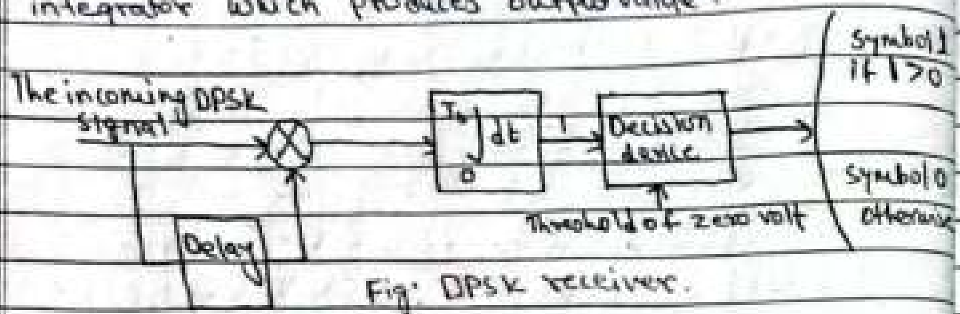


Fig: DPSK receiver.

By comparing the integrator output with decision level of zero volt, the decision device can reconstruct the binary sequence by assigning a symbol '0' for negative output & a symbol '1' for positive output.

Need / Features / Advantages of DPSK

(i) DPSK does not need carrier at the receiver end. This means that the complicated circuitry for generation of local carrier is not required.

(ii) The Bandwidth requirement of DPSK is lower than BPSK.

Quadrature Phase shift keying (QPSK)

- Modulation
- Demodulation
- SS Diagram
- Relevant derivations

} 70ch [8]

In communication systems, we have two main resources, transmission power and channel Bandwidth. Since channel bandwidth depends upon information bit rate, if two or more bits are combined into a symbol then the signalling rate will reduce and thus transmission channel Bandwidth.
In QPSK, two successive bits in data sequence are grouped together. This reduces bit rate or signalling rate and thus reduces channel bandwidth requirement. It has twice the bandwidth efficiency of BPSK.
The phase of the carrier takes on one of four equally spaced values such as $0, \pi/2, \pi$ and $3\pi/2$, where each value of phase corresponds to a unique pair of message bits.

$$S_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + (i-1)\frac{\pi}{2}\right] \qquad 0 \le t \le T_s, \quad i = 1, 2, 3, 4$$

where, T_s = Symbol duration & is twice the bit period.

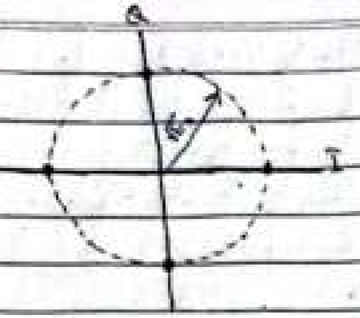


Fig: QPSK Constellation diagram where carrier phases are $0, \pi/2, \pi, 3\pi/2$

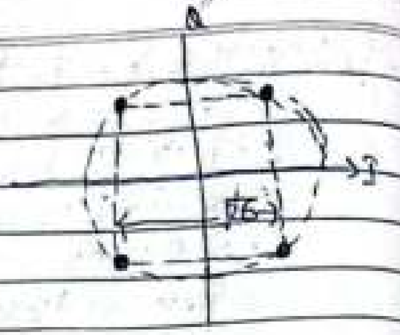


Fig: QPSK constellation where carrier phases are $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

Here, the above are the constellation diagram of QPSK signal. Different QPSK signal sets can be derived by simply rotating the constellation. From the constellation diagram, it can be seen that the distance between adjacent points in the constellation is $\sqrt{2E_s}$.
Since each symbol corresponds to two bits, then $E_s = 2E_b$, thus the distance between two neighboring points in constellation is equal to $2\sqrt{E_b}$.

QPSK Transmission: The unipolar binary message stream has bit rate R_b and is first converted into a bipolar NRZ sequence using unipolar to

bipolar converter.

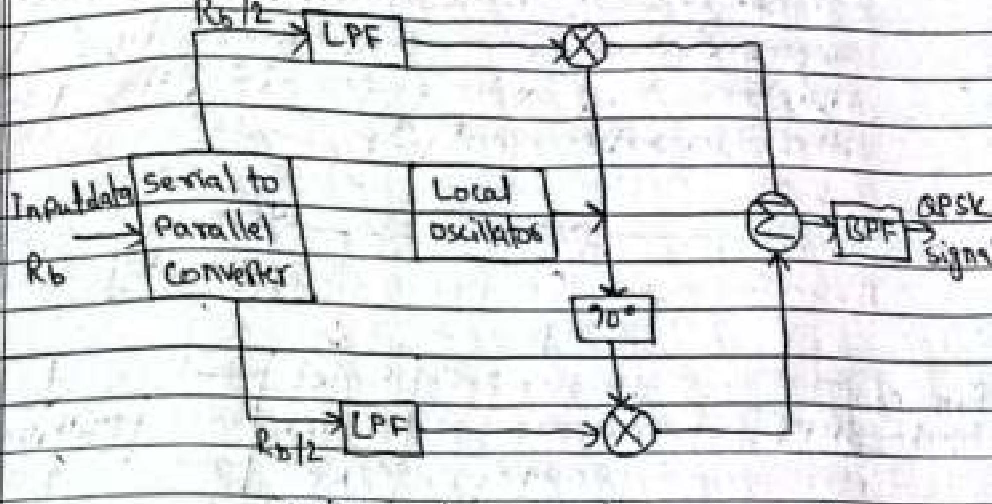


Fig: Block diagram of QPSK Transmitter.

The bit stream $m(t)$ is split into two bit streams $m_I(t)$ & $M_Q(t)$, each having bit rate of $R_s = R_b/2$. The bit stream $m_I(t)$ is called even stream & $M_Q(t)$ is called odd stream. These two binary sequences are separately modulated by two carriers $\phi_1(t)$ and $\phi_2(t)$, which are in quadrature. The two modulated signals, each of which are considered to be BPSK signal, are summed to produce a QPSK signal. The filter at the output of the modulator confines the power spectrum of the QPSK signal within the allocated band. This prevents from spill-over of signal energy into adjacent channels and also removes out of band spurious signals generated during modulation Process. In most implementations, Pulse shaping is done at baseband to provide proper RF filtering at the transmitter output.

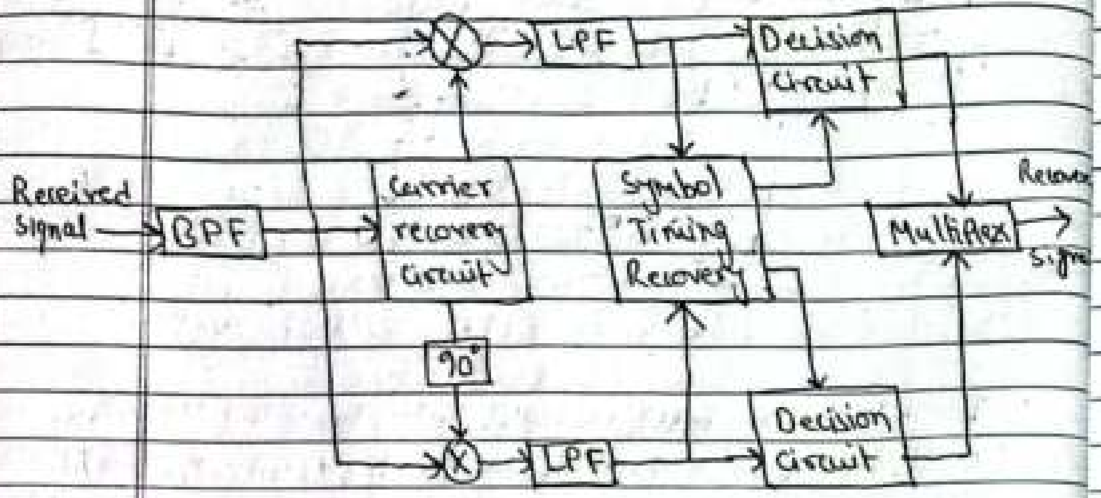


Fig: Block diagram of a QPSK receiver.

The frontend BPF removes the out of band noise and adjacent channel interferenie. The filtered output is split into two parts, & each part is coherently demodulated using the in-phase & quadrature carriers. The coherent carriers used for demodulation are recovered from the received signal using carrier recovery circuits. The O/Ps

of demodulators are passed through decision circuits which generate the inphase and quadrature binary streams. The two components are then multiplexed to reproduce the original binary sequence

M-ARY data communication system: SN: 72Ch[2.5]

In an M-ary signalling scheme, we can send one of the M Possible signals such as $S_1(t), S_2(t), \cdots S_M(t)$, during each signalling interval of duration T seconds. The number of possible signals, ie, M is given by,

$$M = 2^N \quad \text{—(1)}$$

where, N = integer.

Advantages of M-ary scheme over binary schemes is that they achieve better bandwidth efficiency. However they have their own drawbacks like increasing in transmitted power & increase in error probability.

Q) What is MODEM? Discuss the modes of operation of MODEM.: 70ASD[4]

= MODEM (Modulator-Demodulator) is an electronic device used to transmit & receive digital data for transmission over public telephone lines. A modulator translates the data pulses into voice band analog signals at the transmitting end and at the receiving end, the analog signals are demodulated to recover the digital information. Initially, MODEMS were used to connect terminals located in remote places, to a central computer. The following are the modes of operation of MODEM:

(i) Simplex: In this mode, data transmission is done only in one direction & no signalling path is available from receiver to transmitter. Therefore, there is no possibility of retransmission and error connection. This mode has limited use.

(ii) Half-Duplex: In this mode, reverse channel is avai-lable but in turn, not simultaneously. The transmission speed is reduced because of the need to wait for the transmission or reception.

(iii) Full-Duplex: In this mode, data can be transmitted and received in both direction at same time. It requires two Independent channel.

Chapter-6

Random signals and noise in communication systems.

(7 Hours / 12 Marks)

This chapter was not asked in 2076 ch.

Q) Define Moment and central Moment.

= The n^{th} moment of a random variable is defined as the mean value of X^n. Thus, the expression for the n^{th} moment of X is given by,

$$n^{th} \text{ moment of } X = E[X^n] = \overline{X^n} = \int_{-\infty}^{\infty} x^n f_X(x)\,dx$$

'Central moment' is the expected value of the difference between the random variable X & its mean value m_x. Thus, n^{th} central moment is defined as under:

$$n^{th} \text{ central moment of } X = E[(X-m_x)^n]$$
$$= \int_{-\infty}^{\infty} (x-m_x)^n f_X(x)\,dx$$

Show that 1st central moment is always zero.

From eq^n (ii)

m_x = average value of X.

Now, for 1st central moment, $n = 1$

$$1^{st} \text{ central moment of } X = E[(X - m_x)^1]$$

$$= E[(X)] - E[m_x] \quad \text{—— (iii)}$$

Now, since 1st moment of x is given as

$$E[X] = \bar{x} = \int_{-\infty}^{\infty} x f_X(x)\, dx = m_x$$

Thus from eq^n (iii)

$$1^{st} \text{ central moment of } X = E[X] - E(m_x)$$
$$= m_x - E[m_x]$$
$$= m_x - m_x$$
$$= 0.$$

Hence, 1st central moment of continuous random variable is always zero.

72 Kar[2] **Random Process**: Let there be a random experiment
73 Ch[2] E having outcome λ from the sample space S.
69 Ch[4] Thus every time an experiment is conducted, the outcome λ will be one of the sample point in sample space. If this outcome λ is associated with time, then a function of λ and time t is formed i.e. $X(\lambda, t)$. Then the function $X(\lambda, t)$ is known as random process. Hence when any random experiment E is given a time dimension then each outcome appears at some certain time & the random experiment will be converted into Random Process. A random process is the function of two variables λ & t.

Q) Explain Auto-correlation function. 74 Ch[3]

= For carrying out signal analysis, one of the most important statistical characteristics of a random process is its auto-correlation function. This provides the spectral information about the random process. The auto correlation function of a random process is defined as under:

$$R_X(\tau) = E[X(t+\tau)\, X(\tau)] \quad \text{—— (1)}$$

Figure below illustrates the physical significance of the autocorrelation function.

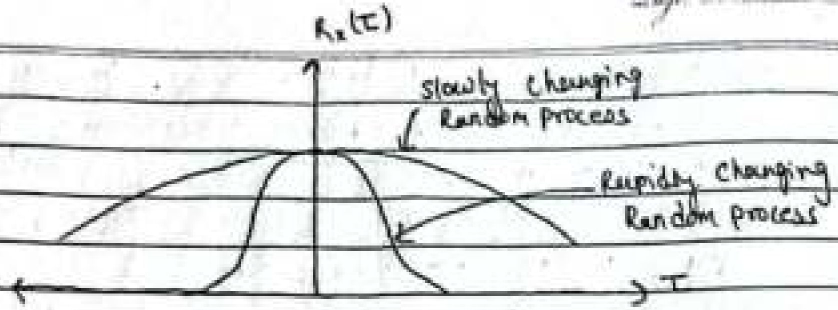


Fig: physical significance of AC function.

The physical significance of $R_x(\tau)$ is that it is a measure of interdependence of two random variables obtained by observing a random process $X(t)$ at times τ seconds apart.
Therefore, if the random process $X(t)$ changes very rapidly with time, the AC function will decrease rapidly to zero as shown in figure above.
The decorrelation time τ_0 characterizes the reduction in $R_x(\tau)$ from $R_x(0)$ to 0. If $\tau > \tau_0$, then, the magnitude of the ac function $R_x(\tau)$ remains below some prescribed value. Therefore, the decorrelation time τ_0 of a wide-sense stationary random process $X(t)$ of a zero mean as the time taken for the magnitude of ac function $R_x(\tau)$ to decrease to 1 percent of the maximum value $R_x(0)$.

sample functions

Q) What do you mean by Ergodic stochastic process? 74Ch[2]
= A random process is known as ergodic process if the time-averages are equal to ensemble averages. Hence for a ergodic process, we have

$$m_x = \langle M_x \rangle$$
$$R_X(t_1, t_2) = \langle R_X(\tau) \rangle$$

For any ergodic process, all ensemble averages will be equal to corresponding time averages for any particular sample functions. The time averages are not a function of time. When time & ensemble averages are same, it implies that ensemble averages also are not a function of time.
Thus an ergodic process is always stationary but converse is not true.

Transmission of a Random process through a linear filter: 74Ch[10], 70Asd[5]

Let us assume that a linear process $X(t)$ is applied as input to a linear time invariant (LTI) filter. Let the impulse response of such filter be $h(t)$. At the output of this filter, let a new random process $Y(t)$ be produced as shown i figure below:

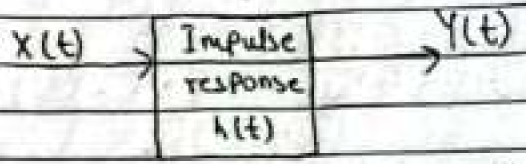


Fig: Transmission of random process through an LTI filter.

Even if the distribution of $X(t)$ is known to us, it is difficult to describe the probability of distribution of $Y(t)$. Let us obtain the mean and autocorrelation function of the output process $Y(t)$. We assume that $X(t)$ is a wide sense stationary process.

* The first mean of $Y(t)$ is defined as under:

$$m_Y(t) = E[Y(t)] = E\left[\int_{-\infty}^{\infty} h(\tau_1)\, X(t-\tau_1)\, d\tau_1\right] \quad \text{--- (i)}$$

where, $\tau_1 \rightarrow$ dummy variable.

we can interchange order of expectation & integration wrt τ_1 provided that $[X(t)]$ is finite for all t & system is stable

$$m_Y(t) = \int_{-\infty}^{\infty} h(\tau_1)\, E[X(t-\tau_1)]\, d\tau_1$$

$$m_Y(t) = \int_{-\infty}^{\infty} h(\tau_1)\, m_X(t-\tau_1)\, d\tau_1 \quad \text{--- (ii)}$$

If i/p $X(t)$ is wide sense stationary, then $m_X(t)$ is a constant equal to m_X.

Thus from (ii)

$$m_Y(t) = m_X \int_{-\infty}^{\infty} h(\tau_1)\, d\tau_1 \quad \text{--- (iii)}$$

$$\therefore m_Y(t) = m_X H(0) \quad \text{--- (iv)}$$

where, $H(0) \rightarrow$ zero frequency (dc) response of system.

Eqn (iv) shows mean of $Y(t)$ is equal to product of mean $X(t)$ & dc response of system.

* Similarly the autocorrelation function of $Y(t)$ is given by,

$$R_Y(t,u) = E[Y(t)\, Y(u)]$$

Where; $t, u \rightarrow$ two values of time at which O/P is observed

The AC function of $Y(t)$ can be obtained by using the convolution integral as under:

$$R_Y(t,u) = E\left[\int_{-\infty}^{\infty} h(\tau_1)X(t-\tau_1)\,d\tau_1 \int_{-\infty}^{\infty} h(\tau_2)X(u-\tau_2)\,d\tau_2\right] \quad \text{— (v)}$$

If $x(t) \rightarrow$ stable &
$E[X^2(t)] \rightarrow$ finite for all t,
we can interchange order of expectation & integration.

$$R_Y(t,u) = \int_{-\infty}^{\infty} d\tau_1\, h(\tau_1) \int_{-\infty}^{\infty} d\tau_2\, h(\tau_2)\, E[X(t-\tau_1)X(u-\tau_2)]$$

$$= \int_{-\infty}^{\infty} d\tau_1\, h(\tau_1) \int_{-\infty}^{\infty} d\tau_2\, h(\tau_2)\, R_X(t-\tau_1, u-\tau_2) \quad \text{— (vi)}$$

when i/p process $X(t) \rightarrow$ wide sense stationery proc. the ac function of $X(t)$ is only a function of the difference $(t-\tau_1)$ & $(u-\tau_2)$. Hence, substituting $\tau = (t-u)$ in eqn (vi), we get

$$R_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)\, h(\tau_2)\, R_X(\tau-\tau_1+\tau_2)\, d\tau_1\, d\tau_2 \quad \text{— (vii)}$$

If we look at the expressions of m_Y and $R_Y(\tau)$, then we can conclude that if the input to a stable LTI filter is a wide sense stationary process, then the output of the filter is also a wide sense stationery random process.

White noise: It is the noise whose power spectral density is uniform over the entire frequency range of interest as shown in figure below:

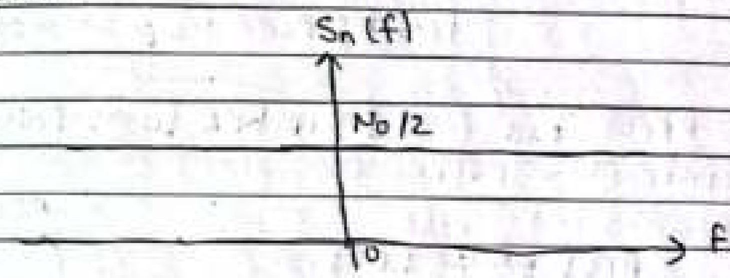


Fig: psd of white noise.

White noise has all the frequency components in equal proportion. It has a gaussian distribution which means PDF of white noise has the shape of gaussian PDF. Thus it is also known as gaussian noise.

From figure above, psd of white noise is given by,

$$S_n(f) = \frac{N_0}{2} \quad \text{——— (1)}$$

This eq^n shows psd of white noise is independent of frequency.
As N_0 is constant, psd is uniform over entire frequency range.

$$N_0 = KT_e$$

where,
K = Boltzmann's constant
T_e = Eq. noise Temperature of system

We know psd & ac function form Fourier transform pair. This means,

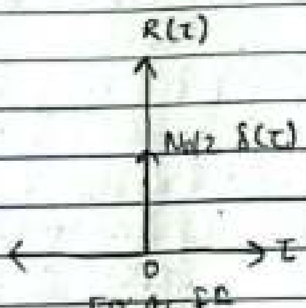


Fig: AC fn

$$R(\tau) \xleftrightarrow{F.T.} S(f)$$

i.e, $FT\{R(\tau)\} = S(f)$

or, $R(\tau) = IFT\{S(f)\}$

for white noise,

$$R(\tau) = IFT\{S_n(f)\} = IFT\{N_0/2\}$$

$$\therefore R(\tau) = \frac{N_0}{2}\delta(t) \quad \Big\{ \text{AC f}^n \text{ of white noise}$$

Noise Equivalent Bandwidth: 73Ch[2], 72Ch[2.5], 70Asd[3]

Assume that a white noise is present at the input of a receiver (filter). Let the filter have a TF H(f) as shown in figure below. This filter is being used to reduce the noise power actually passed on to the receiver. Now consider an ideal (rectangular) filter as shown by dotted plot in figure below. The center frequency of this ideal filter is also fo.

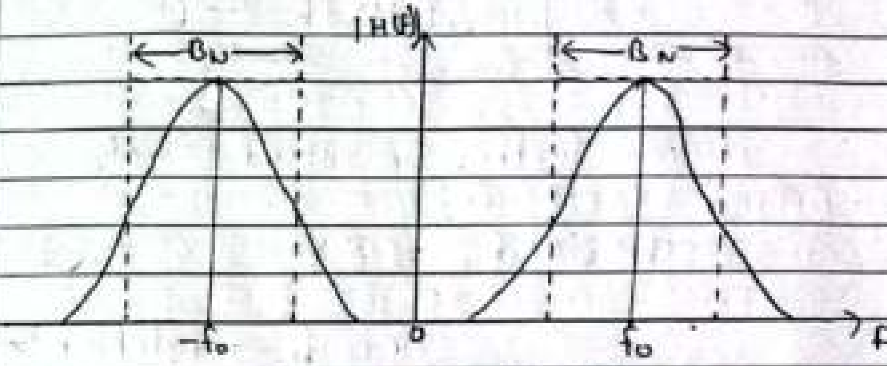


fig: Noise BW of a filter

Let the bandwidth Bn of the ideal filter be adjusted in such a way that the noise output power of the ideal filter is exactly equal to the noise output power of a real R-C filter. Then Bn is called as the noise bw of the real filter.

Q) Determine the noise equivalent bandwidth of RC-LPF and that of ideal LPF of zero frequency response. Also, find o/p noise power of this RC-LPF when input is white noise.

= The noise power at the o/p of a RC LPF is theoretically infinite & is defined only by transfer function of RC filter.

$$P_{NRC} = \frac{N_0}{2} \int_{-\infty}^{\infty} |H_{RC}(f)|^2 \, df$$

$$= N_0 \int_{0}^{\infty} |H_{RC}(f)|^2 \, df \quad \text{—— (i)}$$

It can further be solved to get,

$$P_{NRC} \text{ or } P_{no} = \frac{\pi N_0 f_c}{2} \quad \text{—— (ii)}$$

Here, $f_c \rightarrow$ cutoff filter.

Noise power for ideal LPF is finite & proportional to BW of filter.

$$P_{Nideal} \text{ or } P_{no} = \frac{N_0}{2} \times 2B = N_0 B \quad \text{—— (iii)}$$

To generalize the noise power, we need to define some standard parameter called noise equivalent bandwidth (BW) that can be used to calculate average noise power.

In case of RC filter with an ideal LPF having TF $H(0)$ and Bandwidth equal to B_N. so that the noise power at an output of this idealized filter & RC filter are equal.

$$P_{Nideal} = N_0 B_N \, H^2(0) \quad \text{—— (iv)}$$

$$\& \; P_{Nreal} = N_0 \int_{0}^{\infty} |H(f)|^2 \, df \quad \text{—— (v)}$$

Thus we get,

$$B_N = \frac{\int_{0}^{\infty} |H(f)|^2 df}{|H^2(0)|} \quad \text{—— (vi)}$$

Similarly for Bandpass filter Noise equivalent BW equals,

$$B_N = \frac{\int_{0}^{\infty} |H(f)|^2 \, df}{|H^2(f_c)|^2} \quad \text{—— (vii)}$$

In general equivalent noise bandwidth is expressed as,

$$B_N = \frac{1}{g_a}\int_0^{\infty} |H(f)|^2 \, df \quad \text{— (viii)}$$

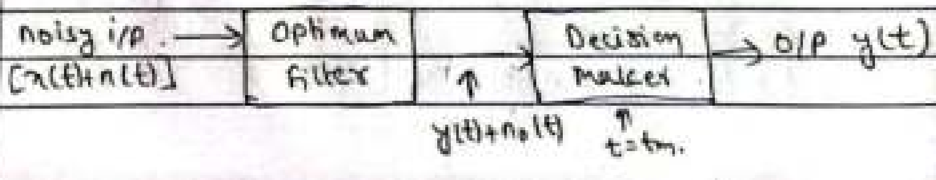
where, $g_a \rightarrow$ max^m available power gain of filter

$\therefore$ In terms of equivalent noise BW the noise power at the o/p of any filter is

$$P_N = N_0 B_N g_a \quad \text{— (ix)}.$$

Optimum detection: Optimum detector of the pulse is a device that has least probability of errors in making decision in favour of 1 or 0.

The basic idea is to pass received signal through the filter that supresses the noise & give sharp peak to the signal at the detection making instance, thus creating max^m SNR at decision making instance.



Matched filter: In optimum filter we consider the noise as generalized Gaussian noise. When this noise is white Gaussian noise, then the optimum filter is known as matched filter.

For white gaussian noise, psd is given as,

$$S_n(f) = \frac{N_0}{2} \quad \text{— (i)}$$

* Realization of matched filter:

Here, f(t) is noisy input signal.

$$f(t) = x(t) + n(t) \quad \text{— (ii)}$$

The output of signal from above figure is,

$$y(t) = f(t) * h(t)$$
$$= \int_{-\infty}^{\infty} f(s)\, h(t-s)\, ds \quad \text{— (iii)}$$

For matched filter, $h(t) = x(t_m - t)$

for time shifted version of $h(t)$;

$$h(t-s) = x(t_m - t + s)$$

$$\therefore y(t) = \int_{-\infty}^{\infty} f(s)\, x(t_m - t + s)\, ds \quad \text{—— (iv)}$$

at $t = t_m$

$$y(t_m) = \int_{-\infty}^{\infty} f(s) \cdot x(s)\, ds \quad \text{—— (v)}$$

This eq^n can be represented in block diagram as;

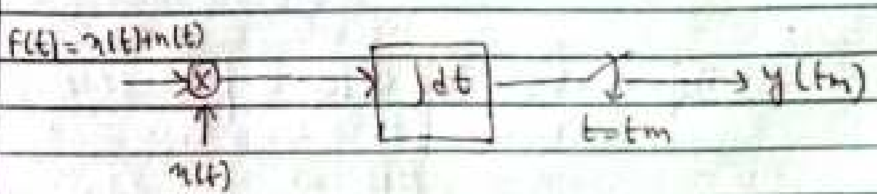


Fig: BD of correlator.

The signal $f(t)$ is multiplied by locally generated replica of i/p signal $x(t)$. Result is integrated & sampled at $t = t_m$. The above arrangement is called time correlator which is synchronous detector.

Matched filter for Rectangular pulse:

- Impulse response of matched filter for rectangular pulse is,

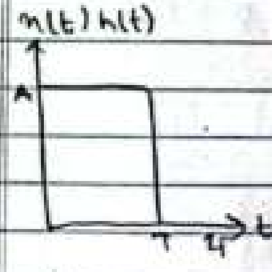
$$H_{mf}(t) = \begin{cases} A & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

It's transfer function is,

$$H_{mf}(f) = \text{sinc}(fT)\, \exp(-j\pi f T)$$

Assuming $AT = 1$

But TF of an ideal LPF is a rectangle.



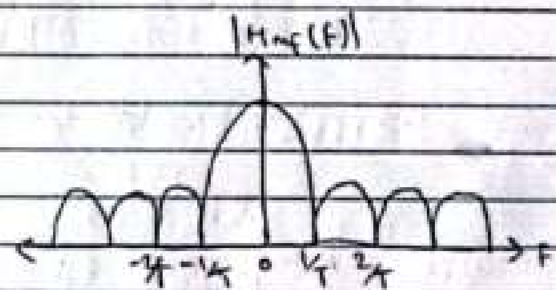


Fig: Amplitude response of filter matched to rectangular pulse

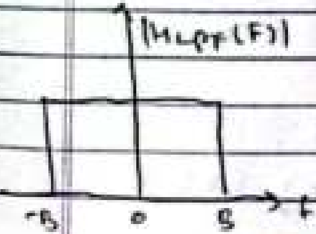


Fig: Amplitude response of ideal LPF approx the MF.

Impulse response of ideal LPF that works as distortionless transmission is.

$$h(t) = \frac{\sin 2\pi B(t-t_0)}{\pi(t-t_0)} = 2B\, \text{sinc}\left[2B(t-t_0)\right]$$

when a rectangular pulse of amplitude A & duration T is passed through an ideal LPF its response is given by.

By convolution,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= 2BA \int_{-T/2}^{T/2} \frac{\sin 2\pi B(t-t_0-\tau)}{2\pi B(t-t_0-\tau)}\, d\tau$$

let, $\lambda = 2\pi B(t-t_0-\tau)$

$$y(t) = \frac{A}{\pi} \int_{-2\pi B(t-t_0-\tau)}^{2\pi B(t-t_0-\tau)} \frac{\sin\lambda}{\lambda}\, d\lambda$$

$y(t)$ is maximum at $t = T/2$ for $BT \geq 1$

$$y(t)\Big|_{max} = \frac{2A}{\lambda}\, \text{sinc}(\pi BT)$$

where, sinc signal is defined by,

$$\text{sinc}(u) = \int_0^u \frac{\sin\lambda}{\lambda}\, d\lambda$$

So if LPF will satisfy first condition of realization aspect of MF (if condition is met)

Noise power at o/p of ideal LPF is $\frac{N_0}{2} \times 2B$

$= BN_0$.

so, max^m SNR at o/p of ideal LPF is,

$$SNR_{LPF} = \frac{\left(\frac{2A}{\pi}\right)^2 S_i^2(\pi BT)}{BN_0}$$

max^m SNR at o/p of MF is,

$$SNR_{MF} = \frac{2A^2T}{N_0}$$

Ratio of SNR: $\frac{SNR_{LPF}}{SNR_{MF}} = \frac{2}{\pi^2 BT} S_i^2(\pi BT)$,

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Impulse response of matched filter,

For white gaussian noise, the power spectral density is given as,

$$S_n(f) = \frac{N_0}{2} \quad \text{—(i)}$$

Now, for the IR of matched filter:

The transfer function of an optimum filter is expressed as,

$$H(f) = K \cdot \frac{X^*(f)}{S_n(f)} e^{-j2\pi fT} \quad \text{—(ii)}$$

Substituting (i) in (ii)

$$H(f) = K \cdot \frac{X^*(f)}{N_0/2} e^{-j2\pi fT}$$

$$= \frac{2K}{N_0} X^*(f) e^{-j2\pi fT} \quad \text{—(iii)}$$

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From the property of fourier transform, we know that,

$$X^*(f) = X(-f)$$

using this property we can write eq^n (iii) as:

$$H(f) = \frac{2K}{N_0} X(-f) e^{-j2\pi fT}$$

Now, the IR can be calculated by taking inverse fourier transfer of above eq^n,

$$h(t) = IFT\,[H(f)]$$

$$= IFT\left[\frac{2K}{N_0} X(-f) e^{-j2\pi fT}\right] \quad \text{—(iv)}$$

The inverse FT of $X(-f)$ is $x(-t)$ and $e^{-j2\pi fT}$ represents time shift of T seconds

Hence, we have:

$$FT[x(-t)] = X(-f)$$

$$\& \quad FT[x(T-t)] = X(-f)\, e^{-j2\pi fT}$$

Using these above properties, the eqn (v) can be written as,

$$h(t) = \frac{2K}{N_0} x(T-t) \quad \text{—— (v)}$$

if $x(t) = x_1(t) - x_2(t)$ then,

$$h(t) = \frac{2K}{N_0} \left[x_1(T-t) - x_2(T-t) \right] \quad \text{—— (vi)}$$

These two eqns (v) & (vi) give reqd ir of matched filter.

Thus this also proves that impulse response of MF is time reverse delayed version of the input signal.

Chapter-7

Noise performance of band-pass (modulated) communication systems. (8 Hours / 14 Marks)

Analysis of DSB-FC, DSB-SC & SSB:

- Detecting gain? : 74Ch [8], 78Bh [1]
- 73Ch [8] - Detection gain of DSB-FC, DSB-SC, SSB / compare.
- Prove that for 100% modulation of DSBAM, the detection gain is less than 1. 74Ch [8]
- 72Ch [8] - Noise performance of DSB AM, DSB-SC, SSB / compare
- Threshold effect in envelop detector for DSB FC. 72 Kar [7]
- CM - DG of SSBSC & compare with DSB-SC : 73sh, 71sh
- Gain parameter in DSB-FC with envelop detector 70 And [5]

Detection gain: Detection gain is the ratio of output signal to noise ratio (SNR_o) to the channel/input signal to noise ratio (SNR_c/SNR_i). It is also known as gain parameter or figure of merit and is written as:

$$\text{Detection gain / Figure of merit } (\gamma) = \frac{SNR_o}{SNR_c} = \frac{SNR_o}{SNR_i}$$

It's value can be less than, greater than or equal to 1 depending on the type of Modulation. The figure of merit should be as high as possible because higher value of figure of merit indicates better noise performance of the receiver.

Noise performance of DSB-AM, SSB-SC & DSB-SC can be compared according to their gain parameter individually. The detection method having high gain Y has better noise performance. So lets find out Y parameter for each.

DSB-AM:

The input to the demodulator is the sum of signal noise.

$$x_i(t) = \{A_c + x_m(t)\}\cos w_c t + n_i(t) \quad \text{--- (1)}$$

Then signal power at i/p is given by

$$P_{Si} = \frac{A_c^2}{2} + \frac{\overline{x_m^2(t)}}{2} \quad \text{--- (a)}$$

Similarly, noise power at input is

$$P_{Ni} = \overline{n_i^2(t)} \quad \text{--- (b)}$$

SNRi

Now, using synchronous detection, the received AM signal is multiplied by $\cos w_c t$ and then passed through LPF.

$$x_i(t)\Big|_{LPF} = x_i(t)\cos w_c t$$

$$= \frac{1}{2} x_m(t) + \frac{1}{2} n_c(t)$$

Now, the O/P signal power is

$$P_{so} = \frac{\overline{x_m^2(t)}}{4} \quad \text{--- (a)}$$

& O/P noise power is,

$$P_{NO} = \frac{\overline{n_c^2(t)}}{4} \quad \text{--- (b)}$$

So, SNR at O/P is,

$$\therefore SNR_o = \frac{P_{so}}{P_{NO}} = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \quad \text{--- (ii)}$$

Similarly from eqn (a) & (b) we can find SNR at input,

$$SNR_i = \frac{P_{si}}{P_{Ni}} = \frac{\frac{A_c^2}{2} + \frac{\overline{x_m^2(t)}}{2}}{\overline{n_i^2(t)}}$$

$$SNR_i = \frac{A_c^2 + \overline{x_m^2(t)}}{2\overline{n_i^2(t)}} \quad \text{—— (iii)}$$

now, the gain parameter is expressed as

$$\gamma = \frac{SNR_o}{SNR_i} = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \Bigg/ \frac{A_c^2 + \overline{x_m^2(t)}}{2\overline{n_i^2(t)}}$$

$$\left\{ \overline{n_c^2(t)} = \overline{n_i^2(t)} \right\}$$

$$\therefore \gamma = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \times \frac{2\overline{n_c^2(t)}}{A_c^2 + \overline{x_m^2(t)}}$$

$$\gamma = \frac{2\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}} \quad \text{—— (iv)}$$

$\therefore$ from above eq^n, it is clear that as γ increases A_c decreases.

since for distortionless transmission,

$$A_c \geq |x_m(t)|_{max}$$

$$\& \ \eta = \frac{\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}}$$

$$\therefore \text{system gain, } \gamma = 2\eta \quad \text{—— (v)}$$

when modulation index is 100% & modulating signal is sinusoidal,

$$\eta = \frac{1}{3}$$

$$\text{or, } \gamma = 2 \times \frac{1}{3} < 1 \quad \text{—— (*)}$$

So, for 100% modulation, γ is less than 1.

The above was for synchronous detection of DSB-AM
now let's study about envelope detection of DSBAM.

The i/p signal can be expressed as

$$x_i(t) = \{A_c + x_m(t)\} \cos w_c t + n_c(t) \cos w_c t + n_s(t) \sin w_c t$$

$$= \{A_c + x_m(t) + n_c(t)\} \cos w_c t + n_s(t) \cos w_c t$$

$x_i(t) = V(t)\cos\{\omega_c t + \phi(t)\}$ —— (vi)

where, $V(t) = \sqrt{(A_c + x_m(t) + n_c(t))^2 + n_s^2(t)}$

$\phi(t) = -\tan^{-1}\left[\dfrac{n_s(t)}{A_c + x_m(t) + n_c(t)}\right]$

Lets assume that noise is very small

$[A_c + x_m(t)] \gg n_i(t)$

or, $[A_c + x_m(t) + n_c(t)]^2 \gg n_s^2(t)$

$\therefore V(t) = A_c + x_m(t) + n_c(t)$

$\&\ \phi(t) = 0$

$\therefore$ The output of the ideal envelope detector is

$V(t) = A_c + x_m(t) + n_c(t)$

LPF filters out the dc component ie. A_c.

so, O/P of LPF is

$x_m(t) + n_c(t)$

$\therefore$ O/P signal power $P_{so} = \overline{x_m^2(t)}$

O/P noise power $P_{no} = \overline{n_c^2(t)}$

$\therefore SNR_o = \dfrac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}}$ —— (vi)

from (vi) & (ii), we can find detection gain,

$\gamma = \dfrac{SNR_o}{SNR_i} = \dfrac{2\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}}$ —— (vii)

Again, we know that,

efficiency, $\eta = \dfrac{\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}}$

$\therefore \gamma = 2\eta$

since Max^m efficiency is $\frac{1}{3}$ for 100% in

DSB-FC AM,

$\therefore \gamma_{max} = 2 \times \frac{1}{3} = \frac{2}{3} < 1.$

$SNR_o < SNR_i \longrightarrow$ degrades the signal.

Detection gain for DSB-SC: The detection method here is always synchronous.

The i/p signal is given by,

$$x_i(t) = x_m(t)\cos w_c t + n_i(t) \text{ —— (viii)}$$

The signal i/p power is,

$$P_{si} = \overline{(x_m(t)\cos w_c t)^2} = \frac{\overline{x_m^2(t)}}{2} \text{ —— (a)}$$

The noise power is

$$P_{Ni} = \overline{n_i^2(t)} \text{ —— (b)}$$

∴ SNR at i/p will be,

$$SNR_i = \frac{\overline{x_m^2(t)}}{2\,\overline{n_i^2(t)}} \text{ —— (ix)}$$

Similarly at the output of demodulator the useful signal component is $\frac{x_m(t)}{2}$

$$\therefore P_{so} = \frac{\overline{x_m^2(t)}}{4} = \frac{P_{si}}{2} \text{ —— (x)}$$

similarly, $n_i(t)$ is multiplied by $\cos w_c t$ during process of detection

$$\therefore n_{det}(t) = n_i(t) \cdot \cos w_c t$$

$$= (n_c(t)\cos w_c t + n_s(t)\sin w_c t)\cos w_c t$$

$$= n_c(t)\cos^2 w_c t + n_s(t)\sin w_c t \cos w_c t$$

$$= \frac{1}{2}\{n_c(t) + n_c(t)\cos 2w_c t + n_s(t)\sin 2w_c t\}$$

All the components except $\frac{n_c(t)}{2}$ is filtered by LPF.

$$\therefore P_{No} = \frac{\overline{n_c^2(t)}}{4} \text{ —— (d)}$$

∴ SNR at o/p will be,

$$SNR_o = \frac{4\,\overline{x_m^2(t)}}{4\,\overline{n_c^2(t)}} = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \text{ —— (x)}$$

∴ gain parameter, γ is expressed as,

$$\gamma = \frac{SNR_o}{SNR_i} = \frac{\overline{x_m^2(t)}}{\overline{n_i^2(t)}} \times \frac{2\overline{n_i^2(t)}}{\overline{x_m^2(t)}} \quad \left\{ \overline{n_o^2(t)} = \overline{n_i^2(t)} \right.$$

$$\therefore \gamma = 2 \quad \text{———} (**)$$

The gain is 2, because the i/p signal consists of 2 sidebands & noise is distributed to both sidebands.

Detection gain for SSB: It also supports only synchronous detection.

The input signal to the demodulator is

$$x_i(t) = x_m(t)\cos\omega_c t + \hat{x}_m(t)\cos\omega_c t + n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t \quad \text{———} (xi)$$

Input signal power is:

$$P_{si} = \frac{\overline{x_m^2(t)}}{2} + \frac{\overline{\hat{x}_m^2(t)}}{2}$$

$$= 2\frac{\overline{x_m^2(t)}}{2} \quad \left\{ \because x_m(t) = \hat{x}_m(t) \right.$$

$$P_{si} = \overline{x_m^2(t)} \quad \text{———} (a)$$

Similarly noise power at input is,

$$P_{Ni} = \overline{n_i^2(t)} \quad \text{———} (b)$$

$$\therefore \text{SNR at input, } SNR_i = \frac{P_{si}}{P_{Ni}} = \frac{\overline{x_m^2(t)}}{\overline{n_i^2(t)}} \quad \text{———} (xii)$$

After synchronous demodulation the output will be,

$$x_o = x_m(t) \text{ or, } \hat{x}_m(t) + n_c(t) \quad \text{———} (xiii)$$

The signal power at o/p is,

$$P_{so} = \overline{x_m^2(t)} \quad \text{———} (x)$$

noise power at output is,

$$P_{No} = \overline{n_c^2(t)} = \overline{n_i^2(t)} \quad \text{———} (y)$$

$$\therefore SNR_o = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \quad \text{———} (xiv)$$

$$\therefore \text{Detection gain, } \gamma = \frac{SNR_o}{SNR_i} = \frac{\overline{x_m^2(t)}}{\overline{n_i^2(t)}} \Big/ \frac{\overline{x_m^2(t)}}{\overline{n_i^2(t)}}$$

$$\therefore \gamma = 1 \quad \text{———} (**)$$

∴ SSB doesn't provide noise improvement in noise performance.

DSBSC has highest value of γ thus it has better noise performance followed by SSB then by DSB-AM which has the worst noise performance out of three.
If we compare DSBSC & SSB, γ is better in DSBSC thus noise performance is better but DSBSC has input noise power twice than in case of SSB. Thus DSB-SC & SSB are identical in noise improvement point of view.

FM Detector:

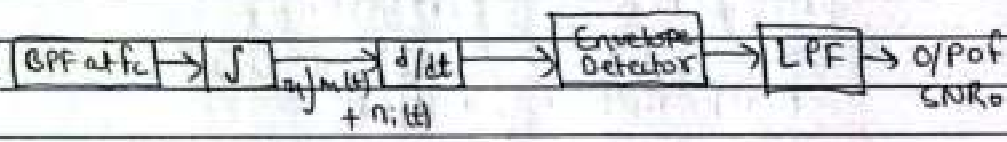
- Capture effect: 78Bh[1], 70Asd[2]
- Gain parameter / Figure of merit of non coherent FM detector: VVI: 73Sh, 71Ch, 74As [6 to 8 M]
- FM threshold effect: 74As[2] 71Ch[4], 69Ch[4], 70Ch[3]
- Comparision of AM & FM in terms of power efficiency, Bandwidth efficiency & system complexity: 70Asd[3]
- Gain parameter of FM system with limiter discrimination method: 78Bh[7]
- Remedy of threshold effect: 70Ch[3], 67Ch[3]

Capture effect:

If there is interference in the form of other stations within compatible carrier frequencies, the FM station will enhance one of the sections, depending upon the intensities of the signal at the receivers. If the level of interference is low compared to the desired signal then FM captures the desired signal & remains locked. But if the level of interference becomes greater than desired signal then the FM captures the interference. This effect of capturing the strong station is called the capture effect.

Gain Parameter of FM detector:

Consider the standard Limiter discriminator FM demodulation.



The input signal of Demodulator is;

$$x_i(t) = x_{FM}(t) + n_i(t)$$

$$x_i(t) = A_c \cos\left[\omega_c t + 2\pi k_f \int_0^t x_m(t)\,dt\right] + n_i(t) \quad \text{—— (i)}$$

so, input signal to noise power is;

$$P_{si} = \frac{A_c^2}{2}$$

similarly $P_{ni} = \overline{n_i^2(t)} = 2B \times N_0$ —— (ii)

where, B = BW of message signal
N_0 = PSDF of white noise

$$N_c(f) = N_s(f) = N_0$$

$$\therefore SNR_i = \frac{P_{si}}{P_{ni}} = \frac{A_c^2}{2} \Big/ 2B \times N_0 = \frac{A_c^2}{4BN_0} \quad \text{—— (iii)}$$

To find output SNR, let's first consider the case when i/p noise is absent. ie,

$$n_c(t) = n_s(t) = 0 \;;\; n_i(t) = 0$$

Then, eq^n (i) will be

$$x_i(t) = A_c \cos(\omega_c t + \phi(t)) \quad \text{—— (iv)}$$

where, $\phi(t) = \left[2\pi k_f \int_0^t x_m(t)\,dt\right]$

output of discriminator is:

$$\frac{dx_i(t)}{dt} = A_c\left(\omega_c + \frac{d\phi}{dt}\right) \sin(\omega_c t + \phi(t))$$

Envelope detector gives envelope of above signal at output and LPF removes dc components centered at ω_c.

so,

O/P of LPF is $A_c \cdot \frac{d\phi(t)}{dt} = A_c\, 2\pi k_f x_m(t)$

$$\therefore P_{so} = 4\pi^2 A_c^2 k_f^2 \overline{x_m^2(t)} \quad \text{—— (v)}$$

Now, assuming message signal is absent in input then $x_m(t) = 0$. so input is only unmodulated carrier and noise then,

$$x_i(t) = A_c \cos\omega_c t + n_i(t)$$
$$= \{A_c + n_c(t)\}\cos\omega_c t + n_s(t)\sin\omega_c t$$

let, $R_n(t) = \sqrt{\{A_c + n_c(t)\}^2 + (n_s(t))^2}$

$$\theta(t) = -\tan^{-1}\left[\frac{n_s(t)}{A_c + n_c(t)}\right]$$

$$\therefore n_i(t) = R_n(t)\cos(w_c t + \theta(t)) \text{ —— (1)}$$

we can assume that $R_n(t)$ will be removed by limiter discriminator.

Assume $A_c >> n_c(t)$

then, $\theta(t) = -\tan^{-1}\left[\frac{n_s(t)}{A_c}\right] \approx \frac{n_s(t)}{A_c}$

so, input to the discriminator is

$$\therefore n_{dis}(t) = \cos\left(w_c t + \frac{n_s(t)}{A_c}\right) \text{ —— (2)}$$

& o/p to the discriminator is

$$\frac{d\, n_{dis}(t)}{dt} = \left\{w_c + \frac{1}{A_c}\frac{dn_s(t)}{dt}\right\}\sin\left(w_c t + \frac{n_s(t)}{A_c}\right)$$

The signal is again passed through envelope detector & LPF so o/p of LPF is:

$$n_o(t) = \frac{1}{A_c}\cdot\frac{dn_s(t)}{dt}$$

But there is no mathematical representation of $n_s(t)$ so we use spectrum method i.e PSDF approach.

$$\begin{matrix} n_s(t) \\ S_{ns}(f) \end{matrix} \longrightarrow \boxed{\begin{matrix}\text{Differentiator} \\ H(f)\end{matrix}} \longrightarrow \begin{matrix} n_o(t) \\ S_{no}(f) \end{matrix}$$

The PSDF of the signal at the o/p of the differentiator is

$$S_{no}(f) = S_{ns}(f)\,|H(f)|^2$$

Transfer function of differentiator

$$H(f) = j\,2\pi f$$

$$\text{so, } S_{no}(f) = N_o\,|j2\pi f|^2 = 4\pi^2 N_o f^2$$

This shows that the noise PSD at the o/p of the discriminator is proportional to f^2

$\therefore$ Noise power within message BW is,

$$P_{N_0} = \frac{1}{A_c^2}\int_{-B}^{B} S_{no}(f)\,df = \frac{1}{A_c^2} \times 4\pi^2 N_0 \int_{-B}^{B} f^2\,df$$

$$= \frac{8\pi^2 N_0 B^3}{3A_c^2}$$

$$\text{So, } SNR_o = \frac{P_{so}}{P_{No}} = 4\pi^2 A_c^2 k_f^2 \overline{x_m^2(t)} \Big/ \frac{8\pi^2 N_0 B^3}{3A_c^2}$$

$$= 4\pi^2 A_c^2 k_f^2 \overline{x_m^2(t)} \times \frac{3A_c^2}{8\pi^2 N_0 B^3}$$

$$= \frac{3A_c^4 k_f^2 \overline{x_m^2(t)}}{2 N_0 B^3}$$

$$\therefore \text{Detection gain}(\gamma) = \frac{SNR_o}{SNR_i}$$

$$= \frac{3A_c^4 k_f^2 \overline{x_m^2(t)}}{2N_0 B^3} \Big/ \frac{A_c^2}{4BN_0}$$

$$= \frac{3A_c^4 k_f^2 \overline{x_m^2(t)}}{2N_0 B^3} \times \frac{4BN_0}{A_c^2}$$

$$\therefore \boxed{\gamma = \frac{6A_c^2 k_f^2 \overline{x_m^2(t)}}{B^2}}$$

FM threshold effect & its Remedy:

The detection gain (γ) in FM is proportional to β^2. So for given SNR, the rise in β will increase γ or SNR_o. But with increase in β will considerably increase the system BW at the Carson's rule rate;

$$B_{FM} = 2(\beta+1) f_m \quad \text{—(i)}$$

Increase in BW will subsequently increase i/p noise power and decrease SNR_i.

As assumed earlier the carrier amplitude A_c is greater than noise so this assumption will no longer remain valid. If we increase β indiscriminately i.e. at some point of increase of β, SNR_i will be so low that the signal as in case of large noise AM signal will be mutilated by the noise causing reception impossible.

In other words there exists threshold effect at less than the threshold level because of which FM system will no more detect message signal.

The threshold level for FM is 10 dB.

Before reaching the threshold level the FM system

produces noise clicks. As SNRi is further decreased then individual clicks are produced which are converted to crackling sound; with further reduced in SNRi & finally threshold level is reached the receiver breaks ie, received signal is completely mutilated by the noise.
FM system owing threshold effect also exhibit locking property. If the SNRi is below the threshold level the receiver locks to the noise.

Remedy:

(i) Using FMFB demodulator: This demodulator has negative feedback & reduces noise threshold. Reduces THL by 5-7dB

(ii) Using PLL Demodulator: It reduces threshold level by 2 to 3 dB.

(iii) Use of Preemphasis & De-emphasis network: It improves system performance by 6dB.

Comparision of AM & FM: There are 3 criteria for comparision:

(i) Bandwidth efficiency: SSB SC has best performance because in this case channel bandwidth is equal to message

bandwidth. Therefore SSB-SB is widely used in applications where the BW is major constraint. such as microwave link, satellite communication, point-to-point communication.
The application which requires near to DC transmission then VSB-SC is used such as TV broadcasting.

(ii) Power efficiency: DSB-AM is least efficient & FM has the highest efficiency due to its high level of noise immunity.
Therefore, in power critical applications such as space vehicle communications, FM broadcasting. FM is preferred over other modulation techniques.

(iii) System complexity: The system complexity plays vital role when number of receiver is tremendous. Since DSB-AM is least complicated it is used for commercial radio broadcasting. In terms of system complexity, SSBSC is slightly more complex than DSB-SC but since DSB-SC is less effective in terms of Bandwidth it is not used mostly in practice

Error probability:

Q) Derive the expression for error probability in binary communication. 70ch[7]

Q) Derive the exp^n for error probability of binary PAM: 72kar[5] and extend it to M-ary: 69ch[6+2]

Q) Error probability in coherent ASK: 72ch[4], 70Asd[4], 69ch[6], 74As[6]

Q) Error probability in binary ASK & extend it to M-ARY: 71ch[6], 73sh, 69ch[6+2],

Q) Error probability of coherent PSK 72h[4], 76ch[6]

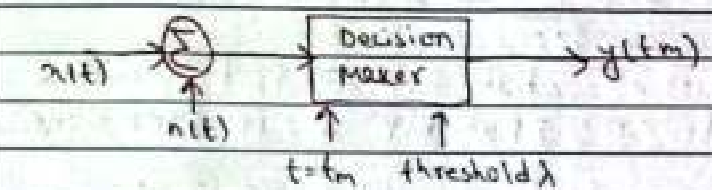
Q) Show ASK requires double the average signal power than PSK for same error probability 72ch[3]

Q) Error probability for M-ary system: 78Bh[6] 71sh[5]

Error probability for binary PAM system & extending it to M-ary system:

Performance of digital communication system is evaluated in terms of error probability. The error probability depends on input SNR (SNRi) and complexity of the decoder.

Let us consider a base-band data communication where no carrier modulation is employed. The most frequently used is the PAM. So we start with analysis of PAM.



The signal output at the decision maker is,

$$y(t_m) = A_m + n_o(t_m) \quad \text{—— (1)}$$

Where, $A_m = +A$ if $B_m = 1$ (Bit 1 is transmitted)
$= -A$ if $B_m = 0$ (Bit 0 is transmitted)

In absence of noise the decision making device compares $y(t)$ against the threshold level λ (which is 0V)

* If $y(t_m) > 0$ then it is decided that bit 1 is transmitted.
* If $y(t_m) < 0$ then decision is favour of 0 bit.

The presence of noise & its strength may introduce errors in the decision making. The error is introduced when the decision is in the favour of 1 & actually 0 is transmitted & vice-versa.

It means, error is introduced when

$y(t_m) > 0$ but bit 0 is transmitted
$y(t_m) < 0$ but bit 1 is transmitted.

Now let us assume noise is present in system. consider bit sequence 0 & 1 are equiprobable and statistically independent. Thus total probability will be sum of each error probability.

Here, $P(b_m = 0) = P(b_m = 1) = \frac{1}{2}$

and, $P[y(t_m) > 0 / b_m = 0]$ is the probability of $y(t_m) > 0$ when binary 0 is transmitted.

similarly, $P[y(t_m) < 0 / b_m = 1]$ is the probability of $y(t_m) < 0$ when binary 1 is transmitted.

Thus, Total probability is:

$$P_e = P[y(t_m) > 0 \cap b_m = 0] + P[y(t_m) < 0 \cap b_m = 1]$$

$$= P[y(t_m) > 0 / b_m = 0] * P(b_m = 0) + P[y(t_m) < 0 / b_m = 1] * P(b_m = 1)$$

$$P_e = \frac{1}{2} \left\{ P[y(t_m) > 0 / b_m = 0] + P[y(t_m) < 0 / b_m = 1] \right\}$$

In terms of noise, the above expression can be written as,

$$P_e = \frac{1}{2} \left\{ P[n_o(t_m) > A] + P[n_o(t_m) < A] \right\}$$

$$P_e = \frac{1}{2} \left\{ P[\,|n_o(t_m)| > A\,] \right\}$$

As input noise $n(t)$ is zero

mean Gaussian process with variance No. so, output noise $n(t)$ is also Gaussian process with variance No.

Therefore,

$$P_e = \frac{1}{2} \int_{x>A}^{\infty} \frac{1}{2\pi N_0} \exp\left(\frac{-x^2}{2N_0}\right) dx$$

let, $z^2 = \frac{x^2}{2\sqrt{N_0}}$

$x = z\sqrt{2N_0}$

$$\therefore P_e = 2 \times \frac{1}{2} \int_{A/\sqrt{2N_0}}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left(-z^2\right) dz \rightarrow \frac{1}{2} \int_{A/\sqrt{2N_0}}^{\infty} \frac{2}{\sqrt{\pi}} \exp(-z^2) dz$$

$$= \frac{1}{2} \times \text{erfc}\left(\frac{A}{\sqrt{2N_0}}\right)$$

$$\therefore P_e = \frac{1}{2} \times \text{erfc}(u)$$

Where, $\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} \exp(-z^2) dz.$

In Mary,

$$P_e = \frac{M-1}{M} \text{erfc}\left(\frac{A}{\sqrt{2N_0}}\right)$$

Remaining in copy - II

Error probability in coherent detection of ASK:

In binary ASK, the i/p signals can be represented as

$$S_1(t) = 0 \quad \text{—— (i)}$$
$$S_2(t) = A\cos w_c t \quad \text{—— (ii)}$$

subtracting (i) from (ii)

$$S_2(t) - S_1(t) = A\cos w_c t \quad \text{—— (iii)}$$

The output signal for each case is

$$S_{o1}(kT_b) = \int_0^{T_b} S_1(t)\,[S_2(t) - S_1(t)]\,dt = 0 \quad \because S_1(t) = 0$$

$$S_{o2}(kT_b) = \int_0^{T_b} S_2(t)\,[S_2(t) - S_1(t)]\,dt$$

$$= \int_0^{T_b} (A\cos w_c t)\cdot(A\cos w_c t)\,dt$$

$$= \int_0^{T_b} A^2\cos^2 w_c t\,dt$$

$$\therefore\ S_{o2}(kT_b) = \frac{A^2}{2}T_b \quad \text{—— (a)}$$

The max^m value of SNR, γ will be,

$$\gamma_{max}^2 = \int_{-\infty}^{\infty} \frac{[S_2(t) - S_1(t)]^2}{N_0/2}\,dt$$

$$= \frac{2}{N_0}\int_0^{T_b} A^2\cos^2 w_c t\,dt$$

$$= \frac{2}{N_0}\,\frac{A^2 T_b}{2}$$

$$\gamma_{max}^2 = \frac{A^2 T_b}{N_0} \quad \text{—— (b)}$$

$\therefore$ The error probability will be

$$P_{emin} = \text{erfc}\left(\frac{\gamma_{max}}{2}\right)$$

$$\text{or,}\quad \boxed{P_e = \text{erfc}\left(\sqrt{\frac{A^2 T_b}{4N_0}}\right)} \quad \text{—— (iv)}$$

Since S1 & S2 are equiprobable the average signal power will be

$$S_{av} = \frac{A^2}{4} \quad \text{--- (v)}$$

so, in terms of average signal power,

$$P_e = erfc\left(\sqrt{\frac{S_{av}T_b}{N_0}}\right) = erfc\sqrt{\frac{E_{av}}{N_0}} \quad \text{--- (vi)}$$

and the average signal energy per bit Information is,

$$E_{av} = S_{av} \cdot T_b \quad \text{--- (vii)}$$

Coherent detection of PSK:

The demodulation used in PSK is always coherent.

The input signal can be expressed as:

$$S_1(t) = -A\cos w_c t \quad \text{--- (viii)}$$
$$S_2(t) = A\cos w_c t \quad \text{--- (ix)}$$

and their respective output signal will be,

$$S_{o1}(KT_b) = -A^2 T_b \quad \text{--- (x)}$$

$$S_{o2}(KT_b) = A^2 T_b \quad \text{--- (xi)}$$

The maximum SNR will be

$$\gamma^2_{max} = \int_{-\infty}^{\infty} \frac{[S_2(t) - S_1(t)]^2}{N_0/2} dt$$

$$= \int \frac{(2A\cos w_c t)^2}{N_0/2} dt$$

$$= \frac{4}{N_0}\int 2A^2\cos^2 w_c t \, dt$$

$$\gamma^2_{max} = \frac{4}{N_0} A^2 T_b \quad \text{--- (a)}$$

$$\text{or, } \gamma/y = 2A\sqrt{\frac{T_b}{N_0}} \quad \text{--- (b)}$$

and, average signal power is, $S_{av} = \frac{A^2}{2}$ --- (c)

Now, error probability in terms of average energy per bit is;

$$P_{e\,min} = erfc\left(\frac{\gamma_{max}}{2}\right)$$

$$= \text{erfc}\ \frac{2A}{2}\sqrt{\frac{T_b}{N_o}}$$

$$P_e = \text{erfc}\sqrt{\frac{A^2T_b}{N_o}} \quad \text{—— (xii)}$$

since $S_{av} = \frac{A^2}{2}$

$$P_e = \text{erfc}\sqrt{\frac{2S_{av}T_b}{N_o}} \quad \text{—— (xiii)}$$

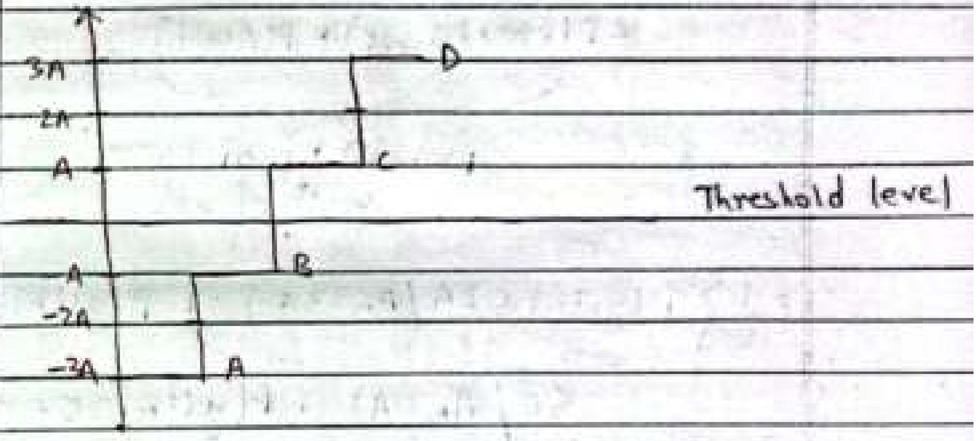
similarly, $E_{av} = S_{av}.T_b$.

$$\therefore P_e = \text{erfc}\sqrt{\frac{2E_{av}}{N_o}} \quad \text{—— (xiv)}$$

Now, comparing (vi) & (xiv); we can see that ASK requires double the average signal power than PSK for same error probability.

Error probability of M-ary system:

Let us consider the case of M=4.



Decoding algorithm is,

$y(t_m) \leq -2A \longrightarrow$ symbol 'A'
$-2A < y(t_m) \leq 0 \longrightarrow$ symbol 'B'
$0 < y(t_m) \leq 2A \longrightarrow$ symbol 'C'
$y(t_m) > 2A \longrightarrow$ symbol 'D'

Here, Symbol A, B, C and D are equiprobable

So, $P(A_{sent}) = P(B_{sent}) = P(C_{sent}) = P(D_{sent}) = 1/4$

Now, probability of error is

$P_e = P(\text{error}/D\text{sent}) * P(D\text{sent}) + P(\text{error}/C\text{sent})$

$* P(C\text{sent}) + P(\text{error}/B\text{sent}) * P(B\text{sent})$

$$+ \underbrace{P(\text{error}/A\text{sent}) * P(A\text{sent})}_{\text{probability of error with Asent.}}$$

$= \frac{1}{4}\{ P(y(t_m) \leq 2A / A_m = 3A) + P(y(t_m) > 2A$ or

$\leq 0 / A_m = A) + P(y(t_m) > 0$ or $\leq -2A /$

$A_m = -A) + P(y(t_m) > -2A / A_m = -3A) \}$

Since we know that,

$y(t_m) = A_m + n_o(t_m)$ so the

expression can be written in terms of noise as,

$P_e = \frac{1}{4}\{ P(n_o(t_m) < -A) + P(n_o(t_m) > A$ or

$n_o(t_m) < -A) + P(n_o(t_m) > A$ or $n_o(t_m) < -A +$

$P(n_o(t_m) > A) \}$

$= \frac{1}{4}\{ P(|n_o(t_m)| > A) + P(|n_o(t_m)| > A)$

$+ P(|n_o(t_m)| > A) \}$

from 1st to 4th term,

$= \frac{3}{4}\{ P(|n_o(t_m)| > A) \}$ —— (1)

Assuming $n_o(t)$ follow gaussian distribution

$$P_e = \frac{3}{4} \times 2 \int_{x=A}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \left(\frac{-x^2}{2N_0} \right) dx$$

$$= \frac{3}{4} \times 2 \int_{A/\sqrt{2N_0}}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-\tau^2)\, d\tau$$

$$= \frac{3}{4} \operatorname{erfc}\left(\frac{A}{\sqrt{2N_0}} \right)$$

In general; $P_e = \frac{(M-1)}{M} \operatorname{erfc}\left(\frac{A}{\sqrt{2N_0}} \right)$ —— (ii)

Chapter: 8

Error control coding techniques (4H/7Marks)

- Define Hamming distance & Hamming code 74ch[2],
- 74ch[6] Explain operation of a $\frac{1}{3}$ convolution encoder.
- Define Hamming weight & Hamming distance with examples. 73sh[3]
- Explain convolution coder with suitable example 72kar[6].
- Define Hamming weight & hamming distance 71ch[2], 70Asd[2], 74AS[2], 72ch[2]
- Importance of hamming distance and hamming weight in coding theory. 71sh[2]
- Explain with example the syndrome decoding method in linear block coding. 71sh[5]
- What is binary cyclic code. 70Asd[1]
- SN: Syndrome calculation in linear systematic block code: 69ch[4]
- Why convolution coder are better suited than block coder. 73ch[2]

Hamming code: Hamming code is a set of error-correction codes that can be used to detect and correct the errors that can occur when the data is moved or stored from the sender to the receiver.

Hamming distance: Let us consider two code words having the same number of elements. The Hamming distance or simply distance between the two code words is defined as the numbers of locations in which their respective elements differ.

Eg: let us consider two code words given below

code word 1: 1 1 0 1 0 1 0 0

codeword 2: 0 1 0 1 1 1 1 0

Hamming dist = 3

The key significance of Hamming distance is that if two codewords have a hamming distance of d between them, then it would take d single bit errors to turn one of them into the other.

Hamming weight: Hamming weight of a code word x is defined as the number of non-zero elements in the code word. Hamming weight of a code word is the distance between that code word and an all-zero code vector (A code having all elements equal to zero).

Eg: let $x = 11010100$.

As number of non zero elements in above code word is 4, the Hamming weight $W(x) = 4$.

It is important as it gives numeric values. It is mostly used in determining path lengths between nodes & error detection.

Syndrome decoding for Block codes:

For the detector to detect or correct errors by sorting bit pattern of all possible valid code words more memory is required for large block lengths. To avoid this problem syndrome decoding is used. For this purpose, parity check matrix H is used.

For an (n,K) linear block code, this parity check matrix H is of size $(n-K)\times n$.

H is given as,

$$H = [P^T : I_{n-K}]_{(n-K)\times n}$$

where,
$P^T \rightarrow$ transpose of P matrix
$I_{n-K} \rightarrow$ identity matrix

Similarly, Transpose of H is given by

$$H^T = \begin{bmatrix} P \\ \cdots \\ I_{n-K} \end{bmatrix}_{n\times(n-K)} \quad \text{—(i)}$$

where, $P \rightarrow$ coefficient matrix.

The transpose of parity check matrix H^T exhibits a very important property i.e,

$$XH^T = (0,0,\ldots 0) \quad \text{—(ii)}$$

This means that the product of any code word X and the transpose of the parity check matrix will always be 0.

We shall use this property for the detection of errors in the received code words as under:

At the receiver, we have

If, $YH^T = (0, 0, \ldots, 0)$, then $Y = X$ ie no error

But if, $YH^T \neq (0, 0, \ldots, 0)$ then $Y \neq X$

ie, error exists in the received code word.

The syndrome is defined as the non-zero output of the product YH^T. Thus, the non-zero syndrome represents the presence of errors in the received code word. The syndrome is represented by S & is mathematically given as,

$$S = YH^T \qquad \text{—(ii)}$$

All zero elements of syndrome represent there is no error, & a non-zero value

of an element in syndrome represents the presence of error. But sometimes, even if all the syndrome elements have zero value, the error exists. This is shown in figure below:

S = 0

Either, $Y = X$, ie there is no error in the received signal

Or, $Y \neq X$, but Y is some other valid codeword (other than X)

Binary cyclic code: It is the special case of linear block codes in which encoding and syndrome calculations can be easily implemented using simple shift registers.

A (n,K) linear code C is called cyclic code if cyclic shifts of the code are also code vectors of C. Thus in polynomial form if $C_1(x)$ is the code vector given by,

$$C_1(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_{n-1} x^{n-1} \qquad \text{—(i)}$$

Then, the shifted vector,

$$c_2(x) = c_{n-1} + c_0 x + c_1 x^2 + c_2 x^3 + \dots + c_{n-2} x^{n-1} \quad \text{— (ii)}$$

is also called the codevector.
Here all addition & multiplication are modulo-2 operation.

Convolution coding: The fundamental hardware unit for the convolutional encoding is a taped shift register with $(L+1)$ stages, as shown in figure(a). Here, $g_0, g_1, \dots$ etc are the tap gains which are nothing but binary digits 0s or 1s. A tap gain of 0 represents an open circuit whereas a tap gain of 1 represents a short circuit.

The message bits enter one by one into the tapped shift register, which are then combined by mod-2 addition to form the encoded bit x.
Therefore, we have

$$x = m_L g_L \oplus \dots \dots \oplus m_1 g_1 \oplus m_0 g_0 \quad \text{— (i)}$$

$$\text{or, } x = \sum_{i=0}^{L} m_i g_i \quad (\text{mod-2 addition})$$

The name convolutional encoding comes from the fact that eqn (i) has a form of binary convolution which is analogous to the convolutional integral. The message bit m_0 in figure (a) represents the current input whereas the bits m_1 to m_L represent the past input or state of the shift register. From eqn (i) it is clear that a message bit x depends on the current message bit m_0 and the state of the shift register defined by the previous L message bits.

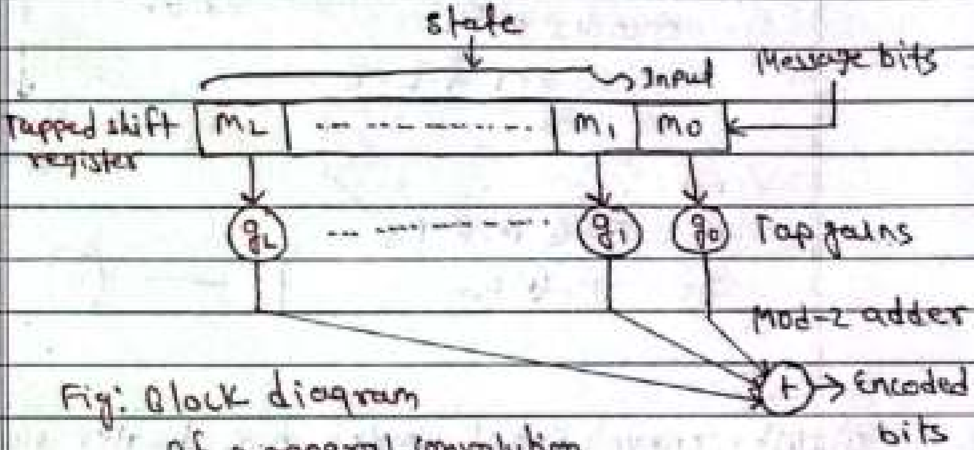


Fig: Block diagram of a general convolution encoder

An example of practical convolution encoder with $n=2$, $K=1$ & $L=2$ is shown in figure below:

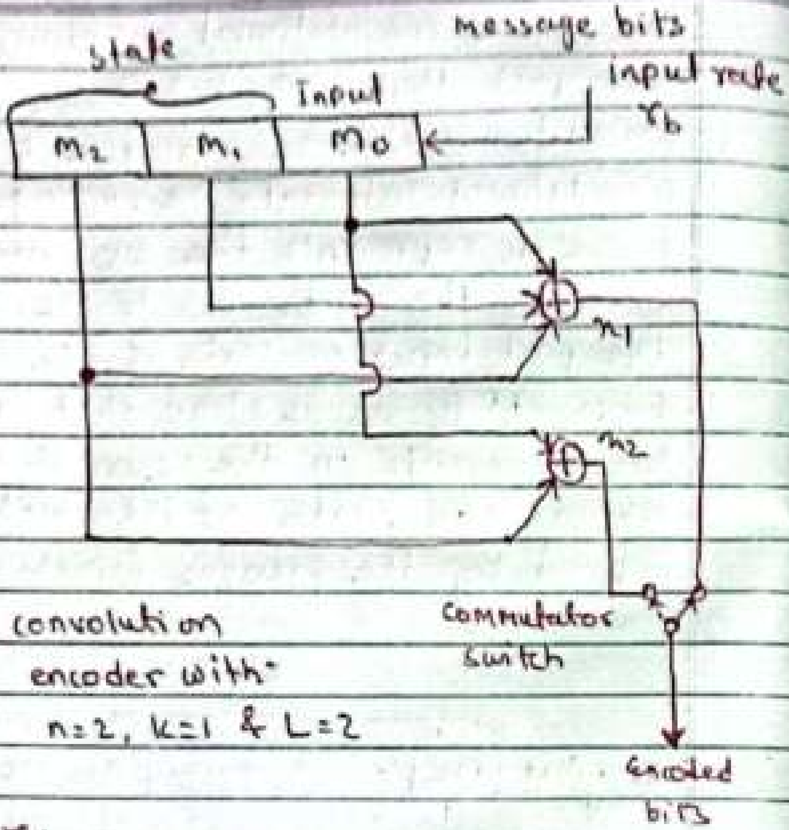


Fig: convolution encoder with $n=2$, $k=1$ & $L=2$

Here,

$$\left.\begin{aligned} x_1 &= m_0 \oplus m_1 \oplus m_2 \\ x_2 &= m_0 \oplus m_2 \end{aligned}\right\} \text{—— (ii)}$$

Q) Why convolutional coders are better suited than Block coder.

= Because of following reasons:

(i) They are preferred in non-systematic form which do not have efficient structure

ie position of parity or check bits is not defined

(ii) They are suitable for detecting & preventing burst errors.

(iii) The hardware component is simpler and encoding process is easy.

Chapter - 1
Numericals:

78 Bhadra: Q 1:

given:

Source, $S = \{A, B, C, D\}$

$P(A) = \frac{1}{2} = 0.5$ | $P(C) = \frac{1}{8} = 0.125$

$P(B) = \frac{1}{4} = 0.25$ | $P(D) = \frac{1}{8} = 0.125$

Encode using: BCD, Shannon-Fano, Huffman | Evaluate coding efficiency.

1) Using BCD coding: BCD is fixed length coding method that usually uses 4 to 8 bits depending on requirement.

Symbol x_i	Probabilities $P(x_i)$	BCD code	n_i fixed length
A	0.5	1100	4
B	0.25	1101	4
C	0.125	1110	4
D	0.125	1111	4

$$\text{Entropy}(H_x) = 0.5 \log_2\left(\frac{1}{0.5}\right) + 0.25 \log_2\left(\frac{1}{0.25}\right) + 2 \times 0.125 \log_2\left(\frac{1}{0.125}\right)$$

$H(x) = 0.5 + 0.5 + 2 \times 0.375$

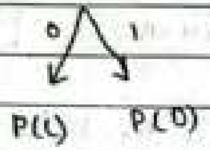
$= 1.75$ bits/symbol

$L_{BCD} = 4 \times (1) = 4$ bits/symbol

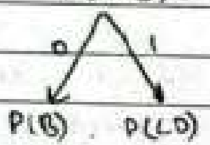
$\eta = \frac{H(x)}{L_{BCD}} \times 100\% = 43.75\%$

2) ~~Shannon Fano coding~~ Huffman:

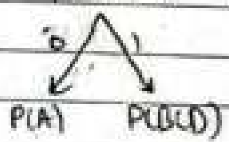
$P(CD) = 0.125 + 0.125 = 0.25$



$P(BCD) = 0.25 + 0.25 = 0.5$



$P(ABCD) = 0.5 + 0.5 = 1$



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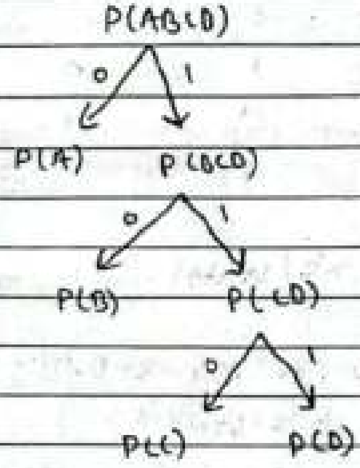
Now, making Huffman tree:



$P(A) = 0$

$P(B) = 10$

$P(C) = 110$

$P(D) = 111$

$H(x) = 1.75$ bits/symbol

$L_H = 1 \times 0.5 + 2 \times 0.25 + 3 \times 0.125 + 3 \times 0.125$

$= 0.5 + 0.5 + 0.375 + 0.375$

$= 1.75$ bits/symbol.

Now, $\eta_H = \frac{1.75}{1.75} \times 100\% = 100\%$

3) Shannon Fano:

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x_i	$P(x_i)$	step 1	step 2	step 3	code
A	0.5	0			0
B	0.25	1	0		10
C	0.125	1	1	0	110
D	0.125	1	1	1	111

$H(x) = 1.75$ bits/symbol

$L_{SF} = 1\times0.5 + 2\times0.25 + 3\times0.125 + 3\times0.125$
$= 1.75$ bits/symbol.

efficiency, $\eta = \frac{1.75}{1.75}\times100\% = 100\%$

$\therefore$ Variable length coding like Shannon-Fano & Huffman coding have greater efficiency than compared to Fixed length coding like BCD coding.

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76th Q no. 1

$P = \{0.36, 0.18, 0.18, 0.12, 0.09, 0.07\}$

Calculate coding efficiency using Shannon fano & huffman coding.

1) Shannon fano.

x_i	P_{xi}	step 1	step 2	step 3	step 4	code
x_1	0.36	0	0			00
x_2	0.18	0	1			01
x_3	0.18	1	0			10
x_4	0.12	1	1	0		110
x_5	0.09	1	1	1	0	1110
x_6	0.07	1	1	1	1	1111

$H(x) = 0.36 \log_2\left(\frac{1}{0.36}\right) + 2\times0.18 \log_2\left(\frac{1}{0.18}\right) + 0.12 \log_2\left(\frac{1}{0.12}\right)$
$+ 0.09 \log_2\left(\frac{1}{0.09}\right) + 0.07 \log_2\left(\frac{1}{0.07}\right)$

$= 0.530 + 0.445 + 0.445 + 0.367 + 0.313 + 0.268$
$= 2.368$ bits/symbol.

$L_{SF} = 2\times0.36 + 2\times0.18 + 2\times0.18 + 3\times0.12 + 4\times0.09 + 4\times0.07 = 0.72 + 0.36 + 0.36 + 0.36 + 0.36 + 0.28$
$= 2.44$ bits/symbol

$\eta_{SF} = \frac{2.368}{2.44} \times 100\% = 0.97 \times 100\% = 97\%$

Huffman coding:

$P(x_5 x_6) = 0.09 + 0.07 = 0.16$

$P(x_5)$ 0.09 $P(x_6)$ 0.07

$P(x_4 x_5 x_6) = 0.16 + 0.12 = 0.28$

$P(x_5 x_6)$ 0.16 $P(x_4)$ 0.12

$P(x_2 x_3) = 0.18 + 0.18 = 0.36$

$P(x_2)$ 0.18 $P(x_3)$ 0.18

$P(x_2 x_3 x_4 x_5 x_6) = 0.36 + 0.28 = 0.64$

$P(x_2 x_3)$ 0.36 $P(x_4 x_5 x_6)$ 0.28

$P(x_1 x_2 x_3 x_4 x_5 x_6) = 0.64 + 0.36 = 1$

$P(x_2 x_3 x_4 x_5 x_6)$ 0.64 $P(x_1)$ 0.36

Creating Huffman tree.

$P(x_1 x_2 x_3 x_4 x_5 x_6)$

$P(x_2 x_3 x_4 x_5 x_6)$ $P(x_1)$

$P(x_2 x_3)$ $P(x_4 x_5 x_6)$

$P(x_2)$ $P(x_3)$ $P(x_5 x_6)$ $P(x_4)$

$P(x_5)$ $P(x_6)$

$P(x_1) = 1$
$P(x_2) = 000$
$P(x_3) = 001$
$P(x_4) = 011$
$P(x_5) = 0100$
$P(x_6) = 0101$

$L_H = 1 \times 0.36 + 3 \times 0.18 + 3 \times 0.18 + 3 \times 0.12 + 4 \times 0.09 + 4 \times 0.07$

$= 0.36 + 0.54 + 0.54 + 0.36 + 0.36 + 0.28$

$= 2.44$ bits / symbol

$\therefore \eta_H = \frac{2.368}{2.44} \times 100\% = 97\%$

Ques: 5

A_0	0.4
A_1	0.3
A_2	0.15
A_3	0.1
A_4	0.05

$$H(x) = 0.4\log_2\left(\frac{1}{0.4}\right) + 0.3\log_2\left(\frac{1}{0.3}\right) + 0.15\log_2\left(\frac{1}{0.15}\right) + 0.1\log_2\left(\frac{1}{0.1}\right) + 0.05\log_2\left(\frac{1}{0.05}\right) = 0.529 + 0.521 + 0.41 + 0.332 + 0.216 = 2.008 \text{ bits/symbol}$$

i) Huffman

$P(A_3A_4) = 0.1 + 0.05 = 0.15$

$P(A_3) \xleftarrow{0} P(A_3A_4) \xrightarrow{1} P(A_4)$

$P(A_3)$ 0.1, $P(A_4)$ 0.05

$P(A_{234}) = 0.15 + 0.15 = 0.3$

$P(A_2) \xleftarrow{0} P(A_{234}) \xrightarrow{1} P(A_{34})$

$P(A_2)$ 0.15, $P(A_{34})$ 0.15

$P(A_{1234}) = 0.3 + 0.3 = 0.6$

$P(A_1) \xleftarrow{0} P(A_{1234}) \xrightarrow{1} P(A_{234})$

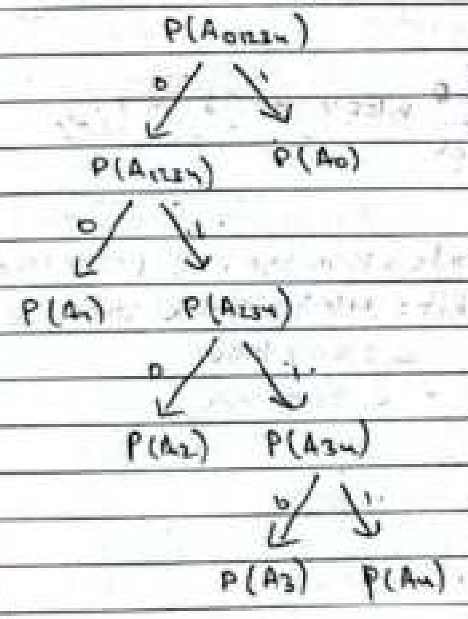
$P(A_1)$ 0.3, $P(A_{234})$ 0.3

$P(A_{01234}) = 0.6 + 0.4 = 1$

$P(A_{1234}) \xleftarrow{0} P(A_{01234}) \xrightarrow{1} P(A_0)$

$P(A_{1234})$ 0.6, $P(A_0)$ 0.4

Huffman tree:



$A_0 = 1$

$A_1 = 00$

$A_2 = 010$

$A_3 = 0110$

$A_4 = 0111$

$$L = 1\times0.4 + 2\times0.3 + 3\times0.15 + 4\times0.1 + 4\times0.05 = 0.4 + 0.6 + 0.45 + 0.4 + 0.2 = 2.05$$

2) Shannon fano

Ai	P(Ai)	step1	step2	step3	step4	code
A0	0.4	0				0
A1	0.3	1	0			10
A2	0.15	1	1	0		110
A3	0.1	1	1	1	0	1110
A4	0.05	1	1	1	1	1111

$\eta = L_H = L_{SF} = \frac{2.00}{2.05} \times 100\% = 97.56\%$

If symbol rate = 1000 symbols persecond.

output bit = H(x) x symbol rate

= 2.00 x 1000

= 2000 bps

72ch:

xi	P(xi)
A	0.3
B	0.1
C	0.02
D	0.15
E	0.4
F	0.03

arranging in decreasing probabilities

xi set	P(xi)
E→A =	0.4
A→B =	0.3
D→C =	0.15
B→D =	0.1
F→E =	0.03
C→F	0.02

Rate R_s = 14.4 K baud = 14400 baud.

Using BCD coding:

xi	P(xi)	code	ni
E→A	0.4	1010	4
A→B	0.3	1011	4
D→C	0.15	1100	4
B→D	0.1	1101	4
F→E	0.03	1110	4
C→F	0.02	1111	4

H(x) = 2.057 bits/symbol. $L_{BCD} = 4$

For Huffman

$P(EF) = 0.03 + 0.02 = 0.05$

0 → $P(E)$ = 0.03, 1 → $P(F)$ = 0.02

$P(DEF) = 0.1 + 0.05 = 0.15$

0 → $P(D)$ = 0.1, 1 → $P(EF)$ = 0.05

$P(CDEF) = 0.15 + 0.15 = 0.3$

0 → $P(C)$ = 0.15, 1 → $P(DEF)$ = 0.15

$P(BCDEF) = 0.3 + 0.3 = 0.6$

0 → $P(B)$ = 0.3, 1 → $P(CDEF)$ = 0.3

$P(ABCDEF)$

0 → $P(BCDEF)$ = 0.6, 1 → $P(A)$ = 0.4

Huffman tree:

$P(ABCDEF)$: 0 → $P(BCDEF)$, 1 → $P(A)$

$P(BCDEF)$: 0 → $P(B)$, 1 → $P(CDEF)$

$P(CDEF)$: 0 → $P(C)$, 1 → $P(DEF)$

$P(DEF)$: 0 → $P(D)$, 1 → $P(EF)$

$P(EF)$: 0 → $P(E)$, 1 → $P(F)$

$P(A) = 1$
$P(B) = 00$
$P(C) = 010$
$P(D) = 0110$
$P(E) = 01110$
$P(F) = 01111$

$L_H = 1\times0.4 + 2\times0.3 + 3\times0.15 + 4\times0.1 + 5\times0.03 + 5\times0.02 = 2.1$ bits/symbol

(i) $R_s = 14.4$ kbaud $= 14400$ baud

O/p bit rate / information rate $= H \times R_s$
$= 2.057 \times 14400$
$= 29620$ bps

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$\eta_{oso} = \frac{2.057}{4} \times 100 = 51.425\%$

$\eta_4 = \frac{2.057}{2.1} \times 100 = 97.95\%$

Chapter 2
sampling

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73Ch 2) = soln

$g(t) = 10\cos(20\pi t)\cos(200\pi t)$
$f_s = 250$ samples/sec

(i) determine the spectrum of resulting sampled signal
(ii) specify the cut-off frequency of the ideal reconstructed filter so as to recover $g(t)$ from its sampled version
(iii) determine the Nyquist rate for $g(t)$.

= soln

$$g(t) = 10\cos(20\pi t)\cdot\cos(200\pi t)$$
$$= 5[2\cos(20\pi t)\cdot\cos(200\pi t)]$$
$$= 5[\cos(220\pi t) + \cos(180\pi t)]$$

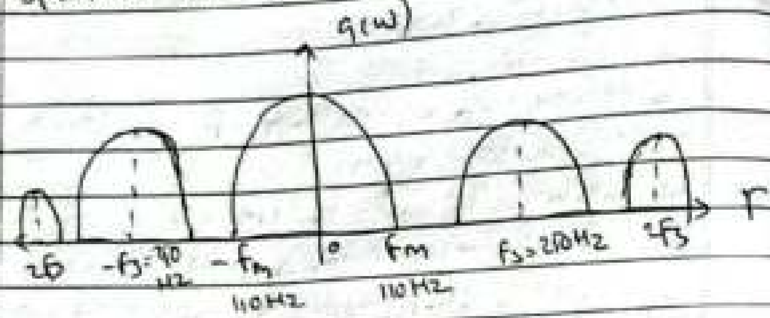
comparing with standard eq2

$\omega_1 = 220\pi$	$\omega_2 = 180\pi$
$2\pi f_1 = 220\pi$	$2\pi f_2 = 180\pi$
$f_1 = 110\,Hz$	$f_2 = 90\,Hz$

since $f_1 > f_2$
$f_m = f_1 = 110\,Hz$

and, sampling frequency $(f_s) = 250\,Hz$.

i) spectrum of sampled



ii) cut off frequency of the ideal reconstruction filter so as to recover $g(t)$;

$F_s = 2f_s$

$= 2\times 110 = 220\,Hz$.

iii) Nyquist Rate $= \frac{1}{2f_m} = \frac{1}{2\times 110}$

$= 4.545\times 10^{-3}\,sec$

$= 4.545\,ms$.

---

ch 2)

2024 2)

Nyquist rate = ?

Nyquist interval = ?

$x(t) = 6\cos 50\pi t + 20\sin 300\pi t - 10\cos 100\pi t$ is sampled & quantized using 512 levels

= soln

$\omega_1 = 50\pi$ $\quad \omega_2 = 300\pi$ $\quad \omega_3 = 100\pi$

$f_1 = 25\,Hz$ $\quad f_2 = 150\,Hz$ $\quad f_3 = 50\,Hz$

Also signal is sampled & quantized using 512 levels.

So, $q = 2^v$

$512 = 2^v$

$2^9 = 2^v$

$\therefore v = 9\,bits$.

$\because f_2 > f_3 > f_1$

$\therefore f_m = f_2 = 150\,Hz$.

Nyquist rate $(f_s) = 2f_m$

$= 2\times 150 = 300\,Hz$.

Nyquist interval $(T_s) = \frac{1}{F_s} = \frac{1}{300} = 0.037\,sec$

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$f_{max} = 3.4 KHz = 3.4 \times 10^3$ Hz per signal.

(i) Total Bandwidth, $B \geq (f_{max} \times 24)$ $(N \times f_m)$
$= 3.4 KHz \times 24$
$B = 81.6$ kHz

(ii) signaling rate of system is given as,

$r = 1.5 \times 10^6$ bits/sec

since there are 24 channels, the bit rate of individual channel is,

$$r(\text{one channel}) = \frac{1.5 \times 10^6}{24} = 62500 \text{ bits/sec}$$

Further, since sample is encoded using 8 bits, the samples per second will be,

$$\text{sample/second} = \frac{r(\text{one channel}) \text{ bits/sec}}{\text{bits/sample}}$$

sample/sec → sampling frequency.

$f_s = 62500$ b/sec / 8 b/sample

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$f_s = 7812.5$ Hz.

738h / 3b)

encoder → 7 bit binary encoder.

bit rate = 50×10^6 bits/sec

$B_{max} = ?$

= 50?

let $BW = f_m$ Hz.

$\therefore f_s \geq 2f_m$

number of bits, $v = 7$ bits.

we know that signalling rate

$r \geq v \cdot f_s$

$r \geq 7 \cdot 2f_m$

substituting for r,

$50 \times 10^6 \geq 7 \cdot 2 f_m$

$f_m \leq 3.57$ MHz

$BW = 3.57$ MHz

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ment
(x) d

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24ch
3) = soln.

given,

$BW = 4.2 MHz$

so, $f_m = 4.2 MHz$

given quantization levels, $q = 512$

(i)

$q = 2^v$

$512 = 2^v$

$2^9 = 2^v$

$\therefore v = 9$

$\therefore$ number of bits/ code word length = 9 bits

(ii) Transmission $BW \geq v f_m$

$BW \geq 9 \times 4.2 MHz$

$BW \geq 37.8 MHz.$

(iii) Final bit rate.

It is equal to signaling rate.

$r = v f_s$

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$r = 9 \times 2 f_m$

$= 9 \times 2 \times 4.2 \times 10^6$

$= 75.6 \times 10^6$ bits/sec.

(iv) O/P SQNR:

$\left(\frac{S}{N}\right)_{dB} \leq (4.8 + 6v) dB.$

$v = 9$

$\left(\frac{S}{N}\right)_{dB} \leq 4.8 + 6 \times 9$

$\left(\frac{S}{N}\right)_{dB} \leq 58.8 dB.$

7.8Bk
3 b)

$BW = 6 MHz.$

$f_m = 6 MHz$

$q = 256.$

$256 = 2^v$

$2^8 = 2^v$

$\therefore v = 8$

(i) code word length = 8 bits.

(ii) Practical BW:

$$BW \geq v \times f_m$$
$$BW > 8 \times 6MHz$$
$$BW \geq 48MHz$$

(iii) Final data rate.

$$r = v f_s$$
$$= v\, 2 f_m$$
$$= 8 \times 2 \times (6MHz)$$
$$= 96 \times 10^6 \text{ bits/sec}$$

(iv) Output SQNR:

$$\left(\frac{S}{N}\right)_{dB} \leq (4.8 + 6v)\, dB.$$
$$\leq 4.8 + 6 \times 8$$
$$\left(\frac{S}{N}\right)_{dB} \leq 52.8\, dB.$$

73(iv)
3) given, $f_m = 4KHz$
$q = 16$

(i)
$$2^v = q$$
$$2^v = 16$$
$$2^v = 2^4$$
$$\therefore v = 4$$

$\therefore$ max bits per sample $= 4$ bits.

(ii)
$$f_s \geq 2 f_m$$
$$\geq 2 \times 4KHz$$
$$\geq 8KHz$$

$\therefore$ min sampling rate $= 8KHz$.

(iii)
Bitrate, $r \geq v f_s$
$$\geq 4 \times 8$$
$$\geq 32k$$
$$\geq 32 \times 10^3 \text{ bits/second}$$

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72/Car 3) given

$f_m = 4KHz$

min dynamic range $= \pm 2.4V$

bit rate $= 64KHz$

bit per sample $= ?$

Quantization noise power $= ?$

$SQNR_{dB} = ?$

minimum $BW = ?$

bit per sample $v = \frac{\log_{10} 64000}{\log_{10} 2}$

$= 16$ bits.

$X_{max} = 2.4V$

Quantization noise power, $P = \frac{A_m^2}{2}$

$= \frac{X_m^2}{2}$

$= \frac{(2.4)^2}{2}$

$= 2.88$

$S/N = \frac{3P2^{2v}}{X_{max}^2}$

$= \frac{3 \times 2.88 \times 2^{2 \times 16}}{(2.4)^2}$

$= 6442450944$

$\left(\frac{S}{N}\right)_{dB} = 10 \times \log_{10}\left(\frac{S}{N}\right)$

$= 10\log_{10}(6442450944)$

$= 98.09 dB$

for min BW,

$f_m = 4KHz.$

$BW \geq v f_m$

since there are 10 audio signals multi

$BW \geq 10 \times v \times f_m$

$\geq 10 \times 16 \times 4$

$BW \geq 640KHz$

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7th 3)

given,

$x(t) = 6\cos(5000\pi t)$

$q = 2^v$

$128 = 2^v$

$2^7 = 2^v$

$v = 7$ bit

(a) $(SQNR) = 4.8 + 6v$

$= 4.8 + 6\times7$

$= 46.8 dB.$

(b) $3\times4^v = 3\times4^7 = 49152.$

we know that,

$$x = \frac{3}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^3$$

$5000\pi t = 2\pi f t$

$f = 2500 Hz$

$$\therefore x = \frac{3}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^3.$$

$$49152 = \frac{3}{8\pi^2}\left(\frac{f_s'}{2500}\right)^3.$$

$f_s' = 272401.75 Hz$

$$\therefore \boxed{f_s' = 272.401 KHz}$$

(c) $$SQNR = \frac{3}{8\pi^2}\left(\frac{2f_x}{f_x}\right)^3$$

$$= \frac{3}{8\pi^2}\times2^3$$

$= 0.30.$

In dB; $10\log_{10}(0.30) = -5.22 dB$

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$\frac{GXh}{3)}$ soln

given,

$f_m = 4 kHz$

$f_s = 2 \times 4 = 8 kHz$.

$\pm A = \pm 1.1V$

Bit rate = 32kbps = νf_s sample/sec.

$32 = 8n$

ν = 4 bits/sample.

$\therefore$ SQNR = $4.8 + 6\nu$

$= 4.8 + 6 \times 4$

$= 28.8 dB$

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$\frac{72Vad}{4)}$ soln

given,

$f_m = 3 kHz$

$f_s = 10 kHz$

Nyquist rate $= 2f_m = 2 \times 3 kHz$

$= 6 kHz$

Sampling interval $(T_s) = \frac{1}{f_s}$

$= \frac{1}{10000}$

$A_{max} = 1$

$\therefore$ step size $(\Delta) = A_m . 2\pi f_m . T_s$

$= \frac{A_m . 2\pi f_m}{f_s}$

$= \frac{1 \times 2\pi \times 3 \times 10^3}{10 \times 10^3}$

$= \frac{6\pi}{10} = 1.88 V$

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$$(SQNR) = \frac{3}{3\pi^2 f_m^2 f_M T_s^3}$$

$f_m = f_M = 3\,kHz$

$$SQNR = \frac{3}{3\times\pi^2\times 3^2\times 3\times 10^7 \times (10^{-4})^3}$$

$= 3.75$

$= 5.74\,dB$

Min transmission BW $= f_m = 3\,kHz$

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72 kan

2)

$x(t) = \text{sinc}(5\pi t)$

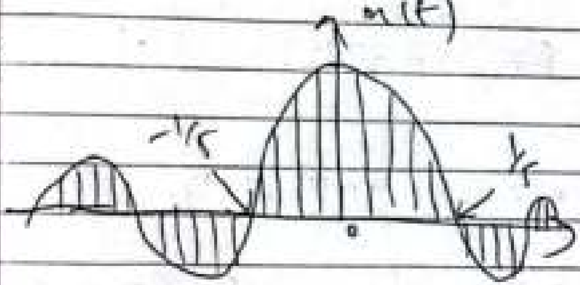
$f_s = 10\,Hz$

1) sketch the sampled of signal
2) sketch the spectrum of the sampled signal for the range $|f| < 30\,Hz$.
3) Explain whether you can recover the signal $x(t)$ from the sampled signal

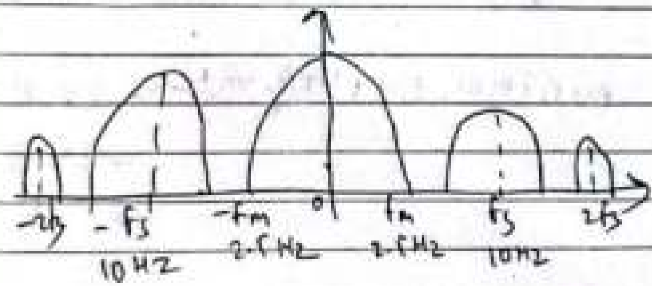
=sol^n given signal,

$x(t) = \text{sinc}(5\pi t) = \text{sinc}(2\times\pi\times 2.5 t)$

1)



(ii)



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(iii) given signal $x(t) = sinc(5\pi t)$.

$f_m = 2.5\ Hz$

$f_s = 10\ Hz$

$f_s \geq 2f_m$

$10 \geq 2 \times 2.5$

$10 \geq 5$

$\therefore x(t)$ can be recovered.

——X——X——

6(ch)
2)

~~fm = 115 KHz~~

Range: 80 to 115 KHZ

$BW = 2f_m = 115 - 80 = 35\ KHZ$

minimum sampling rate $= 2 \times BW$
$= 2 \times 35$
$= 70\ KHZ$

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normally range of min. sampling frequencies lies in between $4f_m$ to $8f_m$ for BP signals.

Range of min. sampling frequencies $= 2 \times BW$ to $4 \times BW$
$= 2 \times 35\ KHZ$ to $4 \times 35\ KHZ$
$= 70\ KHZ$ to $140\ KHZ$

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Chapter - 8

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2074 Q) The generator polynomial of a (7,4) cyclic code is $g(x) = 1+x+x^3$. Find the code for the message ~~signal in~~ vector 1011 in a non systematic & systematic form.

= Sol^n

given, $n=7$ and $k=4$

$$g(x) = 1+x+x^3$$

For message signal 1011,

$$m(x) = m_0 + m_1 x + m_2 x^2 + m_3 x^3$$
$$= 1 + 0.x + 1x^2 + 1.x^3$$
$$= 1+x^2+x^3$$

∴ code vector will be;

$$c(x) = m(x) . g(x)$$
$$= (1+x^2+x^3)(1+x+x^3)$$
$$= 1+x^2+x^3 + x+x^3+x^4 + x^3+x^5+x^6$$
$$= 1+x+x^2+x^3(1\oplus1\oplus1)+x^4+x^5+x^6$$
$$c(x) = 1+x+x^2+x^3+x^4+x^5+x^6.$$

∴ code word will be $c(x) = 1+x+x^2+x^3+x^4+x^5+x^6$
$= 1111111$

which is in non-systematic form.

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For systematic form:

multiply $m(x)$ by x^{n-k}.

$$x^{n-k} m(x) = x^{7-4} m(x)$$
$$x^{n-k} m(x) = x^3(1+x^2+x^3) = x^3+x^5+x^6$$

Divide $x^{n-k} m(x)$ by $g(x)$

$$\frac{x^{n-k} m(x)}{g(x)} = \frac{x^3+x^5+x^6}{1+x+x^3}$$

$$x^3+x+1 \overline{\big) x^6+x^5+x^3 \big(} x^3$$
$$x^6+x^4+x^3$$
$$x \Rightarrow r(x)$$

$$r(x) = [x^{n-k} m(x)] \oplus r(x)$$
$$= [0+0.x+0.x^2+1.x^3+0.x^4+1.x^5+1.x^6] \oplus [0+1.x]$$
$$= 0+1.x+0.x^2+1.x^3+0.x^4+1.x^5+1.x^6$$
$$= 0101011$$

∴ code vector is: $x(x) = \underbrace{010}_{\text{Parity bits}}\underbrace{1011}_{\text{message bits}}$

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$$x^3+x+1 \,\big)\, x^6+x^5+x^3 \,\big(\, x^3+x^2+x+1$$
$$x^6+x^4+x^3$$
$$\overline{x^5+x^4}$$
$$x^5+x^3+x^2$$
$$\overline{x^4+x^3+x^2}$$
$$x^4+x^2+x$$
$$\overline{x^3+x}$$
$$x^3+x+1$$
$$\overline{1 \rightarrow r(x)}$$

$\therefore\ x(x) = r(x) + x^3 M(x)$

$= 1 + x^3 + x^5 + x^6.$

$= \underbrace{100}_{r}\underbrace{1011}_{M}$

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Q) Construct a (7,4) cyclic code using generator polynomial $g(x) = x^3+x^2+1$ with data vector 1011

⇒ sol.

$M(x) = 1.x^3 + 0.x^2 + 1.x + 1.$

$M(x) = x^3+x+1$

$g(x) = x^3+x^2+1$

$x^{n-k}\, m(x) = x^3(x^3+x+1)$

$= x^6+x^4+x^3$

$\dfrac{x^3 M(x)}{g(x)} \rightarrow x^3+x^2+1 \,\big)\, x^6+x^4+x^3 \,\big(\, x^3+x^2$

$$x^6+x^5+x^3$$
$$\overline{x^5+x^4}$$
$$x^5+x^4+x^2$$
$$\overline{x^2 \rightarrow r(x)}$$

$x^6+x^4+x^3 \oplus x^2$

$x^6+x^4+x^3+x^2$

$\underbrace{1011}_{\text{message bits}}\underbrace{100}_{\text{Parity bits}} \rightarrow$ codeword same

71Ch | 72Ch

9.3.8.3 similar

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Q) For a code vector $x = (0111000)$ and the parity check matrix H given below. Prove that the given code is valid.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times7}$$

= sol^n

given code vector, $x = (0111000)$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

To be valid it must satisfy

$xH^T = (0,0 \cdots 0)$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$\therefore xH^T = [0111000]_{1\times7} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{7\times3}$

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$xH^T = (0\times1) \oplus (1\times1) \oplus (1\times1) \oplus (1\times0) \oplus (0\times1) \oplus (0\times0) \oplus (0\times0), (0\times1) \oplus (1\times1) \oplus (1\times0) \oplus (1\times1) \oplus (0\times0) \oplus (0\times1) \oplus (0\times0), (0\times1) \oplus (1\times0) \oplus (1\times1) \oplus (1\times1) \oplus (0\times0) \oplus (0\times0) \oplus (0\times1)$

$= 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0,$
$0 \oplus 1 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0,$
$0 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \oplus 0$

$xH^T = [0,0,0]_{1\times3}$
$= (0,0,0)$

Hence this is valid codeword

79Bh

(6,3)

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

n=6
k=3
message bits = k = 3.
parity bits = n−k
= 3

1) generator matrix:

$G = [I_k | P]$

$\therefore I_k = I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Scanned with CamScanner

$1 \oplus 1 \to 0$
$0 \oplus 1 \to 1$ $\quad 0 \oplus 0 \to 0$
$1 \oplus 0 \to 1$

$$\therefore G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

2) All code words :

Relation bet^n parity bits & message bits

$$[C_0, C_1, C_2] = [M_0, M_1, M_2] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$C_0 = (M_0 \times 1) \oplus (M_1 \times 1) \oplus (M_2 \times 1) = M_0 \oplus M_1 \oplus M_2$$
$$C_1 = (M_0 \times 1) \oplus (M_1 \times 1) \oplus (M_2 \times 0) = M_0 \oplus M_1$$
$$C_2 = (M_0 \times 1) \oplus (M_1 \times 0) \oplus (M_2 \times 1) = M_0 \oplus M_2$$

SNO	Message vector			Parity bits			complete codevector						Wt
	M_0	M_1	M_2	C_0	C_1	C_2	M_0	M_1	M_2	C_0	C_1	C_2	Wt
1)	0	0	0	0	0	0	0	0	0	0	0	0	0
2)	0	0	1	1	0	1	0	0	1	1	0	1	3
3)	0	1	0	1	1	0	0	1	0	1	1	0	3
4)	0	1	1	0	1	1	0	1	1	0	1	1	4
5)	1	0	0	1	1	1	1	0	0	1	1	1	4
6)	1	0	1	0	1	0	1	0	1	0	1	0	3
7)	1	1	0	0	0	1	1	1	0	0	1	1	4
8)	1	1	1	1	0	0	1	1	1	1	0	0	4

3) d_{min} = min weight of any non zero code vector.

$d_{min} = 3$

No. of errors that can be detected is given by,

$$d_{min} \geq S+1$$
$$3 \geq S+1$$
$$\therefore S \leq 2$$

Hence, at the most 2 errors can be detected.

Number of errors that can be corrected is given by,

$$d_{min} \geq 2t+1$$
$$3 \geq 2t+1$$
$$2 \geq 2t$$
$$t \leq 1$$

This means at most one error can be corrected.

optional: correct upto t error & detect $S > t$ errors per word:

$$d_{min} \geq (t+S+1)$$