

Heritage's Engineering Series

Fundamentals of **Thermodynamics** **& Heat Transfer**

for Bachelor of Engineering (BE)



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INTRODUCTION

1.1 Definition and Scope of Engineering Thermodynamics

Engineering thermodynamics is the branch of applied science which deals with energy. We can consider heat engine and refrigerator as the most common thermodynamic applications. In case of heat engine, heat is continuously converted into work while in case of refrigerator, heat is continuously transferred from low temperature to high temperature with the help of work input to produce the desired cooling effect. So in all thermodynamic applications, we either talk about heat or work or both. In this regard, thermodynamics is sometimes also defined as the study of heat and work.

Hence, in basic engineering thermodynamics, we study about

- Definition of energy, different forms of energy and conversion of energy from one form to another to produce certain useful effects.
- Fundamental thermodynamic laws which govern the energy conversion process.
- Properties of common substances which can be used to transfer energy.

Most of the fundamentals in engineering thermodynamics are derived from the physics and we focus or extend our attention on the applications of those fundamentals.

Engineering thermodynamics have wide ranges of applications for different engineering disciplines. Civil engineers may find its application while choosing the layout of a building on a provided land space such that energy requirement for heating and lighting is minimum. Electrical, electronics and computer engineers may find its application while selecting the proper cooling system for the different electrical or electronic devices. Mechanical engineers may find its application while designing different components of power plant system, heat and ventilating units etc.

1.2 Value of Energy to Society

Energy plays a unique and critical role in the world; no activity of any kind can take place without the movement or conversion of energy.

The use of energy has been a key in the development of the human society by helping it to control and adapt to the environment. Managing the use of energy is inevitable in any functional society. In the industrialized world the development of energy resources has become essential for agriculture, transportation, waste collection, information technology, communications that have become prerequisites of a developed society. The increasing use of energy since the Industrial Revolution has also brought with it a number of serious problems, some of which, such as global warming, present potentially grave risks to the world.

In society and in the context of humanities, the word energy is used as a synonym of energy resources, and most often refers to substances like fuels, petroleum products and electricity in general. These are sources of usable energy, in that they can be easily transformed to other kinds of energy sources that can serve a particular useful purpose. Meaning of energy in natural sciences can lead to some confusion, because energy resources are not conserved in nature in the same way as energy is conserved in the context of physics. The actual energy content is always conserved, but when it is converted into heat for example, it usually becomes less useful to society, and thus appears to have been used up.

At a national and international level, it is the lifeblood of modern economies. For developed nations, reliable energy fuels the technologies and services that enrich and extend life. Energy powers advanced computers, improved transportation, expanded communications, cutting-edge medical equipment and procedures, and much more.

For developing nations, expanding reliable and affordable supplies of energy supports and even accelerates changes that improve and save lives. Reliable energy means expanded industry, modern agriculture, increased trade and improved transportation. These are building blocks of economic growth that create the jobs that help people escape poverty and create better lives for their children.

Energy consumed per capita of any country reflects the living standard of the people of those countries.

Total energy consumption per capita of some countries for past four years is listed here for reference.

Energy use (kg of oil equivalent per capita)				
Country Name	2010	2011	2012	2013
Albania	725	770	715	
Algeria	1113	1140	1237	
Angola	639	630	630	
Argentina	1910	1919	1906	
Armenia	838	915	997	
Australia	5560	5500	5644	5592
Austria	4080	3951	3928	3928
Azerbaijan	1280	1369	1473	
Bahrain	9871	9552	9450	
Bangladesh	202	207	214	
Belarus	2898	3098	3223	
Belgium	5544	5369	5029	5020
Benin	384	385	390	
Bolivia	756	779	831	
Bosnia and Herzegovina	1680	1849	1742	
Botswana	1066	1026	1015	
Brazil	1339	1347	1392	
Brunei Darussalam	8239	9696	9526	
Bulgaria	2420	2615	2511	
Cambodia	350	365	370	
Cameroon	338	322	323	
Canada	7390	7367	7226	7149
Chile	1813	1958	2140	2179
China	1889	2044	2143	
Colombia	680	673	674	
Congo, Dem. Rep.	301	310	292	
Congo, Rep.	375	401	400	
Costa Rica	1022	1012	1016	
Cote d'Ivoire	502	563	597	
Croatia	1938	1971	1855	
Cuba	1018	990	1003	
Cyprus	2213	2121	1971	
Czech Republic	4248	4103	4057	3935

1.3 Microscopic and Macroscopic Viewpoint

Any thermodynamic system can be analyzed in two ways: microscopic viewpoint and macroscopic viewpoint. In microscopic viewpoint, system is analyzed with reference to its molecular structure or molecular behavior. In macroscopic viewpoint, system is analyzed with reference certain measurable or observable bulk properties. For example, let us consider pressure of the system. In microscopic analysis, we calculate pressure of the system by calculating force exerted by the molecules on the container wall, whereas, in case of macroscopic analysis, we proceed with the value of pressure measured by a standard calibrated pressure measuring device. With reference to the above mentioned features, we can list the major differences between the microscopic and macroscopic viewpoints as

Microscopic Viewpoint		Macroscopic Viewpoint	
1.	This approach involves large number of variables.	1.	This approach involves less number of variables.
2.	Calculation is time consuming and tedious in microscopic analysis.	2.	Calculation is relatively faster and simpler in macroscopic analysis.
3.	This approach is usually used in statistical thermodynamic and used by pure scientists.	3.	This approach is usually used in applied thermodynamics and used by engineers.

1.4 Concepts and Definitions

Before going to fundamentals of thermodynamics, the terms frequently used in thermodynamics will be discussed.

1.4.1 Thermodynamic System, Surroundings, Boundary and Universe

We encounter infinite number of components around us and every component undergoes an infinite number of activities. So any component or a collection of certain components which is performing certain specific task or activity is called a system. If we analyze this system with thermodynamic perspective (i.e. with reference to thermodynamic parameters), then this system is called a thermodynamic system. We can also define thermodynamic system as a part of the

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universe which is to be studied or analyzed. All the material or medium outside to the system is called surroundings and the medium which separates system from its surroundings is called boundary. System and surroundings taken together is sometimes called universe.

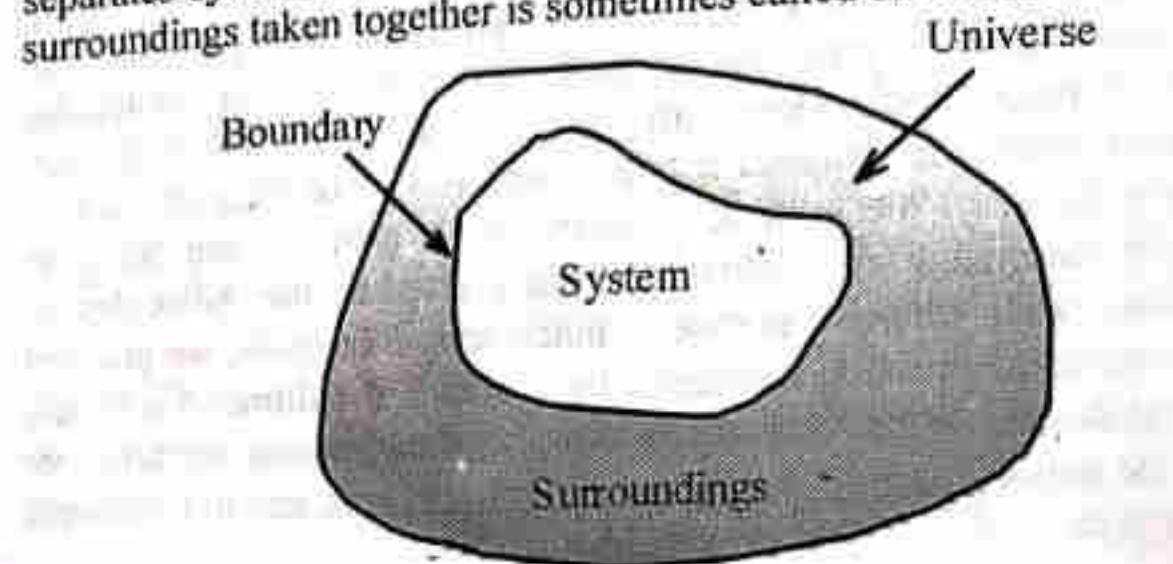


Figure 1.1: System, Surroundings, Boundary and Universe

For example, if we want to study about the temperature of a room we study or focus our attention on the activities occurring inside the room. So for this analysis, room becomes a thermodynamics system; and region outside to the room becomes its surroundings and the four walls, floor and ceiling of the room act as the boundary of the system.

Objective of any thermodynamic analysis is to study the interactions between the system and the surroundings and effects of these interactions on the system or on the surroundings. System and surroundings can interact with each other either by mass transfer or energy transfer or both. According to modes of interaction between the system and the surroundings, systems are classified into three types: closed system, open system and isolated system.

System in which energy transfer can take place but mass cannot is called a closed system. Let us consider an ideal gas contained in a piston cylinder device as shown in **Figure 1.2**. Heat can be supplied to the gas inside the cylinder through surface of the cylinder. Due to the transfer of heat, gas undergoes an expansion and it can displace the piston having certain weight and this phenomenon of displacement of mass (weight) in the direction of force is called work. So in case of piston cylinder device, heat and work can cross the boundary of the system but gas cannot escape out from the system, hence a piston cylinder device is an example for a closed system.

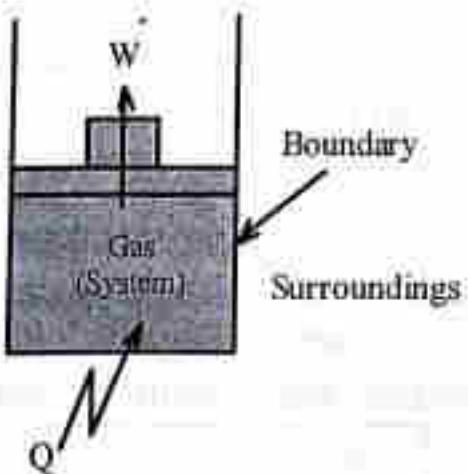


Figure 1.2: Piston-cylinder Device (Closed System)

Total mass of a closed system always remains constant therefore any closed system can be completely described by the properties of particular amount of mass inside the system. Hence, a closed system is also called a control mass (CM).

System in which both mass and energy transfer can take place is called an open system. Let us consider an air compressor, which takes air from the surroundings, compresses it delivers it to the desired application, as shown in **Figure 1.3**. For the compression of air it takes rotational work input from the motor and during compression some heat is rejected from the compressor surface to the surroundings. So in case of an air compressor, both mass and energy (heat and work) can cross the boundary of the system, hence it is an example for an open system.

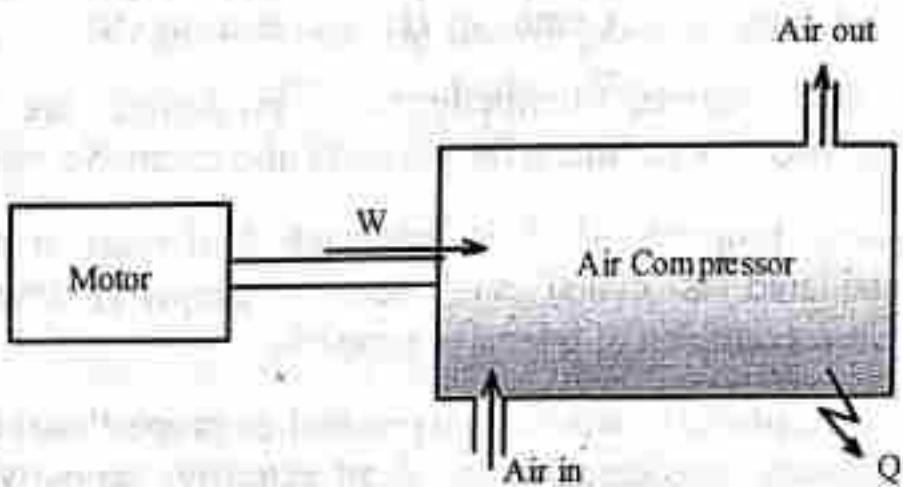


Figure 1.3: Air Compressor (Open System)

During the operation of an air compressor, air continuously flows from inlet to outlet. If we consider a particular molecule of air near to inlet, it goes inside the system, remains there for some interval and again goes out from the system. In case of an open system, properties of particular mass cannot describe the properties of the system for any instant. For the

In the study of an open system, we select some specified region and analyze this region with reference to thermodynamic properties. Hence an open system is sometimes also called a control volume (CV).

System in which neither mass nor energy transfer can take place is called an isolated system. In case of an isolated system there is no interaction between the system and the surroundings. A closed, rigid and insulated container is an example of isolated system, as shown **Figure 1.4**.

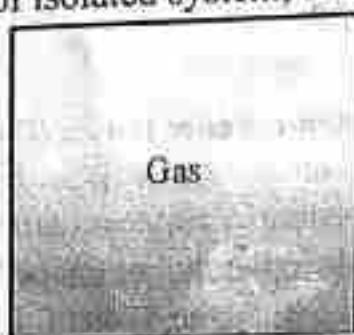


Figure 1.4: Closed Rigid Insulated Container (Isolated System)

As perfect insulator and perfect rigid body are not possible, perfect isolated system is not possible in nature.

1.4.2 Thermodynamic Property

Any parameter or variable used to describe a thermodynamic system is called a thermodynamic property. Most common thermodynamic properties are pressure (P), temperature (T) and volume (V). Other properties which will be dealt in the following chapters are internal energy (U), total energy (E), enthalpy (H) and entropy (S).

According to their nature, thermodynamic properties are generally classified into two groups: intensive property and extensive property.

Thermodynamic property which is independent of mass or part of the system considered is called an intensive property. Pressure and temperature are examples of intensive properties.

Thermodynamic property which is dependent or proportional to mass or part of the system considered is called an extensive property. Volume, internal energy, total energy, enthalpy, entropy are extensive properties.

Since extensive properties are proportional to mass they can be converted into intensive form by dividing its value by mass. The ratio of an extensive property to the mass is called the specific value of that property. Thus, if the total energy of a system is denoted by E and the mass by m , then the specific total energy or total energy per unit mass is given by

$$e = \frac{E}{m} \quad \dots\dots\dots (1.1)$$

In general, capital letters will denote extensive properties, and small letters will be used to represent the specific values of these properties.

1.4.3 Thermodynamic Equilibrium

We can describe the condition of a thermodynamic system with the values of different thermodynamic properties such as pressure, temperature, etc. To describe a system with a certain pressure or temperature, it should have same pressure or temperature throughout the system. Such condition a system is called a thermodynamic equilibrium. Therefore, a system is said to be in a thermodynamic equilibrium if there is no change in any properties of the system when it is isolated from its surroundings.

We can also define thermodynamic equilibrium in another way as: "A system is said to be in a thermodynamic equilibrium if all intensive properties have same value throughout the system."

For a thermodynamic system to be in a thermodynamic equilibrium, it should satisfy the following equilibrium conditions: thermal equilibrium, mechanical equilibrium and chemical equilibrium.

For a system to be in a thermal equilibrium, there should not be any temperature difference within the system. For a system to be in a mechanical equilibrium, there should not be any pressure difference or unbalanced force/moment within the system. Similarly for a system to be in a chemical equilibrium, there should not be any chemical reaction during the observation interval.

1.4.4 Thermodynamic State

Each unique condition at which a thermodynamic system can exist is called a thermodynamic state. Thermodynamic state is specified by a set of two thermodynamic properties. For example, if a room has a temperature of 27°C and a pressure of 1 atm at any instant, this condition of a system defined by $T_1 = 27^{\circ}\text{C}$ and $P_1 = 1\text{ atm}$ can be specified as state 1 of the system. If a room heater is turned on for certain interval, and after some time room might have a temperature of 30°C . Then the condition of the system defined by $T_2 = 30^{\circ}\text{C}$ and $P_2 = 1\text{ atm}$ can be specified as state 2 of the system.

Properties of a Thermodynamic Property

After defining thermodynamic state, we can explain the features of thermodynamic properties as:

A variable is a thermodynamic property, if and only if,

- (a) it has a single value at each equilibrium state, and
- (b) a change in its value between any two prescribed states is independent of path.

Any single variable either satisfies both of these statements or does not satisfy both. Therefore only variables which satisfy both of these statements are thermodynamic properties.

To explain the meaning of the first statement, consider pressure as a variable. Any thermodynamic system cannot have multiple values of pressure at any state if it is in equilibrium condition. But you cannot give a unique value of heat or work for any equilibrium state because heat and work can be defined only for an interval. Hence pressure satisfies the first statement but heat and work do not satisfy it.

Similarly to explain the meaning of the second statement, consider three paths A, B and C between same end states 1 and 2 as shown in **Figure 1.5**. If we check for the change in pressure, we can take reference of any one process, i.e. $(P_2 - P_1)_A = (P_2 - P_1)_B = (P_2 - P_1)_C$. Hence, any variable having such relation is said to be path independent and satisfies second statement also. Again if we consider work as a variable, it will have different values for different path between same initial and final states ($W_A > W_B > W_C$), because work is calculated by evaluating area covered by each path on $P - V$ diagram. Hence, work does not satisfy the second statement. As discussed, pressure which satisfies both statements is a thermodynamic property whereas work which does not satisfy both statements is not a thermodynamic property,

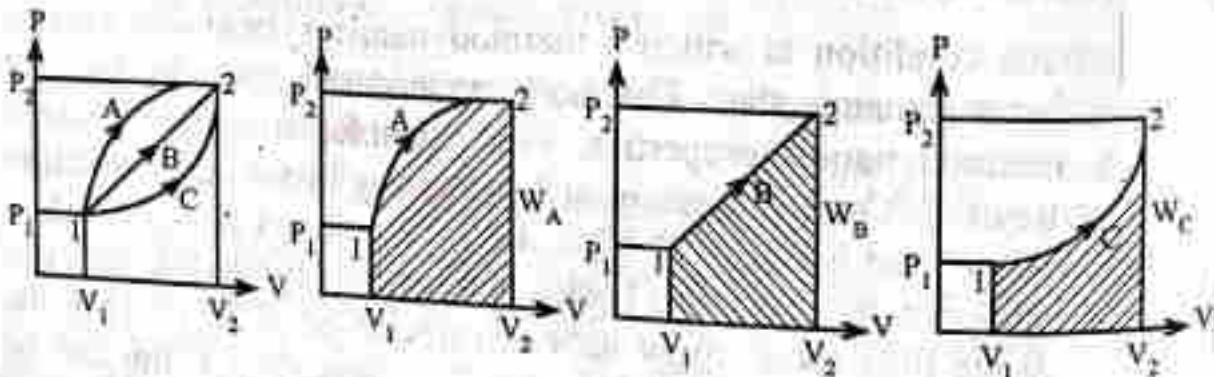


Figure 1.5: Work (A path dependent variable)

Hence, in thermodynamics we should deal with two types of variables. Some variables depend only on the end states but other variables depend both on the end states as well as on the path followed. Any variable which depends only on the end states but not on the path is called a state function whereas any variable which depends not only on the end states but also on the path is called a path function. All thermodynamic properties are the examples state functions. Work and heat are the examples of path functions.

1.4.5 Thermodynamic Process

The path followed by a system when it undergoes from one equilibrium state to another is called a thermodynamic process. Thermodynamic process can also be defined as a locus of states through which system passes while undergoing change from one equilibrium state to another.

Thermodynamic process is described with the help of property diagrams such as $P - V$, $T - V$, $P - T$, $h - s$, $T - s$, $P - h$ diagrams etc.

For example, if an ideal gas contained in a rigid vessel is heated, its pressure increases linearly with temperature while volume remains constant, as shown in **Figure 1.6**. Again, if an ideal gas contained in a freely moving frictionless piston cylinder device is heated, its volume increases linearly with temperature while its pressure remains constant as shown in **Figure 1.7**. Similarly, if an ideal gas contained in a piston cylinder device is heated under constant temperature, its pressure decreases and volume increases as shown in **Figure 1.8**.

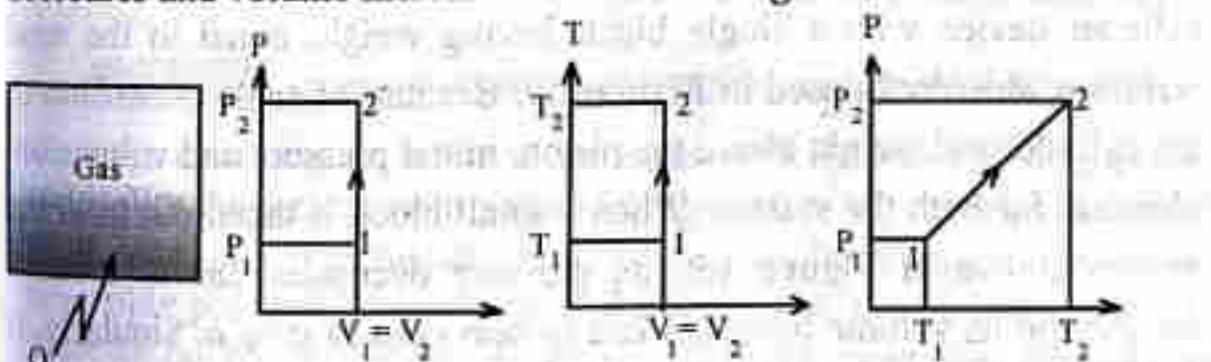


Figure 1.6: Constant volume heating process

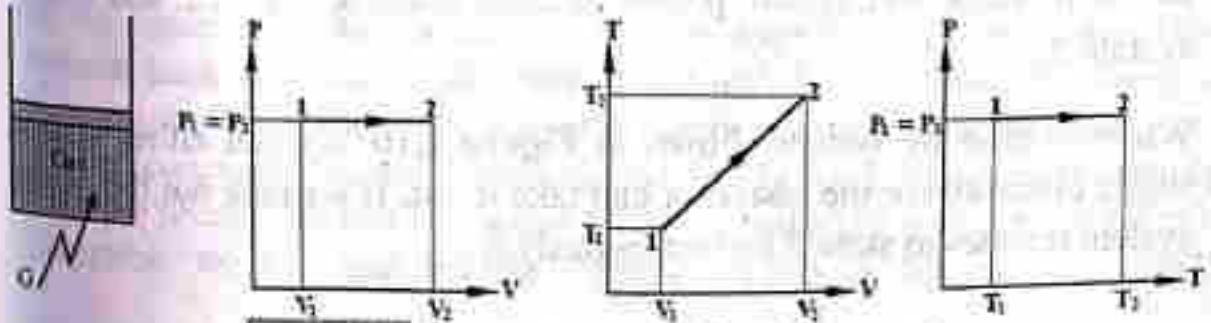


Figure 1.7: Constant pressure heating process

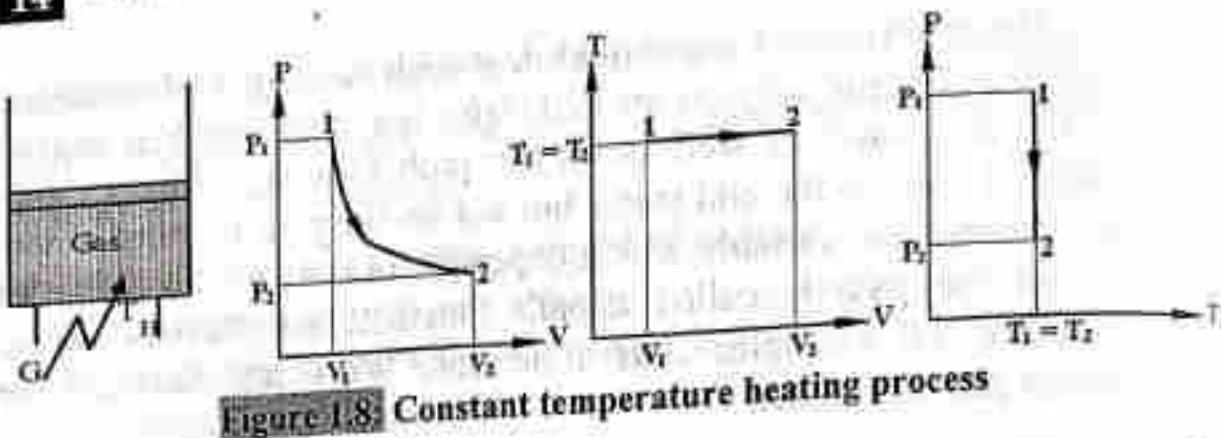


Figure 1.8: Constant temperature heating process

With reference to intermediate states, thermodynamic processes can be classified as: reversible and irreversible process.

Any process which passes through a series of equilibrium states such that each intermediate state can be located on a property diagram is called a reversible process. Reversible process is represented by a continuous line.

Any process which passes through a series of intermediate states which cannot be defined or cannot be located on a property diagram is called an irreversible process. Irreversible process is represented by a broken line.

Consider piston cylinder arrangements shown in **Figure 1.9** and **Figure 1.10**. **Figure 1.9** contains a piston-cylinder device with a number of small weights above the piston. **Figure 1.10** contains a similar piston-cylinder device with a single block having weight equal to the total weight of the blocks used in **Figure 1.9**. Because of same dimension of the cylinder and weight above the piston, initial pressure and volume are identical for both the system. When a small block is taken out from the system shown in **Figure 1.9**, its pressure decreases due to reduced weight and its volume increases and system goes to state *a*. Similarly, if we go on removing weight, its pressure further decreases and volume further increase and system passes through states *b*, *c*, *d*, and finally to state 2.

While in case of system shown in **Figure 1.10**, we can either keep a single block above the piston or can take it out. If we take out the block system reaches to state 2 instantaneously.

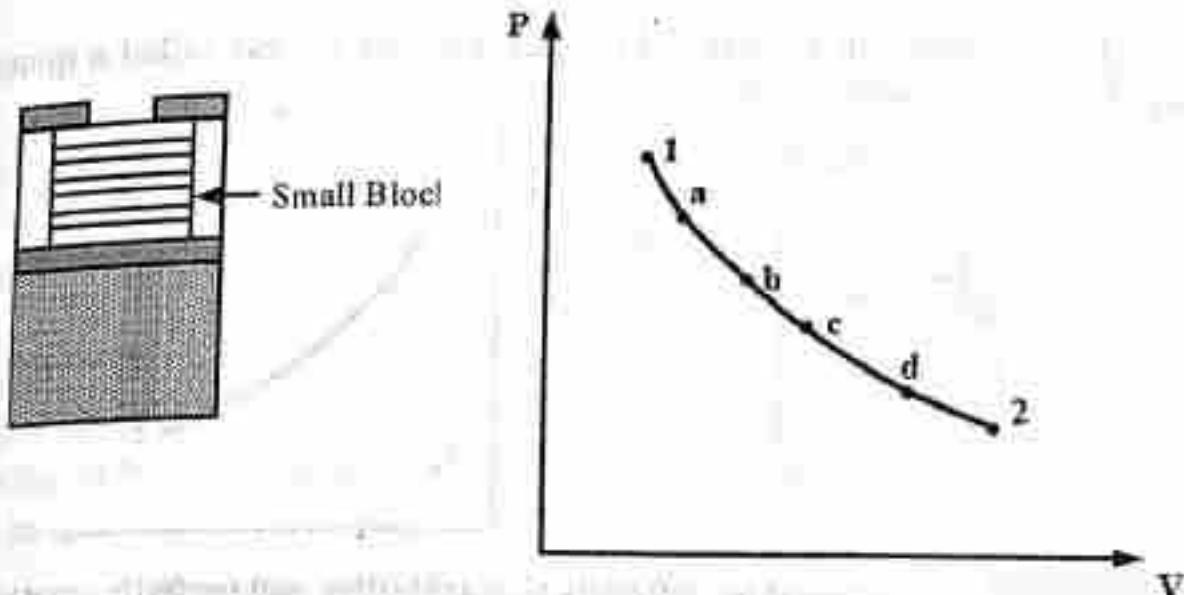


Figure 1.9: Arrangement for slow transition and reversible process

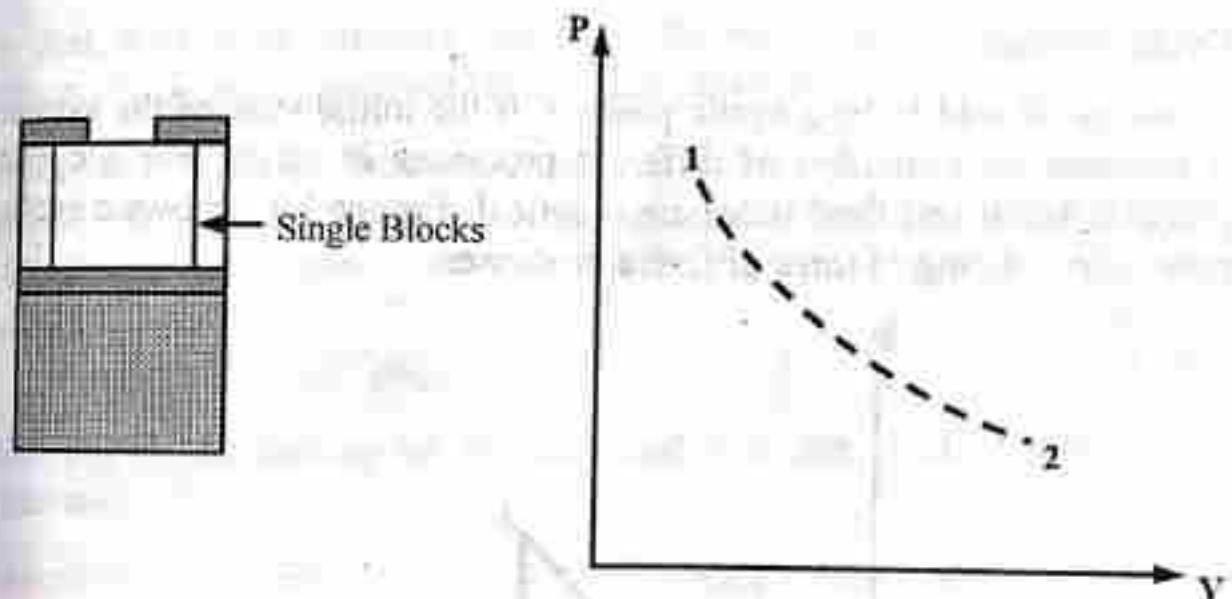


Figure 1.10: Arrangement for rapid transition and irreversible process

Now we can compare the two processes with reference to intermediate states. For the arrangement shown in **Figure 1.9**, we can take the system back from any intermediate state to the reverse direction. So it is a reversible process. While in case of arrangement shown in **Figure 1.10**, there is no possibility of taking system back from any intermediate states; hence it is an irreversible process.

If we again consider process shown in **Figure 1.9**, we cannot take system back from any state between *a* and *b*, or *b* and *c*. Therefore process shown in **Figure 1.9** is also not perfectly reversible. However, we can conclude that the process shown in **Figure 1.9** is more reversible than the process shown in **Figure 1.10**. To make the process completely reversible we can use infinite number of blocks having very small

weights as shown in Figure 1.11. Such process is also called a quasi-equilibrium or quasi-static process.

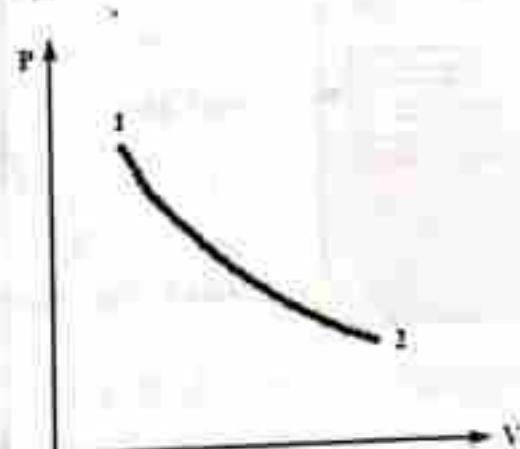


Figure 1.11: Arrangement for infinitely slow transition and perfectly reversible process (quasi-equilibrium process)

Cyclic Process

A process is said to be a cyclic process, if the initial state of the system is restored by a number of different processes in series. For a cyclic process, initial and final states are identical. Figure 1.12 shows a cyclic process consisting of three different processes.

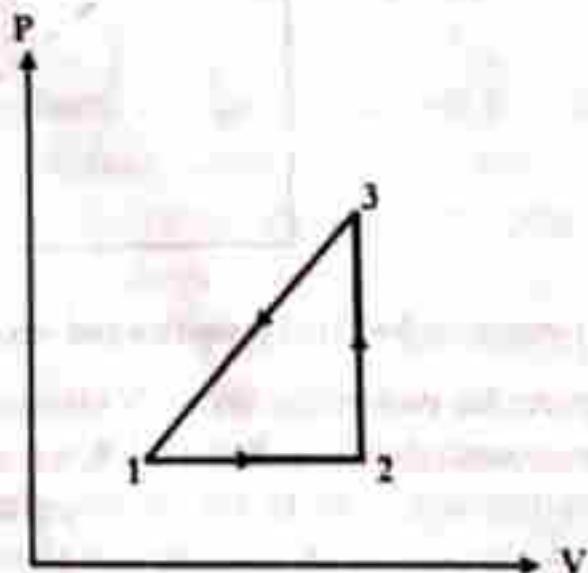


Figure 1.12: Cyclic Process

1.5 Some Common Properties

1.5.1 Specific Volume

Specific volume is defined as volume per unit mass of the substance. Specific volume is reciprocal of commonly used property density denoted by v and it is expressed in m^3/kg .

$$v = \frac{V}{m} \quad \dots \dots \dots (1.2)$$

In thermodynamics, most of the time we deal with gas and vapor which have very low density. Hence it is appropriate to express property or state relationships of such substances in terms of specific volume.

1.5.2 Pressure

Pressure is defined as force per unit area. It is denoted by P and is expressed in N/m^2 or Pascal (Pa). Sometimes pressure is also expressed in bar or atm.

$$1 \text{ bar} = 100 \text{ kPa} = 10^5 \text{ Pa}$$

$$1 \text{ atm} = 101.325 \text{ kPa} = 1.01325 \times 10^5 \text{ Pa}$$

We deal with three pressure terms in thermodynamics: atmospheric pressure (P_{atm}), gauge pressure (P_{gauge}) and absolute pressure (P_{abs}).

Pressure exerted by the ambient air on any surface is called atmospheric pressure. Atmospheric pressure is measured by barometer. Barometer gives atmospheric pressure in terms of height of the mercury column z_{baro} as

$$P_{\text{atm}} = \rho_{\text{Hg}} g z_{\text{baro}} \quad \dots \dots \dots (1.3)$$

where ρ_{Hg} is the density of mercury and g is the local gravitational acceleration.

Pressure of any system measured with reference to atmospheric pressure is called gauge pressure. Gauge pressure is usually measured by pressure gauge or manometer. Pressure gauge gives pressure reading directly in Pa or bar. Manometer gives gauge pressure of a system in terms of height of certain manometric fluid z_{mn} as

$$P_{\text{gauge}} = \rho_{\text{mn}} g z_{\text{mn}} \quad \dots \dots \dots (1.4)$$

where ρ_{mn} is the density of manometric fluid and g is the local gravitational acceleration.

Pressure of a system expressed relative to perfect vacuum is called absolute pressure. Hence P_{atm} , P_{gauge} and P_{abs} can be represented on a pressure scale as shown in **Figure 1.13(a)** and are related as

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}} \quad \dots \dots \dots (1.5)$$

Sometimes pressure of a system may be less than atmospheric pressure, and gauge pressure becomes negative, as shown in **Figure 1.13(b)**. Negative gauge pressure is also called vacuum gauge.

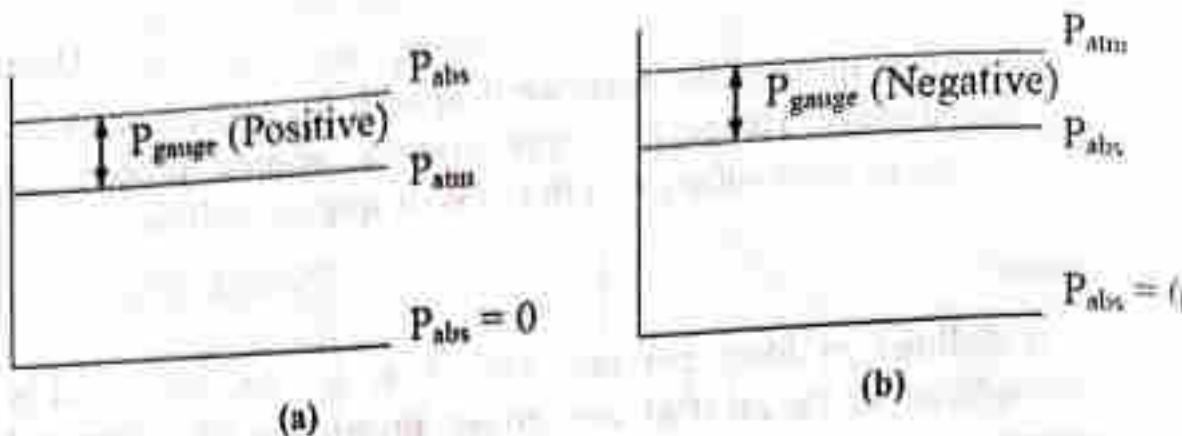


Figure 1.13: Relation between absolute, atmospheric and gauge pressure.

1.5.3 Temperature

Temperature is defined as a degree of hotness or coldness of a system. It is also defined as a property of a system which defines the direction and magnitude of heat transfer. If the temperature of a system is higher than that of the surroundings, heat transfer takes place from the system to the surroundings while if the temperature of the system is less than that of the surroundings, heat transfer takes place from surroundings to the system. Similarly, magnitude of the heat transfer is also proportional to the temperature difference between the system and the surroundings.

Temperature is measured by a thermometer. Commonly used scale for temperature are $^{\circ}\text{C}$, $^{\circ}\text{F}$ and K. Temperature expressed in Kelvin scale is called absolute temperature. We can relate temperature measured on different scales as

$$\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273}{373-273}$$

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5}$$

Scale	Freezing Point	Boiling Point
$^{\circ}\text{C}$	0	100
$^{\circ}\text{F}$	32	212
K	273	373

..... (1.6)

1.6 Equality of Temperature and Zeroth Law of Thermodynamics

1.6.1 Equality of Temperature

Consider a piston cylinder device containing a gas brought in contact with a metal sphere as shown in Figure 1.14. If the temperature of the

sphere is greater than that of the gas, heat flows from sphere to the gas and piston moves upwards due to the expansion of the gas. During the process temperature of the sphere goes on decreasing and that of the gas goes on increasing. After some interval, temperature of both the systems becomes equal and the piston remains stationary. This condition is known as equality of temperature and can be stated as:

Two systems are said to have equal temperatures, if there is no change in any properties of both the system when they are brought in contact with each other.

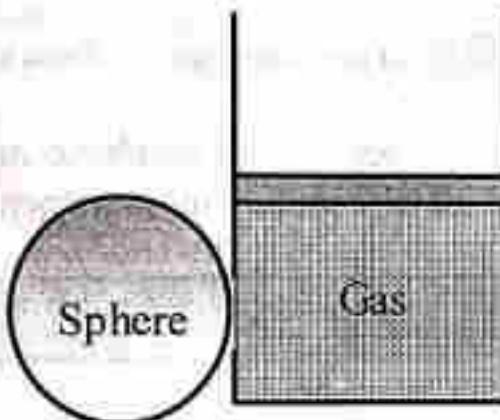


Figure 1.14: An arrangement for equality of temperature

1.6.2 Zeroth Law of Thermodynamics

Consider three systems A, B and C as shown in **Figure 1.13**. If A and C are brought in contact with each other and there is no change in any properties of both the system, then we can conclude that both A and C have same temperatures. Again, if B and C are brought in contact with each other and there is no change in any properties of both the system, then we can conclude that both B and C have same temperatures. Then without performing third experiment, we can conclude that A and B also have same temperatures.

This fundamental is called Zeroth law of thermodynamics which can be stated as:

When two systems are separately in thermal equilibrium with a third system, then these two systems should also be in thermal equilibrium.

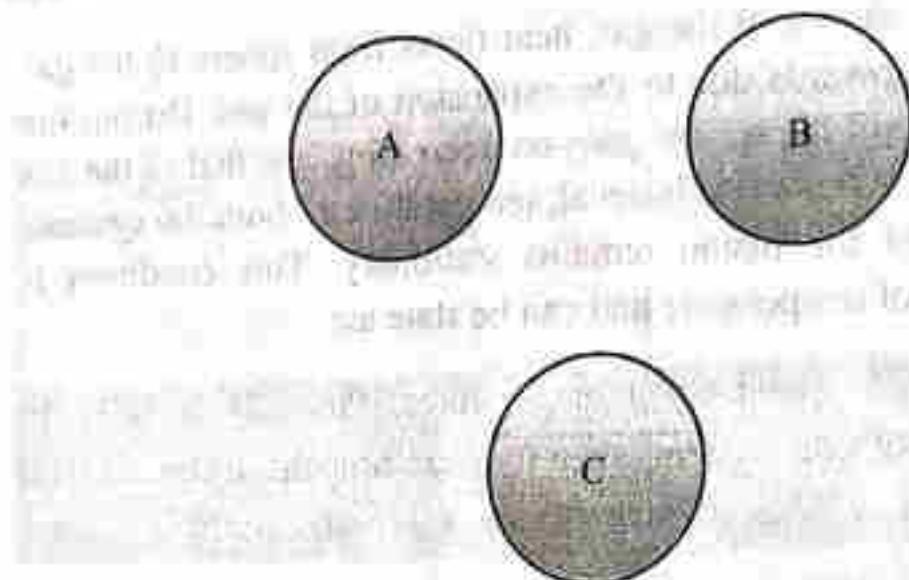


Figure E1.5: An arrangement for Zeroth law

This basic fundamental has been given emphasis and named as Zeroth law because it is used in all temperature measurement process.

SOLVED EXAMPLES

Example E1.1

A container has two compartments separated by a membrane as shown in Figure E1.1. Compartment A has 2 kg of air and a volume of 1 m^3 . Compartment B has 1.5 m^3 of air with a specific volume of $2.5 \text{ m}^3/\text{kg}$. If the membrane is broken, determine the resultant specific volume.



Solution

Figure E1.1

For Initial State;

Initial Volume of compartment A (V_A)	$= 1 \text{ m}^3$
Mass of air in compartment A (m_A)	$= 2 \text{ kg}$
Initial Volume of compartment B (V_B)	$= 1.5 \text{ m}^3$
Initial specific volume of air in compartment B (v_B)	$= 2.5 \text{ m}^3/\text{kg}$

$$\text{Mass of steam in compartment B (m}_B\text{)} = \frac{V_B}{v_B} = \frac{1.5}{2.5} = 0.6 \text{ kg}$$

For Final State;

$$\text{Total volume (V)} = V_A + V_B = 1 + 1.5 = 2.5 \text{ m}^3$$

$$\text{Total mass (m)} = m_A + m_B = 2 + 0.6 = 2.6 \text{ kg}$$

$$\therefore \text{Resulting specific volume (v)} = \frac{V}{m} = \frac{2.5}{2.6} = 0.96154 \text{ m}^3/\text{kg}$$

Example E1.2

Three pressure gauges are connected to a container consisting of two compartments as shown in Figure E1.2. If the local barometer reads 750 mm of Hg and pressure gauges A and B read 300 kPa and 200 kPa respectively. Determine the absolute pressure in each compartment and reading of pressure gauge C.

[Take $\rho_{\text{Hg}} = 13600 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$]

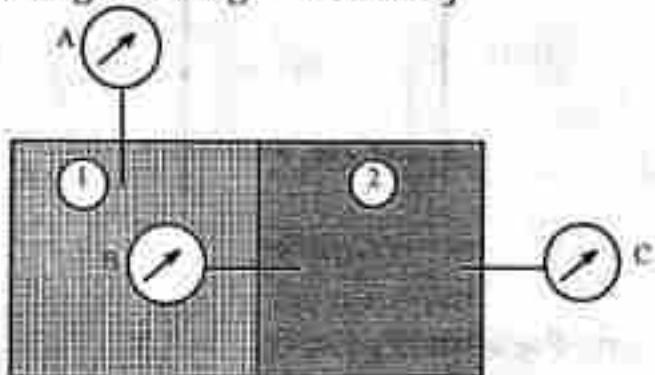


Figure E1.2

Solution

$$\text{Given, Barometer reading (z}_{\text{baro}}\text{)} = 750 \text{ mm}$$

$$\text{Gauge Pressure for Gauge A (P}_A\text{)} = 300 \text{ kPa}$$

$$\text{Gauge Pressure for Gauge B (P}_B\text{)} = 400 \text{ kPa}$$

$$\therefore \text{Atmospheric pressure, P}_{\text{atm}} = \rho_{\text{Hg}} g z_{\text{baro}} = 13600 \times 9.81 \times 750 \times 10^{-3}$$

$$= 100.062 \text{ kPa}$$

Here pressure gauge A measures pressure of compartment 1 relative to atmosphere, pressure gauge B measures pressure of compartment 2 and the pressure gauge C measures the pressure of compartment 2 relative to the atmosphere.

Hence the absolute pressure of the compartment 1 is given by,

$$(P_{\text{abs}})_1 = P_{\text{atm}} + P_A = 100.062 + 300 = 400.062 \text{ kPa}$$

Similarly, the absolute pressure of the compartment 2 is given by,

$$(P_{\text{abs}})_2 = (P_{\text{abs}})_1 + P_B = 400.062 + 200 = 600.062 \text{ kPa}$$

For Pressure Gauge C

$$(P_{\text{abs}})_2 = P_{\text{atm}} + P_C$$

$$\therefore P_C = (P_{\text{abs}})_2 - P_{\text{atm}} = 600.062 - 100.062 = 500 \text{ kPa}$$

Example E1.3

The piston of a vertical piston cylinder device containing gas has mass of 50 kg and a cross sectional area of 0.02 m^2 , as shown in Figure E1.3.

- (a) Determine the pressure inside the cylinder.
- (b) During some process heat is lost by the gas to the surroundings and its volume decreases to $\frac{1}{4}$ th of the initial volume, determine its final pressure. [Take $P_{\text{atm}} = 100 \text{ kPa}$ and $g = 9.81 \text{ m/s}^2$]

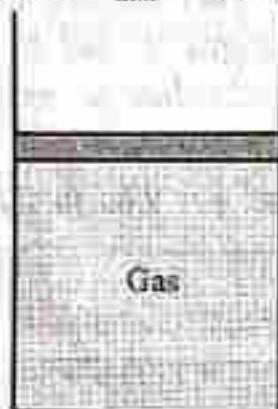


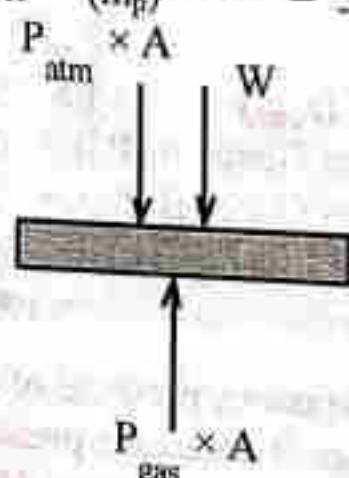
Figure E1.3

Solution

Given, Atmospheric Pressure (P_{atm}) = 100 kPa

Area of the Piston (A) = 0.02 m^2

Mass of the Piston (m_p) = 50 kg



- (a) Referring to the free body diagram of the piston we can write the equilibrium equation as,

$$P_{\text{gas}} \times A + P_{\text{atm}} \times A + W = P_{\text{atm}} \times A + m_p g$$

$$P_{\text{gas}} = P_{\text{atm}} + \frac{m_p g}{A}$$

$$= 100 \times 10^3 + \frac{50 \times 9.81}{0.02} = 124525 \text{ Pa} = 124.525 \text{ kPa}$$

- (b) If we draw free body diagram for the final state there will be no change in any forces, hence pressure remains same.

Example 1.4

A new scale N of temperature is devised in such a way that the freezing point of ice is 20°N and boiling point is 200°N . What will be the temperature reading on ${}^{\circ}\text{C}$, ${}^{\circ}\text{F}$ and K scales when the temperature on the new scale is 160°N ?

Solution

Freezing point and boiling point for the different scales mentioned above are tabulated below:

Scale	Freezing Point	Boiling Point
${}^{\circ}\text{N}$	20	200
${}^{\circ}\text{C}$	0	100
${}^{\circ}\text{F}$	32	212
K	273	373

Then conversion relationship is given by

$$\frac{N - 20}{200 - 20} = \frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{K - 273}{373 - 273}$$

$$\text{or, } \frac{N - 20}{9} = \frac{C}{5} = \frac{F - 32}{9} = \frac{K - 273}{5}$$

Each scales are related to the new scale as

$$C = \frac{5}{9} (N - 20)$$

$$F = N + 12$$

$$K = \frac{5}{9} (N - 20) + 273$$

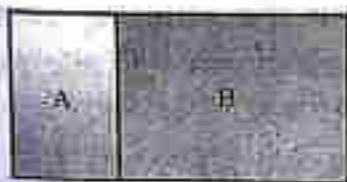
Substituting $N = 160^{\circ}\text{N}$, we get

$$160^{\circ}\text{N} = 77.778 {}^{\circ}\text{C} = 172 {}^{\circ}\text{F} = 350.778 \text{ K}$$

10. Differentiate between state function and path function with examples.
11. Define a thermodynamic process. Sketch P – V, T – V and P – T diagrams for an ideal gas undergoing
 - constant volume cooling process
 - constant pressure cooling process
 - constant temperature heat rejection process.
12. Explain reversible and irreversible processes with reference to intermediate states.
13. Define specific volume and temperature.
14. Define atmospheric pressure, gauge pressure and absolute pressure. Also write down the relationship between them.
15. Define vacuum gauge. Show atmospheric pressure, gauge pressure and absolute pressure on a common pressure scale.
16. State and explain and equality of temperature.
17. State and explain Zeroth law of thermodynamics. Also write down its application.

NUMERICAL PROBLEMS

1. A container has two compartments separated by a membrane as shown in **Figure P1.1**. Specific volume of steam in compartment A and compartment B are $5 \text{ m}^3/\text{kg}$ and $10 \text{ m}^3/\text{kg}$ respectively. If the membrane breaks and steam comes to a uniform state, determine the resulting specific volume.
2. A container having two compartments contains steam as shown in **Figure P1.2**. The specific volume of steam in compartment B is $5 \text{ m}^3/\text{kg}$. The membrane breaks and the resulting specific volume is $8 \text{ m}^3/\text{kg}$. Find the original specific volume of steam in compartment A.
3. A cylinder with a total volume of 1 m^3 has a movable piston as shown in **Figure P1.3**. When the piston is at one fourth of the length, both sides have same specific volume of $4 \text{ m}^3/\text{kg}$. Determine the specific volumes of both sides when the piston is at middle of the cylinder.



$$V_A = 0.2 \text{ m}^3 \quad V_B = 0.4 \text{ m}^3$$

Figure P1.1



$$V_A = 15 \text{ m}^3 \quad V_B = 5 \text{ m}^3$$

Figure P1.2

4. An oxygen cylinder having a volume of 10 m^3 initially contains 5 kg of oxygen. Determine the specific volume of oxygen in the cylinder initially. During certain process 3 kg of oxygen is consumed. Also determine the final specific volume of oxygen in the cylinder. Also plot the amount of oxygen that has been consumed versus the specific volume of the remaining in the cylinder.
5. A large chamber is separated into two compartments which are maintained different pressures, as shown Figure P1.5. Pressure gauge A reads 180 kPa, and pressure gauge B reads 120 kPa. If the barometric pressure is 100 kPa, determine the absolute pressure existing in the compartments and the reading of gauge C.

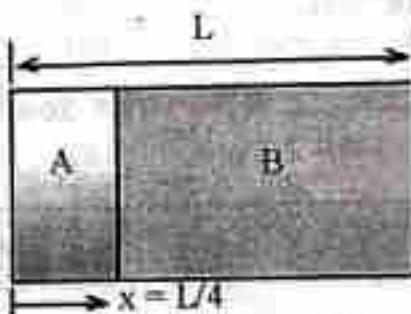


Figure P1.3

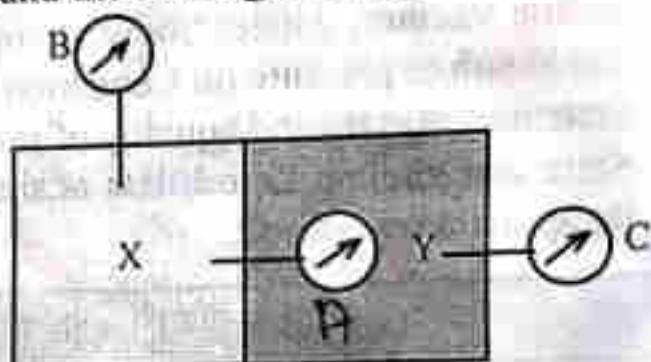


Figure P1.5

6. A pressure gauge connected to a cylinder reads 400 kPa at a location where the atmospheric pressure is 100 kPa. Determine the absolute pressure in the cylinder.
7. The absolute pressure inside a tank is 50 kPa and the surrounding atmospheric pressure is 100 kPa. What reading a gauge mounted in the tank will give? Comment upon the result.
8. A vacuum gauge indicates that pressure of air in a cylinder is 15 kPa (vacuum). The local barometer reads 750 mm of Hg. Determine the absolute pressure inside the cylinder. [Take $\rho_{\text{Hg}} = 13600 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$]
9. At the inlet and exhaust of a turbine the absolute steam pressures are 6000 kPa and 4.0 cm Hg, respectively. Barometric pressure is 75 cm Hg. Calculate the gage pressures for the entering steam and the vacuum gage pressure for the exhaust steam. [Take $\rho_{\text{Hg}} = 13600 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$]
10. During the operation of a lift, it can be subjected to a maximum pressure of 500 kPa. If it is designed to lift a mass upto 900 kg, what should be diameter of the piston/cylinder? [Take $g = 9.81 \text{ m/s}^2$]

11. A piston cylinder arrangement shown in Figure P1.11 has a cross sectional area of 0.01 m^2 and a piston mass of 80 kg. If atmospheric pressure is 100 kPa, what should be the gas pressure to lift the piston? [Take $g = 9.81 \text{ m/s}^2$]

12. A piston cylinder has a diameter of 0.1 m. With an outside atmospheric pressure of 100 kPa, determine the piston mass that will create an inside pressure of 500 kPa. [Take $g = 9.81 \text{ m/s}^2$]

13. For the piston cylinder device shown in Figure P1.13, determine the absolute pressure inside the device. [Take $P_{\text{atm}} = 101.3 \text{ kPa}$]



Figure P1.11

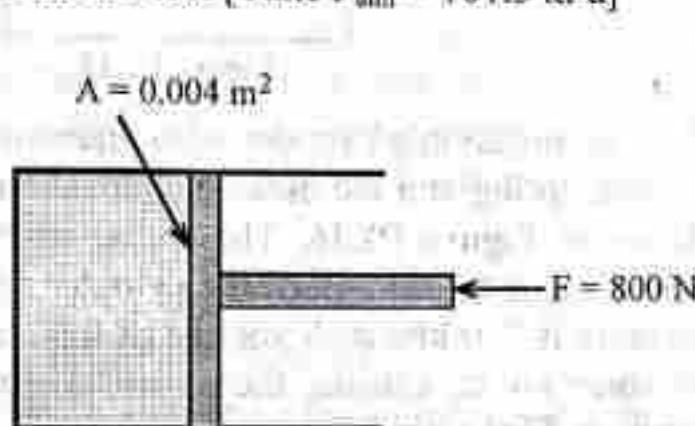


Figure P1.13

14. For the piston cylinder device shown in Figure P1.14, determine the force necessary to produce an absolute pressure of 500 kPa within the device. [Take $P_{\text{atm}} = 100 \text{ kPa}$]

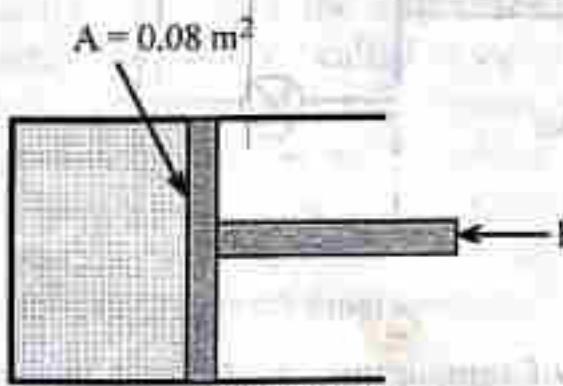


Figure P1.14

15. A piston cylinder device loaded with a linear spring with a spring constant of $k = 100 \text{ kN/m}$ contains a gas initially at a pressure of 100 kPa and a volume of 0.05 m^3 , as shown in Figure P1.15. The cross sectional area of the piston is 0.1 m^2 . Initially spring touches the piston but exerts no force on it. Heat is supplied to the system until its volume doubles, determine the final pressure.

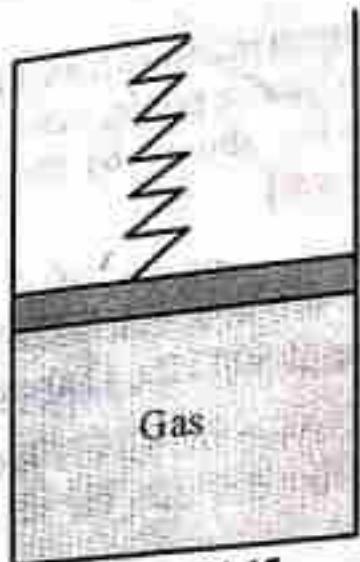


Figure P1.15

16. A 15 kg piston in a cylinder with diameter of 0.15 m is loaded with a linear spring and the outside atmospheric pressure of 100 kPa, as shown in Figure P1.16. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 300 kPa with volume of 0.02 m^3 . The valve is opened to let some air in, causing the piston to rise by 5 cm. Find the new pressure. [Take $g = 9.81 \text{ m/s}^2$]

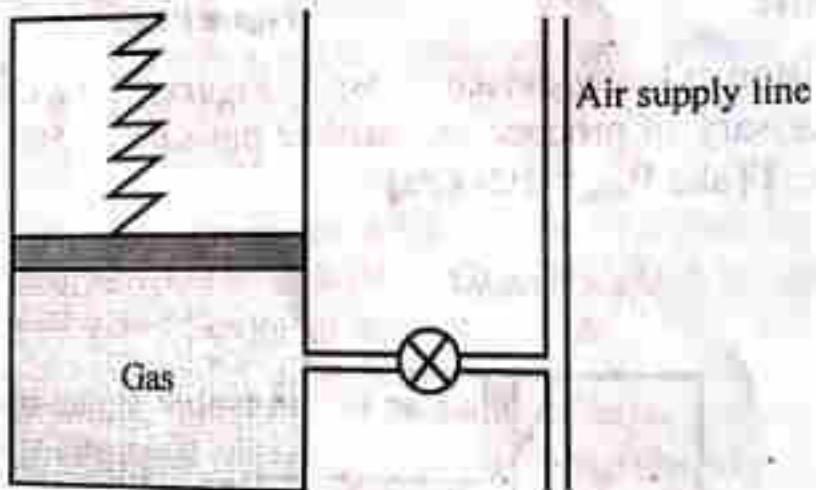


Figure P1.16

17. A new scale N of temperature is devised in such a way that the freezing point of ice is 100°N and boiling point is 400°N . What is the temperature reading on this new scale when the temperature is 150°C ? At what temperature, both the Celsius and new temperature scale reading would be the same?
18. The temperature of a system drops by 36°F during a process. Express this drop in temperatures in $^\circ\text{C}$ and K .

ENERGY AND ENERGY TRANSFER

2

2.1 Energy and its Meaning

In classical mechanics, energy of a system is defined as its capacity to perform work. Energy can also be defined as the capacity of a system to exert (provide) a force for a certain interval. If a system has high energy content, it can provide higher magnitude of force for larger interval of time and if it has less energy content, it can provide lesser magnitude of force for smaller interval of time.

Different forms of energies are classified into two groups in thermodynamics: stored energy and transient energy.

Energy which cannot cross the boundary of the system and remains within the system boundary as inherent property of the system is called stored energy. Internal energy, potential energy and kinetic energy are examples of stored energy. Stored energies have unique value for each equilibrium states and are independent of path and hence are thermodynamic properties.

Energy which can cross the boundary of the system i.e., can flow from the system to surroundings or from the surroundings to the system during a thermodynamic process is called transient energy. Work transfer and heat transfer are examples of transient energy. Transient energies do not have unique values for each equilibrium states as they depend not only on the properties of the system but also on the properties of the surroundings. Furthermore, they are dependent upon the path and hence are not thermodynamic properties.

2.2 Stored Energy

2.2.1 Internal Energy

Internal energy is the energy of the system due to its molecular activity. It can also be defined as the summation of molecular potential energy and molecular kinetic energy. It is due to microscopic phenomena. As the molecular activity increases with temperature, internal energy of the system increases with increase in temperature. It is denoted by U .

2.2.2 Potential Energy

Potential energy is the energy of the system due to its position. Water stored on a reservoir or a compressed spring can store energy in the form of potential energy. In thermodynamics, most of the time we have to deal with gravitational potential energy and the magnitude of gravitational potential energy of the system with respect to a certain reference level is given as

$$PE = mgz \quad \dots \dots \dots (2.1)$$

where m is the mass of the system, g is local gravitational acceleration and z is the elevation of the system. It is due to macroscopic phenomena.

2.2.3 Kinetic Energy

Kinetic energy is the energy of the system due to its motion. Kinetic energy of the system is given as

$$KE = \frac{1}{2}m\bar{V}^2 \quad \dots \dots \dots (2.2)$$

where m is the mass of the system and \bar{V} is the velocity of the system. It is also due to macroscopic phenomena.

2.2.4 Total Energy

Total energy of a system is defined as the summation of its internal energy, potential energy and kinetic energy. It is denoted by E .

$$\cancel{E} = U + PE + KE = U + mgz + \frac{1}{2}m\bar{V}^2 \quad \dots \dots \dots (2.3)$$

We can also define specific total energy of the system, e as

$$e = \frac{E}{m} = u + gz + \frac{1}{2}\bar{V}^2 \quad \dots \dots \dots (2.4)$$

2.3 Energy Transfer

Energy which can be transferred from the system to the surroundings or from the surroundings to the system are classified as heat transfer or work transfer.

2.3.1 Heat Transfer

Transfer of energy, without transfer of mass, because of temperature difference between the system and the surroundings is called heat transfer. Heat transfer is denoted by Q and is expressed in J .

In thermodynamics, heat transferred to (gain by or supplied to) the system is taken as positive heat transfer and heat transferred from (lost by or rejected by) the system is taken as negative heat transfer.

2.3.2 Work Transfer

In classical mechanics, work is said to be done if certain system is displaced by a force in the direction of force. Mathematically,

$$W = \int F \cdot d\vec{s} \quad \dots \dots \dots (2.5)$$

where F is the force applied and \vec{s} is the displacement. This definition is called mechanical definition of work.

This definition of work transfer is not directly applicable for all thermodynamic processes because during some processes energy is transferred without observable macroscopic displacement.

For example, consider an electrical resistor connected to an electric battery as shown in Figure 2.1. During any process in this system, energy is transferred from battery to resistor and then from the resistor to the surroundings. When switch is turned on, current flows through resistor and its temperature increases and then energy is lost from the resistor surface in the form of heat transfer because of temperature difference between the resistor surface and the ambient air. If we analyze the energy transfer from battery to resistor, it is not dependent upon the temperature and hence it is not the heat transfer. So this mode of energy transfer should be work transfer although it does not have any macroscopic displacement.

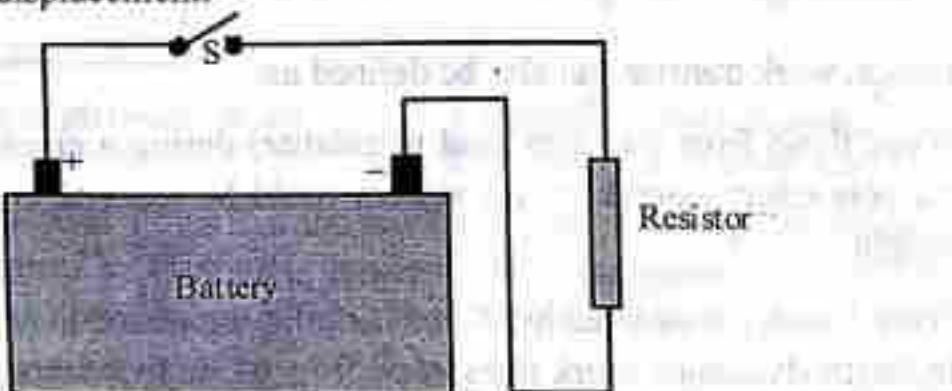


Figure 2.1 Energy Transfer from Battery without Macroscopic Displacement

To explain such kind of thermodynamic processes completely, we redefine work transfer in thermodynamics as:

Work transfer is the transfer of energy, without transfer of mass, because of any property difference other than temperature.

This definition is called thermodynamic definition of work. Common property differences that can produce work transfer are pressure, gravitational potential, electric potential, etc.

To explain the process discussed above with reference to classical definition of work we can replace the electric resistor with a motor having same value of internal resistance, as shown in **Figure 2.2**. So during the same interval it will consume the same amount of energy from the battery. In this case when energy is supplied to motor, it can lift certain weight and macroscopic displacement can be observed.

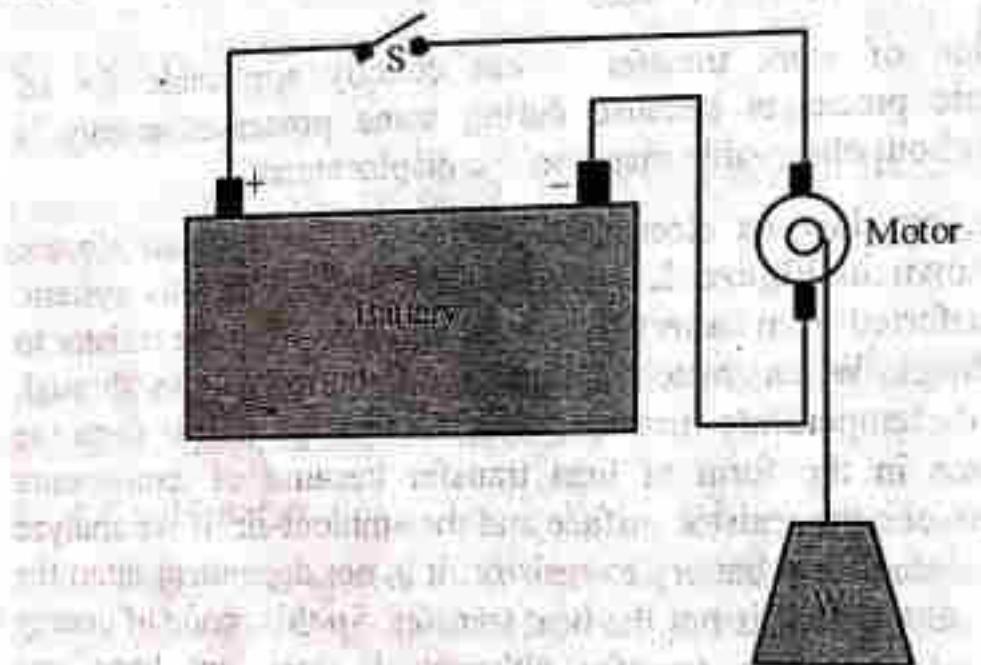


Figure 2.2 Energy Transfer from Battery with Macroscopic Displacement

Hence, work transfer can also be defined as:

Work flows from a system (and to another) during a given operation if the sole effect external to the system could be reduced to the rise of a weight.

Work transfer is denoted by W and it is also expressed in J. In thermodynamics, work transferred from (done by or produced by) the system is taken as positive work transfer and work transferred to (done on or consumed by) the system is taken as negative work transfer.

ASKED QUESTIONS

2.4 Expression for Displacement Work Transfer

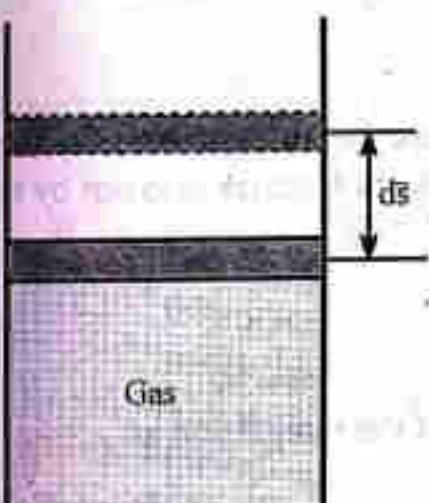
Consider a piston cylinder device containing a gas. During process 1-2, piston is displaced by $d\bar{s}$ from state 1 to state 2. Applying mechanical definition, work transfer is evaluated as

$$W = \int_a^b F \, ds$$

where F is the force provided by the gas pressure, i.e., $F = PA$.

Substituting F into above equation, we get

$$W = \int P A \, dS$$



Module 2-35 Displacement Work Transfer

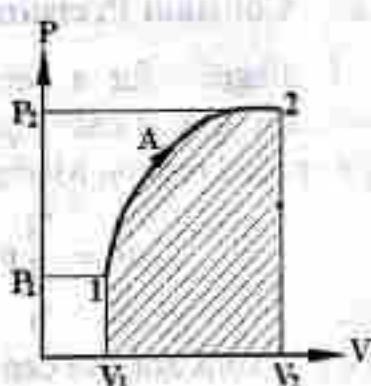


Figure 2-4 Work Transfer Evaluated as Area under Curve

The above equation shows that the work transferred during any process can be determined by evaluating area covered by the process on a $P - V$ diagram (with respect to V axis) as shown in Figure 2.4.

We can further derive expressions for work transfer for different specific conditions.

2.4.1 Constant Volume Process

$P - V$ diagram for a constant volume heating process is shown in Figure 2.5. The area covered by the process 1 – 2 on $P - V$ diagram is zero. Hence, work transfer during a constant volume process is zero. It is obvious because during constant volume process there is no displacement ($dV = 0$) i.e.,

$$\therefore W = \int_{V_1}^{V_2} P dV = 0 \quad \dots \dots \dots (2.7)$$

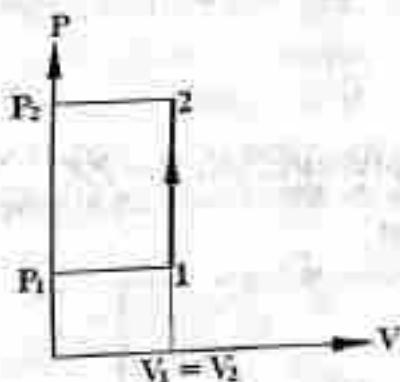


Figure 2.5: Constant Volume Heating Process

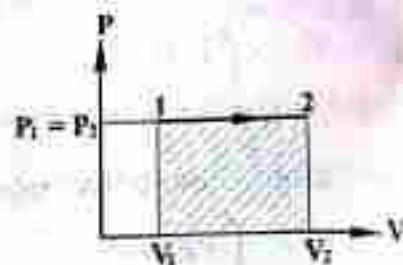


Figure 2.6: Constant Pressure Heating Process

2.4.2 Constant Pressure Process

$P - V$ diagram for a constant pressure heating process is shown in **Figure 2.6**. In this case, magnitude of work transfer is given by the area of the shaded region. Mathematically,

$$W = \int_{V_1}^{V_2} P dV$$

Since P is constant, we can take it out of the integration as

$$\begin{aligned} W &= P \int_{V_1}^{V_2} dV = P [V] \Big|_{V_1}^{V_2} \\ &= P (V_2 - V_1) \quad \dots \dots \dots (2.8) \end{aligned}$$

2.4.3 Constant Temperature Process

Pressure volume relationship for an ideal gas undergoing a constant temperature process is given as

$$P \propto \frac{1}{V} \quad \text{or,} \quad PV = \text{constant} \quad \dots \dots \dots (2.9)$$

$P - V$ diagram for a constant temperature heating (expansion) process is shown in **Figure 2.7**. Applying Equation (2.9) for initial, final and all intermediate states,

$$P_1 V_1 = P_2 V_2 = PV \quad \dots \dots \dots (2.10)$$

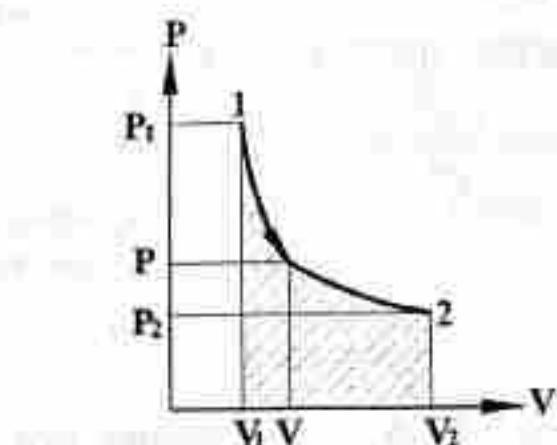


Figure 2.7: Constant Temperature Process

Pressure at any intermediate state is given as

$$P = \frac{P_1 V_1}{V} \quad \dots \dots \dots (2.11)$$

Then work transfer is given as

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{P_1 V_1}{V} dV = P_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\therefore W = P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) \quad \dots \dots \dots (2.12)$$

2.4.4 Polytropic Process

Thermodynamic process which follows the relation $PV^n = \text{constant}$ is called a polytropic process and the index n is called a polytropic index. It is a generalized equation for thermodynamic processes and represents different processes for different values of n , as shown in Figure 2.8.

Value of index (n)	Equation	Process
0	$P = \text{constant}$	Constant pressure or Isobaric
1	$PV = \text{constant}$	Constant temperature or Isothermal
γ	$PV^\gamma = \text{constant}$	Adiabatic
∞	$V = \text{constant}$	Constant volume or Isochoric

Pressure-volume relationship for initial, final and any intermediate state during a polytropic process is given as

..... (2)

$$P_1 V_1^n = P_2 V_2^n = PV^n$$

Pressure at any intermediate state is given as

$$P = \frac{P_1 V_1^n}{V^n}$$

..... (2)

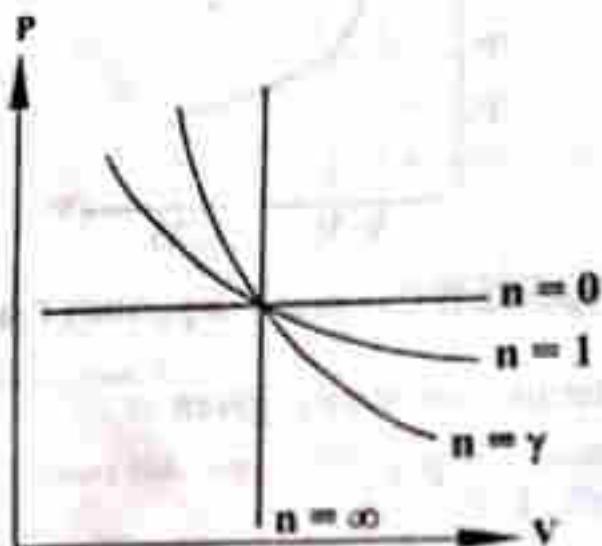


Figure 2.34 Polytropic Process

Then work transfer is given as

$$= \int_{V_1}^{V_2} P dV$$

$$W = \int_{V_1}^{V_2} \frac{P_1 V_1^n}{V^n} dV$$

$$= P_1 V_1^n \int_{V_1}^{V_2} \frac{dV}{V^n}$$

$$= \frac{P_1 V_1^n}{1-n} (V_2^{1-n} - V_1^{1-n})$$

$$= \frac{P_1 V_1^n V_2^{1-n} - P_1 V_1^n}{1-n}$$

Substituting $P_1 V_1^n = P_2 V_2^n$, we get

$$W = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

Equation (2.15) is applicable only when $n \neq 1$.

2.5 Power

Power is defined as a rate of energy transfer. In thermodynamics, we deal with two modes of energy transfer: work transfer and heat transfer.

Power due to work transfer also called mechanical power is defined as the rate of work transfer. Mathematically,

$$\dot{W} = \lim_{\Delta t \rightarrow 0} \frac{\delta W}{\Delta t} \quad \dots \dots \dots (2.16)$$

Power due to heat transfer also called thermal power is defined as the rate of heat transfer. Mathematically,

$$\dot{Q} = \lim_{\Delta t \rightarrow 0} \frac{\delta Q}{\Delta t} \quad \dots \dots \dots (2.17)$$

Power is expressed in J/s or Watt (W).

SOLVED EXAMPLES

Example 2.1

A piston cylinder arrangement contains a gas initially at a pressure of 800 kPa and a volume of 0.1 m³. Determine the work done by the gas when it undergoes the following processes to a final volume of 0.5 m³:

- (a) constant pressure.
- (b) according to linear law to a final pressure of 200 kPa.
- (c) according to law $PV = \text{constant}$.
- (d) according to law $PV^2 = \text{constant}$.
- (e) according to the relation $P = a + \frac{b}{V}$ to a final pressure of 200 kPa,
where a and b are constants.

Solution

Given, Initial State: $P_1 = 800 \text{ kPa}$, $V_1 = 0.1 \text{ m}^3$

Final State: $V_2 = 0.5 \text{ m}^3$

- (a) For constant pressure expansion to final volume of 0.5 m³ work transfer is given by

$$W = P_1(V_2 - V_1) = 800(0.5 - 0.1) = 320 \text{ kJ}$$

- (b) For linear $P - V$ relation process, work done can be evaluated by determining the area under the $P - V$ diagram (Figure E2.1);

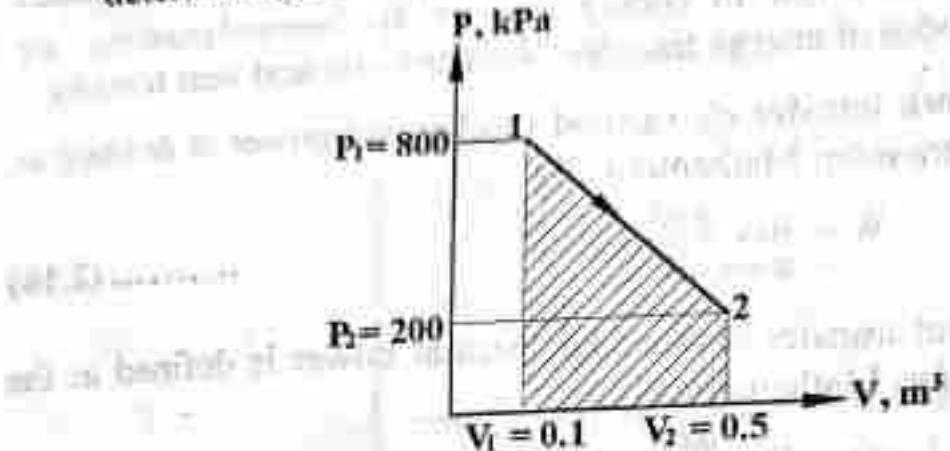


Figure E2.1

$$W = \frac{1}{2} (P_1 + P_2) (V_2 - V_1) = \frac{1}{2} (800 + 200) (0.5 - 0.1) = 200 \text{ kJ}$$

Alternative Method

$P - V$ relation for straight line joining state 1 and state 2 is given as

$$P - P_1 = \frac{P_2 - P_1}{V_2 - V_1} (V - V_1)$$

$$\text{or, } P - 800 = \frac{200 - 800}{0.5 - 0.1} (V - 0.1)$$

$$\therefore P = 950 - 1500V$$

Then work transfer can also be calculated as

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} (950 - 1500V) dV = 200 \text{ kJ}$$

- (c) For $PV = \text{constant}$ i.e., an isothermal process, work done is given by

$$W = P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) = 800 \times 0.1 \times \ln \left(\frac{0.5}{0.1} \right) = 128.755 \text{ kJ}$$

- (d) For a polytropic process,

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

by

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^2 = 32 \text{ kPa}$$

Then work done during the polytropic expansion is given by,

$$W = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{32 \times 0.5 - 800 \times 0.1}{1-2} = 64 \text{ kJ}$$

- (c) Substituting initial and final states on given process equation, we get

$$a + 10b = 800 \quad \dots \dots \dots \text{(i)}$$

$$a + 2b = 200 \quad \dots \dots \dots \text{(ii)}$$

Now, solving equations (i) and (ii),

$$a = 50 \quad \text{and} \quad b = 75$$

Hence, the pressure volume relation for the process is given as

$$P = \left(50 + \frac{75}{V} \right) \text{kPa}$$

Then work transfer during the process is given as

$$\begin{aligned} W &= \int_{V_1}^{V_2} P dV = \int_{V_1=0.1}^{V_2=0.5} \left(50 + \frac{75}{V} \right) dV \\ &= 50 (V_2 - V_1) + 75 \ln \left(\frac{V_2}{V_1} \right) = 140.708 \text{ kJ} \end{aligned}$$

Example 2.2

A piston cylinder device shown in Figure E2.2 contains 2 kg of air initially at a pressure of 200 kPa and a temperature of 50°C. The volume of the air when the piston reaches the upper stops is 1.5 m³. There is heat transfer to the air until its temperature reaches 1200°C. It takes a pressure of 500 kPa to lift the piston from the bottom stops. Sketch the process on P-V and T-V diagrams and determine the final pressure and the total work transfer. [Take R = 287 J/kg K]



Figure E2.2

Solution

Given, Mass of air: $m = 2 \text{ kg}$

Initial State: $P_1 = 200 \text{ kPa}$, $T_1 = 273 + 50 = 323 \text{ K}$

Maximum volume: $V_{\max} = 1.5 \text{ m}^3$

Final Temperature: $T_{\text{final}} = 273 + 1200 = 1473 \text{ K}$

Pressure required to lift the piston: $P_{\text{lif}} = 500 \text{ kPa}$

$$\therefore \text{Volume of air at initial state } V_1 = \frac{mRT_1}{P_1} = \frac{2 \times 287 \times 323}{200 \times 10^3} = 0.92701 \text{ m}^3$$

Initial pressure of the system is 200 kPa and pressure required to lift the piston is 500 kPa. Hence, during initial stage of heating piston remain stationary although heat is supplied to the system, so process is constant volume heating (Process 1-2). During constant volume heating pressure of the system increases from 200 kPa to 500 kPa. Then we can define state 2 as

State 2: $P_2 = 500 \text{ kPa}$, $V_2 = 0.92701 \text{ m}^3$

$$\therefore \text{Temperature of air at state 2 (}T_2\text{)} = \frac{P_2V_2}{mR} = \frac{500 \times 10^3 \times 0.92701}{2 \times 287} = 807.5 \text{ K} = 534.5^\circ\text{C}$$

Here T_2 is less than the final required temperature ($T_{\text{final}} = 1200^\circ\text{C}$), hence the system should be further heated. After reaching 500 kPa, further heating causes displacement of the piston to upward direction and the process occurs at a constant pressure of 500 kPa (Process 2-3) until it just touches the upper stops. Hence, we can define state 2 and state 3 as

State 3: $P_3 = 500 \text{ kPa}$, $V_3 = 1.5 \text{ m}^3$

$$\therefore \text{Temperature of air at state 3 (}T_3\text{)} = \frac{P_3V_3}{mR} = \frac{500 \times 10^3 \times 1.5}{2 \times 287} = 1306.62 \text{ K} = 1033.62^\circ\text{C}$$

Here T_3 is still less than the final required temperature ($T_{\text{final}} = 1200^\circ\text{C}$), hence the system should be further heated. to increase the temperature from 1033.62°C to 1200°C and the process occurs at constant volume (Process 3-4). Hence, we can define state 4 as

State 4: $T_4 = 1473 \text{ K}$, $V_4 = 1.5 \text{ m}^3$

$$\therefore \text{Pressure of air at final state (}P_4\text{)} = \frac{mRT_4}{V_4} = \frac{2 \times 287 \times 1473}{1.5}$$

$P - V$ and $T - V$ diagrams for the process is shown in Figure E2.2 (a) and (b).

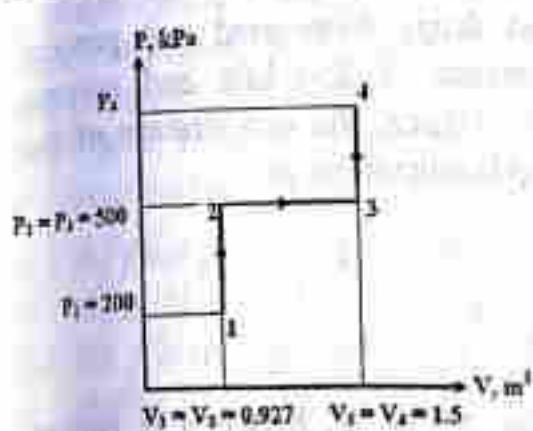


Figure E2.2 (a): P-V diagram

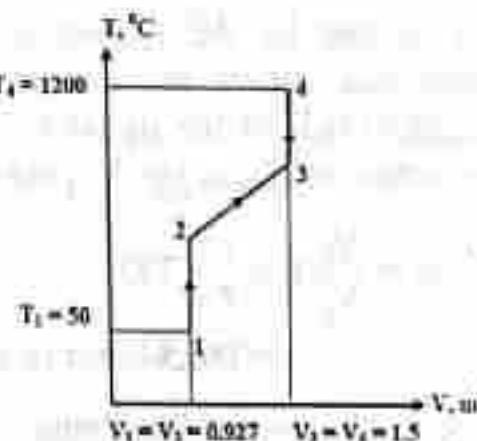


Figure E2.2 (b): T-V diagram

Total work transfer for the process is given as

$$\begin{aligned} W &= W_{12} + W_{23} + W_{34} \\ &= 0 + P_2(V_3 - V_2) + 0 \\ &= 500(1.5 - 0.92701) = 286.495 \text{ kJ} \end{aligned}$$

Detailed Solution

A piston cylinder device shown in Figure E2.3 contains 1 kg of Nitrogen initially at a pressure of 250 kPa and a temperature of 500°C. Heat is lost from the system till its temperature reaches 25°C. Sketch the process on P-V and T-V diagrams and determine the final pressure and the total work transfer. [Take $R = 297 \text{ J/kg K}$]

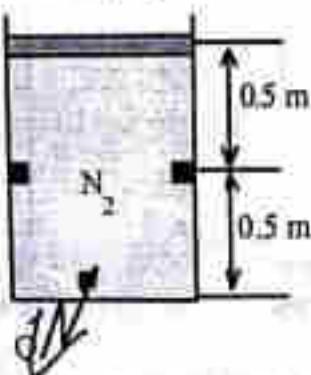


Figure E2.3

Solution

Given, Mass of N_2 : $m = 1 \text{ kg}$

Initial State: $P_1 = 250 \text{ kPa}$, $T_1 = 273 + 500 = 773 \text{ K}$

Final Temperature: $T_{\text{final}} = 273 + 25 = 298 \text{ K}$

$$\therefore \text{Volume of N}_2 \text{ at initial state } V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 297 \times 773}{250 \times 10^3} = 0.918324 \text{ m}^3$$

If heat is lost by the system, piston drops downward and process (Process 1-2) occurs at constant pressure of 250 kPa and volume decreases to half of the initial volume. Hence, the temperature of the system when the piston just hits the stop is calculated as

$$T_2 = \frac{V_2}{V_1} T_1 = \frac{1}{2} \times 773 \\ = 386.5 \text{ K} = 113.5^\circ\text{C}$$

But the final required final temperature is 50 °C, hence it is further cooled to decrease the temperature from 113.5 °C to 25 °C and the process occurs at constant volume (Process 2-3). Hence, we can define state 3 as

$$\text{State 3: } T_3 = 298 \text{ K} \quad V_3 = \frac{V_1}{2} = 0.459162 \text{ m}^3$$

$$\therefore \text{Pressure of N}_2 \text{ at final state } P_3 = \frac{mRT_3}{V_3} = \frac{1 \times 297 \times 298}{0.459162} \\ = 192.756 \text{ kPa}$$

$P - V$ and $T - V$ diagrams for the process is shown in Figure E2.3 (a) and (b).

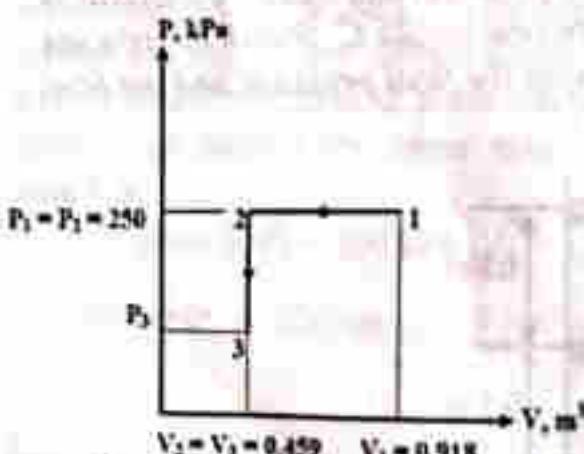


Figure E2.3 (a): P-V diagram

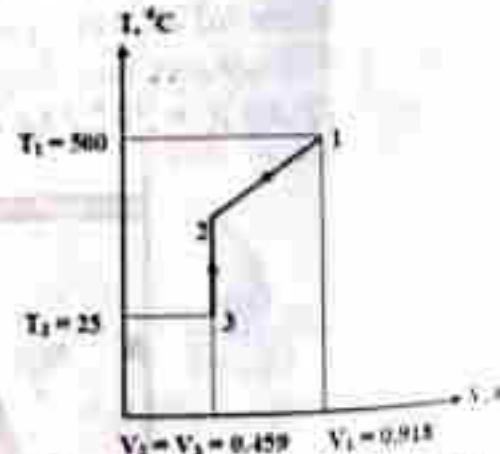


Figure E2.3 (b): T-V diagram

Total work transfer for the process is given as

$$W = W_{12} + W_{23} \\ = P_1(V_2 - V_1) + 0 \\ = 250(0.459162 - 0.918324) \\ = -114.791 \text{ kJ}$$

~~Example 2.4~~

A piston cylinder device restrained by a linear spring ($k = 200 \text{ kN/m}$) shown in Figure E2.4 contains a gas initially at a pressure of 400 kPa and a volume of 0.5 m^3 . Initially spring touches the piston but exerts no force on it. Heat is transferred to the system until its volume doubles. If the cross sectional area of the piston is 0.4 m^2 , determine

- the final pressure inside the cylinder,
- the total work done by the gas, and
- the fraction of total work done consumed for the compression of spring.

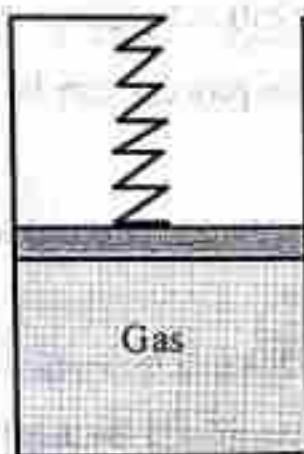


Figure E2.4

Solution

Given, Initial State: $P_1 = 400 \text{ kPa}$,

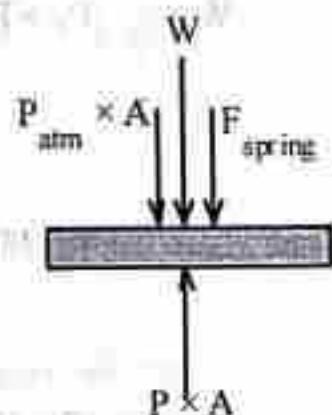
$$V_1 = 0.5 \text{ m}^3$$

$$\text{Final State: } V_2 = 2V_1 = 1 \text{ m}^3$$

$$\text{Spring constant: } k = 200 \text{ kN/m}$$

$$\text{Cross sectional area of the piston:}$$

$$A = 0.4 \text{ m}^2$$



Referring to the free-body diagram of the piston, we can write equation for the pressure inside the cylinder as

$$P = P_{\text{atm}} + \frac{W}{A} + \frac{F_{\text{spring}}}{A}$$

Initially spring touches the piston but exerts no force on it. Substituting initial state ($P_1 = 400 \text{ kPa}$, $F_{\text{spring}} = 0$) on the pressure equation, we get

$$400 = P_{\text{atm}} + \frac{W}{A} + 0$$

$$P_{\text{atm}} + \frac{W}{A} = 400$$

Hence, the pressure equation for any intermediate state is given as

$$P = P_{\text{atm}} + \frac{W}{A} + \frac{F_{\text{spring}}}{A}$$

$$P = 400 + \frac{kx}{A} = 400 + \frac{k(V - V_1)}{A^2} = 400 + \frac{200(V - 0.5)}{(0.4)^2}$$

$$\therefore P = 1250V - 225$$

- (a) Pressure at final state is given as

$$\therefore P_2 = 1250V_2 - 225 = 1250 \times 1 - 225 = 1025 \text{ kPa}$$

- (b) Total work done during the process can be evaluated by integrating pressure equation as

$$W = \int_{V_1}^{V_2} P dV = \int_{0.5}^1 (1250V - 225) dV = 356.25 \text{ kJ}$$

Alternative Method for Total Work Transfer

Work transfer can also be determined by evaluating area under P – V diagram for the process as

$$W = \frac{1}{2} (P_1 + P_2) (V_2 - V_1) = 356.25 \text{ kJ}$$

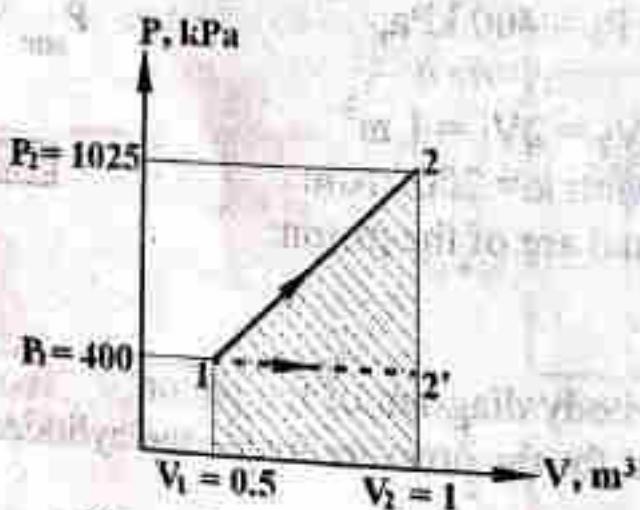


Figure E2.4 (a): P-V diagram

Work done against the spring

$$W_s = \frac{1}{2} k X^2 = \frac{1}{2} k \left(\frac{V_2 - V_1}{A} \right)^2 = 156.25 \text{ kJ}$$

Alternative Method for Spring Work

If there was no spring, the process would occur at constant pressure as shown by line 1-2' on P – V diagram. Work transfer during process 1-2' gives the magnitude of work utilized for the expansion of gas, i.e.

$$W_{\text{eq}} = W_{12'} = P_1 (V'_{2'} - V_1) = 400 \times (1 - 0.5) = 200 \text{ kJ}$$

Then work done utilized for the compression of spring can also be determined by subtracting work done against the compression of gas from the total work done.

$$W_s = W - W_{\text{eq}} = 356.25 - 200 = 156.25 \text{ kJ}$$

Therefore, the fraction of total work done consumed for the compression of spring is

$$\frac{W_s}{W} = \frac{156.25}{356.25} = 0.4386 = 43.86\%$$

OBJECTIVE QUESTIONS

Select the correct answer from the given alternatives.

- Which of the following form of the energy is due to microscopic phenomena?
 - internal energy
 - gravitational potential energy
 - work transfer
 - kinetic energy of the system
- Total energy is defined as the sum of
 - internal energy and enthalpy
 - internal energy, potential energy and flow energy
 - internal energy, potential energy and kinetic energy
 - enthalpy, potential energy and flow energy
- Which of the following statement is not true regarding stored energy?
 - It cannot cross the boundary of the system.
 - It can be defined for a state.
 - It is a state function.
 - It is not a thermodynamic property.

3. Differentiate between the stored energy and transient energy with examples.
4. Define heat transfer. Mention sign convention used for heat transfer in thermodynamics.
5. Define work transfer. Mention sign convention used for work transfer in thermodynamics.
6. Explain the similarities and differences between heat transfer and work transfer.
7. Derive expressions for displacement work transfer for the following processes:
 - (a) constant pressure process
 - (b) constant temperature process
 - (c) polytropic process.
8. Define a polytropic process. Sketch polytropic processes with $n = 0$, 1 , 1.4 and ∞ on a common P - V diagram.

NUMERICAL PROBLEMS

1. ✓ A gas is contained in a piston cylinder device initially at a pressure of 150 kPa and a volume of 0.04 m³. Calculate the work done by the gas when it undergoes the following processes to a final volume of 0.1 m³.
 - (a) constant pressure
 - (b) constant temperature
 - (c) $PV^{1.35} = \text{constant}$.
2. ✓ A spring loaded piston cylinder device contains gas initially at a pressure of 800 kPa and a volume of 0.05 m³. Pressure – volume relationship for the set up is given by $P = a + bV$, where a and b are constants. Heat is added to the system till its final state $P_2 = 2000$ kPa and $V_2 = 0.2$ m³ is reached. Determine the work transfer during the process.
3. ✓ A gas undergoes compression from an initial state of $V_1 = 0.1$ m³, $P_1 = 200$ kPa to a final state of $V_2 = 0.04$ m³, $P_2 = 500$ kPa. If the pressure varies linearly with volume during the process, determine the work transfer.

4. A gas undergoes a polytropic process from an initial state of 50 kPa and 0.02 m^3 to a final state of 100 kPa and 0.05 m^3 . Determine the work transfer.

5. Air is compressed in a cylinder from 1 m^3 to 0.35 m^3 by a piston. The relation between the pressure and volume is given by $P = 5 - 3V$, where P is in bar and V is in m^3 . Compute the magnitude of the work done on the system in kJ.

6. In a non flow process, a gas expands from volume 0.1 m^3 to volume of 0.2 m^3 according to the law

$$P = \frac{2}{V} + 1.5$$

- where P is the pressure in bar, and V is the volume in m^3 . Determine (i) the pressure at the end of the expansion and (ii) the work done by the gas in the expansion process, in kJ.

7. A non flow reversible process occurs for which pressure and volume are correlated by the expression

$$P = V^2 + \frac{6}{V}$$

where P is in bar and V is in m^3 . What amount of work will be done when volume changes from 2 to 4 m^3 ?

8. An ideal gas undergoes an isothermal compression from $V_1 = 3 \text{ m}^3$ to $P_2 = 100 \text{ kPa}$ and $V_2 = 2 \text{ m}^3$. It is further compressed at constant pressure until its volume reduces to 1 m^3 . Determine the total work transfer for the process.
9. An ideal gas undergoes two processes in series

Process 1-2: an expansion from 0.1 m^3 to 0.2 m^3 at constant pressure of 200 kPa

Process 2-3: an expansion from 0.2 m^3 to 0.4 m^3 with a linear rising pressure from 200 kPa to 400 kPa.

Sketch the process on P-V diagram and determine the total work transfer.

10. Two kg of a gas undergoes following process in series to form a cycle:

Process 1-2: constant volume cooling, $V_1 = V_2 = 2 \text{ m}^3$

Process 2-3: constant pressure heating, $P = 100 \text{ kPa}$, $V_3 = 10 \text{ m}^3$

Process 3-1: isothermal compression

11.

12.

13.

14.

15.

16.

Sketch the process on P-V diagram and determine the net work transfer.

11. Air undergoes three process in series to form a cycle:

Process 1-2: compression with $PV^{1.3} = \text{constant}$ from $P_1 = 100 \text{ kPa}$, $V_1 = 0.04 \text{ m}^3$ to $V_2 = 0.02 \text{ m}^3$

Process 2-3: constant pressure process to $V_3 = V_1$

Process 3-1: constant volume

Sketch the process on P-V diagram and determine the total work transfer.

12. A piston cylinder device shown in **Figure P2.12** contains 0.1 kg of air initially at a pressure of 4 MPa and temperature of 200°C . Heat is added to the system until the pressure is 8 MPa and the temperature is 800°C . Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take $R = 287 \text{ J/kg K}$]

13. Argon (1 kg) contained in a piston cylinder device shown in **Figure P2.13** is initially at a pressure of 500 kPa and a temperature of 70°C . Heat is added until the final temperature is 600°C and a pressure of 1 MPa is required to lift the piston from the stops. Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take $R = 208 \text{ J/kg K}$]

14. Air (0.5 kg) in the piston cylinder device shown in **Figure P2.14** has an initial pressure and temperature of 1 MPa and 500°C respectively. The system is cooled until the temperature reaches 50°C . It takes a pressure of 0.5 MPa to support the piston. Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take $R = 287 \text{ J/kg K}$]

15. Oxygen (3.6 kg) contained in a piston cylinder device shown in **Figure P2.15** is initially at a pressure of 200 kPa and a temperature of 50°C . Heat is added until the piston just reaches the upper stops where the total volume is 3 m^3 . It requires a pressure of 500 kPa to lift the piston. Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take $R = 260 \text{ J/kg K}$]

16. A piston cylinder arrangement shown in **Figure P2.16** contains gas initially at $P_1 = P_{\text{atm}} = 100 \text{ kPa}$ and $T_1 = 20^\circ\text{C}$. Piston with a cross sectional area of 0.01 m^2 has a mass of 50 kg and is initially resting on the bottom stops. Heat is added to the system until it touches the upper stops.

(a) Sketch the process on P-V and T-V diagrams.

(b) Determine the total work transfer.

- (c) Determine the final temperature of the gas.
 [Take $g = 9.81 \text{ m/s}^2$]

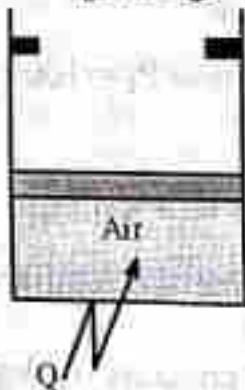


Figure P2.12



Figure P2.13

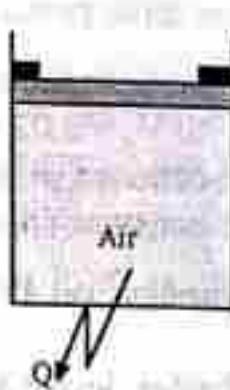


Figure P2.14



Figure P2.15

17. An unstretched spring ($k = 1 \text{ kN/m}$) is attached to a piston cylinder device as shown in Figure P2.17. Heat is added until the pressure inside the cylinder is 400 kPa. If the diameter of the piston is 50 mm, determine the work done by the gas on the piston. [$T_1 = P_{\text{atm}} = 100 \text{ kPa}$]
18. A piston cylinder arrangement loaded with a linear spring ($k = 1 \text{ kN/m}$) as shown in Figure P2.18 contains air. Spring is initially unstretched and undergoes a compression of 40 mm during process. If the mass of the piston is 80 kg and piston diameter is 0.1 m, determine the total work transfer. [Take $P_{\text{atm}} = 100 \text{ kPa}$ and $g = 9.81 \text{ m/s}^2$]

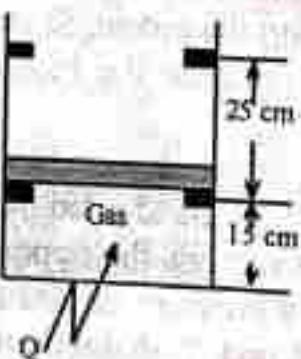


Figure P2.16

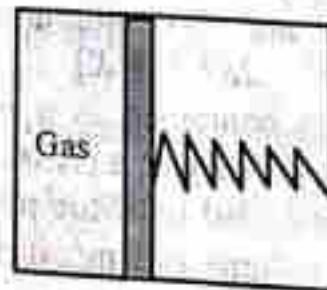


Figure P2.17

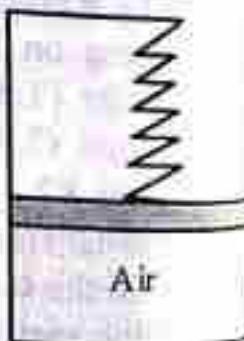


Figure P2.18

19. A piston cylinder arrangement with two set of stops is restrained by a linear spring ($k = 12 \text{ kN/m}$) as shown in Figure P2.19. The cross sectional area of the piston is 0.05 m^2 . The initial pressure of the gas is 500 kPa and the pressure required to lift the piston is 1000 kPa. Heat is supplied to the gas until its pressure reaches 6000 kPa. Sketch the process on P-V diagram and determine the total work transfer.

20. Air (0.01 kg) is contained in a piston cylinder device restrained by a linear spring ($k = 500 \text{ kN/m}$) as shown in Figure P2.20. Spring initially touches the piston but exerts no force on it. Heat is added to the system until the piston is displaced upward by 80 mm . determine
 (a) the temperature at which piston leaves the stops
 (b) work done by the air [Take $R = 287 \text{ J/kg}\cdot\text{K}$, $P_{\text{atm}} = 100 \text{ kPa}$ and $g = 9.81 \text{ m/s}^2$]

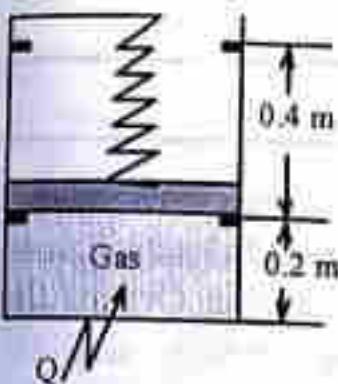


Figure P2.19

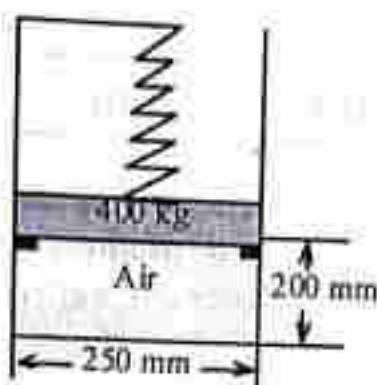


Figure P2.20

21. A gas enclosed by a piston shown in Figure P2.21 starts to expand due to heating. The initial movement of 0.2 m is restrained by a fixed mass of 30 kg and the final 0.05 m is restrained both by the mass and a spring of stiffness 10 kN/m . The cross sectional area of the piston is 0.15 m^2 and the atmospheric pressure is 100 kPa .
- Neglecting the mass of the spring and the piston sketch a P-V diagram of the process.
 - Calculate the work during the initial 0.2 m movement.
 - Calculate the total work done. [Take $g = 9.81 \text{ m/s}^2$]

22. A piston cylinder arrangement shown in Figure P2.22 is restrained by two linear springs as shown. The system contains air initially at a pressure of 150 kPa and a volume of 0.002 m^3 . Heat is added to the system until its volume doubles; determine the total work transfer. Also sketch the process on P-V diagram. Both springs have spring constant of 100 kN/m .

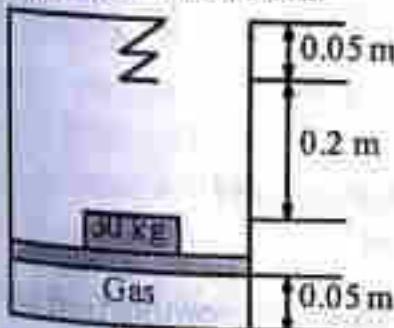


Figure P2.21

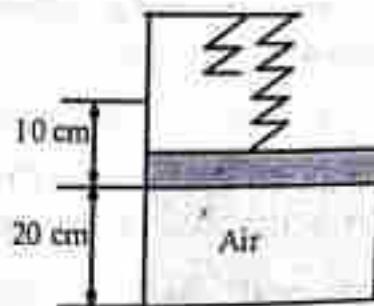


Figure P2.22

PROPERTIES OF COMMON SUBSTANCES

2.1 Introduction

In all thermodynamic applications, energy is transferred with the help of a substance to produce certain useful effect. Substance used in any system or process is called a working substance. One of the major parts of the applied thermodynamics is the property analysis of the common substances that can be used to transfer energy.

For most of the applications, working substances will be in the gaseous state, vapor state or in the form of mixture of liquid and vapor. If the working substance is in gaseous state, it is usually modeled by an ideal gas equation. If the working substance is in liquid state, vapor state (just evaporated) or mixture, its properties cannot be evaluated by using mathematical equation and for this purpose we have to use experimental data.

In the chapter, we briefly revise the ideal gas equations and focus mainly on the properties of two phase (liquid-vapor) mixture.

2.2 Pure Substance and State Postulate

3.2.1 Pure Substance

For most of the thermodynamic analysis, we assume the working substance as a pure substance. A system is said to be composed of pure substance if it is

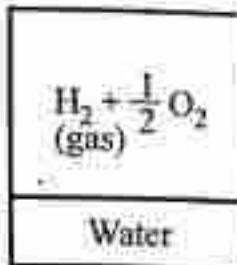
- (a) homogeneous in composition
- (b) homogeneous in chemical aggregation, and
- (c) invariable in chemical aggregation

Homogeneous in composition means that the proportion of the constituent elements of the substance should be same throughout the system. Homogeneous in chemical aggregation means the

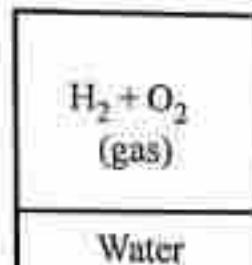
constituent elements of the substance should be combined in the similar manner in all samples of the system. Invariable in chemical aggregation means that the state of homogeneity should not change with time.



(i)



(ii)



(iii)

Figure 3.1 Illustrating the definition of a pure substance

Let us consider three systems, system 1 consisting of homogeneous mixture of liquid water and water vapor, system 2 consisting of a mixture of water vapor and gaseous mixture of hydrogen and oxygen in which H_2 and O_2 present in the proportion of 2:1 and system 3 consisting of a mixture of water vapor and gaseous mixture of hydrogen and oxygen in which H_2 and O_2 present in the proportion of 1:1.

Analyzing system 1, we can observe that proportion of H_2 and O_2 is identical whether the sample is taken from liquid water or water vapor. Similarly, chemical combination ratio of H_2 and O_2 is identical for both liquid water and water vapor. Hence, system 1 satisfies both conditions (a) and (b) and it can be regarded as a pure substance as long as its homogeneity does not change with time (i.e. satisfies condition (c)).

Analyzing system 2, we observe that the proportion of H_2 and O_2 will be the same whether we take a sample from the water vapor or form the gaseous mixture of H_2 and O_2 . So it is homogenous in composition. But if we go on reducing the sample size from the gaseous mixture we can get a single molecule of H_2 or O_2 whereas in case of sample from water vapor the smallest possible sample size is a single water molecule. Hence it is not homogeneous in chemical aggregation and cannot be regarded as a pure substance.

Similarly, if we analyze system 3, we can observe that it is neither homogeneous in composition nor homogeneous in chemical aggregation. Hence, it is not a pure substance.

3.2.2 State Postulate

For a pure substance, we can determine all thermodynamic properties if two independent thermodynamic properties are given. This special feature of a pure substance is called state postulate or two property rule; which can be stated as:

Two independent thermodynamic properties are sufficient to establish a stable thermodynamic state of a system composed of a pure substance.

For example, we can determine the temperature T of a pure substance for any state if its pressure P and volume V are given, i.e.,

$$T = T(P, V) \quad \dots \dots \dots \quad (3.1)$$

State postulate can also be stated in terms of intensive properties as:

Two independent intensive thermodynamic properties are sufficient to establish a stable thermodynamic state of a pure substance.

For example, we can determine the temperature T of a pure substance for any state if its pressure P and specific volume v are given, i.e.,

$$T = T(P, v) \quad \dots \dots \dots \quad (3.2)$$

3.3 Ideal Gas and Ideal Gas Relations

A perfect gas or an ideal gas is defined as a gas having no forces of molecular attraction. A gas which follows the gas laws at all ranges of pressures and temperatures can be considered as an ideal gas but no such gas exists in nature. However, real gases tend to follow these laws at low pressures or high temperature. This is because the molecules are far apart at reduced pressures and elevated temperatures and the force of attraction between them tends to be small. The range of this low pressure and high temperature at which any gas behaves as an ideal gas is different for different gases.

3.3.1 Boyle's Law

This law was enunciated by Robert Boyle on the basis of his experimental results. Boyle's law may be stated as follows:

If the temperature remains constant, the volume of a given mass of a gas is inversely proportional to the pressure.

$$V \propto \left(\frac{1}{P}\right)_T \quad \dots \dots \dots (3.3)$$

3.3.2 Charles Law

This law is also known as Gay-Lussac's law, was and it may be stated in parts:

- (a) If the pressure is held constant, the volume of a given mass of gas varies directly as the absolute temperature.

$$V \propto (T)_P \quad \dots \dots \dots (3.4)$$

- (b) If the volume is held constant, the pressure of a given mass of a gas varies directly with the absolute temperature.

$$P \propto (T)_V \quad \dots \dots \dots (3.5)$$

3.3.3 Equation of state

A relation between all the three variables P , V and T may be obtained by combining Boyle's law and Charles' law. Consider unit mass of an ideal gas undergoing constant pressure process 1 – A followed by constant temperature process A – 2 as shown in Figure 3.2.

Now applying Charle's law for the process 1 – A

$$\begin{aligned} \frac{v_1}{v_A} &= \frac{T_1}{T_A} \\ v_A &= \frac{T_A}{T_1} v_1 = \frac{T_2}{T_1} v_1 \end{aligned} \quad \dots \dots \dots (3.6)$$

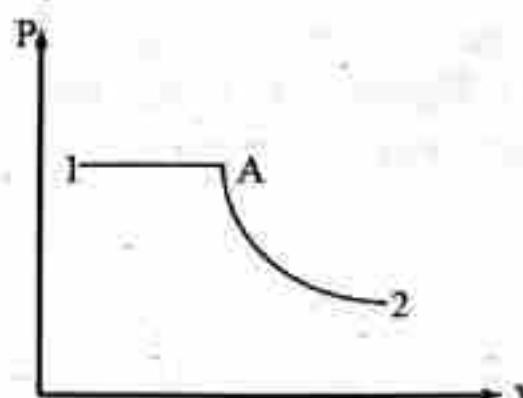


Figure 3.2 Series of Constant Pressure and Constant Temperature Process

Applying Boyle's law for the process A - 2,

$$\frac{v_2}{v_A} = \frac{P_A}{P_2}$$

$$v_A = \frac{P_2}{P_A} v_2 = \frac{P_2}{P_1} v_2 \quad \dots \dots \dots (3.7)$$

Equating the two relations (3.6) and (3.7)

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \quad \dots \dots \dots (3.8)$$

Since states 1 and 2 are selected arbitrarily, it follows that

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} = \frac{P_3 v_3}{T_3} = \frac{P v}{T} = R \quad \dots \dots \dots (3.9)$$

$$Pv = RT$$

where R is a constant for a given gas. The constant R is called the characteristic gas constant and has the units of J/kg.K. Characteristic gas constants for different gases are given in Appendix 1.

For a system consisting of a mass of m kg of gas, Equation (3.9) becomes

$$PV = mRT \quad \dots \dots \dots (3.10)$$

3.4 Two Phase (Liquid and Vapor) Mixture System

3.4.1 Saturation Curve on T - v Diagram

Consider a frictionless piston-cylinder device, shown in Figure 3.3, containing some amount of water at atmospheric condition specified by state A shown in Figure 3.4. If the weight of the piston is negligible, pressure remains constant at 1 atm throughout the process.



Figure 3.3 Arrangement for Constant Pressure Heating of H_2O

Heat is supplied to the system from an external source. The temperature of the water increases with slight increment of specific volume, which is represented by line AB , in **Figure 3.4**. This trend continues until its temperature reaches to 100°C , state B . This particular state of H_2O is called saturated liquid state, because further heat addition causes it to evaporate. Evaporation process is represented by line BC , during which pressure and temperature remains constant and specific volume of the substance increases due to intermolecular expansion. After the absorption of complete latent heat H_2O gets completely converted into vapor, state C , which is called saturated vapor state. When saturated vapor is further heated, its temperature increases with the increase in specific volume as shown by line CD in **Figure 3.4** and this state is called a superheated vapor.

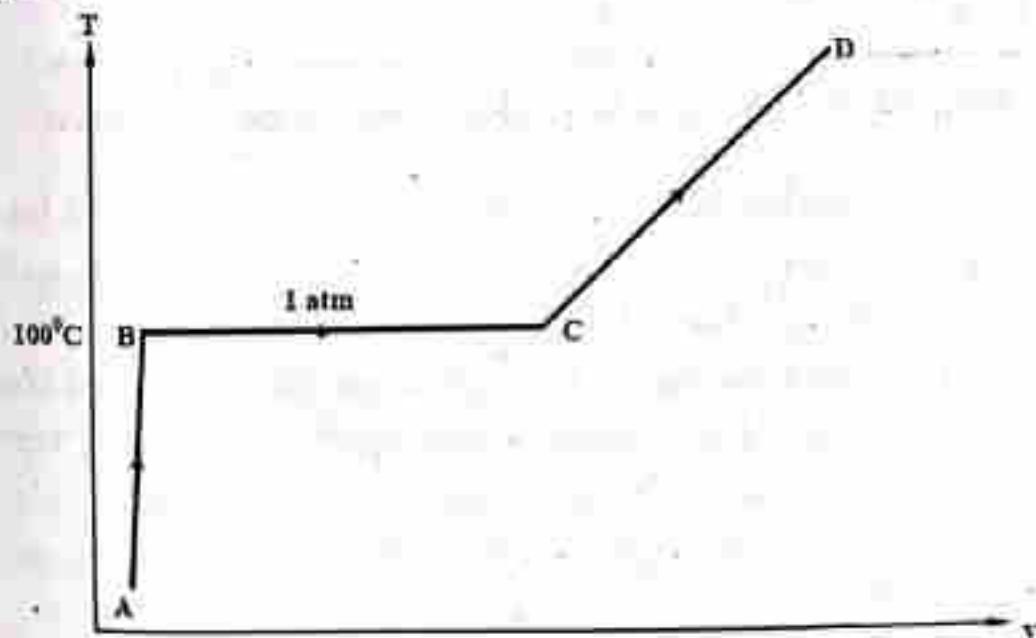


Figure 3.4 $T - v$ Diagram for Constant Pressure Heating of Water

If similar heating process is carried out at high pressures, by putting some weight above the piston, evaporation takes place relatively at higher temperatures, as shown in Figure 3.5. The length of line BC goes on decreasing. If we evaporate water at higher pressure. At some particular pressure length of line BC becomes zero, i.e., saturated liquid state (B_1) and saturated vapor state (C_1) coincide. This particular state of the substance is called the critical point at which both saturated liquid and saturated vapor have identical properties.

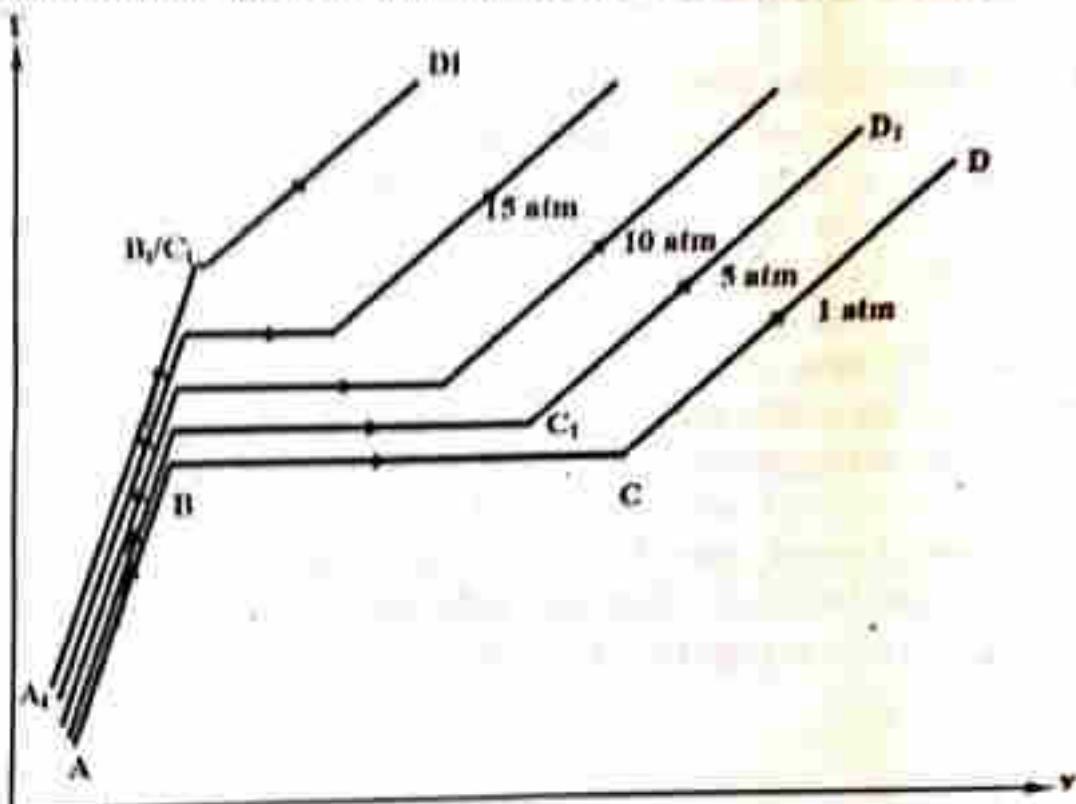


Figure 3.5 T–v Diagram for Heating Water at Different Pressures

The curve joining the saturated liquid states is called saturated liquid line and the curve joining the saturated vapor states is called saturated vapor line. The entire curve so formed as shown in Figure 3.6 is called saturation curve. The region left to the saturated liquid line is called compressed liquid or subcooled liquid region; region right to the saturated vapor line is called superheated vapor region and the region within the curve is called two-phase mixture region.

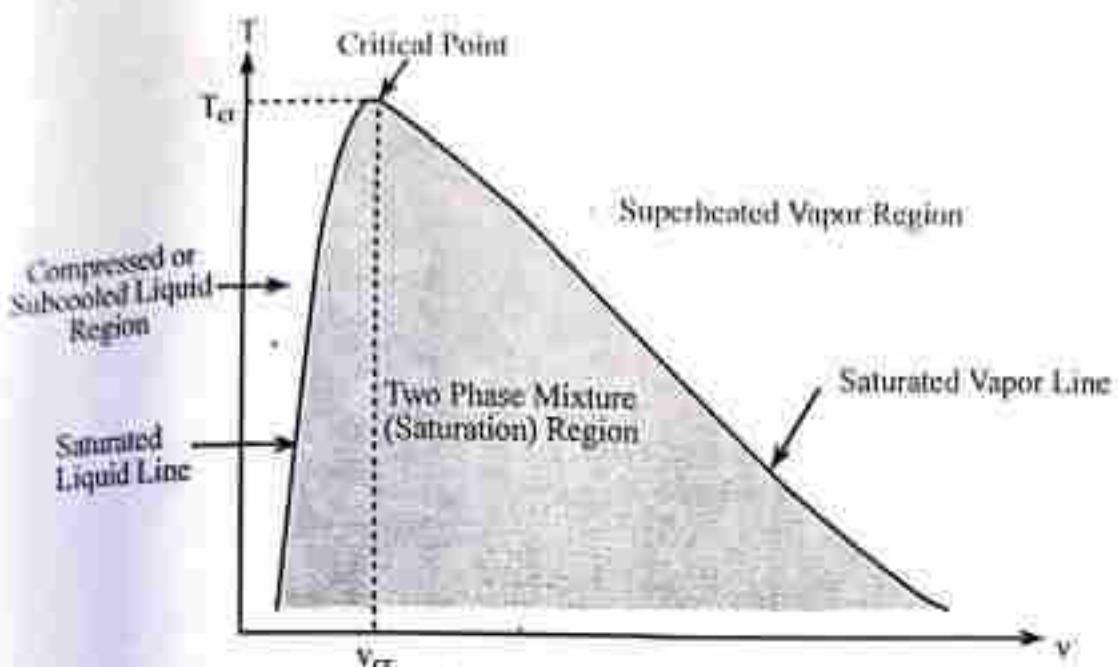


Figure 3.6 Saturation Curve for Two-Phase Mixture on $T - v$ Diagram

3.4.2 Saturation Curve on $P - v$ Diagram

Consider a piston cylinder arrangement containing water at compressed liquid state (say at 25 atm and 100°C) as shown in Figure 3.7. The system is placed in contact with a source which has a temperature slightly higher than that of the H_2O inside the cylinder.

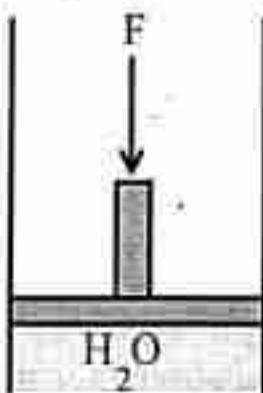


Figure 3.7 Arrangement for Constant Temperature Expansion of H_2O

If the external force F is reduced slowly such that temperature is maintained constant at 100°C, its pressure decreases and there will be slight increment in its specific volume, as shown by line AB in Figure 3.8. This trend continues until its pressure reduces to 1 atm i.e., it reaches to saturated liquid state B . When saturated liquid take heat from the source it gets evaporated and its specific volume increases until it reaches to saturated vapor state C , while its pressure

and temperature remain constant as shown by line BC in Figure 3.8. If pressure is further reduced from state C , its specific volume increases as shown by curve CD in Figure 3.8.

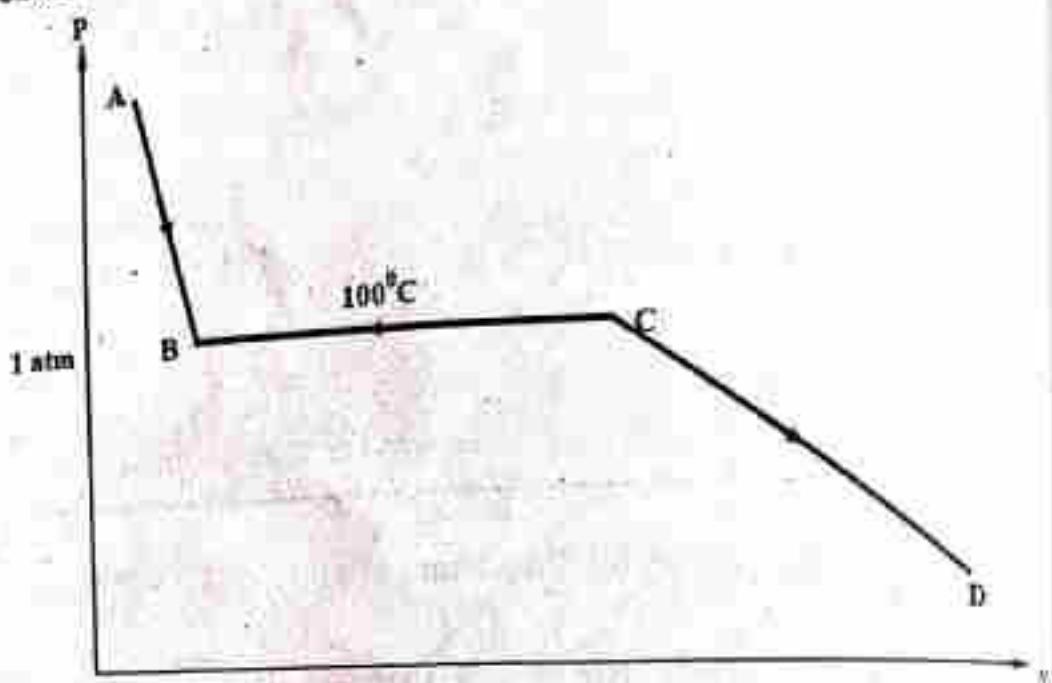


Figure 3.8 P-v Diagram for Constant Temperature Expansion of Water

If we repeat similar expansion process at higher temperatures evaporation takes place correspondingly at higher pressure as shown in Figure 3.9.

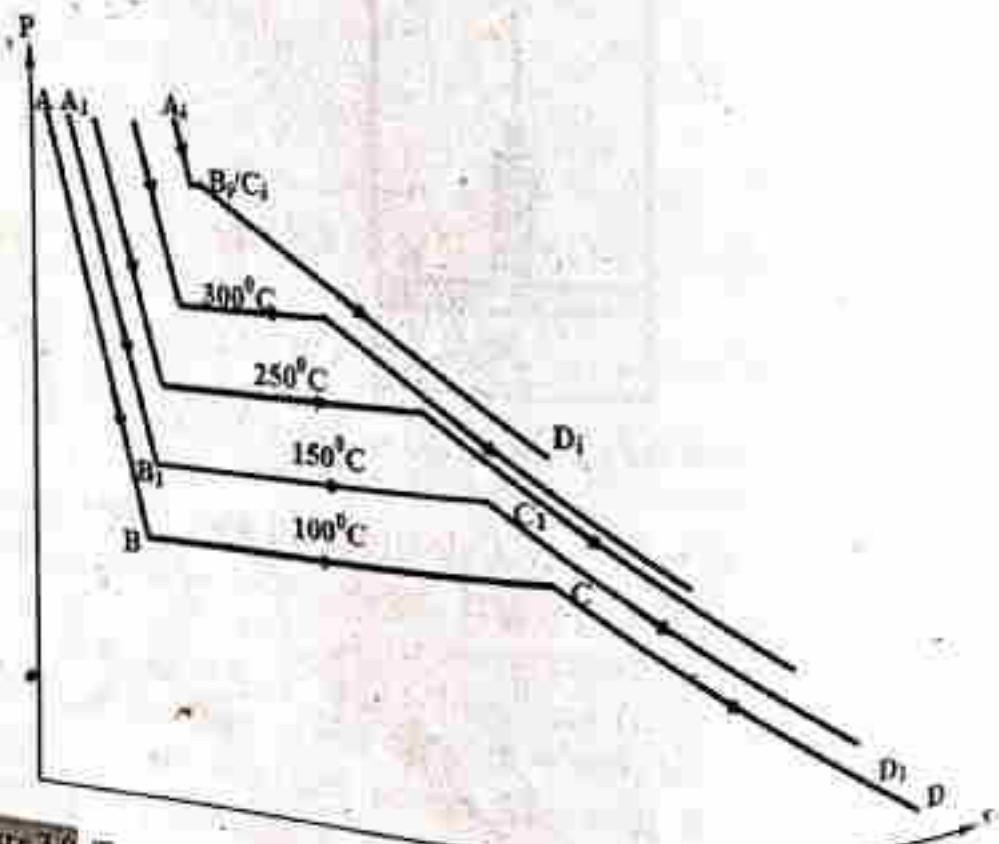


Figure 3.9 T-v Diagram for Expansion of Water

With reference to Figure 3.9, we can draw saturation curve and label different regions, lines and point as shown in Figure 3.10.

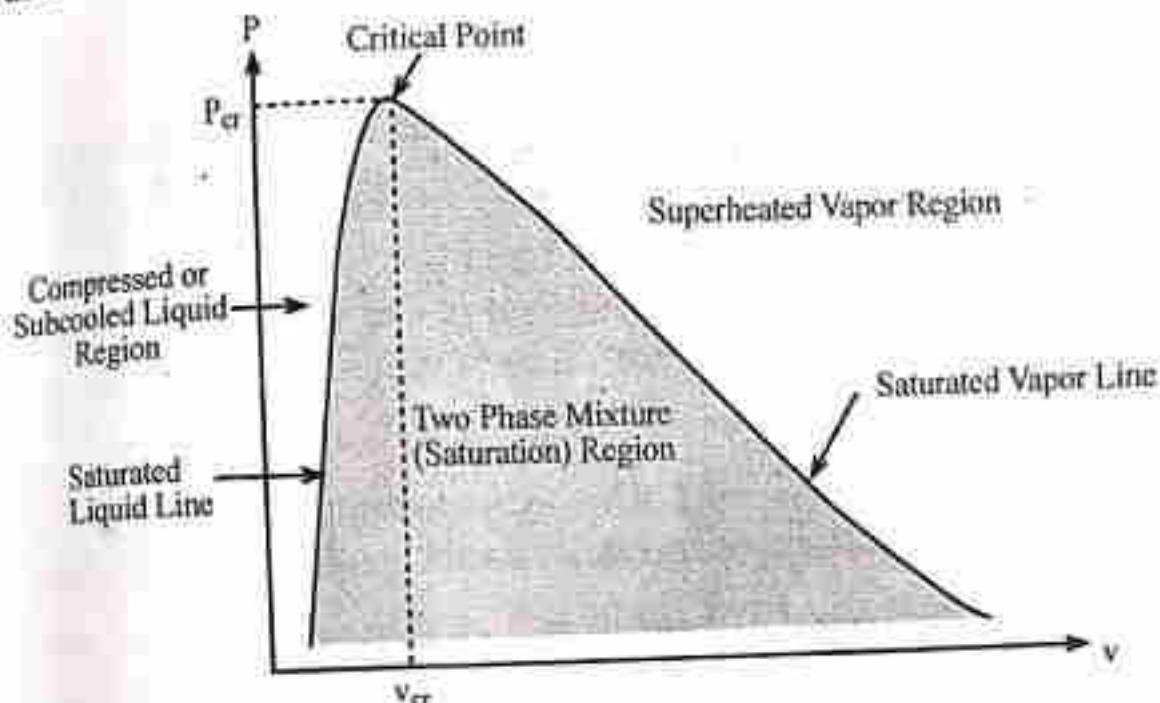


Figure 3.10 Saturation Curve for Two-Phase Mixture on $P - v$ Diagram

3.4.3 Terms Used in Two-Phase Mixture System

Saturation Temperature

For a given pressure, the temperature at which phase change (liquid to vapor or vapor to liquid) occurs is called a saturation temperature. For example, if we heat water at 1 atm, evaporation takes place at 100°C , i.e., saturation temperature for 1 atm is 100°C .

Saturation Pressure

For a given temperature, the pressure at which phase change (liquid to vapor or vapor to liquid) occurs is called a saturation pressure. For example, if we expand water at 100°C , evaporation takes place 1 atm at, i.e., saturation pressure for 100°C is 1 atm.

Saturated Liquid

The state of a substance at which evaporation just starts (during heating) or condensation just completes (during cooling) is called the saturated liquid.

Saturated Vapor

The state of a substance at which evaporation just completes (during heating) or condensation just starts (during cooling) is called the saturated vapor.

Compressed Liquid or Subcooled Liquid

When saturated liquid is further cooled or compressed, it is called compressed liquid or subcooled liquid.

Superheated Vapor

When saturated vapor is further heated or expanded, it is called superheated vapor.

Degree of Superheat

The difference in temperature of a superheated vapor and the corresponding saturation temperature is called a degree of superheat (Figure 3.11).

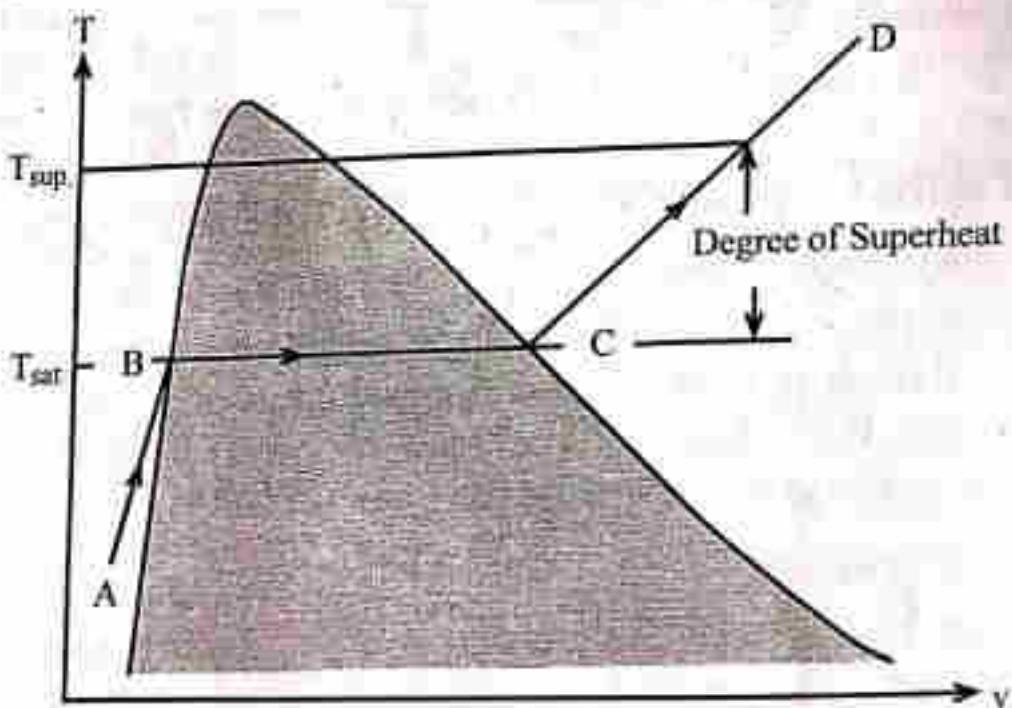


Figure 3.11 Degree of Superheat

Critical Point

The state of a substance at which saturated liquid and saturated vapor have identical properties is called a critical point. The pressure and temperature corresponding to critical point are called critical pressure and critical temperature respectively. For H_2O , critical pressure and critical temperature are 22.055 MPa and 373°C respectively. If we

heat H_2O at 22.055 MPa it suddenly gets converted into saturated vapor from saturated liquid when temperature reaches 373°C . Therefore, when a substance is heated at its critical pressure, there will be sudden phase change from a saturated liquid state to a saturated vapor state and hence no phase transition interval is observed.

3.5 Properties of a Two-Phase Mixture

Let us consider the saturation curves discussed above. Figure 3.12 shows a saturation curve on $T - v$ diagram with a constant pressure line of 1 atm. Let us consider three different states of H_2O as

- State 1: 1 atm, 80°C
- State 2: 1 atm, 100°C
- State 3: 1 atm, 120°C

To fix the states on $T - v$ diagram, we can add two constant temperature lines on Figure 3.12 as shown in Figure 3.13. Then we can fix the state 1 as the intersection of 80°C temperature line and 1 atm pressure line. Similarly, we can fix state 3 as the intersection of 120°C temperature line and 1 atm pressure line. But we cannot fix a unique point for state 2 because the entire line BC satisfy the given condition $P = 1 \text{ atm}$ and $T = 100^{\circ}\text{C}$.

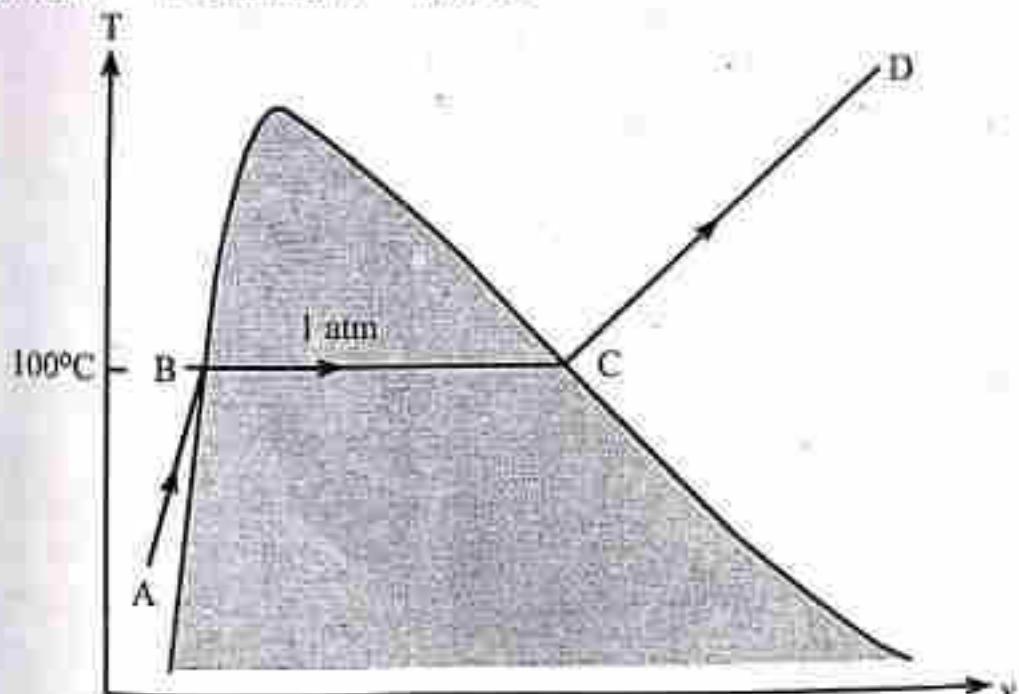


Figure 3.12 Saturation Curve with an Isobaric Line

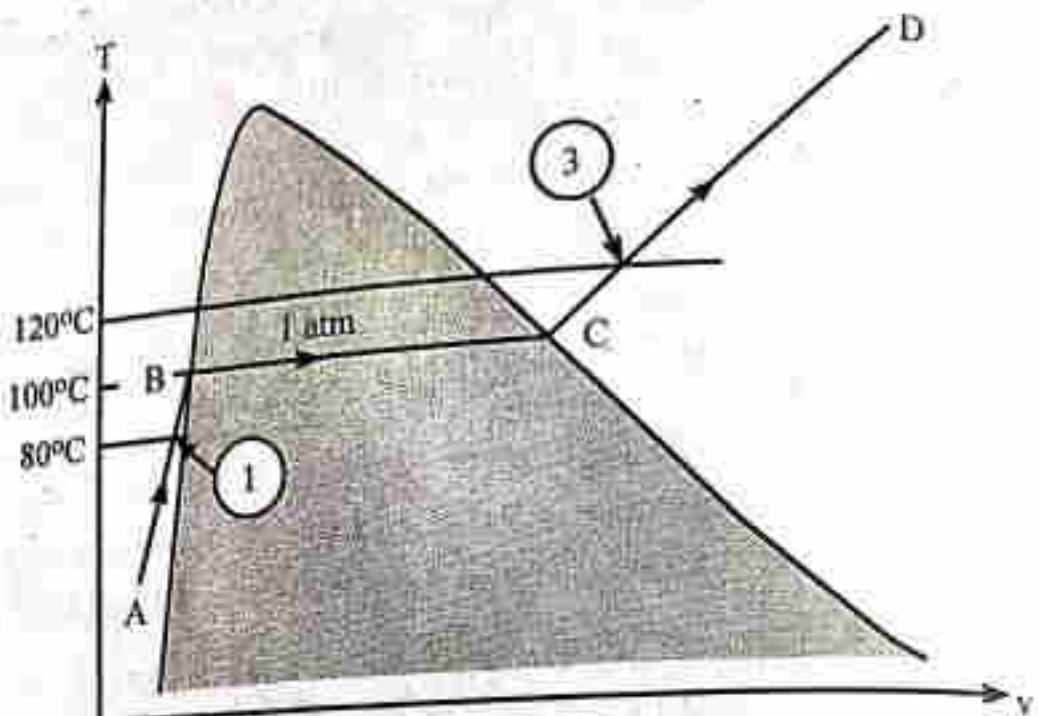


Figure 3.13 Locating States on $T - v$ with reference to Saturation Line

Hence, for a given pressure, if temperature is less than the saturation temperature, its state will be a compressed or subcooled liquid and if temperature is higher than the saturation temperature, the state will be a superheated vapor.

Similarly **Figure 3.14** shows a saturation curve on $P - v$ diagram with a constant temperature line of 100°C . Let us consider the different states of H_2O as

State 4: 1.2 atm, 100°C

State 5: 1 atm, 100°C

State 6: 0.8 atm, 100°C

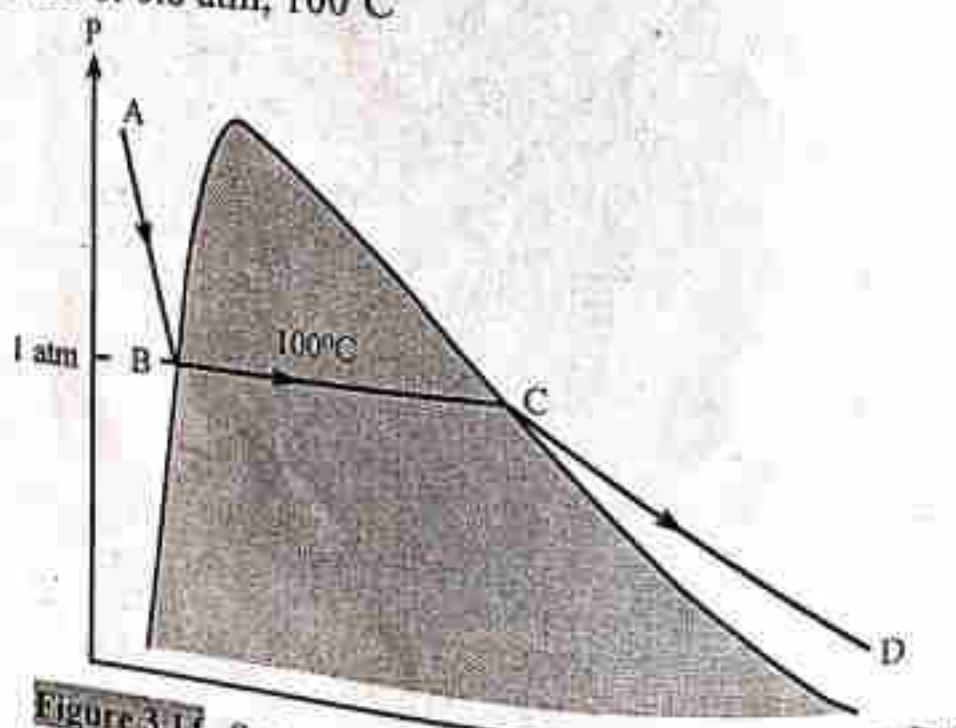


Figure 3.14 Set.

To fix these states on $P-v$ diagram, we should add two constant pressure lines on Figure 3.15. We can then fix the state 4 as intersection of 1.2 atm pressure line and 100°C temperature line and state 6 as intersection of 0.8 atm pressure line and 100°C temperature line. Here again, we cannot fix unique point for state 5, because the entire line BC' satisfy the given condition $P = 1 \text{ atm}$ and $T = 100^{\circ}\text{C}$.

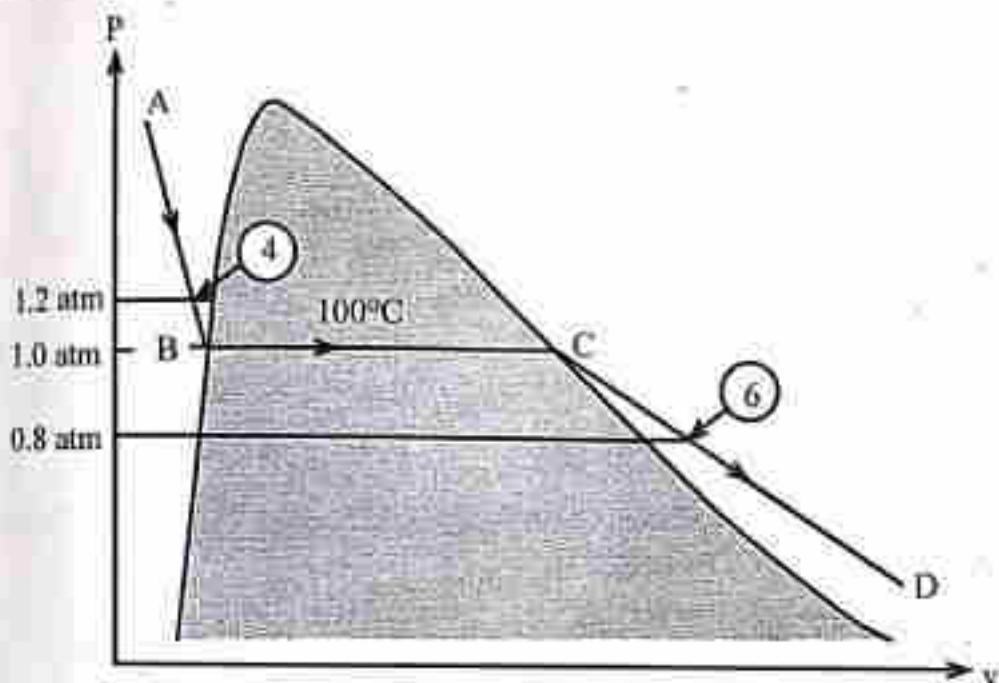


Figure 3.15 Locating States on $P-v$ with reference to Saturation Lines

Hence, for a given temperature, if pressure is higher than the saturation pressure, its state will be a compressed or subcooled liquid and if pressure is less than the saturation pressure, the state will be a superheated vapor.

3.5.1 Quality

From the above examples, we can conclude that states 1, 3, 4 and 6 can be fixed with given pressure and temperature. But states 2 and 5 cannot be located with identical given parameters. We can compare this difference with reference to state postulate. For compressed liquid or superheated vapor state, pressure and temperature are independent, hence we can fix the states with the given pressure and temperature. But for two-phase mixture (saturation) region, pressure and temperature are no more independent, and we cannot fix the state with dependent pressure and temperature.

To fix the state within the saturation region, we should use a new property called quality. Quality is defined as the ratio of mass of saturated vapor and the total mass of the two phase mixture. It is also called dryness fraction and is denoted by x .

$$x = \frac{m_g}{m} = \frac{m_g}{m_l + m_g} \quad \dots \dots \dots (3.11)$$

where

m_g is the mass of saturated vapor

m_l is the mass of saturated liquid, and

m is the total mass of the two phase mixture.

For saturated liquid state, $m = m_l$ or $m_g = 0$

$$\therefore x_{\text{sat liquid}} = 0 \quad \dots \dots \dots (3.12)$$

For saturated vapor state, $m = m_g$ or $m_l = 0$

$$\therefore x_{\text{sat vapor}} = 1 \quad \dots \dots \dots (3.13)$$

Hence, quality varies from 0 to 1 ($0 \leq x \leq 1$) within the saturation region, as shown in Figure 3.16.

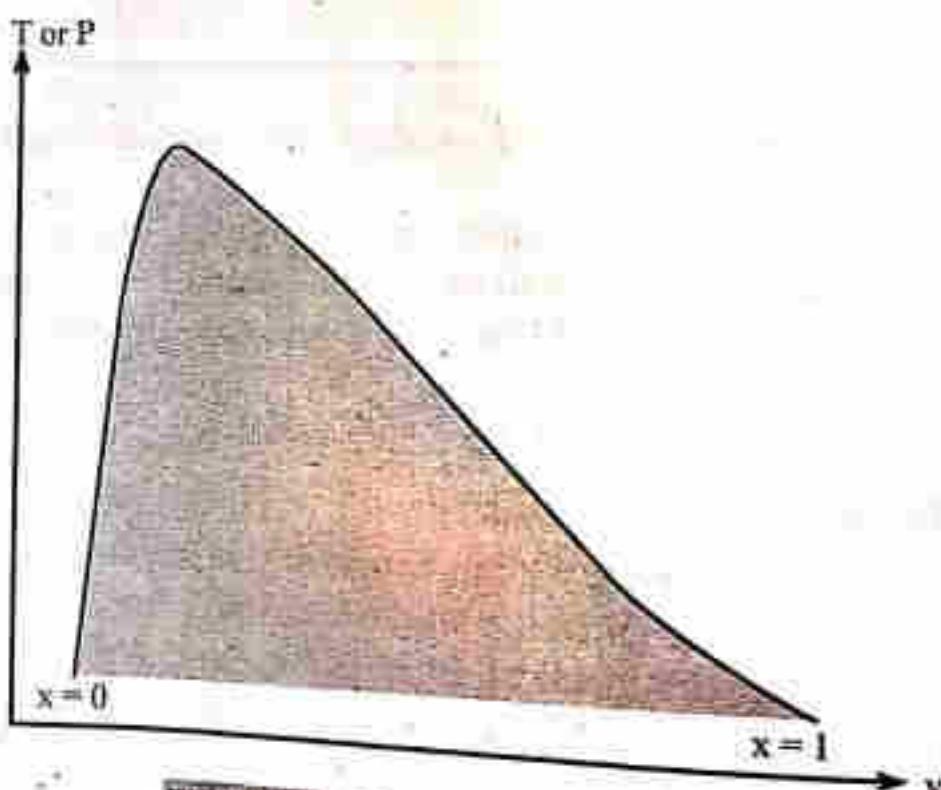


Figure 3.16 Limiting Values of Quality

3.5.2 Moisture Content

To fix the state within the saturation region we can also use the property called moisture content. Moisture content is defined as the

ratio of mass of saturated liquid and the total mass of the two phase mixture. It is denoted by y .

$$y = \frac{m_l}{m} = \frac{m_l}{m_l + m_g} = 1 - x \quad \dots \dots \dots (3.14)$$

~~3.5.3 Specific Properties of a Two-Phase Mixture~~

Specific properties of the two-phase mixture can be expressed in terms of quality. For example, specific volume of a two phase mixture is given by the ratio of total volume of two-phase mixture and the total mass of the two phase mixture, i.e.,

$$v = \frac{V}{m} = \frac{V_l}{m} + \frac{V_g}{m}$$

where

V_l is the volume of saturated liquid, and

V_g is the volume of saturated vapor.

$$\text{or, } v = \frac{V_l}{m_l} \frac{m_l}{m} + \frac{V_g}{m_g} \frac{m_g}{m} = v_l (1 - x) + v_g x$$

where

v_l is the specific volume of saturated liquid, and

v_g is the specific of saturated vapor.

$$\text{or, } v = v_l (1 - x) + v_g x = v_l + x (v_g - v_l)$$

$$\therefore v = v_l + x v_{lg} \quad \dots \dots \dots (3.15)$$

where $v_{lg} = v_g - v_l$, is the change in specific volume during evaporation.

Other specific properties of the two-phase mixture can be expressed in the similar manner as

$$\text{Specific internal energy} \quad u = u_l + x u_{lg} \quad \dots \dots \dots (3.16)$$

$$\text{Specific enthalpy} \quad h = h_l + x h_{lg} \quad \dots \dots \dots (3.17)$$

$$\text{Specific entropy} \quad s = s_l + x s_{lg} \quad \dots \dots \dots (3.18)$$

3.6 Development of Property Data

Properties of a substance within saturation region or near saturation region cannot be determined using the mathematical equations (e.g. ideal gas equation). Hence, properties under such conditions are determined experimentally and presented in the form of tables or charts.

Property tables can be used to determine the property of a substance for a particular state. Property tables for water have been given in this text book (Appendix A2). In this book four separate tables are

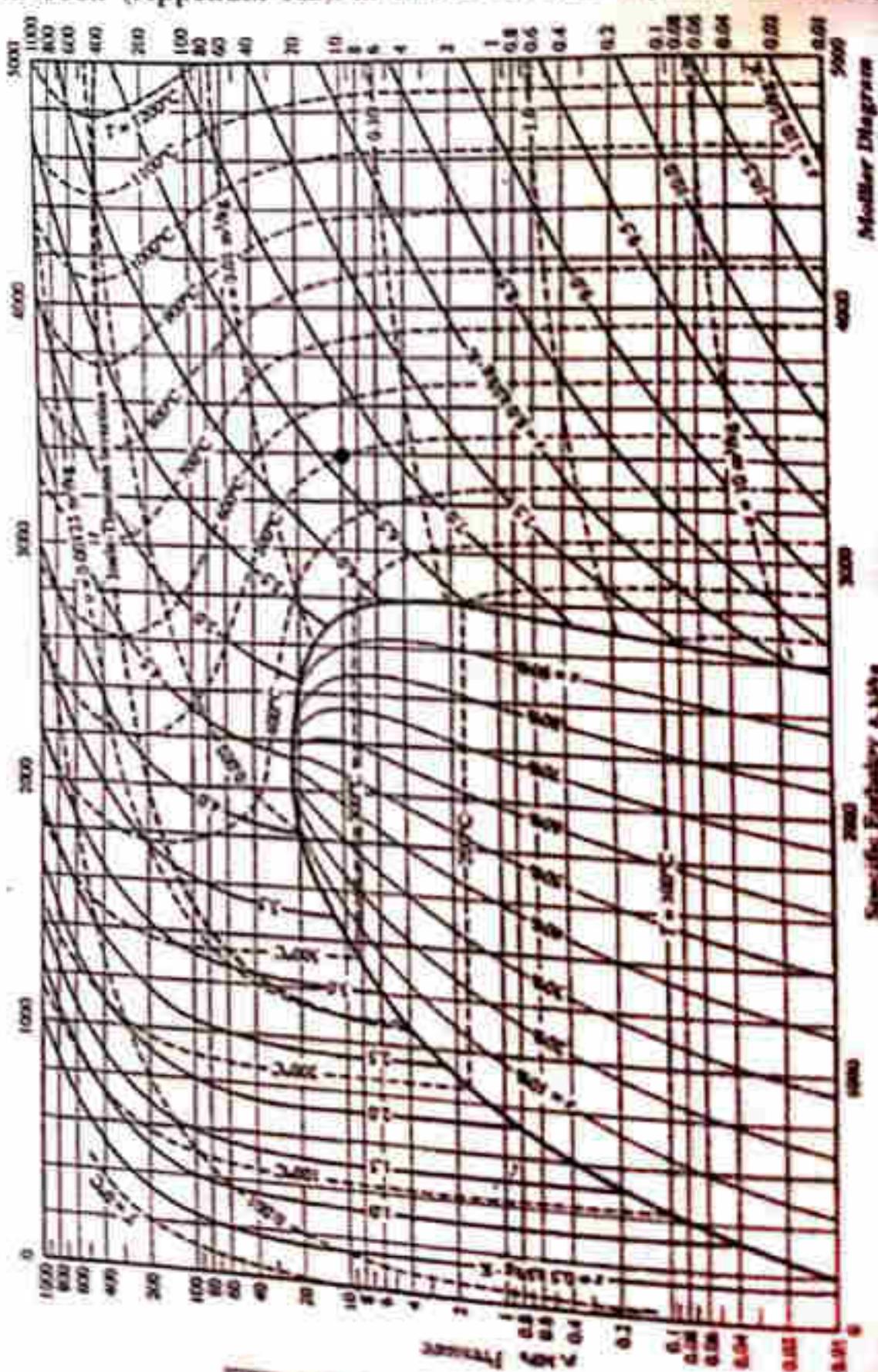


Figure 3.17 P - h Diagram for water

provided for the properties of water (or steam). Table A2.1 gives properties of water at saturation condition tabulated with pressures. Table A2.2 also gives properties of water at saturation condition tabulated with temperatures. Table A2.3 gives properties of water for compressed liquid state. Table A2.4 gives properties of water for superheated vapor state. The intervals between tables entries in complete tabulations are usually chosen so that linear interpolation will provide accurate values for states that fall between entries.

Property charts can be used to determine the properties for a particular state as well as process can also be plotted. $P - h$ diagram for water is shown in **Figure 3.16**.

SOLVED EXAMPLES

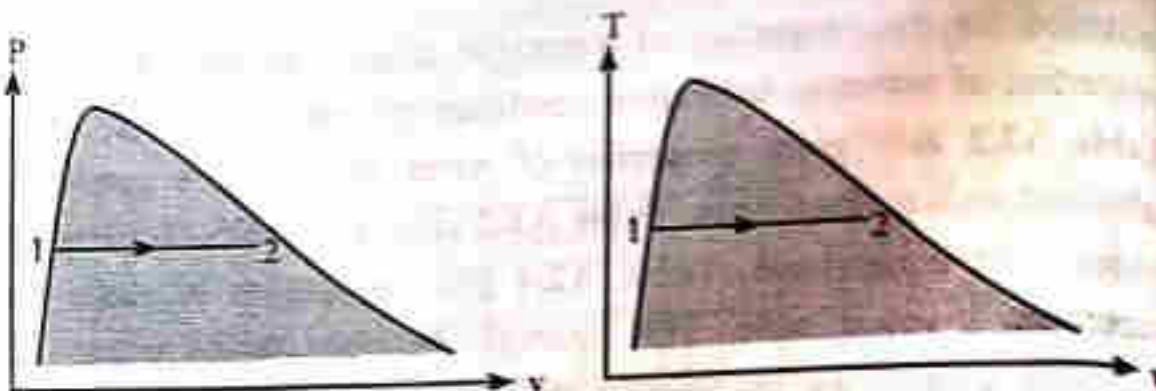
Example 3.1 : Sketch the following processes on $P - v$ and $T - v$ diagrams. Show both initial and final states properly relative to saturation curves:

- A saturated liquid undergoing an isothermal process until it becomes a mixture with a quality of 0.8.
- A saturated vapor in a rigid tank is heated.
- A superheated vapor is condensed isobarically to the saturated liquid state.
- A compressed liquid is heated isobarically to the saturated vapor.
- A two-phase mixture in a rigid tank is heated such that it passes through a critical point.

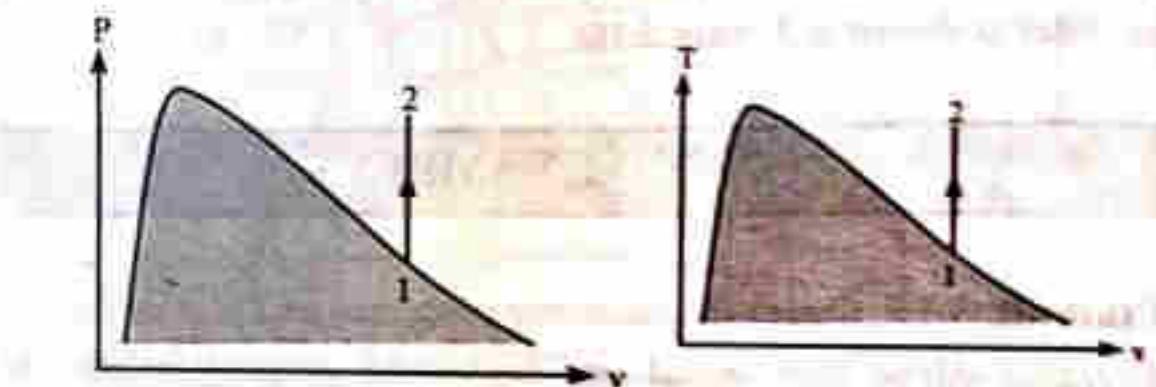
Solution

$P - v$ and $T - v$ diagrams for each of the processes are shown below.

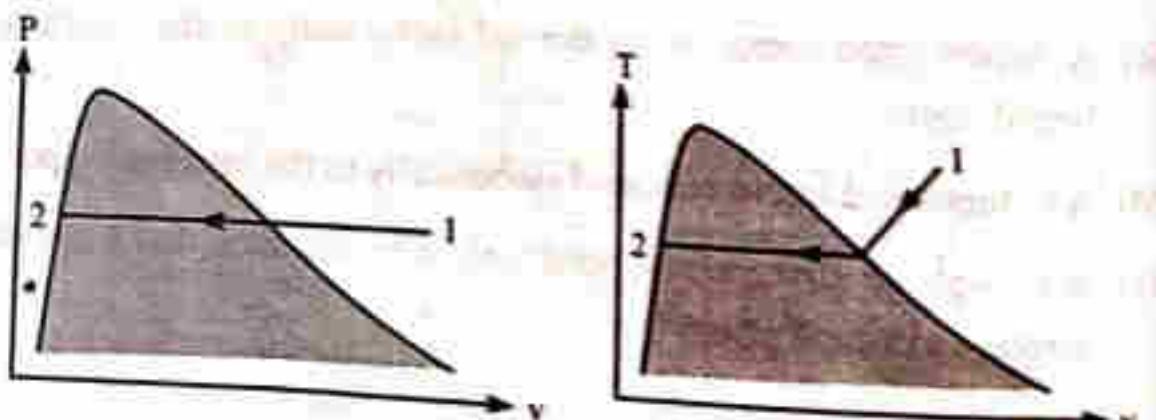
- When a saturated liquid is heated under isothermal condition, evaporation takes place and pressure also remains constant.



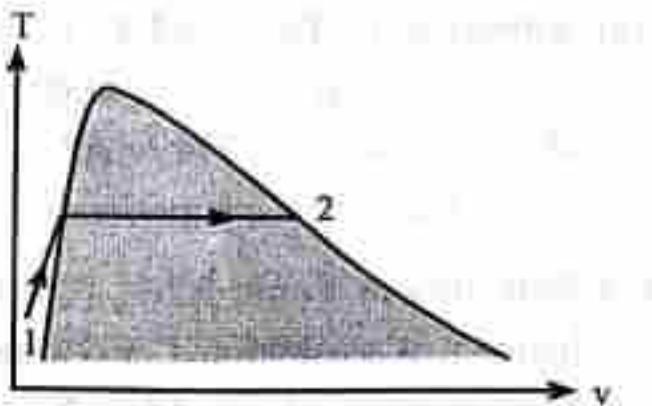
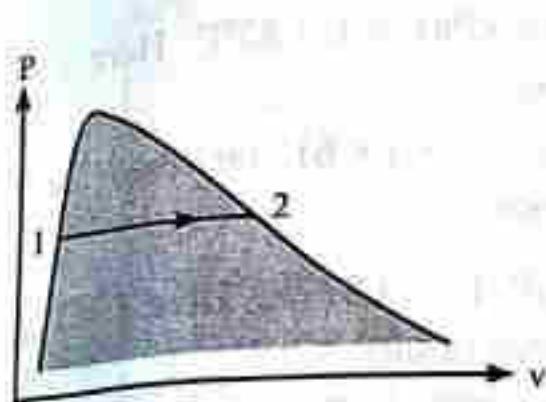
- (b) When a saturated vapor in a rigid tank is heated its specific volume remains constant while its pressure and temperature increase.



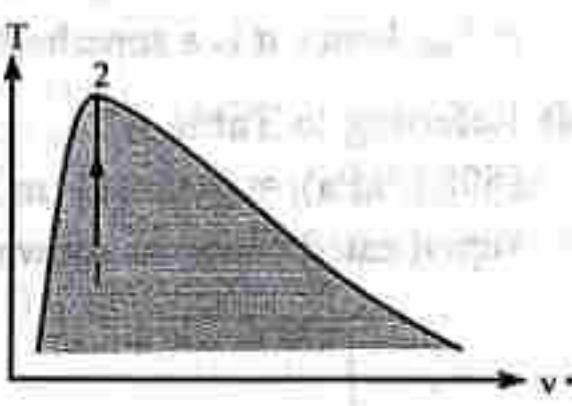
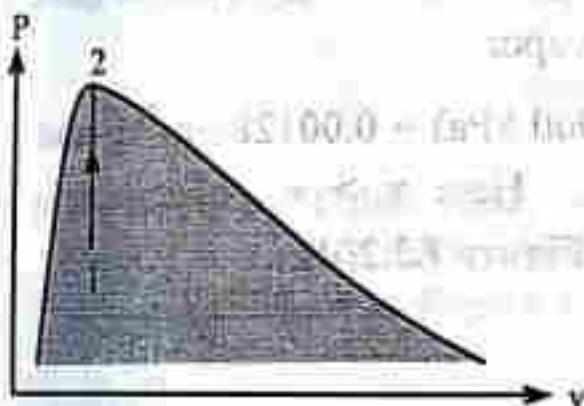
- (c) When a super heated vapor is cooled isobarically its temperature decreases until it becomes a saturated vapor. When the saturated vapor is further cooled condensation takes place and its temperature also remains constant.



- (d) When a compressed liquid is heated isobarically its temperature increases until it becomes a saturated liquid. When the saturated liquid is further heated evaporation takes place and its temperature also remains constant.



- (e) When a two phase mixture in a rigid tank is heated its specific volume remains constant while its pressure and temperature increase such that process line 1-2 intersects saturation curve at critical point at final state.



Example 3.2 : Determine water at each of the following states is a compressed liquid, or a mixture of saturated liquid and saturated vapor or a superheated vapor.

- 500 kPa, 160°C
- 40 kPa, 40°C
- 150°C, 400 kPa
- 5000 kPa, 0.1 m³/kg
- 8000 kPa, 0.01 m³/kg
- 50°C, 0.0005 m³/kg
- 120°C, 0.5 m³/kg
- 300°C, 0.1 m³/kg

Solution

Hint: To determine the physical state of a substance (liquid, vapor or mixture), always refer saturated pressure table if pressure is given and saturated temperature table if temperature is given.

- (a) Referring to Table A2.1, $T_{\text{sat}}(500 \text{ kPa}) = 151.87^\circ\text{C}$. Here $T > T_{\text{sat}}$, hence it is a superheated vapor.
 We can also refer Table A2.2, $P_{\text{sat}}(160^\circ\text{C}) = 617.66 \text{ kPa}$. Here $P < P_{\text{sat}}$, hence it is a superheated vapor.
- (b) Referring to Table A2.1, $T_{\text{sat}}(50 \text{ kPa}) = 75.877^\circ\text{C}$. Here $T < T_{\text{sat}}$, hence it is a subcooled or compressed liquid.
 We can also refer Table A2.2, $P_{\text{sat}}(40^\circ\text{C}) = 7.384 \text{ kPa}$. Here $P > P_{\text{sat}}$, hence it is a subcooled or compressed liquid.
- (c) Referring to Table A2.2, $P_{\text{sat}}(150^\circ\text{C}) = 475.72 \text{ kPa}$. Here $P < P_{\text{sat}}$, hence it is a superheated vapor.
 We can also refer Table A2.1, $T_{\text{sat}}(400 \text{ kPa}) = 143.64^\circ\text{C}$. Here $T > T_{\text{sat}}$, hence it is a superheated vapor.
- (d) Referring to Table A2.1, $v_1(5000 \text{ kPa}) = 0.001286 \text{ m}^3/\text{kg}$ and $v_g(5000 \text{ kPa}) = 0.03944 \text{ m}^3/\text{kg}$. Here $v > v_g$, hence it is superheated vapor, as shown in Figure E3.2(d).

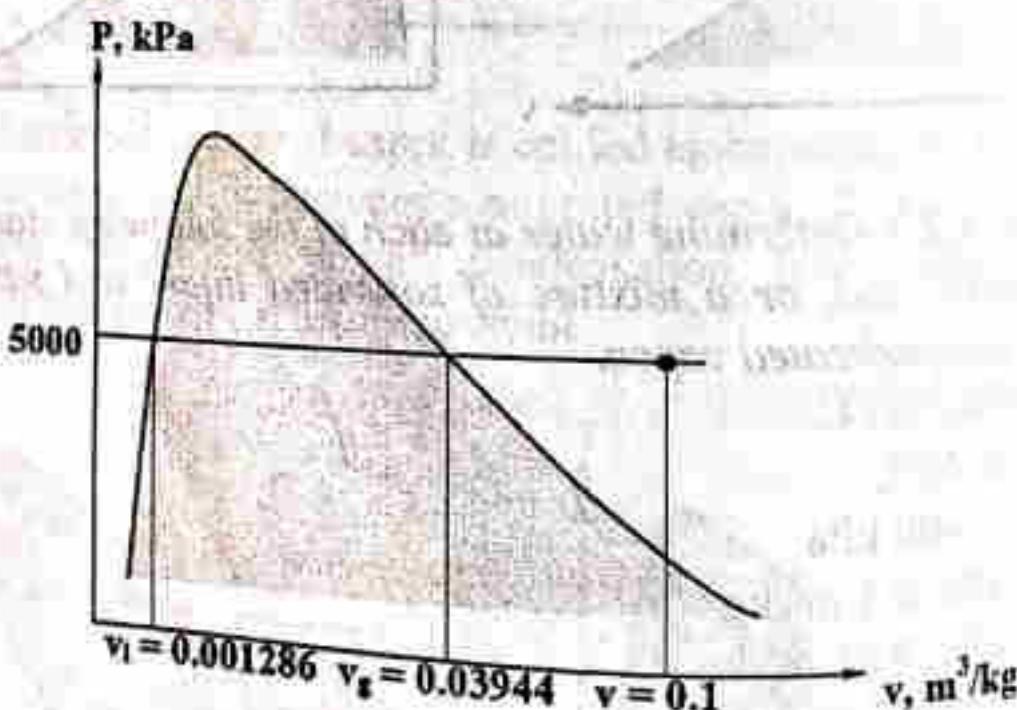


Figure E3.2 (d)

- (e) Referring to Table A2.1, $v_1(8000 \text{ kPa}) = 0.001384 \text{ m}^3/\text{kg}$ and $v_g(8000 \text{ kPa}) = 0.02352 \text{ m}^3/\text{kg}$. Here $v_1 < v < v_g$, hence it is a phase mixture, as shown in Figure E3.2(e).

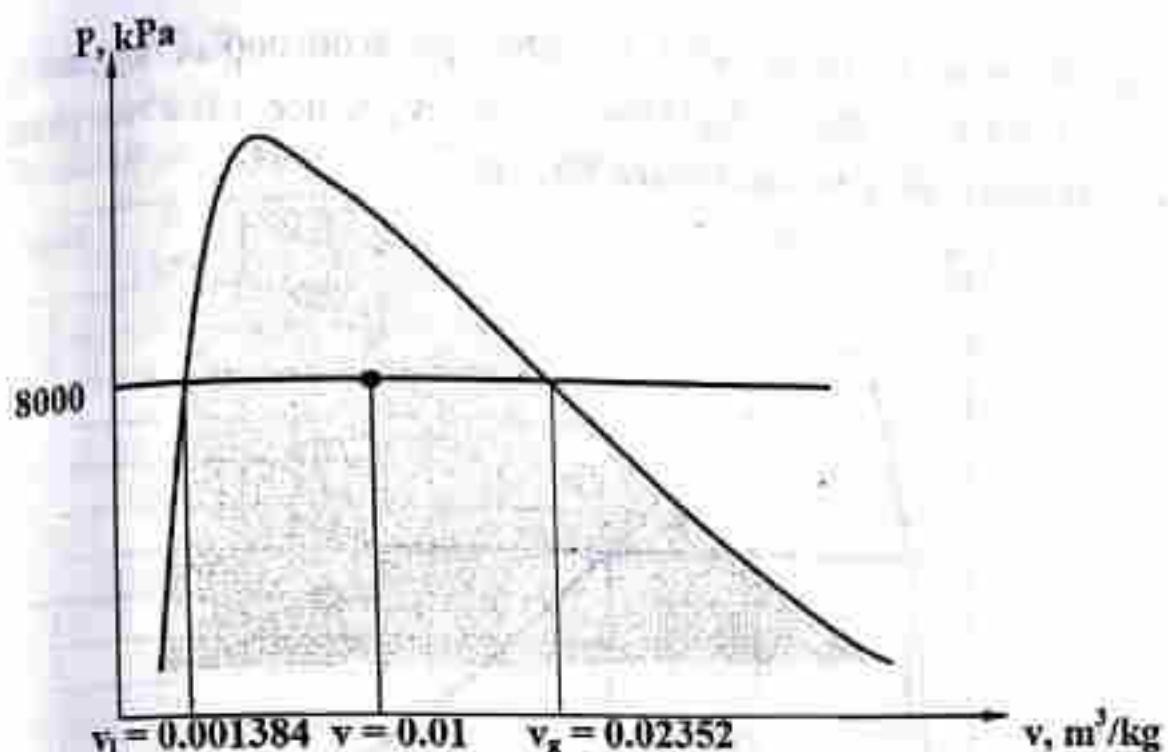


Figure E3.2 (e)

- (f) Referring to Table A2.2, v_1 (50°C) = 0.001012 m^3/kg and v_g (50°C) = 12.037 m^3/kg . Here $v < v_1$, hence it is a compressed liquid, as shown in Figure E3.2(f).

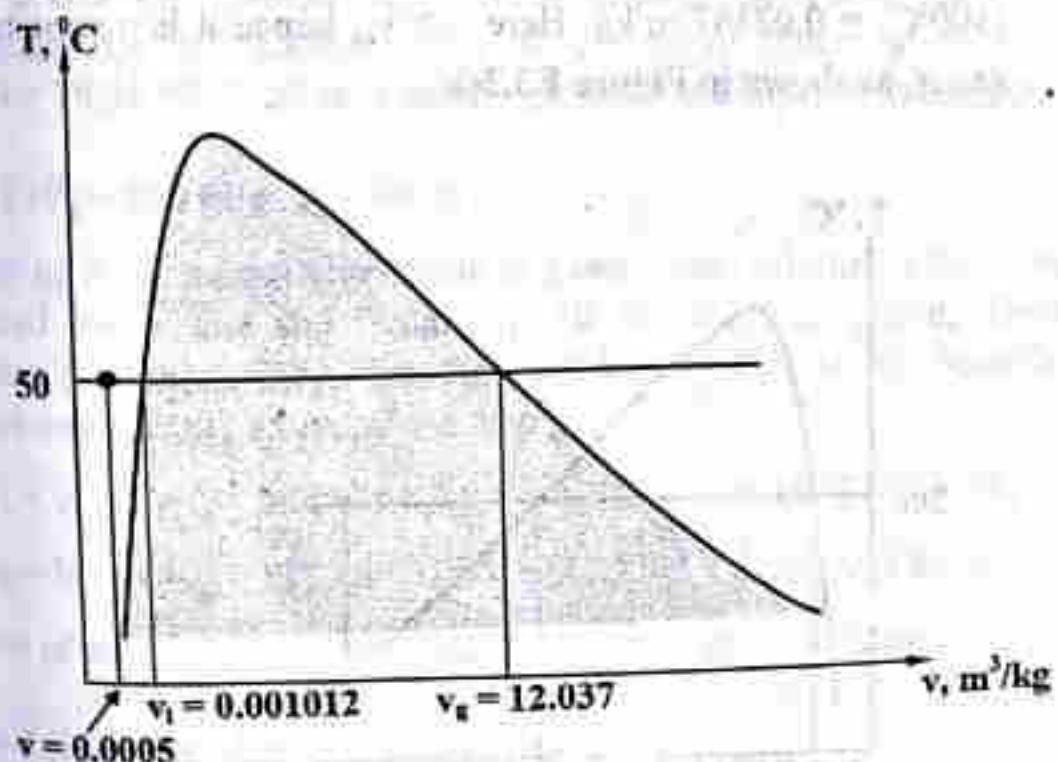


Figure E3.2 (f)

- (g) Referring to Table A2.2, v_1 (120°C) = 0.001060 m³/kg and v_g (120°C) = 0.8922 m³/kg. Here $v_1 < v < v_g$, hence it is a two phase mixture, as shown in Figure E3.2(g).

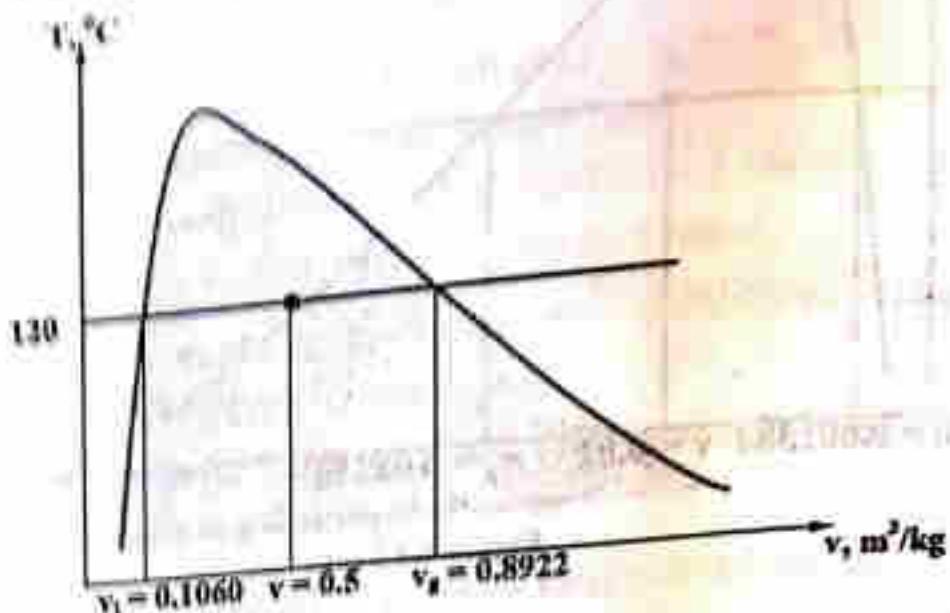


Figure E3.2 (g)

- (h) Referring to Table A2.2, v_1 (300°C) = 0.001404 m³/kg and v_g (300°C) = 0.02167 m³/kg. Here $v > v_g$, hence it is a superheated vapor, as shown in Figure E3.2(h).

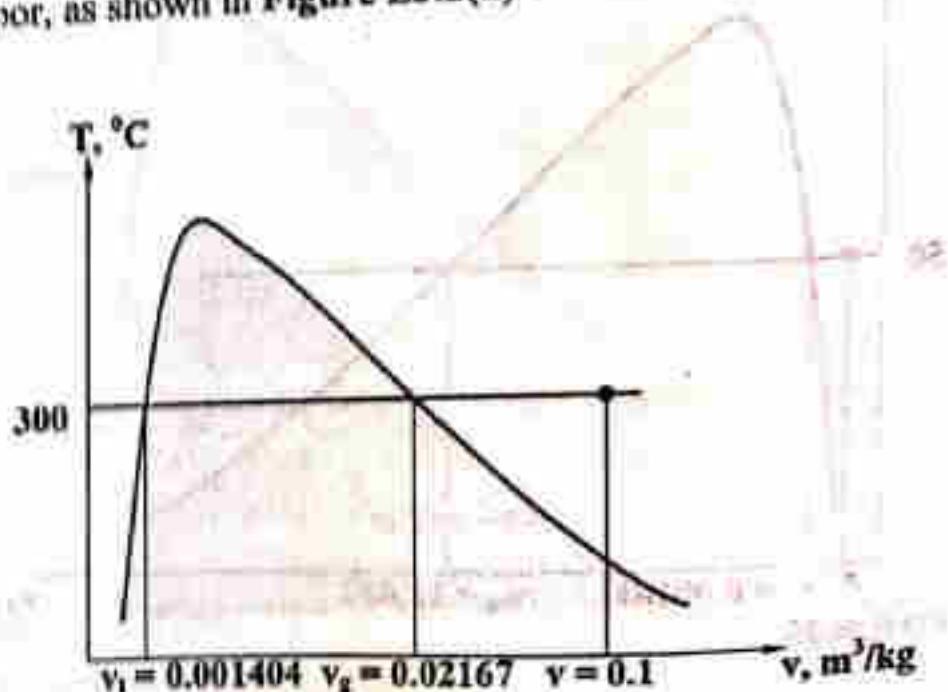


Figure E3.2 (h)

Example 3.3 : Fill in the blanks in the following table with the corresponding properties of water or by the symbol \times , when it is not relevant or meaningless or by the symbol $-$, when it is indeterminate.

State	P, kPa	T, °C	x, %	v, m³/kg	h, kJ/kg	Degree of Superheat
1	200	200				
2	250		80			
3	300			0.5		
4		150	60			
5		200		0.1		
6		250		0.5951		
7	1000				2000	
8		350			2563.5	
9	600			0.001101		
10	2000			0.1757		
11	5000			0.001206		
12		500			3445.4	
13	8000	295.04				

Solution

State 1 ($P_1 = 200 \text{ kPa}$, $T_1 = 200^\circ\text{C}$)

Referring to **Table A2.1**, $T_{\text{sat}}(200 \text{ kPa}) = 120.24^\circ\text{C}$. Here $T > T_{\text{sat}}$, hence it is a superheated vapor. Then referring to **Table A2.4**, $v_1 = 1.0803 \text{ m}^3/\text{kg}$, $h_1 = 2870.0 \text{ kJ/kg}$ and Degree of Superheat = $200 - 120.24 = 79.76^\circ\text{C}$. Quality is meaningless for the superheated vapor.

State 2 ($P_2 = 250 \text{ kPa}$, $x_2 = 80 \%$)

Quality of 80 % means given state is a two phase mixture with 80 % saturated vapor and the remaining 20 % saturated liquid. Then referring to **Table A2.1**, $T_2 = T_{\text{sat}}(250 \text{ kPa}) = 127.44^\circ\text{C}$. Specific volume and specific enthalpy are then given by

$$v_2 = v_1 + x_2 v_{1g} = 0.001067 + 0.8 \times 0.7177 = 0.575227 \text{ m}^3/\text{kg}$$

$$h_2 = h_1 + x_2 h_{1g} = 535.49 + 0.8 \times 2181.3 = 2280.53 \text{ kJ/kg}$$

Degree of superheat is meaningless for the two phase mixture.

State 3 ($P_3 = 300 \text{ kPa}$, $v_3 = 0.5 \text{ m}^3/\text{kg}$)

Referring to **Table A2.1**, $v_1(300 \text{ kPa}) = 0.001073 \text{ m}^3/\text{kg}$, $v_{1g}(300 \text{ kPa}) = 0.6048 \text{ m}^3/\text{kg}$ and $v_g(300 \text{ kPa}) = 0.6059 \text{ m}^3/\text{kg}$. Here $v_1 < v <$

v_g , hence it is a two phase mixture. Then, its temperature, quality and specific enthalpy are given by

$$T_3 = T_{sat}(300 \text{ kPa})$$

$$= 133.56^\circ\text{C}$$

$$x_3 = \frac{v_3 - v_l}{v_{lg}} = \frac{0.5 - 0.001073}{0.6048}$$

$$= 0.82495 = 82.495\%$$

$$h_3 = h_l + x_3 h_{lg} = 561.61 + 0.82495 \times 2163.7$$

$$= 2346.544 \text{ kJ/kg}$$

Degree of superheat is meaningless for the two phase mixture.

State 4 ($T_4 = 150^\circ\text{C}$, $x_4 = 60\%$)

Quality of 60 % means given state is a two phase mixture with 60 % saturated vapor and the remaining 40 % saturated liquid. Then referring to Table A2.2, $P_2 = P_{sat}(150^\circ\text{C}) = 475.72 \text{ kPa}$. Specific volume and specific enthalpy are then given by

$$v_4 = v_l + x_4 v_{lg} = 0.001090 + 0.6 \times 0.3918 = 0.23617 \text{ m}^3/\text{kg}$$

$$h_4 = h_l + x_4 h_{lg} = 632.32 + 0.6 \times 2114.1 = 1900.78 \text{ kJ/kg}$$

Degree of superheat is meaningless for the two phase mixture.

State 5 ($T_5 = 200^\circ\text{C}$, $v_5 = 0.1 \text{ m}^3/\text{kg}$)

Referring to Table A2.2, $v_l(200^\circ\text{C}) = 0.001156 \text{ m}^3/\text{kg}$, $v_{lg}(200^\circ\text{C}) = 0.1261 \text{ m}^3/\text{kg}$ and $v_g(200^\circ\text{C}) = 0.1273 \text{ m}^3/\text{kg}$. Here $v_l < v < v_g$, hence it is a two phase mixture. Then, its pressure, quality and specific enthalpy are given by

$$P_5 = P_{sat}(200^\circ\text{C}) = 1553.6 \text{ kPa}$$

$$x_5 = \frac{v_5 - v_l}{v_{lg}} = \frac{0.1 - 0.001156}{0.1261} = 0.78385 = 78.385\%$$

$$h_5 = h_l + x_5 h_{lg} = 852.38 + 0.78385 \times 1940.1 = 2373.135 \text{ kJ/kg}$$

Degree of superheat is meaningless for the two phase mixture.

State 6 ($T_6 = 250^\circ\text{C}$, $v_6 = 0.5951 \text{ m}^3/\text{kg}$)

Referring to Table A2.2, $v_l(250^\circ\text{C}) = 0.001251 \text{ m}^3/\text{kg}$, $v_{lg}(250^\circ\text{C}) = 0.04886 \text{ m}^3/\text{kg}$, and $v_g(250^\circ\text{C}) = 0.05011 \text{ m}^3/\text{kg}$. Here $v > v_g$, hence it is a superheated vapor. Then, referring to Table A2.4, $P_6 = 400 \text{ kPa}$,

$h_6 = 2963.6 \text{ kJ/kg}$, and Degree of Superheat = $250 - 143.64 = 106.36^\circ\text{C}$. Quality is meaningless for the superheated vapor.

State 7 ($P_7 = 1000 \text{ kPa}$, $h_7 = 2000 \text{ kJ/kg}$)

Referring to Table A2.1, h_l (1000 kPa) = 762.88 kJ/kg, h_{lg} (1000 kPa) = 2014.8 kJ/kg and h_g (1000 kPa) = 2777.7 kJ/kg. Here $h_l < h < h_g$, hence it is a two phase mixture. Then, its temperature, quality and specific volume are given by

$$T_7 = T_{\text{sat}}(1000 \text{ kPa}) = 179.92^\circ\text{C}$$

$$x_7 = \frac{h_7 - h_l}{h_{lg}} = \frac{2000 - 762.88}{2014.8} = 0.61402 = 61.402\%$$

$$v_7 = v_l + x_7 v_{lg} = 0.001127 + 0.61402 \times 0.1933 = 0.11982 \text{ m}^3/\text{kg}$$

Degree of superheat is meaningless for the two phase mixture.

State 8 ($T_8 = 350^\circ\text{C}$, $h_8 = 2563.5 \text{ kJ/kg}$)

Referring to Table A2.2, h_g (250°C) = 2563.5 kJ/kg. Here $h = h_g$, hence it is a saturated vapor. Then, its pressure, quality, specific volume and degree of superheat are given by

$$P_8 = P_{\text{sat}}(350^\circ\text{C}) = 16520 \text{ kPa}$$

$$x_8 = 1.0 = 100\%$$

$$v_8 = v_g(350^\circ\text{C}) = 0.008812 \text{ m}^3/\text{kg}$$

$$\text{Degree of superheat} = 0$$

State 9 ($P_9 = 600 \text{ kPa}$, $v_9 = 0.0011101 \text{ m}^3/\text{kg}$)

Referring to Table A2.2, v_l (600 kPa) = 0.0011101 m³/kg. Here $v = v_l$, hence it is a saturated liquid. Then, its temperature, quality and specific enthalpy are given by

$$T_9 = T_{\text{sat}}(600 \text{ kPa}) = 158.86^\circ\text{C}$$

$$x_9 = 0.0 = 0\%$$

$$h_9 = h_l(600 \text{ kPa}) = 670.71 \text{ kJ/kg}$$

Degree of superheat is meaningless for the saturated liquid.

State 10 ($P_{10} = 2000 \text{ kPa}$, $v_{10} = 0.1757 \text{ m}^3/\text{kg}$)

Referring to Table A2.1, $T_{\text{sat}}(2000 \text{ kPa}) = 212.42^\circ\text{C}$ and v_g (2000 kPa) = 0.0995 m³/kg. Here $v > v_g$, hence it is a superheated vapor. Then referring to Table A2.4, $T_{10} = 500^\circ\text{C}$, $h_{10} = 3467.7 \text{ kJ/kg}$ and

Degree of Superheat = $500 - 212.42 = 287.58^{\circ}\text{C}$. Quality is meaningless for the superheated vapor.

State 11 ($P_{11} = 5000 \text{ kPa}$, $v_{11} = 0.001206 \text{ m}^3/\text{kg}$)

Referring to Table A2.1, v_1 (5000 kPa) = $0.001286 \text{ m}^3/\text{kg}$. Here $v < v_1$, hence it is a compressed liquid. Then referring to Table A2.3, $T_{11} = 230^{\circ}\text{C}$, $h_{11} = 990.43 \text{ kJ/kg}$ and Degree of superheat and quality are meaningless for the compressed liquid.

State 12 ($T_{12} = 500^{\circ}\text{C}$, $h_{12} = 3445.5 \text{ kJ/kg}$)

Since, the given temperature is greater than critical temperature of the water, we directly refer to Table A2.4. We can observe that the given properties match with the properties of the superheated vapor at 4000 kPa, $P_{12} = 4000 \text{ kPa}$, $v_{12} = 0.08642 \text{ m}^3/\text{kg}$ and Degree of Superheat = $500 - 250.39 = 249.61^{\circ}\text{C}$. Quality is meaningless for the superheated vapor.

State 13 ($P_{13} = 8000 \text{ kPa}$, $T_{13} = 295.04^{\circ}\text{C}$)

Referring to Table A2.1, T_{sat} (8000 kPa) = 295.04°C . Here $T = T_{\text{sat}}$, hence it is within saturation region. But within the saturation region we cannot fix the state of a substance with the help of pressure and temperature because they are dependent within the saturation region. Hence, given state and required properties are indeterminate.

Table with complete property values is shown below:

State	P, kPa	$T, ^{\circ}\text{C}$	$x, \%$	$v, \text{m}^3/\text{kg}$	$h, \text{kJ/kg}$	Degree of Superheat, $^{\circ}\text{C}$
1	200	200	x	1.0803	2870.0	79.76
2	250	127.44	80	0.575227	2280.53	x
3	300	133.56	82.495	0.5	2346.544	x
4	475.72	150	60	0.23617	1900.78	x
5	1553.6	200	78.385	0.1	2373.135	x
6	400	250	x	0.5951	2963.6	106.36
7	1000	179.92	61.402	0.11982	2000	x
8	16520	350	100	0.008812	2563.5	0
9	600	158.86	0	0.001101	670.71	x
10	2000	500	x	0.1757	3467.7	287.58
11	5000	230	x	0.001206	990.43	x
12	4000	500	x	0.08642	3445.4	249.61
13	8000	295.04	—	—	—	—

Example 3.4 : Determine the specific volume and specific internal energy of the saturated water vapor at 190 kPa.

Solution

Properties of saturated water at 190 kPa are not given in the provided table (**Table A2.1**). Hence to determine the properties at 190 kPa, we have to use the interpolation technique. For this, we list the required properties for the interval which includes 190 kPa i.e., 175 kPa and 200 kPa.

P, kPa	v _g , m ³ /kg	u _g , kJ/kg	
175	1.0038	2524.7	(a)
200	0.8859	2529.4	(b)

Saturated vapor states of water at 175 kPa and 200 kPa are shown in Figure E3.4. Small portion of the saturation curve (ab) can be assumed as a straight line. Equation of a straight line passing through two given points with respect to X and Y axes are given as

$$Y - Y_a = \frac{Y_b - Y_a}{X_b - X_a} (X - X_a)$$

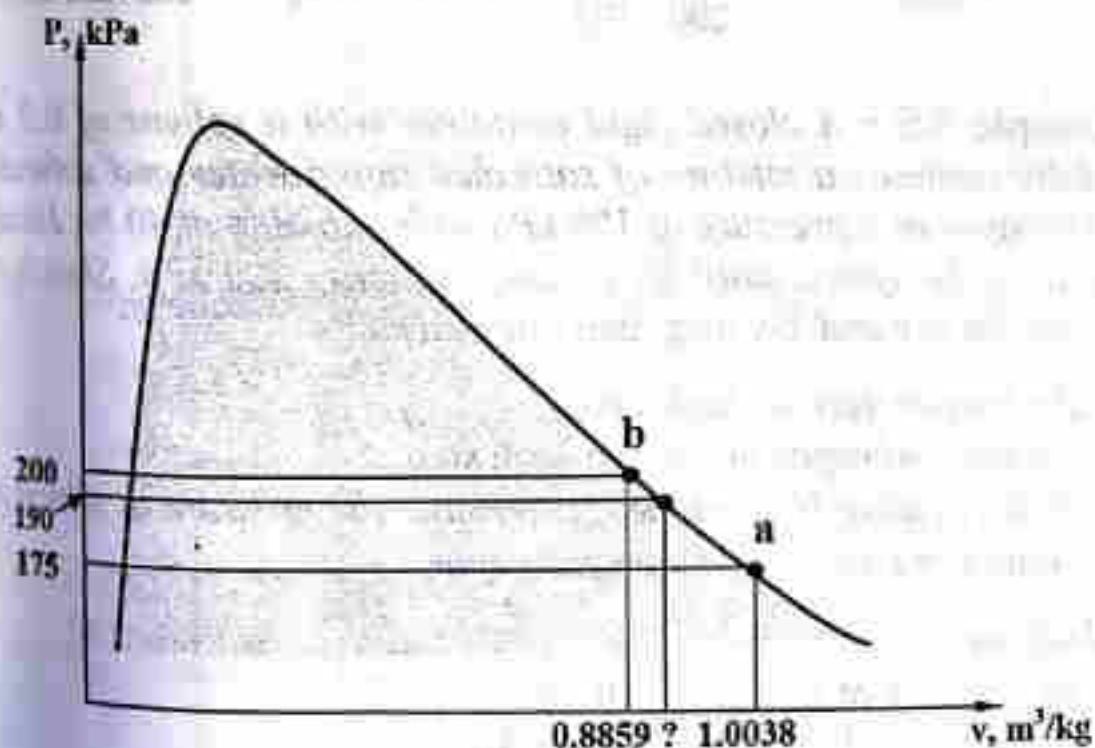


Figure E3.4 Linear Interpolation for Specific Volume

We can modify this equation in terms of given and required properties by replacing Y by the property which is to be determined

and X by the property which is given. Hence for specific volume, modified interpolation equation is given as

$$\begin{aligned} v_g - (v_g)_a &= \frac{(v_g)_b - (v_g)_a}{P_b - P_a} (P - P_a) \\ \therefore v_g &= (v_g)_a + \frac{(v_g)_b - (v_g)_a}{P_b - P_a} (P - P_a) \\ &= 1.0038 + \frac{0.8859 - 1.0038}{200 - 175} (190 - 175) = 0.93306 \text{ m}^3/\text{kg} \end{aligned}$$

Similarly, for specific internal energy, modified interpolation equation is given as

$$\begin{aligned} u_g - (u_g)_a &= \frac{(u_g)_b - (u_g)_a}{P_b - P_a} (P - P_a) \\ \therefore u_g &= (u_g)_a + \frac{(u_g)_b - (u_g)_a}{P_b - P_a} (P - P_a) \\ &= 2524.7 + \frac{2529.4 - 2524.7}{200 - 175} (190 - 175) = 2527.52 \text{ kJ/kg} \end{aligned}$$

Example 3.5 : A closed rigid container with a volume of 0.2 m^3 , initially contains a mixture of saturated liquid water and saturated water vapor at a pressure of 100 kPa with a quality of 40% . Heat is added to the system until its pressure reaches 200 kPa . Sketch the process on P-v and T-v diagrams and determine

- (a) the temperature at each state,
- (b) the mass of vapor present at each state,
- (c) If the heating is continued, determine the pressure at which the container holds only saturated vapor.

Solution

Given, Volume of vessel, $V = 0.2 \text{ m}^3$

State 1: $P_1 = 100 \text{ kPa}$, $x_1 = 0.4$

State 2: $P_2 = 200 \text{ kPa}$

Process: Constant volume heating

Specific volume at initial state,

$$v_1 = v_l + x v_{lg} = 0.001043 + 0.4 \times 1.6933 = 0.678363 \text{ m}^3/\text{kg}$$

Mass of steam inside the vessel

$$m = \frac{V}{v_1} = \frac{0.2}{0.678363} = 0.2948274 \text{ kg}$$

Since, volume is constant specific volume at state 2 is given as

$$v_2 = 0.678363 \text{ m}^3/\text{kg}$$

Referring to Table A2.1, v_1 (200 kPa) = 0.001060 m³/kg, v_{1g} (200 kPa) = 0.8848 m³/kg and v_g (200 kPa) = 0.8859 m³/kg. Here $v_1 < v < v_g$ hence it is a two phase mixture. P - v and T - v diagrams for the process is shown in Figure E3.5(a).

(a) Temperatures at states are given as

$$T_1 = T_{sat}(100 \text{ kPa}) = 99.632^\circ\text{C}$$

$$T_2 = T_{sat}(200 \text{ kPa}) = 120.24^\circ\text{C}$$

(b) Mass of vapor present at state 1,

$$(m_v)_1 = x_1 m = 0.4 \times 0.2948274 = 0.11793 \text{ kg}$$

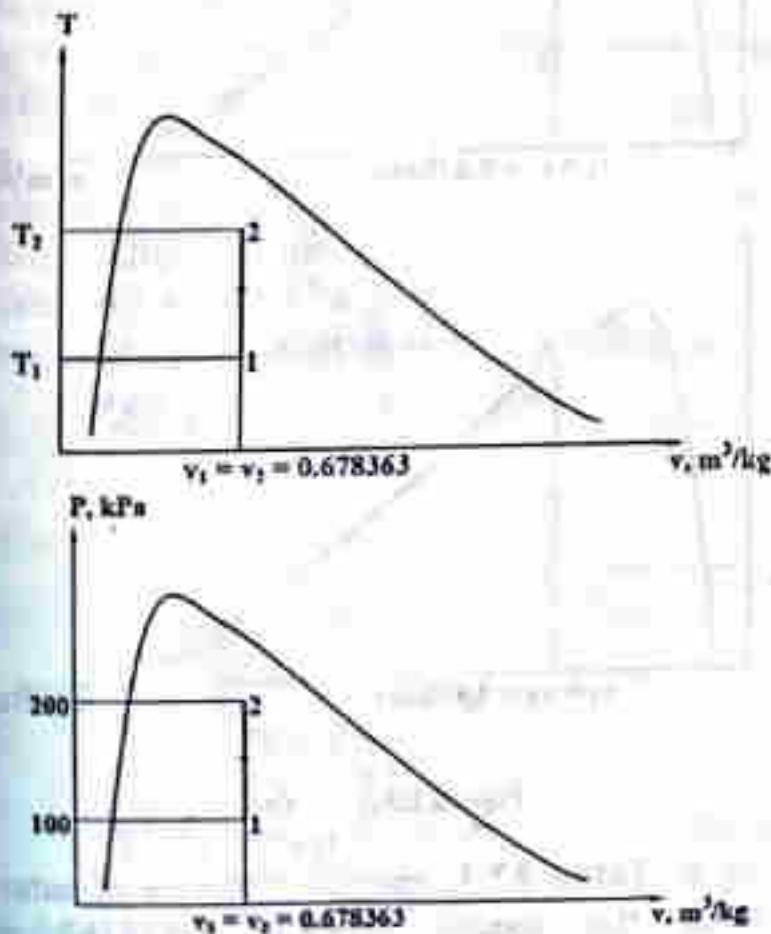


Figure E3.5 (a)

Quality of steam at state 2

$$x_2 = \frac{v - v_1}{v_{lg}} = \frac{0.678363 - 0.001060}{0.8848} = 0.765487$$

∴ Mass of vapor present at state 2

$$(m_g)_2 = x_2 m = 0.765487 \times 0.2948274 = 0.22569 \text{ kg}$$

If heating is continued, the container holds only saturated vapor when the extended process line 1 – 2 intersects the saturated vapor line at state 3, as shown in Figure E3.5(b).

$$v_3 = 0.678363 \text{ m}^3/\text{kg} = v_s$$

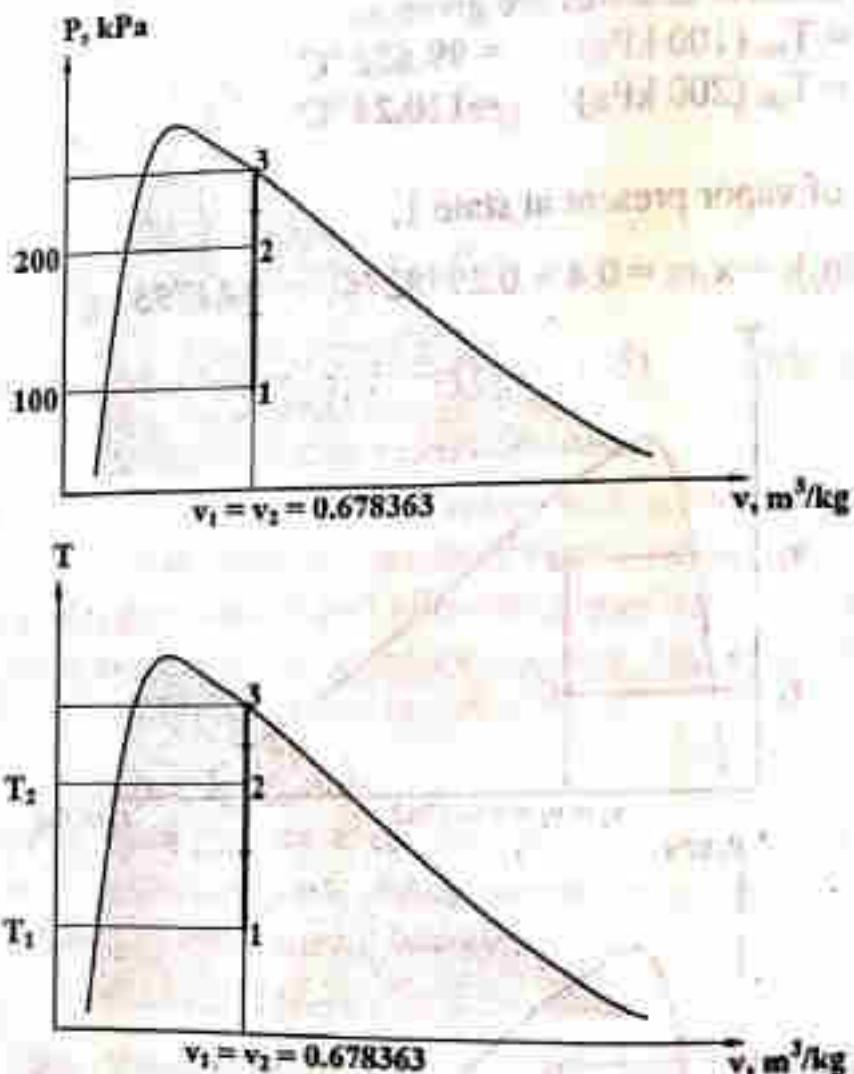


Figure E3.5 (b)

Now referring to Table A2.1, specific volumes of saturated vapor which includes the specific volume $0.678363 \text{ m}^3/\text{kg}$, corresponding pressures are listed as

P, kPa	v _g , m ³ /kg	
250	0.7188	(a)
275	0.6574	(b)

Then applying linear interpolation for pressure,

$$P_3 - P_a = \frac{P_b - P_a}{(v_g)_b - (v_g)_a} [v_3 - (v_g)_a]$$

$$\therefore P_3 = P_a + \frac{P_b - P_a}{(v_g)_b - (v_g)_a} [v_3 - (v_g)_a]$$

$$= 250 + \frac{275 - 250}{0.6574 - 0.7188} (0.768363 - 0.7188) = 266.465 \text{ kPa}$$

Example 3.6 : A rigid vessel having a volume of 0.02 m^3 , initially contains water at its critical state. The vessel is cooled until its pressure drops to 2000 kPa . Determine

- (a) the mass of H_2O present in the vessel,
- (b) the quality at final state, and
- (c) the mass of saturated liquid water and saturated water vapor at the final state.

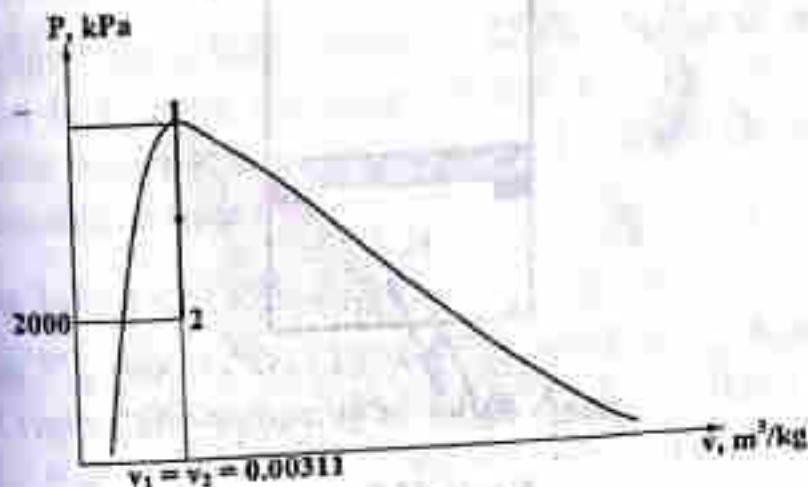
Solution

Given, Volume of vessel, $V = 0.02 \text{ m}^3$

State 1: critical point

State 2: $P_2 = 2000 \text{ kPa}$

Process: Constant volume cooling



Specific volume at initial state,

$$v_1 = v_{fg} = 0.00311 \text{ m}^3/\text{kg}$$

(a) Mass of H_2O ,

$$m = \frac{V}{v} = \frac{0.02}{0.00311} = 6.431 \text{ kg}$$

(b) Specific volume of H_2O at final state,
 $v_2 = v_1 = 0.00311 \text{ m}^3/\text{kg}$

Referring to Table A2.1, v_1 (2000 kPa) = 0.001177 m³/kg and
 v_2 (2000 kPa) = 0.09841 m³/kg. Then quality of H_2O at final state is
given as

$$x_2 = \frac{v_2 - v_1}{v_{lg}} = \frac{0.00311 - 0.001177}{0.09841} = 0.019642$$

(c) Masses of saturated liquid and saturated vapor at final states are
given as

$$(m_g)_2 = x_2 m = 0.019642 \times 6.431 = 0.1263 \text{ kg}$$

$$(m_l)_2 = m - (m_g)_2 = 6.431 - 0.1263 = 6.3047 \text{ kg}$$

Example 3.7 : A piston cylinder device shown in Figure E3.7 contains 2 kg of H_2O with an initial temperature and volume of 80°C and 0.05 m³ respectively. It requires a pressure of 400 kPa to lift the piston from the stops. The system is heated until its temperature reaches 250°C. Sketch the process on P-v and T-v diagrams and determine the total work transfer.

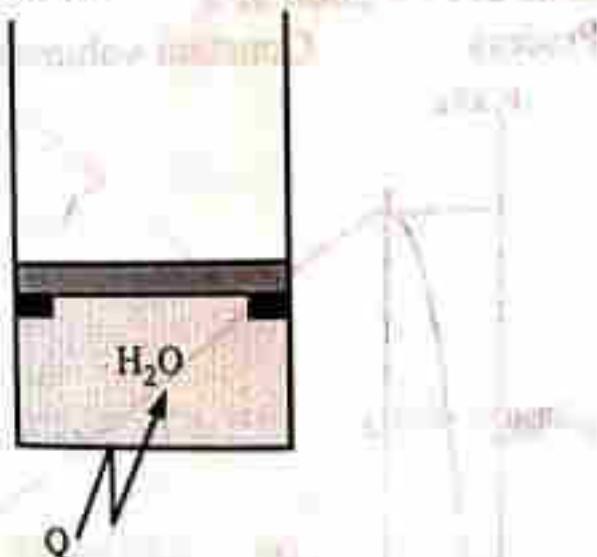


Figure E3.7

Given, Mass of H_2O : $m = 2 \text{ kg}$

Initial State: $T_1 = 80^\circ\text{C}$, $V_1 = 0.05 \text{ m}^3$

Final Temperature: $T_{\text{final}} = 250^\circ\text{C}$

Pressure required to lift the piston: $P_{\text{lift}} = 400 \text{ kPa}$

$$\therefore \text{Specific volume of H}_2\text{O at initial state } v_1 = \frac{V_1}{m} = \frac{0.05}{2} \\ = 0.025 \text{ m}^3/\text{kg}$$

Referring to **Table A2.2**, v_1 (80°C) = $0.001029 \text{ m}^3/\text{kg}$, v_{1g} (80°C) = $3.4078 \text{ m}^3/\text{kg}$ and v_g (80°C) = $3.4088 \text{ m}^3/\text{kg}$. Here $v_1 < v < v_g$, hence it is a two phase mixture.

\therefore Pressure of H_2O at initial state $P_1 = P_{\text{sat}}$ (80°C) = 47.373 kPa

Initial pressure of the system is 47.373 kPa and pressure required to lift the piston is 400 kPa . Hence, during initial stage of heating piston remain stationary although heat is supplied to the system, so process is constant volume heating (Process 1-2). During constant volume heating pressure of the system increases from 47.373 kPa to 400 kPa . Hence, we can define state 2 as

State 2: $P_2 = 400 \text{ kPa}$, $V_2 = 0.05 \text{ m}^3$

\therefore Specific volume of H_2O at state 2 $v_2 = 0.025 \text{ m}^3/\text{kg}$

Now referring to **Table A2.1**, v_1 (400 kPa) = $0.001084 \text{ m}^3/\text{kg}$, v_{1g} (400 kPa) = $0.4614 \text{ m}^3/\text{kg}$ and v_g (400 kPa) = $0.4625 \text{ m}^3/\text{kg}$. Here $v_1 < v < v_g$, hence it is also a two phase mixture.

\therefore Temperature of H_2O at state 2 (T_2) = T_{sat} (400 kPa) = 143.64°C

But the required final temperature is 250°C , hence it should be further heated to increase the temperature from 143.64°C to 250°C and the process occurs at constant pressure of 400 kPa (Process 2-3). Hence, we can define state 3 as

State 3: $P_3 = 400 \text{ kPa}$, $T_3 = 250^\circ\text{C}$

As we know, T_{sat} (400 kPa) = 143.64°C . Here $T > T_{\text{sat}}$, hence it is a superheated vapor. Then referring to **Table A2.4**, $v_3 = 0.5951 \text{ m}^3/\text{kg}$. $P - v$ and $T - v$ diagrams for the process is shown in Figure E3.7 (a) and (b).

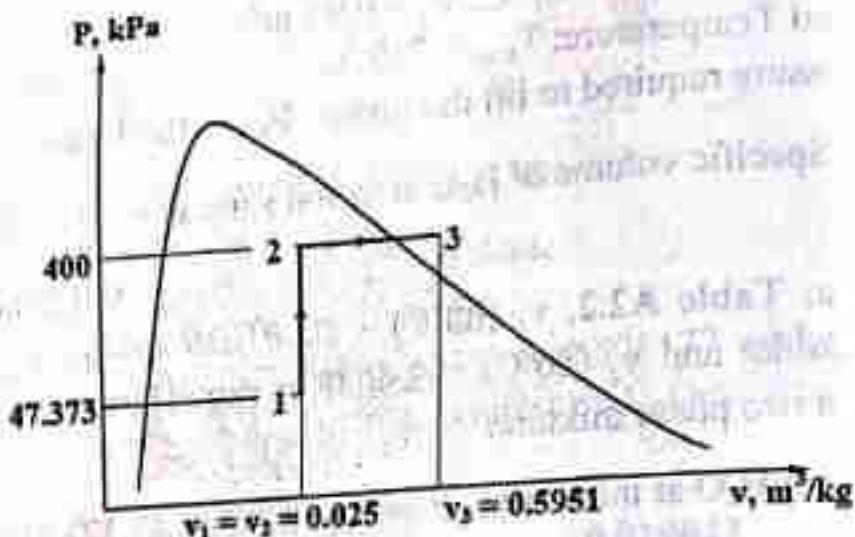


Figure E3.7 (a)

P - V diagram

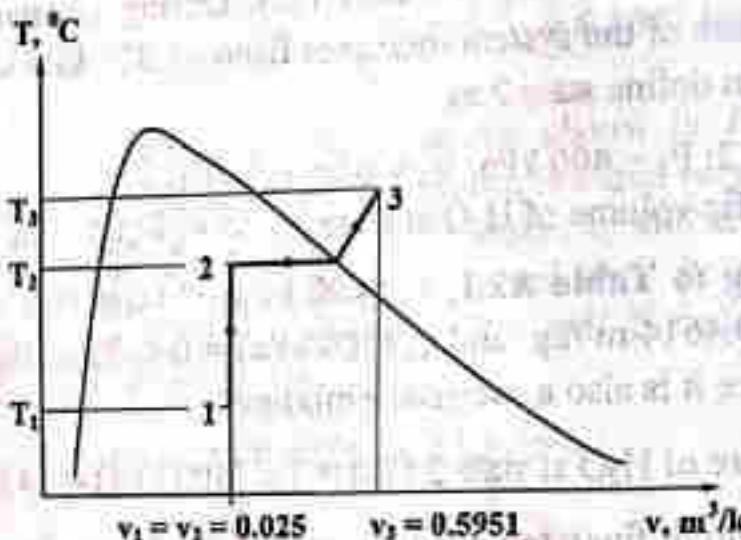


Figure E3.7 (b)

T - V diagram

Total work transfer for the process is given as

$$\begin{aligned}
 W &= W_{12} + W_{23} \\
 &= 0 + P_2(V_3 - V_2) \\
 &= mP_2(v_3 - v_2) = 2 \times 400(0.5951 - 0.025) \\
 &= 456.08 \text{ kJ}
 \end{aligned}$$

NUMERICAL PROBLEMS

1. Fill in the blanks in the following table with the corresponding properties of water or by the symbol \times , when it is not relevant or meaningless or by the symbol $-$, when it is indeterminate.

S.N.	P, kPa	T, $^{\circ}\text{C}$	x, %	v, m^3/kg
1	300	200		
2	300		65	
3		200		0.1050
4	10000			0.04863
5	20000	120		
6	5000			0.03944
7		150		0.00109
8	101.32	100		

2. Determine the pressure for water at 250°C with specific volume of $0.25 \text{ m}^3/\text{kg}$.
3. Determine the temperature and quality (if needed) for water at a pressure of 200 kPa and having a specific volume of
 (a) $0.8 \text{ m}^3/\text{kg}$
 (b) $1.25 \text{ m}^3/\text{kg}$.
4. A rigid vessel contains 8 kg of water at 120°C . If 5 kg of the water is in the liquid form and the rest in the vapor form. Determine:
 (a) the pressure in the vessel,
 (b) the volume of the tank,
 (c) the volume of saturated liquid and saturated vapor respectively, and
 (d) the specific enthalpy of H_2O .
5. A two phase mixture of H_2O has a temperature of 200°C . The mass of saturated liquid is 1 kg and saturated vapor is 3 kg. Determine the pressure and volume of the mixture.
6. A 0.3 m^3 rigid vessel contains 5 kg of water at 150 kPa. Determine:
 (a) the temperature,
 (b) the mass of each phase, and
 (c) the specific enthalpy.

7. A closed rigid vessel contains 0.1 m^3 of saturated liquid and 0.9 m^3 of saturated vapor of water in equilibrium at 200°C . Determine its quality.
8. A vessel contains 2 kg of saturated liquid water and saturated water vapor mixture at a temperature of 150°C . One third of the volume is saturated liquid and two third is saturated vapor. Determine the pressure, quality and volume of the mixture.
9. 2 kg of water is contained in a rigid vessel of volume 0.5 m^3 . Heat is added until the temperature is 150°C . Determine:
- the final pressure,
 - the mass of the vapor at the final state, and
 - the volume of the vapor at the final state.
10. Saturated water vapor at 200 kPa is in a freely moving piston cylinder device. At this state piston is 0.1 m from the bottom. Determine the height of the piston when the temperature is
- 250°C
 - 150°C
 - 100°C
11. Water in a piston cylinder device evaporates at a temperature of 120°C . If the diameter of the piston is 0.15 m and the local atmospheric pressure is 101 kPa, what is the mass of the piston? [Take $g = 9.81 \text{ m/s}^2$]
12. A piston cylinder device containing water has a piston mass of 50 kg and a cross sectional area of 0.01001 m^2 . If the atmospheric pressure is 101 kPa, determine the temperature at which water will start boiling. [Take $g = 9.81 \text{ m/s}^2$]
13. 5 kg of H_2O is contained in a closed rigid container with an initial pressure and quality of 1000 kPa and 40 % respectively. Heat is added to the system until the container holds only saturated vapor. Sketch the process on P-v and T-v diagrams and determine:
- the volume of the container, and
 - the final pressure.
14. A rigid vessel with volume of 0.4 m^3 contains 2 kg of water in the form of saturated liquid and saturated vapor mixture. Heat is

supplied to the system from an external source. Determine the temperature at which the water in the vessel is completely vaporized.

15. Water initially at saturated vapor state is heated in a closed rigid vessel from an initial temperature 150°C to a final temperature of 600°C . Sketch the process and P-v and T-v diagrams and determine the initial and final pressure.
16. Steam contained in a closed container initially at a pressure of 2 MPa and a temperature of 400°C . The temperature drops as a result of heat transfer to the surroundings until the temperature reaches 80°C . Determine:
 - (a) the pressure at which the condensation first occurs,
 - (b) the pressure and quality at final state, and
 - (c) the percentage of volume occupied by the saturated liquid at the final state.
17. Water is contained in a rigid vessel of 5 m^3 at a quality of 0.8 and a pressure of 2 MPa. If it is cooled to a pressure of 400 kPa, determine the mass of saturated liquid and saturated vapor at the final state.
18. A rigid container with a volume of 0.170 m^3 is initially filled with steam at 200 kPa, 300°C . It is cooled to 90°C .
 - (a) At what temperature does a phase change start to occur?
 - (b) What is the final pressure?
 - (c) What mass fraction of the water is liquid in the final state?
19. A 0.05 m^3 rigid vessel initially contains a mixture of saturated liquid and saturated vapor at 100 kPa. The water is now heated until it reaches the critical state. Determine the mass and volume of the saturated liquid water at the initial state.
20. A piston cylinder device shown in Figure P3.20 contains 0.5 kg of water initially at a pressure of 1 MPa and 250°C . Heat is lost by the system until it becomes saturated liquid. Sketch the process on P-v and T-v diagrams and determine the work transfer.

21. A piston cylinder device shown in **Figure P3.21** contains 0.2 kg of a mixture of saturated liquid water and saturated water vapor at a temperature of 50°C and a volume of 0.03 m^3 . The mass of the piston resting on the stops is 50 kg and the cross sectional area of the piston is 12.2625 cm^2 . The atmospheric pressure is 100 kPa. Heat is transferred until it becomes saturated vapor. Sketch the process on P-v and T-v diagrams and determine:
- the temperature at which the piston just leaves the stops.
 - the final pressure, and
 - the total work transfer. [Take $g = 9.81 \text{ m/s}^2$]
22. A piston cylinder device shown in **Figure P3.22** contains water initially at a pressure of 125 kPa with a quality of 50 %. Heat is added to the system until it reaches to a final temperature of 800°C . It takes a pressure of 600 kPa to lift the piston from the stops. Sketch the process on P-v and T-v diagrams and determine:
- the mass of H_2O in the system, and
 - the total work transfer.
23. A piston cylinder device shown in **Figure P3.23** contains 2 kg of water initially at a pressure of 500 kPa with a quality of 20 %. The water is heated until it becomes a saturated vapor. The volume of the system when the piston is at the upper stops is 0.4 m^3 . Sketch the process on P-v and T-v diagrams and determine:
- the final pressure, and
 - the total work transfer.



Figure P3.20



Figure P3.21

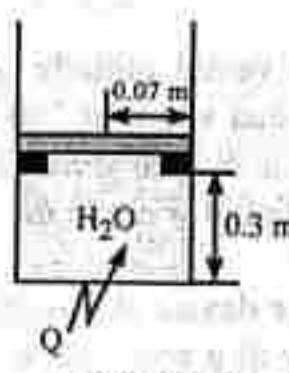


Figure P3.22

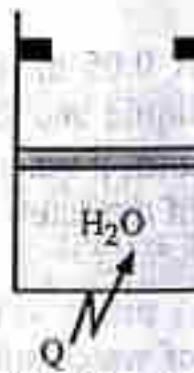


Figure P3.23

24. The frictionless piston shown in **Figure P3.24** has a mass of 20 kg and a cross sectional area of 78.48 cm^2 . Heat is added until the

temperature reaches 400°C . If the quality of the H_2O at the initial state is 0.2, determine:

- the initial pressure,
- the mass of H_2O ,
- the quality of the system when the piston hits the stops,
- the final pressure, and
- the total work transfer.

[Take $P_{\text{atm}} = 100 \text{ kPa}$, $g = 9.81 \text{ m/s}^2$]

25. A piston cylinder arrangement shown in Figure P3.25 contains 0.2 kg of water initially at a pressure of 150 kPa with a quality of 40 %. The system is heated to a position where the piston is locked, and then cooled till it becomes a saturated vapor at a temperature of 60°C . Sketch the process on P-v and T-v diagrams and determine the total work transfer.
26. A piston cylinder arrangement shown in Figure P3.26 contains 1 kg of water initially at a pressure of 1 MPa and a temperature of 500°C . The water is cooled until it is completely converted into the saturated liquid. It requires a pressure of 400 kPa to support the piston. Sketch the process on P-v and T-v diagrams and determine the total work transfer.
27. A piston cylinder arrangement shown in Figure P3.27 contains water initially at a pressure of 1 MPa and a temperature of 400°C . Heat is transferred from the system to the surroundings until its pressure drops to 100 kPa. Sketch the process on P-v and T-v diagrams and determine:
- the mass of H_2O in the system, and
 - the total work transfer.

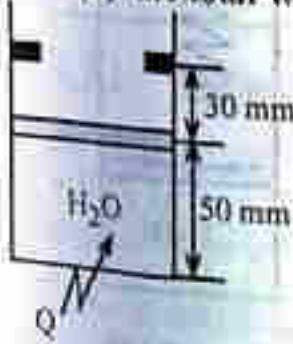


Figure P3.24

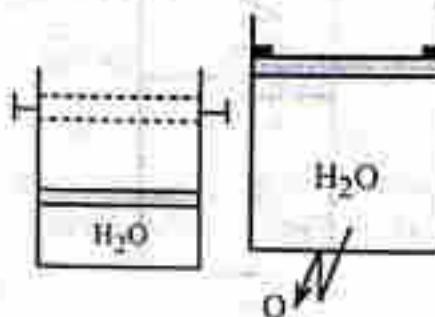


Figure P3.25

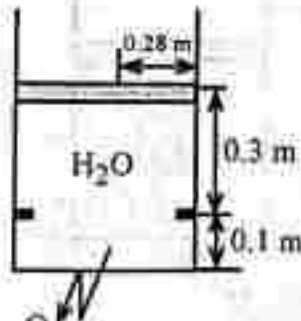


Figure P3.26

Figure P3.27

28. A piston cylinder arrangement shown in **Figure P3.28** contains 2 kg of water initially at a pressure of 200 kPa and a temperature of 50°C . Heat is added until the piston reaches the upper stop, where the total volume is 1.5 m^3 . It takes a pressure of 600 kPa to lift the piston. Sketch the process on P-v and T-v diagrams and determine the final temperature and the work transfer.
29. A piston cylinder device with a linear spring initially contains water at a pressure of 4 MPa and 500°C with the initial volume being 0.1 m^3 , as shown in **Figure P3.29**. If the piston is at the bottom, the system pressure is 300 kPa. The system now cools until the pressure reaches 1000 kPa. Sketch the process on P-v diagram and determine
- the mass of H_2O
 - the final temperature and volume, and
 - the total work transfer.
30. A piston cylinder arrangement shown in **Figure P3.30** contains water initially at $P_1 = 100 \text{ kPa}$, $x_1 = 0.8$ and $V_1 = 0.01 \text{ m}^3$. When the system is heated, it encounters a linear spring ($k = 100 \text{ kN/m}$). At this state volume is 0.015 m^3 . The heating continues till its pressure is 200 kPa. If the diameter of the piston is 0.15 m, determine
- the final temperature, and
 - the total work transfer.
- Also sketch the process on P-v diagram.



Figure P3.28

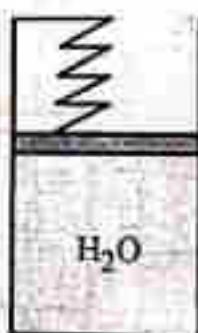


Figure P3.29



Figure P3.30

FIRST LAW OF THERMODYNAMICS

4

4.1 Introduction

First law of thermodynamics is based on the conservation principles. It gives the mathematical expression for the effect of the interactions between the system and surrounding on the stored energy (total energy) of the system. In closed system (control mass), interactions can take place only in the form of energy transfer (work transfer and heat transfer). Therefore first law of thermodynamics for a control mass can be explained with reference to conservation of energy only. Whereas in an open system (control volume), interactions can take place both by the mass transfer and energy transfer. Hence the first law of thermodynamic for a control volume is explained with reference to both mass conservation and energy conservation principles.

However, for the comparative study, we analyze both the control mass and control volume with reference to conservation of mass as well as conservation of energy principle.

4.2 First Law of Thermodynamics for a Control Mass

4.2.1 Conservation of Mass for a Control Mass

As defined earlier, a control mass (closed system) is a system in which energy transfer can take place but mass transfer cannot. Hence, conservation of mass for a control mass can be stated as

Total mass of a control mass always remains constant.

Mathematically,

$$dm = 0 \quad \dots \dots \dots (4.1)$$

For any process between state 1 and 2,

$$\begin{aligned} m_2 - m_1 &= 0 \\ \therefore m_2 &= m_1 \end{aligned} \quad \dots \dots \dots \quad (4.2)$$

Equation (4.1) can also be expressed in terms rate as

$$\frac{dm}{dt} = 0 \quad \dots \dots \dots \quad (4.3)$$

4.2.2 Conservation of Energy for a Control Mass

Consider a control mass undergoing a process during which δQ amount of heat is supplied to the control mass and it produces δW amount of work. If heat supplied during the process is greater than the work produced by the system then total energy of the system increases; whereas if heat supplied during the process is less than the work produced by the system then the total energy of the system decreases.

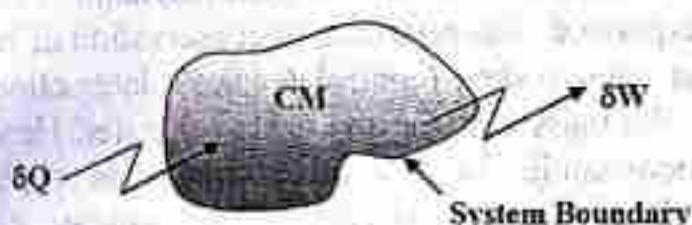


Figure 4.1 Control Mass undergoing a Process

We can generalize it as conservation of energy for a control mass as:

The change in total energy of a control mass is equal to the heat supplied to the control mass minus the work produced by the control mass.

Mathematically,

$$dE = \delta Q - \delta W \quad \dots \dots \dots \quad (4.4)$$

which can also be expressed in terms of rates as

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \quad \dots \dots \dots \quad (4.5)$$

For any process 1 – 2 between state 1 and 2,

$$E_2 - E_1 = Q_{12} - W_{12} \quad \dots \dots \dots \quad (4.6)$$

(point function) = 0
 point fn \rightarrow temp., pressure, density, mass
 volume, entropy, internal energy.

First Law of Thermodynamics

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where Q_{12} and W_{12} are the total heat transferred to the control mass and total work transferred by the control mass during the process, respectively.

Substituting Equation (2.3) into Equation (4.6),

$$(U + PE + KE)_2 - (U + PE + KE)_1 = Q_{12} - W_{12}$$

$$\text{or, } \left(U + mgz + \frac{1}{2} m \bar{V}^2 \right)_2 - \left(U + mgz + \frac{1}{2} m \bar{V}^2 \right)_1 = Q_{12} - W_{12} \quad \dots \quad (4.7)$$

Most common example of a control mass is a piston cylinder device and for a stationary piston cylinder device the changes in potential energy and kinetic energy are negligible in comparison to the change in internal energy. Therefore, for the piston cylinder devices, Equation (4.7) reduces to

$$U_2 - U_1 = Q_{12} - W_{12} \quad \dots \quad (4.8)$$

Equation (4.8) can also be rearranged for the heat transfer as

$$Q_{12} = (\Delta U)_{12} + W_{12} \quad \dots \quad (4.9)$$

4.2.3 First Law of Thermodynamics for a Control Mass Undergoing Cyclic Process (CAQ)

Taking cyclic integral of Equation (4.4).

$$\oint dE = \oint \delta Q - \oint \delta W \quad \dots \quad (4.10)$$

As defined earlier, for a cyclic process, initial and final states are identical, i.e., $\oint dE = 0$, $\oint \delta Q \neq 0$ and $\oint \delta W \neq 0$. Then Equation (4.10) reduces to

$$\oint \delta Q = \oint \delta W \quad \dots \quad (4.11)$$

Equation (4.11) can also be expressed in the equivalent form as

$$\sum Q = \sum W \quad \dots \quad (4.12)$$

For a cycle shown in Figure 1.11, Equation (4.12) can be expanded as

$$Q_{12} + Q_{23} + Q_{31} = W_{12} + W_{23} + W_{31} \quad \dots \quad (4.13)$$

Hence, first law of thermodynamics for a control mass undergoing a cyclic process can be stated as

Whenever a control mass is taken through a cycle then the heat transferred to the control mass is equal to the net work done by the control mass.

OR

Whenever a control mass is taken through a cycle then the heat rejected by the control mass is equal to the net work done on the control mass.

We can also explain this statement directly with reference to conservation energy principle. During a cyclic process, there is no storage or consumption of total energy of the system and therefore incoming energy should be equal to outgoing energy.

4.3 Applications of First Law of Thermodynamics for Non-flow Processes

We can apply Equation (4.9) to derive the expression for heat transfer for different processes on a control mass.

4.3.1 Constant Volume (Isochoric) Process

For a constant volume process, there is no displacement work transfer, i.e., $W_{12} = 0$. Substituting into Equation (4.9), we get

$$Q_{12} = (\Delta U)_{12} \quad \dots \quad (4.14)$$

Hence, heat transferred to a control mass during a constant volume process is equal to the increase in internal energy of the system.

4.3.2 Constant Pressure (Isobaric) Process

For a constant pressure process, displacement work transfer is given as

$$W_{12} = P(V_2 - V_1)$$

where, $P_1 = P_2 = P$.

Substituting into Equation (4.9), we get

$$\begin{aligned} Q_{12} &= (\Delta U)_{12} + P(V_2 - V_1) \\ &= (U_2 - U_1) + (P_2 V_2 - P_1 V_1) \\ &= (U_2 + P_2 V_2) - (U_1 + P_1 V_1) \\ \therefore Q_{12} &= (U + PV)_2 - (U + PV)_1 \quad \dots \quad (4.15) \end{aligned}$$

The expression $U + PV$ occurs so frequently in thermodynamics that it has been given a special name and symbol; enthalpy and H , i.e.,

$$H = U + PV \quad \dots \quad (4.16)$$

Hence, enthalpy is defined as the summation of internal energy and the product of pressure and volume. Likewise other thermodynamic properties it depends only on end states not on the path. It is expressed in Joule (J).

Substituting Equation (4.16) into Equation (4.15),

$$Q_{12} = H_2 - H_1 = (\Delta H)_{12} \quad \dots \quad (4.17)$$

Hence, heat transferred to a control mass during a constant pressure process is equal to the increase in enthalpy of the system.

4.3.3 Specific Heats of an Ideal Gas

Specific heat of a substance is defined as a heat required for raising the temperature of a unit mass of the substance by 1°C . This definition is applicable for solid and liquid substances. But in case of a gas, heat required to raise temperature of unit mass of the gas by 1°C is different for the constant volume and the constant pressure process. Therefore, for a gas, two specific heats are defined: specific heat at constant volume (c_v) and specific heat at constant pressure (c_p).

Specific Heat at Constant Volume

Heat supplied to an ideal gas during a constant volume process is equal to the change in internal energy. Hence, specific heat of an ideal gas at constant volume can be defined in terms of change in internal energy.

For this let us start with the state postulate, i.e., specific internal energy of an ideal gas can be determined if its temperature and specific volume are given.

$$u = u(T, v) \quad \dots \quad (4.18)$$

Then the change in specific internal energy is given as

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv \quad dQ = mC_V dT = du \\ \therefore (C_V = \frac{du}{dT}) \quad \dots \quad (4.19)$$

For a constant volume process, substituting $dv = 0$, we get

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT \quad C_p = \left(\frac{dh}{dT}\right) \quad \dots \quad (4.20)$$

which can be expressed in simpler form as

$$du = c_v dT \quad \dots \dots \dots (4.21)$$

where,

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v \quad \dots \dots \dots (4.22)$$

is called specific heat at constant volume.

Hence, specific heat at constant volume for an ideal gas is defined as the change in specific internal energy per degree change in temperature during a constant volume.

Specific Heat at Constant Pressure

Heat supplied to an ideal gas during a constant pressure process is equal to the change in enthalpy. Hence, specific heat of an ideal gas at constant pressure can be defined in terms of change in enthalpy.

For this let us start with the state postulate for enthalpy as

$$h = h(T, P) \quad \dots \dots \dots (4.23)$$

Then the change in specific enthalpy is given as

$$dh = \left(\frac{\partial h}{\partial T} \right)_P dT + \left(\frac{\partial h}{\partial P} \right)_T dP \quad \dots \dots \dots (4.24)$$

For a constant pressure process, substituting $dP = 0$, we get

$$dh = \left(\frac{\partial h}{\partial T} \right)_P dT \quad \dots \dots \dots (4.25)$$

which can be expressed in simpler form as

$$dh = c_p dT \quad \dots \dots \dots (4.26)$$

where,

$$c_p = \left(\frac{\partial h}{\partial T} \right)_P \quad \dots \dots \dots (4.27)$$

is called specific heat at constant pressure.

Hence, specific heat at constant pressure for an ideal gas is defined as the change in specific enthalpy per degree change in temperature during a constant pressure.

4.3.4 Constant Temperature (Isothermal) Process for an Ideal Gas

For a constant temperature process, displacement work transfer is given as

$$W_{12} = P_1 V_1 / n \left(\frac{V_2}{V_1} \right)$$

The change in internal energy of an ideal gas during an isothermal process is given as

$$(\Delta U)_{12} = m c_V (T_2 - T_1) = 0$$

Substituting $(\Delta U)_{12}$ and W_{12} into Equation (4.9),

$$Q_{12} = W_{12} = P_1 V_1 / n \left(\frac{V_2}{V_1} \right) \quad \dots \dots \dots (4.28)$$

Hence, heat transferred to a control mass consisting of an ideal gas during a constant temperature process is equal to the work done by the system.

4.3.5 Polytropic Process for an Ideal Gas

Work transfer and the change in internal energy of an ideal gas during a polytropic process are given as

$$\begin{aligned} W_{12} &= \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{m R (T_2 - T_1)}{1-n} \\ (\Delta U)_{12} &= m c_V (T_2 - T_1) \end{aligned} \quad \begin{matrix} \text{Some } \frac{\partial T_2}{\partial T_1} \text{ Const} \\ \text{Vol...H1...Q130} \end{matrix} \quad \dots \dots \dots (4.30)$$

Substituting $(\Delta U)_{12}$ and W_{12} into Equation (4.9),

$$Q_{12} = m \left(c_V + \frac{R}{1-n} \right) (T_2 - T_1) \quad \dots \dots \dots (4.31)$$

Equation (4.31) can also be expressed in simpler form as

$$Q_{12} = m c_n (T_2 - T_1) \quad \dots \dots \dots (4.32)$$

where,

$$c_n = c_V + \frac{R}{1-n}$$

is the polytropic specific heat.

4.4 First Law of Thermodynamics for a Control Volume

4.4.1 Conservation of Mass for a Control Volume

Any control volume (open system) can interact with its surroundings by energy as well as mass transfer. Therefore, mass of the control

volume may change or under some specific condition, may remain constant. In general, conservation of mass for a control volume can be stated as

The change in mass within a control volume is equal to the mass entering into the control volume minus the mass leaving the control volume.

Mathematically,

where \dot{m}_{in} is given by the summation of mass flow at each inlets and \dot{m}_{out} is given by the summation of mass flow at each outlets, i.e.,

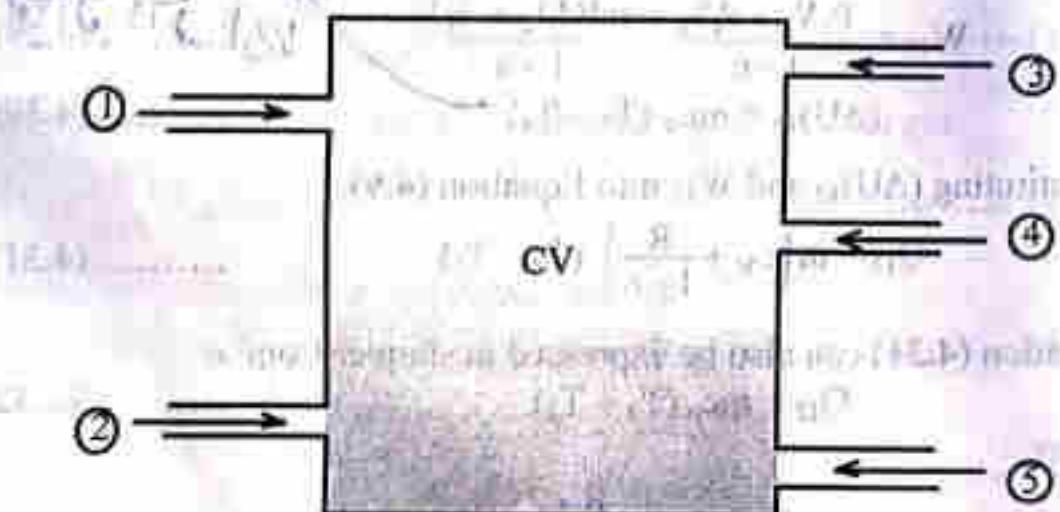


Figure 4.2 Mass Transfer in a Control Volume

For example, consider a control volume shown in Figure 4.1 consisting of two inlets and three outlets. Total mass flow rates \dot{m}_{in} and \dot{m}_{out} for the system are given as

$$\dot{m}_{in} = \dot{m}_1 + \dot{m}_2$$

$$\dot{m}_{out} = \dot{m}_3 + \dot{m}_4 + \dot{m}_5$$

Expression for Mass Flow Rate

Consider fluid flowing through a port shown in Figure 4.3 having a uniform cross sectional area A , where it crosses ΔL distance in time interval Δt .

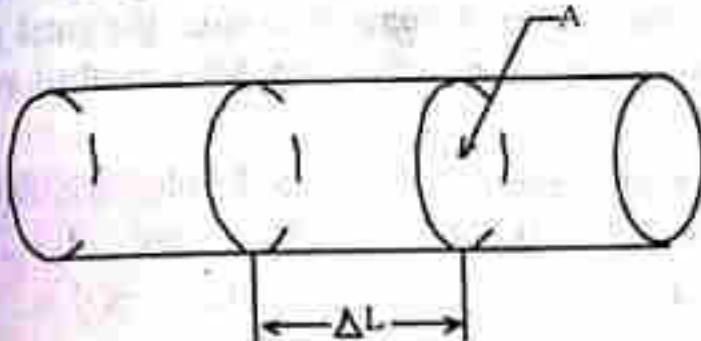


Figure 4.3 Mass flow at a port

Total mass of the fluid crossing the section of length ΔL is given by

$$m = \rho V_{\text{swept}} \quad \dots \dots \dots (4.36)$$

where, ρ is the density of the fluid and V_{swept} is the volume swept away by the fluid in the given time interval Δt .

Substituting $V_{\text{swept}} = A\Delta L$, into Equation (4.36), we get

$$m = \rho A \Delta L \quad \dots \dots \dots (4.37)$$

The mass flow rate is then given by

$$\dot{m} = \rho A \frac{\Delta L}{\Delta t} = \rho A \bar{V} \quad \dots \dots \dots (4.38)$$

where \bar{V} is the velocity of fluid.

Equation (4.38) also be expressed in terms of specific volume as

$$\dot{m} = \frac{A \bar{V}}{v} \quad \boxed{\text{Ans - } 1/3} \quad \dots \dots \dots (4.39)$$

Equation (4.33) can then be expressed in terms of section properties as

$$\frac{dm_{\text{CV}}}{dt} = \sum_{\text{in}} \left(\frac{A \bar{V}}{v} \right) - \sum_{\text{out}} \left(\frac{A \bar{V}}{v} \right) \quad \dots \dots \dots (4.40)$$

4.4.2 Conservation of Energy for a Control Volume

Along with heat transfer and work transfer, mass transfer also affects the total energy of a control volume. Mass entering into the control volume increases the total energy of the system whereas mass leaving from the control volume decreases the total energy of the system. Hence conservation of energy for a control volume can be stated as:-

The change in total energy of a control volume is equal to the net energy transported by the fluid into the control volume plus the heat transferred to the control volume minus the work done by the control volume.

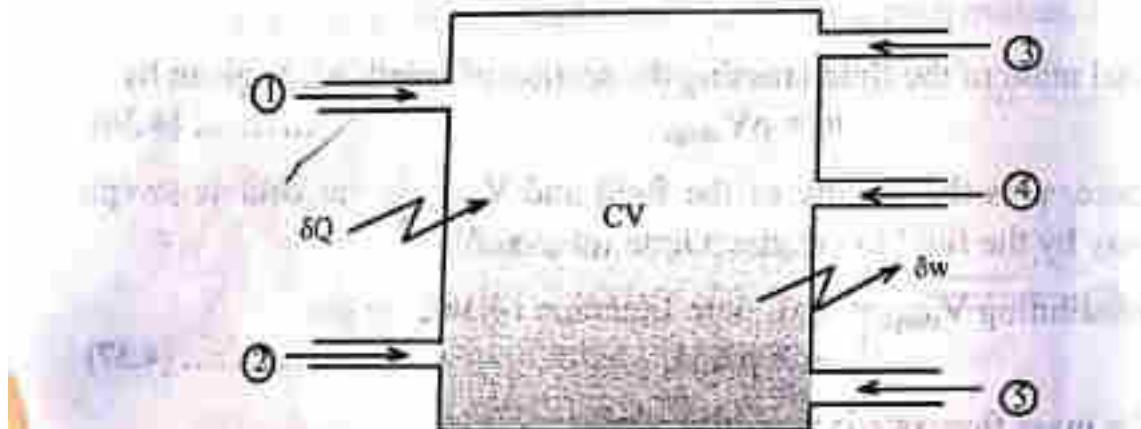


Figure 4.4 Interaction between a Control Volume and Surroundings

Mathematically,

$$\frac{dE_{CV}}{dt} = \dot{E}_{\text{net}} + \dot{Q} - \dot{W} \quad \dots \dots \dots (4.41)$$

Substituting $\dot{E}_{\text{net}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}}$, we get

$$\frac{dE_{CV}}{dt} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{Q} - \dot{W} \quad \dots \dots \dots (4.42)$$

where

$$\dot{E}_{\text{in}} = \sum_{\text{in}} \left\{ \dot{m} \left(u + \frac{1}{2} \vec{V}^2 + gz \right) \right\}$$

$$\dot{E}_{\text{out}} = \sum_{\text{out}} \left\{ \dot{m} \left(u + \frac{1}{2} \vec{V}^2 + gz \right) \right\}$$

Heat transfer always occurs due to the difference in temperature between the system and the surroundings whether it is a control mass or control volume, i.e.,

$$\dot{Q} = \dot{Q}_{CV} \quad \dots \quad (4.43)$$

But the total work transfer associated with a control volume includes various modes of work transfer such as flow work, shaft work, expansion/compression work, etc, i.e.,

$$\dot{W} = \dot{W}_{\text{flow}} + \dot{W}_{\text{shaft}} + \dot{W}_{\text{general}} \quad \dots \quad (4.44)$$

Flow work is the energy required to get the flowing fluid into the control volume or work performed by the fluid coming out from the control volume. In this regard, flow work at the inlet is taken as negative and flow work at the outlet is taken as positive.

To derive an expression for flow work for any section, consider fluid flowing through an inlet section, shown in Figure 4.5, with a cross sectional area of A , where it crosses ΔL distance in time interval Δt .

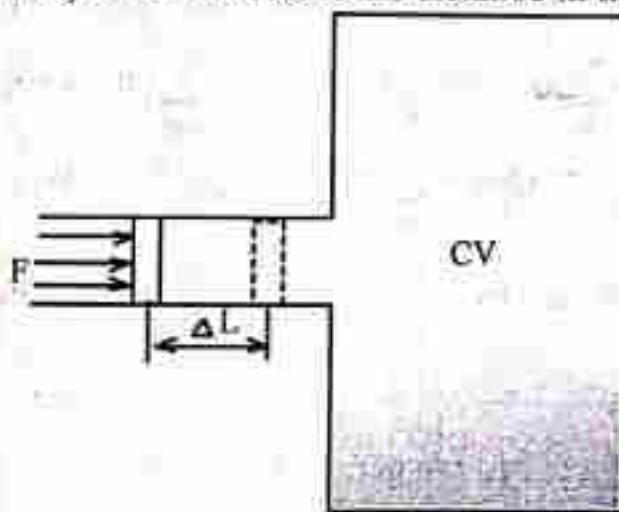


Figure 4.5 Flow Work at Inlet

Energy (work) required for the displacement of the fluid is given as

$$W_{\text{flow}} = F \cdot \Delta L \quad \dots \quad (4.45)$$

where F is the force acting on the fluid particle, which is given as the product of pressure P at the section and the cross section area A of the section.

Substituting $F = PA$ into Equation (4.45),

$$W_{\text{flow}} = PA \cdot \Delta L = PV_{\text{swept}} \quad \dots \quad (4.46)$$

Specific flow work or flow work per unit mass of the flowing fluid is then given as

$$w_{\text{flow}} = Pv \quad \dots \dots \dots \quad (4.47)$$

Therefore, the rate of flow work is evaluated as

$$\dot{W}_{\text{flow}} = \dot{m}w_{\text{flow}} = \dot{m}Pv \quad \dots \dots \dots \quad (4.48)$$

Shaft work is associated with the work produced by the shaft (turbine) by consuming energy carried by a fluid or work consumed by the shaft (compressor or pump) to increase the fluid energy.

Substituting \dot{W}_{flow} into Equation (4.44),

$$\dot{W} = \sum_{\text{in}} (\dot{m}Pv) + \sum_{\text{out}} (\dot{m}Pv) + \dot{W}_{\text{CV}} \quad \dots \dots \dots \quad (4.49)$$

where,

$$\dot{W}_{\text{CV}} = \dot{W}_{\text{shaft}} + \dot{W}_{\text{general}}$$

Substituting Equation (4.43) and (4.49) into Equation (4.41),

$$\begin{aligned} \frac{dE_{\text{CV}}}{dt} &= \sum_{\text{in}} \left\{ \dot{m} \left(u + \frac{1}{2} \bar{V}^2 + gz \right) \right\} - \sum_{\text{out}} \left\{ \dot{m} \left(u + \frac{1}{2} \bar{V}^2 + gz \right) \right\} \\ &\quad + \dot{Q}_{\text{CV}} + \sum_{\text{in}} (\dot{m}Pv) - \sum_{\text{out}} (\dot{m}Pv) - \dot{W}_{\text{CV}} \end{aligned}$$

$$\begin{aligned} \frac{dE_{\text{CV}}}{dt} &= \sum_{\text{in}} \left\{ \dot{m} \left(u + Pv + \frac{1}{2} \bar{V}^2 + gz \right) \right\} - \sum_{\text{out}} \left\{ \dot{m} \left(u + Pv + \frac{1}{2} \bar{V}^2 + gz \right) \right\} \\ &\quad + \dot{Q}_{\text{CV}} - \dot{W}_{\text{CV}} \quad \dots \dots \dots \quad (4.50) \end{aligned}$$

Substituting $h = u + Pv$, we get

$$\frac{dE_{\text{CV}}}{dt} = \sum_{\text{in}} \left\{ \dot{m} \left(h + \frac{1}{2} \bar{V}^2 + gz \right) \right\} - \sum_{\text{out}} \left\{ \dot{m} \left(h + \frac{1}{2} \bar{V}^2 + gz \right) \right\} + \dot{Q}_{\text{CV}} - \dot{W}_{\text{CV}} \quad \dots \dots \dots \quad (4.51)$$

Equation (4.51) is the general energy equation for a control volume where the expression $\left\{ \dot{m} \left(h + \frac{1}{2} \bar{V}^2 + gz \right) \right\}$ represents energy carried by the fluid and is called flow energy.

4.5 Control Volume Analysis

Any control volume can be analyzed either with reference to space (coordinates) or with reference time. While analyzing control volume with reference to space, it can be classified as a uniform system or a non-uniform system. If the properties of the system at a particular instant of time do not vary with space (coordinates), it is called a uniform system. If the properties of the system at a particular instant of time vary with space (coordinates), it is called a non-uniform system.

While analyzing a control volume with reference to time, it can be classified as a steady state system or an unsteady state system. If the properties of the system at a particular point do not vary with time, it is called a steady state system and if the properties of the system at a particular point vary with time it is called an unsteady state system.

In this book, we will consider only uniform steady state system and uniform unsteady state system.

4.5.1 Steady State Analysis

For the steady state operation of a control volume, its properties (total mass and total energy) should not change with time. Mathematically,

$$\frac{dm_{CV}}{dt} = 0 \quad \dots \dots \dots \quad (4.52)$$

and

$$\frac{dE_{CV}}{dt} = 0 \quad \dots \dots \dots \quad (4.53)$$

Substituting Equation (4.52) into Equation (4.33),

$$0 = \dot{m}_{in} - \dot{m}_{out}$$

$$\therefore \dot{m}_{in} = \dot{m}_{out} \quad \dots \dots \dots \quad (4.54)$$

Substituting Equation (4.53) into Equation (4.51),

$$0 = \sum_{in} \left\{ \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right) \right\} - \sum_{out} \left\{ \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right) \right\} + \dot{Q}_{CV} - \dot{W}_{CV}$$

$$\therefore \sum_{in} \left\{ \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right) \right\} + \dot{Q}_{CV} = \sum_{out} \left\{ \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right) \right\} + \dot{W}_{CV}$$

$$\dots \dots \dots \quad (4.55)$$

Equations (4.54) and (4.55) show that for the steady state operation, incoming mass should be equal to outgoing mass and incoming energy should be equal to outgoing energy.

Most common examples of steady state devices are turbine, compressor, nozzle, etc. These devices have single inlet and single outlet. If we denote inlet section by 1 and outlet section by 2, i.e.,

$$\dot{m}_{in} = \dot{m}_1 \quad \text{and} \quad \dot{m}_{out} = \dot{m}_2$$

Then Equation (4.54) reduces to

$$\dot{m}_1 = \dot{m}_2 = \dot{m} \quad \dots \quad (4.56)$$

Similarly Equation (4.55) reduces to

$$\begin{aligned} \dot{m}_1 \left(h_1 + \frac{1}{2} \bar{V}_1^2 + gz_1 \right) + \dot{Q}_{CV} &= \dot{m}_2 \left(h_2 + \frac{1}{2} \bar{V}_2^2 + gz_2 \right) + \dot{W}_{CV} \\ \text{or, } \dot{m} \left(h_1 + \frac{1}{2} \bar{V}_1^2 + gz_1 \right) + \dot{Q}_{CV} &= \dot{m} \left(h_2 + \frac{1}{2} \bar{V}_2^2 + gz_2 \right) + \dot{W}_{CV} \\ \therefore \dot{Q}_{CV} - \dot{W}_{CV} &= \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \end{aligned} \quad \dots \quad (4.57)$$

~~4.5.2~~ Unsteady State Analysis

During the unsteady state operation of a control volume, its properties (total mass and total energy) change with time, i.e., total mass and total energy of the system is function of time. Mathematically,

$$\frac{dm_{CV}}{dt} \neq 0 \quad \text{or} \quad m_{CV} = f_1(t) \quad \dots \quad (4.58)$$

and

$$\frac{dE_{CV}}{dt} \neq 0 \quad \text{or} \quad E_{CV} = f_2(t) \quad \dots \quad (4.59)$$

Therefore the generalized mass conservation and energy conservation equations for the unsteady state devices can be derived by integrating Equations (4.58) and (4.59) respectively with reference to time for the required interval.

For any process 1 – 2 between state 1 (instant t_1) and state 2 (instant t_2), mass conservation equation reduces to

$$\int_{t_1}^{t_2} \left(\frac{dm_{CV}}{dt} \right) dt = \int_{t_1}^{t_2} \dot{m}_{in} dt - \int_{t_1}^{t_2} \dot{m}_{out} dt$$

$$\therefore m_2 - m_1 = m_{in} - m_{out} \quad \dots \dots \dots (4.60)$$

where m_1 and m_2 are the masses of the control volume at state 1 and state 2, m_{in} is the total mass that has entered into the control volume and m_{out} is the total mass that has gone out from the control volume during the interval t_1 to t_2 .

Similarly, integrating energy conservation equation, we get

$$\int_{t_1}^{t_2} \left(\frac{dE_{CV}}{dt} \right) dt = \int_{t_1}^{t_2} \sum_{in} \left\{ \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right) \right\} dt - \int_{t_1}^{t_2} \sum_{out} \left\{ \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right) \right\} dt$$

$$+ \int_{t_1}^{t_2} \dot{Q}_{CV} dt - \int_{t_1}^{t_2} \dot{W}_{CV} dt -$$

$$\therefore (E_{CV})_2 - (E_{CV})_1 = m_{in} \left(h_{in} + \frac{1}{2} \vec{V}_{in}^2 + gz_{in} \right) - m_{out} \left(h_{out} + \frac{1}{2} \vec{V}_{out}^2 + gz_{out} \right) + Q_{12} + W_{12} \quad \dots \dots \dots (4.61)$$

4.6 Control Volume Applications

Operation and performance of common devices such as turbine, compressor, fan, nozzle, heat exchanger etc can be studied with reference to mass conservation and energy conservation equations discussed earlier. Depending upon their operation and function, common control volume applications can be classified into four groups: steady state work applications, steady state flow applications, unsteady state work applications and unsteady state flow applications.

4.6.1 Steady State Work Applications

Devices which operate under steady state conditions and either produce or consume work are called steady state work applications. Common examples of steady state work applications are turbine, compressor, pump, fan, etc.

A turbine is a device which produces power by consuming energy carried by a fluid. A turbine has generally a single inlet and a single outlet, therefore energy equation for the steady state operation of the turbine is given as

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \quad \dots (4.62)$$

Schematic representation of a turbine is shown in **Figure 4.6(a)**. In thermodynamics, most of the time we have to deal with steam turbine or gas turbine. In case of these turbines, some part of the energy carried by the fluid at inlet is lost as a heat transfer from the turbine surface. If turbine surface is insulated and heat transfer loss is negligible, it is called an adiabatic turbine. Therefore, for an adiabatic turbine energy equation reduces to

$$\dot{W}_{CV} = \dot{m} \left[(h_1 - h_2) + \frac{1}{2} (\bar{V}_1^2 - \bar{V}_2^2) + g(z_1 - z_2) \right] \quad \dots (4.63)$$

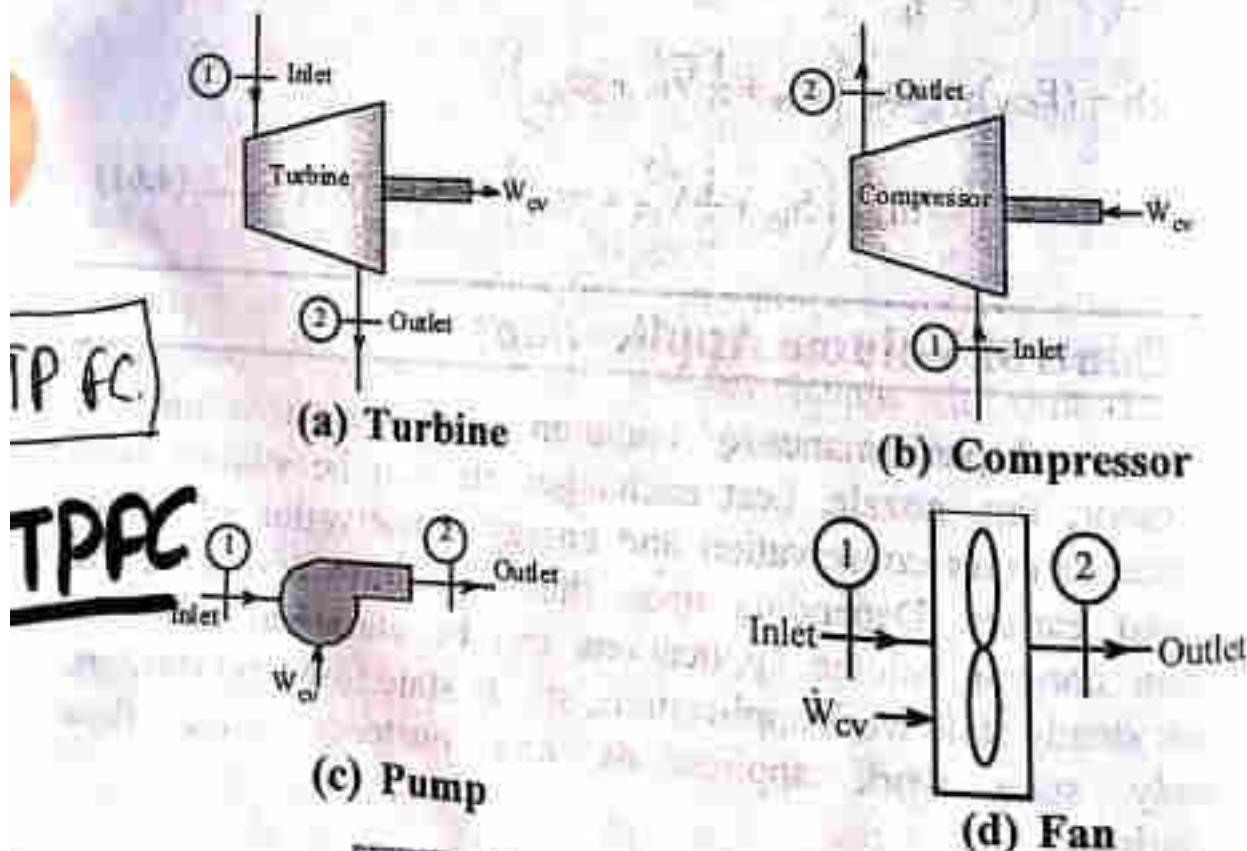


Figure 4.6 Steady State Work Applications
Compressor, pump and fan increase fluid energy by consuming mechanical work. Schematic representations of these devices are shown in **Figure 4.6(b)**, **Figure 4.6(c)** and **Figure 4.6(d)** respectively.

Compressor usually increases pressure energy of the gaseous substance, pump increases pressure or potential energy of the liquid substance and fan increases kinetic energy (by increasing velocity) of the fluid. Hence energy equations for these devices are also given by Equation (4.62).

Again, if these devices operate under the adiabatic condition, energy equation is given by Equation (4.63).

4.6.2 Steady State Flow Applications

Devices which operate under steady state conditions and do not produce or consume work are called steady state flow applications. Common examples of steady state flow applications are nozzle, diffuser, heat exchanger, evaporator, condenser, throttling valve, etc.

Nozzle is a device with decreasing cross sectional area and is used to increase fluid velocity whereas diffuser is a device with increasing cross sectional area and is used to decrease fluid velocity. Schematic representation of nozzle and diffuser is shown in Figure 4.7(a) and (b). These devices have a single inlet and a single outlet and therefore general energy equation for these devices is given as

$$\dot{Q}_{CV} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \quad \dots \dots (4.64)$$

In these devices, change in potential energy is usually negligible in comparison to change in kinetic energy or change in enthalpy. If the nozzle or diffuser is operating under adiabatic condition energy equation reduces to

$$(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) = 0 \quad \dots \dots \dots (4.65)$$

which can also be expressed as

$$h_1 + \frac{1}{2} \bar{V}_1^2 = h_2 + \frac{1}{2} \bar{V}_2^2 \quad \dots \dots \dots (4.66)$$

Heat exchanger is a device used to transfer heat from one fluid to another. Schematic representation of a heat exchanger is shown in Figure 4.7 (c), where 1_h is inlet of hot fluid, 2_h is outlet of hot fluid, 1_c is inlet of cold fluid and 2_c is outlet of cold fluid. To determine the magnitude of net heat exchange \dot{Q} , we can analyze either control volume A or control volume B. If we have to study the effect of heat

exchange on the properties of hot or cold fluid, we can analyze the control volume C. In case of heat exchanger, changes in potential energy and kinetic energy are negligible in comparison to the change in enthalpy. Therefore, energy equation for the control volume A is given as

$$\dot{Q} = \dot{m}_h (h_{2h} - h_{1h}) \quad \dots \dots \dots (4.67)$$

where \dot{m}_h is the mass flow rate of hot fluid, h_{1h} is the enthalpy of hot fluid at the inlet and h_{2h} is the enthalpy of hot fluid at the outlet.

Similarly, energy equation for control volume C is given as

$$\dot{m}_c h_{1c} + \dot{m}_h h_{1h} = \dot{m}_h h_{2h} + \dot{m}_c h_{2c} \quad \dots \dots \dots (4.68)$$

where \dot{m}_c is the mass flow rate of cold fluid, h_{1c} is the enthalpy of cold fluid at the inlet and h_{2c} is the enthalpy of cold fluid at the outlet.

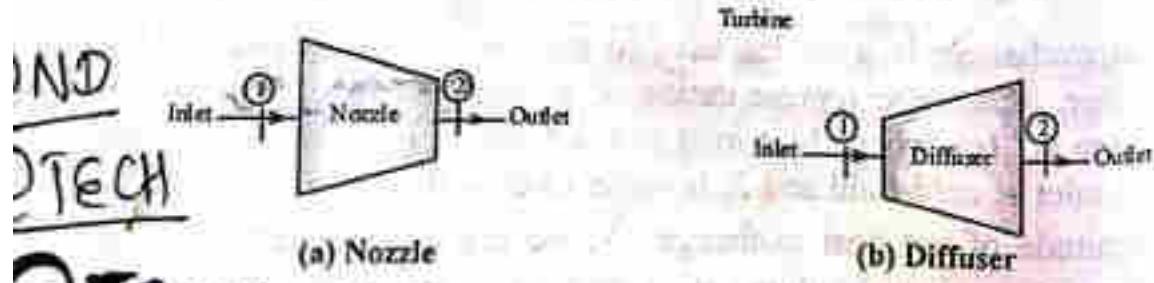
Evaporator and condenser are special types of heat exchangers. Evaporator converts liquid into vapor by absorbing heat from the surroundings whereas condenser converts vapor into liquid by rejecting heat to the surroundings. Schematic representations of evaporator and condenser are shown in Figure 4.7(d) and (e). Energy equation for the evaporator or condenser is given as

$$\dot{Q} = \dot{m}(h_2 - h_1) \quad \dots \dots \dots (4.69)$$

Throttling valve reduces pressure of the fluid without performing work. Heat transfer, change in potential energy and kinetic energy are also negligible. Schematic representation of a throttling valve is shown in Figure 4.7(f). Energy equation for the throttling valve is given as

$$h_2 - h_1 = 0 \\ \therefore h_2 = h_1 \quad \dots \dots \dots (4.70)$$

Hence, enthalpy remains constant during a throttling process.



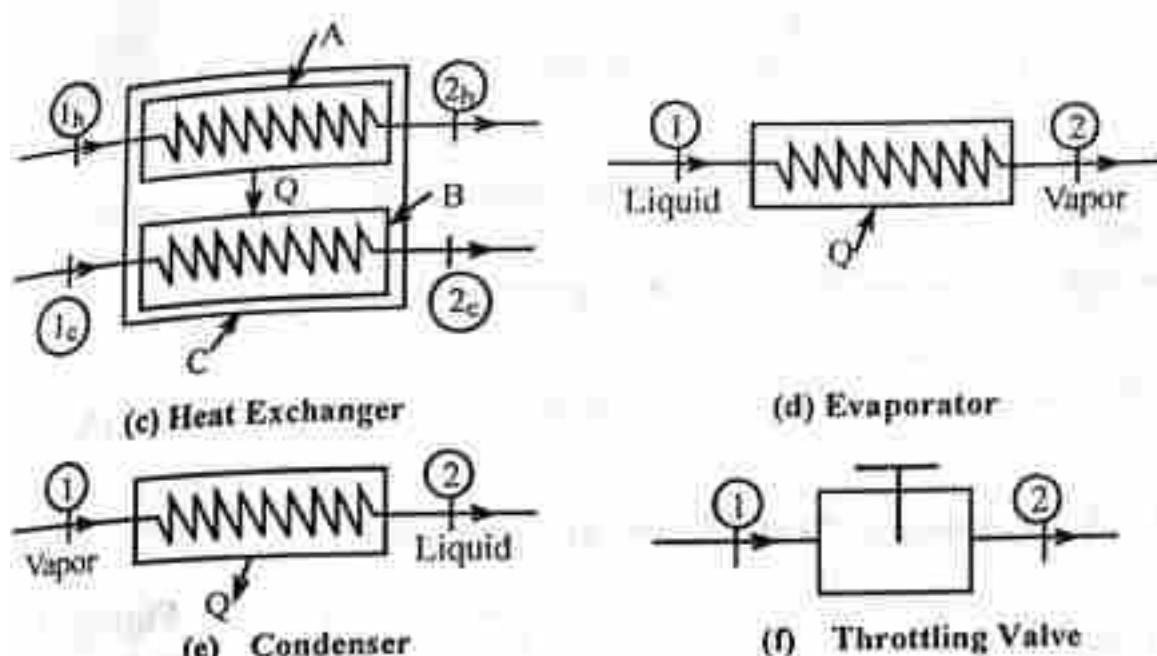


Figure 4.7 Steady State Flow Applications

6.3 Unsteady State Work Applications

Turbine, compressor, pump etc operate at steady state at their normal operation. But the same devices operate at unsteady state during the start up and shut down period.

To have general idea about the unsteady state work applications, let's consider a system shown in Figure 4.8, where gas is supplied to the piston cylinder device through a valve. When gas is supplied to it, it can produce some work by displacing the piston.

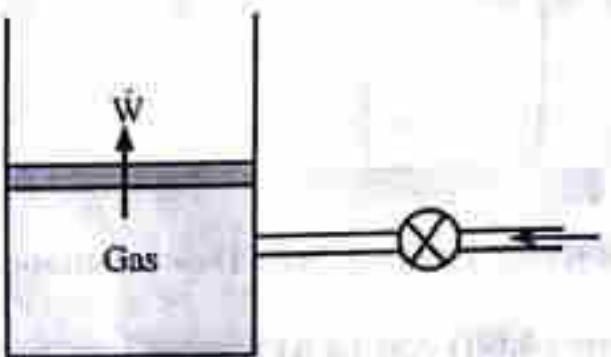


Figure 4.8 Unsteady State Work Application

Applying Equations (4.60) and (4.61), mass conservation and energy conservation equations for the device are given as

$$m_2 - m_1 = m_{in} \quad \dots\dots\dots (4.71)$$

$$(E_{cv})_2 - (E_{cv})_1 = m_{in} \left(h_{in} + \frac{1}{2} \bar{V}_{in}^2 + gz_{in} \right) + Q_{12} - W_{12} \quad \dots \dots (4.72)$$

In case of piston cylinder device, changes in potential energy and kinetic energy are negligible in comparison to change in internal energy. Therefore, Equation (4.72) reduces to

$$m_2 u_2 - m_1 u_1 = m_{in} \left(h_{in} + \frac{1}{2} \bar{V}_{in}^2 + gz_{in} \right) + Q_{12} - W_{12} \quad \dots \dots (4.73)$$

4.6.4 Unsteady State Flow Applications

Let us consider a cooking gas cylinder as shown in Figure 4.9. During the cooking of a certain food, gas is consumed and mass of the system continuously decreases and it does not produce any boundary work. Therefore it is an unsteady state flow application.

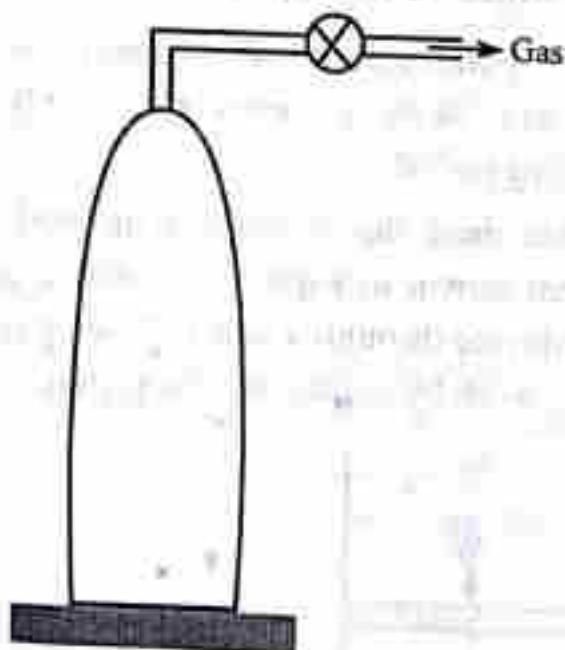


Figure 4.9 Unsteady State Flow Application

Applying Equations (4.60) and (4.61), mass conservation and energy conservation equations for the device are given as

$$m_2 - m_1 = -m_{out} \quad \dots \dots \dots (4.74)$$

$$(E_{cv})_2 - (E_{cv})_1 = -m_{out} \left(h_{out} + \frac{1}{2} \bar{V}_{out}^2 + gz_{out} \right) + Q_{12} \quad \dots \dots (4.75)$$

In this case also, changes in potential energy and kinetic energy are negligible in comparison to change in internal energy. Therefore, Equation (4.75) reduces to

$$m_2 u_2 - m_1 u_1 = -m_{out} \left(h_{out} + \frac{1}{2} \vec{V}_{out}^2 + gz_{out} \right) + Q_{in} \quad \dots \dots \dots (4.76)$$

4.7 Other Statements of First Law

We can derive the mathematical expressions as well as statements for the first law of thermodynamics for different specific processes with reference to energy equations derived earlier.

(a) First Law of Thermodynamics for an Isolated System

First law of thermodynamics for a control mass undergoing any process is given as

$$dE = \delta Q - \delta W$$

If the control mass is isolated from its surroundings, i.e., there is no interaction between the system and the surroundings ($\delta Q = \delta W = 0$), then the energy equation reduces to

$$\begin{aligned} dE &= 0 \\ \text{or, } E_2 - E_1 &= 0 \\ \therefore E_2 &= E_1 \end{aligned} \quad \dots \dots \dots (4.77)$$

Hence, total energy of an isolated system always remains constant.

(b) First Law of Thermodynamics for a Control Mass Undergoing an Adiabatic Process

If the boundary of the control mass is insulated and there is no heat transfer ($\delta Q = 0$), energy equation reduces to

$$dE = -\delta W \quad \dots \dots \dots (4.78)$$

Therefore, increase in total energy of a control mass during an adiabatic process is equal to the work done on the control mass.

(c) Perpetual Motion Machine of the First Kind (PMM-I) is not possible.

Perpetual motion or continuously running machine is not possible in nature because of friction. Hence, we cannot take continuous useful output effect without corresponding supply of input energy.

SOLVED EXAMPLES

Example 4.1 : A piston cylinder device contains 0.2 kg of a gas initially at $P_1 = 1000 \text{ kPa}$ and $V_1 = 0.02 \text{ m}^3$. It undergoes polytropic expansion to a final pressure of 200 kPa during which the relation between pressure and volume is $PV^n = \text{constant}$. If the specific internal energy of the gas decreases by 160 kJ/kg during the process, determine the heat transfer for the process.

Solution

Given, Mass of gas: $m = 0.2 \text{ kg}$

Initial State: $P_1 = 1000 \text{ kPa}$, $V_1 = 0.02 \text{ m}^3$

Final Pressure: $P_2 = 200 \text{ kPa}$

Process Relation: $PV^n = \text{constant}$

Change in Specific Internal Energy: $\Delta u = -160 \text{ kJ/kg}$

$$\therefore \text{Volume of gas at final state } V_2 = \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}} V_1 = (5)^{\frac{1}{3}} \times 0.02 \\ = 0.034 \text{ m}^3$$

Work transfer during the polytropic expansion is given as

$$W = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{1000 \times 0.02 - 200 \times 0.034}{1-3} = 6.58 \text{ kJ}$$

Change in internal energy during the process

$$\Delta U = m(\Delta u) = 0.2 \times -160 = -32 \text{ kJ}$$

\therefore Heat transfer during the process

$$Q = \Delta U + W = -32 + 6.58 = -25.42 \text{ kJ}$$

Example 4.2 : A piston cylinder device shown in Figure E4.2 contains 3.06 kg of air initially at a temperature of 34°C. Heat is supplied to the system until it reaches to a final temperature of 950°C and a final pressure of 5 MPa. Sketch the process on $P-V$ and $T-s$ diagrams and determine the total work transfer and total heat transfer. [Take $R = 287 \text{ J/kgK}$ and $c_V = 718 \text{ J/kgK}$]

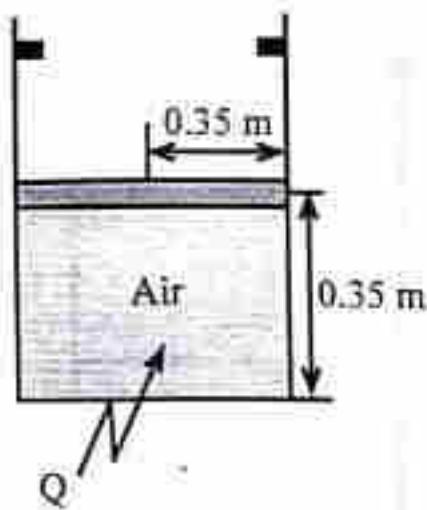


Figure E4.2

SolutionGiven, Mass of air: $m = 3.06 \text{ kg}$

$$\text{Initial State: } V_1 = \pi \times (0.35)^2 \times 0.35 = 0.13475 \text{ m}^3,$$

$$T_1 = 273 + 34 = 307 \text{ K}$$

$$\text{Final State: } P_{\text{final}} = 5 \text{ MPa}, T_{\text{final}} = 273 + 950 = 1223 \text{ K}$$

Pressure of the air at the initial state

$$P_1 = \frac{mRT_1}{V_1} = \frac{3.06 \times 287 \times 307}{0.13475} = 2.00 \text{ MPa}$$

Volume of the air at the final state

$$V_{\text{final}} = \frac{mRT_{\text{final}}}{P_{\text{final}}} = \frac{3.06 \times 287 \times 1223}{5 \times 10^6} = 0.21481 \text{ m}^3$$

Initial pressure of the system is 2 MPa and when heat is supplied to the system piston moves upward at constant pressure till it touches the stop (Process 1 – 2). The final required pressure is 5 MPa hence it should be further heated to increase the pressure from 2 MPa to 5 MPa and the process occurs at constant volume (Process 2 – 3). Hence, we can define state 2 and state 3 as

$$\text{State 2: } P_2 = 2 \text{ MPa}, V_2 = 0.21481 \text{ m}^3$$

$$\text{State 3: } P_3 = 5 \text{ MPa}, V_3 = 0.21481 \text{ m}^3$$

$P - V$ and $T - V$ diagrams for the process is shown in Figure E4.2 (a) and (b).

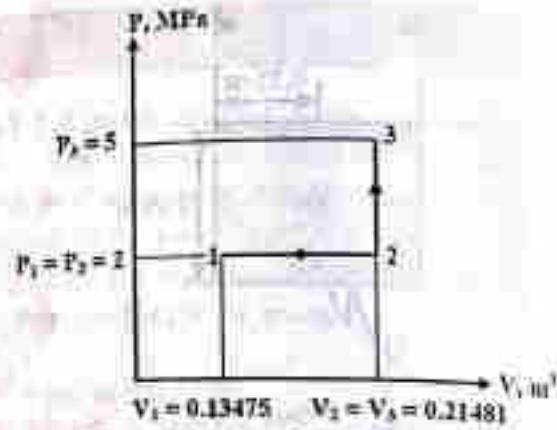


Figure E4.2 (a) P-V diagram

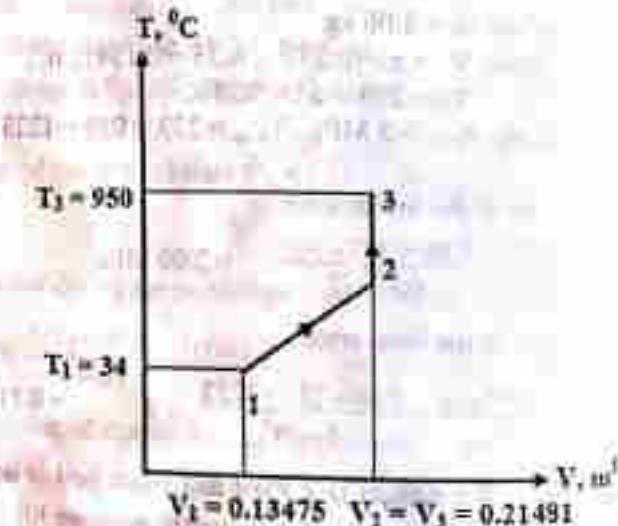


Figure E4.2 (b) T-V diagram

Total work transfer for the process is given as

$$\begin{aligned} W &= W_{12} + W_{23} \\ &= P_1(V_2 - V_1) + 0 \\ &= 2000(0.21481 - 0.13475) = 160.193 \text{ kJ} \end{aligned}$$

Change in total energy for the process is given as

$$\begin{aligned} \Delta U &= mc_v(T_3 - T_1) \\ &= 3.06 \times 0.718 \times (1223 - 37) = 2012.525 \text{ kJ} \end{aligned}$$

Total heat transfer for the process is given as

$$\begin{aligned} Q &= \Delta U + W \\ &= 2012.525 + 160.193 = 2172.718 \text{ kJ} \end{aligned}$$

Example 4.3 : Water (2 kg) is contained in a piston cylinder device shown in Figure E4.3. The mass of the piston is such that the H_2O exists at a pressure of 10 MPa and a temperature of 800°C . There is a heat transfer from the device until the piston just rests on stops at which time the volume inside the cylinder is $2.45 \times 10^{-3} \text{ m}^3$. Sketch the process on $P-v$ and $T-v$ diagrams and determine the total work transfer and total heat transfer.

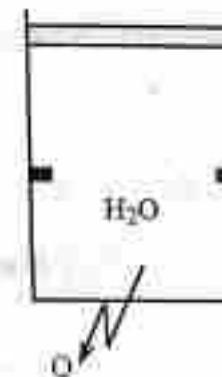


Figure E4.3

Solution

Given, Mass of H_2O : $m = 2 \text{ kg}$
 Initial State: $P_1 = 10 \text{ MPa}$, $T_1 = 800^\circ\text{C}$
 Final State: $V_2 = 2.45 \times 10^{-3} \text{ m}^3$

Referring to Table A2.1, T_{∞} (10000 kPa) = 311.03°C . Here $T > T_{\infty}$ hence it is a superheated vapor. Then referring to Table A2.4, $v_1 = 0.04863 \text{ m}^3/\text{kg}$ and $u_1 = 3627.2 \text{ kJ/kg}$.

When heat is transferred from the system, piston slowly drops downward and cooling process occurs at constant pressure until it touches the stops. Hence, we can redefine state 2 as $P_2 = 10 \text{ MPa}$ and

$$V_2 = \frac{V_1}{2} = \frac{2.45 \times 10^{-3}}{2} = 0.001225 \text{ m}^3/\text{kg}$$

Referring to Table A2.1, v_1 (10000 kPa) = 0.001452 m³/kg. Since v_1 , hence it is a compressed liquid. Then referring to Table A2.1, we can list the following properties which includes the given v_2 and volume of 0.001225 m³/kg as:

T°C	v, m ³ /kg	u, kJ/kg
230	0.001199	975.55
250	0.001241	1073.0

Now applying linear interpolation equation for specific internal energy,

$$u_2 - u_1 = \frac{u_b - u_a}{v_b - v_a} (v_b - v_2)$$

$$\begin{aligned} u_2 &= u_1 + \frac{u_b - u_a}{v_b - v_a} (v_b - v_2) \\ &= 975.55 + \frac{1073.0 - 975.55}{0.001241 - 0.001199} (0.001225 - 0.001199) \\ &= 1037.4 \text{ kJ/kg} \end{aligned}$$

P-v and T-v diagrams for the process is shown in Figures E4.3(a) and (b).

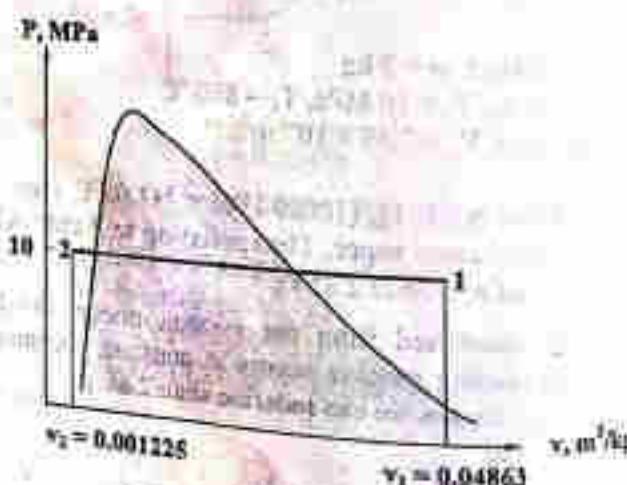


Figure E4.3 (a) P-V diagram

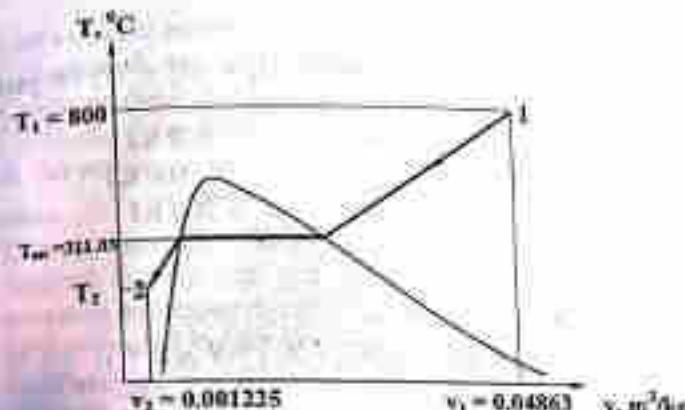


Figure E4.3 (b) T-V diagram

Total work transfer for the process is given as:

$$\begin{aligned} W &= P_1 (V_2 - V_1) \\ &= m P_1 (v_2 - v_1) \\ &= 2 \times 10000 \times (0.001225 - 0.04863) = -948.1 \text{ kJ} \end{aligned}$$

Change in total energy for the process is given as:

$$\begin{aligned} \Delta U &= m (u_2 - u_1) \\ &= 2 \times (1037.4 - 3627.2) = -5179.6 \text{ kJ} \end{aligned}$$

Total heat transfer for the process is given as:

$$\begin{aligned} Q &= \Delta U + W \\ &= -948.1 - 5179.6 = -6127.7 \text{ kJ} \end{aligned}$$

Example 4.4 : A system undergoes a cycle consisting of four processes. Complete the missing table entries.

Process	$\Delta U, \text{kJ}$	W, kJ	Q, kJ
1 → 1	-500		0
2 → 3	0	-100	
3 → 4			400
4 → 1		300	500

Solution

Applying control mass energy equation ($Q = \Delta U + W$) for processes 1 – 2, 2 – 3, and 4 – 1 for the missing parameters as

$$\begin{aligned} W_{12} &= Q_{12} - \Delta U_{12} = 0 + 500 &= 500 \text{ kJ} \\ Q_{23} &= \Delta U_{23} + W_{23} = 0 - 100 &= -100 \text{ kJ} \\ \Delta U_{41} &= Q_{41} - W_{41} = 500 - 300 &= 200 \text{ kJ} \end{aligned}$$

Now for the complete cycle

$$\begin{aligned} \sum Q &= \sum W \\ \text{or, } Q_{12} + Q_{23} + Q_{34} + Q_{41} &= W_{12} + W_{23} + W_{34} + W_{41} \\ \text{or, } 0 - 100 + 400 + 500 &= 500 - 100 + W_{34} + 300 \\ \therefore W_{34} &= 100 \text{ kJ} \end{aligned}$$

Applying control mass energy equation, we get

$$\Delta U_{34} = Q_{34} - W_{34} = 400 - 100 = 300 \text{ kJ}$$

Alternative Method

For a complete cycle,

$$\begin{aligned} (\Delta U)_{\text{cycle}} &= 0 \\ \text{or, } (\Delta U)_{12} + (\Delta U)_{23} + (\Delta U)_{34} + (\Delta U)_{41} &= 0 \\ -500 + 0 + 200 + (\Delta U)_{41} &= 0 \\ \therefore \Delta U_{34} &= 300 \text{ kJ} \end{aligned}$$

The complete table is given below.

Process	$\Delta U, \text{kJ}$	W, kJ	Q, kJ
1 – 2	-500	500	0
2 – 3	0	-100	-100
3 – 4	300	100	400
4 – 1	200	300	500

Example 4.5 : A gas undergoes a thermodynamic cycle consisting of the following three processes:

Process 1 – 2: constant volume, $V_1 = 0.08 \text{ m}^3$, $P_1 = 100 \text{ kPa}$

$$U_2 - U_1 = 140 \text{ kJ}$$

Process 2 – 3: expansion with $PV = \text{constant}$, $U_3 = U_1$

$$P_2 V_2 = P_3 V_3$$

(a) Sketch the process on P – V and T – V diagrams.

- Calculate the net work for the cycle.
- Calculate the net heat for the cycle.
- Calculate the heat transfer for process 2 – 3.
- Calculate the heat transfer for process 3 – 1.
- Is this power cycle or a refrigeration cycle?

Solution

(a) During constant volume process 1 – 2, there is an increase in internal energy, hence it is a heating process. During which pressure and temperature of the gas increases. During expansion ($PV = \text{constant}$) process 2 – 3, temperature of the gas remains constant, volume increases and pressure decreases. During constant pressure process 3 – 1, work transfer is negative, hence it is a cooling process. Therefore, its volume and temperature decreases. P – V and T – V diagrams for the cycle are shown below:

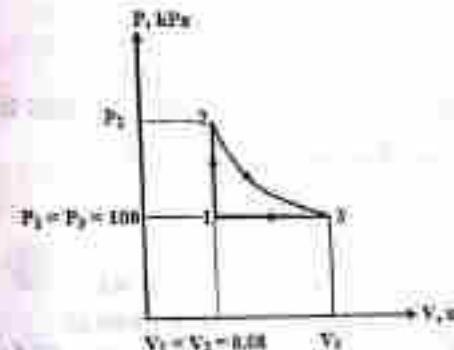


Figure E4.5 (a) P-V diagram

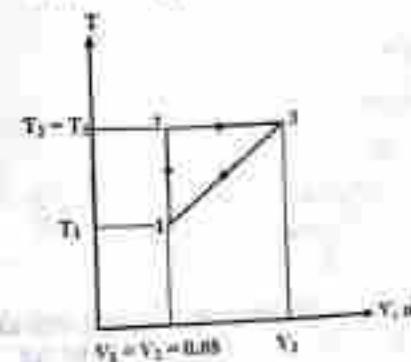


Figure E4.5 (b) T-V diagram

(b) Work transfer during process 1 - 2 is given as

$$W_{12} = P_1 (V_1 - V_2)$$

$$\therefore V_2 = V_1 - \frac{W_{12}}{P_1} = 0.08 - \frac{(-56)}{100} = 0.64 \text{ m}^3$$

Similarly, work transfer during process 2 - 3 is given as

$$W_{23} = P_2 V_2 \ln \left(\frac{V_3}{V_2} \right) = P_2 V_2 \ln \left(\frac{V_1}{V_2} \right)$$

$$= 100 \times 0.64 \times \ln \left(\frac{0.64}{0.08} \right) = 133.084 \text{ kJ}$$

Then, net work for the cycle is given by

$$\sum W = W_{12} + W_{23} + W_{31} = 0 + 133.084 - 56 = 77.084 \text{ kJ}$$

(c) For a cycle, net work transfer is equal to net heat transfer, therefore

$$\sum Q = \sum W = 77.084 \text{ kJ}$$

(d) Heat transfer during process 2 - 3 is given as

$$Q_{23} = (\Delta U)_{23} + W_{23} = 0 + 133.084 = 133.084 \text{ kJ}$$

(e) Net heat transfer for the cycle is given as

$$\sum Q = Q_{12} + Q_{23} + Q_{31}$$

where

$$Q_{12} = (\Delta U)_{12} + W_{12} = 140 + 0 = 140 \text{ kJ}$$

Then heat transfer for the process 3 - 1 is given as

$$Q_{31} = \sum Q - (Q_{12} + Q_{23}) = 77.084 - (140 + 133.084) = -196 \text{ kJ}$$

Alternative Method

For a complete cycle,

$$(\Delta U)_{cycle} = 0$$

$$\text{or, } (\Delta U)_{12} + (\Delta U)_{23} + (\Delta U)_{31} = 0$$

$$140 + 0 + (\Delta U)_{31} = 0$$

$$\therefore (\Delta U)_{31} = -140 \text{ kJ}$$

Then heat transfer for the process 3 - 1 is given as

$$Q_{31} = (\Delta U)_{31} + W_{31} = -140 - 56 = -196 \text{ kJ}$$

(f) Since net work is positive, given cycle is a power cycle.

Example 4.6 : Air flows at a rate of 1.2 kg/s through a compressor, entering at 100 kPa, 25 °C, with a velocity of 60 m/s and leaving at 500 kPa, 150 °C, with a velocity of 120 m/s. Heat loss by the compressor to the surrounding is estimated to be 20 kJ/kg. Calculate the power required to drive the compressor and diameters of inlet and exhaust pipes. [Take R = 287 J/kgK and c_p = 1005 J/kgK]

Solution

Given, Mass flow rate of air: $\dot{m} = 1.2 \text{ kg/s}$

Properties of air at inlet: $P_1 = 100 \text{ kPa}$, $T_1 = 25 + 273 = 298 \text{ K}$
 $V_1 = 60 \text{ m/s}$

Properties of air at outlet: $P_2 = 500 \text{ kPa}$
 $T_2 = 150 + 273 = 423 \text{ K}$
 $V_2 = 120 \text{ m/s}$

Heat loss per unit mass of air: $q_{cv} = -20 \text{ kJ/kg}$

∴ Heat transfer rate, $\dot{Q}_{cv} = \dot{m} q_{cv} = 1.2 \times -20 = -24 \text{ kW}$

Now, applying steady state energy equation

$$\dot{Q}_{cv} = \dot{W}_{cv} = \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\vec{V}_2^2 - \vec{V}_1^2) \right]$$

For an ideal gas using $h_2 - h_1 = c_p (T_2 - T_1)$ and rearranging the above equation for \dot{W}_{cv} ,

$$\begin{aligned} \dot{W}_{cv} &= \dot{Q}_{cv} - \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\vec{V}_2^2 - \vec{V}_1^2) \right] \\ &= -24 \times 10^3 - 1.2 \times \left[1005 (423 - 298) + \frac{1}{2} (\vec{V}_2^2 - \vec{V}_1^2) \right] \\ &= -181.23 \text{ kW} \end{aligned}$$

Specific volumes of air at the inlet and outlet are given by

$$V_1 = \frac{RT_1}{P_1} = \frac{287 \times 298}{100 \times 10^3} = 0.85526 \text{ m}^3/\text{kg}$$

$$V_2 = \frac{RT_2}{P_2} = \frac{287 \times 423}{500 \times 10^3} = 0.242802 \text{ m}^3/\text{kg}$$

Inlet area and exit area are given by

$$A_1 = \frac{\dot{m} V_1}{\dot{V}_1} = \frac{1.2 \times 0.85526}{60} = 0.0171052 \text{ m}^2$$

$$A_2 = \frac{\dot{m}V_2}{V_2} = \frac{1.2 \times 0.242802}{120} = 0.00242802 \text{ m}^2$$

Then, inlet and exit diameters are given by

$$D_1 = 2\sqrt{\frac{A_1}{\pi}} = 2\sqrt{\frac{0.0171052}{\pi}} = 0.1476 \text{ m}$$

$$D_2 = 2\sqrt{\frac{A_2}{\pi}} = 2\sqrt{\frac{0.00242802}{\pi}} = 0.0556 \text{ m}$$

Example 4.7 : A steam turbine develops 60 MW of power output. Mass flow rate of steam is found to be 80 kg/s. Properties of steam at inlet and exit of the turbine are as follows:

Pressure	8 MPa	0.4 MPa
Temperature	500 °C	
Quality		80 %
Velocity	50 m/s	150 m/s
Elevation above the reference level	10 m	5 m

- (a) Determine the rate at which heat is lost from the turbine surface.
 (b) Determine the inlet and outlet areas.

Solution

Given, Mass flow rate of steam: $\dot{m} = 80 \text{ kg/s}$

Properties of steam at inlet: $P_1 = 8000 \text{ kPa}$, $T_1 = 500 \text{ °C}$,

$$\bar{V}_1 = 50 \text{ m/s}, z_1 = 10 \text{ m}$$

Properties of steam at outlet: $P_2 = 400 \text{ kPa}$, $x_2 = 0.8$,

$$\bar{V}_2 = 150 \text{ m/s}, z_2 = 5 \text{ m}$$

Power output of the turbine: $W_{CV} = 60 \text{ MW}$

For the other properties of steam at inlet, referring to Table A2.4,

$$v_1 = 0.04174 \text{ m}^3/\text{kg} \text{ and } h_1 = 3398.5 \text{ kJ/kg.}$$

For the other properties of steam at outlet, referring to Table A2.4

v_1 (400 kPa) = 0.001084 m³/kg, v_{1g} (400 kPa) = 0.4614 m³/kg = h_1 (400 kPa) = 604.91 kJ/kg, h_{1g} (400 kPa) = 2133.6 kJ/kg. Therefore specific volume and specific enthalpy of steam at exit are given by

$$V_2 = v_1 + x_2 v_{1g} = 0.001084 + 0.8 \times 0.4614$$

$$= 0.370204 \text{ m}^3/\text{kg}$$

$$h_2 = h_1 + x_2 h_{1g} = 604.91 + 0.8 \times 2133.6 = 2311.79 \text{ kJ/kg}$$

(a) Now, applying steady state energy equation

$$\dot{Q}_{CV} = \dot{W}_{CV} - \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right]$$

Rearranging the above equation for \dot{Q}_{CV} ,

$$\dot{Q}_{CV} = \dot{W}_{CV} + \dot{m} \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right]$$

$$= 60000 + 80 \times \left[(2311.79 - 3398.5) + \frac{1}{2000} (150^2 - 50^2) + \frac{9.81 \times (5 - 10)}{1000} \right]$$

$$= -26140.724 \text{ kW} = -26.14 \text{ MW}$$

(b) Inlet area and exit area are given by

$$A_1 = \frac{\dot{m}V_1}{V_1} = \frac{80 \times 0.04174}{50} = 0.066784 \text{ m}^2$$

$$A_2 = \frac{\dot{m}V_2}{V_2} = \frac{80 \times 0.370204}{150} = 0.197442 \text{ m}^2$$

Example 4.8 : Steam at 0.4 MPa and 200 °C enters into an adiabatic nozzle with a velocity of 50 m/s and leaves the nozzle at 0.1 MPa and with a velocity of 750 m/s. Determine

- (a) the exit temperature of the steam.
 (b) the ratio of inlet diameter to the exit diameter.

Solution

Given, Properties of steam at inlet: $P_1 = 400 \text{ kPa}$, $T_1 = 200 \text{ °C}$,

$$\bar{V}_1 = 50 \text{ m/s}$$

Properties of steam at outlet: $P_2 = 100 \text{ kPa}$, $\bar{V}_2 = 750 \text{ m/s}$

For the other properties of steam at inlet, referring to Table A2.4, $v_1 = 0.5142 \text{ m}^3/\text{kg}$ and $h_1 = 2860.1 \text{ kJ/kg}$.

Now applying energy equation for an adiabatic nozzle,

$$h_1 + \frac{1}{2} \bar{V}_1^2 = h_2 + \frac{1}{2} \bar{V}_2^2$$

Rearranging the above equation for h_2 ,

$$h_2 = h_1 + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2)$$

$$= 2860.1 + \frac{1}{2000}(50^2 - 750^2) = 2580.1 \text{ kJ/kg}$$

Referring to Table A2.1,

$$\begin{aligned} v_1(100 \text{ kPa}) &= 0.001043 \text{ m}^3/\text{kg}, v_{fg}(100 \text{ kPa}) = 1.6933 \text{ m}^3/\text{kg} \\ v_1(100 \text{ kPa}) &= 1.6933 \text{ m}^3/\text{kg} \text{ and } h_1(100 \text{ kPa}) = 417.51 \text{ kJ/kg} \\ v_f(100 \text{ kPa}) &= 2257.6 \text{ kJ/kg}, h_f(100 \text{ kPa}) = 2675.1 \text{ kJ/kg} \end{aligned}$$

Here, $h_1 < h_f < h_g$, therefore condition of the steam at the nozzle exit is a two phase mixture. Therefore, temperature of the steam at the exit is then given by

$$T_3 = T_{\text{sat}}(100 \text{ kPa}) = 99.632^\circ\text{C}$$

Then quality of the two phase mixture is given by

$$x_2 = \frac{h_2 - h_1}{h_{fg}} = \frac{2580.1 - 417.51}{2257.6} = 0.9579$$

Therefore, specific volume of steam at the nozzle exit is given by
 $v_2 = v_1 + x_2 v_{fg} = 0.001043 + 0.9579 \times 1.6933 = 1.62308 \text{ m}^3/\text{kg}$

Applying conservation of mass equation

$$\dot{m} = \frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2}$$

Then ratio of inlet and exit area is given by

$$\frac{A_1}{A_2} = \frac{\bar{V}_2}{\bar{V}_1} \frac{v_1}{v_2} = \frac{750}{50} \times \frac{0.5342}{1.62308} = 4.9369$$

Therefore ratio of inlet and exit diameter is given by

$$\frac{D_1}{D_2} = \sqrt{\frac{A_1}{A_2}} = \sqrt{4.9369} = 2.222$$

Example 4.9 : In a water heater operating under steady condition, water at 50°C flowing with a mass flow rate of 5 kg/s is mixed with the saturated vapor at 120°C. The mixture then leaves the heater as saturated liquid water at 100°C. Determine the rate at which saturated water vapor must be supplied to the heater.

Solution

Schematic diagram for the heating arrangement is shown in Fig E4.9.

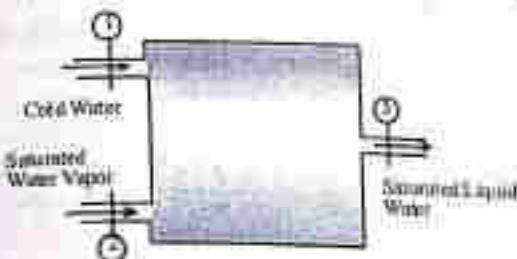


Figure E4.9

Properties of water at the inlets and outlet are given as

Properties at inlet 1 $\dot{m}_1 = 5 \text{ kg/s}$, $h_1 = h_f(50^\circ\text{C}) = 209.33 \text{ kJ/kg}$

Properties at inlet 2 $h_2 = h_g(120^\circ\text{C}) = 2706.2 \text{ kJ/kg}$

Properties at outlet 3 $h_3 = h_f(100^\circ\text{C}) = 419.06 \text{ kJ/kg}$

Applying mass conservation and energy equation for the device

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 \quad \dots \dots \dots \text{(a)}$$

$$\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2 \quad \dots \dots \dots \text{(b)}$$

Substituting Equation (a) into Equation (b), we get

$$(\dot{m}_1 + \dot{m}_2)h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$$

Rearranging the above equation for \dot{m}_2

$$\dot{m}_2 = \dot{m}_1 \left(\frac{h_3 - h_1}{h_2 - h_3} \right) = 5 \left(\frac{419.06 - 209.33}{2706.2 - 419.06} \right) = 0.4585 \text{ kg/s}$$

OBJECTIVE QUESTIONS

Select the correct answer from the given alternatives.

1. First law of thermodynamics is based on
 - (a) conservation of linear momentum
 - (b) conservation of energy
 - (c) conservation of work
 - (d) conservation of angular momentum

NUMERICAL PROBLEMS

- A control mass containing 0.5 kg of a gas undergoes a process in which there is a heat transfer of 120 kJ from the system to the surroundings. Work done on the system is 60 kJ. If the initial specific internal energy of the system is 400 kJ/kg, determine the final specific internal energy.
- A gas contained in a piston cylinder device undergoes a polytropic process for which pressure volume relationship is given by $Pv^{1.2} = \text{constant}$. The initial pressure is 400 kPa, the initial volume is 0.2 m³ and the final volume is 0.4 m³. The internal energy of the gas decreases by 20 kJ during the process. Determine the work transfer and heat transfer for the process.
- A piston cylinder arrangement contains 0.01 m³ air at 150 kPa and 27°C. The air is now compressed in a process for which pressure and volume relationship is given by $PV^{1.25} = \text{constant}$ to a final pressure 600 kPa. Determine the work transfer and heat transfer for the process. [Take R = 287 J/kgK and c_v = 711 J/kgK]
- A closed system undergoes a process A from state 1 to state 2 as shown in Figure P4.4; which requires a heat input of $Q_A = 185$ kJ. The system returns adiabatically from state 2 to state 1 through process B. Determine the work transfer for process B. (Take $\gamma = 1.4$)

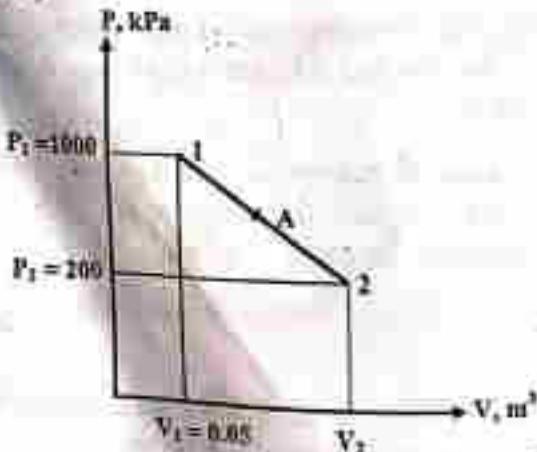


Figure P4.4

A ~~gas~~ undergoes a thermodynamic cycle consisting of the following three processes:

- Process 1–2: expansion with $PV = \text{constant}$, $P_1 = 300$ kPa, $U_2 = U_1$
- Process 2–3: constant volume with $V_2 = V_3 = 2$ m³, $U_3 - U_2 = 300$ kJ
- Process 3–1: constant pressure, $W_{12} = -1200$ kJ
- Sketch the process on P–V and T–V diagrams.
 - Calculate the net work for the cycle.
 - Calculate the net heat for the cycle.
 - Calculate the heat transfer for process 1–2.
 - Calculate the heat transfer for process 3–1.
 - Is this power cycle or a refrigeration cycle?
6. A rigid vessel having a volume of 0.4 m³ initially contains a two-phase mixture at a pressure of 100 kPa with 2% of its volume occupied by saturated liquid and the remaining by the saturated vapor. Heat is supplied to the vessel until it holds only saturated vapor. Determine the total heat transfer for the process.
7. A rigid vessel with a volume of 0.1 m³ contains water initially at 400 kPa with a quality of 60%. A heater is turned on heating the water at a rate of 2 kW. Determine the time required to vaporize all the liquid.
8. A closed rigid tank contains 2 kg of a saturated water vapor initially at 160°C. Heat transfer occurs from the system and the pressure drops to 150 kPa. Determine the amount of heat lost by the system.
9. A rigid vessel initially contains 4 kg of a saturated water vapor. It is cooled to a final state where the temperature is 150°C and quality is 0.1. Determine the initial temperature and the heat transfer from the system.
10. A rigid vessel of volume 0.2 L contains water at its critical state. It is cooled down to room temperature of 25°C. Determine the heat loss from the water.
11. A piston cylinder device shown in Figure P4.11 restrained by a linear spring contains 2 kg of air initially at 150 kPa and 27°C. It is now heated until its volume doubles at which time temperature reaches 527°C. Sketch the process on P–V and determine the

- total work and heat transfer in the process. [Take $R = 287 \text{ J/kgK}$ and $c_v = 718 \text{ J/kgK}$]
12. A piston cylinder device shown in Figure P4.12 loaded with a linear spring ($k = 20 \text{ kN/m}$) contains 0.5 kg of H_2O initially at a pressure of 200 kPa and a volume of 0.4 m^3 . Heat is transferred to the H_2O until a final pressure of 400 kPa is reached. If the cross sectional area of the piston is 0.05 m^2 , determine the final temperature and the heat transfer for the process.

13. Oxygen (4 kg) is contained in a piston cylinder device shown in Figure P4.13 initially at a pressure of 1000 kPa and a temperature of 77°C . There is a heat transfer to the system until the piston reaches the upper stops, at which time volume inside the cylinder is 0.6 m^3 . The oxygen is further heated until the pressure reaches to 2000 kPa . Sketch the process on $P - V$ and $T - V$ diagrams and determine the total work and heat transfer in the process. [Take $R = 260 \text{ J/kgK}$ and $c_v = 660 \text{ J/kgK}$]



Figure P4.11



Figure P4.12



Figure P4.13

14. Nitrogen (5 kg) is contained in a piston cylinder device shown in Figure P4.14 initially at a pressure of 800 kPa and a temperature of 127°C . There is a heat transfer to the system until the temperature reaches to 527°C . It takes a pressure of 1500 kPa to lift the piston. Sketch the process on $P - V$ and $T - V$ diagram and determine the total work and heat transfer in the process. [Take $R = 297 \text{ J/kgK}$ and $c_v = 743 \text{ J/kgK}$]

15. Air (0.4 kg) is contained in a piston cylinder device shown in Figure P4.15 initially at a pressure of 1500 kPa and 800 K . The cylinder has stops such that the minimum volume of the system is 0.04 m^3 . The air in the cylinder is cooled to 300 K . Sketch the process on $P - V$ and $T - V$ diagrams and determine

- (a) the final volume and pressure of the air, and
 (b) the total work and heat transfer in the process. [Take $R = 287 \text{ J/kgK}$ and $c_v = 718 \text{ J/kgK}$]

16. Air is contained in a piston cylinder device shown in Figure P4.16 initially at a pressure and temperature of 1000 kPa and 300°C . Heat is lost by the system until its pressure drops to 750 kPa . Sketch the process on $P - V$ and $T - V$ diagrams and determine the total work and heat transfer. [Take $R = 287 \text{ J/kgK}$ and $c_v = 718 \text{ J/kgK}$]

17. Air (0.1 kg) is contained in piston/cylinder assembly as shown in Figure P4.17. Initially, the piston rests on the stops and is in contact with the spring, which is in its unstretched position. The spring constant is 100 kN/m . The piston weighs 30 N and atmospheric pressure is 101 kPa . The air is initially at 300 K and 100 kPa . Heat transfer occurs until the air temperature reaches the surrounding temperature of 700 K .

- (a) Find the final pressure and volume.
 (b) Find the process work.
 (c) Find the heat transfer.
 (d) Draw the $P - V$ diagram of the process. [Take $R = 287 \text{ J/kgK}$ and $c_v = 718 \text{ J/kgK}$]

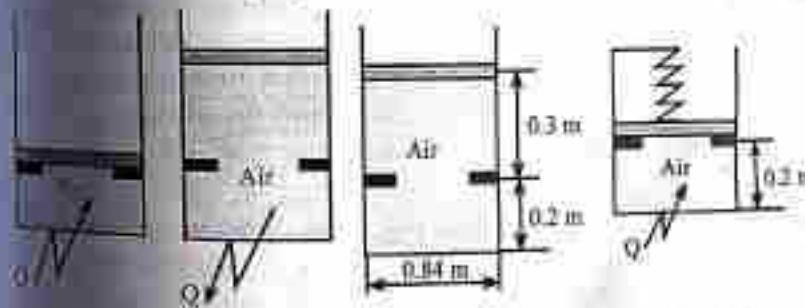


Figure P4.14 Figure P4.15 Figure P4.16 Figure P4.17

18. Water (1.5 kg) is contained in a piston cylinder device shown in Figure P4.18 initially at a pressure of 400 kPa with a quality of 50% . There is a heat transfer to the system until it reaches a final temperature of 500°C . It takes a pressure of 800 kPa to lift the piston. Sketch the process on $P - v$ and $T - v$ diagrams and determine the total work and heat transfer.

19. Water (0.5 kg) is contained in a piston cylinder device shown in Figure P4.19 initially at a pressure of 200 kPa with a quality of 80%. Mass of the piston is such that a pressure of 300 kPa is required to lift it. Heat is transferred to the system until its volume doubles. Sketch the process on $P-v$ and $T-v$ diagrams and determine

- the final temperature
- the total work transfer, and
- the total heat transfer.

20. Water (4 kg) is contained in a piston cylinder device shown in Figure P4.20 initially at an atmospheric pressure of 100 kPa with a quality of 10%. The piston has a mass of 100 kg and a cross sectional area of 24.525 cm^2 . Heat is now added until it reaches a saturated vapor state. Sketch the process on $P-v$ and $T-v$ diagrams and determine

- the initial volume
- the final pressure
- the total work transfer, and
- the total heat transfer. [Take $g = 9.81 \text{ m/s}^2$]

21. A piston cylinder device shown in Figure P4.21 contains 4 kg of water initially at saturated liquid state at 5 MPa. There is a heat transfer to the system until it hits the stops at which time its volume is 0.08 m^3 . There is further heat transfer to the device until water is completely vaporized. Sketch the process on $P-v$ and $T-v$ diagrams and determine the total work and heat transfer.

22. A piston cylinder device shown in Figure P4.22 contains water initially at $P_1 = 1 \text{ MPa}$ and $T_1 = 500^\circ\text{C}$. A pressure of 500 kPa is required to support the piston. There is a heat transfer from the device until its temperature drops to 30°C . Sketch the process on $P-v$ and $T-v$ diagrams and determine the total work and heat transfer.

23. A piston is free to move between two sets of stops in a piston cylinder device shown in Figure P4.23. When the piston rests on the bottom stop, the volume is 0.3 m^3 and when the piston reaches the upper stop, the volume is 0.6 m^3 . The system initially contains water at 100 kPa with a quality 10%. Heat is supplied to the system until it contains only saturated vapor. The

area of the piston is such that a pressure of 250 kPa is required to lift the piston. Sketch the process on $P-v$ and $T-v$ diagrams and determine

- the final pressure
- the total work transfer, and
- the total heat transfer.



Figure P4.19



Figure P4.21

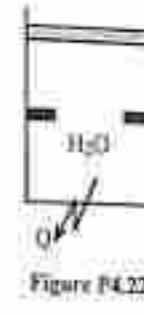


Figure P4.22



Figure P4.23

steam enters through a turbine operating under steady state conditions at a rate of 5 kg/s. The properties of the steam at turbine inlet are $P_1 = 1 \text{ MPa}$ and $T_1 = 500^\circ\text{C}$ and at the turbine exit $P_2 = 50 \text{ kPa}$ and $x_2 = 0.9$. If the heat loss from the turbine surface occurs at a rate of 500 kW, determine the power output from the turbine.

Steam enters a turbine operating at steady state with a mass flow rate of 1.2 kg/s. Properties of the steam at the inlet are $P_1 = 5 \text{ MPa}$, $T_1 = 450^\circ\text{C}$, $\dot{V}_1 = 10 \text{ m/s}$ and at the exit are $P_2 = 100 \text{ kPa}$, $x_2 = 90\%$, $\dot{V}_2 = 50 \text{ m/s}$. If the power output of the turbine is 1200 kW, determine the rate of heat transfer from the turbine.

Air enters a compressor operating at steady state at 100 kPa, 300 K and leaves at 1000 kPa, 400 K, with a volumetric flow rate of 1.5 m³/min. The work consumed by the compressor is 250 kJ per kg of air. Neglecting the effects of potential and kinetic energy, determine the heat transfer rate, in kW. [Take $R = 287 \text{ J/kgK}$, and $c_p = 1005 \text{ J/kgK}$]

Air expands through an adiabatic turbine from 1000 kPa, 400 K to 100 kPa, 400 K. The inlet velocity is 10 m/s whereas exit velocity is 100 m/s. The power output of the turbine is 3600 kW. Determine the mass flow rate of air and the inlet and exit areas. [Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$]

28. Air enters the turbine at 1 MPa and 327°C with a velocity of 100 m/s and exits at 100 kPa and 27°C with a low velocity. Heat transfer loss from the turbine surface is 1200 kJ/min and the power output of the turbine is 240 kW. Determine the mass flow rate of air through the turbine. [Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$]

29. Air flows steadily through an adiabatic compressor entering at 150 kPa, 150°C and with a velocity of 200 m/s and leaving at 1000 kPa, 500°C and with a velocity of 100 m/s. The exit area of the compressor is 100 cm². Determine
 (a) the mass flow rate of air through the compressor, and
 (b) the power required to drive the compressor.
 [Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$]

30. An adiabatic turbine operating under steady state conditions develops 12 MW of power output for a steam mass flow rate of 15 kg/s. The steam enters at 4 MPa with a velocity of 20 m/s and exits at 60 kPa with a quality of 85% and velocity of 100 m/s. Determine the inlet temperature of steam.

31. Air enters an adiabatic nozzle steadily at 300 kPa, 150°C and with a velocity of 20 m/s and leaves at 100 kPa and with a velocity of 200 m/s. The inlet area of the nozzle is 0.01 m². Determine
 (a) the mass flow rate of air through the nozzle,
 (b) the exit temperature of the air, and
 (c) the exit area of the nozzle.
 [Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$]

32. Air at 100 kPa and 127°C enters an adiabatic diffuser at a rate of 1.5 kg/s and leaves at a pressure of 150 kPa. The velocity of the air is decreased from 250 m/s to 50 m/s as it passes through the diffuser. Determine
 (a) the exit temperature of the air, and
 (b) the exit area of the diffuser.
 [Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$]

33. Air enters a nozzle steadily at 300 kPa, 127°C and with a velocity of 40 m/s and leaves at 100 kPa and with a velocity of 300 m/s. The heat loss from the nozzle surface is 20 kJ/kg of the air. The inlet area of the nozzle is 100 cm². Determine

- (a) the exit temperature of the air, and
 (b) the exit area of the nozzle.
 [Take $R = 287 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$]

34. Steam enters an adiabatic nozzle with $P_1 = 2.5 \text{ MPa}$, $T_1 = 250^\circ\text{C}$ and with very low velocity. The steam exits the nozzle with $P_2 = 0.1 \text{ MPa}$ and velocity of 650 m/s. The mass flow rate of steam is 2 kg/s. Determine the exit area of the nozzle.

35. Steam at 4 MPa, 450°C enters a nozzle operating at steady state with a velocity of 50 m/s. Steam leaves the nozzle at 2 MPa and 300°C. The inlet area of the nozzle is 80 cm² and heat loss from the nozzle surface occurs at the rate of 100 kW. Determine
 (a) the mass flow rate of steam,
 (b) the exit velocity of the steam, and
 (c) the exit area of the nozzle.

36. Steam enters into a nozzle at $P_1 = 1000 \text{ kPa}$, $T_1 = 300^\circ\text{C}$ and with a velocity of 75 m/s. Steam leaves the nozzle at $P_2 = 80 \text{ kPa}$, $T_2 = 200^\circ\text{C}$ and with a velocity of 350 m/s. Determine the heat loss per unit mass of the steam from the nozzle surface.

37. A vapor heating arrangement operates at steady state with liquid water entering at inlet 1 with $P_1 = 500 \text{ kPa}$ and $T_1 = 50^\circ\text{C}$. Steam at $P_2 = 500 \text{ kPa}$ and $T_2 = 200^\circ\text{C}$ enters at inlet 2. Saturated liquid water exits with a pressure of $P_3 = 500 \text{ kPa}$ from the outlet 3. Determine the ratio of mass flow rates m_1/m_2 .

38. Atmospheric air containing water vapor enters the dehumidifier with an enthalpy of 90 kJ/kg at a rate of 210 kg/h. Heat is removed from the air as it passes over a bank of tubes through which cold water flows. Atmospheric moisture that condenses on the tube drains from the dehumidifier with an enthalpy of 34 kJ/kg at a rate of 4 kg/h. Air leaving has an enthalpy of 23.8 kJ/kg. Velocities through the dehumidifier are very low. Determine the rate of heat removal, in kW, from the air stream through the dehumidifier.

39. Steam enters into a well insulated throttling valve at 10 MPa, 300°C and exits at 5 MPa. Determine the final temperature of the steam.

SECOND LAW OF THERMODYNAMICS

5

5.1 Limitations of the First Law (Necessity of the Second Law)

First law of thermodynamics explains thermodynamic processes with reference to mass conservation and energy conservation. It deals with the quantitative aspect of the energy and gives the only condition that any process is possible provided that the total energy remains constant. But some processes in nature cannot occur although energy conservation principle is satisfied. In this regard, second law of thermodynamics deals with quality or nature of energy and defines the direction of the process in which the system can proceed.

The following examples give some idea about the inability of the first law of thermodynamics to explain any process completely.

- According to the first law of thermodynamics, for a cyclic process net heat transfer is equal to net work transfer. But any real device, even operating on a cycle, cannot convert heat supplied to it completely into output work. Hence, the second law of thermodynamics explains why any real engine cannot operate without heat loss.
- Consider an insulated container having two compartments as shown in Figure 5.1(a) (State 1). One of the compartments contains gas and another is vacuum. Due to the pressure difference between the two compartments, gas continuously exerts force on the partition wall. If the gas pressure is sufficient to break the partition wall, gas expands and fills up the entire container as shown in Figure 5.1(b) (State 2). If we analyze the process 1 → 2, total energy is same in both State 1 and State 2, hence the first law of thermodynamics is obviously satisfied. But if we assume that process occurs in reverse direction i.e., from State 2 and State 1, first law of thermodynamics is still satisfied. But system can never proceed in this direction.

We can also compare the nature of two modes of energy transfer i.e., work transfer and heat transfer with the help of second law of thermodynamics. Work transfer occurs due to relative displacement between the system and surroundings. Whenever there is a displacement, friction comes into play and some part of the work is converted into heat. Hence, work always produces some heat itself. But the reverse process i.e., the self conversion of heat to work (without any device) cannot occur. For example, when a shaft rotates on a bearing heat is produced in the bearing due to the friction. But if we heat the bearings of a stationary shaft, it cannot rotate due to heating effect.

The second and third examples discussed above present the basic nature of inherent tendency of a system (or a process), that most of the natural processes in nature proceed in only direction. Such directional feature of the processes cannot be explained only with the first law of thermodynamics.

One of the main features of the second law of thermodynamics is that it defines the directions of the processes. Second law of thermodynamics defines the direction of the process with reference to the system property called entropy. To have an idea about the entropy, let us again consider the example discussed above. If we analyze the samples having same size (volume) from both systems of Figure 5.1(a) and Figure 5.1(b); sample from the system of Figure 5.1(a) will contain relatively larger number of molecules than that of the sample from the system of Figure 5.1(b) and it represents the system in better way. Hence, the system at state 1 has more reliability or less uncertainty than that at state 2.

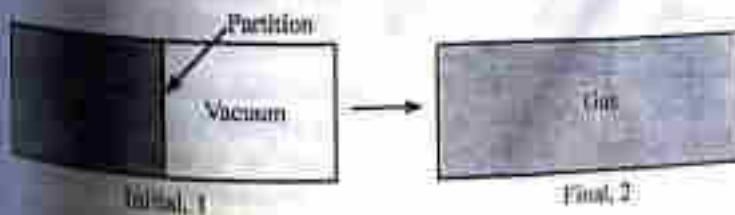
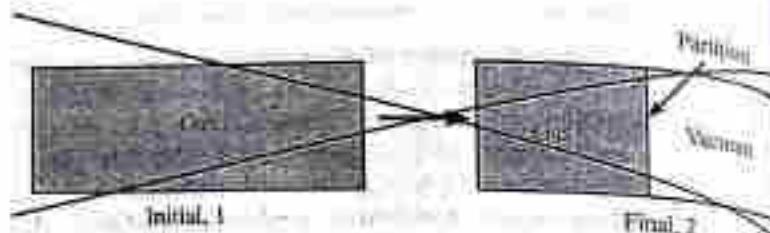


Figure 5.1(b): Possible Direction for a Process

**IMPOSSIBLE Impossible Direction for a Process**

Again gas molecules have same energy in both the states. As there more space for gas molecules in state 2, gas molecules at state 1 have less randomness as compared to that at state 2.

We can also compare the basic nature of work transfer and heat transfer process. Work transfer is produced due to the relative displacement of the system boundary. Hence, during the work transfer process all molecules or particles have uniform orientation (all particles have same velocity). Whereas when heat is supplied to the system, its molecular randomness increases. In this regard, work transfer is called high grade of energy and heat transfer is called low grade of energy.

Now, if we compare the direction of the process in terms of the factors discussed above, we can conclude that:

- System itself tends to undergo a process from less uncertain or less random state to more uncertain or more random state. But the reverse direction is not possible.
- High grade of energy (work) itself get converted into low grade of energy (heat) and the reverse direction is not possible (without any device).

The property of a system which gives a measure of molecular randomness, disorder or uncertainty existing in a system is called entropy. It is an extensive property and is denoted by S .

5.2 Second Law of Thermodynamics for Isolated System

With reference to an isolated system shown Figure 5.1(a), we can state and explain second law of thermodynamics for an isolated system. As explained earlier, any isolated system can process in the direction in which its randomness or uncertainty i.e., entropy increases. We can state this feature of the isolated system as second law of thermodynamics as:

Entropy of an isolated system always increases or may remain constant

$$\Delta S_{\text{gen}} > 0 \quad \dots\dots\dots (5.1)$$

$$\text{Final } S_{\text{gen}} > \text{Initial } S_{\text{gen}} \quad \dots\dots\dots (5.2)$$

Equation (5.1) shows that entropy of an isolated system at the final state of any real process is always greater than that at the initial state. The difference between entropies at the final state and the initial state during any process is called entropy production or entropy generation and is denoted by ΔS_{gen} .

Equation (5.1) is equivalent form to avoid inequality

$$\Delta S_{\text{gen}} = 0 \quad \dots\dots\dots (5.3)$$

$$\Delta S_{\text{gen}} > 0 \quad \dots\dots\dots (5.4)$$

$$\frac{\partial S_{\text{gen}}}{\partial T} > 0 \quad \dots\dots\dots (5.5)$$

$$-\dot{S}_{\text{gen}} = 0 \quad \dots\dots\dots (5.6)$$

5.3 Reversible and Irreversible Processes

We have already discussed (Chapter 1.4.5) about reversible and irreversible processes differentiated with reference to the nature of intermediate states. We can also completely differentiate reversible and irreversible processes with reference to entropy. According to second law of thermodynamics, during any real process net entropy change is always greater than or equal to zero, i.e., for any process between state 1 and state 2

$$S_2 \geq S_1$$

During any process if $S_2 = S_1$, the reverse process is also possible because entropy is same for both forward and reverse direction. Hence the process is called reversible process. But during any process, if $S_2 > S_1$, i.e., entropy increases in forward direction, then the reverse direction is not possible because any real process can not result in decrease in entropy. Hence the process is called an irreversible process.

A process is said to be a reversible process if the initial conditions of both the system and surroundings can be restored by the reverse action such that net change in entropy is zero for both forward and reverse process.

A process is said to be an irreversible process, if the initial conditions of both the system and surroundings cannot be restored (certain effects are left either on the system or on the surroundings) by the reverse action. In this case entropy of the system increases in forward process and the reverse direction is not possible.

A process is said to be an internally reversible if the initial condition of only the system is restored (but certain effects are left on the surroundings) when the system is taken through reverse direction.

Similarly a process is said to be an externally reversible if the initial condition of only the surroundings is restored (but certain effects are left on the system) when the system is taken through the reverse direction.

Therefore, for a process to be a complete reversible process, it should be internally as well as externally reversible.

Let us consider an example shown in Figure 5.2, to explain how irreversibility arises in real processes. A solid block is placed on a horizontal surface. When a force F is applied to the block during the process 1 → 2, it is displaced from position 1 (state 1) to position 2 (state 2). When the same force F is applied in the opposite direction, the block will be displaced from position 2 to position 1. During the reverse process, the position of the block is restored, but its thermodynamic properties such as temperature cannot be restored as long as there is friction between the block and the surface.

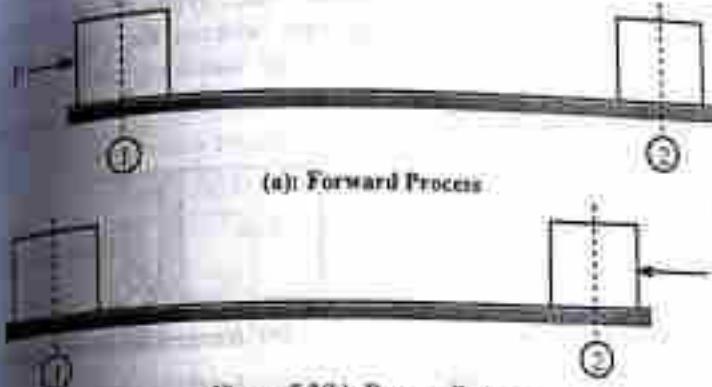


Figure 5.2(b): Reverse Process

Irreversibility due to Friction

This example clearly highlights the fact that most of the real processes involving work transfer are irreversible due to the friction. Another cause of irreversibility is heat transfer because heat always increases the entropy of the system.

The major factors causing irreversibility in any process are classified as mechanical irreversibility and thermal irreversibility. Mechanical irreversibility is associated with the friction between the system and the surroundings and friction within the system. Thermal irreversibility is associated with heat transfer between the system and the surroundings or within the system.

These factors causing irreversibility can also be classified as internal irreversibility and external irreversibility. Internal irreversibility is associated with friction within the system and heat transfer within the system. External irreversibility is associated with the friction between the system and the surroundings and heat transfer between the system and the surroundings.

Most of the real processes in nature are irreversible and we can model these processes as reversible process with certain simplifying assumptions. Few examples are presented here to explain how we can approximate real processes as reversible processes.

(i) Processes involving Friction

Consider a piston-cylinder device containing a gas as shown in Figure 5.3. During a certain process in the system piston can move and gas can

undergo either compression or expansion. Such types of processes are usually irreversible because of the friction between the piston and the cylinder wall. But most of the time, we consider such processes occurring in a piston cylinder device as a reversible process assuming the surface to be frictionless.



(a) Irreversible Process



(b) Reversible Process

Figure 5.4 Process involving Friction

(b) Process involving Nonlinear Spring or Inelastic Behavior

If the piston cylinder device is restrained by a spring as shown in Figure 5.4, the process will be reversible if spring characteristic is linear and will be irreversible if spring characteristic is nonlinear. Similarly, if certain part of the system undergoes inelastic deformation during the process, the process will be irreversible because system cannot regain its original position even after the removal of the load. Process in which deformations are within the elastic range can be assumed as a reversible process.

(c) Free Expansion Process

Consider an insulated container having two compartments consisting of a gas and a vacuum separated by a wall connected with a valve as shown in Figure 5.5 (a). When the valve is open gas expands spontaneously and we cannot reverse it back from intermediate state. Hence, free expansion or unrestricted expansion is an irreversible process. We can assume expansion of a gas as a reversible process if it is restrained by some external forces as shown in Figure 5.5 (b). During any process in this system, if the valve is open gas molecules exert force on the piston surface but piston cannot undergo spontaneous expansion because of restraining force F . When the restraining force F is reduced the gas undergoes an expansion and when F is increased the gas undergoes compression. Hence such type of expansion against a restraining force (restricted expansion) can be assumed as a reversible process.



(a) Irreversible Process



(b) Reversible Process

Figure 5.5 Process involving Nonlinear Spring

(a) Irreversible Process

(b) Reversible Process

Figure 5.6 Free expansion and Unrestricted Expansion

Heat Transfer with Finite Temperature Difference

When any system having a temperature T_L is brought in contact with a source having a temperature of T_H as shown in Figure 5.6(a), heat flows from the source to the system. During the heat transfer there will be temperature gradient within the system and it passes through a series of non-equilibrium states and the process becomes irreversible. We can make the process less irreversible by reducing the temperature gradient within the system. For this we first bring the system with the temperature T_L with a source the source having a temperature of $T_L + dT$ as shown in Figure 5.6(b). Heat is transferred from the source to the system and after some time temperature of the system also reaches $T_L + dT$. Then the system is brought in contact with a source having a temperature $T_L + 2dT$, such that a temperature gradient of only dT exists over short interval. Therefore heat transfer with infinitesimally small temperature difference can be approximated as a reversible heat transfer.



(a) Irreversible Process



(b) Reversible Process

Figure 5.6 Heat Transfer Process

5.4 Entropy Relations

As entropy is a thermodynamic property, it can be used with any other property to completely describe the state of a system. Applying the postulate, we can determine the internal energy of a system if its volume and entropy are given as

$$U = U(S, V)$$

The change in internal energy during any process is then given as

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV \quad (5.9)$$

The two partial derivatives in the above equations are the ratios of extensive properties, hence are intensive parameters. The first partial derivative is equal to the temperature and the second partial derivative is equal to the negative of the pressure, i.e.,

$$T = \left(\frac{\partial U}{\partial S}\right)_V \quad \text{and} \quad P = -\left(\frac{\partial U}{\partial V}\right)_S \quad (5.10)$$

Substituting into Equation (5.9), we get

$$dU = TdS - PdV \quad (5.11)$$

This equation is commonly known as Gibbs Equation. Rearranging Gibbs equation, we can get the expression for the change in entropy as

$$dS = \frac{dU}{T} + \frac{P}{T} dV \quad (5.12)$$

Entropy equation can also be expressed in terms of enthalpy. For this, let us start with the definition of enthalpy as

$$H = U + PV \quad (5.13)$$

The change in enthalpy during any process is then given as

$$dH = dU + PdV + Vdp \quad (5.14)$$

Substituting Equation (5.11) ($dU + PdV = TdS$) into Equation (5.14),

$$dH = TdS + Vdp \quad (5.15)$$

Rearranging Equation (5.15), we can get the expression for the change in entropy as

$$dS = \frac{dH}{T} - \frac{V}{T} dp \quad (5.16)$$

5.4.1 Entropy Relations for an Ideal Gas

For an ideal gas, the state properties P , V and T are related by the equation of state as

$$PV = mRT$$

$$\frac{P}{T} = \frac{mR}{V} \quad (5.17)$$

The change in internal energy of an ideal gas is given by

$$dU = mc_v dT \quad (5.18)$$

Substituting Equations (5.17) and (5.18) into Equation (5.12), we get

$$dS = mc_v \frac{dT}{T} + mR \frac{dV}{V} \quad (5.19)$$

Assuming c_v and R as constants, we can get the expression for the change in entropy for an ideal gas during any process between state 1 and state 2 in terms of temperature and volume ratios, as

$$S_2 - S_1 = mc_v \ln \left(\frac{T_2}{T_1} \right) + mR \ln \left(\frac{V_2}{V_1} \right) \quad (5.20)$$

Similarly, equation of state can also be rearranged as

$$\frac{V}{T} = \frac{mR}{P} \quad (5.21)$$

The change in enthalpy of an ideal gas is given by

$$dH = mc_p dT \quad (5.22)$$

Substituting Equations (5.21) and (5.22) into Equation (5.16), we get

$$dS = mc_p \frac{dT}{T} - mR \frac{dp}{P} \quad (5.23)$$

Assuming c_p and R as constants, we can get the expression for the change in entropy for an ideal gas during any process between state 1 and state 2 in terms of temperature and pressure ratios, as

$$S_2 - S_1 = mc_p \ln \left(\frac{T_2}{T_1} \right) - mR \ln \left(\frac{P_2}{P_1} \right) \quad (5.24)$$

5.4.2 Entropy Relation for an Incompressible Substance

Volumes of the solid and liquid substances do not change appreciably with the pressure or temperature and hence are assumed as

incompressible substances. Substituting $dV = 0$ in Equation (5.12), we get entropy relation for an incompressible substance as

$$dS = \frac{dU}{T} = \frac{mc\delta T}{T} \quad \dots\dots\dots (5.25)$$

where c is the specific heat of the incompressible substance.

Then the change in entropy for an incompressible substance during a process between state 1 and state 2 is given as

$$S_2 - S_1 = mc\ln\left(\frac{T_2}{T_1}\right) \quad \dots\dots\dots (5.26)$$

5.5 Control Mass Formulation of Second Law of Thermodynamics

As we know that any control mass can interact with its surroundings either by heat transfer or by work transfer; therefore the control mass formulation of the second law of thermodynamics gives the expression for the change in entropy of the control mass because of heat transfer and work transfer. For this formulation, we first find out the effects of heat transfer and work transfer separately on the entropy of the control mass and then we can develop the general expression and statement for the second law of thermodynamics for a control mass.

Consider an isolated system shown in Figure 5.7 consisting of a control mass interacting with a number of heat transfer reservoirs and work transfer reservoirs.



FIGURE 5.7 Isolated system consisting of Control Mass, Heat Transfer Reservoirs and Work Transfer Reservoirs

Contribution of Heat Transfer on Entropy

To determine the effect of heat transfer on entropy, we consider a system having infinite heat capacity such that its temperature is unaffected by the heat transfer. Such an idealized system which can interact with its surroundings only by heat transfer (but not work transfer) is called a reversible heat transfer reservoir and is specified by its temperature (T).

Substituting $\delta W = PdV = 0$, into Equation (5.12), we get

$$dS = \frac{dU}{T} \quad \dots\dots\dots (5.27)$$

Applying first law of thermodynamics, we get

$$\delta Q - dU + PdV = dU + 0 = dU \quad \dots\dots\dots (5.28)$$

Substituting Equation (5.28) into Equation (5.27), we get an expression for the change in entropy due to a reversible heat transfer process as

$$dS = \frac{\delta Q}{T} \quad \dots\dots\dots (5.29)$$

Equation (5.29) shows that entropy of a system increase if heat is transferred and decrease if it loses heat.

Contribution of Work Transfer on Entropy

To determine the effect of work transfer on entropy, we consider a system having infinite work capacity such that its pressure is unaffected by the work transfer. Such an idealized system which can interact with its surroundings only by work transfer (but not heat transfer) is called a reversible work transfer reservoir and is specified by its pressure (P).

Applying first law of thermodynamics for a reversible work transfer process, we get

$$\delta Q (=0) = dU + PdV \\ dU = -PdV \quad \dots\dots\dots (5.30)$$

Substituting Equation (5.30) into Equation (5.12), we get an expression for the change in entropy due to a reversible work transfer process as

$$dS = \frac{-PdV}{T} + \frac{PdV}{T} = 0 \quad \dots\dots\dots (5.31)$$

Equation (5.31) shows that work transfer does not have any contribution on the entropy of the system.

Now, applying second law of thermodynamics for the isolated system shown in Figure 5.7,

$$(dS)_{\text{isolated}} \geq 0$$

Since the total change in entropy of the isolated system is given by the sum of the change in entropy of the control mass, the change in entropy of the reversible heat transfer reservoirs and the change in entropy of the reversible work transfer reservoirs, i.e.,

$$(dS)_{CM} + \sum(dS)_{RHTRs} + \sum(dS)_{RWTRs} \geq 0 \quad \dots \dots \dots (5.32)$$

Since there is no effect of reversible work transfer on the entropy, substituting $\sum(dS)_{RWTRs} = 0$, into Equation (5.32),

$$(dS)_{CM} + \sum(dS)_{RHTRs} \geq 0 \quad \dots \dots \dots (5.33)$$

Substituting Equation (5.29) into Equation (5.33),

$$(dS)_{CM} + \sum\left(\frac{\delta Q_i}{T_i}\right)_{RHTRs} \geq 0 \quad \dots \dots \dots (5.34)$$

If $(\delta Q)_{RHTRs}$ is the heat supplied to the reservoir at temperature T_i , that is supplied by the control mass, therefore,

$$(\delta Q)_{RHTRs} = -(\delta Q)_{CM} \quad \dots \dots \dots (5.35)$$

Substituting Equation (5.35) into Equation (5.34), we get

$$(dS)_{CM} - \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM} \geq 0$$

$$(dS)_{CM} \geq \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM} \quad \dots \dots \dots (5.36)$$

With reference to Equation (5.36), second law of thermodynamics for a control mass can be stated as

The change in entropy of a control mass is always equal to the sum of heat transferred to the control mass boundary and the product of temperatures.

Equation (5.36) can be expressed separately as:

$$(dS)_{CM} = \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM}, \text{ if the process is reversible.} \quad \dots \dots \dots (5.37)$$

$$(dS)_{CM} > \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM}, \text{ if the process is irreversible.} \quad \dots \dots \dots (5.38)$$

Equation (5.37) shows that if heat transfer on any control mass occurs under reversible condition, then the change in its entropy is equal to entropy contributed by the heat transfer where as Equation (5.38) shows that if the same amount of heat is transferred under irreversible condition then the change in entropy of the system is greater than the entropy contributed by the heat transfer. Hence the difference in values of left hand side and right hand side of Equation (5.38) represents the entropy produced or entropy produced by the system itself.

Therefore, Equation (5.36) can also be expressed in terms of entropy generation to avoid inequality sign as

$$(dS)_{CM} - \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM} - (dS)_{gen} = 0 \quad \dots \dots \dots (5.39)$$

where

$$(dS)_{gen} = (dS)_{CM} - \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM} \quad \dots \dots \dots (5.40)$$

Equations (5.36) and (5.39) can also be expressed in terms of ratio as

$$\left(\frac{dS}{dT}\right)_{CM} \geq \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM} \quad \dots \dots \dots (5.41)$$

$$\left(\frac{dS}{dT}\right)_{CM} - \sum\left(\frac{\delta Q_i}{T_i}\right)_{CM} - \dot{S}_{gen} = 0 \quad \dots \dots \dots (5.42)$$

For example reservoir undergoing reversible, Equation (5.37) reduces to

$$\begin{aligned} dS &= \frac{\delta Q}{T} \\ \delta Q &= T dS \end{aligned} \quad \dots \dots \dots (5.43)$$

Equation (5.43) shows that area under $T-S$ diagram gives the magnitude of heat transfer as shown in Figure 5.8.

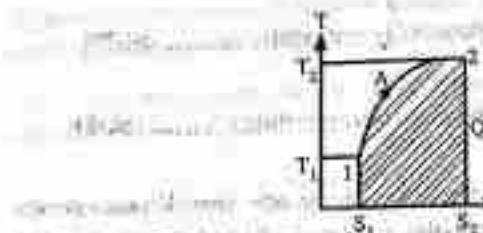


Figure 5.10 Heat Transfer Evaluated as Area under Curve on $T-S$ diagram

5.6 Control Volume Formulation of Second Law of Thermodynamics

Control volume formulation of second law of thermodynamics gives the expression for the change of entropy of a control volume due to mass transfer as well as energy transfer. The effect of mass transfer on the entropy can be determined by evaluating properties of working substance at the inlet and outlet. The effects heat transfer and work transfer on the entropy of the control volume is similar to that for the control mass. Therefore, we can state the second law of thermodynamics for a control volume as

The change in entropy of a control volume minus the net entropy change of work and balance due to mass transfer in general must be equal to the entropy generation produced by the control volume boundary, absent irreversibilities.

Mathematically,

$$\left(\frac{dS}{dt}\right)_{CV} - \dot{S}_{out} \geq \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{CV} \quad \dots \dots \dots (5.46)$$

Substituting $\dot{S}_{gen} = \dot{S}_{in} - \dot{S}_{out}$,

$$\left(\frac{dS}{dt}\right)_{CV} + \dot{S}_{in} + \dot{S}_{out} \geq \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{CV} \quad \dots \dots \dots (5.47)$$

Equation (5.47) can be expressed separately as

$$\left(\frac{dS}{dt}\right)_{CV} = \dot{S}_{in} - \dot{S}_{out} + \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{CV}, \text{ if the process is reversible.} \quad \dots \dots \dots (5.48)$$

$$\left(\frac{dS}{dt}\right)_{CV} > \dot{S}_{in} - \dot{S}_{out} + \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{CV}, \text{ if the process is irreversible.} \quad \dots \dots \dots (5.49)$$

Equation (5.46) shows that the change entropy of a control volume for a reversible process is equal to entropy contributed by the heat transfer and mass transfer whereas Equation (5.47) shows that if the same amount of heat is transferred under irreversible condition then the entropy of the system is greater than the entropy contributed by the heat transfer and mass transfer. Hence the difference in values of left hand side and right hand side of Equation (5.47) represents the generated or entropy produced by the system itself.

Therefore, Equation (5.45) can also be expressed in terms of entropy generation to avoid inequality sign as

$$-\dot{S}_{in} + \dot{S}_{out} - \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{CV} = \dot{S}_{gen} = 0 \quad \dots \dots \dots (5.48)$$

$$\dot{S}_{gen} = \left(\frac{dS}{dt} \right)_{CV} - \dot{S}_{in} + \dot{S}_{out} - \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{CV} \quad \dots \dots \dots (5.49)$$

5.7 Isentropic Process

Change in entropy of any system during a reversible process is given as

$$dS = \frac{\delta Q}{T} \quad \dots \dots \dots (5.50)$$

If this process is reversible as well as adiabatic ($\delta Q = 0$), Equation (5.50) reduces to

$$\begin{aligned} dS &= 0 \\ S_2 - S_1 &= 0 \\ S_2 &= S_1 \end{aligned} \quad \dots \dots \dots (5.51)$$

Hence, during a reversible and adiabatic process, entropy of the system remains constant and the process is called an isentropic process.

5.7.1 Isentropic Relations for an Ideal Gas

Substituting $S_2 - S_1 = 0$ in Equations (5.20) and (5.24), we get

$$0 = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right) \quad \dots\dots\dots (5.52)$$

$$0 = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right) \quad \dots\dots\dots (5.53)$$

Simplifying Equation (5.52),

$$c_v \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left(\frac{V_2}{V_1}\right)$$

$$\text{or, } \ln\left(\frac{T_2}{T_1}\right)^{CV} = \ln\left(\frac{V_1}{V_2}\right)^{-R}$$

Taking antilog on both sides, we get

$$\left(\frac{T_2}{T_1}\right)^{CV} = \left(\frac{V_1}{V_2}\right)^{-R} = \left(\frac{V_1}{V_2}\right)^R$$

$$\text{or, } \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R}{CV}} = \left(\frac{V_1}{V_2}\right)^{\frac{CP-CV}{CV}} = \left(\frac{V_1}{V_2}\right)^{\frac{CP}{CV}-1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{1-\frac{1}{R}} \quad \dots\dots\dots (5.54)$$

Equation (5.54) gives the temperature – volume relationship for an ideal gas undergoing an isentropic process.

Simplifying Equation (5.53),

$$c_p \ln\left(\frac{T_2}{T_1}\right) = R \ln\left(\frac{P_2}{P_1}\right)$$

$$\text{or, } \ln\left(\frac{T_2}{T_1}\right)^{CP} = \ln\left(\frac{P_1}{P_2}\right)^R$$

Taking antilog on both sides, we get

$$\left(\frac{T_2}{T_1}\right)^{CP} = \left(\frac{P_1}{P_2}\right)^R$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{R}{CP}} = \left(\frac{P_1}{P_2}\right)^{\frac{CP-CV}{CV}} = \left(\frac{P_1}{P_2}\right)^{\frac{CV}{CP}} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1-\frac{1}{R}}} \quad \dots\dots\dots (5.55)$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1-\frac{1}{R}}} \quad \dots\dots\dots (5.55)$$

Equation (5.55) gives the temperature – pressure relationship for an ideal gas undergoing an isentropic process.

5.7.2 Isentropic Relations for an Incompressible Substance

The change in specific entropy of an incompressible substance is given by

$$ds = \frac{du}{T} \quad \dots\dots\dots (5.56)$$

Substituting $ds = 0$ into Equation (5.56), we get the relation for an incompressible substance undergoing an isentropic process as

$$\begin{aligned} ds &= \frac{du}{T} = 0 \\ du &= 0 \end{aligned} \quad \dots\dots\dots (5.57)$$

Isentropic relation for an incompressible substance can also be expressed in terms of specific enthalpy. For this let us start with the definition of the specific enthalpy as

$$h = u + Pv \quad \dots\dots\dots (5.58)$$

Then the change specific enthalpy during any process is given by

$$dh = du + Pdv + vdp \quad \dots\dots\dots (5.59)$$

Substituting $dv = 0$ for an incompressible substance and $du = 0$ [from Equation (5.57)], into Equation (5.59), we get

$$dh = vdp \quad \dots\dots\dots (5.60)$$

Integrating Equation (5.60) for any process 1 – 2 between state 1 and state 2, we get isentropic relation for an incompressible substance as

$$h_2 - h_1 = v(P_2 - P_1) \quad \dots \dots \dots (5.4)$$

5.7.3 Isentropic Efficiency of Steady Flow Devices

The process occurring in any steady flow device will be isentropic if it does not involve any kind of losses (frictional loss, heat loss, etc.) therefore isentropic process is an ideal process. But the process occurring in any real device involves losses and the real process differs from the idealized isentropic condition. The performance of the real device is compared with the idealized device (isentropic) with reference to isentropic efficiency.

Isentropic Efficiency of a Turbine

In case of work producing device real work is always less than its isentropic work output because of losses. Hence, the isentropic efficiency of a turbine is defined as the ratio of work output from a turbine and the work that would have been produced when the turbine operates under isentropic condition, i.e.,

$$\eta_{IT} = \frac{W_{real}}{W_{isentropic}} \quad \dots \dots \dots (5.5)$$

Isentropic Efficiency of a Pump/Compressor

In case of work consuming device real work is always more than its isentropic work input because we have to increase work input to overcome the losses to get the same desired output effect. Hence, the isentropic efficiency of a pump or compressor is defined as the ratio of the work that would have been required when the pump/compressor operates under isentropic condition to work required for the real pump/compressor and, i.e.,

$$\eta_{IP} = \frac{W_{isentropic}}{W_{real}} \quad \dots \dots \dots (5.6)$$

Isentropic Efficiency of a Nozzle

Similarly, the isentropic efficiency of a nozzle is defined as the ratio of the kinetic energy of the fluid at the real nozzle exit to the kinetic energy value at the exit of an isentropic nozzle for the same inlet state and exit pressure, i.e.,

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$$\eta_{IN} = \frac{(\bar{V}_{real})^2 / 2}{(\bar{V}_{isentropic})^2 / 2} = \frac{(\bar{V}_{real})^2}{(\bar{V}_{isentropic})^2} \quad \dots \dots \dots (5.64)$$

5.8 Carnot Cycle, Heat Engine, Heat Pump and Refrigerator

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5.8.1 Carnot Cycle

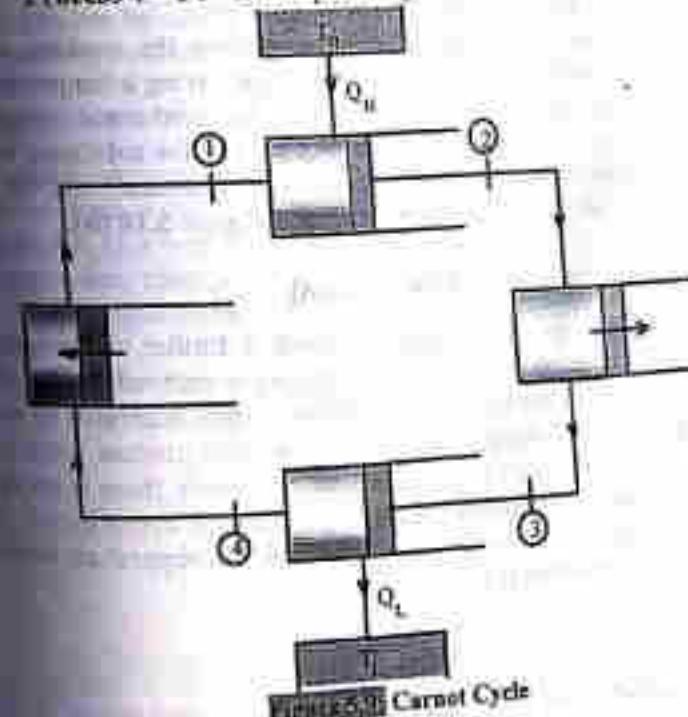
Carnot cycle is an ideal cycle devised by Carnot which will have efficiency equal to that of a reversible cycle (theoretical maximum) without considering its mechanical details. This cycle is not practically possible. However, it is used as a reference standard for the comparison of different practical cycles. Carnot cycle consists of the following processes in series as shown in Figure 5.9 and Figure 5.10.

Process 1 – 2 : Isothermal heat addition

Process 2 – 3 : Isentropic expansion

Process 3 – 4 : Isothermal heat rejection

Process 4 – 1 : Isentropic compression



Process 1 – 2 (Isothermal Heat Addition)

During process 1 – 2, heat is added to the working substance inside the cylinder from a high temperature source having a temperature of T_H and the working substance undergoes an isothermal expansion. During the isothermal expansion, pressure of the substance decreases, its volume increases, its temperature remains constant and the entropy increases as shown in Figure 5.10 (a) and Figure 5.10 (b).

Process 2 – 3 (Isentropic Expansion)

During process 2 – 3, working substance undergoes an isentropic expansion during which its pressure further decreases and its volume further increases as shown in Figure 5.10 (a). During isentropic (reversible and adiabatic) expansion process no heat is supplied to the system and hence it produces work due to expense of its internal energy. Therefore, during the isentropic expansion process, entropy of the system remains constant and its temperature decreases as shown in Figure 5.10 (b).

Process 3 – 4 (Isothermal Heat Rejection)

During process 3 – 4, heat is transferred from the working substance inside the cylinder to a low temperature sink having a temperature of T_L and the working substance undergoes an isothermal compression. During the isothermal compression, pressure of the substance increases, its volume decreases, its temperature remains constant and the entropy decreases as shown in Figure 5.10 (a) and Figure 5.10 (b).

Process 4 – 1 (Isentropic Compression)

During process 4 – 1, working substance is further compressed under isentropic condition such that its initial state is restored during which its pressure further increases and its volume further decreases as shown in Figure 5.10 (a). During isentropic compression process work supplied to the system increases its internal energy because there is no heat loss from the system. Therefore, during the isentropic compression process, entropy of the system remains constant and its temperature increases as shown in Figure 5.10 (b).

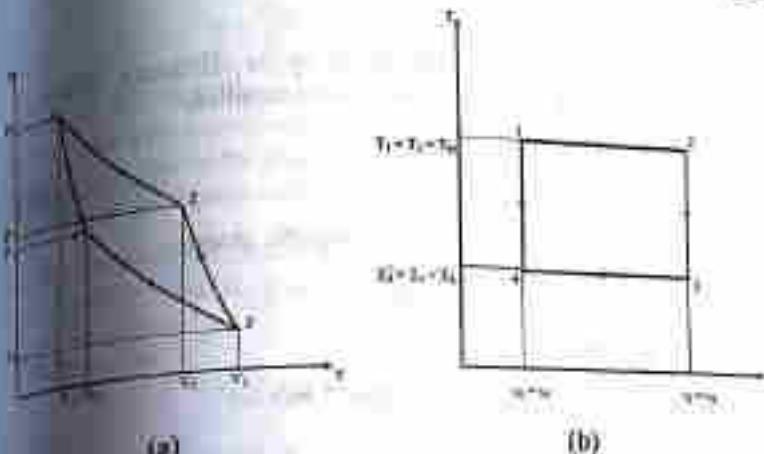


Figure 5.10: P – V and T – S Diagrams for Carnot Cycle

5.3 Heat Engine, Heat Pump and Refrigerator

Devices which operate on cyclic process and produce work are called heat engines whereas devices which are used for heating or cooling of surroundings are called heat pump and refrigerator. Performances of all these devices can be analyzed with the help of first law as well as second law of thermodynamics.

Heat Engine

Device which operates on a cyclic process and converts heat energy into useful work is called a heat engine. Figure 5.11 shows a schematic representation of a heat engine which takes Q_H amount of heat from a high temperature reservoir at T_H (source) converts some part of it into useful work (W) and rejects remaining part Q_L to a low temperature reservoir at T_L (sink).

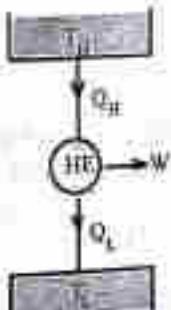


Figure 5.11: Schematic Representation of a Heat Engine

Performance of a heat engine is measured by its efficiency, which is defined as the ratio of work output and the heat supplied,

$$\eta = \frac{W}{Q_H} \quad \dots \dots \dots (5.65)$$

Applying first law of thermodynamics for a cyclic process,

$$\oint \delta Q = \oint \delta W$$

$$\therefore Q_H - Q_L = W \quad \dots \dots \dots (5.66)$$

Substituting Equation (5.66) into Equation (5.65),

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \quad \dots \dots \dots (5.67)$$

Now applying second law of thermodynamics for the cyclic process,

$$\oint dS_{CM} \geq \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM} \quad \dots \dots \dots (5.68)$$

Since entropy is a property of the system and for a complete cycle, change in entropy is always zero, i.e.,

$$0 \geq \frac{Q_H - Q_L}{T_H - T_L} \quad \dots \dots \dots (5.69)$$

For a reversible heat engine, Equation (5.69) reduces to

$$0 = \frac{Q_H - Q_L}{T_H - T_L} \quad \dots \dots \dots (5.70)$$

or,

$$\frac{Q_H}{T_H} = \frac{Q_L}{T_L}$$

$$\therefore \frac{Q_L}{Q_H} = \frac{T_L}{T_H} \quad \dots \dots \dots (5.70)$$

Hence, efficiency of a reversible engine which is also called the Carnot efficiency can also be expressed in terms of temperature ratio as

$$\eta_{Carnot} = \eta_{rev} = 1 - \frac{T_L}{T_H} \quad \dots \dots \dots (5.71)$$

Similarly, for an irreversible engine, Equation (5.69) reduces to

$$0 > \frac{Q_H - Q_L}{T_H - T_L} \quad \dots \dots \dots (5.72)$$

Whenever irreversibility in any engine increases work output decrease for the same heat input because of frictional losses and the heat rejection to surroundings. Hence for an irreversible engine, the expression at the right hand side of the Equation (5.7) becomes always negative.

As the work output from a real engine decreases with the increase in irreversibility, efficiency of a real engine is always than that of a reversible engine, i.e.,

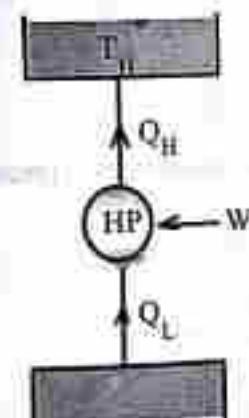
$$\eta_{irrev} < 1 - \frac{T_L}{T_H} \quad \dots \dots \dots (5.73)$$

Therefore, we can combine Equations (5.71) and (5.73) as

$$\eta \leq 1 - \frac{T_L}{T_H} \quad \dots \dots \dots (5.74)$$

Heat Pump

Heat pump is a device, operating on a cyclic process, which takes heat from a low temperature reservoir at T_L (surroundings) and delivers it to a high temperature T_H (desired space) with the help of external work. Heat pump maintains the temperature of a desired space higher than that of the surroundings. Figure 5.12 shows a schematic representation of a heat pump.



Schematic Representation of a Heat Pump

Performance of a heat pump is measured by its coefficient of performance, which is defined as the ratio of desired effect and the work supplied.

$$\text{COP} = \frac{\text{Desired effect}}{\text{Work input}} \quad \dots \dots \dots (5.7)$$

In case of heat pump, desired effect is the amount of heat supplied to the desired space (Q_H), i.e.,

$$(\text{COP})_{HP} = \frac{Q_H}{W} \quad \dots \dots \dots (5.8)$$

Applying first law of thermodynamics for the heat pump,

$$\oint \delta Q = \oint \delta W$$

$$-Q_H + Q_L = -W$$

$$\therefore W = Q_H - Q_L \quad \dots \dots \dots (5.7)$$

Substituting Equation (5.77) into Equation (5.76),

$$(\text{COP})_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}} \quad \dots \dots \dots (5.7)$$

Now applying second law of thermodynamics for the cyclic process,

$$\oint dS_{CM}(=0) \geq \oint \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM}$$

$$0 \geq -\frac{Q_H}{T_H} + \frac{Q_L}{T_L} \quad \dots \dots \dots (5.7)$$

For a reversible heat pump, Equation (5.79) reduces to

$$0 = -\frac{Q_H}{T_H} + \frac{Q_L}{T_L}$$

$$\text{or, } \frac{Q_H}{T_H} = \frac{Q_L}{T_L} \quad \dots \dots \dots (5.8)$$

$$\therefore \frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

which is identical to Equation (5.70).

Substituting Equation (5.80) into Equation (5.78), COP for a reversible heat pump can also be expressed in terms of temperature ratio as

$$(\text{COP})_{rev, HP} = \frac{1}{1 - \frac{T_L}{T_H}} \times \frac{T_H}{T_H - T_L} \quad \dots \dots \dots (5.81)$$

In a reversible heat pump, for the same heating effect, we should supply the work input to overcome the frictional losses and therefore the COP of the heat pump decreases. Hence COP of an irreversible heat pump is always less than that of a reversible heat pump, i.e.,

$$(\text{COP})_{irrev, HP} < \frac{T_H}{T_H - T_L} \quad \dots \dots \dots (5.82)$$

In general, we can combine Equations (5.81) and (5.82) as

$$(\text{COP})_{HP} \leq \frac{T_H}{T_H - T_L} \quad \dots \dots \dots (5.83)$$

Refrigerator (CAQ)

A refrigerator is a device, operating on a cyclic process, which takes heat from a low temperature reservoir at T_L (desired space) and delivers it to a high temperature T_H (surroundings) with the help of external work. It thus maintains the temperature of a desired space lower than that of the surroundings. Figure 5.13 shows a schematic representation of a refrigerator.

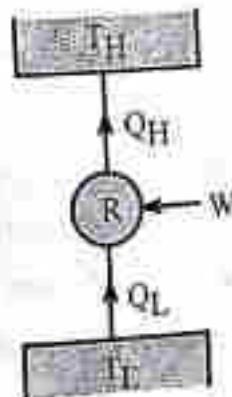


Figure 5.13: Schematic Representation of a Refrigerator

Performance of a refrigerator is also measured by its coefficient of performance, and in case of refrigerator, desired effect is the amount of heat taken out from the desired space (Q_L), i.e.,

$$(COP)_R = \frac{Q_L}{W} \quad \dots \dots \dots (5.84)$$

Going through similar analysis applying first law and second law of thermodynamics as done for a heat pump, we get an expression for the COP of a refrigerator as

$$(COP)_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1} \quad \dots \dots \dots (5.85)$$

Then, COP for a reversible refrigerator can also be expressed in terms of temperature ratio as

$$(COP)_{rev,R} = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L} \quad \dots \dots \dots (5.86)$$

COP of an irreversible refrigerator is always less than that of a reversible refrigerator, i.e.,

$$(COP)_{irrev,R} < \frac{T_L}{T_H - T_L} \quad \dots \dots \dots (5.87)$$

In general, we can combine Equations (5.86) and (5.87) as

$$(COP)_R \leq \frac{T_L}{T_H - T_L} \quad \dots \dots \dots (5.88)$$

Comparison of Equations (5.78) and (5.85) gives

$$COP_{hp} - COP_R = 1 \quad \dots \dots \dots (5.89)$$

5.8.3 Clausius Inequality

With reference to Equations (5.69) and (5.79), we can conclude that for both power producing and power consuming cycles, $\oint \left(\frac{\delta Q}{T} \right)$ is zero if the cycle is reversible and negative if irreversible, i.e. in general

$$\oint \left(\frac{\delta Q}{T} \right) \leq 0 \quad \dots \dots \dots (5.90)$$

The statement is also known as Clausius Inequality. Hence, for any cycle

$\oint \left(\frac{\delta Q}{T} \right) < 0$, then the cycle is irreversible.

$\oint \left(\frac{\delta Q}{T} \right) = 0$, then the cycle is reversible.

$\oint \left(\frac{\delta Q}{T} \right) > 0$, then the cycle is impossible.

5.9 Classical Statements of Second Law of Thermodynamics and their Equivalence

Statement of the second law of thermodynamics is to describe the direction or reversibility of a process. Direction or possibility of a process in nature can be defined without using property entropy. These statements of second law of thermodynamics are called classical statements of second law of thermodynamics.

First statement is popularly known as Kelvin-Planck statement of the second law of thermodynamics, which is stated as

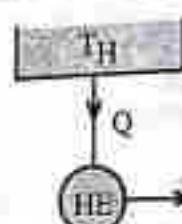
"It is impossible to construct a device to work in a cyclic process which receives heat energy from a single source and converts all the heat supplied to it into an equivalent amount of work."

Other statement of the second law of thermodynamics known as Clausius statement is stated as:

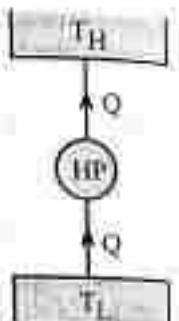
"It is impossible to construct a device to work in a cyclic process which receives heat energy from a body at a lower temperature and rejects heat to a body at a higher temperature."

Both the statements are shown diagrammatically in Figure 5.14. These statements seemed to be different but for any process if Kelvin-Planck statement is satisfied, Clausius statement will also be satisfied. In this way, these two statements are equivalent.

AQ imp



(a) Impossible Device defined by Kelvin-Planck Statement



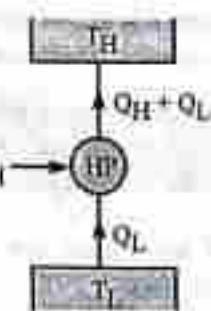
(b) Impossible Device defined by Clausius Statement

Figure 5.14: Models of Classical Statements of Second Law

To prove the equivalence of these statements, consider a system as shown in **Figure 5.15**. It is obvious that, both the heat engine and heat pump obey the first law of thermodynamics but the pump obeys Clausius statement and engine violates Planck statement. If we consider this system as a whole, we find that Q_L units of heat are continuously transferred from the heat sink at T_L to heat source at T_H without any external input to the system, which is against Clausius statement. Thus violation of Kelvin-Planck statement leads to the violation of Clausius statement.



Violation of Kelvin-Planck statement



Confirms to Clausius statement



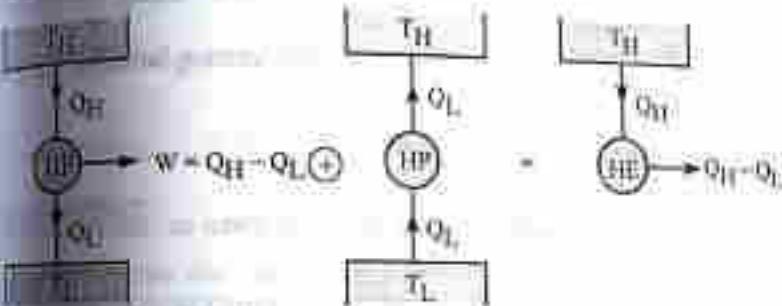
Violation of Clausius statement

Figure 5.15: Violation of Kelvin-Planck statement leads to the violation of Clausius statement

Considering the system shown in **Figure 5.16**, the heat pump violates the Clausius statement while the engine satisfies the Kelvin-Planck statement. But taken together the net effect is that an amount of heat $(Q_H - Q_L)$ is taken from the source at T_H and is being completely

continuously converted into work (as no net exchange takes place with sink) and this is against Kelvin-Planck statement. Thus a violation of Clausius statement leads to the violation of Kelvin-Planck statement.

Therefore it is obvious that the two statements are complementary to each other. The truth of the first implies the truth of the second and vice-versa.



Confirms to Kelvin-Planck statement

Violation of Clausius statement

Violation of Kelvin-Planck statement

ANSWER: Violation of Clausius statement leads to the violation of Kelvin-Planck statement.

SOLVED EXAMPLES

Example 5.1

An inventor makes the following claims. Determine whether the claims are true or not and explain why or why not.

A steam engine operating between temperatures 200°C and 50°C will produce 1.2 kW of power output consuming $0.15 \text{ kg}/\text{h}$ of steam having a calorific value of 42500 kJ/kg .

A heat pump supplies heat to a room maintained at 22°C at a rate of 20000 kJ/h . The inventor claims a work input of 3000 kJ/h is required when the surroundings is at -7°C .

A refrigerator maintains -5°C in the refrigerator which is kept in a room where the temperature is 30°C and has a COP of 8.

Solution

(a) Given,

Higher Temperature: $T_H = 2000 + 273 = 2273 \text{ K}$
 Lower Temperature: $T_L = 500 + 273 = 773 \text{ K}$

Power Output: $\dot{W} = 1.2 \text{ kW}$

Fuel Consumption Rate: $m_f = 0.15 \text{ kg/h}$
 Calorific value of fuel: $CV = 42500 \text{ kJ/kg}$

Maximum possible efficiency of the engine operating between the given temperature limits is given by

$$\eta_{\text{max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{773}{2273} = 65.992 \%$$

Rate at which heat is supplied to the engine is given as

$$Q_H = m_f CV = \frac{0.15}{3600} \times 42500 = 1.7708 \text{ kW}$$

Therefore, efficiency of the engine according to the inventor's claim given as

$$\eta_{\text{inventor}} = \frac{\dot{W}}{Q_H} = \frac{1.2}{1.7708} = 67.765 \%$$

Here, $\eta_{\text{inventor}} > \eta_{\text{max}}$, hence given statement is not valid.

(b) Given,

Higher Temperature: $T_H = 22 + 273 = 295 \text{ K}$ Lower Temperature: $T_L = -2 + 273 = 271 \text{ K}$ Heating Rate: $Q_H = 50000 \text{ kJ/h}$ Power Input: $\dot{W} = 8000 \text{ kJ/h}$

Maximum possible COP of the heat pump operating between the given temperature limits is given by

$$(\text{COP})_{\text{rev}, \text{HP}} = \frac{T_H}{T_H - T_L} = \frac{295}{295 - 271} = 12.29$$

COP of the heat pump according to the inventor's claim is given as

$$(\text{COP})_{\text{inventor}} = \frac{Q_H}{\dot{W}} = \frac{50000}{8000} = 6.25$$

Here, $(\text{COP})_{\text{inventor}} < (\text{COP})_{\text{rev}, \text{HP}}$, hence the given statement is valid if the heat pump is running under the irreversible conditions.

(c) Given,

Higher Temperature: $T_H = 30 + 273 = 303 \text{ K}$
 Lower Temperature: $T_L = -5 + 273 = 268 \text{ K}$
 COP according to the inventor's claim: $(\text{COP})_{\text{inventor}} = 8$

Maximum possible COP of the refrigerator operating between the given temperature limits is given by

$$(\text{COP})_{\text{rev}, R} = \frac{T_L}{T_H - T_L} = \frac{268}{303 - 268} = 7.657$$

$(\text{COP})_{\text{inventor}} > (\text{COP})_{\text{rev}, R}$, hence the given statement is not valid.

Example 5.2

An ideal engine has an efficiency of 25 %. If the source temperature is increased by 200°C its efficiency gets doubled, determine its source and sink temperatures.

Solution

Let the source temperature and the sink temperature of the engine be T_H and T_L . Then its efficiency is given by

$$\eta_1 = 1 - \frac{T_L}{T_H}$$

$$0.25 = 1 - \frac{T_L}{T_H}$$

$$0.75 = \frac{T_L}{T_H}$$

$$T_L = 0.75 T_H \quad \dots \text{(E5.2a)}$$

When the source temperature is increased by 200°C ($= 200\text{K}$), its efficiency gets doubled, i.e.,

$$\eta_2 = 1 - \frac{T_L}{T_H + 200}$$

$$0.5 = 1 - \frac{T_L}{T_H + 200}$$

$$0.5 = \frac{T_L}{T_H + 200} \quad \dots \text{(E5.2b)}$$

Substituting Equation (E5.2 a) into Equation (E5.2b), we get

$$0.5 = \frac{0.75T_H}{T_H + 200}$$

$$\text{or, } 0.5T_H + 100 = 0.75T_H \\ \text{or, } 0.25T_H = 100 \\ \therefore T_H = 400 \text{ K}$$

Substituting T_H into Equation (E5.2 a), we get

$$T_L = 0.75 \times 400 \text{ K} = 300 \text{ K}$$

Example 5.3

A heat pump has a coefficient of performance that is 80% of the theoretical maximum. It maintains a hall at 20°C , which leaks energy kW per degree temperature difference to the ambient. For a maximum 1.5 kW power input, determine the minimum outside temperature for which the heat pump is sufficient.

Solution

Given, Higher Temperature: $T_H = 20 + 273 = 293 \text{ K}$

Heating Rate: $Q_H = 1 \times (T_H - T_L) = (T_H - T_L) \text{ kW}$

Power Input: $\dot{W} = 1.5 \text{ kW}$

Actual COP of the heat pump is 80% of the theoretical maximum (reversible COP), i.e.,

$$(\text{COP})_{\text{actual HP}} = 0.8 (\text{COP})_{\text{rev HP}}$$

$$\text{or, } \frac{Q_H}{\dot{W}} = 0.8 \frac{T_H}{T_H - T_L}$$

$$\text{or, } \frac{T_H - T_L}{1.5} = 0.8 \frac{T_H}{T_H - T_L}$$

$$\text{or, } (T_H - T_L)^2 = 1.2T_H = 1.2 \times 293 = 351.6$$

$$\text{or, } T_H - T_L = 18.751$$

$$\therefore T_L = T_H - 18.751 = 274.249 \text{ K} = 1.249^\circ\text{C}$$

Example 5.4

4 kg of water at 25°C is mixed with 1 kg of ice at 0°C in an adiabatic system. Calculate the change in entropy due to mixing process.

Latent heat of ice $L = 336 \text{ kJ/kg}$ and specific heat of water $c = 4.18 \text{ kJ/kg}\cdot\text{K}$

Solution

Mass of water: $m_w = 4 \text{ kg}$

Initial Temperature of water: $T_{w1} = 25 + 273 = 298 \text{ K}$

Mass of ice: $m_i = 1 \text{ kg}$

Initial Temperature of ice: $T_{i1} = 0 + 273 = 273 \text{ K}$

If T_2 be the final equilibrium temperature, then the heat lost by water is absorbed by the ice, i.e.,

$$\begin{aligned} m_w c (T_{w1} - T_2) &= m_i L + m_i c (T_2 - 273) \\ 4.18 \times (298 - T_2) &= 1 \times 336 + 1 \times 4.18 \times (T_2 - 273) \\ 4892.56 - 16.72 T_2 &= 336 + 4.18 T_2 - 1141.14 \\ 20.9 T_2 &= 5787.7 \\ T_2 &= 276.9234 \text{ K} \end{aligned}$$

The change in entropy of the water is given by

$$(AS)_w = m_w c \ln \left(\frac{T_2}{T_{w1}} \right) = 4 \times 4.18 \times \ln \left(\frac{276.9234}{298} \right) = -1.22645 \text{ kJ/K}$$

The change in entropy of the ice is given by the summation of change in entropy of the ice during melting of ice and the change entropy of water when its temperature increases from 273 K to T_2 , i.e.,

$$\begin{aligned} (\Delta S)_i &= \frac{m_i L}{273} + m_i c \ln \left(\frac{T_2}{273} \right) \\ &= \frac{1 \times 336}{273} + 1 \times 4.18 \times \ln \left(\frac{276.9234}{273} \right) = 1.29041 \text{ kJ/K} \end{aligned}$$

The change in entropy due to the mixing process is then given by

$$(AS)_w + (\Delta S)_i = -1.22645 + 1.29041 = 0.06396 \text{ kJ/K}$$

Example 5.5

Steam at 1 MPa and 300°C is flowing with a velocity of 50 m/s adiabatically through a nozzle and leaves the nozzle at 10°C . Determine the exit velocity of steam from the nozzle.

Solution

Given: Properties of steam at inlet: $P_1 = 1000 \text{ kPa}$, $T_1 = 300^\circ\text{C}$, $\bar{V}_1 = 50 \text{ m/s}$

Properties of steam at outlet: $P_2 = 150 \text{ kPa}$

Process: Reversible and adiabatic (isentropic)

For the other properties of steam at inlet, referring to Table A2.4, $h_1 = 3050.6 \text{ kJ/kg}$ and $s_1 = 7.1219 \text{ kJ/kgK}$.

Since entropy remains constant during isentropic process, entropy at the nozzle exit is $s_2 = 7.1219 \text{ kJ/kgK}$.

Referring to Table A2.1, s_1 (150 kPa) = 1.4338 kJ/kgK, s_{1g} (150 kPa) = 5.7894 kJ/kgK, s_1 (150 kPa) = 7.2232 kJ/kgK and h_1 (150 kPa) = 467.18 kJ/kg, h_{1g} (150 kPa) = 2226.2 kJ/kg.

Here, $s_1 < s_2 < s_{1g}$, therefore condition of the steam at the nozzle exit is a two phase mixture. Therefore, quality of the two phase mixture is given by

$$x_2 = \frac{s_2 - s_1}{s_{1g} - s_1} = \frac{7.1219 - 1.4338}{5.7894} = 0.9823$$

Therefore, specific volume of steam at the nozzle exit is given by

$$h_2 = h_1 + x_2 h_{1g} = 467.18 + 0.9823 \times 2226.2 = 2654.427 \text{ kJ/kg}$$

Now applying energy equation for an adiabatic nozzle,

$$h_1 + \frac{1}{2} \bar{V}_1^2 = h_2 + \frac{1}{2} \bar{V}_2^2$$

Rearranging the above equation for \bar{V}_2 ,

$$\begin{aligned} \bar{V}_2 &= \sqrt{2(h_1 - h_2) + \bar{V}_1^2} \\ &= \sqrt{2000(3050.6 - 2654.427) + (50)^2} = 891.54 \text{ m/s} \end{aligned}$$

Example 5.6

Steam enters into a turbine at 2 MPa, 400°C and with a velocity of 200 m/s and saturated vapor exits from the turbine at 100 kPa with a velocity of 80 m/s. The power output of the turbine is 800 kW when the

mass flow rate of steam is 1.5 kg/s. Turbine rejects heat to the surroundings at 300 K. Determine the rate at which the entropy is generated within the turbine.

Solution

Given: Properties of steam at inlet: $P_1 = 2000 \text{ kPa}$, $T_1 = 400^\circ\text{C}$, $\bar{V}_1 = 200 \text{ m/s}$

Properties of steam at outlet: $P_2 = 100 \text{ kPa}$, saturated vapor, $\bar{V}_2 = 80 \text{ m/s}$

Mass flow rate of steam: $m = 1.5 \text{ kg/s}$

Power output of the turbine, $W_{CV} = 800 \text{ kW}$

Temperature of the surroundings: $T_{sw} = 300 \text{ K}$

For the other properties of steam at inlet, referring to Table A2.4, $h_1 = 3247.5 \text{ kJ/kg}$ and $s_1 = 7.1269 \text{ kJ/kgK}$.

Similarly, for the other properties of steam at outlet, referring to Table A2.1, $h_2 = h_g$ (100 kPa) = 2675.1 kJ/kg and $s_2 = s_g$ (100 kPa) = 7.3589 kJ/kgK.

Now applying energy equation for the turbine,

$$Q_{CV} - W_{CV} = m \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right]$$

Rearrange the above equation for Q_{CV} ,

$$Q_{CV} = W_{CV} + m \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right]$$

$$\begin{aligned} &= 800 + 1.5 \times \left[(2675.1 - 3247.5) + \frac{1}{2000} (80^2 - 200^2) \right] \\ &= -10.8 \text{ kW} \end{aligned}$$

The rate of entropy generation during steady operation of any turbine is given by

$$\dot{S}_{gen} = \dot{S}_{int} - \dot{S}_{in} - \sum \left(\frac{\dot{Q}_i}{T_i} \right)_{CV} = m(s_2 - s_1) - \frac{Q_{CV}}{T_{sw}}$$

$$= 1.5 (7.3589 - 7.1269 - \frac{(-83.8)}{300}) = 0.6273 \text{ kW/K}$$

Example 5.57

Steam enters into a turbine at a rate of 1.5 kg/s with $P_1 = 2 \text{ MPa}$, $T_1 = 750^\circ\text{C}$ and exits at $P_2 = 10 \text{ kPa}$.

- If the turbine is isentropic, what is the power output of the turbine?
- If the isentropic efficiency of the turbine is 80%, what is the power output?
- What is the outlet enthalpy of the steam from the real turbine?

Solution

Given, Properties of steam at Inlet: $P_1 = 2000 \text{ kPa}$, $T_1 = 750^\circ\text{C}$

Properties of steam at outlet: $P_2 = 10 \text{ kPa}$

- For the other properties of steam at inlet, referring to Table A.2.4, $h_1 = 4033.5 \text{ kJ/kg}$ and $s_1 = 8.0651 \text{ kJ/kgK}$.

Since entropy remains constant during isentropic process, entropy at the turbine exit is $s_2 = 8.0651 \text{ kJ/kgK}$.

Referring to Table A2.1, $s_1 (10 \text{ kPa}) = 0.6493 \text{ kJ/kgK}$, $s_{fg} (10 \text{ kPa}) = 7.4989 \text{ kJ/kgK}$, $s_1 (10 \text{ kPa}) = 8.1482 \text{ kJ/kgK}$ and $h_1 (10 \text{ kPa}) = 191.83 \text{ kJ/kg}$, $h_{fg} (10 \text{ kPa}) = 2392.0 \text{ kJ/kg}$.

Here, $s_1 < s_2 < s_f$, therefore condition of the steam at the nozzle exit is two phase mixture. Therefore, quality of the two phase mixture is given by

$$x_2 = \frac{s_2 - s_1}{s_{fg}} = \frac{8.0651 - 0.6493}{7.4989} = 0.9889$$

Therefore, specific volume of steam at the nozzle exit is given by

$$b_2 = b_1 + x_2 b_{fg} = 191.83 + 0.9889 \times 2392.0 = 2557.323 \text{ kJ/kg}$$

Now applying energy equation for an isentropic turbine,

$$W_{\text{isent}} = m(h_1 - h_2) = 1.5 \times (4033.5 - 2557.323) = 2214.266 \text{ kW}$$

Isentropic efficiency of the turbine is given by

$$\eta_{IT} = \frac{\dot{W}_{\text{real}}}{\dot{W}_{\text{isent}}}$$

Therefore, power output from the real turbine is given by

$$\dot{W}_{\text{real}} = \eta_{IT} \times \dot{W}_{\text{isent}} = 0.8 \times 2214.266 = 1771.413 \text{ kW}$$

Power output from the real turbine can also be given as

$$W_{\text{real}} = m(h_1 - h_2)$$

Therefore, power output from the real turbine is given by

$$h_2 = h_1 - \frac{W_{\text{real}}}{m} = 4033.5 - \frac{1771.413}{1.5} = 2852.56 \text{ kJ/kg}$$

OBJECTIVE QUESTIONS

Select the correct answer from the given alternatives.

- Second law of thermodynamics does not deal with the
 - (a) direction of process
 - (b) quality or nature of energy
 - (c) entropy of the system
 - (d) total energy of the system
- Second law mainly deals with
 - (a) internal energy
 - (b) enthalpy
 - (c) entropy
 - (d) temperature
- Second law of thermodynamics gives the expression for change in
 - (a) entropy
 - (b) enthalpy
 - (c) system
 - (d) surroundings
- (a) total energy
 - (b) enthalpy
 - (c) entropy
 - (d) pressure
- Which of the following variables is a path function?
 - (a) entropy generation
 - (b) pressure
 - (c) internal energy
 - (d) enthalpy

NUMERICAL PROBLEMS

1. During an experiment a student claims that based on his measurements, a heat engine receives 300 kJ from a source at 500 K, converts 160 kJ of it into work and rejects heat to the sink at 300 K. Are these data reasonable? Why?
2. A heat engine receives 400 kJ from a source at a temperature of 1000 K. It rejects 150 kJ of heat to sink at a temperature of 300 K. Is this a reversible cycle, an irreversible cycle or an impossible cycle?
3. An inventor claims that a heat pump can maintain a room $\approx 20^\circ\text{C}$ when the surroundings is at 0°C . The heat loss from the room occurs at a rate of 1000 kJ/min and the heat pump requires 1 kW of power input. Evaluate his claim.
4. During an experiment conducted in a room at 27°C , a student measures that a refrigerator consumes 2 kW of power and removes 56000 kJ of heat from the desired space at -23°C . The running time for the refrigerator during the experiment was 30 min. Are these data reasonable? Why?
5. A power cycle operating between two reservoirs receives Q_H from a high temperature source at $T_H = 1000\text{ K}$ and rejects energy Q_L to a low temperature sink at $T_L = 300\text{ K}$. For each of the following case, determine whether the cycle operates reversibly, irreversibly or is impossible.
 - (a) $Q_H = 800\text{ kJ}$, $W = 600\text{ kJ}$,
 - (b) $Q_H = 800\text{ kJ}$, $Q_L = 240\text{ kJ}$,
 - (c) $W = 690\text{ kJ}$, $Q_L = 640\text{ kJ}$,
 - (d) $\eta = 50\%$
6. A heat pump cycle operating between two reservoirs takes energy from the source at $T_L = 270\text{ K}$ and supplies Q_H to a room at $T_R = 300\text{ K}$. For each of the following case, determine whether the cycle operates reversibly, irreversibly or impossible.
 - (a) $Q_H = 1000\text{ kJ}$, $Q_L = 200\text{ kJ}$,
 - (b) $Q_H = 2000\text{ kJ}$, $Q_L = 1800\text{ kJ}$,
 - (c) $W = 200\text{ kJ}$, $Q_L = 2000\text{ kJ}$
 - (d) COP = 8.6
7. Find the efficiency, power output and heat rejection rate for a heat engine operating on a Carnot cycle which receives heat at a rate of 6 kW at 327°C and rejects heat to 27°C .

1. An ideal heat engine has the same efficiency for source and sink at 500 K and 400 K respectively as for source and sink at T_H and 800 K. Determine T_H in K.

A heat engine operates between a high temperature source T_H and a low temperature sink at 300 K. The engine develops 60 kW of power and rejects heat to the sink at the rate of 72 MJ/h. Determine the minimum theoretical value for T_H in K.

3. A heat engine takes heat at a rate of 1200 kW from a high temperature source at 600°C rejects heat to the ambient at 25°C . Power output from the engine is 700 kW. Determine the engine efficiency and the energy rejected to the ambient. Compare both of these for a Carnot engine operating between the same temperature limits.

4. A engine burns 0.5 kg. of a fuel at 1800 K and rejects energy at an average temperature of 600 K. If the calorific value of the fuel is 42000 kJ/kg, determine the amount of work output that the engine can provide.

5. A heat engine having a thermal efficiency of 40% produces 40 kW of power output. Determine the fuel consumption rate in kg/h, if the calorific value of the fuel is 42000 kJ/kg.

6. A heat engine consumes fuel at a rate of 30 L/h and delivers 80 kW of power output. If the calorific value of the fuel is 42000 kJ/kg and density of 0.8 g/cm^3 , determine the efficiency of the engine.

7. A heat engine has a solar collector receiving 0.25 kW/m^2 , and provides a high temperature source at 400 K. The heat engine rejects heat to the ambient at 30°C . If the required power output is 2 kW, what is the minimum size of the solar collector?

8. An ideal engine can develop 27 kW power output while rejecting 15 kJ of heat per cycle. The engine operates between $T_H = 1200\text{ K}$ and $T_L = 300\text{ K}$. Determine the minimum theoretical number of cycles per minute.

9. An ideal engine have a efficiency of 25%. If the sink temperature is reduced by 100°C , its efficiency gets doubled, determine its source and sink temperatures.

10. The difference between source and sink temperatures of an ideal engine is 450°C . If the work output of the engine is 1.5 times

- the heat rejected, determine its thermal efficiency, temperature and sink temperature.
18. An ideal heat pump requires a power input of 2.5 kW and maintains the temperature of a room at 25°C which loses heat at a rate of 20 kW to the colder ambient. What is the coefficient of performance of the heat pump? What is the surroundings temperature?
19. A building is maintained at 23°C by a heat pump when the surrounding temperature drops to -7°C. The rate at which heat is lost from the building is found to be 30 kW. What is the minimum power required to drive the heat pump?
20. A heat pump having a COP of 5 maintains a building at a temperature of 24°C by supplying heat at a rate of 72000 kJ/h when the surroundings is at 0°C. The heat pump runs 12 hours in a day and the electricity costs Rs 10/kWh.
- Determine the actual and minimum theoretical cost per day.
 - Compare the actual operating cost with the cost of direct electric resistance heating.
21. A building is maintained at a temperature of 25°C by a heat pump having a coefficient of performance of 2.5. It loses heat at a rate of 1 kW per degree temperature difference between the inside and outside. If the outside temperature is -10°C, determine the power required to drive the heat pump.
22. A heat pump having a coefficient of performance 50% of the theoretical maximum maintains a house at a temperature of 20°C. The heat leakage from the house occurs at a rate of 0.8 kW per degree temperature difference. For a maximum power input of 1.5 kW, determine the minimum surroundings temperature for which the heat pump will be sufficient?
23. A heat pump maintains a room at a temperature of 20°C when the surroundings is at 5°C. The rate of heat loss from the room is estimated to be 0.6 kW per degree temperature difference between the inside and outside. If the electricity costs Rs 10/kWh, determine the minimum theoretical cost per day.
24. An air conditioning unit rejects 5 kW to the ambient surroundings and requires a power input of 1.2 kW. Determine the rate of cooling and the coefficient of performance.
25. An air conditioner operates in a room at 22°C. Heat must be taken from the desired space at a rate of 2.5 kW to maintain its temperature at -7°C. What is the minimum power required to drive the refrigerator?
26. A refrigerator takes heat from a desired space maintained at -5°C at a rate of 100 kJ/min and rejects heat to the surroundings at 25°C. If the coefficient of performance of the refrigerator is 50% of that of a reversible refrigerator cycle operating between the same temperatures. Determine the power required to drive the cycle.
27. An air conditioning unit having a COP of 4 maintains a house at 20°C on a day when the outside temperature is 35°C. The thermal load consists of heat energy entering through the walls at a rate of 600 kWh and from the occupants, computers and lighting at a rate of 120 kWh/min. Determine the power required to drive the unit and compare it with the minimum theoretical power required.
28. A refrigerator having a COP of 4 maintains the freezer compartment at -3°C by removing heat at a rate of 1000 kJ/h and rejects heat to the surroundings at 27°C.
- Determine the power input to the refrigerator and compare it with minimum theoretical power input.
 - If the electricity costs Rs 10/kWh, determine the actual and minimum theoretical cost per day for effective operation of 12 hours.
29. An air conditioning unit having a COP 50% of the theoretical maximum maintains a house at a temperature of 20°C by cooling it to the surroundings temperature. The house gains energy at a rate of 0.8 kW per degree temperature difference. For a maximum power input of 1.8 kW, determine the maximum surroundings temperature for which it provides sufficient cooling.
30. An air conditioning unit heats a house in the winter and then cools it to cool it in the summer. The room temperature should be 20°C in the winter and 26°C in the summer. Heat transfer through walls and ceilings is estimated to be 3000 kWh per degree temperature difference between the inside and outside.
- Determine the power required to run it in the winter when the outside temperature decrease to 0°C.
 - If the unit is run by the same power as calculated in (a) throughout the year, determine the maximum outside temperature for which the house can be maintained at 20°C.

31. An air conditioning unit with a power input of 1.5 kW works as a cooling unit in summer and works as heating unit in winter. It maintains a hall at 20°C year around, which exchanges heat at a rate of 0.8 kW per degree temperature difference with the surroundings. Determine the maximum and the minimum outside temperature for which this unit is sufficient.
32. A rigid vessel consists of 0.4 kg of hydrogen initially at 200 kPa and 27°C . Heat is transferred to the system from a reservoir at 600 K until its temperature reaches 450 K. Determine the heat transfer, the change in entropy of hydrogen and the amount of entropy produced [Take $c_v = 10.183 \text{ J/kgK}$]
33. A piston cylinder device shown in Figure P5.33 contains 2 kg of water at 2 MPa and 300°C . Heat is added from the source at 800 K to the water until its temperature reaches 800°C . Determine the total entropy generated during the process.
34. A piston cylinder device shown in Figure P5.34 contains 1 kg of water at saturated vapor state 500 kPa. It is cooled so that its volume reduces to half of the initial volume because of heat transfer to the surroundings at 20°C . Determine the total entropy generated during the process.
35. A piston cylinder device loaded with a linear spring as shown in Figure P5.35 contains 0.5 kg of water at 100 kPa and 25°C . Heat is transferred from a source at 750°C until water reaches to a final state of at 100 kPa and 600°C . Determine the total entropy generated during the process.
36. A piston cylinder device shown in Figure P5.36 contains 1.5 kg of water initially at 100 kPa with 10% of quality. The mass of the piston is such that a pressure of 500 kPa is required to lift the piston. Heat is added to the system from a source at 500°C until its temperature reaches 400°C . Determine the total entropy generation during the process.
37. Water is contained in a piston cylinder device with two set of stops as shown in Figure P5.37 is initially at 1 MPa and 400°C . The limiting volumes are $V_{\min} = 1 \text{ m}^3$ and $V_{\max} = 2 \text{ m}^3$. The weight of the piston is such that a pressure of 400 kPa is required to support the piston. The system is cooled to 100°C by allowing system to reject heat to the surroundings at 25°C . Sketch the process on $P-v$ and $T-v$ diagrams and determine the total entropy generated during the process.

 $T-v$ diagrams

Figure P5.34

process



Figure P5.35

process



Figure P5.36

process



Figure P5.37

38. 2 kg water at 100°C is mixed with 4 kg of water at 20°C in an isolated system. Calculate the net change in entropy due to the mixing process. [Take specific heat of water $c = 4.18 \text{ kJ/kgK}$]

39. Block A ($m_A = 0.5 \text{ kg}$, $c_A = 1 \text{ kJ/kgK}$) and block B ($m_B = 1 \text{ kg}$, $c_B = 0.5 \text{ kJ/kgK}$) which are initially at 100°C and 500°C respectively are brought in contact inside an isolated system. Determine the change in entropy when they reach to a final state of thermal equilibrium.

40. A lump of steel ($c_s = 0.5 \text{ kJ/kgK}$) of mass 10 kg at 727°C is dropped in 100 kg of oil ($c_o = 3.5 \text{ kJ/kgK}$) at 27°C . Determine the net change in entropy.

41. 1 kg of air enclosed in an isolated box with volume V_1 , pressure P_1 and temperature T_1 is allowed to expand freely until its volume increases to $V_2 = 2V_1$. Determine the change in entropy. [Take $R = 287 \text{ J/kgK}$]

42. A rigid cylinder contains nitrogen initially at 100 kPa, 300 K , and 0.005 m^3 . It is heated reversibly until its temperature reaches 400 K . Determine the entropy change of the nitrogen during the process. [Take $R = 297 \text{ J/kgK}$, $c_v = 743 \text{ J/kgK}$]

43. 1 kg of air initially at 150 kPa and 27°C is heated reversibly at constant pressure to 227°C . Determine the entropy change of the air during the process. [Take $c_p = 1005 \text{ J/kgK}$]

44. 1 kg of air initially at 400 kPa and 500 K expand polytropically ($PV^{\gamma} = \text{constant}$) until its pressure reduces to 100 kPa. Determine the entropy change of the nitrogen during the process. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$.]
45. 0.5 m³ of air at 600 kPa and 500 K expands reversibly to 100 kPa. Determine the change in entropy when it undergoes the following processes:
- $PV = \text{constant}$
 - $PV^3 = \text{constant}$
 - Adiabatic process [Take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$.]
46. Air at a pressure of 100 kPa and 27°C is compressed by an air compressor to a pressure of 1500 kPa. Determine the work required per kg of air for the compressor assuming process to be reversible and adiabatic. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$.]
47. Steam at 2.5 MPa and 500°C and with a velocity of 100 m/s enters into an well insulated turbine and exits at 100 kPa and with a velocity of 150 m/s. The work developed per kg of steam is claimed to be
- 650 kJ/kg
 - 680 kJ/kg
- Evaluate each claim.
48. Steam at 5 MPa and 400°C enters into a turbine at a rate of 2 kg/s and exit at a pressure of 400 kPa. Assuming the process to be reversible and adiabatic, determine the power output.
49. Steam enters a turbine at 1.5 MPa and 300°C and with a velocity of 60 m/s, expands in a reversible adiabatic process and exits at 20 kPa with a velocity of 150 m/s. Determine the specific work output.
50. Steam enters into a turbine at 2 MPa and 300°C and exits at 20 kPa. If the power output of the turbine is 1 MW, determine the mass flow rate of steam. Assume reversible adiabatic process.
51. Steam enters a nozzle at 1.5 MPa and 300°C and with a velocity of 50 m/s, undergoes a reversible adiabatic process and exits at 20 kPa. Determine the exit velocity.

52. A compressor receives air at 100 kPa and 27°C and requires a power input of 60 kW. If the mass flow rate of the air is 0.1 kg/s, determine the maximum exit pressure of the compressor. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$.]
53. Air at 100 kPa and 25°C enters into a diffuser at a velocity of 150 m/s and exits with a velocity 40 m/s. Assuming the process to be reversible and adiabatic, determine the exit pressure and temperature of the air. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$.]
54. Air at 200 kPa and 1000 K with very low velocity enters into a nozzle and exits at a pressure of 100 kPa. Assuming the process to be isentropic, determine the exit velocity. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$.]
55. Air enters into an insulated turbine at 500 kPa and 527°C and exits at 100 kPa and 267°C. Determine the work developed per kg of air and whether the process is internally reversible, irreversible or impossible. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$.]
56. Determine whether it is possible to compress air adiabatically from 100 kPa and 27°C to
- 50 kPa, 150°C and
 - 400 kPa, 200°C [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$.]
57. Air is compressed isothermally from 100 kPa and 27°C to 1000 kPa by applying 175 kJ/kg of work. Determine whether it is a reversible, irreversible or an impossible process. [Take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$.]
58. Air enters an adiabatic nozzle at 4 MPa, 400°C and with a velocity of 50 m/s and exits at 2 MPa and with a velocity of 300 m/s. If the nozzle has an inlet area of 8 cm², determine
- exit temperature of steam from the nozzle, and
 - the rate of entropy generation for the process.
59. Air enters a compressor operating steadily at 100 kPa, 27°C and with a volumetric flow rate of 1.2 m³/min and exits at 400 kPa, 177°C. The power required to drive the compressor is 3.6 kW. Determine

- (a) the heat transfer rate from the compressor surface and
 (b) the rate of entropy generation if heat is transferred to the surroundings at 20°C. [Take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$]
60. Air enters a nozzle operating steadily at 2 MPa, 327°C and with a velocity of 50 m/s and exits at 100 kPa, 27°C and with a velocity of 500 m/s. Determine
 (a) the heat loss per kg of air from the nozzle surface and
 (b) the rate of entropy generation per kg of air if heat is transferred to the surroundings at 20°C. [Take $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$]
61. Steam enters into a well insulated throttling valve at 10 MPa and 600°C and exits at 5 MPa. Determine the change in entropy per unit mass of the steam.
62. Air at 1 MPa and 327°C is throttled to a pressure of 100 kPa. Determine the change in entropy per unit mass of the air. [Take $R = 287 \text{ J/kgK}$]
63. Steam enters a turbine at 2 MPa and 300°C and exits at 20 kPa. If the specific work output from the turbine is 650 kJ/kg of steam, determine the isentropic efficiency of the turbine.
64. Steam enters an adiabatic turbine at 5 MPa, 500°C, and with a velocity of 50 m/s and exits at 50 kPa, 100°C and with a velocity of 150 m/s. If the power output of the turbine is 5 MW, determine
 (a) the mass flow rate of steam flowing through the turbine, and
 (b) the isentropic efficiency of the turbine.
65. Air enters a gas turbine at 1 MPa and 1500 K and exits at 100 kPa. If its isentropic efficiency is 80 %, determine the turbine inlet temperature. [Take $\gamma = 1.4$]

THERMODYNAMIC CYCLES

6

6.1 Introduction

Most of the devices which are used for energy conversion operate on cyclic processes. These devices are used either to produce work or heat by supplying heat from the combustion of fuel or to provide heating or cooling effect by supplying work input usually through electrical power. In this chapter, we study the working principles of different practical cycles and analyze their performances.

6.1.1 Classification of Practical Cycles

Practical cycles are classified according to different basis. Some of these bases are explained below.

(a) According to Power (Work)

According to power, cycles are classified as power cycle and refrigeration cycle.

Power Cycle: A cycle is said to be a power cycle, if it produces heat or delivers work to the surroundings when the cycle is executed. Power cycle operates the heat engine. Hence for a power cycle, net work is always positive. Common examples of power cycles are Brayton cycle, Otto cycle, Diesel cycle, Rankine cycle, etc.

Refrigeration Cycle: A cycle is said to be a refrigeration cycle, if heat should be supplied to execute the cycle. Refrigeration cycle includes the heat pump and refrigeration. Hence for a refrigeration cycle, net work is always negative. Most common example of refrigeration cycle is a vapor compression refrigeration cycle.

(b) According to Working Substance

According to working substance, cycles are classified as vapor compression cycle and vapor cycle.

Gas Cycle: A cycle is said to be a gas cycle, if the working substance remains in gaseous state throughout the cycle. Common examples of gas power cycles are Brayton cycle, Otto cycle, Diesel cycle, etc.

Vapor cycle: A cycle is said to be vapor cycle, if the working substance undergoes phase change (from liquid to vapor or vapor to liquid) during the cycle. Most common example of vapor power cycle is Rankine cycle.

(c) According to Combustion Location

This classification is applicable only for power cycles. According to combustion location, cycles are classified as internal combustion cycle and external combustion cycle.

Internal Combustion Cycle: A cycle is said to be an internal combustion cycle, if the combustion takes place inside any one component of the system. In internal combustion cycles, the working substance and fuel come in contact with each other. Examples of internal combustion cycles are Otto cycle and Diesel cycle.

External Combustion Cycle: A cycle is said to be an external combustion cycle, if the combustion takes place outside the components of the system. In external combustion cycles, the working substance and fuel do not come in contact with each other. Examples of external combustion cycles are Brayton cycle and Rankine cycle.

6.1.2 Common Features of Power Cycles

All practical cycles undergo the following processes in sequence: compression, heat addition, expansion and heat rejection. For ideal analysis of all power cycles, expansion and compression processes are assumed to be isentropic processes. Hence the practical power cycles can be differentiated only with reference to the conditions at which heat is supplied to the system and heat is rejected by the system. The table given below gives the comparison between different cycles with reference to constituent processes.

Table 6.1: Comparison between Different Power Cycles

Process/Cycle	Ideal Cycle	Gas cycles			Vapor Cycle
	Carnot	Otto	Diesel	Brayton	Rankine
Compression	Isentropic	Isentropic	Isentropic	Isentropic	Isentropic
Heat Addition	Isothermal	Isochoric	Isochoric	Isochoric	Isochoric
Expansion	Isentropic	Isentropic	Isentropic	Isentropic	Isentropic
Heat Rejection	Isothermal	Isochoric	Isochoric	Isochoric	Isochoric

6.2 Brayton Cycle

Brayton cycle is an ideal model for the power cycle consisting of a gas turbine. It consists of two isentropic and two isobaric processes. The major components (devices) are shown in Figure 6.1.

Process 1 – 2 (Isentropic Compression)

Low pressure and low temperature air at the low temperature heat exchanger exit is supplied to the compressor where it is compressed and delivered to the high temperature heat exchanger. Process occurring in the compressor is assumed to be an isentropic process. Since, during isentropic compression, pressure of the working substance increases and its specific volume decreases as shown in Figure 6.2 (a); while entropy of the system remains constant and its temperature increases as shown in Figure 6.2 (b).

Process 2 – 3 (Isobaric Heat Addition)

Heat is supplied to the high temperature heat exchanger from an external source and the working substance is heated under constant pressure. Hence, during the constant pressure heating, specific volume, temperature and entropy of the air increase as shown in Figure 6.2 (a) and Figure 6.2 (b).

Process 3 – 4 (Isentropic Expansion)

High temperature and high pressure air leaving the high temperature heat exchanger is supplied to the turbine. The turbine produces work by converting energy carried by the air at the turbine inlet. The process occurring in the turbine is assumed to be an isentropic process.

expansion. Hence, during an isentropic expansion, pressure of the air decreases and its specific volume further increases as shown in Figure 6.2 (a) whereas entropy of the system remains constant and its temperature decreases as shown in Figure 6.2 (b).

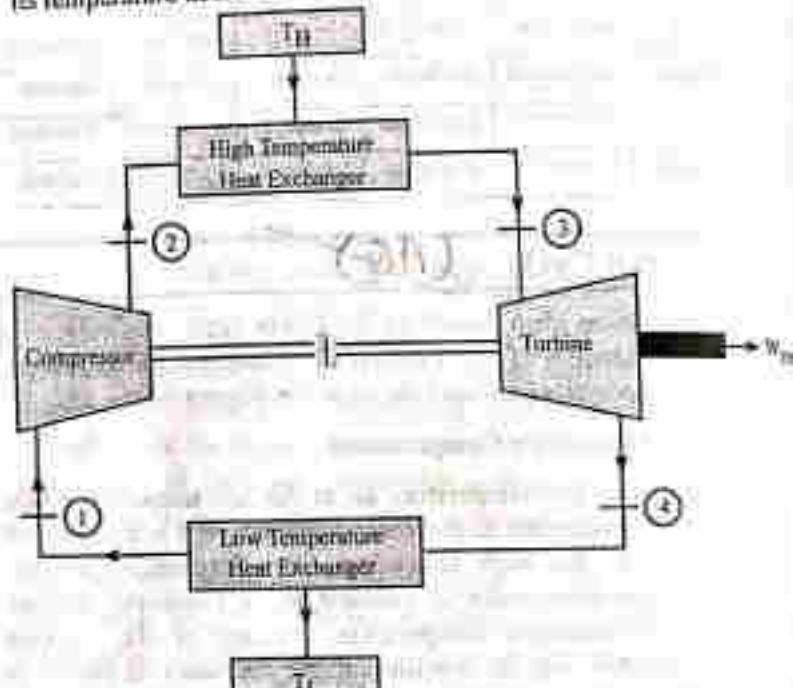


Figure 6.1 Brayton Cycle

Process 4 – 1 (Isobaric Heat Rejection)

Exit air from the turbine is delivered to the low temperature heat exchanger where it rejects heat to the low temperature sink such that its initial state is restored. Heat rejection process occurs at constant pressure during which, specific volume, temperature and entropy of the air decrease as shown in Figure 6.2 (a) and Figure 6.2 (b).

Efficiency of Brayton Cycle

Work produced by the turbine per kg of air is given by

$$w_t = w_{34} = h_3 - h_4 = c_p (T_3 - T_4) \quad \dots \dots \dots (6.1)$$

Work consumed by the compressor per kg of air is given by

$$w_c = w_{12} = h_2 - h_1 = c_p (T_2 - T_1) \quad \dots \dots \dots (6.2)$$

During steady state operation of the cycle, some part of the work produced by the turbine is utilized to run the compressor. Hence, the work delivered to the surroundings is given by

$$w_{\text{out}} = w_t - w_c = c_p [(T_3 - T_4) - (T_2 - T_1)] \quad \dots \dots \dots (6.3)$$

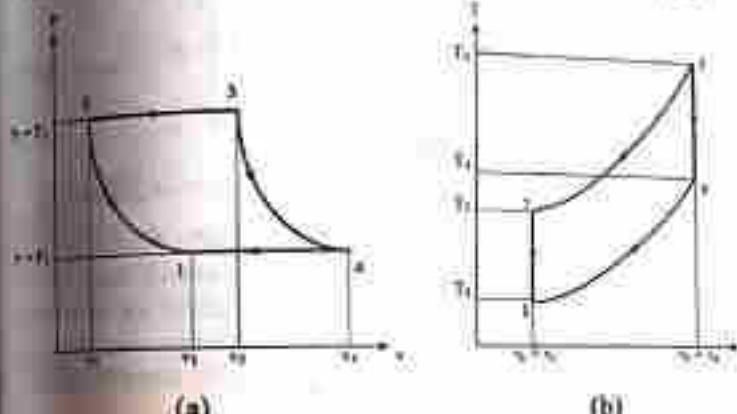


Figure 6.2 P – v and T – s Diagrams for Brayton Cycle

Heat supplied to the air in the high temperature heat exchanger is given by

$$q_{hi} = q_{32} = h_3 - h_2 = c_p (T_3 - T_2) \quad \dots \dots \dots (6.4)$$

Efficiency of the Brayton cycle is then given by

$$\eta_B = \frac{c_p [T_3 - T_4] - [T_2 - T_1]}{c_p (T_3 - T_2)} = \frac{[(T_3 - T_4) - (T_2 - T_1)]}{(T_3 - T_2)} \quad \dots \dots \dots (6.5)$$

Equation (6.5) can also be expressed as

$$\eta_B = \frac{T_4 - T_1}{T_3 - T_2} \quad \dots \dots \dots (6.6)$$

Equation (6.6) requires temperatures of each state to determine the efficiency of the cycle. We can further simplify it to reduce the number of required variables. For this, applying pressure – temperature relationship for isentropic compression process 1–3 and isentropic expansion process 3–4,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{1-\gamma}{\gamma}} \quad \dots\dots\dots (6.7)$$

$$\frac{T_2}{T_4} = \left(\frac{P_2}{P_4} \right)^{\frac{1-\gamma}{\gamma}} \quad \dots\dots\dots (6.8)$$

Substituting $P_2 = P_1$ and $P_4 = P_1$, into Equation (6.8), we get

$$\frac{T_2}{T_4} = \left(\frac{P_2}{P_1} \right)^{\frac{1-\gamma}{\gamma}} \quad \dots\dots\dots (6.9)$$

Equating Equations (6.7) and (6.9),

$$\frac{T_2}{T_1} = \frac{T_1}{T_4} \quad \text{or,} \quad \frac{T_4}{T_2} = \frac{T_1}{T_1} \quad \dots\dots\dots (6.10)$$

Now using componendo and dividendo,

$$\frac{T_4}{T_3} = \frac{T_1}{T_2} = \frac{T_4 - T_1}{T_3 - T_2} \quad \dots\dots\dots (6.11)$$

Substituting Equation (6.11) into Equation (6.6), we get

$$\eta = 1 - \frac{T_4}{T_3} = 1 - \frac{T_1}{T_2} \quad \dots\dots\dots (6.12)$$

Substituting Equations (6.7) and (6.9) into Equation (6.12), we can express efficiency in terms of pressure ratio as

$$\eta = 1 - \left(\frac{P_1}{P_2} \right)^{\frac{1-\gamma}{\gamma}} = 1 - \left(\frac{1}{r_p} \right)^{\frac{1-\gamma}{\gamma}} \quad \dots\dots\dots (6.13)$$

where $r_p = \frac{P_2}{P_1}$ is called pressure ratio.

Equation (6.13) shows that efficiency of the Brayton cycle increases with the increase in pressure ratio as shown in Figure 6.3.

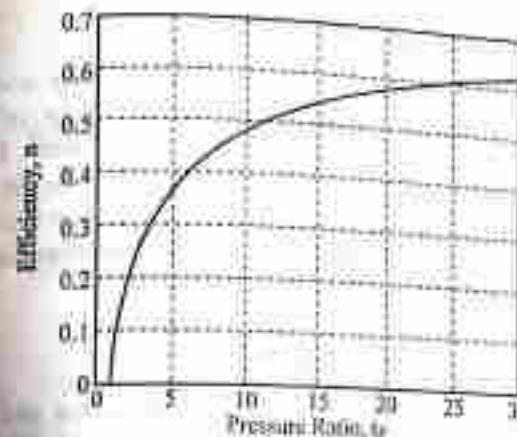


FIGURE 6.3 Variation of Efficiency of Brayton Cycle with Pressure Ratio

Although high pressure ratio is desirable for high efficiency but in reality high pressure ratio causes high temperature and there is a limit to which a real turbine blade can withstand maximum temperature or pressure. In real gas turbine cycle pressure ratio is usually between 10 to 16.

6.3 Rankine Cycle

Rankine cycle is an ideal model for the power cycle consisting of a steam turbine. The major components (devices) of the Rankine cycle are shown in Figure 6.4.

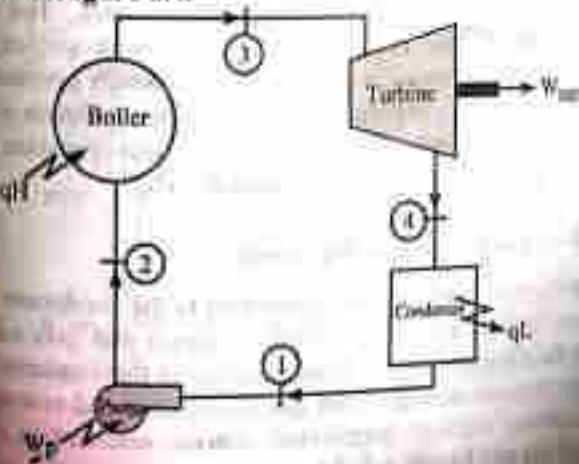


FIGURE 6.4 Rankine Cycle

Process 1 - 2 (Isentropic Pumping Process)

Low pressure saturated liquid from the low pressure condenser is supplied to the high pressure boiler with the help of a pump. Pumping process is assumed to be an isentropic process. During the pumping process, working substance remains in liquid state, its specific volume remains almost constant and pressure increases and similarly temperature increases and entropy remains constant as shown in Figure 6.5 (a) and Figure 6.5 (b).

Process 2 - 3 (Isobaric Heat Addition)

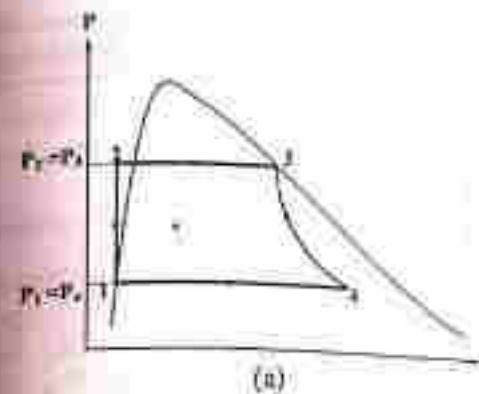
Heat is supplied to the boiler from an external source and the boiler produces steam by heating the liquid water. Heat addition in the boiler occurs at constant pressure. Hence, during the constant pressure heating, specific volume, temperature and entropy of the steam increases. The state of the steam is generally assumed at a saturated vapor, as shown in Figure 6.5 (a) and Figure 6.5 (b). But if the rate of heat supply to the boiler is increased, the steam at the boiler outlet may be superheated, as shown in Figure 6.6 (a) and Figure 6.6 (b).

Process 3 - 4 (Isentropic Expansion)

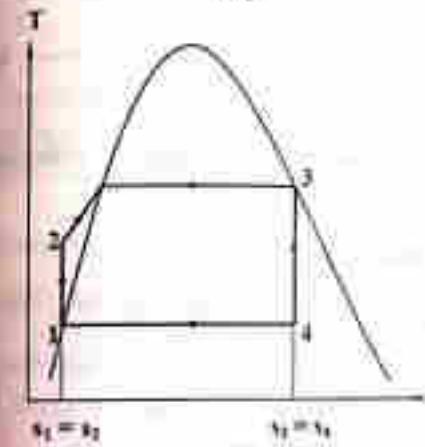
High temperature and high pressure steam produced by the boiler is supplied to the steam turbine which produces work by consuming energy carried by the steam at the turbine inlet. The process occurring in the turbine is assumed to be an isentropic expansion. Hence, during an isentropic expansion during which pressure of the steam decreases and its specific volume increases as shown in Figure 6.5 (a) whereas entropy of the system remains constant and its temperature decreases as shown in Figure 6.5 (b).

Process 4 - 1 (Isobaric Heat Rejection)

The steam from the turbine exit is delivered to the condenser where it rejects heat to its surroundings (cooling water) and gets condensed back into the saturated liquid. Heat rejection in the condenser occurs at constant pressure during which specific volume and entropy of the steam decreases while its temperature remains constant as shown in Figure 6.5 (a) and Figure 6.5 (b).

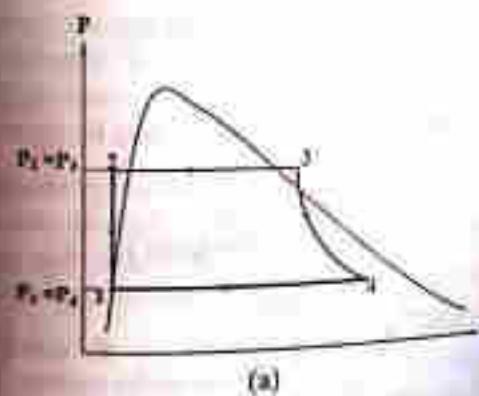


(a)



(b)

FIGURE 6.5 P-v and T-s Diagrams for Rankine Cycle with Saturated Vapor at Boiler Outlet



(a)

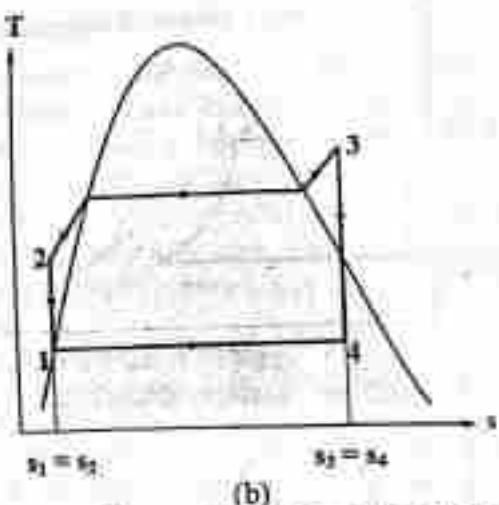


Figure 6.20 P-v and T-s Diagrams for Rankine Cycle with Superheated Vapor at Boiler Outlet

Efficiency of Rankine Cycle

Work produced by the turbine per kg of steam is given by

$$w_T = w_{34} = h_3 - h_4 \quad \dots \dots \dots (6.14)$$

Work consumed by the pump per kg of steam is given by

$$w_p = w_{12} = h_2 - h_1 \quad \dots \dots \dots (6.15)$$

During steady-state operation of the cycle, some part of the work produced by the turbine is utilized to run the compressor. Hence, the work delivered to the surroundings is given by

$$w_{ss} = w_T - w_p = [(h_3 - h_4) - (h_2 - h_1)] \quad \dots \dots \dots (6.16)$$

Heat supplied to the steam in the boiler is given by

$$q_B = q_{23} = h_3 - h_2 \quad \dots \dots \dots (6.17)$$

Efficiency of the Rankine cycle is then given by

$$\eta = \frac{w_{net}}{q_B} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2} \quad \dots \dots \dots (6.18)$$

6.4 Air Standard Internal Combustion Cycles

Air standard internal combustion cycles are the idealized models for the operation of petrol and diesel engines.

6.4.1 Operation of Four Stroke Engines

Schematic diagram of an internal combustion engine is shown in figure 6.7(a). Piston is connected to the crank through the connecting rod. Crank of the engine is connected to the wheel of the vehicle through a flywheel and when the piston travels up and down between TDC (Top Dead Center) and BDC (Bottom Dead Center), crank rotates in a circular slot and thereby drives the wheel of the vehicle. The operating cycle completes through the following processes in series: suction stroke, compression stroke, power (expansion) stroke and exhaust stroke.

Operation of Four Stroke Petrol Engine

In case of petrol engines, fuel is ignited by the electric spark induced by the spark plug. Therefore, petrol engine is also called spark ignition (SI) engine.

Suction Stroke: During the suction stroke of the petrol engine, the piston travels from TDC to BDC and the mixture of air and petrol enters into the engine cylinder through the inlet valve.

Compression Stroke: At the end of the suction stroke inlet valve is closed and when the piston travels from BDC to TDC, the mixture inside the cylinder is compressed isentropically. Due to the compression, pressure and temperature of the mixture inside the cylinder increases.

Power (Expansion) Stroke: At the end of the compression stroke, ignition produces electric spark and the mixture of petrol and air at high pressure and temperature gets burned. Due to the impact of burning, piston expands isentropically from TDC to BDC.

Exhaust Stroke: At the end of the power stroke, the cylinder pressure drops. The mixture of air and burnt products of petrol. When the piston travels from BDC to TDC, the burnt products flow out of the cylinder through exhaust valve.

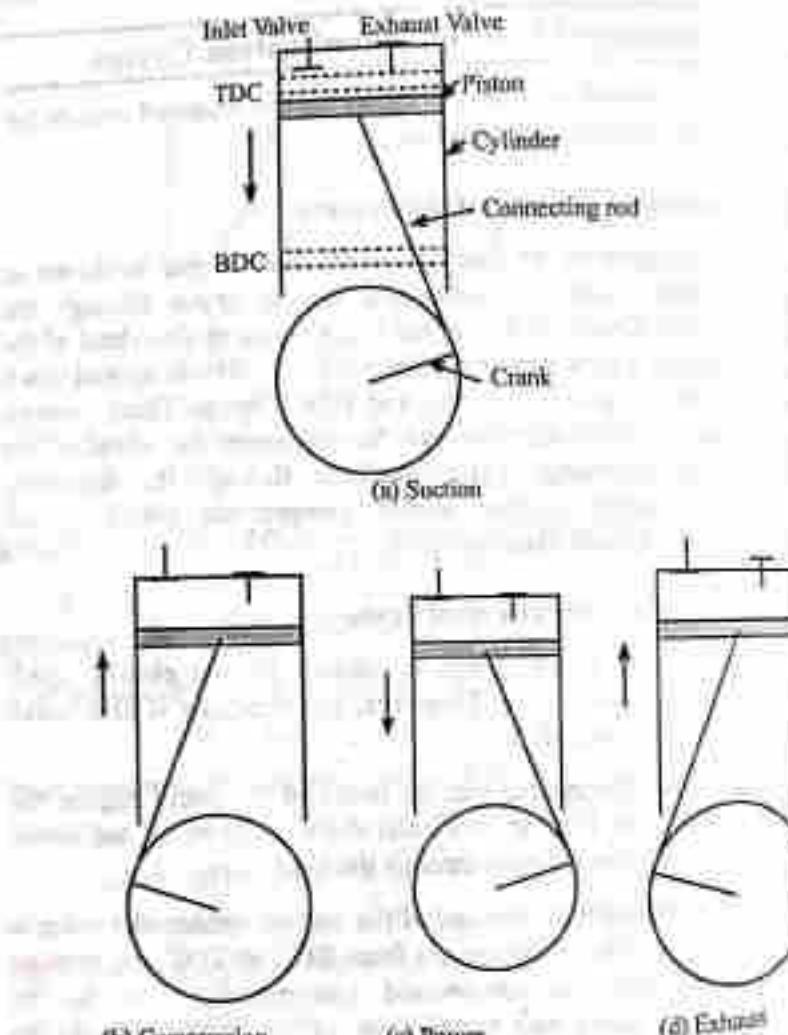


Figure 4.1 Operation of Four-Stroke Petrol Engine

Operation of Four Stroke Diesel Engine

In case of diesel engines, diesel is exposed to high temperature air at the end of the compression stroke, gets self burnt. Therefore, diesel engine is also called compression ignition (CI) engine.

Suction Stroke: During the suction stroke of the diesel engine, the piston travels from TDC to BDC and the air enters into the engine cylinder through the inlet valve.

Compression Stroke: At the end of the suction stroke inlet valve is closed and when the piston travels from BDC to TDC, the air inside the cylinder is compressed isentropically. Due to the compression, pressure and temperature of the mixture inside the cylinder increases.

Power (Expansion) Stroke: At the end of the compression stroke, fuel injector injects fuel into compressed air for certain interval. The temperature of air at this rate will be greater than self ignition temperature of the diesel and therefore diesel gets self burnt due to its exposure to the high temperature air. Due to the impact of burning, piston expands isentropically from TDC to BDC.

Exhaust Stroke: At the end of the power stroke, the cylinder contains the mixture of air and burnt products of diesel. When the piston travels from BDC to TDC, the burnt products flow out of the cylinder through exhaust valve.

4.2 Air Standard Analysis

During the normal operation of the petrol and diesel engines, the ratio of the fuel per unit mass of air i.e. fuel-air ratio, is very small (say 0.025), and therefore the properties of the fuel-air mixture during combustion approximates closely to those of air. The properties of the products of combustion also do not differ much from those of pure air. Therefore, simplified analysis of such cycles can be carried out by considering only air as its working substance. The simplified analysis of the cycle considering only air as the working substance is called an air standard analysis. Efficiency obtained from such analysis is called an air standard efficiency.

Assumptions of air standard analysis are listed below:

1. The cycle consists of fixed mass of air.
2. The expansion and compression processes are isentropic.
3. The combustion process is replaced by an equivalent heat addition process from an external source.

- (d) The exhaust process is replaced by an equivalent heat rejection process to an external sink.
- (e) The properties of air (ϵ_p , c_v , R) remains constant.

Although the air standard analysis differs from the actual operation, it gives basic idea about the variation of efficiency of the cycle with different parameters.

6.4.3 Air Standard Otto Cycle

It is an idealized model for the operation of a petrol engine. It consists of two isentropic and two isochoric processes as shown in Figure 6.8.

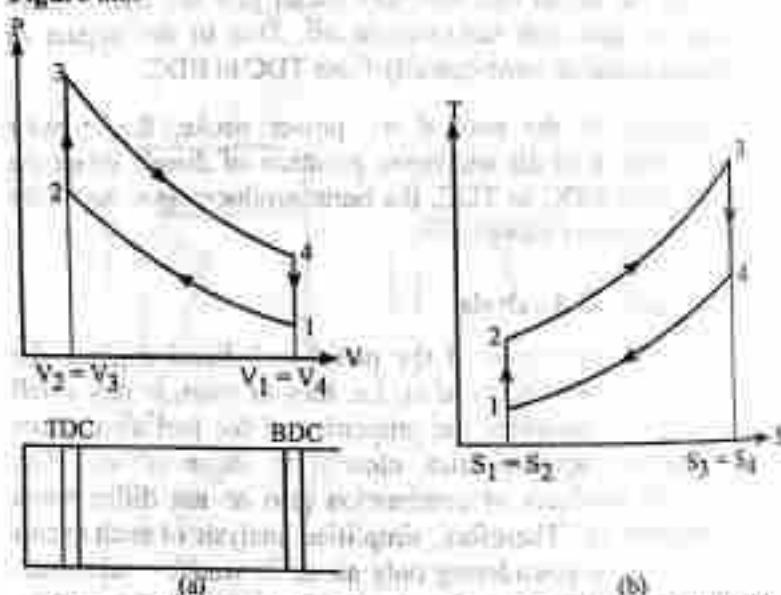


Figure 6.8: P-V and T-S Diagrams for an Air Standard Otto Cycle

Process 1 – 2 (Isentropic Compression)

At the beginning of the compression, the piston will be at the TDC and when the piston travels from BDC to TDC, the working substance inside the cylinder is compressed under isentropic condition. During the isentropic compression, pressure of the system increases, its volume decreases, temperature increases and entropy remains constant, as shown Figure 6.8.

Process 2 – 3 (Isochoric Heat Addition)

At the end of the compression stroke, spark plug provides an electric spark and petrol gets burn instantaneously. Therefore, heat addition for an ideal Otto cycle is considered as an isochoric process. During the heat addition process, pressure, temperature and entropy of the system increases as shown in Figure 6.8.

Process 3 – 4 (Isentropic Expansion)

Due to the impact of burning, piston gets expanded from TDC to BDC and the process is assumed to be an isentropic expansion. During the expansion process, pressure of the system decreases, volume increases, temperature decreases and entropy remains constant, as shown in Figure 6.8.

Process 4 – 1 (Isochoric Heat Rejection)

Heat is rejected by the system to the surroundings through the exhaust pipe and this process is replaced by an equivalent constant volume heat rejection process to complete the cycle. Hence, during isochoric heat rejection process, temperature, pressure and entropy of the system decrease as shown in Figure 6.8.

Efficiency of an Air Standard Otto Cycle

Efficiency of an internal combustion cycle can be determined as

$$\eta = 1 - \frac{q_L}{q_H} \quad \dots\dots\dots (6.19)$$

where q_L is the heat rejected per kg of air per cycle and q_H is the heat added per kg of air per cycle.

Heat rejected and heat added during the cycle are given as

$$q_L = q_{41} = c_V (T_4 - T_1) \quad \dots\dots\dots (6.20)$$

$$q_H = q_{23} = c_V (T_2 - T_3) \quad \dots\dots\dots (6.21)$$

Substituting Equations (6.20) and (6.21) into Equation (6.19), we get

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

(6.22)

Equation (6.22) requires temperatures of each state to determine the efficiency of the cycle. We can further simplify it to reduce the number of required variables. For this, applying temperature - volume relationship for isentropic compression process 1 - 2 and isentropic expansion process 3 - 4,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

(6.23)

$$\frac{T_4}{T_1} = \left(\frac{V_4}{V_1} \right)^{\gamma-1}$$

(6.24)

Substituting $V_3 = V_2$ and $V_4 = V_1$ into Equation (6.24), we get

$$\frac{T_2}{T_4} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

(6.25)

Equating Equations (6.23) and (6.25),

$$\frac{T_2}{T_1} = \frac{T_1}{T_4} \quad \text{or,} \quad \frac{T_4}{T_1} = \frac{T_1}{T_2}$$

(6.26)

Now using componendo and dividendo,

$$\frac{T_4}{T_3} = \frac{T_1}{T_2} = \frac{T_4 - T_1}{T_3 - T_2}$$

(6.27)

Substituting Equation (6.27) into Equation (6.22), we get

$$\eta = 1 - \frac{T_4}{T_3} = 1 - \frac{T_1}{T_2}$$

(6.28)

Substituting Equations (6.23) and (6.25) into Equation (6.28), we can express efficiency in terms of volume ratio as

$$\eta = 1 - \left(\frac{V_2}{V_1} \right)^{\gamma-1} = 1 - \left(\frac{1}{r} \right)^{\gamma-1}$$

(6.29)

where $r = \frac{V_1}{V_2}$, is called compression ratio.

Equation (6.29) shows that efficiency of the air standard Otto cycle increases with the increase in compression ratio as shown in Figure 6.4. Although high compression ratio is desirable for high efficiency but in reality high pressure ratio causes high temperature and there is a limitation to which a real engine cylinder can withstand maximum temperature or pressure. In real petrol engine cycle compression ratio is usually between 8 to 12.

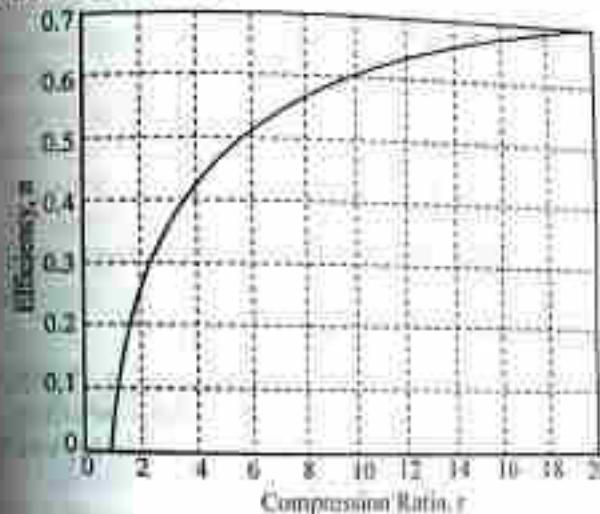


Fig. 6.4 Variation of Efficiency of an Air Standard Otto Cycle with Compression Ratio

Expression for Compression Ratio

Different parameters of an internal combustion engine are shown in Figure 6.10. The distance between the BDC and TDC of an engine cylinder is called stroke length (L_s) and the volume occupied in this space is called stroke volume (V_s). The space between the engine cylinder and the TDC is called clearance space and the volume occupied in this space is called clearance volume (V_c). The compression ratio is given by

$$r = \frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = 1 + \frac{V_s}{V_c} \quad \dots \dots \dots (6.30)$$

Where the stroke volume is given by

$$V_S = \frac{\pi}{4} D_p^2 L_S \quad \dots \dots \dots (6.3)$$

where D_p is the piston diameter.

Substituting Equation (6.31) into Equation (6.30), we get an expression for compression ratio in terms of engine dimensions as

$$r = 1 + \frac{\pi D_p^2 L_S}{4 V_c} \quad \dots \dots \dots (6.3)$$

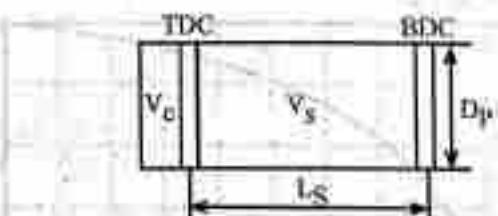


Figure 6.10 Different Engine Parameters

6.4.4 Air Standard Diesel Cycle

It is an idealized model for the operation of a diesel engine. It consists of isentropic compression, isobaric heat addition, isentropic expansion and isochoric heat rejection as shown in Figure 6.11.

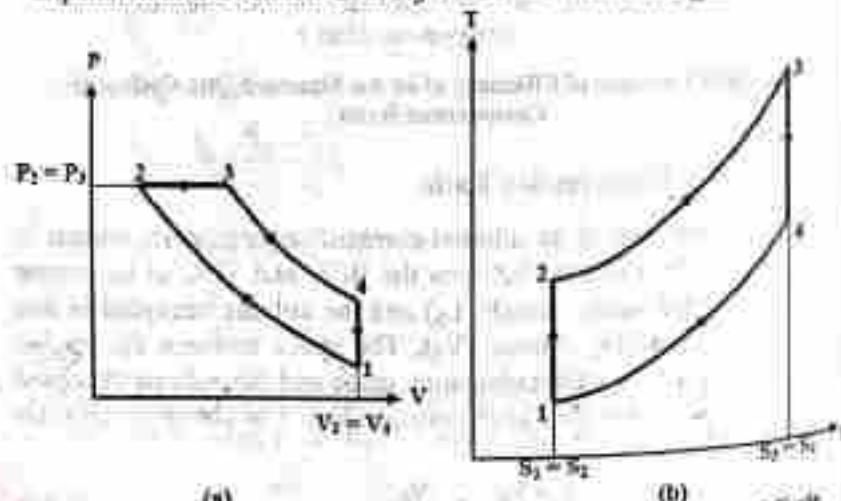


Figure 6.11 P-V and T-S Diagrams for an Air Standard Diesel Cycle

Process 1-2 (Isentropic Compression)

At the beginning of the compression, piston will be at the BDC and the cylinder contains air. When the piston travels from BDC to TDC, air inside the cylinder is compressed under isentropic condition. During the isentropic compression, pressure of the system increases, volume decreases, temperature increases and entropy remains constant, as shown Figure 6.11.

Process 2-3 (Isobaric Heat Addition)

At the end of the compression stroke, the temperature of the air reaches greater than the self ignition temperature of the diesel. At this stage, fuel injector nozzle supplies diesel into the cylinder and the diesel gets self burnt. However in this case, combustion of the diesel cannot complete instantaneously and the piston moves from TDC to BDC simultaneously with heat addition. Therefore, heat addition for an ideal diesel cycle is modeled as an isobaric process. Hence, during the constant pressure heat addition process, volume, temperature and entropy of the system increase as shown in Figure 6.11.

Process 3-4 (Isentropic Expansion)

Due to the impact of burning, piston gets further expansion isentropically. During the expansion process, pressure of the system decreases, volume increases, temperature decreases and entropy remains constant, as shown in Figure 6.11.

Process 4-1 (Isochoric Heat Rejection)

Heat is rejected by the system to the surroundings through the exhaust gas and this process is replaced by an equivalent constant volume heat rejection process to complete the cycle. Hence, during isochoric heat rejection process, temperature, pressure and entropy of the system decrease as shown in Figure 6.11.

Efficiency of an Air Standard Diesel Cycle

The efficiency of an ideal diesel cycle is given as

$$\eta = 1 - \frac{q_1}{q_3} \quad \dots \dots \dots (6.33)$$

Where q_1 is heat rejected by the system during constant volume process and q_3 is heat added.

$$q_L = q_{H1} = c_V(T_4 - T_1) \quad \dots \dots \dots (6.34)$$

Similarly, heat added to the system during constant pressure process 4 – 1 is given as

$$q_H = q_{H2} = c_p(T_3 - T_2) \quad \dots \dots \dots (6.35)$$

Substituting Equations (6.34) and (6.35) into Equation (6.33), we get

$$\eta = 1 - \frac{c_V}{c_p} \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{\gamma} \frac{\frac{T_4 - 1}{T_1}}{\frac{T_3 - 1}{T_2}} \quad \dots \dots \dots (6.36)$$

Applying temperature – volume relationship for isentropic compression process 1 – 2,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (\tau)^{\gamma-1} \quad \dots \dots \dots (6.37)$$

where $\tau = \frac{V_1}{V_2}$ is called compression ratio.

Similarly, applying temperature – volume relationship for an isobaric heat addition process 2 – 3,

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = \alpha \quad \dots \dots \dots (6.38)$$

where $\alpha = \frac{V_3}{V_2}$ is called cut-off ratio because heat addition & fuel supply gets out of from state 3. Therefore state 3 is also called cut-off state.

Again, applying temperature – volume relationship for isentropic expansion process 3 – 4,

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)^{\gamma-1} = \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_4} \right)^{\gamma-1} = \left(\frac{\alpha}{\tau} \right)^{\gamma-1} \quad \dots \dots \dots (6.39)$$

Substituting $V_4 = V_1$ and compression ratio and cut-off ratio in Equation (6.39), we get

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_2} \frac{V_2}{V_1} \right)^{\gamma-1} = \left(\frac{\alpha}{\tau} \right)^{\gamma-1} \quad \dots \dots \dots (6.40)$$

Multiplying Equations (6.37), (6.38) and (6.40),

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \frac{T_3}{T_2} \frac{T_2}{T_1} = \left(\frac{\alpha}{\tau} \right)^{\gamma-1} \cdot \alpha \cdot (\tau)^{\gamma-1} = (\alpha)^{\gamma} \quad \dots \dots \dots (6.41)$$

Substituting Equation (6.37), (6.38) and (6.41) into Equation (6.36), we get an expression for the efficiency of diesel cycle as

$$\eta = 1 - \frac{1}{\gamma} \left(\frac{1}{\tau} \right)^{\gamma-1} \left[\frac{\alpha^{\gamma} - 1}{\alpha - 1} \right] \quad \dots \dots \dots (6.42)$$

6.4.5 Mean Effective Pressure

Mean effective pressure is one of the parameter used for the comparison of different internal combustion cycles. It is defined as the magnitude of the constant pressure that would produce same work as that produced by the actual pressure varying cycle, i.e.,

$$W = P_{MEP} (V_{max}) = P_{MEP} (V_1 - V_2)$$

$$P_{MEP} = \frac{W}{V_1 - V_2} = \frac{w}{v_1 - v_2} \quad \dots \dots \dots (6.43)$$

6.5 Vapor Compression Refrigeration Cycle

Vapor compression refrigeration cycle is the most widely used cycle for heating and cooling of the desired space. Major components of the vapor compression refrigeration cycle are shown in Figure 6.12. The working substance used to run the cycle is called a refrigerant.

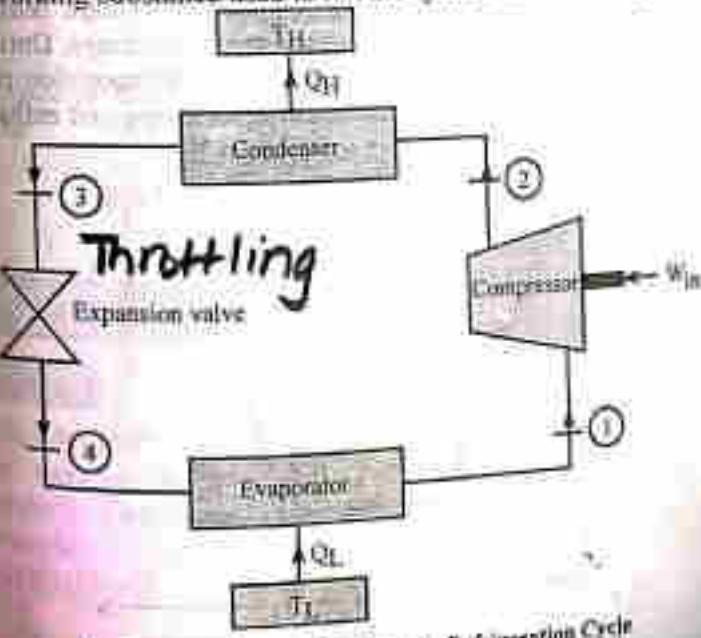


FIGURE 6.12 Vapor Compression Refrigeration Cycle

Exit vapor from the evaporator is supplied to the compressor, where it is compressed with the help of work input usually provided by the electricity. The compression process is assumed to be an isentropic process, during which pressure, temperature and enthalpy of the refrigerant increases while its entropy remains constant as shown in Figure 6.13.

The high pressure and high temperature refrigerant from the compressor outlet is delivered to the condenser. Condenser can interact with its surroundings and the high temperature refrigerant rejects heat to the surroundings. While rejecting heat vapor at the condenser inlet gets condensed. Condensation process occurs at constant pressure, during which entropy and enthalpy of the refrigerant decreases as shown in Figure 6.13.

The high pressure liquid at the exit of the condenser is delivered to the expansion (throttling) valve. During the throttling process, enthalpy of the refrigerant remains constant, its pressure and temperature decreases and entropy increases as shown in Figure 6.13.

The low pressure and low temperature refrigerant from the expansion valve enters into the evaporator. The low temperature refrigerant inside the evaporator can take heat from its surroundings. During the absorption of heat, refrigerant gets evaporated. Evaporation process takes place at constant pressure during which entropy and enthalpy of the refrigerant increases as shown in Figure 6.13.

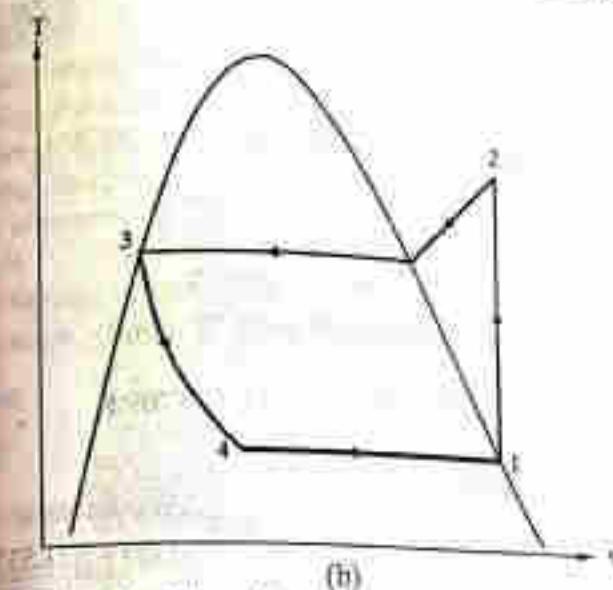
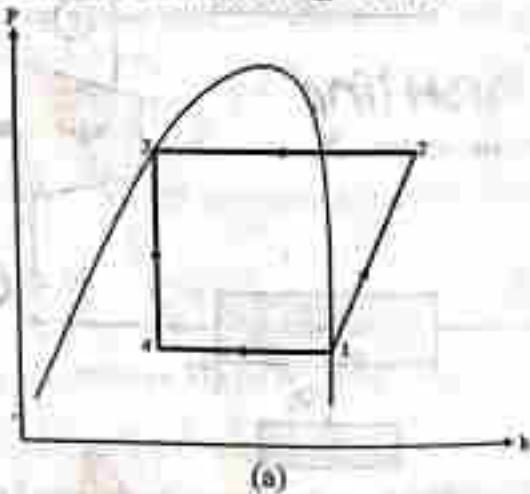


FIGURE 6.13 P-h and T-s Diagrams for Vapor Compression Refrigeration Cycle

If the cycle is used for the heating of a desired space then its COP is given as

$$(COP)_{HP} = \frac{q_{12}}{w} = \frac{h_2 - h_1}{h_2 - h_3} \quad \dots \dots \dots (6.44)$$

If the cycle is used for the cooling of a desired space then its COP is given as

$$(COP)_C = \frac{q_{12}}{w} = \frac{h_1 - h_4}{h_2 - h_1} \quad \dots \dots \dots (6.45)$$

SOLVED EXAMPLES

Example 6.1

An ideal Brayton cycle has pressure ratio of 10. The temperature of air at compressor and turbine inlets are 300 K and 1200 K respectively. Determine its thermal efficiency and the mass flow rate of air required to produce net power output of 80 MW. [Take $\gamma = 1.4$, $c_p = 1005 \text{ J/kgK}$]

Solution

Given,

$$\text{Pressure ratio: } r_p = \frac{P_2}{P_1} = 10$$

Compressor inlet temperature: $T_1 = 300 \text{ K}$ Turbine inlet temperature: $T_3 = 1200 \text{ K}$ Power output of the cycle: $W = 80 \text{ MW}$

Temperature at the compressor exit is given by

$$T_2 = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} T_1 = (r_p)^{\frac{1}{\gamma}} T_1 = (10)^{\frac{1.4-1}{1.4}} \times 300 = 579.209 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} T_3 = \left(\frac{1}{r_p} \right)^{\frac{1}{\gamma}} T_3 = \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} \times 1200 = 621.537 \text{ K}$$

Work produced by the turbine per kg of air is then given by

$$W_T = c_p (T_3 - T_4) = 1.005 \times (1200 - 621.537) = 581.355 \text{ kJ/kg}$$

Work consumed by the compressor per kg of air is then given by

$$W_C = c_p (T_2 - T_1) = 1.005 \times (579.209 - 300) = 280.605 \text{ kJ/kg}$$

Heat supplied per kg of air is then given by

$$q_H = c_v (T_3 - T_1) = 1.005 \times (1200 - 300) = 623.895 \text{ kJ/kg}$$

Net work produced by the cycle per kg of air is then given by

$$W_{net} = W_T - W_C = 581.355 - 280.605 = 300.75 \text{ kJ/kg}$$

Efficiency of the cycle is then given by

$$\eta_i = \frac{W_{net}}{q_H} = \frac{300.75}{623.895} = 48.205\%$$

Alternatively,

$$\eta = 1 - \left(\frac{1}{r_p} \right)^{\frac{1}{\gamma}} = 1 - \left(\frac{1}{10} \right)^{\frac{1.4-1}{1.4}} = 48.205\%$$

Mass flow rate of air is given by

$$\dot{m} = \frac{W}{W_{net}} = \frac{80 \times 10^3}{300.75} = 266.002 \text{ kg/s}$$

Solution

The compression ratio of an air standard Otto cycle is 8. At the beginning of the compression process, the pressure and temperature of air are 100 kPa and 20°C respectively. The heat added per kg of air during the cycle is 2000 kJ/kg. Determine the pressure and temperature at the end of each process of the cycle.

- (a) thermal efficiency, and
- (b) mean effective pressure. [Take $\gamma = 1.4, c_v = 718 \text{ J/kgK}$]

Solution

Given

$$\text{Compression ratio: } r = \frac{V_1}{V_2} = 8$$

Initial state 1 : $P_1 = 100 \text{ kPa}, T_1 = 100 \text{ kPa}, T_1 = 293 \text{ K}$ Heat added per cycle: $q_H = q_{23} = 2000 \text{ kJ}$

Applying P – V relation for an isentropic compression 1 – 2, pressure at state 2,

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = P_1 (r)^{\gamma-1} = 100 \times (8)^{1.4-1} = 1837.917 \text{ kPa}$$

Similarly, applying T – V relation for an isentropic compression 1 – 2, temperature at state 2,

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = T_1 (r)^{\gamma-1} = 293 \times (8)^{1.4-1} = 673.137 \text{ K}$$

Heat added during the cycle is given by

$$q_H = q_{23} = c_v (T_3 - T_2)$$

Temperature at state 3,

$$T_3 = \frac{q_H}{c_v} + T_2 = \frac{2000}{0.718} + 673.137 = 3458.653 \text{ K}$$

Applying P – T relation for an isochoric heat addition process 2 – 3, pressure at state 3,

$$P_3 = \frac{T_3}{T_2} P_2 = \frac{3458.653}{673.137} \times 1837.917 = 9443.419 \text{ kPa}$$

Finally, applying P – V and T – V relations for an isentropic expansion 3 – 4, pressure and temperature at state 4 are given by

$$P_4 = P_3 \left(\frac{V_1}{V_4} \right)^{\gamma-1} = P_3 \left(\frac{1}{r} \right)^{\gamma-1} = 9443.419 \times \left(\frac{1}{8} \right)^{1.4} = 513.8109 \text{ kPa}$$

$$T_4 = T_3 \left(\frac{V_1}{V_4} \right)^{\gamma-1} = P_3 \left(\frac{1}{r} \right)^{\gamma-1} = 3458.653 \times \left(\frac{1}{8} \right)^{1.4} = 1505.465 \text{ K}$$

(b) Heat rejected during the cycle,

$$q_L = q_{41} = c_v (T_4 - T_1) = 0.718 \times (1505.465 - 293) \\ = 870.55 \text{ kJ/kg}$$

∴ Efficiency of the cycle,

$$\eta = 1 - \frac{q_L}{q_H} = 1 - \frac{870.55}{2000} = 56.472\%$$

Alternatively,

$$\eta = 1 - \left(\frac{1}{r} \right)^{1/\gamma-1} = 1 - \left(\frac{1}{8} \right)^{1/1.4-1} = 56.472\%$$

(c) Work output per kg of air per cycle,

$$w = q_H - q_L = 2000 - 870.55 = 1129.45 \text{ kJ/kg}$$

Alternatively,

$$w = \eta q_H = 0.56472 \times 2000 = 1129.45 \text{ kJ/kg}$$

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 293}{100} = 0.84091 \text{ m}^3/\text{kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.84091}{8} = 0.10511 \text{ m}^3/\text{kg}$$

Alternatively,

$$v_2 = \frac{RT_2}{P_2} = \frac{0.287 \times 673.137}{1837.917} = 0.10511 \text{ m}^3/\text{kg}$$

∴ Mean effective pressure of the cycle,

$$P_{M.E.P} = \frac{w}{v_1 - v_2} = \frac{1129.45}{0.84091 - 0.10511} = 1535.003 \text{ kPa}$$

The pressure and temperature at the end of suction stroke of an air cooled diesel cycle are 100 kPa and 27°C respectively. Maximum pressure during the cycle is 1600°C and the compression ratio is 16. Determine

- (a) Percentage of stroke at which cut-off takes place,
- (b) Temperature at the end of the expansion stroke, and
- (c) Thermal efficiency.

Solution:

$$\text{Given: } \text{Compression ratio: } r = \frac{V_1}{V_2} = 16$$

Properties at state 1 : $P_1 = 100 \text{ kPa}$; $T_1 = 300 \text{ K}$

Maximum temperature during the cycle:

$$T_{max} = T_3 = 1600 + 273 = 1873 \text{ K}$$

Specific volume of air at state 1,

$$v_1 = \frac{T_1}{P_1} = \frac{0.287 \times 300}{100} = 0.861 \text{ m}^3/\text{kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.861}{16} = 0.0538125 \text{ m}^3/\text{kg}$$

Using $T - V$ relation for an isentropic compression 1 → 2.

Temperature at state 2,

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = T_1 (r)^{\gamma-1} = 300 \times (16)^{1.4-1} = 909.429 \text{ K}$$

Applying $T - v$ relation for an isobaric heat addition process 2 → 3.

Temperature at state 3,

$$T_3 = \frac{T_2}{v_2} v_1 = \frac{1873}{909.429} \times 0.0538125 = 0.110829 \text{ m}^3/\text{kg}$$

Percentage of stroke at which cut-off takes place,

$$\frac{v_3 - v_2}{v_1 - v_2} = \frac{0.110829 - 0.0538125}{0.861 - 0.0538125} = 0.0706355 = 7.06355\%$$

(d) Cut-off ratio for the cycle is given by,

$$\frac{v_1 - v_2}{v_2} = \frac{v_1}{v_2} - 1 = \frac{0.861}{0.0538125} - 1 = 2.0595$$

Similarly, applying $T - V$ relations for an isentropic expansion, temperature at (the end of expansion stroke) state 4 is given by,

$$\begin{aligned} T_4 &= T_3 \left(\frac{V_3}{V_4} \right)^{\gamma-1} = T_3 \left(\frac{V_3}{V_2} \frac{V_2}{V_4} \right)^{\gamma-1} = T_3 \left(\frac{V_3}{V_2} \frac{V_2}{V_1} \right)^{\gamma-1} \\ &= T_3 \left(\frac{a}{r} \right)^{\gamma-1} = 1873 \times \left(\frac{2.0595}{16} \right)^{0.4} = 824.892 \text{ K} \end{aligned}$$

(c) Efficiency of the cycle is then given by,

$$\eta = 1 - \frac{1}{r} \left(\frac{1}{r} \right)^{\gamma-1} \left[\frac{a^{\gamma}-1}{a-1} \right] = 1 - \frac{1}{1.4} \left(\frac{1}{16} \right)^{0.4} \left[\frac{2.0595^{1.4}-1}{2.0595-1} \right] = 61.89\%$$

Example E6.4

Steam at 1 MPa and 400°C is expanded on a steam turbine working on a Rankine cycle to 10 kPa. Determine the net work per kg of steam and the cycle efficiency.

Solution

Given,

Properties of steam at turbine inlet : $P_1 = 1000 \text{ kPa}$, $T_1 = 400^\circ\text{C}$

Properties of steam at turbine exit : $P_4 = 10 \text{ kPa}$

With reference to $T - S$ diagram of the cycle shown in Figure E6.4, properties of steam at each states are evaluated as follows:

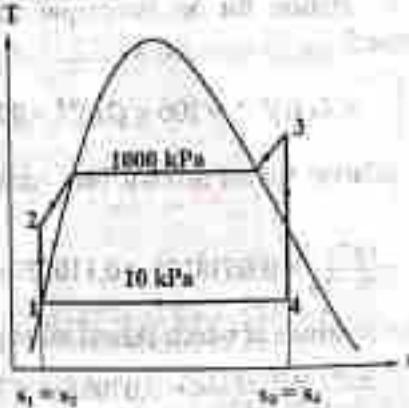


Figure E6.4: $T - s$ Diagram for the Rankine Cycle given in Example E6.4

State 1: $P_1 = 10 \text{ kPa}$, saturated liquid

According to Table A.1, $h_1 = h_2 (10 \text{ kPa}) = 191.83 \text{ kJ/kg}$
 $s_1 = s_2 (10 \text{ kPa}) = 0.00101 \text{ m}^2/\text{kg}$

State 3: $P_3 = 1000 \text{ kPa}$, compressed liquid

Using isentropic relation for an incompressible substance,

$$\begin{aligned} h_3 &= v_3 (P_3 - P_1) \\ h_3 &= v_3 (P_2 - P_1) = 191.83 + 0.00101 \times (1000 - 10) \\ h_3 &= 192.8299 \text{ kJ/kg} \end{aligned}$$

State 4: $P_4 = 1000 \text{ kPa}$, $T_4 = 400^\circ\text{C}$, superheated vapor

According to Table A.1, $h_4 = 3263.8 \text{ kJ/kg}$, $s_4 = 7.4648 \text{ kJ/kgK}$

At $P_4 = 10 \text{ kPa}$,

Isentropic expansion process 3-4, $s_4 = s_3 = 7.4648 \text{ kJ/kgK}$

According to Table A.1, $s_1 < s_4 < s_3$, hence it is a two phase mixture.

Quality of steam at state 4,

$$x_4 = \frac{s_4 - s_3}{s_{4L} - s_3} = \frac{7.4648 - 6.6493}{7.4989} = 0.90887$$

Specific enthalpy of steam at state 4 is then given by,

$$\begin{aligned} h_4 &= h_{4L} + x_4 h_{4V} = 191.83 + 0.90887 \times 2392.0 \\ &= 2365.8389 \text{ kJ/kg} \end{aligned}$$

Work produced by the turbine per kg of steam is given by

$$w_t = w_{32} = h_3 - h_4 = 3263.8 - 2365.8389 = 897.961 \text{ kJ/kg}$$

Work consumed by the pump per kg of steam is given by

$$w_p = w_{21} = h_2 - h_1 = 192.8299 - 191.83 = 0.9999 \text{ kJ/kg}$$

The net work delivered to the surroundings is given by

$$w_{net} = w_t - w_p = 897.961 - 0.9999 = 896.96 \text{ kJ/kg}$$

Heat supplied to the steam in the boiler is given by

$$q_{in} = q_{12} = h_3 - h_2 = 3269.8 - 192.8299 = 3070.97 \text{ kJ/kg}$$

Efficiency of the Rankine cycle is then given by

$$\eta_{Rankine} = \frac{w_{net}}{q_{in}} = \frac{896.97}{3070.97} = 29.2077\%$$

Example 6.5

In a vapor compression refrigeration system, the condenser is 20°C and the evaporator temperature is -10°C. Saturated liquid enters the expansion valve and saturated vapor enters the compressor. For a refrigeration effect of 3.5 kW, determine COP, mass flow rate of the refrigerant and the power input if the refrigerant is ammonia.

Solution

Given,

Temperature of refrigerant at condenser : $T_2 = 20^\circ\text{C}$

Temperature of refrigerant at evaporator : $T_1 = -10^\circ\text{C}$

With reference to T-s diagram of the cycle shown in Figures 1.65, properties of refrigerant at each states are evaluated as follows.

State 1: $T_1 = -10^\circ\text{C}$, saturated vapor

Referring to Table A4.1, $h_1 = h_2(-10^\circ\text{C}) = 1450.5 \text{ kJ/kg}$,
 $s_1 = s_2(-10^\circ\text{C}) = 5.7564 \text{ kJ/kgK}$

State 2: $T_2 = 20^\circ\text{C}$, superheated vapor

For an isentropic compression process 1-2, $s_2 = s_1 = 5.7564 \text{ kJ/kgK}$

Referring to Table A4.3, other properties of superheated ammonia at 20°C can be listed as

P, kPa	h, kJ/kg	s, kJ/kgK	
450	1511.8	5.7749 (a)
500	1508.1	5.7138 (b)

Now applying linear interpolation,

$$\begin{aligned} h_2 &= h_a + \frac{h_b - h_a}{s_b - s_a} (s_2 - s_a) \\ &= 1511.8 + \frac{1508.1 - 1511.8}{5.7138 - 5.7749} (5.7564 - 5.7749) \\ &= 1510.6797 \text{ kJ/kg} \end{aligned}$$

State 3: $T_3 = 20^\circ\text{C}$, superheated liquid

Referring to Table A4.1,

$$h_3 = h_1(20^\circ\text{C}) = 294.3 \text{ kJ/kg}$$

State 4: $T_4 = -10^\circ\text{C}$, Two phase mixture

For the throttling process 3-4, $h_3 = h_4 = 294.3 \text{ kJ/kg}$

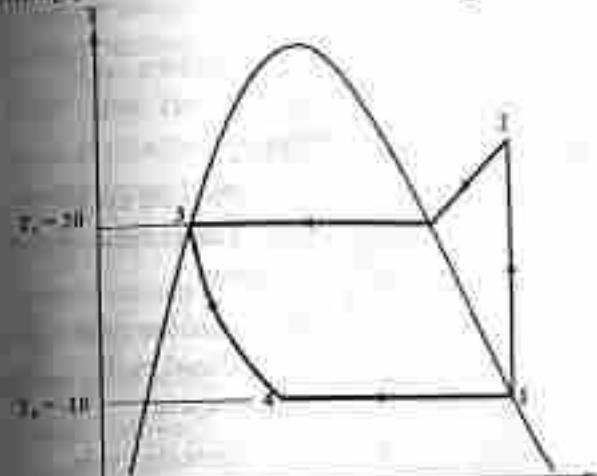


Figure 6.4: T-s Diagram for the Refrigeration Cycle given in Example 6.5

Heat removed per kg of refrigerant from the desired space

$$q = h_2 - h_3 = 1450.5 - 294.3 = 1156.2 \text{ kJ/kg}$$

Work required per kg of refrigerant

$$w_{in} = h_1 - h_4 = 1510.6797 - 1450.5 = 60.1797 \text{ kJ/kg}$$

Therefore, COP of the system is given by

$$\text{COP} = \frac{q}{w_{in}} = \frac{1156.2}{60.1797} = 19.2125$$

Mass flow rate of the refrigerant is given by

$$\dot{m} = \frac{q}{w_{in}} = \frac{3.5}{1156.2} = 3.027 \times 10^{-3} \text{ kg/s}$$

Power required to drive the system is given by

$$P = \dot{m}(w_{in}) = 3.027 \times 10^{-3} \times 60.1797 = 0.18217 \text{ kW}$$

12. Derive an expression for compression ratio for an internal combustion engine. Discuss the effect of different parameters involved in the expression on the compression ratio.
13. Explain the working principle of a vapor compression refrigeration cycle. Sketch the cycle on P-h and T-s diagrams. Also derive expressions for its COP when it is used as
 - (a) a heat pump, and
 - (b) a refrigerator.

NUMERICAL PROBLEMS

Take $\gamma = 1.4$, $R = 287 \text{ J/kgK}$, $c_v = 718 \text{ J/kgK}$ and $c_p = 1005 \text{ J/kgK}$ for all gas power cycles.

1. Air at 100 kPa and 25°C enters into a compressor of an ideal Brayton cycle and exits at 1000 kPa. The maximum temperature during the cycle is 1127°C. Determine
 - (a) the pressure and temperature at each states of the cycle,
 - (b) the compressor work, turbine work and net work per kg of air, and
 - (c) the cycle efficiency.
2. Air at the compressor inlet of an ideal gas turbine cycle is at 100 kPa and 20°C. The heat added to the cycle per kg of air is 800 kJ/kg. The maximum temperature during the cycle is limited to 1400 K. Determine
 - (a) the pressure ratio,
 - (b) the net work output per kg of air, and
 - (c) the cycle efficiency.
3. Air enters the compressor of an ideal Brayton cycle at 100 kPa, 290 K with a volumetric flow rate of 4 m³/s. The pressure ratio for the cycle is 10 and the maximum temperature during the cycle is 1500 K. Determine
 - (a) the thermal efficiency of the cycle,
 - (b) the fraction of work output that is consumed by the compressor, and
 - (c) the net power output.
4. An ideal Brayton cycle has a pressure ratio of 12. The pressure and temperature at the compressor inlet are 100 kPa and 27°C

respectively. The maximum temperature during the cycle is 1127°C. If the mass flow rate of air is 8 kg/s, determine the power output and efficiency of the cycle.

A power plant operating on an ideal Brayton cycle delivers a power output of 80 MW. The minimum and maximum temperatures during the cycle are 300 K and 1500 K respectively. The pressure at the compressor inlet and exit are 100 kPa and 1000 kPa respectively.

- (a) Determine the thermal efficiency of the cycle.
- (b) Determine the power output from the turbine.
- (c) What fraction of the turbine power output is required to drive the compressor?

An ideal gas turbine cycle produces 15 MW of power output. The properties of air at the compressor inlet are 100 kPa and 30°C. The pressure ratio for the cycle is 15 and the heat added per kg of air per cycle is 800 kJ/kg. Determine

- (a) the efficiency of the cycle,
- (b) the maximum temperature in the cycle, and
- (c) the mass flow rate of air.

In an ideal Brayton cycle, air enters the compressor at 100 kPa and 300 K and the turbine at 1000 kPa and 1200 K. Heat is transferred to the air at a rate of 30 MW. Determine the efficiency and the power output of the cycle.

The minimum and maximum temperatures during an ideal Brayton cycle are 300 K and 1200 K respectively. The pressure ratio is such that the net work developed is maximized. Determine

- (a) the compressor and turbine work per unit mass of air, and
- (b) the thermal efficiency of the cycle.

The compressor and turbine of an ideal gas turbine each have isentropic efficiencies of 80%. The pressure ratio is 10. The minimum and maximum temperatures are 300 K and 1200 K respectively. Determine

- (a) the net work per kg of air,
- (b) the thermal efficiency of the cycle, and
- (c) compare both of these for a cycle with ideal compressor and turbine.

10. Determine the efficiency of an ideal Rankine cycle operating between the boiler pressure of 1.5 MPa and a condenser pressure of 8 kPa. The steam leaves the boiler as saturated vapor.
11. A Rankine cycle has a boiler working at a pressure of 2 MPa. The maximum and minimum temperatures during the cycle are 400°C and 50°C respectively. Determine the efficiency of the cycle and compare it with that of the Carnot cycle operating between the same temperature limits.
12. A steam power plant operates on a simple Rankine cycle between the pressure limits of 2 MPa and 20 kPa. The temperature of the steam at the turbine inlet is 400°C, and the mass flow rate of steam is 50 kg/s. Determine:
- the thermal efficiency of the cycle, and
 - the net power output of the plant.
13. An ideal Rankine cycle operates between a boiler pressure of 1.5 MPa and a condenser pressure of 10 kPa. The exit steam from the turbine should have a quality of 96% and the power output of the turbine should be 80 MW. Determine:
- the minimum boiler exit temperature,
 - the efficiency of the cycle, and
 - the mass flow rate of steam.
14. Saturated vapor enters into a turbine of an ideal Rankine cycle at 10 MPa and saturated liquid exits the condenser at 10 kPa. The power output of the cycle is 120 MW. Determine:
- the thermal efficiency of the cycle,
 - the back work ratio,
 - the mass flow rate of steam,
 - the rate at which heat is supplied to the boiler,
 - the rate at which heat is rejected from the condenser, and
 - the mass flow rate of condenser cooling water, if the cooling water enters at 20°C and exits at 35°C. [Take specific heat of water as 4.18 kJ/kgK].
15. Superheated steam at 8 MPa, 500°C enters into turbine of a 500-MW power plant working on a Rankine cycle. The steam leaves the condenser as saturated liquid at 8 kPa. The turbine and pump have isentropic efficiencies of 90% and 80% respectively. For the cycle, determine:
- the net work per kg of steam, and
 - the heat supplied into the boiler per kg of steam, and
 - the thermal efficiency.
16. An air standard Otto cycle has a compression ratio of 10. At the beginning of the compression stroke, the pressure and temperature are 100 kPa and 20°C respectively. The peak temperature during the cycle is 2000 K. Determine:
- the pressure and temperature at the end of each process of the cycle,
 - the thermal efficiency, and
 - the mean effective pressure.
17. An air standard Otto cycle has a compression ratio of 8 and its compression stroke has air at 100 kPa and 300 K. The pressure during the cycle is 6000 kPa. Determine:
- the peak temperature in the cycle,
 - the temperature at the end of expansion stroke, and
 - the cycle efficiency.
18. An ideal Otto cycle has a compression ratio of 8. The minimum and maximum temperatures during the cycle are 300 K and 1500 K respectively. Determine:
- the heat added per kg of air,
 - the thermal efficiency, and
 - the efficiency of a Carnot cycle operating between the same temperature limits.
19. The compression ratio of an ideal Otto cycle is 8.5. At the beginning of the compression stroke, air is at 100 kPa and 27°C. The pressure is doubled during the constant volume heat addition process. Determine:
- the heat added per kg of air,
 - the net work output per kg of air,
 - the thermal efficiency, and
 - the mean effective pressure.
20. At the beginning of the compression stroke in an air standard Otto cycle is at 100 kPa and 300 K. The temperature of the air before and after the expansion stroke are 1550 K and 650 K respectively. If the air circulation rate is 3 kg/min, determine the compression ratio, air standard efficiency and the power output.
21. The properties of air at the beginning of an air standard Otto cycle are $P_1 = 100 \text{ kPa}$, $T_1 = 300 \text{ K}$ and $V_1 = 0.5 \times 10^{-3} \text{ m}^3$. The maximum temperature during the cycle is 1400 K, and the compression ratio is 8. Determine:

- (a) the heat added during the cycle,
 (b) the net work output,
 (c) the thermal efficiency, and
 (d) the mean effective pressure.
22. The following data are obtained for a four stroke petrol engine:
 Cylinder bore = 14 cm
 Stroke length = 15 cm
 Clearance volume = 231 cm³
- Determine:
 (a) the ratio of clearance volume and swept volume,
 (b) the compression ratio, and
 (c) the thermal efficiency.
23. In an ideal Otto cycle, heat added to the system due to combustion is twice the heat rejected through the exhaust gas. Determine the thermal efficiency and compression ratio of the engine.
24. At the beginning of the compression stroke of an air standard Diesel cycle having a compression ratio of 16, the temperature is 300 K and the pressure is 100 kPa. If the cut off ratio for the cycle is 2, determine:
 (a) the pressure and temperature at the end of each process of the cycle,
 (b) the thermal efficiency, and
 (c) the mean effective pressure.
25. An air standard Diesel cycle has a compression ratio of 16. At the beginning of the compression stroke, the pressure and temperature are 100 kPa and 27°C respectively. The heat added per kg of air during the cycle is 2000 kJ/kg. Determine
 (a) the pressure and temperature at the end of each process of the cycle,
 (b) the thermal efficiency, and
 (c) the mean effective pressure.
26. An air standard diesel cycle has a compression ratio of 22 and expansion ratio of 11. Determine its cut off ratio and thermal efficiency.
27. The pressure and temperature at the beginning of the compression stroke of an air standard Diesel cycle are 100 kPa and 300 K. The peak pressure and temperature during the cycle are 5000 kPa and 3000 K respectively. Determine the compression ratio and the cycle efficiency.

A Diesel cycle has a compression ratio of 20. The air at the beginning of the compression stroke is at $P_1 = 100 \text{ kPa}$, $T_1 = 290 \text{ K}$, and $V_1 = 0.5 \times 10^{-3} \text{ m}^3$. The maximum temperature during the cycle is 2000 K. Determine:

- (a) the maximum pressure during the cycle,
 (b) the cycle efficiency, and
 (c) the work output.

The properties of air at the beginning of compression stroke in an air standard Diesel cycle are 100 kPa and 300 K. The air at the beginning of the expansion stroke is at 6500 kPa and 2000 K. Determine:

- (a) the compression ratio,
 (b) the thermal efficiency, and
 (c) the mean effective pressure.

An engine working on a diesel cycle has a compression ratio of 16 and the cut off takes place at 8 % of the stroke. Determine its air standard efficiency.

The following data are given for a four stroke diesel engine:

Cylinder bore	= 14 cm
Stroke length	= 25 cm
Clearance volume	= 350 cm ³

Determine the air standard efficiency, if fuel injection takes place at constant pressure for 5 % of the stroke.

2. Air at the beginning of compression stroke in an ideal Diesel cycle is at 100 kPa and 295 K and the compression ratio is 20. Determine the maximum temperature during the cycle to have an efficiency of 65 %.

3. An engine with bore of 8 cm and stroke of 12 cm has a compression ratio of 6. To increase the compression ratio 1.5 mm is machined off the cylinder head face to increase stroke volume while the clearance volume remains same. Determine the new compression ratio.

HEAT TRANSFER

7

7.1 Introduction

As defined earlier in Chapter 2, heat transfer is the transfer of energy because of temperature difference. Heat always flows from high temperature to low temperature. Hence, the heat transfer processes involve the transfer of energy, and obey the first as well as the second law of thermodynamics. It may appear that the principles of heat transfer are derived from the basic laws of thermodynamics. But there are certain differences between heat transfer and thermodynamics. The science of thermodynamics deals with the amount of heat transfer as a system undergoes a process from an equilibrium state to another, and makes no reference to how long the process will take. But in engineering, we are often interested in the rate of heat transfer and spatial variation of temperature, which is the topic of the science of heat transfer. The subject of heat transfer is based on foundation of thermodynamics and fluid mechanics.

Heat transfer, in general, is three-dimensional and time dependent i.e., the temperature in a medium varies with position as well as time. Heat transfer in a medium is said to be steady if the temperature at any particular point does not vary with time, and unsteady or transient when it varies with time.

Most of the engineering disciplines have application of heat transfer to design appropriate cooling systems such as transformer, integrated circuits etc. Mechanical engineers may find its application for the design of boilers, refrigerators or other types of heat exchangers for the storage, transfer and use of cryogenic substances.

7.2 Modes of Heat Transfer

According to the physical mechanism and the governing law associated with them, heat transfer is classified into three major modes: conduction, convection and radiation.

7.2.1 Conduction

Heat conduction is due to the property of matter which allows the transfer of heat energy even its parts are not in motion relative to one another. Magnitude of conduction heat transfer is given by the following equation,

$$Q = -kA \frac{dT}{dx} \quad \dots\dots\dots(7.1)$$

Q is the heat transfer rate (W),

k is the thermal conductivity of the material ($\frac{W}{mK}$),

A is the cross sectional area of the material normal to the direction of heat flow (m^2), and

$\frac{dT}{dx}$ is the temperature gradient within the material ($\frac{K}{m}$).

The negative sign in Equation (7.1) indicates that heat flows in the direction of decreasing temperature.

Heat is conducted in solids in two ways: transport of energy by free electrons and lattice vibration. In good conductors, a large number of electrons move about in the lattice structure of the material which transport heat from high temperature region to the low temperature region. The portion of energy transported by free electrons is larger than that by lattice vibration. An increase in temperature causes increase in both lattice vibration and speed of electrons, but increased vibration of lattice disturbs the movement of electrons causing reduction in transport of energy by free electrons which means the overall conduction is reduced. In ceramics and alloys, the transport of energy is mainly due to lattice vibration and an increase in temperature increases conduction.

Conduction in liquid and gases are based on the movements of atoms and molecules. In case of liquids, random translatory motion is dominant and it appears that translation of energy occurs by longitudinal vibrations similar to the propagation of sound.

7.2.2 Convection

Convection heat transfer occurs in fluid medium and heat is transferred by the actual movement of the molecules. Magnitude of heat transfer is given by Newton's law of cooling,

$$Q = h.A(T_s - T_e) \quad \dots\dots\dots(7.2)$$

where, \dot{Q} is the heat transfer rate (W),

h is the convection heat transfer coefficient ($\frac{W}{m^2 K}$)

A is the surface area of the solid confining the liquid (m^2).

T_s is the solid surface temperature, and

T_∞ is the free stream temperature of the fluid.

The heat transfer coefficient h depends upon the thermodynamic and transport properties (e.g. density, viscosity, specific heat and thermal conductivity of the fluid), the geometry of the surface, the nature of fluid flow, and the prevailing thermal conditions.

7.2.3 Radiation

Radiation heat transfer can occur without any medium and heat is transferred in the form of electromagnetic wave (EMW). Magnitude of radiation heat exchange between two practical bodies at temperatures T_1 and T_2 is given by Stefan-Boltzmann law,

$$\dot{Q} = \epsilon \sigma A (T_1^4 - T_2^4) \quad \dots \dots \dots (7.3)$$

where, \dot{Q} is the heat transfer rate (W),

ϵ is the emissivity,

A is the surface area (m^2), and

σ is the Stefan-Boltzmann constant ($= 6.67 \times 10^{-8} W/m^2 K^4$).

7.3 One Dimensional Steady State Heat Conduction through a Plane Wall

Consider a plane wall made up of a material having a thermal conductivity of k , thickness L , cross-sectional area of A as shown in Figure 7.1. Its opposite faces are exposed to uniform temperatures of T_1 and T_2 respectively. Here, the temperature gradient exists only in x -direction, hence we can model it as an one dimensional steady state heat conduction problem by using Fourier equation [Equation (7.1)]. Rearranging Equation (7.1), we get

$$\dot{Q} dx = -kAdT \quad \dots \dots \dots (7.4)$$

Integrating with associated boundary conditions,

$$\dot{Q} \int_0^L dx = - \int_{T_1}^{T_2} kAdT \quad \dots \dots \dots (7.5)$$

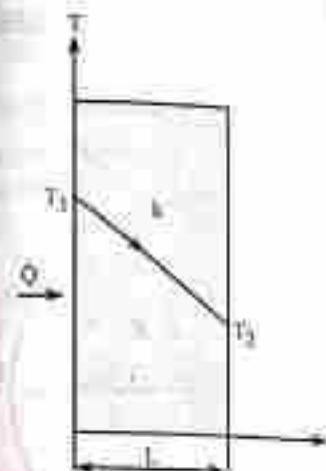


Figure 7.1 One Dimensional Heat Conduction through a Plane Wall

assuming k as a constant and for a plane with a uniform cross section, Equation (7.5) reduces to

$$\dot{Q}L = -kA(T_2 - T_1) = kA(T_1 - T_2)$$

Hence, the magnitude of conduction heat transfer through a plane wall is given by

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) \quad \dots \dots \dots (7.6)$$

7.4 Radial Steady State Heat Conduction through a Hollow Cylinder

Consider a hollow cylinder having a length L and inside and outside radii r_i and r_o respectively made up of a material having a thermal conductivity of k as shown in Figure 7.2. Inside and outside curved surfaces of the cylinder are exposed to uniform temperatures of T_1 and T_2 respectively. Here, the temperature gradient exists in radial direction only, hence we can apply Fourier equation as

$$\dot{Q} = -kA \frac{dT}{dr}$$

In this case, mean normal to the direction of heat flow is given by the curved surface area of the cylinder. Therefore, substituting $A = \frac{2\pi rL}{2\pi L}$ into Equation (7.7), we get

$$\dot{Q} = -k(2\pi rL) \frac{dT}{dr}$$

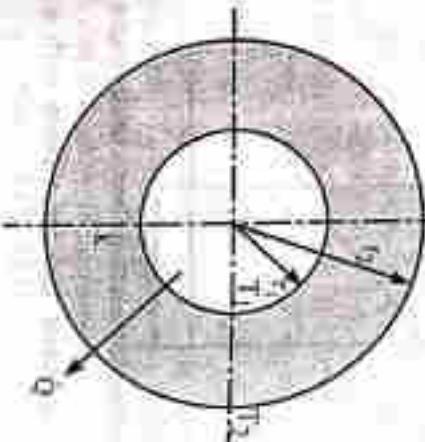


FIGURE 7.2 Radial Heat Conduction through a Hollow Cylinder

Rearranging Equation (7.8),

$$\dot{Q} \frac{dt}{r} = -k(2\pi L) dT \quad \text{.....(7.9)}$$

Integrating with associated boundary conditions,

$$\dot{Q} \int_{r_1}^{r_2} \frac{dt}{r} = - \int_{T_1}^{T_2} k(2\pi L) dT$$

$$\text{or, } \dot{Q} \ln \left(\frac{r_2}{r_1} \right) = -2\pi kL(T_2 - T_1) \quad \text{.....(7.10)}$$

Hence, the magnitude of radial heat conduction through a hollow cylinder is given by

$$\dot{Q} = \frac{2\pi kL(T_2 - T_1)}{\ln \left(\frac{r_2}{r_1} \right)} \quad \text{.....(7.11)}$$

7.3 Heat Conduction through Composite Structures

Arrangements of different materials designed for a specific application is called a composite structure.

7.3.1 Composite Plane Wall

Consider a composite plane wall consisting of three layers of three different materials shown in Figure 7.3. Thicknesses of each layers are L_1 , L_2 and L_3 , respectively and thermal conductivities of each layer are k_1 , k_2 and k_3 , respectively. Two opposite faces of the composite wall subjected to temperatures of T_1 and T_3 and interface temperatures are T_2 and T_4 , respectively.

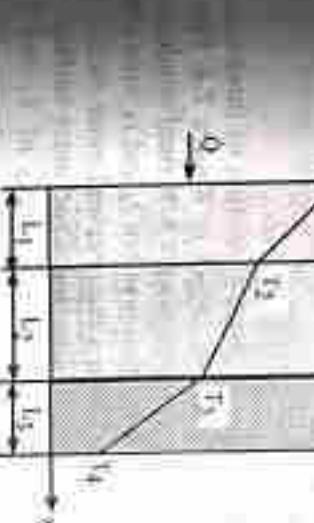


FIGURE 7.3 Heat Conduction through a Composite Plane Wall

For unidirectional heat transfer, heat flowing through each layer should

$$\dot{Q} = \frac{k_i A (T_i - T_{i+1})}{L_i}$$

$$\dot{Q} = \frac{k_1 A (T_1 - T_2)}{L_1} \quad \text{.....(7.12)}$$

$$\dot{Q} = \frac{k_2 A (T_2 - T_3)}{L_2} \quad \text{.....(7.12)}$$

Hence, the above equations for the temperature difference

$$T_1 - T_2 = \frac{\dot{Q}}{A} \frac{L_1}{k_1}$$

$$T_2 - T_3 = \frac{\dot{Q}}{A} \frac{L_2}{k_2} \quad \dots \dots \dots (7.13)$$

$$T_3 - T_4 = \frac{\dot{Q}}{A} \frac{L_3}{k_3}$$

Adding above equations, we get

$$T_1 - T_4 = \frac{\dot{Q}}{A} \left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \right) \quad \dots \dots \dots (7.14)$$

Rearranging Equation (7.14), we get an expression for conduction heat transfer through a composite plane wall as,

$$\dot{Q} = \frac{A(T_1 - T_4)}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \right)} \quad \dots \dots \dots (7.15)$$

7.5.2 Composite Hollow Cylinder

Consider a composite hollow cylinder consisting of three cylindrical layers of three different materials shown in Figure 7.4. First layer with inner and outer radii of r_1 and r_2 have a thermal conductivity of k_1 , second layer with inner and outer radii of r_2 and r_3 have a thermal conductivity of k_2 and third layer with inner and outer radii of r_3 and r_4 have a thermal conductivity of k_3 . Inner and outer surfaces of the composite layers are subjected to temperatures of T_1 and T_4 and interface temperatures are T_2 and T_3 respectively.

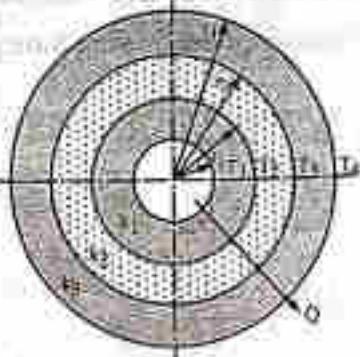


Figure 7.4 Heat Conduction through a Composite Cylindrical Layer

For steady state heat transfer, heat flowing through each layer should be same i.e.,

$$\begin{aligned} \dot{Q} &= \frac{2\pi k_1 L (T_1 - T_2)}{\ln \left(\frac{r_2}{r_1} \right)} \\ \dot{Q} &= \frac{2\pi k_2 L (T_2 - T_3)}{\ln \left(\frac{r_3}{r_2} \right)} \quad \dots \dots \dots (7.16) \\ \dot{Q} &= \frac{2\pi k_3 L (T_3 - T_4)}{\ln \left(\frac{r_4}{r_3} \right)} \end{aligned}$$

Rearranging the above equations for the temperature difference,

$$\begin{aligned} T_1 - T_2 &= \frac{\dot{Q}}{2\pi L} \frac{\ln \left(\frac{r_2}{r_1} \right)}{k_1} \\ T_2 - T_3 &= \frac{\dot{Q}}{2\pi L} \frac{\ln \left(\frac{r_3}{r_2} \right)}{k_2} \quad \dots \dots \dots (7.17) \\ T_3 - T_4 &= \frac{\dot{Q}}{2\pi L} \frac{\ln \left(\frac{r_4}{r_3} \right)}{k_3} \end{aligned}$$

Adding above equations, we get

$$T_1 - T_4 = \frac{\dot{Q}}{2\pi L} \left(\frac{\ln \left(\frac{r_2}{r_1} \right)}{k_1} + \frac{\ln \left(\frac{r_3}{r_2} \right)}{k_2} + \frac{\ln \left(\frac{r_4}{r_3} \right)}{k_3} \right) \quad \dots \dots \dots (7.18)$$

Rearranging Equation (7.18), we get an expression for conduction heat transfer through a composite cylinder as,

$$Q = \frac{2\pi L (T_1 - T_3)}{\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \right)} \quad \dots \dots \dots (7.19)$$

7.6 Electric Analogy for Heat Transfer and Thermal Resistance

Complex heat transfer problems can also be solved by converting the given problem into an equivalent electric circuit and applying the appropriate network theorem. To compare the analogous parameters, we can start with Ohm's law as

$$I = \frac{\Delta V}{R} \quad \dots \dots \dots (7.20)$$

which shows that current (I) flows due to the difference in electric potential (ΔV) and the property of the substance by virtue of which it opposes the flow of current through it is called the electric resistance (R).

We can express heat transfer equation in the similar manner as

$$Q = \frac{\Delta T}{R_{th}} \quad \dots \dots \dots (7.21)$$

which shows that heat (Q) flows due to the difference in temperature (ΔT) and the property of the substance by virtue of which it opposes the flow of heat through it is called the thermal resistance (R_{th}). Thermal resistance is expressed in k/W or °C/W.

Thermal Resistance of a Plane Wall

Heat transfer through a plane wall given by Equation (7.6) can also be expressed as

$$\dot{Q} = \frac{T_1 - T_3}{L/Ak} \quad \dots \dots \dots (7.22)$$

Comparing Equations (7.21) and (7.22), we get an expression for the thermal resistance of a plane wall as

$$(R_{th})_{\text{plane wall}} = \frac{1}{Ak} \quad \dots \dots \dots (7.23)$$

Thermal Resistance of a Hollow Cylinder

Heat transfer through a hollow cylinder given by Equation (7.11) can also be expressed as

$$\dot{Q} = \frac{T_1 - T_3}{\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \right) 2\pi L} \quad \dots \dots \dots (7.24)$$

Comparing Equations (7.21) and (7.24), we get an expression for the thermal resistance of a hollow cylinder as

$$(R_{th})_{\text{hollow cylinder}} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} \quad \dots \dots \dots (7.25)$$

Thermal Resistance of a Convective Layer

Convection heat transfer through fluid layer given by Equation (7.2) can also be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{\frac{1}{hA}} \quad \dots \dots \dots (7.26)$$

Comparing Equations (7.21) and (7.26), we get an expression for the thermal resistance of a convective layer as

$$(R_{th})_{\text{convective layer}} = \frac{1}{hA} \quad \dots \dots \dots (7.27)$$

7.6.1 Heat Transfer through a Composite Plane Wall Using Electric Analogy Approach

Equivalent electric circuit for a composite plane wall shown in Figure 7.1 is shown in Figure 7.5.

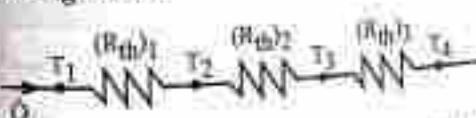


Figure 7.5: Equivalent Electric Circuit for a Composite Plane Wall

In this case same amount of heat flows through each layer; hence all thermal resistances are arranged in series. Whenever heat flows

through a thermal resistance, there will a temperature drop (analogous to voltage drop in case of electric circuit). Thermal resistances for each layers of the plane wall are given as

$$(R_{th})_1 = \frac{L_1}{Ak_1} \quad , \quad (R_{th})_2 = \frac{L_2}{Ak_2} \quad \text{and} \quad (R_{th})_3 = \frac{L_3}{Ak_3}$$

Equivalent thermal resistance is then given as

$$R_{eq} = (R_{th})_1 + (R_{th})_2 + (R_{th})_3 = \frac{1}{A} \left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \right) \quad (7.20)$$

Then overall heat transfer for a composite plane wall is given by

$$\dot{Q} = \frac{\Delta T}{R_{eq}} = \frac{T_1 - T_4}{\frac{1}{A} \left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \right)} = A \left(\frac{T_1 - T_4}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}} \right) \quad (7.21)$$

which is identical to Equation (7.15).

7.6.2 Heat Transfer through a Composite Cylinder Using Electric Analogy Approach

Equivalent electric circuit for a composite cylinder shown Figure 7.4 is shown in Figure 7.6.

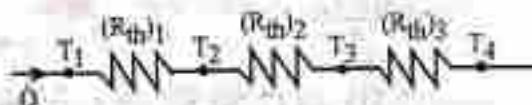


Figure 7.6: Equivalent Electric Circuit for a Composite Cylinder

In this case also same amount of heat flows through each layer; hence all thermal resistances are arranged in series. Thermal resistances for each layers of the composite cylinder are given as

$$(R_{th})_1 = \frac{\ln \left(\frac{r_2}{r_1} \right)}{2\pi k_1 L} \quad , \quad (R_{th})_2 = \frac{\ln \left(\frac{r_3}{r_2} \right)}{2\pi k_2 L} \quad \text{and} \quad (R_{th})_3 = \frac{\ln \left(\frac{r_4}{r_3} \right)}{2\pi k_3 L}$$

Equivalent thermal resistance is then given as

$$R_{eq} = (R_{th})_1 + (R_{th})_2 + (R_{th})_3 = \frac{1}{2\pi L} \left(\frac{\ln \frac{r_2}{r_1}}{k_1} + \frac{\ln \frac{r_3}{r_2}}{k_2} + \frac{\ln \frac{r_4}{r_3}}{k_3} \right) \quad (7.20)$$

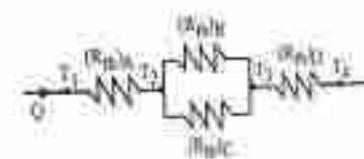
Then overall heat transfer for a composite cylinder is given by

$$\begin{aligned} \frac{\Delta T}{R_{eq}} &= \frac{T_1 - T_4}{\frac{1}{2\pi L} \left[\frac{\ln \left(\frac{r_2}{r_1} \right)}{k_1} + \frac{\ln \left(\frac{r_3}{r_2} \right)}{k_2} + \frac{\ln \left(\frac{r_4}{r_3} \right)}{k_3} \right]} \\ Q &= \frac{2\pi L \left(T_1 - T_4 \right)}{\left[\frac{\ln \left(\frac{r_2}{r_1} \right)}{k_1} + \frac{\ln \left(\frac{r_3}{r_2} \right)}{k_2} + \frac{\ln \left(\frac{r_4}{r_3} \right)}{k_3} \right]} \end{aligned} \quad (7.21)$$

Identical to Equation (7.19).

7.6.3 Application of Electric Analogy Approach

Electric analogy approach becomes more appropriate when the configuration of the wall becomes more complex. For example, for a complex plane wall shown in Figure 7.7(a), the equivalent electric circuit will be as shown in Figure 7.7(b).



(a)
Complex Plane Wall and its Equivalent Electric Circuit

Equivalent thermal resistance for the given circuit is then given by

$$R_{eq} = (R_{th})_B + \frac{(R_{th})_B \times (R_{th})_C}{(R_{th})_B + (R_{th})_C} + (R_{th})_D$$

7.7 Combined Heat Transfer and Overall Heat Transfer Coefficient

In real case, all modes of heat transfer will present, whenever there exists a temperature difference. Among these modes, we will consider the problems involving conduction and convection only.

7.7.1 Plane Wall Subjected to Convective Medium on both Sides

Consider a plane wall having a thickness of L and a thermal conductivity of k as shown in Figure 7.8. One of its face is exposed to a hot fluid with temperature T_A and convective heat transfer coefficient h_A while the opposite face is exposed to a cold fluid with temperature T_B and convective heat transfer coefficient h_B .

For steady state heat transfer, heat flow through each layer should be same, i.e.,

$$\begin{aligned} Q &= h_A A (T_A - T_1) \\ Q &= \frac{kA}{L} (T_1 - T_2) \\ Q &= h_B A (T_2 - T_B) \end{aligned} \quad \dots\dots\dots (7.21)$$

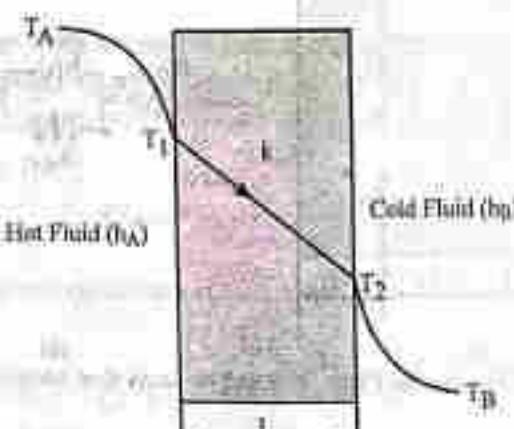


Figure 7.8: Plane Wall Subjected to Convective Medium on both Sides

Arranging the above equations for the temperature differences,

$$\begin{aligned} T_A - T_1 &= \frac{Q}{A} \frac{1}{h_A} \\ T_1 - T_2 &= \frac{Q}{A} \frac{L}{k} \\ T_2 - T_B &= \frac{Q}{A} \frac{1}{h_B} \end{aligned} \quad \dots\dots\dots (7.22)$$

Adding above equations, we get

$$T_A - T_B = \frac{Q}{A} \left(\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B} \right) \quad \dots\dots\dots (7.23)$$

Rearranging Equation (7.15), we get an expression for overall heat transfer coefficient for combined conduction and convection heat transfer as,

$$Q = \frac{A(T_A - T_B)}{\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B}} \quad \dots\dots\dots (7.24)$$

Equation (7.24) can also be expressed in simpler form as

$$Q = UA(T_A - T_B) \quad \dots\dots\dots (7.25)$$

$$\text{where } U = \frac{1}{\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B}} \quad \dots\dots\dots (7.26)$$

and overall heat transfer coefficient,

Alternative Method (Electric Analogy Approach)

Equivalent circuit for the heat transfer problem shown in Figure 7.8 is shown in Figure 7.9.



Figure 7.9: Equivalent Circuit for a Plane Wall Subjected to Convective Medium on both Sides

Overall thermal resistance for the circuit is then given by

$$R_{eq} = \frac{1}{A} \left(\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B} \right) \quad \dots\dots\dots (7.27)$$

Overall heat transfer for a composite plane wall is given by

$$Q = \frac{\Delta T}{R_{eq}} = \frac{(T_A - T_B)}{\frac{1}{A} \left(\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B} \right)} = \frac{A(T_A - T_B)}{\left(\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B} \right)} \quad \dots \dots (7.39)$$

which is identical to Equation (7.35).

7.7.2 Hollow Cylinder Subjected to Convective Medium on both Sides

Consider a hollow cylinder with inner and outer radii of r_1 and r_2 , length L , and thermal conductivity k , as shown in Figure 7.10. The inner surface of the cylinder is exposed to a hot fluid with temperature T_A and convective heat transfer coefficient of h_A , whereas the outer surface of the cylinder is exposed to a cold fluid with temperature T_B and convective heat transfer coefficient of h_B .

For steady state heat transfer, heat flow through each layer should be same, i.e.,

$$Q = h_A A_1 (T_A - T_1)$$

$$Q = \frac{2\pi k L}{\ln\left(\frac{r_2}{r_1}\right)} (T_1 - T_2) \quad \dots \dots (7.40)$$

$$Q = h_B A_2 (T_2 - T_B)$$

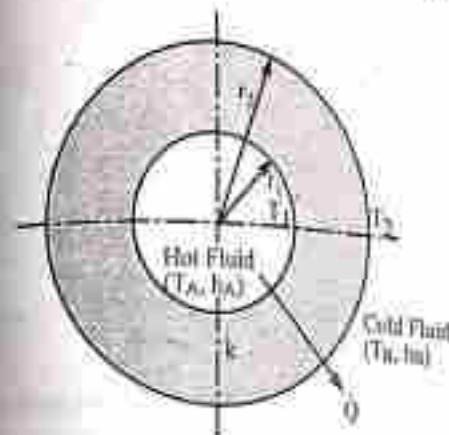
where A_1 and A_2 are the inside and outside curved surface areas respectively.

Rearranging the above equations for the temperature differences,

$$T_A - T_1 = \frac{Q}{A_1 h_A} \quad \dots \dots (7.41)$$

$$T_1 - T_2 = Q \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} \quad \dots \dots (7.42)$$

$$T_2 - T_B = \frac{Q}{A_2 h_B} \quad \dots \dots (7.43)$$



Hollow Cylinder Subjected to Convective Medium on both Sides

Using above equations, we get

$$T_A - T_B = Q \left(\frac{1}{h_A A_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} + \frac{1}{h_B A_2} \right) \quad \dots \dots (7.44)$$

Combining Equation (7.42), we get an expression for overall heat transfer coefficient for combined conduction and convection heat transfer as,

$$\frac{1}{U} = \left(\frac{1}{h_A A_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} + \frac{1}{h_B A_2} \right)^{-1} \quad \dots \dots (7.45)$$

Equation (7.43) can also be expressed as

$$Q = \frac{A_1 (T_A - T_B)}{\left(\frac{1}{h_A} + \frac{A_1}{2\pi k L} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_B A_2} \right)} \quad \dots \dots (7.46)$$

$$Q = \frac{A_2 (T_A - T_B)}{\left(\frac{A_2}{A_1} \frac{1}{h_A} + \frac{A_2}{2\pi k L} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_B} \right)} \quad \dots \dots (7.47)$$

Equations (7.44) and (7.45) can also be expressed in the simpler form

and
where,

$$\dot{Q} = U_1 A_1 (T_A - T_B)$$

$$\dot{Q} = U_2 A_2 (T_A - T_B)$$

and
and

$$U_1 = \frac{1}{\frac{1}{h_A} + \frac{\Delta_1}{2\pi kL} \ln\left(\frac{D_2}{D_1}\right) + \frac{\Delta_1}{A_2} \frac{1}{h_B}}$$

$$U_2 = \frac{1}{\frac{1}{h_A} + \frac{\Delta_2}{2\pi kL} \ln\left(\frac{D_2}{D_1}\right) + \frac{1}{h_B}}$$

are called inside overall heat transfer coefficient and outside overall heat transfer coefficient respectively.

Substituting $A_1 = 2\pi r_1 L$ and $A_2 = 2\pi r_2 L$, Equations (7.48) and (7.49) reduce to

$$U_1 = \frac{1}{\frac{1}{h_A} + \frac{r_1}{k} \ln\left(\frac{D_2}{D_1}\right) + \frac{r_1}{r_2} \frac{1}{h_B}} \quad \dots \dots \dots (7.50)$$

and

$$U_2 = \frac{1}{\frac{r_2}{h_A} + \frac{r_2}{k} \ln\left(\frac{D_2}{D_1}\right) + \frac{1}{h_B}} \quad \dots \dots \dots (7.51)$$

Alternative Method (Electric Analogy Approach)

Equivalent circuit for the heat transfer problem shown in Figure 7.10 is shown in Figure 7.11.



Figure 7.11 Equivalent Circuit for a Hollow Cylinder Subjected to Convective Medium on both Sides

Equivalent thermal resistance for the circuit is then given by

$$R_{eq} = \frac{1}{h_A A_1} + \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi k L} + \frac{1}{h_B A_2} \quad \dots \dots \dots (7.52)$$

Then overall heat transfer for a composite plane wall is given by

$$\dot{Q} = \frac{\Delta T}{R_{eq}} = \frac{(T_A - T_B)}{\frac{1}{h_A A_1} + \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi k L} + \frac{1}{h_B A_2}} \quad \dots \dots \dots (7.53)$$

which is identical to Equation (7.43).

7.8 Nature of Heat Convection

As mentioned earlier, convection heat transfer occurs in fluid medium. According to the mechanism of fluid flow, convection heat transfer is classified into two types: free convection and forced convection.

Free Convection

Convection heat transfer process in which flow of fluid is caused by density gradient is called free convection or natural convection. Most common example of free convection is the cooling of a room without fan by natural circulation of air. During the free convection, flow rate of fluid is usually low and heat transfer take place at very low rate.

Forced Convection

Convection heat transfer process in which flow of fluid is caused by some external devices such as pump, fan, blower, etc. Example of forced convection is the cooling of a room by a fan. Heat transfer rate is relatively higher in forced convection due to increased mass flow rate of fluid.

7.9 Introduction to Heat Radiation

7.9.1 Nature of Heat radiation

Radiation is an electromagnetic phenomenon of varying wavelengths closely allied to the transmission of light and radio. It proceeds in straight lines at the speed of light, (3×10^8 m/s). This speed is the product of the wavelength and frequency of the radiation

$$C = \lambda v$$

where C is the speed of light, λ is wavelength and v is the frequency.

Unlike conduction, radiation requires no transfer medium between the emitting and receiving surfaces. In fact, any material medium between such surfaces could impede radiation transfer of energy. The classic example of energy transmission by radiation is the sun, which transmits abundant energy to the earth by this means.

The amount of energy (heat) transferred by conduction largely depends upon temperature difference rather than temperature level. But in radiation it is the temperature of the emitting surface that controls the quality of the energy transmitted. Radiation is very much a surface phenomenon and will leave the transmitting surface through a wide wavelength band. A surface will emit or absorb radiant energy without a temperature difference, but in order for the energy transfer to occur there must be a temperature difference between the exchange surfaces.

Figure 7.12 shows the general shape of the spectromagnetic curve of radiant power against wavelength for a black body. A black body which is defined as being black is a perfect radiator of energy.

In Figure 7.12 the height of the curve defines the quantity, and the general shape shows the quality of the emitted radiant energy. At the lower radiation temperature there is a shift towards emission of lower wavelength.

A radiating surface at a high temperature, perhaps over 800°C, will emit some wavelengths which are within the visible light spectrum, approximately 10^{-6} to 10^{-7} meters. At lower temperatures, less than 800°C, the radiation will be in infrared range, approximately 10^{-4} to 10^{-6} meters.

If a body emits energy across the black body wavelength spectrum but only a fraction of the power of the black body, then it is called a grey body. The term black and grey do not necessarily refer to the color of the body; they merely describe its effectiveness as a radiator. A black body is a perfect radiator, a grey body is not perfect radiator.

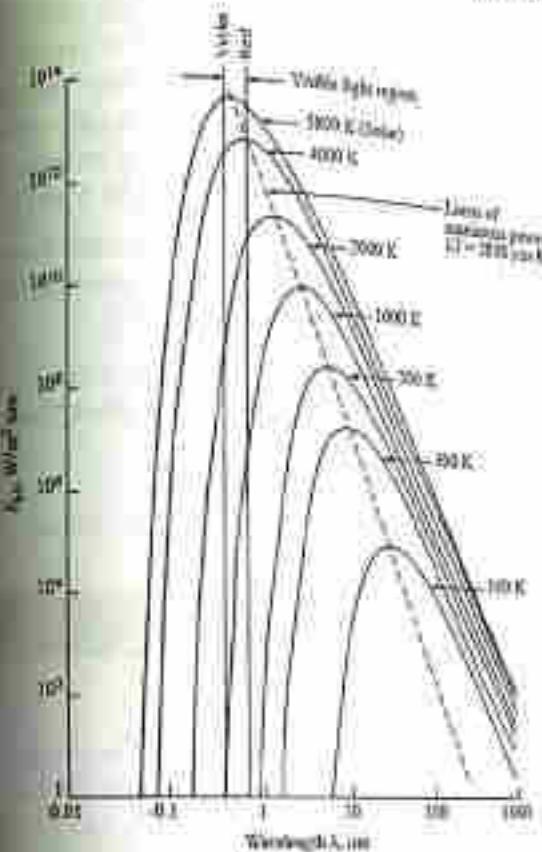


FIGURE 7.12 Spectromagnetic curves

7.43 Radiation Surface Properties

Surfaces are capable of emitting, absorbing, reflecting or transmitting radiant energy. It can be shown that the emissivity and absorptivity of a given surface are of equal value. For any given surface, the radiation received must be reflected, absorbed or transmitted through the material, i.e.,

$$E_{reflected} + E_{absorbed} + E_{transmitted} = E_{incident} \quad \dots \dots \dots (7.55)$$

Dividing both sides of Equation (7.55) by $E_{incident}$, we get

$$\frac{E_{reflected}}{E_{incident}} + \frac{E_{absorbed}}{E_{incident}} + \frac{E_{transmitted}}{E_{incident}} = 1 \quad \dots \dots \dots (7.56)$$

$$\alpha + \rho + \tau = 1$$

where α is the absorptivity which is defined as the ratio of absorbed energy to the total incident energy, ρ is the reflectivity which is defined as the ratio of reflected energy to the total incident energy and τ is the transmissivity which is defined as the ratio of transmitted energy to the total incident energy. Hence for any surface, the sum of absorptivity, reflectivity and transmissivity must be equal to unity.

For a black body which absorbs all the energy incident upon it, $\alpha = 1$ and hence $\rho = \tau = 0$. Similarly, for a white body which reflects all the energy incident upon it, $\rho = 1$, and hence $\alpha = \tau = 0$.

For a grey surface the emissivity, e_g , which is a ratio of the grey surface emission to the black surface emission, must therefore be of the form

$$e_g = \frac{E_g}{E_B} < 1 \quad \dots \dots \dots (7.5)$$

SOLVED EXAMPLES

Example 7.1

A furnace wall is made of 20 cm of magnesite brick and 20 cm of common brick. The magnesite brick is exposed to hot gases at 1200°C and common brick outer surface is exposed to 35°C room air. The surface heat transfer coefficient of the inside wall is $40 \text{ W/m}^2\text{K}$ and that of the outside wall is $20 \text{ W/m}^2\text{K}$ respectively. Thermal conductivities of magnesite and common brick are 4 and 0.5 W/mK respectively. Determine:

- heat loss per m^2 area of the furnace wall and
- maximum temperature to which common brick is subjected.

Solution

Given,

Thickness of magnesite brick:

$$L_1 = 20 \text{ cm} = 0.2 \text{ m}$$

Thickness of common brick:

$$L_2 = 20 \text{ cm} = 0.2 \text{ m}$$

Thermal conductivity of magnesite brick: $k_1 = 4 \text{ W/mK}$

Thermal conductivity of common brick: $k_2 = 0.5 \text{ W/mK}$

Heat transfer coefficient of the inner surface: $h_A = 40 \text{ W/m}^2\text{K}$

Heat transfer coefficient of the outer surface: $h_B = 20 \text{ W/m}^2\text{K}$

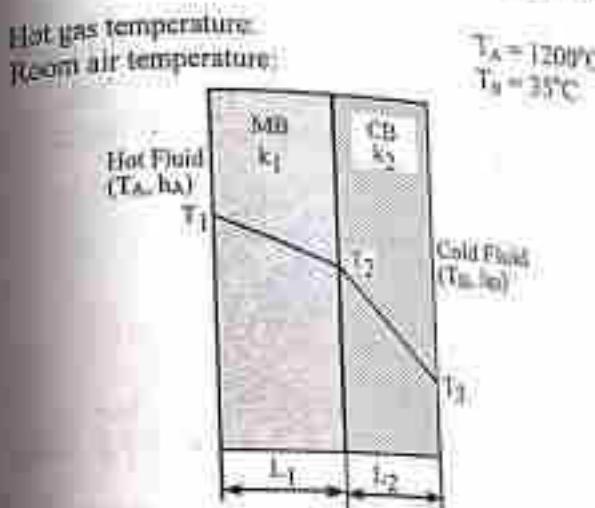


Figure E7.1 (a): Composite Plane wall for Example 7.1

Heat transfer per unit area for a composite plane wall subjected to convection on both sides is given as

$$\frac{\dot{Q}}{A} = \frac{T_A - T_B}{\frac{1}{h_A} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_B}}$$

$$= \frac{1200 - 35}{\frac{1}{40} + \frac{0.2}{4} + \frac{0.2}{0.5} + \frac{1}{20}} = \frac{1165}{0.525} = 2219.05 \text{ W/m}^2$$

The maximum temperature within the common brick is T_2 . Therefore, considering heat transfer by convection on hot gas and a magnesite brick layer,

$$\frac{\dot{Q}}{A} = \frac{T_A - T_2}{\frac{1}{h_A} + \frac{L_1}{k_1}}$$

Rearranging the above equation for T_2 ,

$$T_2 = T_A - \frac{\dot{Q}}{A} \left(\frac{1}{h_A} + \frac{L_1}{k_1} \right)$$

$$= 1200 - 2219.05 \left(\frac{1}{40} + \frac{0.2}{4} \right)$$

$$= 1200 - 166.429 = 1033.57^\circ\text{C}$$

Alternative Method (Electric Analogy Approach)

Equivalent circuit for the heat transfer problem shown in Figure E7.1(a) is shown in Figure E7.1(b). Equivalent thermal resistance for the circuit shown is given as

$$R_{eq} = \frac{1}{h_A A} + \frac{L_1}{Ak_1} + \frac{L_2}{Ak_2} + \frac{1}{h_B A}$$

$$= \frac{1}{A} \left(\frac{1}{h_A} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_B} \right) = \frac{1}{A} \left(\frac{1}{40} + \frac{0.2}{4} + \frac{0.2}{0.5} + \frac{1}{20} \right)$$

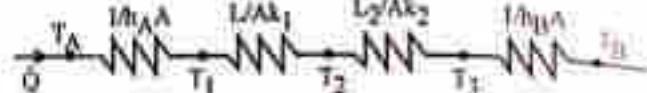


Figure E7.1 (b): Equivalent Circuit for the Composite Plane Wall for Example 7.1

Then overall heat transfer is given by

$$\dot{Q} = \frac{\Delta T}{R_{eq}} = \frac{(T_A - T_B)}{R_{eq}} = \frac{(1200 - 35)}{0.525/A}$$

$$\therefore \frac{\dot{Q}}{A} = \frac{1165}{0.525} = 2219.05 \text{ W/m}^2$$

Example 7.2

A steel pipe with ID and OD as 80 mm and 120 mm is covered with two layers of insulation, 25 mm and 40 mm thick. The thermal conductivities of insulating materials are 0.2 W/mK and 0.1 W/mK respectively while that of steel is 50 W/mK. The inner surface of the pipe is 200°C while surface temperature of insulation is 40°C. Determine the heat loss from the unit length of the pipe and layer contact temperatures.

Solution

Given,

Inner radius of pipe:

$$r_1 = 40 \text{ mm}$$

Outer radius of pipe:

$$r_2 = 60 \text{ mm}$$

Outer radius of first insulator:

$$r_3 = 60 + 25 = 85 \text{ mm}$$

Outer radius of second insulator: $r_4 = 85 + 40 = 125 \text{ mm}$

Thermal conductivity of pipe: $k_p = 50 \text{ W/mK}$

Thermal conductivity of first insulator: $k_1 = 0.2 \text{ W/mK}$

Thermal conductivity of second insulator: $k_2 = 0.1 \text{ W/mK}$

Inner surface temperature: $T_1 = 200^\circ\text{C}$

Outer surface temperature: $T_2 = 40^\circ\text{C}$

Heat transfer per unit length for the composite cylinder is then given as

$$\frac{\dot{Q}}{L} = \frac{2\pi (T_1 - T_2)}{\frac{1}{k_1} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}}$$

$$\frac{\dot{Q}}{L} = \frac{2\pi (200 - 40)}{\frac{1}{50} + \frac{\ln(85/60)}{0.2} + \frac{\ln(125/85)}{0.1}} = 378.139 \text{ W/m}$$

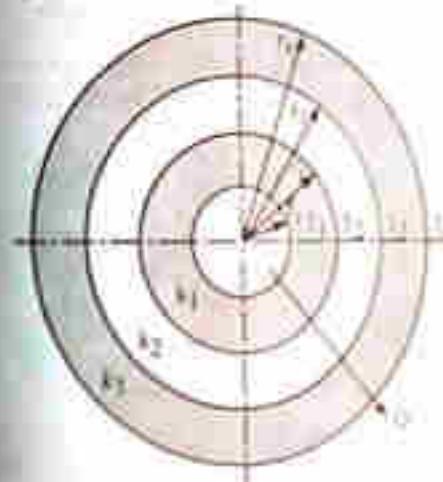


Figure E7.2 (a): Composite Cylinder for Example 7.2

Applying heat transfer equation for steel pipe we get

$$\dot{Q} = \frac{2\pi k_p (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

Assuming the above equation for \dot{Q} ,

$$T_2 - T_1 = \frac{(Q/L) \ln(r_2/r_1)}{2\pi k_1}$$

$$= 200 - \frac{179.3189 \times \ln(60/40)}{2\pi \times 50} = 199.769^\circ\text{C}$$

Again, applying heat transfer equation for first insulating layer,

$$\frac{Q}{L} = \frac{2\pi k_2 (T_2 - T_3)}{\ln\left(\frac{r_2}{r_3}\right)}$$

Rearranging the above equation for T_3 ,

$$T_3 = T_2 - \frac{(Q/L) \ln(r_3/r_2)}{2\pi k_2}$$

$$= 188.769 - \frac{179.3189 \times \ln(85/60)}{2\pi \times 0.2} = 150.066^\circ\text{C}$$

Alternative Method (Electric Analogy Approach)

Equivalent circuit for the heat transfer problem shown in Figures E7.2(a) is shown in Figure E7.2(b). Equivalent thermal resistance for each layer are given as

$$(R_{th})_1 = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_1 L} = \frac{\ln\left(\frac{60}{40}\right)}{2\pi \times 50 \times L} = \frac{1.29064 \times 10^{-3}}{L}$$

$$(R_{th})_2 = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_2 L} = \frac{\ln\left(\frac{85}{60}\right)}{2\pi \times 0.2 \times L} = 0.27717$$

$$(R_{th})_3 = \frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi k_3 L} = \frac{\ln\left(\frac{125}{85}\right)}{2\pi \times 0.1 \times L} = 0.61380$$



Figure E7.2 (b): Equivalent Circuit for the Composite Cylinder for Example 7.2

Equivalent thermal resistance is then given as

$$= (R_{th})_1 + (R_{th})_2 + (R_{th})_3$$

$$= 1.29064 \times 10^{-3} + 0.27717 + 0.61380 = \frac{0.892265}{L}$$

The overall heat transfer for a composite cylinder is given by

$$Q = \frac{\Delta T}{R_{eq}} = \frac{T_1 - T_4}{R_{eq}} = \frac{200 - 40}{0.892265} = 179.3189 \text{ W/m}$$

Applying heat transfer equation for the first thermal resistance only,

$$Q = \frac{T_1 - T_2}{(R_{th})_1}$$

$$T_2 = T_1 - Q(R_{th})_1 = 200 - Q \times \frac{1.29064 \times 10^{-3}}{L}$$

$$= 200 - \frac{Q}{L} \times 1.29064 \times 10^{-3}$$

$$= 200 - 179.3189 \times 1.29064 \times 10^{-3} = 199.769^\circ\text{C}$$

Again, applying heat transfer equation for the second thermal resistance only,

$$Q = \frac{T_1 - T_3}{(R_{th})_2}$$

$$T_3 = T_1 - Q(R_{th})_2 = 199.769 - Q \times \frac{0.27717}{L}$$

$$= 199.769 - \frac{Q}{L} \times 0.27717$$

$$= 199.769 - 179.3189 \times 0.27717 = 150.066^\circ\text{C}$$

PROBLEMS

- 4.23 A $10\text{ cm} \times 10\text{ cm}$ square plate ($k = 20 \text{ W/mK}$) 50 cm by 75 cm is maintained at 100°C . Heat is lost from the plate surface by convection and radiation to the ambient air at 20°C . If the emissivity of the surface is 0.9 and the convection heat transfer coefficient is $20 \text{ W/m}^2\text{K}$, determine the inside plate temperature.

Solution

Given,

Thickness of plate:

Cross sectional area of plate:

Thermal conductivity of plate:

Outside surface temperature of plate:

Ambient air temperature:

Emissivity of plate surface:

Convection coefficient:

$$L = 2.5 \text{ cm}$$

$$A = 50 \times 75 \text{ cm}^2$$

$$k = 50 \text{ W/mK}$$

$$T_2 = 300^\circ\text{C}$$

$$T_b = 20^\circ\text{C}$$

$$\epsilon = 0.9$$

$$h = 20 \text{ W/m}^2\text{K}$$

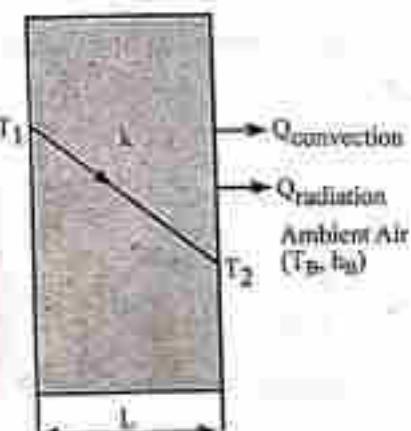


Figure E7.3: Plate for Example E7.3

For steady state heat transfer,

$$Q_{\text{radiation}} = Q_{\text{convection}} + Q_{\text{conduction}}$$

$$\frac{kA}{L}(T_1 - T_2) = hA(T_2 - T_b) + \epsilon\sigma A(T_2^4 - T_b^4)$$

$$\text{or, } \frac{k}{L}(T_1 - T_2) = h(T_2 - T_b) + \epsilon\sigma(T_2^4 - T_b^4)$$

Rearranging the above equation for T_1 ,

$$\begin{aligned} T_1 &= T_2 + \frac{L}{k} [h(T_2 - T_b) + \epsilon\sigma(T_2^4 - T_b^4)] \\ &= 300 + \frac{2.5 \times 10^{-2}}{50} [20(300 - 20) + 0.9 \times 5.67 \times 10^{-8} \times (573^4 - 293^4)] \\ &= 305.3625^\circ\text{C} \end{aligned}$$

Example E7.4

A pipe having an outside diameter of 2 cm is to be covered with two layers of insulation, each having a thickness of 1 cm. The outer conductivity of one material is 5 times that of the other. Assuming that the inner and outer surface temperatures of the outer insulation are fixed, calculate by what percentage the heat loss will be reduced when the better insulating material is used to be closer than it is away from the pipe.

Solution

Outside radius of the pipe:

$$r_2 = 1 \text{ cm}$$

Outer radius of first insulator:

$$r_3 = 1 + 1 = 2 \text{ cm}$$

Outer radius of second insulator:

$$r_4 = 2 + 1 = 3 \text{ cm}$$

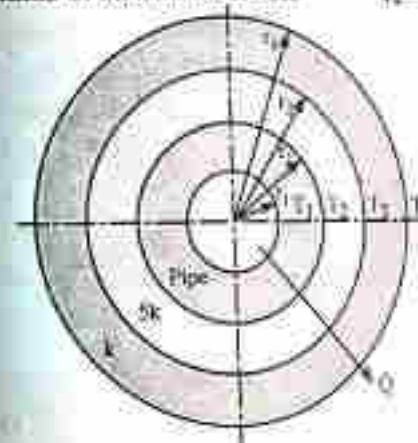


Figure E7.4(a): Heat transfer through composite cylinder when better insulating material is away from the pipe

Let conductivities of the insulating materials are k and $5k$. When better insulating material is away from the pipe as shown in Figure E7.4(a), heat transfer is given by

$$\begin{aligned} \left(\frac{Q}{A}\right) &= \frac{2\pi(T_2 - T_1)}{\ln(r_3/r_2) + \frac{\ln(r_4/r_3)}{5k}} \\ &= \frac{2\pi(T_2 - T_1)}{\ln(2/1) + \frac{\ln(3/2)}{5k}} = 11.54797k(T_2 - T_1) \end{aligned}$$

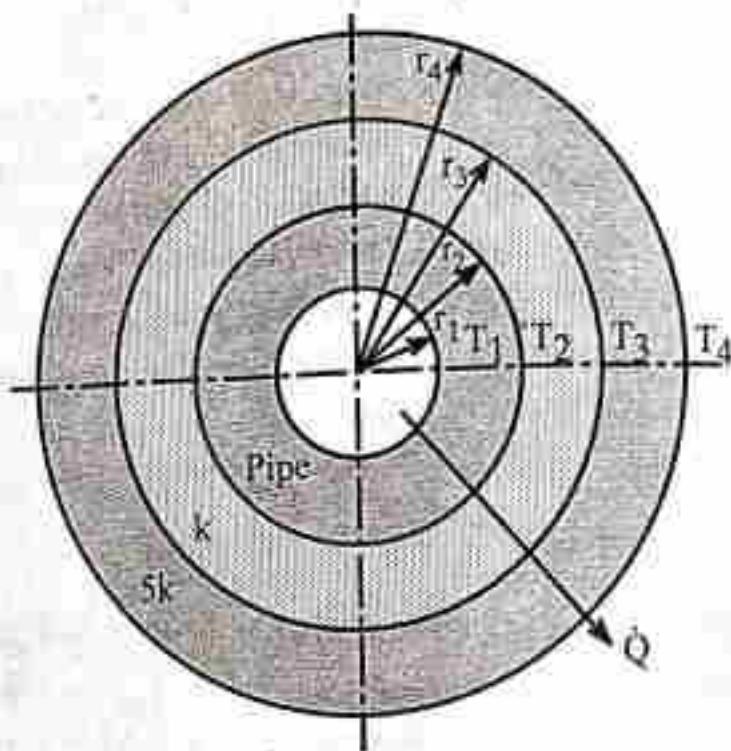


Figure E7.4(b): Heat transfer through composite cylinder when better insulating material is next to the pipe

Let conductivity of the insulating materials are k and $5k$. When better insulating material is next to the pipe as shown in Figure E7.4(b), heat transfer is given by

$$\begin{aligned} \left(\frac{\dot{Q}}{L} \right) &= \frac{2\pi(T_2 - T_4)}{\frac{\ln(r_3/r_2)}{k} + \frac{\ln(r_4/r_3)}{5k}} = \frac{2\pi(T_2 - T_4)}{\frac{\ln(2/1)}{k} + \frac{\ln(3/2)}{5k}} \\ &= 8.11529k(T_2 - T_4) \end{aligned}$$

Percentage reduction in heat transfer is then given by

$$\begin{aligned} &= \left[\frac{\left(\frac{\dot{Q}}{L} \right)_1 - \left(\frac{\dot{Q}}{L} \right)_2}{\left(\frac{\dot{Q}}{L} \right)_1} \right] \times 100\% \\ &= \left[1 - \frac{\left(\frac{\dot{Q}}{L} \right)_2}{\left(\frac{\dot{Q}}{L} \right)_1} \right] \times 100\% = \left[1 - \frac{8.11529k(T_2 - T_4)}{11.54797k(T_2 - T_4)} \right] \times 100\% \\ &= 29.725\% \end{aligned}$$

NUMERICAL PROBLEMS

1. An insulating material having a thermal conductivity of 0.05 W/mK is used to limit the heat transfer to 80 W/m² for a temperature difference of 150°C across the opposite faces. Determine the required thickness of the material.
 2. A brick wall 12 m thick and 5 m² surface area is exposed to 250°C at one face and 50°C to another face. If the thermal conductivity of the material is 1.5 W/mK, determine the heat transfer rate.
 3. The heat transfer rate through a wooden board of 3 cm thick for a temperature difference of 24°C between the inner and outer surface is 80 W/m². Determine the thermal conductivity of the board.
 4. Magnitude of conduction heat transfer through an insulating layer of 0.8 m² surface area, 5 cm thick and having a thermal conductivity of 0.25 W/mK is found to be 1600 W. Determine the temperature difference existing across the material.
 5. Determine the rate of heat loss from a brick wall ($k = 0.7 \text{ W/mK}$) of length 5 m, height 4 m and 0.25 m thick. The temperature of the inner surface is 120°C and that of outer surface is 30°C. Also calculate the distance from the inner surface at which temperature is 90°C.
 6. A hollow cylinder with inner and outer diameters of 8 cm and 12 cm respectively has an inner surface temperature of 200°C and an outer surface temperature of 50°C. If the thermal conductivity of the cylinder material is 60 W/mK, determine the heat transfer from the unit length of the pipe. Also determine the temperature at the surface at a radial distance of 5 cm from the axis of the cylinder.
 7. The inside and outside surface temperatures of a window are 20°C and 0°C, respectively. If the window is 80 cm by 50 cm and

is 6 cm thick and has a thermal conductivity of 0.8 W/mK . Determine the heat loss through the glass in 1 hour.

The roof of an electrically heated home is 10 m long, 8 m wide, 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is $k = 0.8 \text{ W/mK}$. The temperatures of the inner and the outer surfaces of the roof one night are measured to be 18°C and 5°C , respectively, for a period of 12 hours. Determine:

- (a) the rate of heat loss through the roof that night and
 (b) the cost of that heat loss to the home owner if the cost of electricity is Rs 10/kWh.

A plate having a surface area of 4 m^2 and temperature of 80°C is exposed to air at 25°C . If the heat transfer coefficient between the surface and air is $20 \text{ W/m}^2\text{K}$, determine the heat transfer rate from the plate to the air.

A 1.2 m long tube with outer diameter of 4 cm having outside temperature of 120°C is exposed to the ambient air at 20°C . If the heat transfer coefficient between the tube surface and the air is $20 \text{ W/m}^2\text{K}$, determine the heat transfer rate from the tube to the air.

An electric current is passed through a wire 2 mm in diameter and 10 cm long. The wire is submerged in the liquid water. During boiling of water temperature of water is 100°C and convection heat transfer coefficient is $4500 \text{ W/m}^2\text{K}$. Determine the power supplied to the wire to maintain the wire surface temperature at 120°C .

The heat flux at the surface of an electrical heater is 3500 W/m^2 . The heater surface temperature is 120°C when it is cooled by air at 30°C . What is the average convective heat transfer coefficient? What will the heater temperature be if the power is reduced so that the heat flux is 2500 W/m^2 ?

A 2 m long, 0.35 cm diameter electrical wire extends across a room at 20°C. Heat is generated in the wire as a result of joule heating, and the surface temperature of the wire is measured to be 150°C in steady operation. Also, the voltage drop across the electric current through the wire are measured to be 50 V and 1 A, respectively. Neglecting the effect heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the

14. Two very large plates are maintained at 1200°C and 40°C respectively. Calculate the heat transfer rate due to radiation per unit area. Assume black body properties.
15. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 5°C in winter and 26°C in summer. Determine the rate of radiation heat transfer between a person inside the house and the surrounding surfaces if the exposed surface area, the average outer surface temperature and the surface emissivity of the person are 1.4 m^2 and 30°C and 0.95, respectively.
16. A room is maintained at 22°C by an air conditioning unit. Determine the total rate of heat transfer from the person standing in the room if the exposed surface area and the average outer surface temperature of the person are 1.5 m^2 and 30°C , respectively, and the convection heat transfer coefficient is $10\text{ W/m}^2\text{K}$. Take surface emissivity as 0.95.
17. A steel pipe having an outer diameter of 4 cm is maintained at a temperature of 80°C in a room where the ambient temperature is 25°C . The emissivity of the surface is 0.8 and the convection heat transfer coefficient between the surface and air is $10\text{ W/m}^2\text{K}$. Determine the total heat loss from the unit length of the pipe.
18. A hot plate of length 80 cm, width 50 cm and thickness 4 cm is placed in air stream at 20°C . It is estimated that a total of 300 W is lost from the plate surface by radiation when it has a outer surface temperature of 250°C at steady state. If the convective heat transfer coefficient is $25\text{ W/m}^2\text{K}$ and the thermal conductivity of the plate is 50 W/mK , determine the inside surface temperature of the plate.
19. A flat plate solar collector is insulated at the back surface and exposed to solar radiation at the front surface. The front surface absorbs solar radiation at a rate of 800 W/m^2 and losses heat by convection to the ambient air at 25°C . If the heat transfer coefficient between the plate and the air is $20\text{ W/m}^2\text{K}$, determine the surface temperature of the plate.
20. A flat plate collector having collection efficiency of 80% is insulated at the back surface and exposed to solar radiation at the front surface. The front surface receives solar radiation at a rate of 850 W/m^2 and dissipates heat to the ambient air at 20°C both by convection and radiation. If the convection heat transfer coefficient between the plate and air is $16\text{ W/m}^2\text{K}$, determine the surface temperature of the plate.
21. A glass plate is insulated on the back and exposed to solar radiation at the front surface. The exposed surface of the plate has an emissivity of 0.7. If solar radiation is incident on the plate at a rate of 750 W/m^2 and the surrounding air temperature is 20°C , determine the surface temperature of the plate. Assume the convection heat transfer coefficient to be $40\text{ W/m}^2\text{K}$.
22. The inner surface of a 2 cm thick $50\text{ cm} \times 50\text{ cm}$ plate ($k = 10\text{ W/mK}$) is at 400°C . The outer surface dissipates heat by forced convection and radiation to the ambient air at 27°C . If the outer surface has an emissivity 0.85 and the convection heat transfer coefficient between the outer plate surface and the ambient air is $20\text{ W/m}^2\text{K}$, determine the outer surface temperature of the plate.
23. The inner surface of a 0.2 m thick wall ($k = 1\text{ W/mK}$) is exposed to combustion gas and its outer surface is exposed to ambient air at 20°C . The emissivity of the wall surface is 0.8 and the maximum heat transfer coefficient for the wall surface and air is $10\text{ W/m}^2\text{K}$. Under steady state condition, temperature at the outer surface of the wall is found as 75°C . Determine the wall inner surface temperature.
24. An oven covered with asbestos ($k = 0.2\text{ W/mK}$) has the inner and outer dimensions of $45\text{ cm} \times 60\text{ cm} \times 75\text{ cm}$ and $50\text{ cm} \times 65\text{ cm} \times 80\text{ cm}$ respectively. The inside wall temperature of the oven is 300°C and the outside wall temperature is 35°C . Determine the power input required to maintain the steady state conditions.
25. A furnace compartment consists of a cubical cavity of 1 m side. The bottom of the compartment is completely insulated. Determine the minimum thickness of the insulation ($k = 0.025\text{ W/mK}$) that must be applied to the top and side walls to ensure a heat loss of less than 400 W when the inner and outer surfaces are at -5°C and 30°C respectively.
26. Walls of a furnace $4\text{ m} \times 3\text{ m}$ are constructed from an inner fire brick ($k = 0.4\text{ W/mK}$) wall 30 cm thick, a layer of ceramic fiber insulation ($k = 0.2\text{ W/mK}$) 10 cm thick and steel protective layer ($k = 50\text{ W/mK}$) 4 mm thick. The inside temperature of the fire brick layer was measured as 500°C and

the temperature of the outside of the insulation at 50°C. Determine:

- the rate of heat loss through the wall,
- the temperature at the interface between fire brick layer and insulation layer, and
- the temperature at the outside surface of the steel layer.

27. A furnace is made of fireclay brick of thickness 0.3 m and thermal conductivity of 1.2 W/mK. The outside surface is to be insulated by an insulating material with the thermal conductivity of 0.05 W/mK. Determine the thickness of the insulating layer in order to limit the heat loss per unit area of the furnace wall to 1200 W/m² when the inside surface of the wall is at 900°C and the outside surface is at 25°C.

28. A furnace wall 300 mm thick is made up of an inner layer of fire brick ($k = 1$ W/mK) covered with a layer of insulation ($k = 0.05$ W/mK). The inner surface of the wall is at 1300°C and the cold surface is at 30°C. Under steady state condition, temperature at the interface is measured to be 1100°C. Determine:

- heat loss per unit area of the wall, and
- the thickness of each layer.

29. Find the heat transfer through the composite wall as shown in Figure P7.29. Assume one dimensional flow. The thermal conductivities of wall materials are $k_A = 150$ W/mK, $k_B = 30$ W/mK, $k_C = 65$ W/mK and $k_D = 50$ W/mK. All dimensions are in cm.

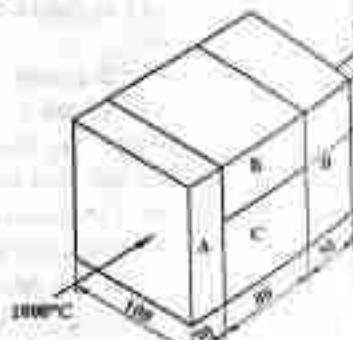


figure P7.29

30. An exterior wall of a house consists of 10 cm of common brick ($k = 0.8$ W/mK) followed by a 4 cm layer of gypsum plaster ($k = 0.5$ W/mK). What thickness of rock wool insulation ($k = 0.05$ W/mK) should be added to reduce the heat transfer through the wall by 50%?

A composite wall consists of 12 cm thick layer of common brick of thermal conductivity 0.8 W/mK and 4 cm thick plaster of thermal conductivity 0.5 W/mK. An insulating material of thermal conductivity 0.1 W/mK is to be added to reduce the heat transfer through wall by 75%. Determine the required thickness of the insulating layer.

31. A furnace wall is made of a layer of fire clay ($k = 0.5$ W/mK) 120 mm thick and a layer of red brick ($k = 0.8$ W/mK) 50 mm thick. If the wall temperature inside the furnace is 1200°C and 200°C on the outside wall is 100°C. Determine the heat loss per unit area of the wall. If it is desired to reduce the thickness of the firebrick layer by filling the space between the two layers by insulation ($k = 0.1$ W/mK) such that total thickness remains same. Determine the required thickness of the filling to ensure the amount of heat transfer for the same temperature difference is reduced by 60%.

32. A pipe ($k = 20$ W/mK) with inner and outer diameters of 3 cm and 4 cm respectively is covered with 2 cm layer of insulation ($k = 0.2$ W/mK). If the inside and outside surfaces of the insulation are at 500°C and 100°C respectively, determine the heat loss from the unit length of the pipe. Also determine the insulation interface temperature.

33. A cast iron pipe ($k = 25$ W/mK) with inner and outer diameters of 50 mm and 70 mm respectively is covered by an insulator ($k = 0.05$ W/mK). Under steady state condition, temperature between the pipe and insulator interface is found to be 250°C. The allowable heat loss from the unit length of the pipe is 500 W and outer surface temperature of the insulator should not exceed 300°C. Determine:

- minimum thickness of the insulation required, and
- temperature at the inner surface of the pipe.

34. A 200 mm diameter 50 m long pipe carrying steam is covered with 40 mm thick high temperature insulation ($k = 0.1$ W/m) and 20 mm thick low temperature insulation ($k = 0.05$ W/m). The inner and outer surfaces of the insulating layers are at 400°C and 30°C, respectively. Determine:

- the rate of heat loss from the pipe,
- the temperature at the interface of two insulating layers.

- (c) the rate of heat transfer from unit area of the pipe surface, and
 (d) the rate of heat transfer from unit area of the outer surface of the composite insulation.
36. A steel pipe ($k = 45.5 \text{ W/mK}$) with outer diameter of 90 mm and thickness 3 mm is used for flow of brine at -22°C . The pipe may be insulated by any one of the two types of insulation. Insulation I has $k_1 = 0.037 \text{ W/mK}$ and insulation II has $k_2 = 0.047 \text{ W/mK}$. If one of these insulations has to be used for pipe insulation so that maximum heat transfer is to be limited to 11.6 W/m of pipe and the temperature of insulation at the outer surface could be maintained not less than 15°C , determine the required thickness of insulation for each case.
37. A flat plate ($k = 100 \text{ W/mK}$) of 5 mm thick is exposed to a gas at a temperature of 150°C on one side and cooling water at 20°C on the opposite side. The heat transfer coefficients for inside and outside surfaces are $4000 \text{ W/m}^2\text{K}$ and $2000 \text{ W/m}^2\text{K}$ respectively. Determine the heat transfer per unit area of the plate and the temperatures at inner and outer surfaces of the plate.
38. A mild steel tank ($k = 45 \text{ W/mK}$) of wall thickness 15 mm contains water at 100°C . The heat transfer coefficients for the inside and outside surfaces of the tank wall are $2500 \text{ W/m}^2\text{K}$ and $20 \text{ W/m}^2\text{K}$ respectively. If the ambient air temperature is 20°C . Determine:
 (a) the rate of heat loss per unit area of the wall, and
 (b) the temperatures at the inner and outer surfaces of the tank.
39. The inner dimensions of a freezer compartments are $50 \text{ cm} \times 30 \text{ cm} \times 40 \text{ cm}$. Its wall consists of two 4 mm thick enameled steel sheet ($k = 45 \text{ W/mK}$) separated by 5 cm layer of fiber glass insulation ($k = 0.05 \text{ W/mK}$). The inside temperature is maintained at -10°C and the outside temperature on a hot summer day is 40°C . Calculate the rate at which heat should be thrown out if convective heat transfer coefficients for inner and outer surfaces are $20 \text{ W/m}^2\text{K}$ and $10 \text{ W/m}^2\text{K}$ respectively. Also calculate interface temperatures.
40. The interior of a refrigerator having inside dimensions of $0.4 \text{ m} \times 0.4 \text{ m} \times 0.8 \text{ m}$ is to be maintained at 0°C . The walls of the refrigerator are constructed of two mild steel sheets ($k = 50 \text{ W/mK}$) 2.5 mm thick with 40 mm of glass wool insulation ($k = 0.04 \text{ W/mK}$) between them. If the heat transfer coefficients at the inner and outer surfaces are $10 \text{ W/m}^2\text{K}$ and $20 \text{ W/m}^2\text{K}$ respectively. Determine:
 (a) the rate of heat removal from the interior of the refrigerator when the kitchen temperature is 25°C .
 (b) the temperatures at both sides of the glass wool insulation.
 (c) the maximum operating temperature of the kitchen oven is set at 500°C . Due to seasonal variations, the kitchen temperature varies from 5°C to 35°C . If the average heat transfer coefficient between the oven outer surface and the kitchen air is $20 \text{ W/m}^2\text{K}$, determine the required thickness of fiber glass ($k = 0.04 \text{ W/mK}$) insulation to ensure that the outside surface temperature of the oven does not exceed 50°C .
41. A turbine blade is modeled as a flat plate. The thermal conductivity of the blade material is 15 W/mK and its thickness is 5 mm. The upper surface of the blade is exposed to hot gases at 1000°C and the lower surface is cooled by air bleed of the compressor. The heat transfer coefficients at the upper and lower surfaces of the blade are $2500 \text{ W/m}^2\text{K}$ and $1500 \text{ W/m}^2\text{K}$ respectively. Under steady state conditions, the temperature at the upper surface of the blade is measured as 850°C , determine the temperature of the coolant air.
42. The inside surface of an insulating layer is at 300°C and the outside surface is dissipating heat by convection into air at 25°C . The insulating layer has a thickness 5 cm and thermal conductivity of 0.8 W/mK . What is the minimum heat transfer coefficient at the outside surface if the outside surface temperature should not exceed 100°C ?
43. A $1 \text{ m} \times 1 \text{ m}$ long steel plate ($k = 50 \text{ W/mK}$) is well insulated on its top and bottom sides, while its left section is maintained at 100°C and the right section is exposed to ambient air at 20°C . Under steady state conditions, a thermocouple inserted at the middle of the plate shows a temperature of 80°C . Determine the value of convection heat transfer coefficient for convection heat transfer between the right section of the plate and air.

45. A lake surface is covered by a 8 cm thick layer of ice ($k = 2.23 \text{ W/mK}$) when the ambient air temperature is -12.5°C . A thermocouple embedded on the upper surface of the layer indicates a temperature of -5°C . Assuming steady state conduction in ice and no liquid subcooling at the bottom surface of the ice layer. Find the heat transfer coefficient at the upper surface. Also work out the heat loss per unit area.
46. A composite wall is made up of three layers of thicknesses 20 mm, 100 mm and 120 mm with thermal conductivities of 15 W/mK , 3 W/mK , $k \text{ W/mK}$ and respectively. The inside surface is exposed to hot gas at 1250°C and the outside surface is at 30°C which is exposed to ambient air at 25°C . The heat transfer coefficients for inside and outside surfaces are $20 \text{ W/m}^2\text{K}$ and $10 \text{ W/m}^2\text{K}$ respectively. Determine:
- the unknown thermal conductivity k , and
 - the interface temperatures.
47. In a coal-fired power plant, a furnace wall consists of a 125 mm wide refractory brick and a 125 mm wide insulating firebrick separated by an air gap. The outside wall is covered with a 12 mm thickness of plaster. The inner surface of the wall is at 1100°C , and the room temperature is 10°C . The heat-transfer coefficient from the outside wall surface to the air in the room is $17 \text{ W/m}^2\text{K}$, and the resistance to heat flow of the air gap is 0.14 k/W . The thermal conductivities of the refractory brick, the insulating firebrick, and the plaster are 1.6, 0.3, and 0.14 W/mK , respectively. Calculate
- The rate of heat loss per unit area of wall surface.
 - The temperature at each interface throughout the wall.
 - The temperature at the outside surface of the wall.
48. A square plate heater ($10\text{cm} \times 10\text{cm}$) is inserted between two slabs having same cross-sectional areas. The left slab is 100 mm thick ($k = 50 \text{ W/mK}$) and the right slab is 50 mm thick ($k = 0.25 \text{ W/mK}$). The heat transfer coefficients for left and right slab outer surfaces are $250 \text{ W/m}^2\text{K}$ and $50 \text{ W/m}^2\text{K}$ respectively. The ambient air temperature is 25°C . If the rating of the heater is 1 kW ; determine:
- (i) temperature at the heater surface, and
(ii) outer surface temperatures of each slab.
49. A long steel pipe ($k = 50 \text{ W/mK}$) having an inside diameter 40 mm and outside diameter 120 mm is covered with two layers of insulation. The layer in contact with pipe is 30 mm thick asbestos ($k = 0.15 \text{ W/mK}$) and the layer next to it is 20 mm thick magnesium ($k = 0.1 \text{ W/mK}$). The heat transfer coefficients for the inner and outside surfaces are $240 \text{ W/m}^2\text{K}$ and $10 \text{ W/m}^2\text{K}$ respectively. If the temperature of the steam inside the pipe is 100°C and the ambient air temperature is 25°C . Determine:
- the inside overall heat transfer coefficient U_i ,
 - the outside overall heat transfer coefficient U_o ,
 - the heat transfer rate using U_i , and
 - the heat transfer rate using U_o .
50. A 140 mm diameter pipe carrying steam is covered by a layer of insulation ($k = 0.5 \text{ W/mK}$) of 30 mm thick. Later, an extra layer of another insulation ($k = 1 \text{ W/mK}$) having a thickness 20 mm is added. If the surrounding temperature remains constant and heat transfer coefficient for both insulating layers is $10 \text{ W/m}^2\text{K}$, determine the percentage change in heat transfer rate due to extra insulation.
51. A steam pipe ($k = 45 \text{ W/mK}$) has inside diameter of 100 mm and outside diameter of 140 mm. It is insulated at the outside with asbestos ($k = 1 \text{ W/mK}$). The steam temperature is 200°C and the outer temperature is 25°C . The heat transfer coefficients for inner and outer surfaces are $120 \text{ W/m}^2\text{K}$ and $40 \text{ W/m}^2\text{K}$ respectively. Determine the required thickness of the asbestos in order to limit the heat losses to 1250 W/m^2 .
52. A 100 mm diameter pipe carrying steam is covered by a layer of insulation ($k = 0.05 \text{ W/mK}$) having a thickness of 40 mm. The heat transfer coefficient between the outer surface of insulation and the ambient air is $20 \text{ W/m}^2\text{K}$. Determine the required thickness of another insulating layer ($k = 0.08 \text{ W/mK}$) that must be added to reduce the heat transfer rate by 40% assuming heat transfer coefficient remains same.