

# CSCD84 ①

AI - planning dynamic problems (adapting)

Applications of AI:

Robotics → Planning  
↳ Control  
↳ Mapping

• Language Processing

↳ Translation → { structure  
semantics  
distributions

↳ Question Parsing - Wolfram Alpha.  
↳ Search by Similarity - Google.

Games → Competitive playing against AI chess

Logic → Answering Questions

↳ Theorem Proving

↳ Automatic software verification (formal meth)

↳ Deduction

↳ Constraint satisfaction.

Decision Making → Expert Systems

Classification → spam/ham

↳ event recognition.

↳ Recognition

Agent

↑ Intelligent → An entity

↳ sense environment.

↳ carry out actions

↳ achieve given goal

Utility function ( )

maximize utility

- Reactive (Reflex)
- Simulate / Predict

Reactive



Cat & Mouse  $\leftarrow$  see a cat run away  
Smell choose - follow

Simulate & predict  $\rightarrow$  set of possible actions.  
utility function.

- look at maze choose safer path.

model of environment and other agents

SEARCH

Looking for optimal actions  
promising/good

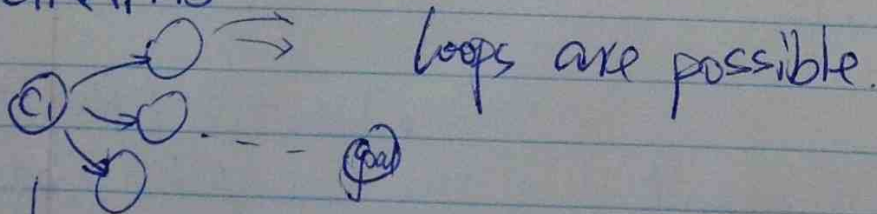
Scheduling  $\rightarrow$  try all possible solutions.

Components of Search problems  $\swarrow$  valid agent actions  
\* State Space  $\rightarrow$  All possible configurations of the  
environment & any agents  
eg: All possible places cats & mice can be.

\* function that determines goal

How to represent S.S

GRAPHS



**Tree**  $\rightarrow$  you don't miss any good (optimal) paths solution



## OSCD84 ②

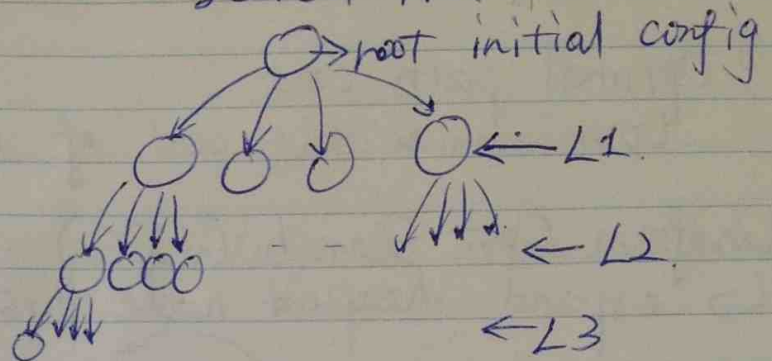
Successor function

- for any config, determines possible succeeding configs.
- Don't forget the path.

model  $\rightarrow$  agents



Search Tree



Search  
BFS. DFS.

Overview of general search alg  
while unexpanded nodes remain

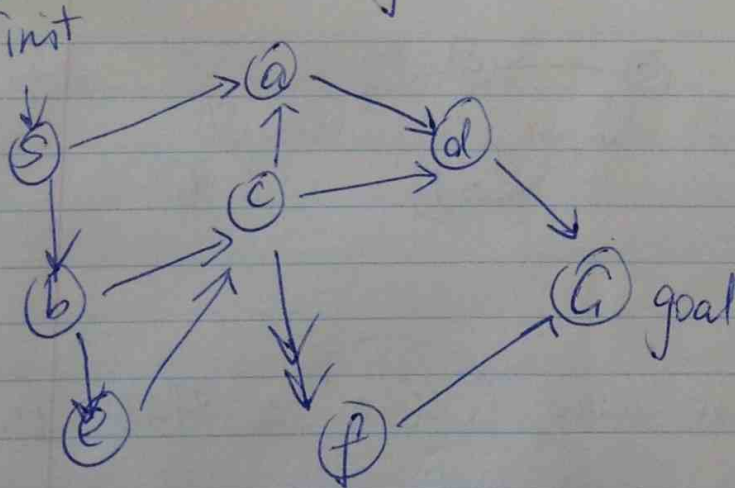
state/configuration  
possible sol'n.

← select node to expand

↳ check if it is a goal

↳ if not, add unexpanded children to list of nodes to search.

\* Policy  
DFS  
BFS



get a goal as fast as possible, get a better goal.

- multiple goal states
- some goals can be better than others.

→ what does it mean better goal?

- configuration/state is better
- often cost of path

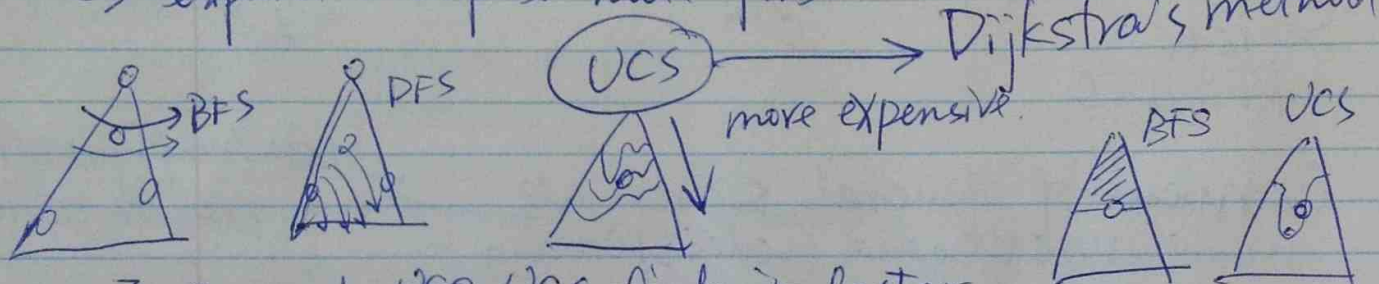
BFS/DFS don't care about the cost of actions

? Optimal path cost

↳ minimize the cost of agent's actions

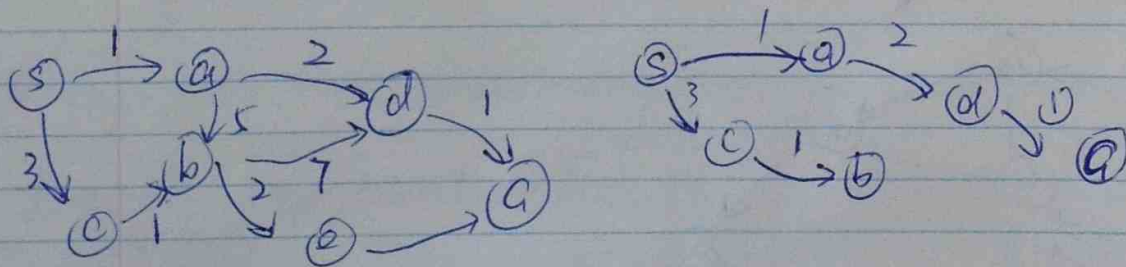
Uniform Cost Search (UCS)

↳ expand cheapest node first



In general, ~~use~~ UCS finds it faster.

UCS → cost distance from (S)  
cost from (S)



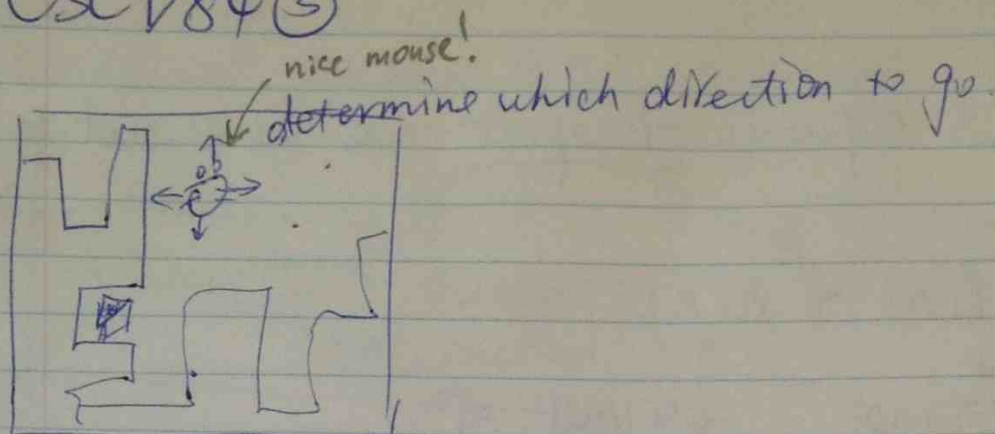
Heuristic Search

↳ Attempt to measure how "close" a state is to a goal  
↑  
cost of actions needed to get to

→ heuristic search cost  
Value of a node →  $f(n) = g(n) + h(n)$  → heuristic cost of getting a solution  
cost of getting to node from S



# CSC D84 ③



A\*

→ UCS → using a heuristic  $h(n)$  to estimate nearness to goal

Admissible heuristic

$h(n) \leq h^*(n)$  → don't over estimate  
estimated cost of actual cost

If  $h(n) > h^*(n)$  miss opt sol'n.

~~$h(n) \neq \phi$~~   $h(n) = \phi$   
 $h^*(n)$

Search BFS/DFS

→ opt sol'n (action taken)

• actions have a cost (different)

UCS ← Dijkstra's Method ← shortest path on graph

They missing → how close to the are we to goal

Search first down promising paths.

→ Question: less search means

- less nodes expanded (time)
- cost of agent actions!

A\* → like UCS expands cheapest available action first.

$$g(n) = d(n) + h(n)$$

$\downarrow$  distance (cost) from start       $\uparrow$  estimate of distance to goal

$$\begin{matrix} n_1 & d(n_1) < d(n_2) \\ n_2 & h(n_2) < h(n_1) \end{matrix}$$

$h(n)$  - heuristic

• admissible:

$h^*(n) \leftarrow$  actual cost of getting to Goal from  $n$ .

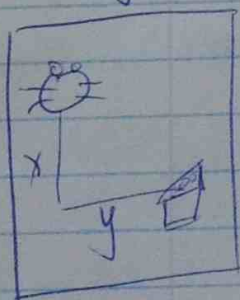
$$h(n) \leq h^*(n)$$

if  $h(n) > h^*(n) \leftarrow$  missing optimal solution.

• How to prove admissibility?

say  $h(n) =$  your function of state

argue  $h(n)$  (whatever it may be) can't be  $< h(n)$



$$h(n) = x(n) + y(n)$$

$$g(n) = \underline{d(n) + h(n)} \quad \text{add two parts.}$$

$$\max(h_1(n), h_2(n)) \checkmark$$

$h_1, h_2$  admissible.

$$h^*(n)$$

lowest  $h_1(n) = \phi$

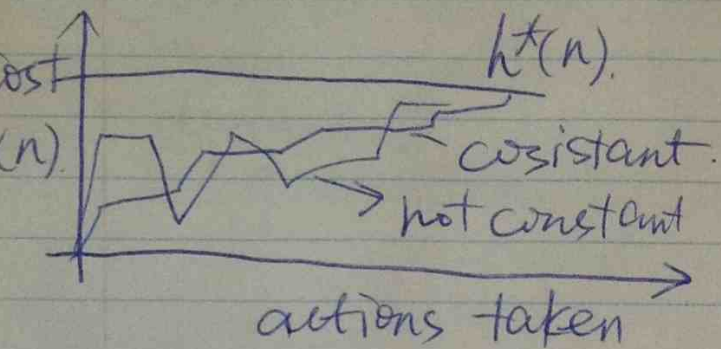
largest  $h_2(n) = h^*(n)$



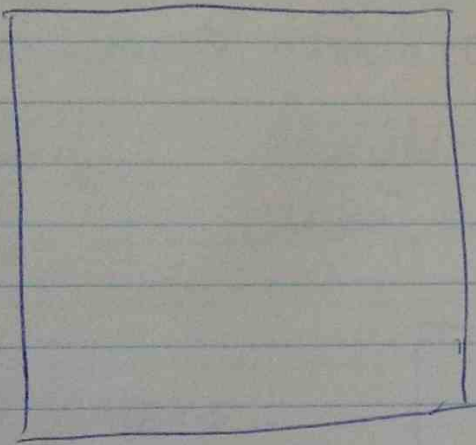


CSCD84 (4)

consistent means  $g(n)$  increases monotonically



still admis.  
not consistent



A\* no kitty.

# CSCD18 (5)

## Constraint Satisfaction

- Subset of search problems
- Set of variables  $x_i$  — can take values from some domain  $D$
- Set of constraints — single, pairs, sets

Ex:

### Scheduling

variables: times for  $i$  course lectures to take place

constraints: which course can't be at same time.

Domain

9:00  
10:00

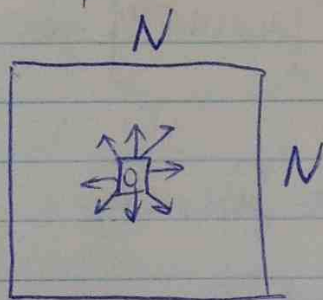
Variables

CSCB07, B08

Ex:

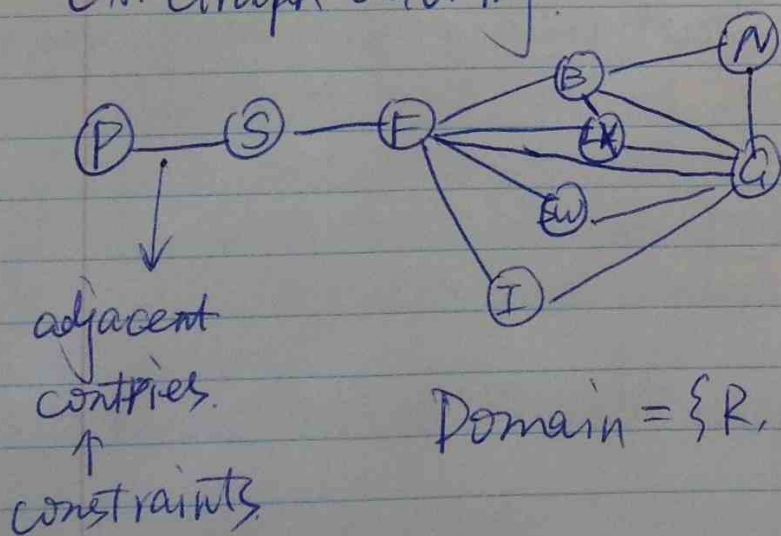
N-queens

N queens, can't  
attack each other



solve with search.

### Ex. Graph coloring



Domain = {R, G, B, Y}



## Types of CSP

a) Finite Domain — discrete.  
if  $n$  variables, if domain size  $d$ .  $O(d^n)$   
• Boolean SAT.

b) Infinite domains

c) Continuous Vars.  $\rightarrow$  Linear programming

## Types of constraints

Unary  $\rightarrow$  Spain is Red

binary  $\rightarrow S \neq F$

higher order  $\rightarrow$  hard.

Soft constraints  $\rightarrow$  penalty for breaking

## Solving CSPs

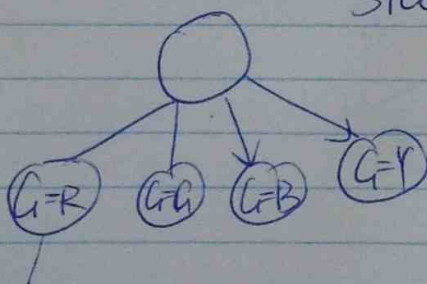
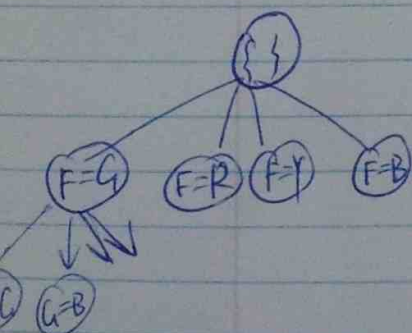
$\hookrightarrow$  Backtracking ~~tree~~ Search.

[Full] State (DFS)

— Full state is a solution.

Start with empty assignment.

next Full state.



Do we ever create entire search tree?

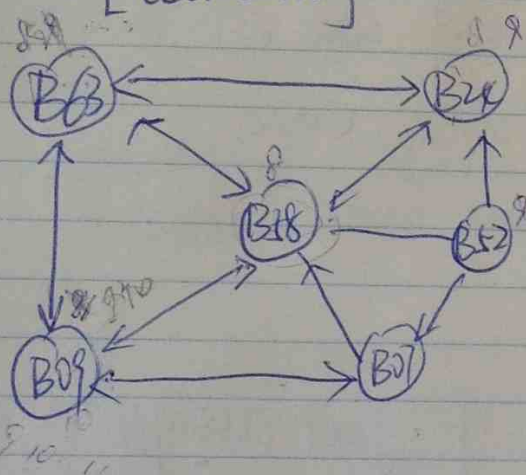
# CSCD84 ⑥

## CSPs - Recursive Backtracking

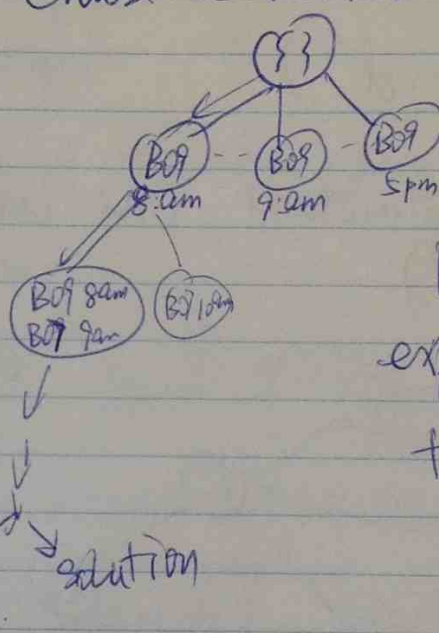
variables  $\rightarrow$   $x_i$  nodes  
edges  $\rightarrow$  constraints

values from a  
 $\rightarrow$  [domain]

$\{\}$   $\leftarrow$  tree  
 $\rightarrow$  contains some partial  
valid assignment of  
variables



Choose a variable



DFS

BFS is going to  
expand the whole  
tree.

## Improving Backtracking Search

② Choose order of variables carefully  
• which variable has most constraints

$\rightarrow$  places more constraints on remaining variables

① Choose first variable with fewest possible values left.

③ Once you choose a variable  $\rightarrow$  Choose the least  
constraining value first  
max chance of success.



## Arc Consistency

→ Once you assign a value to one variable, go check that all remaining variables have at least one valid value left.

$O(d^n)$  <sup># of vars</sup> Standard CSP.

size of ~~vars~~ domain

break problem into subproblems of size  $C$ .

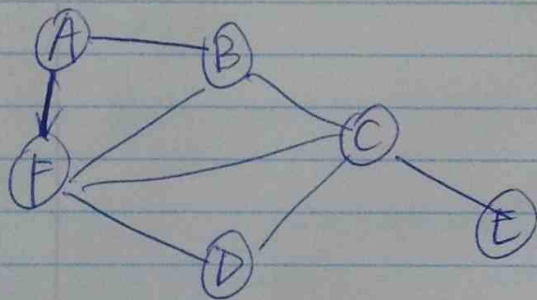
$$O\left(\frac{n}{C} \cdot d^C\right)$$

$$n=80, d=2, C=20.$$

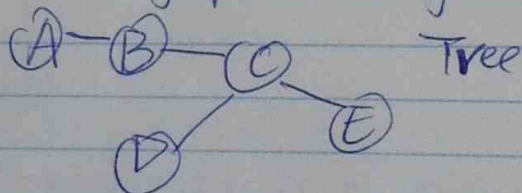
CSP - tree structure (no loops)

$$O(nd^2)$$

nearly  
T.S. CSP.



what if just assign  $F$  a value?  
 $F = \text{RED}$ . (graph coloring)



For each value of  $F$ , solve remaining T.S. CSP.

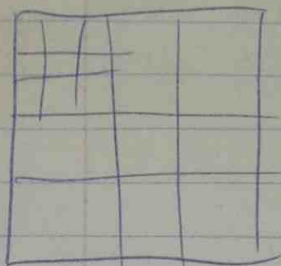
$$O(nd^2) \times d.$$

## Iterative Solutions

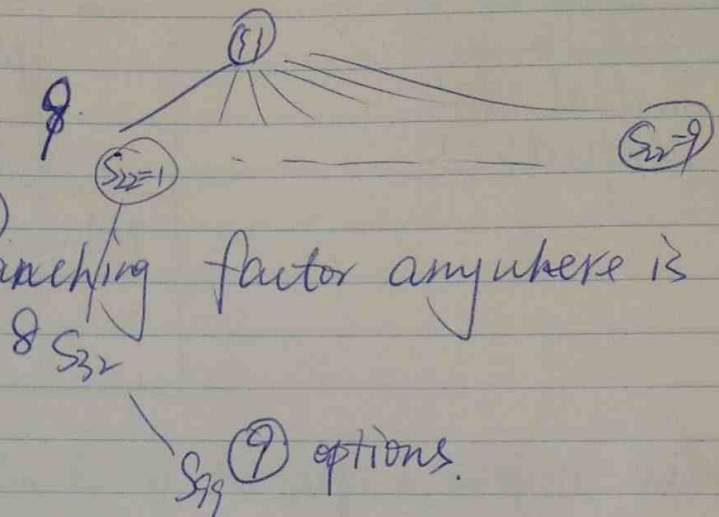
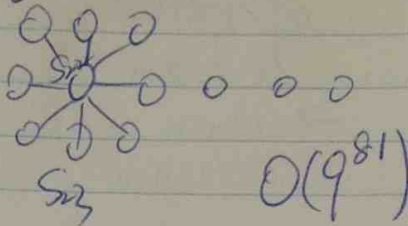
\* sub optimal — good enough.

CSCD84 (7)

Variables: empty squares (81). domain 1-9.



backtracking search.



Step ① choose var  $S(2,2)$

worst case branching factor anywhere is 9.

Reasonable solution in a reasonable time

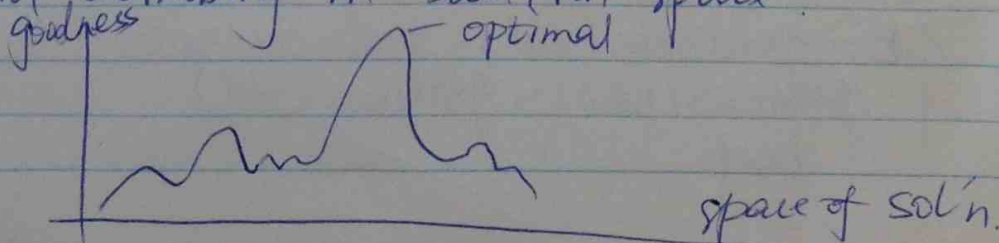
Iterative Method [Local Search]

- Always complete assignment (may not satisfy constraints)  
(Random Initial state)
- Operators — Reassign values to variables

at each step

choose 1 variable randomly  
choose different value / randomly  
check constraints  
if new better than old : keep  
otherwise — move on

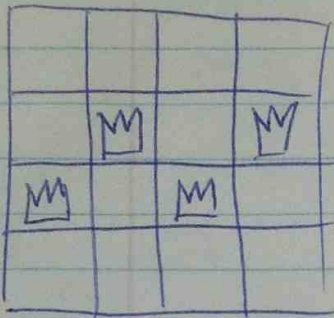
- Hill climbing in solution space.



Local max are a problem.

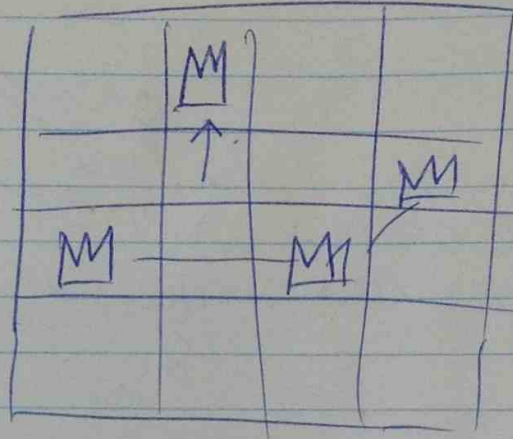


## 4 Queens



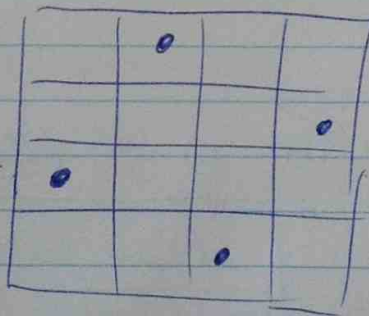
initial state  
conflicts = 5

operator  
↓  
choose 1 Queen randomly,  
→ then move it on the same  
column  
↓



conflicts = 2

solve n-queens for  
 $n = 10,000,000$   
in nearly constant time



→ Dealing with local max/min

→ Random re-starts, keep best sol'n

→ Simulated Annealing  
(Deterministic) "

D.A Search: → Initially random state  
→ start a Temperature =  $k$

Loop

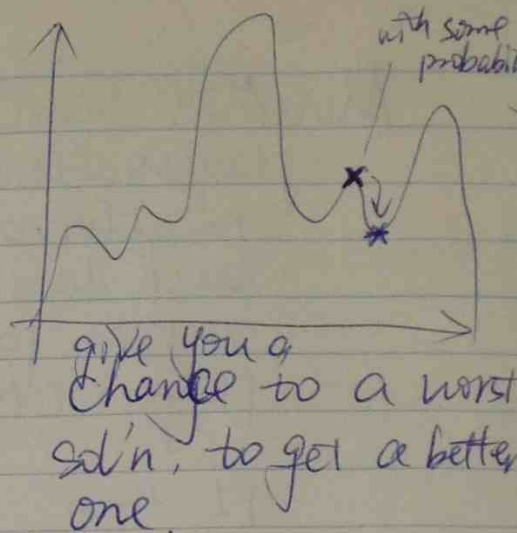
change 1 variable at random  
 $\Delta E = \text{goodness}(\text{old}) - \text{goodness}(\text{new})$   
if  $\text{goodness}(\text{new}) > \text{goodness}(\text{old})$   
keep

CSCD84 ⑧

otherwise:

Accept with probability  
proportional to  $e^{-\Delta E/T}$

← make  $T = T * \text{decay} \rightarrow [0, 1]$   
↓  
happen outside "if"



## GAME PLAYING

↳ Adversarial 2 player game

- pong • chess
- checkers • M vs C.

• Utility function

→ how do we represent game configurations?

x		o
x		o
x		

player 1 +100  
2 -100

game states

scoring game configurations

General Strategy


P1 - choose sequence of moves that maximize utility

P2 - Choose sequence of moves that minimize utility




# MiniMax Search


- players alternate
  - ↳ alternating min/max
- each level in search tree contains all possible moves for 1 player at 1 turn.
- players choose move with largest minimax value.

P1 turn  tic-tac-toe

9 choices

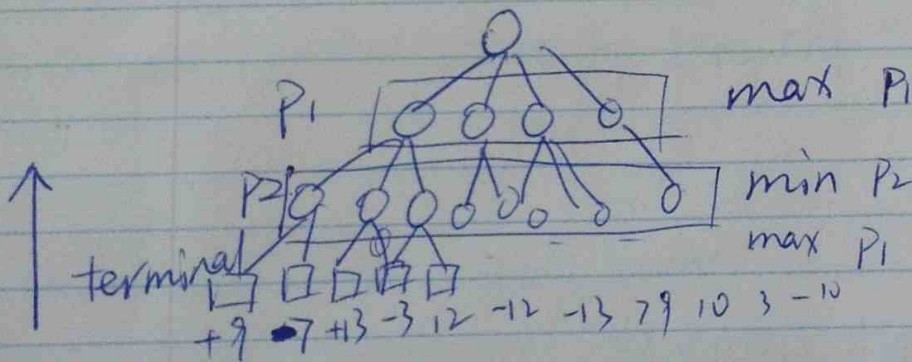
P2 turn 

8 choices



terminal node

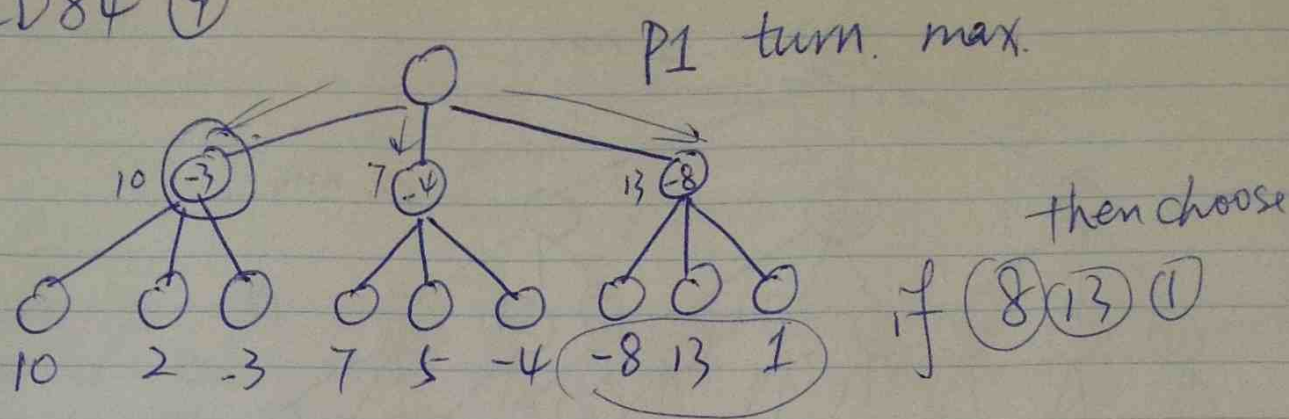
draw  
+ - 0



## Minimax

- Create starting node
- During the game
  - Determine turn
    - $\max()$   $\min()$
  - Search for successors to the current node up to a pre-defined depth
  - Evaluate each of the leaf nodes
- Utilities propagate up the tree alternating min/max

CSCD84 (9)



A3

minimax(s)

if terminal(s)

return utility(s)

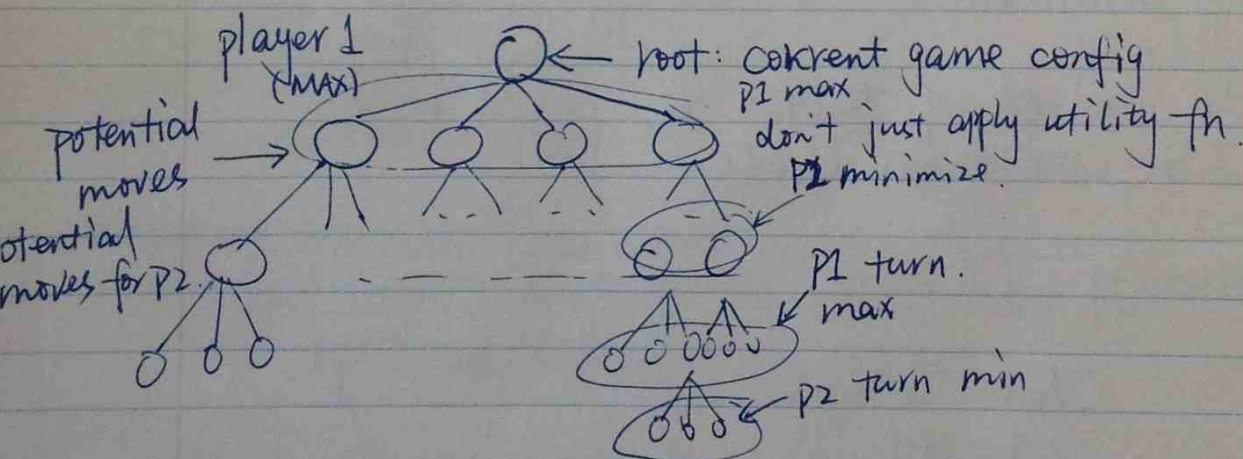
[for i in successors(s)  
v[i] = minimax(i)]

if s.type = max

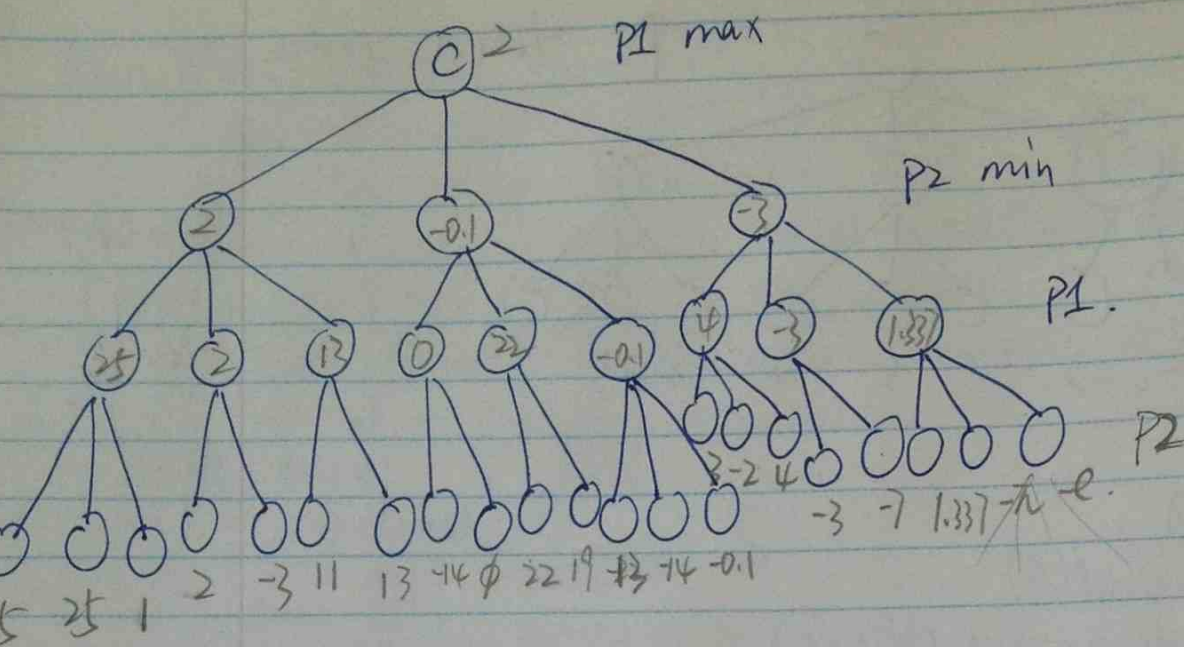
return max(v[i])

else

return min(v[i])







Search depth makes a difference

# of turns

1-ply \* # of players

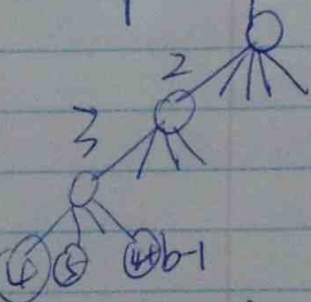
Properties of Minimax

- Time complexity

Tree with branching factor  $b$ ,  
and I'm looking  $m$  levels

$$O(b^m)$$

- Space Complexity  $O(mb)$



Chess  $b \approx 35$

$m = 100$

$$35^{100}$$

novice player = 4 ply

master  $\rightarrow$  8 ply

Grandmaster  $\rightarrow$  12 ply

## CSCD84 ⑩

Utility function:  $\rightarrow$  return goodness of configuration  
(+) favors player 1  
(-) favors player 2.

In practice, the only "certain" utility is at end-nodes  
partial games  $\rightarrow$  tricky (don't know how it

think of typical approach.  
— Select a number of features

— Linear combination

$$U = \sum_{i=1}^{N-F} w_i f_i$$

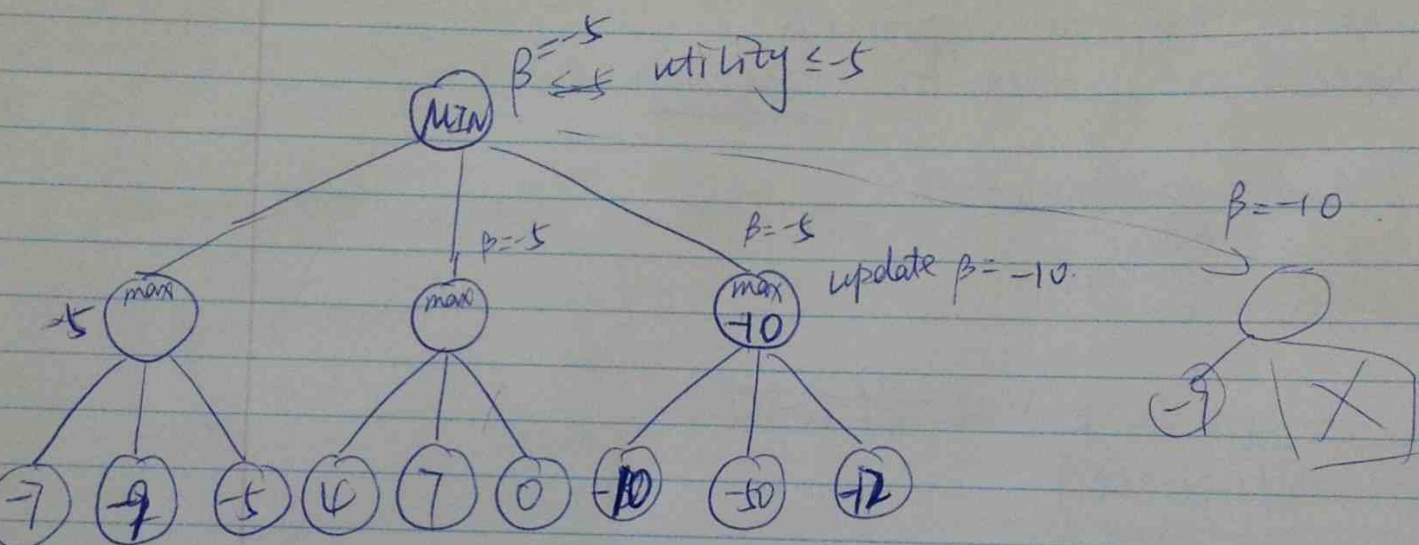
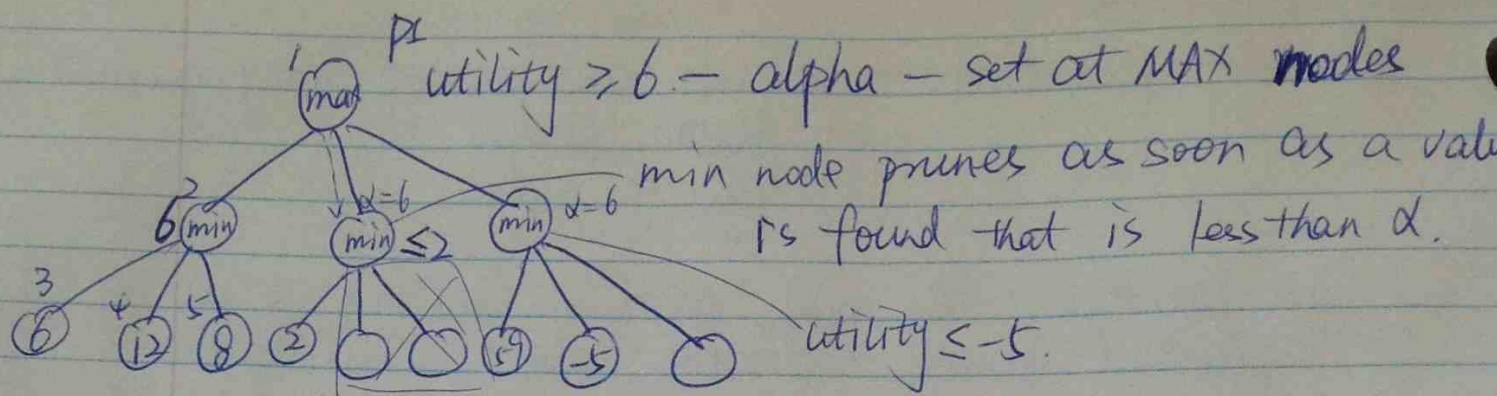
$$U = 200(K - K') + 9(Q - Q') + 5(P - P') + 3(B - B') + 3(K_n - K_n') + 1(P - P') + \text{other stuff.}$$

Reducing Search Tree complexity

- Prune (cut) parts of the tree that cannot be part of the optimal sequence of moves.

$\alpha$ - $\beta$  pruning — Branch & Bound.





- chop branching factor.

$$b \rightarrow x\sqrt{b}$$

Search twice as deep.

# CSCD84 (11)

## Minimax.

- Players are rational / play optimally.

- Modify Minimax, instead of [utility]

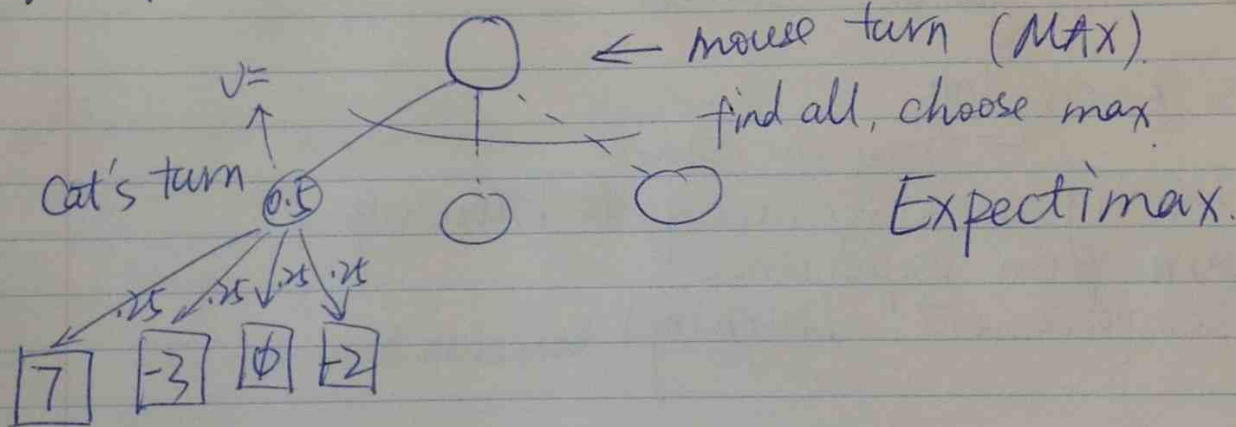
↳ Expected Utility.

Maximizing Expected

Utility = Rational agent

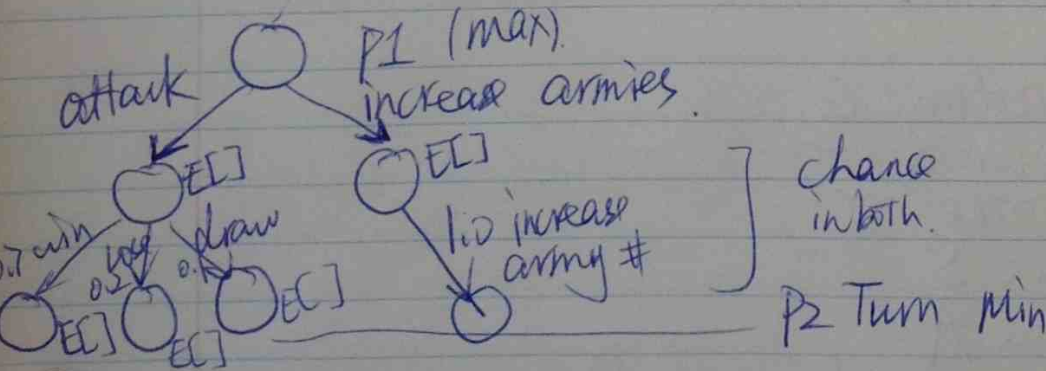
for games with chance  
 Actions map to multiple results.  
 ↳ randomness (dice, card shuffling)  
 ↳ insufficient evidence  
 ↳ unmodeled variables  
 ↳ noise

Example - mouse vs random cat.



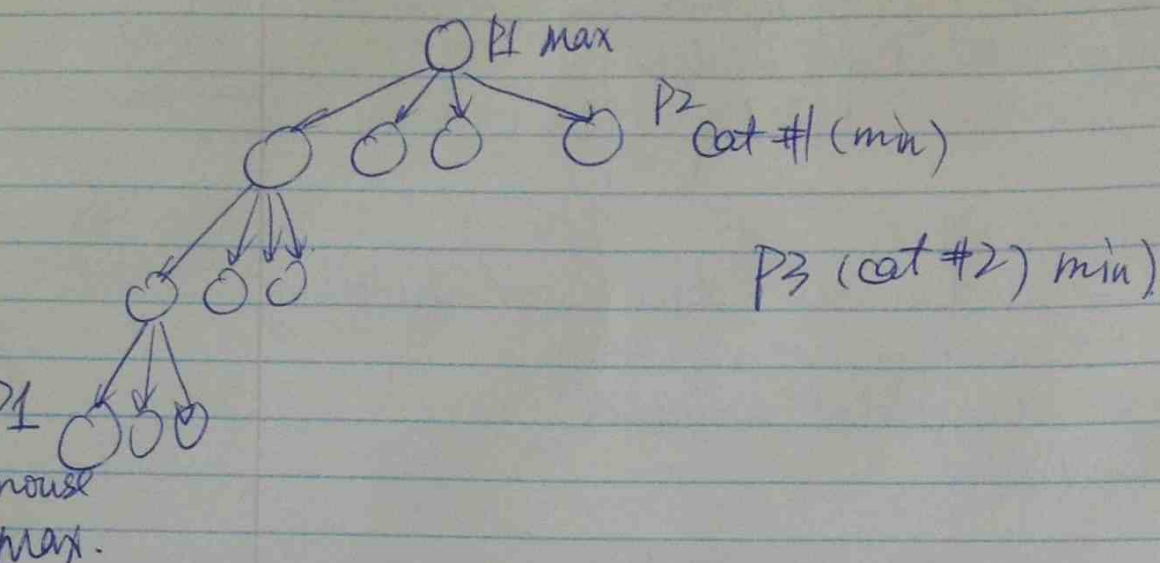
$$U = 0.25(7 - 3 + 0 - 2) = 0.5$$

Risk → ExpectiMinimax.





1 mouse 2 cats



Non zero-sum games — Scrabble

Agents know lots

↳ from the start

↳ Possibly missing / incomplete knowledge

→ Learn from experience  
(Reinforcement Learning)

Idea:

→ keep track of statistics of the utility (reward) of  
performing certain actions in certain states

• Agent receives input  $i$   
(state  $S$ )

• Agent carries out action  $a$ .

• Agent receives reward (+/-)  
↳ new state  $S'$

Model — set of states  $S$

set of actions  $A$

reinforcement signal (reward)

## CSCD84 (12)

Task for agent?

→ Come up with a policy  $\pi$  mapping states to actions, s.t. rewards are maximized over time

bad: Environment changing stats not apply.

↓  
Encode to  
apply stats.

Transition  $(s, a) \rightarrow s'$  - (chance is involved)

Transition function  $T(s, a, s') = p$  (probability)

Stats: on states ✓

on action ✓

on transitions → assume given.

on rewards ✓

Maximize Expected rewards.

$$E\left(\sum_{t=0}^{\infty} \gamma^t r_t\right)$$

discount factor  $(\gamma, 1)$

(I don't know whether can survive that long.  $\gamma^t \downarrow$  when  $t \uparrow$ )

↓  
Reward obtained at time  $t$ .