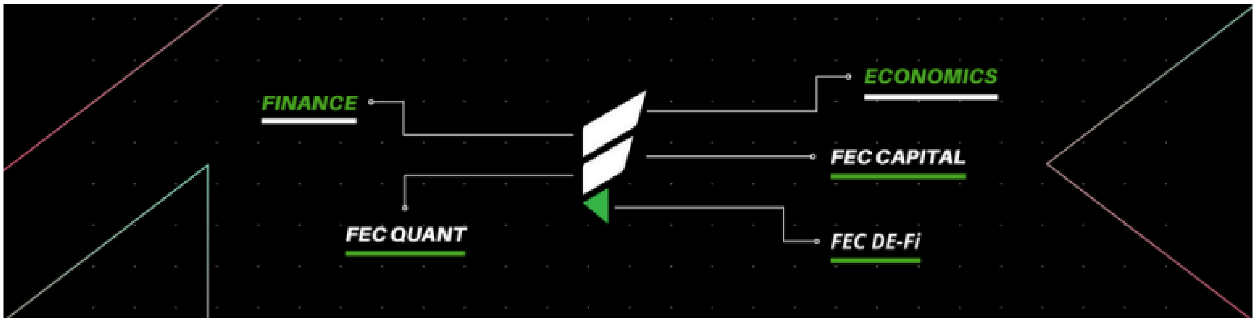


Game Theory Project

Abstract

This study examines two game theory scenarios: The Resource Allocation Challenge and Monopoly vs. Competition. In the Resource Allocation Challenge, participants take on roles as Water Manager or Water Users, engaging in three bidding rounds for limited water resources. The objective is to meet water needs at the lowest cost, demonstrating the strategic dynamics of resource allocation.

Monopoly vs. Competition simulates market dynamics, with one player as a monopolist and others as competitors. The game explores strategies like price-setting and market entry, highlighting the balance between maintaining monopoly power and fostering competition. Both games provide insights into strategic decision-making in resource management and market competition.

Introduction

Game theory is a mathematical framework for analyzing strategic interactions where the outcome depends on the actions of multiple decision-makers. It explores how individuals or groups choose strategies to maximize their payoffs, considering the potential actions of others. Key concepts include Nash Equilibrium, where no player can benefit from changing their strategy alone, and dominant strategies, which consistently offer better outcomes.

With applications in economics, politics, biology, and beyond, game theory explains competitive behaviors, negotiations, and resource allocations. It offers valuable insights into decision-making in complex scenarios.

Game 1: The Resource Allocation Challenge

Game Overview:

Roles:

- Water Manager: Allocates water to Farmers, Factories, and Homes.
- Water Users: Farmers, Factories, and Homes, who need water to operate and stay satisfied.

Objective:

- Water Users: Get enough water to meet your needs at the lowest cost over three rounds.Maximum quantity of water that can be taken in one round should be predefined and also minimum price of bidding should be revealed.

Assumptions

Rational Decision-Making: Water Users aim to minimize costs while meeting their needs, adjusting strategies based on round outcomes.

Fixed Supply and Bid Limits: The water supply, maximum allocation, and minimum bid prices are predefined and consistent throughout the game.

How to Play:

Round 1: Initial Allocation

- **Bidding:**
  - Water Users: Decide how much water they need and how much they are willing to pay. Submit bids to the Water Manager**.**
- **Water Allocation:**
  - Water Manager: Distributes the available water based on the bids. The highest bidders get their water first.
- **Check Results:**
  - Everyone sees how much water they received. Water Users see if they met their needs, and the Water Manager checks how much money they made.

Round 2: Adjust and Allocate

- **Bidding:**
  - Water Users: After seeing the results from Round 1, adjust your bid if needed and submit it.
- **Water Allocation:**
  - Water Manager: Distributes water again based on the new bids.
- **Check Results:**

    - Everyone checks their water allocation and adjusts their strategy for the final round.

Round 3: Final Allocation

- **Final Bids:**
    - Water Users: Make your final bid for water, considering how much you need to meet your goals.
- **Final Allocation:**
    - Water Manager: Distributes the last round of water.
- **Final Results:**
    - Water Users: See if you got enough water over all three rounds to meet your needs.

Winning the Game:

- **Water Users:** The winners are those who met their water needs at the lowest cost.

Payoff Matrix Construction:

Key Variables:

1. Water Supply (W): Total amount of water available in each round.
2. Bid Price (P): Minimum bid price for water.
3. Water Needs (N): The amount of water each Water User needs.
4. Cost Function: The total cost for each Water User, calculated as the bid price multiplied by the amount of water received.
5. Satisfaction Function: A Water User is satisfied if they receive at least their water needs. Satisfaction leads to a payoff.

Define the Parameters:

- Assume a fixed total water supply W= 100 units per round.
- Minimum bid price P= 10 per unit of water.
- Water needs are predetermined for each Water User. For example, Farmer needs 30 units, Factory needs 40 units, and Home needs 20 units.

Bid Range:

- Bids range from the minimum price P to some upper limit L. For simplicity, let’s assume the upper limit is 50 per unit of water.

Payoff Calculation:

- Cost for each Water User: Cost= Bid Price × Amount of Water Received

- Satisfaction: If a Water User receives at least their required amount of water, their payoff is, Satisfaction Payoff = Water Needs − Cost.

If not satisfied, their payoff is negative and equals −Cost.

Bidding Strategy:

- Water Users will need to balance their bids to secure enough water while minimizing their total cost.

Assumptions for simplicity:

- There are 3 Water Users: Farmer (F), Factory (Fac), and Home (H).
- Water needs are as follows: Farmer needs 30 units, Factory needs 40 units, Home needs 20 units.
- Bidding range is from 10 to 50.

Bids (Units of Water)	Bid Price	Farmer's Cost	Factory's Cost	Home's Cost	Farmer's Payoff	Factory's Payoff	Home's Payoff
30 units, 40 units, 20 units	20	600	800	400	0	0	0
25 units, 40 units, 30 units	25	625	800	750	-250	0	-730
35 units, 35 units, 30 units	30	1050	1225	900	-1050	-1225	-880

Identifying Best Responses

1. Best Response for the Farmer:

Scenario Analysis:

- Bid: 20, Others Bids: 30 units, 40 units, 20 units (Bid Price:₹ 20)
  - Farmer’s Cost: 30 units × ₹20 = ₹600
  - Payoff: 0 (meets needs without excess cost)
- Bid: 25, Others Bids: 40 units, 30 units, 20 units (Bid Price: ₹ 25)
  - Farmer’s Cost: 25 units × ₹25 = ₹ 625
  - Payoff: -250 (higher cost, possibly unmet needs)
- Bid: 30, Others Bids: 35 units, 35 units, 30 units (Bid Price: ₹ 30)
  - Farmer’s Cost: 30 units × ₹ 30 = ₹ 900
  - Payoff: -1050 (high cost, potentially unmet needs)

Best Response: Based on the payoffs, bidding ₹ 20 (where the Farmer meets their needs without extra cost) is the best response as it results in the highest payoff (zero cost).

2. Best Response for the Factory:

Scenario Analysis:

- Bid: 20, Others Bids: 30 units, 40 units, 20 units (Bid Price: ₹ 20)
  - Factory's Cost: 40 units × ₹ 20 = ₹ 800
  - Payoff: 0 (meets needs)
- Bid: 25, Others Bids: 30 units, 40 units, 30 units (Bid Price: ₹ 25)
  - Factory's Cost: 40 units × ₹ 25 = ₹ 1000
  - Payoff: -250 (higher cost but meets needs)
- Bid: 30, Others Bids: 35 units, 35 units, 30 units (Bid Price: ₹ 30)
  - Factory's Cost: 40 units × ₹ 30 = ₹ 1200
  - Payoff: -1225 (high cost)

Best Response: Bidding ₹ 20 is optimal if it ensures meeting needs at the minimum cost. This results in zero net cost.

3. Best Response for the Home:

Scenario Analysis:

- Bid: 15, Others Bids: 30 units, 40 units, 20 units (Bid Price: ₹ 15)
  - Home's Cost: 20 units × ₹ 15 = ₹ 300
  - Payoff: 0 (meets needs)
- Bid: 20, Others Bids: 30 units, 40 units, 30 units (Bid Price:₹ 20)
  - Home's Cost: 20 units × ₹ 20 = ₹ 400
  - Payoff: -730 (higher cost, less favorable)
- Bid: 25, Others Bids: 35 units, 35 units, 30 units (Bid Price: ₹ 25)
  - Home's Cost: 20 units × ₹ 25 = ₹ 500
  - Payoff: -880 (high cost)

Best Response: Bidding ₹ 15 is optimal if it allows the Home to meet their needs at the lowest cost, resulting in zero net cost.

Summary of Best Responses

- **Farmer**: The best response is to bid ₹ 20. This allows the Farmer to meet their needs without incurring excess costs.
- **Factory**: The best response is to bid ₹ 20, as it minimizes costs and meets the need for 40 units of water.
- **Home**: The best response is to bid ₹ 15, which meets the water needs with the lowest cost.

Strategic Implications

1. **Conservative Bidding**: Each Water User's best response involves conservative bidding to minimize costs while ensuring they meet their needs. Bids at the minimum price are preferred if they are sufficient to meet needs.
2. **Adjusting Strategies**: If any Water User expects others to bid conservatively and get enough water, they might also adopt a similar strategy. Conversely, if there is an expectation of higher bids, they may need to increase their bids to secure their required water.

The Nash equilibrium in this scenario is:

- Farmer bids ₹ 20
- Factory bids ₹ 20
- Home bids ₹ 15

In this equilibrium:

1.Each Water User's strategy is the best response to the strategies of the others.

2. No Water User can improve their payoff by unilaterally changing their bid.

Google Collab Link for the game:

https://colab.research.google.com/drive/1GBRKPaZ2uXS3UG48mtMpiogUOEt0M3ug?usp=sharing

2. Game Theory: Monopoly vs. Competition

Players:

1. Potential New Entrant (Firm A) : A firm considering entering the market.
2. Monopolist (Firm B): A single firm that dominates the market.

Strategies:

- **Firm A:**
  - Enter: Enter the market.
  - Do Not Enter: Stay out of the market.
- **Firm B:**
  - Accommodate: Keep prices the same, accommodating Firm A.
  - Fight: Cut prices, leading to a price war.

Assumptions:

- The market is controlled by one dominant firm (the monopolist).
- Barriers to entry are significant, making it challenging for new firms to enter.

Gameplay:

  - The monopolist sets the initial price for the product.
  - Competitors can enter the market by setting up their production and choosing their prices.
  - Demand is split between the monopolist and competitors based on price and brand loyalty.
  - The game evolves as competitors try to undercut the monopolist or differentiate their products to gain market share.
  - The monopolist can adjust prices, invest in barriers to entry, or try to buy out competitors.
- Learning Outcome: This game demonstrates the dynamics between monopolies and competitive markets, showing how monopolies can maintain power or how competitors can disrupt them.

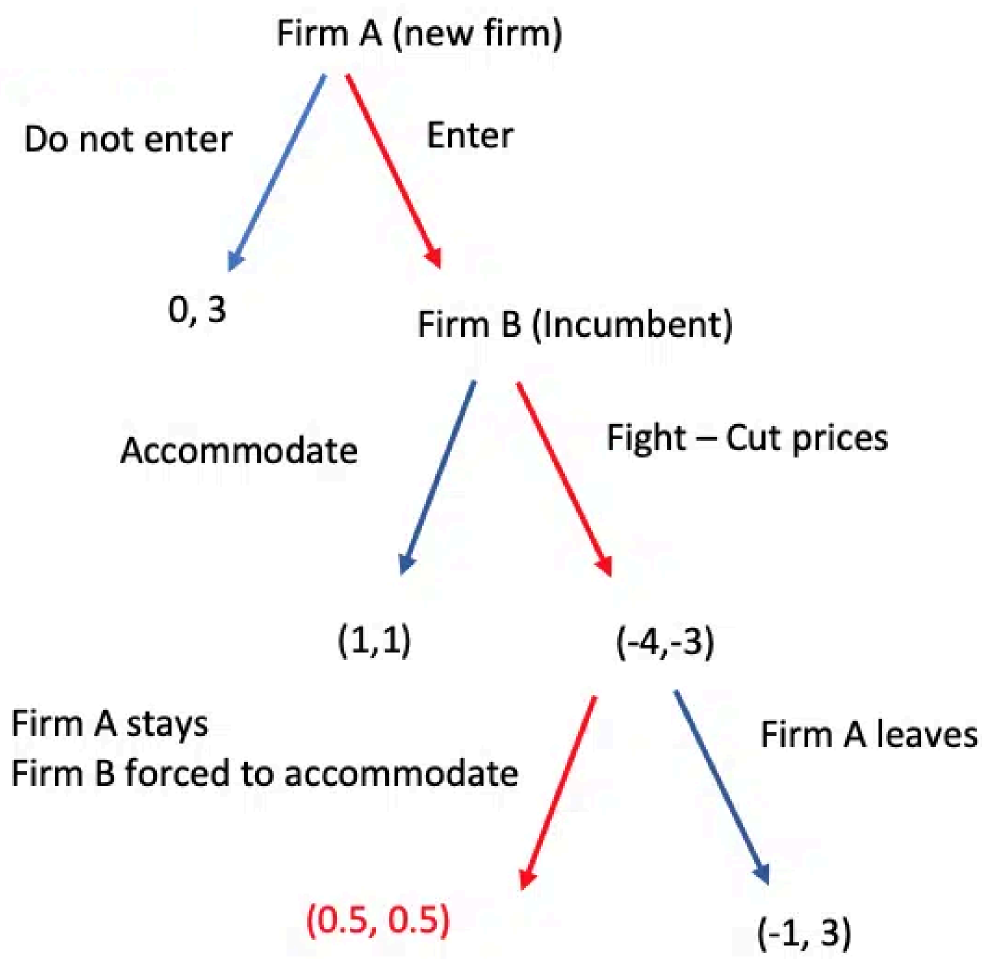
Game Setup

Payoffs:

1. If Firm A Chooses "Do Not Enter":
   - Firm B (Accommodate or Fight): Firm B makes ₹3, Firm A makes ₹0.
2. If Firm A Chooses "Enter":
   - Firm B Chooses "Accommodate":
     - Both firms earn ₹1 each. (Payoff: (1, 1))

- Firm B Chooses "Fight":
    - Firm A loses ₹4, Firm B loses ₹3. (Payoff: (-4, -3))

Game Representation:



Analysis:

1. **Firm A's Best Response:**
    - If Firm B Accommodates, Firm A will make ₹1 by entering vs. ₹0 by staying out. So, entering is better.
    - If Firm B Fights, Firm A will lose ₹4 by entering vs. ₹0 by staying out. So, staying out is better.
2. **Firm B's Best Response:**
    - If Firm A Does Not Enter, Firm B will earn ₹3 regardless of whether it accommodates or fights.
    - **If Firm A Enters:**

        - Accommodating leads to both firms earning ₹1.
        - Fighting leads to losses of ₹4 and ₹3.
3. **Therefore, Firm B's dominant strategy is to Accommodate to avoid losses, as it maximizes profit or minimizes loss compared to fighting.**

Nash Equilibrium:

- The Nash Equilibrium is when both firms' strategies lead to a stable outcome where no player benefits from unilaterally changing their strategy. In this case:
    - Firm A will Enter the market.
    - Firm B will Accommodate.
- The outcome is (1, 1), where both firms earn ₹1. Neither firm can improve its payoff by changing its strategy given the other's choice, so this is indeed a Nash Equilibrium.

Google Collab Link for the game:

https://colab.research.google.com/drive/1QOobYVGi9p_KROZ_8FE5hXsIzMtX8u8O?usp=sharing

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