

Representing Points

In a 2D image, each point can be represented as a vector: (x, y).

These vectors can be considered as arrows starting from the origin (0,0) and pointing to the coordinates of the point.

The Rotation Matrix

A rotation matrix, denoted by R, is a 2x2 matrix used to perform rotations.

The general form of a rotation matrix for an angle θ is given by:

R =

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \end{bmatrix}$$

$$\begin{bmatrix} \sin(\theta) & \cos(\theta) \end{bmatrix}$$

θ is the angle of rotation

This matrix rotates the image counterclockwise direction around the origin.

Multiplying by the Rotation Matrix

To rotate a point (x, y), it is multiplied by the rotation matrix R:

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \end{bmatrix} \begin{bmatrix} x \end{bmatrix}$$

$$\begin{bmatrix} \sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} y \end{bmatrix}$$

Result:

$$\begin{bmatrix} \cos(\theta)x - \sin(\theta)y \end{bmatrix}$$

$$\begin{bmatrix} \sin(\theta)x + \cos(\theta)y \end{bmatrix}$$

Example:

Rotation by 90 Degrees

Rotating a point (x, y) by 90 degrees clockwise.

The rotation matrix for 90 degrees is:

$$\begin{bmatrix} 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$

Multiplying this matrix by (x, y):

$$\begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} \begin{bmatrix} -y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} * \begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} x \end{bmatrix}$$

So, a point (x, y) rotated by 90 degrees becomes (-y, x).

Applying to the Entire Image

To rotate an entire image, this transformation is applied to each point in the image.

For multiple points (x_1, y_1) , (x_2, y_2) , ..., (x_n, y_n) , each point is multiplied by the rotation matrix to get their new coordinates.

Conclusion

- Linear transformations for rotating 2D images are performed using rotation matrices.
- The rotation matrix R rotates each point by an angle θ .
- Multiplying the point's vector by R gives the new coordinates after rotation.
- This process can be applied to all points in the image to achieve the desired rotation.