

**Assignment 2:****Due Date: Wednesday January 28th, 11:59 p.m.****Objectives:**

- Divide and Conquer
- Solving recurrence relations

**Submission Instruction**

- **Start early**
- You are allowed to work in groups of at most two students. It is okay if you want to work on your own.
- Write your answers in a file named **A2.pdf**.
- Make sure to put your names (as well as your partner's name) at the top of both the **A2.pdf**
- Submit **A2.pdf**. **Do not** create any folder
- **Make sure to have one submission per group**
- **Make sure that you are typing or your writing is neat. You will lose marks if something is not readable**

**Problems**

**Q1.** The following algorithm finds the maximum element in the given array. Prove the correctness of the algorithm. In addition, analyze its time complexity in the worst-case and best-case scenarios.

```
def max(A)
    m = A[0]
    for i = 1 to n-1 do
        if A[i] > m then m = A[i]
    return(m)
```

**Q2.** The following algorithm returns the product of two numbers, a and b. The parameters x and y are natural numbers. First, prove the correctness of the algorithm. Then, analyze the time complexity of the algorithm in the worst case scenario.

```
function mult(a, b)
    if b = 0:
        return 0
    else if b is odd:
        return(mult(2a, b/2 )+a)
    else:
        return(mult(2a, b/2 ))
```

**Q3:** Solve the following recurrence using substitution method and then prove the correctness using induction:

$$T(1) = 1 \quad n \leq 2$$

$$T(n) = 2T(n-1) + n - 1 \quad n > 2$$

**Q4:** Solve the following recurrence relation using recursion tree and master theorem:

$$T(n) = c \quad n \leq 2$$

$$T(n) = 7T(n/2) + n^2 \quad n > 2$$

**Q5:** In the merge-sort algorithm we studied in the class, a problem is divided into two subproblems. Design and analyze a new version where each problem is divided into  $n$  subproblems. Write the pseudocode of the algorithm and analyze its time complexity.

**Q6:** For the polynomial multiplication algorithm we did in the class, write the pseudocode of the most efficient algorithm we discussed in class. In addition, prove the correctness of the algorithm?