

National Institute of Technology, Jamshedpur

Mid Semester Examination, October, 2023

Branch: B.Tech 1st Semester, All branch
Subject: Engineering Mathematics - I
Subject Code: MA1101

Full Marks : 30
Time: 2 hours

Answer all questions. The figures on the right hand margin indicate marks.

1. Let $f(x)$ be continuous on $[a-1, a+1]$ and differentiable in $(a-1, a+1)$. Show that (5)
 there exists a θ , where $0 < \theta < 1$, such that the following relation holds

$$\left| \frac{\phi(a-1) - 1}{\phi(a) - 1} \right| = \left| \frac{f'(a+\theta)}{f'(a-\theta)} - 1 \right|$$

where $\phi(x) = f(x) - f(x+1)$.

2. Expand the function $\log[1 - \log(1-x)]$ in powers of x by Taylor's theorem up to the (5)
 terms contain x^3 . Hence find the series of $\log[1 + \log(1+x)]$ in powers of $\frac{x}{x+1}$.

3. Show that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ where $0 < a < b$ and hence prove (5)
 that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

4. Find all the asymptotes of the curve $x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$. (5)

5. Show that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature $\rho = \frac{a^2 b^2}{p^3}$, where p (5)
 is the length of the perpendicular from the center upon the tangent at (x, y) to the ellipse.

6. Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$, where (5)
 a is a constant.

***** All the Best *****