Roll Number: Name of Student

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR

Department of Electronics and Communication Engineering

Autumn Mid Semester Examination, October 2024

B. Tech. (2^{nd} Year): 3^{rd} Semester — Course Code: EC1303

(Electronics and Communication Engineering) — Course Name: Probability and Stochastic Process

Date of Exam: 04/10/2024

Time: 2 Hours — M. Marks: 30 — Name of Faculty: Dr. Nagendra Kumar

Note: The question paper consists of 04 questions. The marks are indicated in the right margin. Attempt all Questions.

Q. (1)

(a) Given that $P(A) = 0.9, P(B) = 0.8$, and $P(A \cap B) = 0.75$, find (i) $P(A \cap \overline{B})$; (ii) $P(\overline{A} \cap \overline{B})$ (02)

(b) Two manufacturing plants produce similar parts. Plant 1 produces 1,000 parts, 100 of which are defective. Plant 2 produces 2,000 parts, 150 of which are defective. A part is selected at random and found to be defective. What is the probability that it came from plant 1? (02)

Q. (2)

Recheck

(a) An information source generates symbols at random from a four-letter alphabet $\{a, b, c, d\}$ with probabilities $P(a) = \frac{1}{2}, P(b) = \frac{1}{4}$, and $P(c) = P(d) = \frac{1}{8}$. A coding scheme encodes these symbols into binary codes as follows: $a \to 0, b \to 10, c \to 110, d \to 111$. Let X be the random variable denoting the length of the code that is, the number of binary symbols (bits). (04)

(i) Sketch the cumulative distribution function (cdf) $F_X(x)$ of X

(ii) Find $P(X \le 1), P(1 < X \le 2), P(X > 1), P(1 \le X \le 2)$

(b) A random variable is defined by the cdf (03)

$$F_X(x) = \begin{cases} 0; & x < 0 \\ \frac{1}{2}x; & 0 \le x < 1 \\ k; & 1 \le x \end{cases}$$

(i) Find the value of k

Recheck. (ii) Find $P\left(\frac{1}{2} < X \le 1\right)$ and $P(\frac{1}{2} < X < 1)$

(c) The joint probability density function (pdf) of a bivariate random variable (X, Y) is given by (04)

$$f(x,y) = \begin{cases} Axy; & 0 < x < 1, 0 < y < 1 \\ 0; & \text{otherwise} \end{cases}$$

where A is a constant.

(i) Find the value of A. 4

Recheck (ii) Are X and Y independent?

(iii) Find $P(X + Y < 1)$. 1/6

Q. (3)

(a) Let $Y = X^2$. Find and sketch the pdf of Y if X is a uniform random variable over (-1, 2). (04)

Sketch

check (b) Consider the random process $X(t)$ is given as (03)

$$X(t) = Y \cos \omega t \qquad t \ge 0$$

where ω is a constant and Y is a uniform random variable over (0, 1).

(i) Find $E[X(t)]$.

(ii) Find the autocorrelation function $R_X(t, s)$ of $X(t)$.

(iii) Find the autocovariance function $K_X(t, s)$ of $X(t)$.

Q. (4)

(a) Consider a random process $X_0(t) = \alpha X(t) \cos(\omega_0 t + \varphi)$ where $X(t)$ is zero mean, stationary random process. $X(t)$ and φ are assumed to be independent. α and ω_0 are constants. φ is a random variable uniformly distributed in the interval $(-\pi, \pi)$. Show that $X_0(t)$ is wide sense stationary (WSS) process. (04)

(b) The process X(t) is wide-sense stationary and normally distributed with mean $E[X(t)] = 0$ and auto-correlation $R_X(\tau) = 4e^{-2|\tau|}$. (04)

(i) Find $E\{[X(t+1) - X(t-1)]^2\}$. 7.8534

(ii) Find pdf of $X(t)$.

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR

Department of Electronics and Communication Engineering

Autumn End Semester Examination, Dec 2024

B. Tech. (2^{nd} Year): 3^{rd} Semester
(Electronics and Communication Engineering)

Course Code: EC1303
Course Name: Probability and Stochastic Process

Date of Exam: 07/12/2024

Time: 3 Hours | M. Marks: 50 | Name of Faculty: Dr. Nagendra Kumar

Note: The question paper consists of 05 questions. The marks are indicated in the right margin. Attempt all Questions.

Q. (1) (a) Consider the experiment of throwing a dart onto a circular plate with unit radius. Let X be the random variable representing the distance of the point where the dart lands from the origin of the plate. Assume that the dart always lands on the plate and that the dart is equally likely to land anywhere on the plate. **(4)**
Find (i) $P(X < a)$ (ii) $P(a < X < b)$, where $a < b \leq 1$ (iii) Sketch the CDF $F_X(x)$ of the random variable X.

(b) Let $Y = e^X$. Find the probability density function of Y if X is uniform random variable over (0, 1). Also find mean of Y. **(4)**

1.5 **Q. (2)** 1 (a) Let $Y = X^2$. Find and sketch the pdf of Y if X is a Gaussian distributed random variable with mean zero and variance 1. **(3)**

(b) The joint probability mass function (pmf) of a bivariate random variable (X, Y) is given by **(4)**

$$p_{XY}(x_i, y_j) = \begin{cases} k(2x_i + y_j) & x_i = 1, 2;\ y_j = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. (i) Find the value of k. (ii) Find the marginal pmfs of X and Y. (iii) Are X and Y independent?

Q. (3) (a) Two random processes $X(t)$ and $Y(t)$ are given by **(4)**

$$X(t) = A\cos(\omega t + \theta) \qquad Y(t) = A\sin(\omega t + \theta)$$

where A and ω are constants and θ is a uniform random variable over $(0, 2\pi)$. Find the cross-correlation function of $X(t)$ and $Y(t)$ and verify $R_{XY}(-\tau) = R_{YX}(\tau)$.

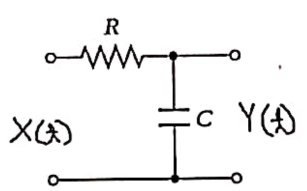
(b) Consider a random process $X(t) = \cos(\omega t + \theta)$ where ω is constant and θ is a random variable with uniform distribution over $(0, 2\pi)$. **(6)**

Determine whether $X(t)$ is an ergodic random process. Check it for mean and autocorrelation.

4) 1 (a) Let $X(t) = A\cos(\omega_0 t + \theta)$, where A and ω_0 are constants, θ is a uniform random variable over $(-\pi, \pi)$. Find the power spectral density of $X(t)$. **(5)**

(b) A WSS random process $X(t)$ is applied to the input of an LTI system with impulse response $h(t) = 3e^{-2t}u(t)$. Find the mean value of output $Y(t)$ of the system if $E[X(t)] = 2$. **(3)**

(c) The input $X(t)$ to the RC filter shown in the figure below is a white noise with power spectral density σ^2. Find the mean-square value of $Y(t)$. **(4)**



Q. (5) check (a) Let $X_1, X_2, \ldots, X_n$ be a random sample of a normal random variable X with known mean μ and unknown variance σ^2. Find the maximum likelihood estimator of σ^2. **(4)**

(b) Find the minimum mean square error estimate of a random variable Y by a constant C. **(3)**

(c) Let $X_1, X_2, \ldots, X_n$ be a random sample of a Bernoulli random variable X with probability mass function given by **(6)**

$$f(x; p) = p^x(1-p)^{1-x} \qquad x = 0, 1$$

where p, $0 \leq p \leq 1$, is unknown. Assume that p is a uniform random variable over (0, 1). Find the Baye's estimator of p.