1. P(16, 8, 20, 4)

i) Translate the point to $t_x = 6$, $t_y = 2$, $t_z = 3$

ii) Scaled the point at $S_x = 0.5$, $S_y = 2$, $S_z = 0.25$

iii) Now rotate the point around Y-axis where $\theta_y = 60°$

iv) After that rotated the rotated point around x-axis where $\theta_x = 45°$

Solution: Here, P(16, 8, 20, 4) or (4, 2, 5, 1)

i) $$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+6 \\ 2+2 \\ 5+3 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 8 \\ 1 \end{bmatrix}$$

∴ After translation the point is (10, 4, 8, 1)

ii) $$\begin{bmatrix} x'' \\ y'' \\ z'' \\ w'' \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 2 \\ 1 \end{bmatrix}$$

∴ After scale the point is (5, 8, 2, 1)

iii) $$\begin{bmatrix} x''' \\ y''' \\ z''' \\ w''' \end{bmatrix} = \begin{bmatrix} \cos 60 & 0 & -\sin 60 & 0 \\ 0 & 1 & 0 & 0 \\ \sin 60 & 0 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.5 - 1.732 \\ 8 \\ 4.33 + 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.768 \\ 8 \\ 5.33 \\ 1 \end{bmatrix}$$

∴ After ~~rotate~~ the point is (0.768, 8, 5.33, 1)

iv) $$\begin{bmatrix} x'' \\ y'' \\ z'' \\ w'' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 45 & -\sin 45 & 0 \\ 0 & \sin 45 & \cos 45 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.768 \\ 8 \\ 5.33 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.768 \\ 5.657 - 3.769 \\ 5.657 + 3.769 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.768 \\ 1.888 \\ 9.426 \\ 1 \end{bmatrix}$$

∴ After rotate the point is (0.768, 1.888, 9.426, 1)