Assignment 3

Nishan Dhoj Karki

09/10/2022

1 Let $\sum = \{$a, b$\}$. Consider the language of all strings that contain exactly twice as many occurrences of b as occurrences of a. Call this language L_{abb}. For example, "abb", "bababb" and ϵ are in L_{abb}, but "bbab", "aba", and "bbbabbb" are not.

Proof by Contradiction: Pumping Lemma

Let us start our proof with the assumption that L_{abb} is regular. If L_{aab} is regular then the pumping lemma holds for L_{aab}.

Let p be the pumping length and s be a string such that $|s| \geq p$ and $s \in L_{aab}$.
Let us choose a string $a^p b^{2p}$ to be s.

The pumping lemma states that s can be divided into three pieces xyz such that:
1. for each $i \geq 0$, $xy^i z \in L_{aab}$,
2. $|y| > 0$, and
3. $|xy| \leq p$

From condition 3 of pumping lemma,
Since there are no b's in xy, $|xy| \leq p$.

From condition 2 of pumping lemma,
y contains at least one a's. So, $|y| > 0$.

From condition 1 of pumping lemma,
$xyyz = a^{p+k} b^{2p}$. Here, $k = |y| \geq 1$.

The string $xyyz \notin L_{abb}$ as the occurrence of a does not reflect exactly twice occurrence of b.

This contradicts our assumption as the condition (1) of the pumping lemma is not met. Hence, the language does not satisfy the pumping lemma and is not regular.

2 1.46 Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection,and complement. c. $\{w|w \in \{0,1\}*$ is not a palindrome$\}$

Proof by Contradiction: Pumping Lemma

Let us assume that $L = \{w|w \in \{0,1\}*$ is not a palindrome$\}$ is regular.

The complement closure property of regular language states that the complement of regular language is also regular. Hence, $\overline{L}$ is also regular. We can define the complement of L as:
$\overline{L} = \{w|w \in \{0,1\}*$ is a palindrome$\}$.

If $\overline{L}$ is regular then the pumping lemma must hold for $\overline{L}$. Let p be the pumping length and s be a string such that $|s| \geq p$ and $s \in \overline{L}$.

The pumping lemma states that s can be divided into three pieces xyz such that:
1. for each $i \geq 0$, $xy^iz \in L_{aab}$,
2. $|y| > 0$, and
3. $|xy| \leq p$

Let s be the string 0^p10^p such that $|s| \geq p$ and $s \in \overline{L}$

From condition 3 of pumping lemma,
Since there are no 1's in xy, $|xy| \leq p$.

From condition 2 of pumping lemma,
y contains at least one 0's. So, $|y| > 0$.

From condition 1 of pumping lemma,
$xyyz = 0^{p+k}10^p$. Here, $k = |y| \geq 1$.

The resulting string $xyyz \notin \overline{L}$ as pumping any number of 0's will not validate the string as palindrome. This contradicts the statement that $\overline{L}$ is regular as condition (1) of pumping lemma is not met. Hence, through the complement closure property of regular language we can state that the language L cannot be regular as $\overline{L}$ is not regular. Thus, we have proved by contradiction that L is not regular.