Assignment 9

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1   3SAT ∈ NP (50 points)
The 3SAT problem is described in chapter 7.4 of your textbook. Give two proofs that 3SAT ∈ NP. In your first proof, use a poly-time verifier. In your second proof, use a poly-time non-deterministic Turing machine. This should not be overly complicated, but be careful and complete.

We have,
3SAT=$\{\langle\phi\rangle|\phi$ is a satisfiable 3-CNF formula$\}$.

1.1 Proof: Using a Poly-time verifier.

Let V be a poly-time verifier for $3SAT$:

V = "On input $\langle\langle\phi\rangle, c\rangle$, here ϕ is a CNF and c is the collections of values for the variables of ϕ i.e. is either a value $TRUE$ or $FALSE$.

    1 Check if each variable of ϕ is either $TRUE$ or $FALSE$ i.e. boolean value.

    2 Check if c has values for all variables in ϕ.

    3 For each literal in ϕ, evaluate its value using c.

    4 Evaluate the value of each clause using the literals.

    5 If any clause returns a value $FALSE$, $REJECT$.

    6 If all clauses return a value $TRUE$, $ACCEPT$.

Here, V accepts $\langle\langle\phi\rangle, c\rangle$ for some c, if ϕ is satisfiable. The runtime of the verifier V is polynomial as all check steps are running in polynomial times. The runtime of V is primarily dependent on the number of clauses in ϕ. Since, the described verifier verifies $3SAT$ in polynomial time, 3SAT ∈ NP.

1.2 Proof: Using Poly-time NDTM

Constructing a poly-time NDTM T for $3SAT$:

T = "On input $\langle \phi \rangle$, here ϕ is 3-CNF

- Assign N to the count for all the distinct variables in ϕ.

- Non-deterministic-ally select boolean values for distinct value of ϕ.
  - Evaluate ϕ for those selection for each branch.
  - If any one branch evaluates ϕ to be $TRUE$, $ACCEPT$.

- If no evaluation outputs $TRUE$, $REJECT$.

Here the NDTM T accepts if any one branch evaluates ϕ to be $TRUE$ and rejects if none on the branch evaluates ϕ to be $TRUE$. The running time of NDTM is the order of the length of its longest branch. The length of the longest branch for selecting boolean values is the total number of distinct variables in ϕ i.e N. Since each branch has a factor of two i.e. $TRUE$ or $FALSE$. Here, NDTM T runs in polynomial time. Thus, 3SAT $\in$ NP.