Fuzzy Learning

- In 1965 **Lotfi Zadeh**, published his famous paper "fuzzy sets". Zadeh extended the work on possibility theory into a formal system of mathematical logic and introduced a new concept for applying natural language terms. This new logic for representing and manipulating fuzzy terms was called fuzzy logic.

-**Traditional Logic:** Traditional Boolean logic uses sharp distinctions. For instance Tom with height 181cm is tall. If we draw a line at 180 cm, David with height 179cm is small. Is David really small?

Fuzzy logic: It is a form of knowledge representation suitable for notions that can't be defined precisely but which depend upon their contexts. A way to represent variation or Imprecision in logic
- **Fuzzy** means "not clear, distinct or precise or blurred". It is a concept of partial truth, where truth value may range between completely true or completely false.
- In contrast with traditional logic theory where binary sets have two valued logic (True/False), fuzzy logic variables may have value that ranges in degree from 0 to 1.
- Fuzzy logic is a form of multi-valued logic.
- Fuzzy logic reflects how people think. It attempts to model our sense of our decision making and our common sense.
- Example: Temperature, Height, Speed, Distance, Beauty
    - Motor is running really hot
    - Tom is a very tall guy

Crisp (Traditional) Variables

Crisp variables represent precise quantities. It denotes sharp distinctions.

- X = 3.1415
- A Î {0,1}
- Men Î {Tall, short}
- Speed Î { slow, fast}

Crisp and Fuzzy sets

In a **crisp set**, an element is either a member of the **set** or not. For example, a jelly bean belongs in the class of food known as candy. Mashed potatoes do not. Fuzzy **sets**, on the other hand, allow elements to be partially in a **set**.

In fuzzy theory, fuzzy set A of universe X is defined by function μA (x) called membership function of set A.

μA **(x) :** X -> [0,1] ,

Where, μA (x) = 1, if x is totally in A

= 0, if x is not in A

0 < μA (x) < 1, if x is partially in A.

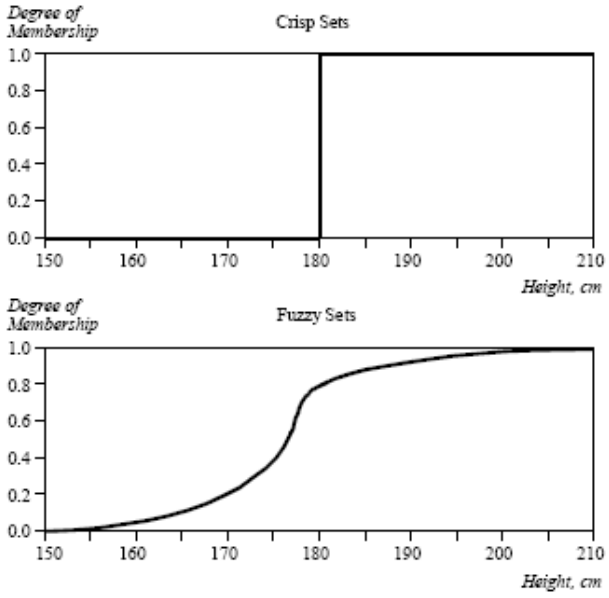
- For any element x of X, membership function μA (x) equals the degree to which x is an element of set A. This degree ranges from 0 to 1, represents degree of membership, also called membership value of element x in set A.

μA : X -> [0,1] the membership function of A.

μA (x) ϵ [0,1] is the degree of membership x in A.

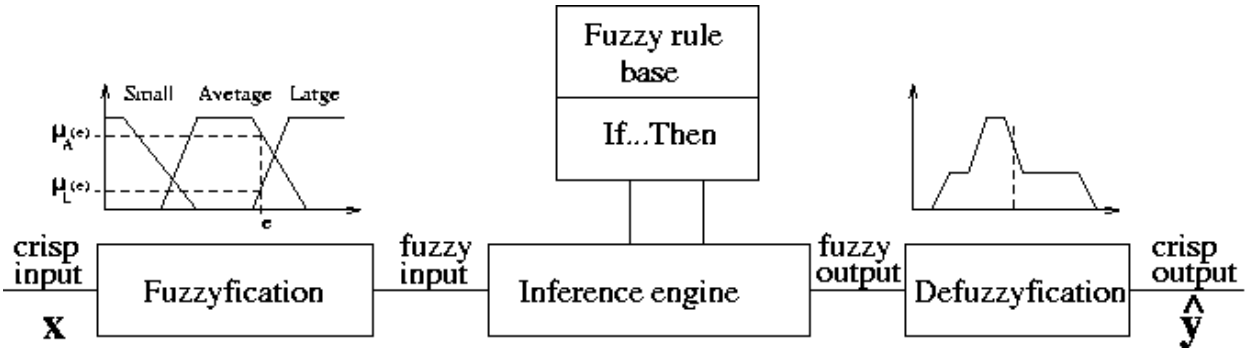
- A fuzzy variable is often denoted by its membership function.

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00



Fuzzy Inferences

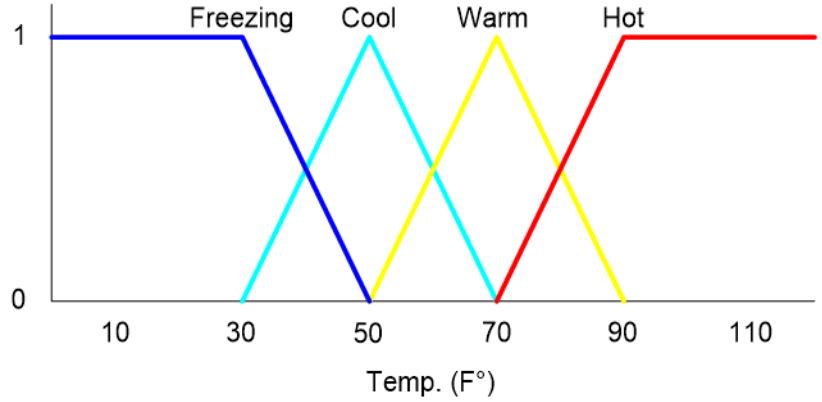
- Two approaches of fuzzy inference are
  - Mamdani Inference
  - Sugeno fuzzy inference



Mamdani inference is applied in four stages:

i. Fuzzyfication of input variables:

- Determines an input's membership in overlapping sets.
- **Fuzzification** is the process of changing a real scalar value into a fuzzy value. This is achieved with the different types of fuzzifiers (membership functions). Fuzzy Linguistic Variables are used to represent qualities spanning a particular spectrum.
- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic



ii. Rule Evaluation

- The second step is to take the fuzzified inputs, (such as m(x=$A$1) = 0.5, m(x=$A$2) = 0.2, m(y=$B$1) = 0.1 and m(y=B 2) = 0.7), and apply them to the antecedents of the fuzzy rules.
- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.
- This number (the truth value) is then applied to the consequent membership function.

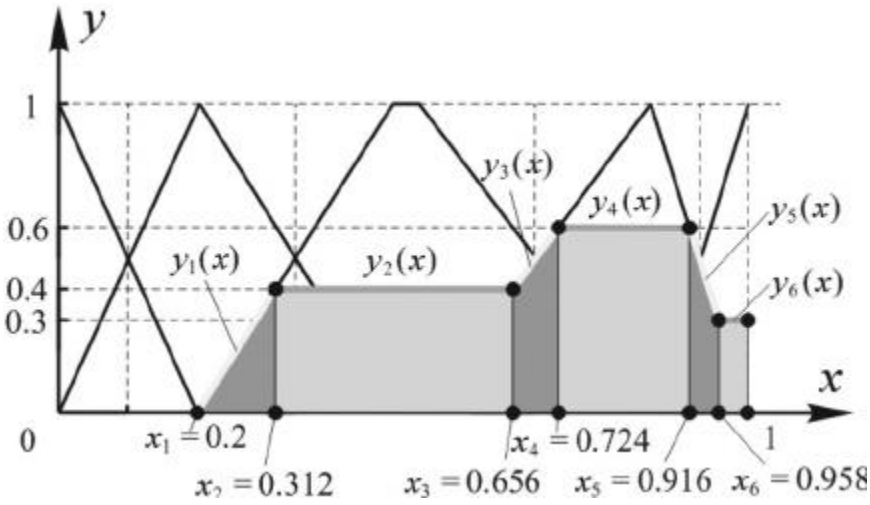
iii. Aggregation of the rule outputs:

- Aggregation is the process of unification of the outputs of all rules.
- Take the membership functions of all rule consequents previously scaled and combine them into a single fuzzy set.
- Determine outputs based on inputs and rules.
- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.

iv. Defuzzification:

-Defuzzification is the process of producing a quantifiable result in Crisp logic, given fuzzy sets and corresponding membership degrees. It is the process that maps a fuzzy set to a crisp set.

- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number



Drawbacks

- Fuzzy logic deals with imprecision, and vagueness, but not uncertainty
- Requires tuning of membership functions
- Fuzzy Logic control may not scale well to large or complex problems