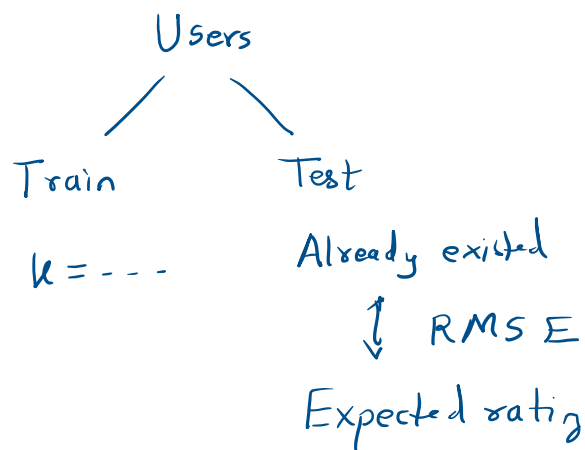
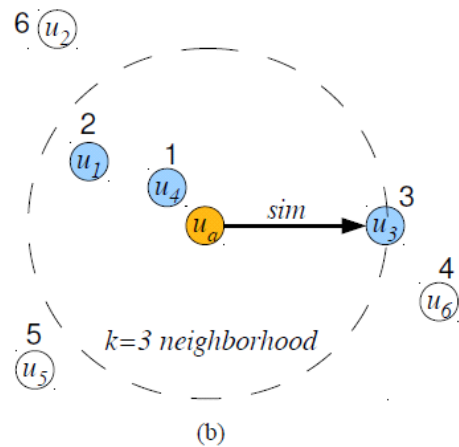


## User Based Collaborative Filtering

User x Item matrix

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$
$u_1$	?	4.0	4.0	2.0	1.0	2.0	?	?
$u_2$	3.0	?	?	?	5.0	1.0	?	?
$u_3$	3.0	?	?	3.0	2.0	2.0	?	3.0
$u_4$	4.0	?	?	2.0	1.0	1.0	2.0	4.0
$u_5$	1.0	1.0	?	?	?	?	?	1.0
$u_6$	?	1.0	?	?	1.0	1.0	?	1.0
$u_a$	?	?	4.0	3.0	?	1.0	?	5.0
$\hat{r}_a$	3.5	4.0	0	0	1.3	0	2.0	

(a)



## Item Based Collaborative Filtering

Item Similarity Matrix

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$		
$i_1$	-	0.1	0	0.3	0.2	0.4	0	0.1	-	
$i_2$	0.1	-	0.8	0.9	0	0.2	0.1	0	0.0	
$i_3$	0	0.8	-	0	0.4	0.1	0.3	0.5	4.6	$= (4 \cdot 0.4 + 5 \cdot 0.5) / (0.4 + 0.5)$
$i_4$	0.3	0.9	0	-	0	0.1	0	0.2	3.2	$= (2 \cdot 0.3 + 5 \cdot 0.2) / (0.3 + 0.2)$
$i_5$	0.2	0	0.4	0	-	0.1	0.2	0.1	-	-
$i_6$	0.4	0.2	0.1	0.3	0.1	-	0	0.1	2.0	$= (2 \cdot 0.4) / 0.4$
$i_7$	0	0.1	0.3	0	0.2	0	-	0	4.0	$= (4 \cdot 0.2) / 0.2$
$i_8$	0.1	0	0.5	0.2	0.1	0.1	0	-	-	
$u_a$	2	-	-	-	4	-	-	5		

$u_a$  row: ? ? ? ? ? ? ?

$$\bar{r} = \frac{\sum r_i s_i}{\sum s_i}$$

$s_i$ :  
similarities

Ua	2	-	-	-	4	-	-	5
	?	?	?		?	?		

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 7 & 8 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 3(3) + 4(2) + 2(1) \\ 1(3) + 7(2) + 8(1) \end{bmatrix}$$

$$35 = 7 \times 5$$

Ratings matrix

	$I_1$	$I_2$	$\dots$	$I_n$
$U_1$	3	? ✓		?
$U_2$	? ✓	2		2
$U_3$	1	? ✓		4
$\vdots$				?
$U_m$	?	2		? ✓

$$\hat{r}_{ui} = \mu + \underline{b_u} + \underline{b_i} + \underline{q_i^T p_u}$$

$$3 = \mu + \underbrace{b_u + b_i + q_i^T p_u}_{\hat{r}_{ui}}$$

$$2 = \hat{r}_{ui}$$

$$\text{error} = r_{ui} - \hat{r}_{ui}$$

$$Z = \sum (r_{ui} - \hat{r}_{ui})^2$$

$$R_{m \times n} = P_{m \times f} Q_{f \times n} = \hat{R}$$

Optimal  
Values are found by  
Gradient Descent Method

R

Users-Movies Rating Matrix

		Movies				
		M1	M2	M3	M4	M5
Users	U1	3	4	2	5	1
	U2	2	4	1	2	4
	U3	3	3	5	2	2

3x5

$p_u$

Users-Factors Matrix

		Factors		
		F1	F2	F3
Users	U1	0.73	3.22	0
	U2	0	1.57	2.53
	U3	1.62	0	1.44

Factors-Movies Matrix

		Movies				
		M1	M2	M3	M4	M5
Factors	F1	1.47	1	2.73	1.73	0
	F2	0.6	1.01	0	1.27	0.31
	F3	0.42	0.95	0.39	0	1.39

$q_i$

1-

Users-Movies Rating Matrix

		Movies				
		M1	M2	M3	M4	M5
Users	U1	3	4	2	5	1
	U2	2	4	1	2	4
	U3	3	3	5	2	2

3x5

$r_i$

$p_u$

Users-Factors Matrix

		Factors		
		F1	F2	F3
Users	U1	0.73	3.22	0
	U2	0	1.57	2.53
	U3	1.62	0	1.44

3x3

$P$

Factors-Movies Matrix

		Movies				
		M1	M2	M3	M4	M5
Factors	F1	1.47	1	2.73	1.73	0
	F2	0.6	1.01	0	1.27	0.31
	F3	0.42	0.95	0.39	0	1.39

3x5

$Q$