

Convolution Concept

By Example

Digit Image

- Consider a 9 x 9 pixel image
- Black and White
- Containing only 1s and 0s

0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Filter

- We now consider a 3 x 3 filter

0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	1	1	0	0	0
0	0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

1	1	1
0	0	1
0	1	0

The figure illustrates a 2D convolution operation. A 3x3 kernel is applied to a 7x7 padded input matrix to produce a 5x5 output feature map.

Kernel:

1	1	1
1	2	3
0	0	0

Padded Input Matrix (7x7):

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Output Feature Map (5x5):

3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10
7	8	9	10	11

Convolution

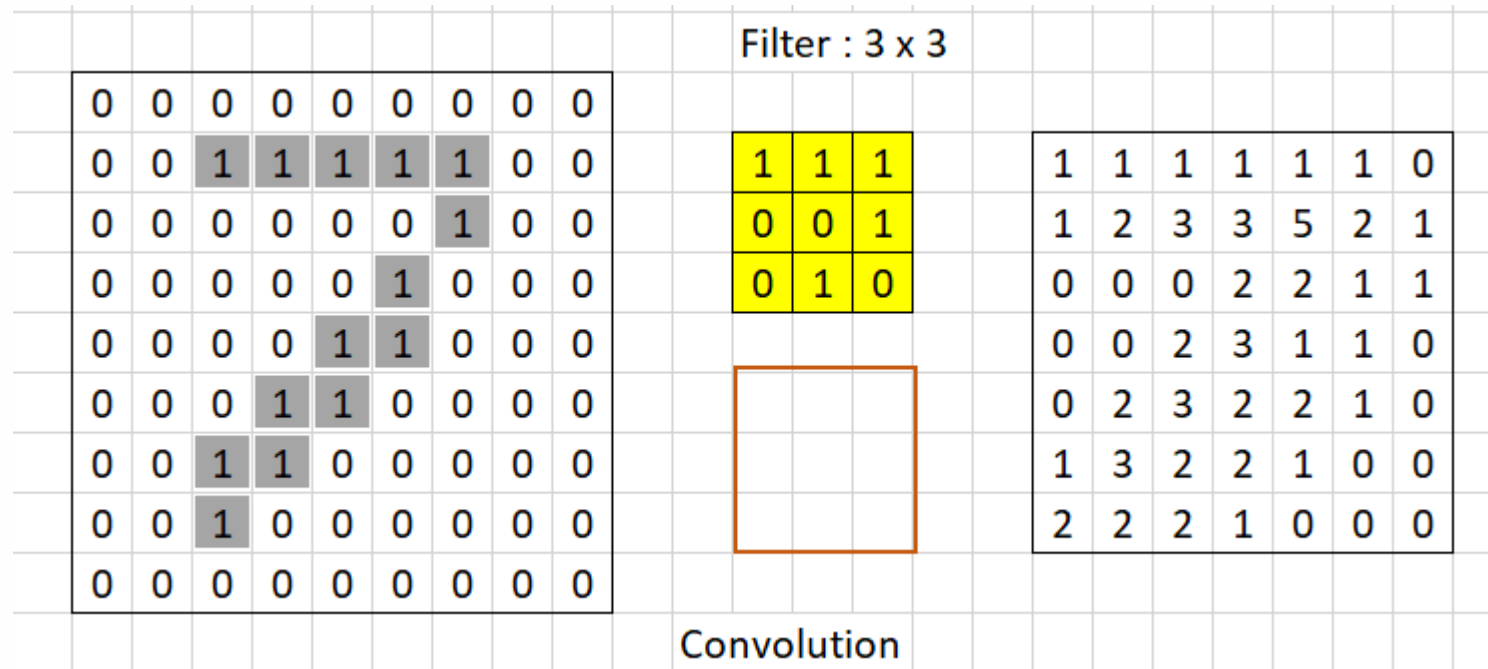
Convolution in Practice

- There are several types of filters. For more information you may go to link: <https://lodev.org/cgtutor/filtering.html>
- The filter values in the example may not be specific hand-picked values, but can be any set of parameters namely w_1, w_2, \dots which need to be tuned for the image

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

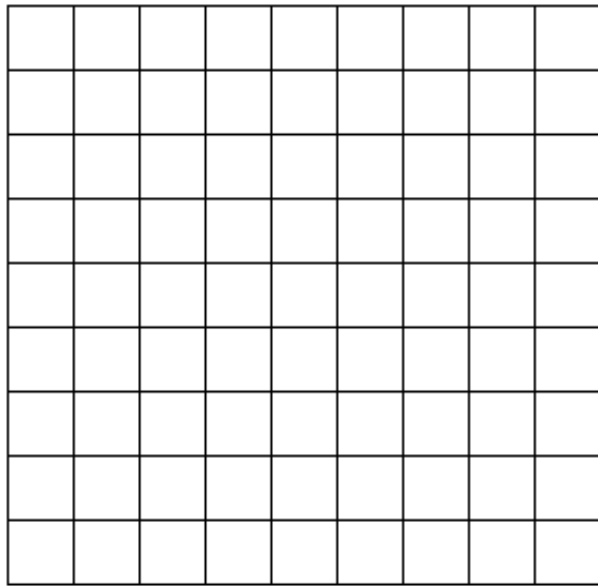
Shrinking of Image

- Each time when you apply the convolution filter, the image shrinks
e.g. 9 x 9 image getting reduced to 7 x 7 image

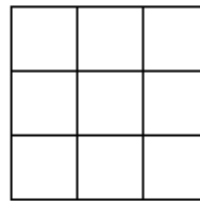


Shrinking of Image

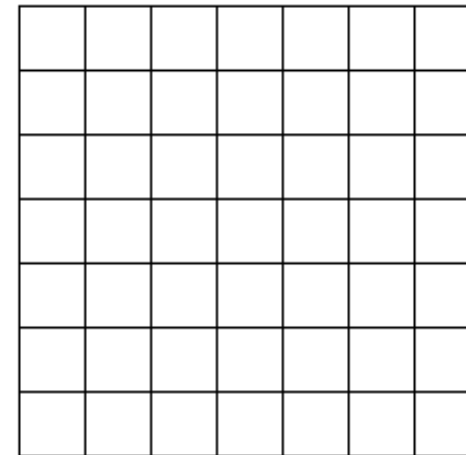
- The image of $n \times n$ when convolved by $f \times f$ filter gets reduced to $(n-f+1) \times (n-f+1)$ image



$n \times n$



$f \times f$



$(n-f+1) \times (n-f+1)$

Padding

- Problem with shrinking of the image is that, the cells which not on the edge have less participation in calculation of convolution
- Also for each convolution operation, the image gets shrunk
- If we pad the image before we convolve then both of the above issues get avoided.
- We can have a padding of any number of cells say p

Padding

- The image of $n \times n$ when convolved by $f \times f$ filter, with padding p gets reduced to $(n+2p-f+1) \times (n+2p-f+1)$ image, which generates the image of same size as original one provided f is odd numbered.

0	0	0	0	0	0	0	0	0	0	0
0										0
0										0
0										0
0										0
0										0
0										0
0										0
0										0
0										0
0	0	0	0	0	0	0	0	0	0	0

$n \times n$

$f \times f$

with p padding

$(n+2p-f+1) \times (n+2p-f+1)$

Strides

- Instead of going one cell, we can go 2 cells or more cells ahead while we convolve

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$n \times n$

1	1	1
0	0	1
0	1	0

$f \times f$

0				

$(n+2p-f+1)/s \times (n+2p-f+1)/s$

with p padding, s strides

Strides

- With Padding $p = 1$, $s = 2$, we will have output image as 5×5 , by flooring the term $(n+2p-f+1)/s$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$n \times n$

1	1	1
0	0	1
0	1	0

$f \times f$

0	1			

$(n+2p-f+1)/s \times (n+2p-f+1)/s$

with p padding, s strides

Strides

- With Padding $p = 1$, $s = 2$, we will have output image as 5×5 , by flooring the term $(n+2p-f+1)/s$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$n \times n$

1	1	1
0	0	1
0	1	0

$f \times f$

with p padding, s strides

0	1	1	1	0
0				

$(n+2p-f+1)/s \times (n+2p-f+1)/s$

Strides

- Finally, we have the following output

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$n \times n$

1	1	1
0	0	1
0	1	0

$f \times f$

0	1	1	1	0
0	2	3	2	0
0	0	3	1	0
0	3	2	0	0
0	1	0	0	0

$(n+2p-f+1)/s \times (n+2p-f+1)/s$

with p padding, s strides