Conditional Game Theory

Combines Bayesian Probability, Game Theory and Network Theory

$$X_1 \longrightarrow X_2$$

Assume directionality
- The child activates the relationship
- The chid modulates behaviour according to the parent

Assume conditionality
- The child does not require knowledge of the preferences of the parent

Question : How does the child respond to the influence of the parent without knowledge of the preferences of the parent?

Answer : Preferences are modelled according to **conditionalisation** logic. The child responds by defining a conditional *preference*

Conditionalisation is a mathematical expression of ‘virtual bargaining’.
Behaviour is based on anticipated agreements.

Definitions

conjecture profile
(set of hypothetical actions for group by X_i)

Conditioning conjecture set

$$\mathcal{H}_{I|\mathrm{pa}(i)}(a_i|\alpha_{\mathrm{pa}(i)}) : \mathrm{pa}(X_i) \models \alpha_{\mathrm{pa}(i)} \implies X_i \models a_i$$

The parent set (all individuals that influence X_i)

Hypothetical proposition : If there is a conditioning conjecture set for the parent set, then the agent (child) will conjecture a conjecture profile

The **conditional utility** of a hypothetic proposition is

The set of action profiles
(possible actions)

$$u_{I|\mathrm{pa}(i)}(a_i|\alpha_{\mathrm{pa}(i)} : \mathcal{A} \to \mathbb{R})$$

Note, if pa(i) = {} then this is a categorical utility

Conditional utility - whether there is significant information for the individual to act

Categorical utility - context/situational dependent relationships

Normalised utilities are then **utility mass functions**

Distinction between epistemological uncertainty and behavioural uncertainty:

Epistemological uncertainty => a lack of complete knowledge about the realisation of proposition

Behavioural uncertainty => a lack of certainty regarding whether an action should be carried out

However - there is a connection between the epistemological domain of Bayesian networks and the behavioural domain of social networks

$$Y_1 \xrightarrow{\quad p_{2|1} \quad} Y_2$$

This is the **joint probability mass function**, given by

$$p_{12}(Y_1, Y_2) = p_1(Y_1)p_{2|1}(Y_2|Y_1)$$

The joint probability mass functions is the means by which a notion of **strict individual belief** and an expanded notion of **conditional belief** are combined:

$$u_{12}(a_1, a_2) = u_1(a_1)u_{2|1}(a_2|a_1)$$

This does not imply a group belief. Instead an innately individual concept of belief

$$u_{12}(a_1, a_2) = u_1(a_1)u_{2|1}(a_2|a_1)$$

This function gives an ordering of all joint conjecture profiles with respect to their compatibility.

It provides an assessment of disputes and possibilities for compromise - a measure of *how conjecture sets coordinate* - the **coordination function**:

$$u_{1:n}(a_i, ..., a_n) = \prod_{i=1}^{n} u_{i|\mathrm{pa}(i)}(a_i|\alpha_{\mathrm{pa}}(i))$$

So far, described only how X_i is influenced…

Question: How do individuals conjectures (a_{ii}) fit together to generate the coordinated outcome?

Answer : Introduce a **coordinate decision rule** to capture this.

$$w_i(a_{ii}) = \sum_{\sim a_{ii}} w_{1:n}(a_{11}, ..., a_{1n})$$

The summation of all coordinate functions except for the iith element.

The coordinate decision rule is a measure of how the parts of a profile fit together to form the social behaviour.

If the system is cooperative => this rule will be high when in a team.

If the system is conflictive => this rule will be high when in war.

Note : *The individual is still not required to maximise self interest.*

The individual decision rules are a compromise between individual interests and group coordination.

The coordination utility and coordinated decision rules appear <u>endogenously</u>

Note:

One of the necessary ingredients for using probability syntax to describe social influence is **invariance**.

$$p_{12}(y_1, y_2) = p_1(y_1)p_{2|1}(y_2|y_1) = p_{1|2}(y_1|y_2)p_2(y_2) = p_{21}(y_2, y_1)$$

This clearly holds for objective events such as a coin flip… Not so clear that it holds for subjective events.

Quantification

If Y_1 and Y_2 are statistically independent then conditioning Y_1 will have no effect on Y_2 (as $p_{2|1} = p_2$)

Also, if Y_1 and Y_2 are statistically dependent then it is impossible to synthesis probability mass functions as before

Question: If we assume statistical independence can we still produce joint mass functions? How different is $p_1(y_1)p_2(y_2)$ and $p_{12}(y_1, y_2)$?

The concept of mutual information can answer this.

$$\mathcal{I}(Y_1, Y_2) = \sum_{Y_1, Y_2} p_{12}(y_1, y_2) \log_2 \frac{p_{12}(y_1, y_2)}{p_1(y_1)p_2(y_2)}$$

This is a measure of the statistical independence of Y_1 and Y_2:

If the mutual information is greater or equal to zero then Y_1 and Y_2 are statistically independent

*If it equals zero then all of the individual preferences are **<u>socially uncoordinated</u>***

This motivates a notion of entropy,

$$\mathcal{H}(Y_i) = -\sum_{Y_i} p_i(y_i)\log_2(p_I(y_i))$$

And joint entropy,

$$\mathcal{H}(Y_1, Y_2) = -\sum_{Y_1, Y_2} p_{12}(y_1, y_2)\log_2(p_{12}(y_1, y_2))$$

Here, entropy serves as a numerical measure for the average *epistemic* uncertainty associated with the random event.

If it equals zero then all the probability mass will be concentrated around one outcome.

If it is maximised then all the probability mass will be distributed between two outcomes

We have,

$$\mathcal{H}(Y_1, Y_2) = \mathcal{H}(Y_1) + \mathcal{H}(Y_2)$$

And it can be shown that,

$$\mathcal{I}(Y_1, Y_2) = \mathcal{H}(Y_1) + \mathcal{H}(Y_2) - \mathcal{H}(Y_1, Y_2)$$

For behaviours, entropy is a measure of the average behavioural uncertainty regarding a preference (i.e. indecisiveness) when choosing amongst alternatives:

It is a measure of the average number of opportunities lost

We can define the dispersion relation,

$$d(X_1, X_2) = \mathcal{H}(X_1, X_2) - \mathcal{I}(X_1, X_2)$$

This is a metric fo the distances between X_1 and X_2 as a function of their social relationships.

For a minimised distance, X_1 is said to be *slaved* to X_2

For a maximised distance, X_1 and X_2 are socially uncoordinated and hence, have no social influence on one another.

By normalising this relation by the entropy of the two pairs, we can summarise this with a coordination index

$$\mathcal{C}(X_i, X_j) = 1 - \mathcal{D}(X_i, X_j)$$

This is a measure of the intrinsic ability of individuals to align their interest as a result of direct social influence

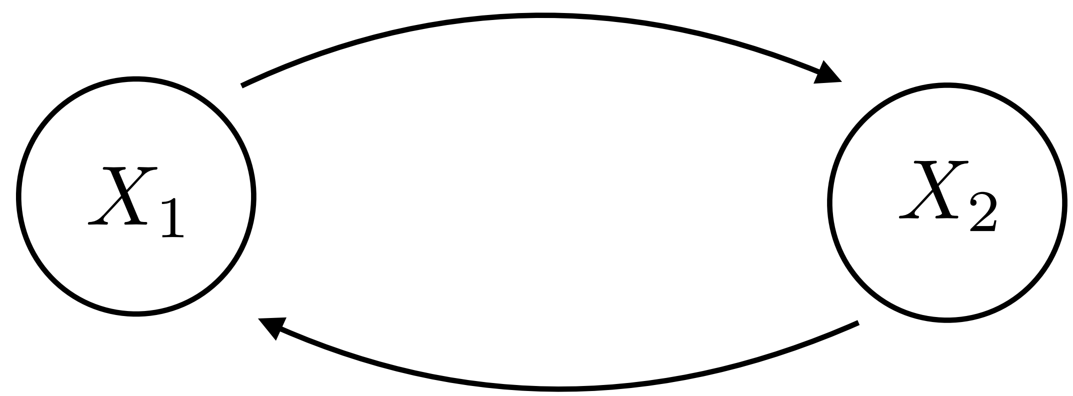
The index can be considered as a measure of the ecological fitness of a given network to function appropriately in its environment

Dynamics

So far, we have considered all relations to be unidirectional, moving from parent to child without a means for the child to influence the parent

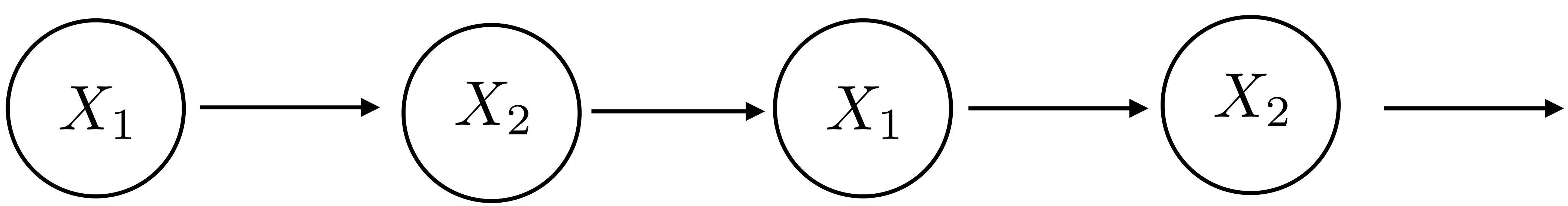
Question: Is it possible to introduce a mechanism for dialogue?

A **dynamic** process by which individuals can work together?



Probability prevents this due to the coupling of conditional relationships (*invariance*). It does not support independent specification of the conditional probabilities

This can be resolved by considering unwrapped series of cyclic networks



This gives us a means to temporally update the system and investigate when the updates converge

This dynamic phenomena can be captured through a Matrix Form Dynamic Model (MFDM)