# Practical-1

1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - \sqrt{2x}}\right]$

Soln $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - \sqrt{2x}} \times \frac{\sqrt{3a+x} + \sqrt{2x}}{\sqrt{3a+x} + \sqrt{2x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}}\right]$

$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + \sqrt{2x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{\sqrt{3a+a} + 2\sqrt{a}}{} \quad \frac{(a-x)}{(a-x)} \frac{(\sqrt{3a+x} + 2\sqrt{x}}{(\sqrt{a+2x} + \sqrt{3x}}$

$\frac{1}{3} \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}} \Rightarrow \frac{1}{3} \frac{\cancel{4}\sqrt{a}}{\cancel{2}\sqrt{3}\cancel{a}}$

$= \cancel{\frac{2}{3\sqrt{3}}} \quad \frac{2}{3\sqrt{3}}$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{y\sqrt{a+y}\,\sqrt{a+y} + \sqrt{a}}\right]$

$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

putting value $y = 0$

$\frac{1}{2(\sqrt{a})(\sqrt{a})} = \frac{1}{2a}$

Scanned with CamScanner

3) $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$ $\quad x = h + \pi/6$ where $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

using $\cos(A+B)$

$\cos A.\cos B - \sin A \sin B$

$\sin(A+B)$

$\sin A \cos B + \cos A \sin B$

$$\lim_{h \to 0} \frac{\cos h.\cos\pi/6 - \sin h \sin\pi/6 - \sqrt{3}\sin h \cos\pi/6 + \cos h \sin\pi/6}{\pi - \cancel{6}(\cancel{6h + \pi/6})}$$

$\cos\pi/6 = \cos 30^\circ = \sqrt{3}/2$

$\sin\pi/6 = 1/\sqrt{2}$

$$\lim_{h \to 0} \frac{\cos h\frac{\sqrt{3}}{2} - \frac{1}{2}\sin h - \sqrt{3}\left(\sin h\frac{\sqrt{3}}{2} + \cos h\frac{1}{\sqrt{2}}\right]}{-6h}$$

$$\lim_{h \to 0} \frac{\cancel{-}\sin 4h/2}{\cancel{-}6h}$$

$$\frac{1}{3}\lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

4) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

$$\lim_{x \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3} \times (\sqrt{x^2+5} + \sqrt{x^2-3}) \times (\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+3} - \sqrt{x^2+1})(\sqrt{x^2+5} + \sqrt{x^2-3} \times \sqrt{x^2+3} + \sqrt{x^2+1})}$$

$$\lim_{x \to \infty} \frac{4\sqrt{x^2+3} + \sqrt{x^2+1}}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4\lim_{x \to \infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2(1-3/x^2)}} = \underline{4}$$

5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \le \pi/2$ } at $x = \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$ for $\pi/2 < x < \pi$

Soln

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ for at $x = \pi/2$ define

$f(x) = \lim_{x \to \pi/2^+} f(x) = \lim \dfrac{\cos x}{\pi - 2x}$

By substituting $x - \pi/2 = h$ $x = h + \pi/2$

$\lim_{h \to 0} \dfrac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)} \Rightarrow \lim_{h \to 0} \dfrac{\cos(h+\pi/2)}{-2h}$

$\Rightarrow \lim_{h \to 0} \dfrac{\cos h \cos \pi/2 - \sin h \sin \pi/2}{-2h}$

$\lim = \dfrac{-\sin h}{-2h} = \dfrac{1}{2}$

$f(x) = \lim_{x \to \pi/2^-} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x \to \pi/2} = \dfrac{2 \sin x \cdot \cos x}{\sqrt{2} \sin x}$

$= \sqrt{2} \lim_{x \to \pi/2^-} \cos x$

LHL ≠ RHL

f(x) not continuous at $\pi/2$

ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$x+3$ $\quad 3 \le x < 6$
$\frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$
at $x = 3$ & $x = 6$

at $x = 3$

① $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x = 3$ define.

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 6.$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{(x-3)} = 6$

LHL = RHL so continous at 3.

for $x = 6$

$f(6) = \frac{x^2-9}{x-3} = \frac{36-9}{6+3} = \underline{3}$

2) $\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x+3)(x-3)}{(x+3)}$

$\lim_{x \to 6^+} (x-3) = 6-3 = \underline{3}$

$\lim_{x \to 6^-} x+3 = 3+6 = 9$

LHL ≠ RHL function is not continous

16

Q(i) $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ k & x = 0 \end{cases}$ at $x=0$

Solⁿ $f$ is continous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \dfrac{1-\cos 4x}{x^2} = k \Rightarrow \lim_{x\to 0} \dfrac{2\sin^2 2x}{x^2} = k$

$2\lim_{x\to 0} \dfrac{\sin^2 2x}{x^2} = k \Rightarrow 2\lim_{x\to 0} \left(\dfrac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k \qquad k = 8$

II) $f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ k & x = 0 \end{cases}$ at $x = 0$

Solⁿ $f(x) = (\sec^2 x)^{\cot^2 x}$

using

$\cancel{\sec^2}\ \tan^2 x - \sec^2 x = 1$

$\cot^2 x = \dfrac{1}{\tan^2 x}$

$\lim_{x\to 0} (\sec^2 x) \cot^2 x$

$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$

We knows that

$\lim_{x\to 0} (1+px)^{1/px} = e$

$= e$

$k = e$

iii) $f(x) = \begin{cases} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & x \neq \pi/3 \\ k & x = \pi/3 \end{cases}$ at $x = \pi/3$

Sol$^n$ $x - \pi/3 = h$

$x = h + \pi/3$ where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} \cdot \tan(\pi/3 + h)}{\pi \cdot 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

using $\tan(A+B)$ formula

$$\lim \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 - \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}(1 - \tan \pi/3 \cdot \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\,\cancel{\sin}\,\sqrt{3}\tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - 3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\tan h}}{3h}$$

$$\lim_{h \to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{-4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$\lim \frac{4\tan h}{3h(1 + \sqrt{3}\tan h)}$$

$$4/3 \cdot \frac{1}{1 - \sqrt{3}/0} = \underline{\underline{4/3}}$$

Q. Discuss the continuity of the following function which of the function have removable discontinuity & Redefine function

(i) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$   $x \neq 0$
$\frac{\pi}{60}$   $x = 0$   ] $x = 0$

Solⁿ $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(e^{3x}-1)\sin x}{x^2}$

$= \lim_{x\to 0} \frac{(e^{3x}-1)}{3x} \times 3 \lim_{x\to 0} \frac{\sin x^\circ}{x}$

$\therefore \lim_{x\to 0} \frac{\sin x^\circ}{x} = \frac{\sin(\pi x/180)}{\frac{\pi x}{180}} \times \pi/180$

$3\log e \times \pi/180 = \pi/60$

$f(x)$ is continous at $x = 0$

(ii) $f(x) = \frac{1-\cos 3x}{x\tan x}$   $x \neq 0$
$x = 0$   } $x = 0$

Solⁿ $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos 3x}{x\tan x}$

$\lim_{x\to 0} \frac{2\sin^2 3/2 x}{x\tan x}$

$\lim_{x\to 0} \frac{\frac{2\sin^2 3/2 x}{x^2}}{\frac{x\tan x}{x^2}} = 2\times 9/4 = 9/2$

$f(x) = 9/2$ is not continous $x = 0$

Redefine $f x$   $x = 0$   $f(0) = 9/2$

removable $f(x) \neq f(0)$ discontinous $x = 0$

(8) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x \neq 0$ is continuous $x = 0$ find $f(0)$

Solⁿ:- $f(x) = \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1 - \cos x + 1}{x^2}$

$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\lim_{x \to 0} \frac{(e^{x^2} - 1)}{x} + \lim_{x \to 0} \frac{(1 - \cos x)}{x}$

$\log 2 + 2 \times 1/2_4 = 3/2 = f(0).$

(8) If $f(x) = \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ $x \neq \pi/2$ continuous $x = \pi/2$ find $f(\pi/2)$.

Solⁿ:- $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$

put $x - \pi/2 = h$ $x = \pi/2 + h$ $h \to 0$.

$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cancel{\cos^2 x}} \times \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}} \Rightarrow \frac{1 + 2 \sin x}{(1 - \sin^2 x)(\sqrt{2} - \sqrt{1 + \sin x})}$

$\frac{1}{2(\sqrt{2} + \sqrt{2})} = \frac{1}{4\sqrt{2}}$ $f(\pi/2)$.

Scanned with CamScanner

18

# Pratical - 2

## Topic ⇒ Derivatives

1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

i) $\cot x$

Ans $f(x) = \cot x$

$$= \lim_{x \to 0} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$\lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a} \quad = \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

$(x-a) = h \qquad x = a+h \qquad x \to a \quad h \to 0.$

$$f(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \cdot \tan(a+h)\tan a}$$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$.

$\tan A - \tan B = \tan(A-B)(1 + \tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan A + \tan(a+h))}{\cancel{h} \tan(a+h)\tan a}$$

$$= \frac{-\tan h}{h} \times \frac{1 + \tan^2 a}{\tan^2 a} \quad = \frac{-\sec^2 a}{\tan^2 a}$$

$$= 1/\cos^2 a \times \frac{\cos^2 a}{\sin^2 a} \quad = -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\operatorname{cosec}^2 a$

$f$ is differential $\forall a \in \mathbb{R}$.

(iii) Cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cos} a}{x-a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{(x-a) \sin a \cdot \sin x}$$

put $x - a = h$
$x = a + h$
$x \to a \quad h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \cdot \sin(a+h)}$$

formula :- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \sin a \cdot \sin(a+h)}$$

$$\lim_{h \to 0} -\frac{\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a.$$

(ii) sec x

Sol$^n$:- $f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x-0}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x-a}$$

$$\lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cos a \cos x}$$

put $x - a = h$
$x \to a \quad h \to 0$

$$Df(x) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

formula $= -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a+h}{2}\right)}{h\cos a \cdot \cos(a+h)}$$

$$\lim_{h \to 0} \frac{-2\sin(2a+h)\sin h/2}{\cos a \cos(a+h) \times -h/2} \quad h/2$$

$$= \frac{-\frac{1}{2} \times -2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= -1/2 \times \frac{-2\sin a}{\cos a \cos a} = \underline{\tan a \times \sec a}$$

**Q.2** If $f(x) = 4x+1 \quad x \le 2$
$\qquad x^2+r \quad x > 0$ at $x=2$ then find function is differentiable or not

Sol$^n$:- L.H.D

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$\lim_{x \to 2^-} \frac{4x+1-(4\times2+1)}{x-2}$$

$$\lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(\cancel{x-2})}{\cancel{x-2}} = \underline{4}$$

R.H.D

$$Df(2^+) = \lim \frac{x^2+5-9}{x-2}$$

$$\lim_{x \to 2^+} \frac{x^2-4}{x-2} = 2+2 = \underline{4}$$

R.H.D = L.H.D
f is differentrable at $x=2$

Q.3 If $f(x) = 4x+7$    $x<3$    $x=3$ then
$x^2+3x+1$    $x \geq 3$

find f is differentiable or not?

Sol$^n$- R.H.D

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x+3-(3^2+3+3+1)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+13x-18}{x-3}$$

$$\lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$\lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = \underline{9}$$

L.H.D

$$Df(3^-) = \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$\lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$\lim_{x \to 3} \frac{4x-12}{x-3}$$

$$Df(3^-) = 4$$

RHD ≠ LHD

Q.4 If $f(x) = 8x - 5$
$3x^2 - 4x + 7 \quad x > 2 \quad$ at $x = 2$
then
find $f$ is differentiable or not.

Sol$^n$:-

$f(2) = 8 \times 2 - 5 = 16 - 5 = \underline{11}$

R.H.D

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$= \lim \frac{(3x+2)(x-2)}{(x-2)}$$

$$3 \times 2 + 2 = 8.$$

$Df(2^+) = 8$

L.H.D $\quad Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$$\lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$Rf'(x^-) = 8$

$L.H.D = R.H.D$

$\therefore f$ is differentiable at $x = 3$

# Practical-3

## Application of Derivative

1) Find the interval in which function is increasing or decreasing

1) $f(x) = x^3 - 5x - 11$
2) $f(x) = x^2 - 4x$
3) $f(x) = 2x^3 + x^2 - 20x + 4$
4) $f(x) = x^3 - 27x + 5$
5) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2). Find the interval in which function is concave upward.

1) $y = 3x^2 - 2x^3$
2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
3) $y = x^3 - 27x + 5$
4) $y = 69 - 24x - 9x^2 + 2x^3$
5) $y = 2x^3 + x^2 - 20x + 4$.

**Sol$^n$**

1 (1) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

f is increasing $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and $f$ is decreasing iff $f'(x) < 0$.

$\therefore 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$.

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x \in (2, \infty)$

$f$ is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x \in (-\infty, 2)$.

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)\ (3x-5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$.

and $f$ is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$= 3x^2 + x - 10 < 0$

$3x^2+6x-5x-10<0$

$3x(x+2)-5(x+2)<0$

$(x+2)(3x-5)<0$

$x \in (-2, 5/3)$

-2  5/3

4) $f(x)=x^3-27x+5$

$f'(x)=3x-27$

$\therefore$ f is increasing iff $f'(x)>0$

$\therefore 3(x^2-9)>0$

$(x-3)(x+3)>0$  $x \in (-\infty,-3)\cup(3,\infty)$

-3  3

and f is decreasing iff $f'(x)<0$

$3x^2-27<0$

$3(x^2-9)<0$

$\therefore (x-3)(x+3)<0$

$x \in (-3,3)$

-3  3

5) $f(x)=2x^3-9x^2-24x+9$

$f'(x)=6x^2-18x-24$

$\therefore$ f is increasing iff $f'(x)>0$

$\therefore 6x^2-18x-24>0$

$6(x^2-3x-4)>0$

$x^2-4x+x-4$

$x(x-4)+1(x-4)>0$

$(x-4)(x+1)$

$x \in (-\infty,-1)\cup(4,\infty)$

-1  4

and f is increasing iff $f'(x)<0$

$\therefore 6x^2-18x-24<0$

$\therefore 6(x^2-3x-4)<0$

$x^2-4x+x-4<0$

$x(x-4)+1(x-4)<0$

$(x-4)(x+1)<0$

$x \in (-1,4)$.

-1  4

Scanned with CamScanner

Q.2

1) $y = 3x^2 - 2x^3$

$f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - 1/2 > 0$ $\quad \because f''(x) > 0$

$x > 1/2$ $\quad x \in (1/2, \infty)$.

(2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x+2) > 0$

$(x-2)(x-1) > 0$

$+ \quad - \quad +$ ; 1, 2 $\quad x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$x > 0$ $\quad x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore \quad 6x > 0 \quad 12x - 18 > 0$

$x > 0 \quad 12(x - 18/12)$

$x - 3/2 > 0 \qquad x > 3/2 \qquad x \in (3/2, \infty)$

**5)** $y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$f$ is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore \quad 12x + 2 > 0$

$12(x + 1/6) > 0$

$x < -1/6$

$\because f''(x) > 0$

$\therefore$ There exist interval $(-1/6, \infty)$.

# Practical-4

Topic :- Application of derivative & Newton's method.

Q.1 → Find maximum & minimum value of following

1) $f(x) = x^2 + 16/x^2$

2) $f(x) = 3 - 5x^2 + 3x^5$

3) $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$

4) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

Q.2 → find the root of the following equation by Newton's (take 4 iteration only) correct upto 4 decimal.

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

2) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Soln

Q.1 $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

Now consider $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$ $\quad x^2 = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4 \Rightarrow 2 + 96/16 = 8 > 0.$

$f$ has minimum value at $x = 2$

$f(2) = 2^2 + 16/2^2 \Rightarrow 4 + 4 = 8 > 0$

$f''(-2) = 2 + 96/(-2)^4$

$2 + 96/16 = 2+6 = 8 > 0$ 40

$\therefore$ $f$ has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x=2$, and $x=-2$

(2) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30+60 = 30 > 0$ $\therefore$ $f$ has minimum value at $x = +1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$6 - 5 = 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$-30 < 0$ $f$ has maximum value at $x = -1$

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ $f$ has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

(3) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$ $x = 0$ $x = 2$

$\therefore f''(x) = 6x - 6$

$f''(x)\ 6(0) - 6 = -6 < 0$ $\therefore$ $f$ has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6 = 12 - 6 = 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$f(2) \quad 8 - 12 + 1 = -3$

$f$ has maximum value 1 at $x = 0$ and $f$ has minimum value $-3$ at $x = 2$.

iv). $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider, $f(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$x = 2$ or $x = 1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$24 - 6 = 18 > 0$

$\therefore f$ has minimum value at $x = 2$.

$\therefore f(2) = 2(2)^3 - 3(2)^3 - 12(2) + 1$

$16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$-18 < 0$

$f$ has maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$-2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has maximum value 8 at $x = -1$ and $f$ has minimum value $-19$ at $x = 2$

Q.2

① $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_0 = 0 \rightarrow$ given

By Newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = \underline{0.1727}$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$
$= 0.0051 - 0.0895 - 9.4985 + 9.5$
$= -0.0829.$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= 0.0895 - 1.0362 - 55$
$= -55.9461$

$x_2 = x_1 - f(x_1) / f'(x_1)$
$= 0.1727 - 0.0829 / 55.9467$
$= 0.1712$

$f(x_2) = \cancel{3}(0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$0.0050 - 0.0879 - 9.416 + 9.5$
$0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$

$x_3 = x_2 - f(x_2) / f'(x_2)$
$= 0.1712 + 0.0011 / 55.9393$
$= 0.1712$

$\therefore$ The root of the eqn is $= 0.1712$

(ii) $f(x) = x^3 - 4x - 9$ $\quad [2, 3]$
$f'(x) = 3x^2 - 4$
$f(2) = \cancel{3}(2)^3 - 4(2) - 9 = -9$

$f(3) = 3^3 - 4(3) - 9$
$27 - 12 - 9 = 6$

let $x_0 = 3$ be the initial appoximation
$\therefore$ By Newton's method.

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23 = 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$20.5528 - 10.9568 - 9$

$0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$22.5096 - 4 = 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$2.7392 - 0.5961/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$19.8386 - 10.8284$

$0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$21.9851 - 4$

$= 17.9851$

$= 2.7071 - \dfrac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$19.7158 - 10.806 - 9 = 0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0050$

$= 2.7065$

Scanned with CamScanner

$f(x) = x^3 - 1.8x^2 - 10x + 7 \quad [1,2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1) - 10(1) + 7 = \underline{6.2}$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$ be initial approximation. By Newton's Method.

$x_{n+1} = x_n - f(x_n) / f'(x_n).$

$x_1 = x_0 - f(x_2) / f'(x_2)$

$2 - 2.2/5.2 = 2 - 0.4230 = \underline{1.577}$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577)^2 + 17$

$3.9219 - 4.4764 - 15.77 + 17$

$\underline{6.6755}$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$1.577 + 0.6755 / 8.2164$

$1.577 + 0.0822 = \underline{1.6592}$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$4.5677 - 4.9553 - 16.592 + 17$

$\underline{0.0204}$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$1.6592 + 0.0204 / 7.7143$

$1.6592 + 0.0026$

$\underline{1.6618}$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$4.5892 - 4.9708 - 16.618 + 17$

$\underline{0.0004}$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$8.2847 - 5.9824 - 10$

$\underline{-7.6977}$

$x^4 = x_3 - f(x_3) / f'(x_3)$

$1.6618 + \frac{0.0004}{7.6977}$

$x_4 = \underline{\underline{1.6618}}$

Scanned with CamScanner

# Practical-5

## Topic :- Integration.

Q.1 Solve the following integration.

1. $\int \frac{dx}{\sqrt{x^2+2x-3}}$

2. $\int (4e^{3x}+1)\,dx$

3. $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

4. $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

5. $\int t^7 \sin(2t^4)\,dt$

6. $\int \sqrt{x}\,(x^2-1)\,dx$

7. $\int \frac{1}{x^3} \sin(1/x^2)\,dx$

8. $\int \frac{\cos x}{3\sqrt{\sin^2 x}}\,dx$

9. $\int e^{\cos^2 x} \sin 2x\,dx$

10. $\int \frac{x^2-2x}{(x^3-3x^2+1)}\,dx$

## Sol$^n$

① $\int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$\# \; a^2+2ab+b^2 = (a+b)^2$

$\int \frac{1}{\sqrt{x^2+2x+1-4}}$

$\int \frac{1}{\sqrt{(x+1)^2-4}}$

substitute
put $x+1 = t$
$dx = \frac{1}{t} \, dt$
where $t=1$ $\quad t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$

using

$\# \; \int \frac{1}{\sqrt{x^2-a^2}} \, dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$= \ln(|x+1+\sqrt{(x^2+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|) + c.$

2) $\int (4e^{3x}+1 \, dx$

$= \int 4e^{3x} \, dx + \int 1 \, dx$

$4\int e^{3x} \, dx + \int 1 \, dx.$ $\quad \# \; \int e \, dx = \frac{1}{x} e^{ax}$

$\frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$\int 2x^2 - 3\sin x + 5x^{1/2}\,dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$\int 2x^2\,dx - \int 3\sin(x)\,dx + 5\int x^{1/2}\,dx$

$\frac{2x^3}{3} + 3\cos x + \frac{10\sqrt{x}}{3} + c$

$\frac{2x^3 + 10\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

# split the denominator.

$\int \frac{x^3}{x^{1/2}}\,dx + \int \frac{3x}{x^{1/2}}\,dx + \frac{4}{x^{1/2}}\,dx$

$\int x^{5/2}\,dx \int 3x^{1/2}\,dx \int \frac{4}{x^{1/2}}\,dx$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 8t^3$

$\int t^7 \sin(2t^4) \times \frac{1}{8t^3}\,du$

$\int t^4 \sin(u) \times \frac{1}{8}\,du$

substitute $t^4$ with $u/2$

$\int \frac{u/2\,\sin(u)}{8}\,dt = \int \frac{u \times \sin u}{16}\,du.$

$1/16 \int u \sin u\,du$

$\int u\,dv = uv - \int v\,du$
where $u = u$
$dv = \sin(u) \times du$
$du = 1\,du$ ; $v = -\cos(u)$

$\frac{1}{16}\; u - \cos u - \int -\cos(u)\,du$

$\int \cos x\,dx = \sin x$

$= \frac{1}{16}\int (u \times -\cos u + \sin u)$

Return the substitution $u = 2t^4$

$= \frac{1}{16}\left(2t^4 \times (-\cos(2t^4)) + \sin(2t^4)\right)$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

vi) $\int \sqrt{x}\,(x^2 - 1)\,dx$

$\int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$\int x^{1/2} x^2 - x^{1/2}\, dx$

$\int x^{5/2} - x^{1/2}\, dx$

$\int x^{5/2} - \int x^{1/2}\, dx$

$\frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3} + c$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

put $t = \sin(x)$
$dt = \cos(x)$

$= \int \frac{\cancel{\cos x}}{(\sin x)^{3/2}} \times \frac{1}{\cos x}\,dx$

$\int \frac{1}{\sin^{3/2} x^2}\,dt$ $\qquad \sin x = t$

$\int \frac{1}{(t)^{3/2}} = 3\,\sqrt[3]{t}$

Retur
$3\sqrt[3]{}$

10) $\int \frac{x^2}{x^3}$

pu

$\int x$

$\frac{1}{3}$

Return substitution $t = \sin(x)$

$3\sqrt[3]{\sin(x)} + c$

(10) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = t$

$3x^2-6x\,dx = dt$

$\int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dt$

$\frac{1}{3}\int \frac{1}{x^3-3x^2+1}\,dt$

$\frac{1}{3}\int \frac{1}{t} = \frac{1}{3}\ln(t)$

$t = x^3-3x^2+1$

$\frac{1}{3}\ln(x^3-3x^2+1) + c.$

## Practical no-6

Topic:- Application of Integration & numeric integration.

Q.1 Find the length of the following curve.

1. $x = t\sin t$, $y = 1-\cos t$ $t \in [0, 2\pi]$
2. $y = \sqrt{4-x^2}$ $x \in [-2, 2]$
3. $y = x^{3/2}$ in $[0, 4]$
4. $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$
5. $x = \frac{1}{6}y^3 + \frac{1}{2y}$ $y \in [1, 2]$

Q.2 Using Simpson's Rule solve the following

1. $\int_0^2 e^{x^2}\,dx$ with $n=4$
2. $\int_0^4 x^2\,dx$ with $n=4$
3. $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n=6$

Sol$^n$



① $l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$

$\int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$

$\int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$\int_0^{2\pi} \sqrt{2(1-\cos t)}\, dt$

$\int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt = \int_0^{2\pi} 2\sin t/2\, dt$

$= \left[-4\cos(t/2)\right]_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$

$4 + 4 = \underline{\underline{8}}$

② $y = \sqrt{4-x^2} \quad x \in [-2, 2]$

$\frac{dy}{dx} = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$

$l = 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$l = 2^2 \cdot \int_0^2 \sqrt{\frac{1+x^2}{4-x^2}}\, dx$

$l = 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx = 4\left[\sin^{-1}(x/2)\right]_0^2 = \underline{\underline{2\pi}}$

(3) $y = x^{3/2}$ in $[0,4]$

$f'(x) = 3/2\, x^{1/2}$

$[f'(x)]^2 = \frac{9}{4}x$

$l = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

$\int_0^3 \sqrt{1+\frac{9}{4}x}\,dx$

put $u = 1+9/4x$ $du = \frac{9}{4}dx$

$l = \int_1^{1+9/4x} 4/9 \sqrt{u}\,du = \left[4/9 \cdot 2/3 (u^{3/2})\right]^{1+9/4x}$

$= 8/27\left[\left(1+\frac{9x}{4}\right)-1\right]$

(4) $x = 3\sin t$, $y = -3\cos t$ $t \in [0, 2\pi]$

Soln:- $\frac{dx}{dt} = 3\cos t$ $\frac{dy}{dt} = -3\sin t$.

$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}\,dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 + 9\cos^2 t}\,dt$

$\int_0^{2\pi} 3\sqrt{x}\,dt$

$-3[x]_0^{2\pi} = -3(2\pi - 0)$

$L = \underline{6\pi \text{ unit}}$

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1,2]$

$\Rightarrow \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy = \int_1^2 \sqrt{1+\frac{(y^4-1)}{4y^2}}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy = \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$\frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$

$\frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right] = \frac{17}{12}$ units.

Q2

(1) $\int_0^2 e^{x^2}\, dx$ with $n = 4$

$\int_0^2 e^{x^2}\, dx = 16.4526$

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

By Simpsons rule.

$\int_0^2 dx = \frac{1/2}{3}(y_0 + 4y_1 + 2 \times y_2 + 4 \times y_3 + y_4)$

$\frac{1}{2}\left(e_0^{2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + 7e^{(2)^2}\right)$

$\approx \underline{\underline{17.3536}}$

(2) $\int_0^4 x^2\,dx \quad n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$

$\frac{1}{3}\left(y(0) + 4(1) + 2(2) + 4(3) + (4)\right]$

$\frac{1}{3}\left(0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$

$\frac{64}{3} \approx \underline{\underline{21.333}}$

(3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx \quad n = 6$

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 0 | $\pi/18$ | $2\pi/18$ | $3\pi/18$ | $4\pi/18$ | $5\pi/18$ |
| $y$ | 0 | 0.4167 | 0.584 | 0.707 | 0.801 | 0.875 |
| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ |

$\int_0^{\pi/3} \sqrt{\sin x}\,dx = \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right)$

$\approx \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.875)\right\}$

$\approx 2(0.584 + 0.801) + 0.932$

$\approx \underline{\underline{0.6801}}$

AK 09/01/2020

Scanned with CamScanner

# Practical = 7

Topic ⇒ Solve the following differential eqn (Differential eqn).

① $x\frac{dy}{dx} + y = e^x$

dividing by $x$

$\frac{dy}{dx} + y/x = \frac{e^x}{x}$

by comparing with

$\frac{dy}{dx} + p(x)y = Q(x)$

$I.f = e^{\int p\,dx}$

$e^{\int 1/x\,dx}$ $\quad\quad \int e^{\log x} dx = x$

$y(I.f) = \int Q\,(I.f)\,x\,dx + c$

$y(x) = \int \frac{e^x}{x} \cdot x\,dx + c$

$y(x) = \int e^x + c$

$y(x) = e^x + c$

② $e^x\frac{dy}{dx} + 2e^xy = 1$

dividing by $e^x$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

By comparing with

$\frac{dy}{dx} + P(x)y = Q(x)$ $\quad Q(x) = \frac{1}{e^x}$

$I.f = e^{\int p\,dx}$

$e^{\int 2\,dx} = \cancel{e^{2x}}\, e^{2x}$

~~sol^n~~ $y(I.f) = \int Q(I.f)\,dy + c.$

$y(e^{2x}) = \int \frac{1}{e^x}(e^{2x})\,dy + c$

$ye^{2x} = e^x + c$

$= \underline{ye^{2x} - e^x + c}$

Q.3 $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

divide ~~multiply~~ by $x$

$x^2\frac{dy}{dx} = \frac{\cos x}{x^2} - \frac{2\cancel{x}y}{x^2}$

$\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{2}{x}$ $\quad I.f = e^{\int p\,dx} = e^{\int 2/x\,dx} = \underline{\cancel{x}x^2}$

$y(I.f) = \int Q(I.f)\,dx + c$

$y(2x) = \int \frac{\cos x}{x^2} \times 2x^2\,dx + c$

$y(2x) = \sin x + c \quad = \underline{2yx - \sin e - c}$

Q.5 $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

divide by $e^{2x}$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$\frac{dy}{dx} + p(x)y = Q(x)$ $\quad p(x) = 2$

$I.f = e^{\int p(x)dx} = e^{\int 2\,dx} = e^{2x}$

$y(I.f) = \int Q(I.f)\,dy + c$

$ye^{2x} = \int \frac{2x}{e^{2x}} \times e^{2x} + c$

$ye^{2x} = x^2 + c$

Q.6 $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} - \frac{3y}{x} = \frac{\sin x}{x^3}$

$I.f = e^{\int p\,dx} = e^{\int 3/x} = x^3$

$y(I.f) = \int Q(I.f)\,dx + c$

$x^2$ $\quad y(x^3) = \int \frac{\sin x}{x^3} \times x^3 + c$

$x^3y = -\cos x + c$

Q.6 $\sec^2x\tan y\,dx + \sec^2y\tan x\,dy = 0$

$\sec^2x\tan y\,dx = -\sec^2y\tan x\,dy$

$\int \frac{\sec^2x\,dx}{\tan x} = -\frac{\sec^2y\,dy}{\tan y}$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\Rightarrow \tan x \cdot \tan y = e^c$

vi) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$ differentiation

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 x$

$\int \frac{dv}{\cos^2 x} = \int dx \Rightarrow \tan v = x + c$

$\tan(x+y-1) = x + c.$

vii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = v$

$2 + 3\frac{dy}{dv} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v+1+2v+4}{v+2}$

$= \frac{3v+3}{v+2} = \frac{3(v+1)}{v+2}$

$\int \frac{v+2}{v+1} dv = 3\,dx$

$$\int \frac{u+1+1}{u+1}\, du = \int 3\, dx$$

$$\int \frac{u+1}{u+1}\, du + \int \frac{1}{u+1}\, du = \int 3\, dx$$

$$u + \log(u+1) = 3x + c$$

$$u + \log(u+1) - 3x - c$$

$$2x + 3y + \log(2x + 3y + 1) - 3x - c = 0$$

AK
15/01/2020

# Practical = 8

Aim:- Using Euler's method find following

1) $\frac{dy}{dx} = y+e^{x}-2$   $y(0)=2$, $h=0.5$   find $y(2)$

2) $\frac{dy}{dx} = 1+y^2$   $y(0)=0$, $h=0.2$, find $y(1)$.

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$   $y(0)=1$, $h=0.2$   find $y(1)$.

4) $\frac{dy}{dx} = 3x^2+1$, $y(1)=2$   find $y(2)$ for $h=0.5$
& $h = 0.25$

5. $\frac{dy}{dx} = \sqrt{xy}+2$   $y(1)$; find $y(1.2)$ with $h=0.2$

Soln :

1) $\frac{dy}{dx} = ye^{x}-2$

$f(x,y) = y+e^{x}-2$   $y_0=2$   $x_0=0$   $h=0.5$

| $n$ | $x_0$ | $y_0$ | $f(x_0,y_0)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 2.5 |
| 1 | 0.5 | 2.5 | 2.847 | 3.57435 |
| 2 | 1 | 3.5743 | 4.2925 | 5.3615 |

$y_{n+1} = y_n + hf(x_n\, y_n)$

| $n$ | $x_n$ | $y_n$ | $f(x_n\, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 3 | 1.5 | 5.3615 | 7.8437 | |
| 4 | 2 | 9.2831 | | |

By Euler's formula
$y(2) = 9.2831$

2) $\frac{dy}{dx} = 1+y^2$

$f(x,y) = 1+y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's Iteration formulae.

$y_{n+1} = y_n + h f(x_n, y_n)$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0.2 |
| 1 | 0.2 | 0.2 | 0.104 | 0.408 |
| 2 | 0.4 | 0.408 | 1.1665 | 0.6413 |
| 3 | 0.6 | 0.6413 | 1.4113 | 0.9236 |
| 4 | 0.8 | 0.9236 | 1.8503 | 1.2942 |
| 5 | 1 | 1.2942 | | |

By Euler's formulae.

$y(1) = 1.2492$

3) $\frac{dy}{dx} = \sqrt{x}/4$ $y(0) = 1$ $x_0 = 0$, $h = 0.2$

Using Euler's Iteration formulae,

$y_{n+1} = y_n + n h f(x_n y_n)$

| $n$ | $x_n$ | $y_n$ | $f(x_n y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 0.2 | 1 | 0.4472 | 1.0894 |
| 2 | 0.4 | 1.0894 | 0.6059 | 1.2105 |
| 3 | 0.6 | 1.2105 | 0.7046 | 1.3513 |
| 4 | 0.8 | 1.3513 | 0.7699 | 1.5031 |
| 5 | 1 | 1.5051 | | |

$y(1) = 1.5031$

4) $\frac{dy}{dx} = 3x^2+1 \quad y_0 = 2 \quad x_0 = 1 \quad h = 0$

for $h = 0.5$

using Euler's formula.

| $n$ | $x_n$ | $y_n$ | $f(x_n\ y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 4 | 4 |
| 1 | 1.5 | 4 | 7.75 | 7.875 |
| 2 | 2 | 7.875 | | |

$y(2) = 7.875$

$h = 0.25$

| $n$ | $x_n$ | $y_n$ | $f(x_n\ y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 4 | 3 |
| 1 | 1.25 | 3 | 5.6875 | 4.4218 |
| 2 | 1.50 | 4.4218 | 59.6569 | 14.3360 |
| 3 | 1.25 | 19.3360 | 1122.6424 | 249.9966 |
| 4 | 2 | 299.9966 | | |

$y(2) = 299.9966$

5) $\frac{dy}{dx} = \sqrt{xy}+2$

$y(0)=1 \quad y_0 = 1 \quad x_0 = 1 \quad h = 0.2$ find (1.2)

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 3.6 |
| 1 | 1.2 | 3.6 | | |

# Practical = 9

## Limits and Partial Order Derivative

Q.1

① $\lim_{(x,y)\to(-4,1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5} = -\dfrac{61}{4}$

2) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(0+1)((2)^2+0-(2))}{2+0} = \dfrac{1(4+0-8)}{2} = -\dfrac{4}{2} = -2$

③ $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^2-x^2yz}$

At $(1,1,1)$ Denominator $\neq 0$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \cancel{\dfrac{x^2-y^2z^2}{x^3-x^2yz}}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{(x-yz)(x+yz)}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} = \dfrac{x+yz}{x^2}$

on Applying limit

$\dfrac{1+1(1)}{(1)^2} = 2$

**Q.2**

① $f(x,y) = xy e^{x^2+y^2}$

$\therefore f_x = \frac{\partial}{\partial x} f(x,y))$

$\frac{\partial}{\partial x}(xye^{x^2+y^2})$

$ye^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$\frac{\partial}{\partial y}(xye^{x^2+y^2})$

$xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

② $f(x,y) = e^x \cos y$

$F_x = \frac{\partial}{\partial x}(f(x,y)$

$\frac{d}{dx}(e^x \cos y)$

$\therefore F_x - e^x \cos y$

$F_y = \frac{d}{dy}(f(x,y))$

$= \frac{d}{dy}(e^x \cos y)$ $\quad F_y = -e^x \sin y$

③ $F(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$F_x = \frac{d}{dx} f(x,y))$

$\frac{d}{dx}(x^3y^2 - 3x^2y + y^3 + 1)$

$F_x = 3x^2y^2 - 6xy$

$f_y = \frac{d}{dy}(F(x,y))$

$\frac{d}{dy}(x^3y^2 - 3x^2y + y^3 - 1)$

$\therefore f_y = 2x^3y - 3x^2 + 3y^2$

Q.3

(i) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{d}{dx}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{d}{dx}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2-0}{(1+y^2)^2} = \frac{2(1+y^2)}{(1+y^2)(1+y^2)} = \frac{2}{1+y^2}$

At $(0,0)$

$= \frac{2}{1+0} = \underline{\underline{2}}$

$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{d}{dx}(2x) - 2x \frac{\partial}{dx}(1+y^2)}{(1+y^2)^2}$

$\frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2} = \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$

$= \frac{-4(0)(0)}{(1+0)^2} = \underline{\underline{0}}$

Q.4

(i) $F(x,y) = \frac{y^2-xy}{x^2}$

$F_x = \frac{x^2 \frac{d}{dx}(y^2-xy) - (y^2-xy)\frac{d}{dx}(x^2)}{(x^2)^2}$

$\frac{-x^2y - 2x(y^2-xy)}{x^4}$

$F_y = \frac{2y-x}{x^2}$

$$f_{xx} = \frac{d}{dx}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4 \frac{d}{dx}(-x^2y - 2xy^2 + 2x^2y) - (-x^2y - 2xy^2 + 2x^2y)\frac{d}{dx}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy^2 + 2x^2y)}{x^8} \quad ①$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^3}\right)$$

$$\frac{2-0}{x^3} = \frac{2}{x^3} \quad - ②$$

$$f_{xy} = \frac{d}{dy}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad - ③$$

$$f_{yx} = \frac{d}{dx}\left(\frac{2y - x}{x^3}\right)$$

$$\frac{x^3\frac{d}{dx}(2y - x) - (2y - x)\frac{d}{dx}(x^3)}{x^6} \quad - ④$$

$f_{xx}$ [illegible]

$f_{xy} = f_{yx}$

⑤ $f(x,y) = x^3 + 3x^2y^2 - 2y(x^2+1)$

$f_x = \frac{d}{dx}(x^3 + 3x^2y^2 - 2y\,x^2+1)$ $\quad f_y = \frac{d}{dy}(x^3 + 3x^2y^2 - 2y\,(x^2+1))$

$3x^2 + 6xy^2 - \frac{2x}{x^2+1}$ $\quad = 0 + 6x^2y - 0$

$$f_{xx} = 6x + 6y^2 - \frac{\left(x^2+1\left(\frac{d(2x)}{dx}\right) - 2x\frac{d}{dx}(x^2+1)\right)}{(x^2+1)^2}$$

Scanned with CamScanner

$6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$

$f_{yy} = \frac{d}{dy}(6x^2y) \quad = 6x^2$

$f_{xy} = \frac{d}{dy}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$0 + 12xy - 0 \quad = 12xy \quad$ - (iii)

$f_{yx} = \frac{d}{dx}(6x^2y) \quad = \quad 12xy \quad$ - (iv)

from iii and iv

$\therefore f_{xy} = f_{yx}$

(ii) $f(x,y) = \sin(xy) + e^{x+y}$

$F_x = y\cos(xy) + e^{x+y}$ (1) $\qquad f_y = x\cos(xy) + e^{x+y}$ (1)

$= y\cos(xy) + e^{x+y} \qquad x\cos x(xy) + e^{x+y}$

$\therefore f_{xx} = \frac{d}{dx}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\,(y) + e^{x+y}$

$\cancel{-y\sin(xy) + e^{x+y}}$

$\therefore f_{yy} = \frac{d}{dy}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)\,(x) + e^{x+y}$

$-x^2\sin(xy) + e^{x+y}$

$\therefore f_{xy} = \frac{d}{dy}(y\cos(xy) + e^{x+y})$

$-y^2\sin(xy) + \cos(xy) + e^{x+y}$

Scanned with CamScanner

$f(y\,x) = \frac{\partial f_y}{\partial x} = \frac{d(x\cos xy + e^{x+y})}{dx}$

$= \cos xy(1) + x(-\sin(xy))\,y + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$

$\therefore f(xy) = f(yx) = xy\sin xy + \cos xy + e^{x+y}$

Q.5 (i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \times 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \times 2y = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$\frac{2+x-1+y-1}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

(ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$\rightarrow f(\pi/2, 0) = 1 - \pi/2 + 0$

$= 1 - \pi/2$

$f(x) = -1 + y\cos x$

$f(y) = 1$

$f_x(\pi/2, 0) = -1 + 0\cos(\pi/2) = -1$

$f_y(\pi/2, 0) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 + \pi/2 + (-1)(x - \pi/2) + 1(y-0)$

$1 - \pi/2 - x + \pi/2 + y$

$= y - x + 1$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$\Rightarrow f(1,1)$ $\log(1) + \log(1)$

$0 + 0 = 0$

$f(x) = 1/x$ $\quad f(y) = 1/y$ $\quad f_x(1,1) = 1$ $\quad f_y(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$0 + 1(x-1) + 1(y-1)$

$\underline{x + y - 2}$

# Practical - 10

Q.1

① $f(x,y) = x+2y-3$ at $(1,-1)$ $u = 3i - j$

Here $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{10}$

Unit vector along $u$ is $\frac{u}{|\bar{u}|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1, \frac{-h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$\lim_{h\to 0} = \frac{-4 + h/\sqrt{10} + 4}{h}$

$= \frac{1}{\sqrt{10}}$.

(11) $f(x) = y^2 - 4x + 1$  $a = (3,4) = u = i + 5j$

Here $v = i + 5j$ is not a unit vector
$|\vec{v}| = \sqrt{1+25} = \sqrt{26}$.

unit vector along $u$ is $\frac{v}{|v|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$16 + \frac{25h^2}{\sqrt{26}} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$\frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_uf(a) = \lim_{h \to 0} \frac{\frac{25h^2}{\sqrt{26}} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)$

$\therefore D_uf(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii) $2x+3y$ at $(1,2)$, $u=(3i+4j)$

Here $u = 3i+4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

Unit Vector along u is $\frac{v}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1)+3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$

$f(a+hv) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h\to 0} \frac{\frac{18\cancel{h}}{5} + \cancel{8} - \cancel{8}}{\cancel{h}}$

$= \frac{18}{5}$

Q. Find gradient vector for the following func of given

i) $f(x,y) = x^y + y^x$ , $a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y , x^y \log x + xy^{x-1})$

$f(1,1) = (1+, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1}x) y^2$ $a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2} , 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2} , \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2} , \frac{\pi}{\cancel{4}_2}(-\cancel{2})\right)$

$= \left(\frac{1}{2} , -\frac{\pi}{2}\right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$ , $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

At $\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z} , xz - e^{x+y+z} , xy - e^{x+y+z}$

$f((1,-1),0) = \Big((-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{x+(-1)+0}, (1)(-1) - e^{1+(-1)+0}$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q3) Find the equation of tangent and normal to each of the following using curves at given Points.

(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$\Rightarrow$ $f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1, y_0 = 0$

eq$^n$ of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ $\rightarrow$ It is two required eq$^n$ of tangent

eq$^n$ of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$$1(1) + 2(y) + d = 0$$
$$\therefore 1 + 2y + d = 0 \quad \text{at } (1,0)$$
$$1 + (2(0) + d = 0$$
$$d + 1 = 0$$
$$\therefore d = -1$$

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$\Rightarrow$ $f_x = 2x + 0 - 2 + 0 + 0$
$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$
$= 2y + 3$

$(x_0, y_0) = (2, -2) \quad \therefore x_0 = 2,\ y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

$eq^n$ of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + ((-1)(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ $\rightarrow$ It is required $eq^n$ of tangent.

$eq^n$ of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + (2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q4) Find the eq$^n$ of tangent and normal line to each of the following Surface:-

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eq$^n$ of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ → This is required eq$^n$ of tangent.

eq$^n$ of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

(ii) $3xy^z - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1$, $y_0 = -1$, $z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq$^n$ of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → This is required eq$^n$ of tangent

eq$^n$ of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q5) Find the local maxima and minima for the following

(i) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\Rightarrow$ $f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \longrightarrow (2)$

Multiply eq$^n$ 1 with 2

$\therefore \quad 4x - 2y = -4$

$\underline{2y - 3x = 4}$

$x = 0$

Substitute value of $x$ in eq$^n$ (1)

$2(0) - y = -2$

$\cancel{-}y = \cancel{-}2$

$\therefore y = 2$

$\therefore$ Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 62$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$\therefore 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

(ii) $f(x, y) = 2x^4 + 3x^2y - y'$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \rightarrow (1)$

$f_y = 0$

$3x^2 - 2y = 0 \rightarrow (2)$

Multiply eqn (1) with 3

(2) with 4

$12x^2 + 9y = 0$

$-12x^2 + 8y = 0$

$17y = 0 \qquad \therefore y = 0$

73

Substitute value of y in $eq^n$ ①

$$4x^2 + 3(0) = 0$$
$$4x^2 = 0$$
$$x = 0$$

Critical point is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 6$

$r$ at $(0, 0)$

$= 2y(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (5)^2$

$= 0 - 0$

$= 0$

$r = 0$ and $rt - 5^2 = 0$

(nothing to Say)

$f(x, y)$ at $(0, 0)$

$2(0)^4 + 3(0)^{2/0}) - (0)$

$= 0 + 0 - 0$

$= 0$

(iii) $f(x, y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore \quad 2x + 2 = 0$

$x = \frac{-2}{2} \quad \therefore \quad x = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2} \; 4$

$\therefore y = 4$

$\therefore$ Critical point is $(-1, 4)$

Scanned with CamScanner

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 + -2 + 32 - 70$

$= 14 + 30 - 70$

$= 34 - 70$

$= 33$,,

07/2/2024