2302T005

Roll No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Program: B. Tech, Course: Computer Science and Engineering (Artificial Intelligence & Machine Learning)

Subject: Engineering Mathematics, Code: ETMT109

Semester: I

Time: 03 Hours **Max Marks: 70**

Instructions to the Students:

1. This Question paper consists of two Sections. All sections are compulsory.
2. Section A comprises 10 questions of short answer type. All questions are compulsory. Each question carries 02 marks.
3. Section B comprises 8 long answer type questions out of which students must attempt any 5. Each question carries 10 marks.
4. Do not write anything on the question paper.

Q.No.		SECTION –A (SHORT ANSWER TYPE QUESTIONS)	Marks
1.	a	Prove that $\tanh\left(\log\sqrt{3}\right)=\frac{1}{2}$	(2)
	b	Value of $(1-i)^{100}$ is: (i) $2^{100}(\cos 100\pi - i\sin 100\pi)$ (ii) $2^{100}(\cos 25\pi - i\sin 25\pi)$ (iii) $2^{50}(\cos 100\pi - i\sin 100\pi)$ (iv) $2^{50}(\cos 25\pi - i\sin 25\pi)$	(2)
	c	All the four entries of the a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are non-zero, and one of the Eigen Values is zero. Then, i. $\frac{a}{b}=\frac{c}{d}$ ii. $ad+bc=0$ iii. $\frac{a}{b}-\frac{c}{d}=1$ iv. $ad+bc=1$	(2)

d	Find the rank of the matrix $\begin{bmatrix} -2 & 3 & 0 & 0 \\ 1 & 4 & 3 & -1 \\ 3 & 1 & 3 & -1 \end{bmatrix}$	(2)
e	By using a suitable Maclaurin series, find the sum to the infinity of : $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \ldots$ sin π	(2)
f	Find the asymptotes parallel to the x-axis for the curve $x^2y^2 = a^2\left(x^2 + y^2\right)$	(2)
g	The series $\sum_{n=1}^{\infty} \frac{(-1)^n n^{500}}{(1.0001)^n}$ is: i. Converges absolutely ii. Converges to $-\infty$ iii. Bounded but divergent iv. Divergent	(2)
h	Find the value of x for which the series $n^{\log x}$ is convergent?	(2)
i	The product of order and degree of the differential equation $\sqrt{1 + \frac{d^2y}{dx^2}} = x\frac{dy}{dx}$ is: i. 3 ii. 2 iii. 4 iv. 1	(2)
j	The differential equation $7ydx - (4y + 9x)dy = 0$ is: i. Exact and Homogeneous but not Linear ii. Exact and Linear but not Homogeneous iii. Exact, Homogeneous and Linear iv. Homogeneous and Linear but not Exact	(2)

SECTION –B (LONG ANSWER TYPE QUESTIONS)

2. If $\alpha, \alpha^2, \alpha^3, \alpha^4$ are the roots of $x^5 - 1 = 0$. Find them and show that (10)

$$(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$$

3.
   i. Separate $\left(\sqrt{i}\right)^{\sqrt{i}}$ in to real and imaginary parts. (10)

   ii. Find the radius of curvature of the Folium $x^3 + y^3 = 3axy$ at $(3a/2, 3a/2)$.

4. Test the convergence of the series $\sum \frac{(n!)^2}{(2n)!} x^{2n}$ (10)

5.
   i. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and hence find it's inverse. (10)

   ii. Solve $(r + \sin\theta - \cos\theta)dr + r(\sin\theta + \cos\theta)d\theta = 0$

6.
   i. Discuss the convergence of the series $\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \ldots.$ (10)

   ii. Show that the Matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{bmatrix}$ is unitary matrix.

7. Find the Eigen Values and Eigen Vectors of $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$ (10)

8. Solve the differential equation $\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 14\frac{dy}{dx} - 8y = e^x \cos 2x$ (10)

9.
   i. If $y = \left(\sin^{-1} x\right)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ (10)

   ii. Consider the graph of $y = x^3$ on the interval $0 \le x \le 2$. Compute the Area of the Surface of Revolution formed by revolving this graph about the x-axis.

==END OF PAPER==