

**Course B. Tech Computer Science and Engineering**  
**(Artificial Intelligence and Machine Learning)**  
**Subject: Discrete Mathematics, Code: ETCS-201**  
**Semester: III**

**Time: 03 Hours**

**Max Marks: 70**

**Instructions to the Students:**

1. This Question paper consists of two Sections. All sections are compulsory.
2. Section A comprises 10 questions of short answer type. All questions are compulsory. Each question carries 2 marks.
3. Section B comprises 8 long answer type questions out of which students must attempt any 5. Each question carries 10 marks.
4. Do not write anything on the question paper.

Q.No.	SECTION -A (SHORT ANSWER TYPE QUESTIONS)	Marks
1. a.	Show that the function $f : R \rightarrow R$ , defined as $f(x) = x^2$ , is neither one-one nor onto.	(2)
b.	Explain transitive relation on a set with example.	(2)
c.	State "Well-Ordering Principle".	(2)
d.	What is recurrence relations? Also, solve the following recurrence relations: $a_r + 5a_{r-1} + 6a_{r-2} = 0.$	(2)
e.	State pigeon-hole principle.	(2)
f.	Explain "Satisfiable" and "valid propositional logic" with counter example	(2)
g.	What is permutation group?	(2)
h.	Define semi- group and group. Give an example of a finite semi-group which is not a group.	(2)
i.	What is Chromatic number of a graph?	(2)
j.	What is planar graph? Describe with an example.	(2)



## SECTION -B (LONG ANSWER TYPE QUESTIONS)

2. If  $f$  be a real function defined by  $f(x) = \sqrt{x-1}$ . Find  $(f \circ f \circ f)(x)$ . Also, show that  $f \circ f \neq f^2$ . (10)
3. If  $R$  and  $S$  are two equivalence relations on a set  $A$ , then  $R \cap S$  is also an equivalence relations on set  $A$ . Is union of two equivalence relations on a set is an equivalence relations on a set. Justify your answer. (10)
4. (a) If  ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$ , then find the value of  $n^2 + n + 15$ . (2×5=10)  
 (b) Find the number of 6 digit even integers which can be formed such that  
 (i) Repetition of digits is allowed.  
 (ii) Repetition of digits is not allowed.
5. (a) It is known that at the university 60% Professors play tennis, 50% Professors play Football, 70% hockey, 20% tennis and football, 30% tennis and hockey, 40% football and hockey. If someone claimed that 20% Professors play all 3 sports. Would you believe the claim? (2×5=10)  
 (b) Solve  $a_r - 2a_{r-1} - 3a_{r-2} = 0$ ,  $a_0 = 3$ ,  $a_1 = 1$ , by using generating functions.
6. (a) Prove the following statement:  
 "If  $3n+2$  is odd, then  $n$  is odd". (2×5=10)  
 (b) Prove that  
 "If  $x, y$  are integers such that  $xy$  is odd, then both  $x$  and  $y$  are odd".
7. If  $G$  is an abelian group and  $N$  is a normal subgroup of  $G$ , then prove that  $G/N$  is abelian. Show by an example that the converse need not be true. (10)
8. Prove that every field is an integral domain. Is converse true? Justify your answer. (10)
9. Let  $G$  be a connected Planar simple graph with  $e$ -edges and  $v$ -vertices. Let  $r$  be number of regions in planar representation of  $G$ , then prove that  $r = e - v + 2$ . (10)

==END OF PAPER==