INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

DEPARTMENT OF MECHANICAL ENGINEERING

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ME 670 Advance Computational Fluid Dynamics (Advance CFD)

Assignment – 3

Guided By

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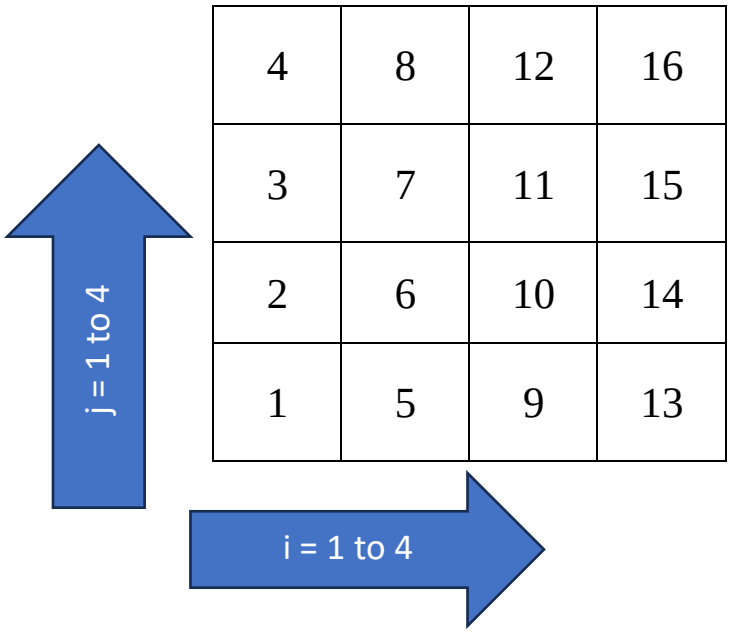
April 2024

Table of Contents

Conjugate Gradient Method (CG) & Pre-Conjugate Gradient Method (PCG)

The discretized equations detail and the boundary condition implementation detail are explained in Code and exactly similar to analysis done in class

Lexicographic Ordering Details



Discretized Equation

$$\frac{\partial T^2}{\partial x^2} + \frac{\partial T^2}{\partial y^2} = 0$$

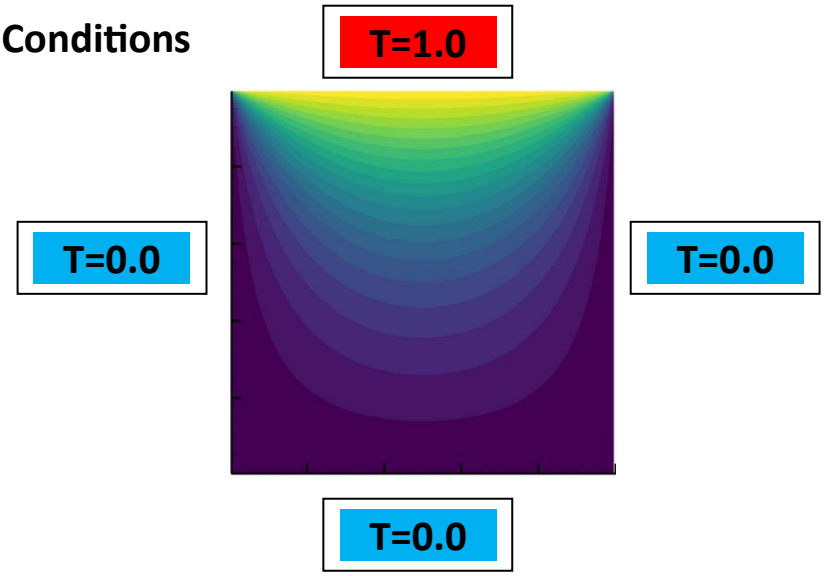
$$\frac{T_{i+1,j} - 2 * T_{i,j} + T_{i-1,j}}{(\Delta x)^2} + \frac{T_{i,j+1} - 2 * T_{i,j} + T_{i,j-1}}{(\Delta y)^2} = 0$$

$$N_i = 128 \quad ; \quad N_j = 128$$

$$i = 1 \; to \; N_i \quad ; \quad j = 1 \; to \; N_j$$

$$N = N_i * N_j = 16384$$

Boundary Conditions



ILU & SIP Equations

ILU	
$\mathrm{L}_{\mathrm{W}}^{n} = \begin{cases} 0 & 1 \le n \le N_j \\ \mathrm{A}_{\mathrm{W}}^{n} & 1 + N_j \le n \le N \end{cases}$	$\mathrm{L}_{\mathrm{S}}^{n} = \begin{cases} 0 & n = 1 \\ \mathrm{A}_{\mathrm{S}}^{n} & 2 \le n \le N \end{cases}$
$\mathrm{L}_{\omega}^{n} = \begin{cases} \mathrm{A}_{\mathrm{P}}^{n} & n = 1 \\ \mathrm{A}_{\mathrm{P}}^{n} - L_S^n U_N^{n-1} & 2 \le n \le N_j \\ \mathrm{A}_{\mathrm{P}}^{n} - L_S^n U_N^{n-1} - L_W^n U_E^{n-N_j} & 1 + N_j \le n \le N \end{cases}$	
$\mathrm{U}_{N}^{n} = \begin{cases} A_N^n / L_P^n & 1 \le n \le N-1 \\ 0 & n = N \end{cases}$	$\mathrm{U}_{E}^{n} = \begin{cases} A_E^n / L_P^n & 1 \le n \le N - N_j \\ 0 & 1 + N - N_j \le n \le N \end{cases}$

SIP ($\alpha = 0.5$)

$$L_W^n = \begin{cases} 0 & 1 \le n \le N_j \\ \dfrac{A_W^n}{1 + \alpha U_N^{n-N_j}} & 1 + N_j \le n \le N \end{cases}$$

$$L_S^n = \begin{cases} 0 & n = 1 \\ \dfrac{A_S^n}{1 + \alpha U_E^{n-1}} & 2 \le n \le N \end{cases}$$

$$L_\omega^n = \begin{cases} A_P^n & n = 1 \\ A_P^n - L_S^n (U_N^{n-1} - \alpha U_E^{n-1}) & 2 \le n \le N_j \\ A_P^n - L_S^n (U_N^{n-1} - \alpha U_E^{n-1}) - L_W^n (U_E^{n-N_j} - \alpha U_N^{n-N_j}) & 1 + N_j \le n \le N \end{cases}$$

$$U_N^n = \begin{cases} \dfrac{A_N^n - \alpha L_W^n U_N^{n-N_j}}{L_P^n} & N_j \le n \le N - 1 \\ A_N^n / L_P^n & 1 \le n \le N_J \\ 0 & n = N \end{cases}$$

$$U_E^n = \begin{cases} \dfrac{A_E^n - \alpha L_S^n U_E^{n-1}}{L_P^n} & 1 \le n \le N - N_j \\ 0 & 1 + N - N_j \le n \le N \end{cases}$$

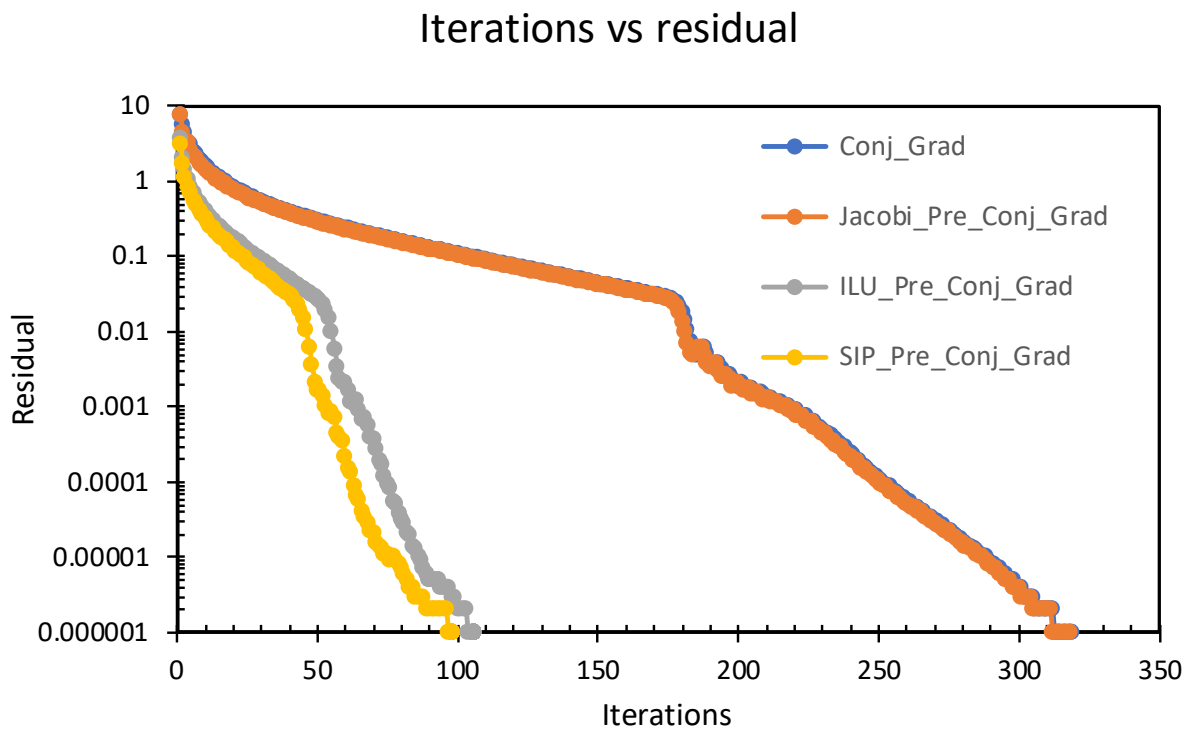
Results

Find All the Results & Comparisons in attached files

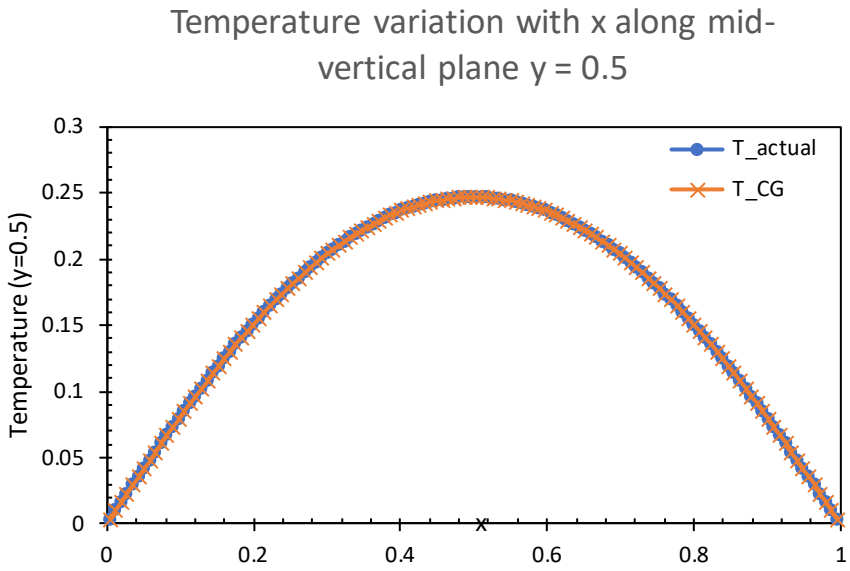
1.0 Number of Iterations

Method	Iterations
Conjugate Gradient Method	319
Jacobi - Pre Conditioner Conjugate Gradient Method	318
ILU - Pre Conditioner Conjugate Gradient Method	106
SIP - Pre Conditioner Conjugate Gradient Method	98

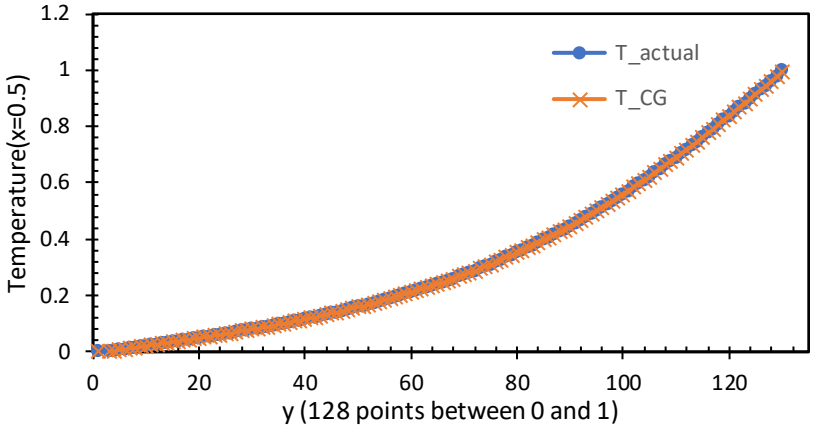
2.0 Iterations vs Residual

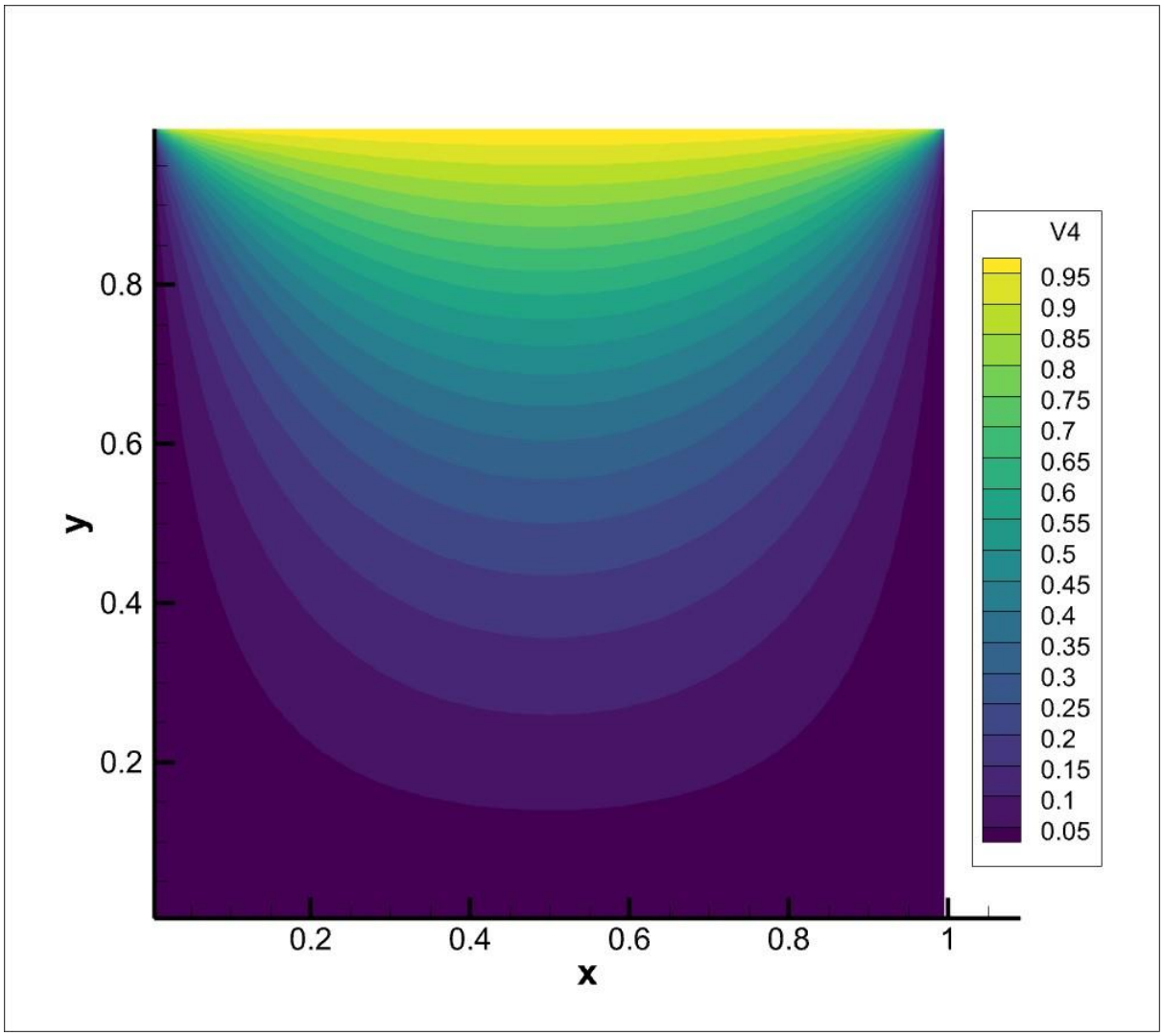


3.0 Conjugate Gradient Method

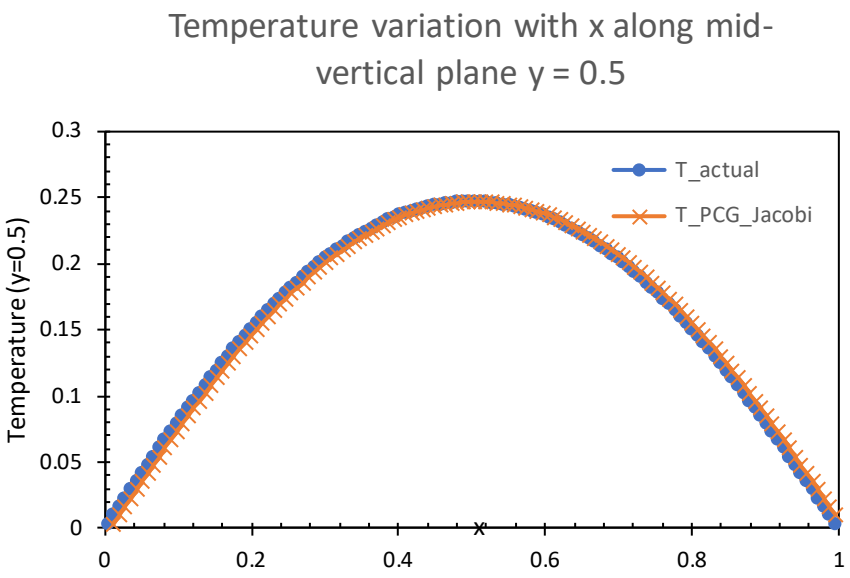


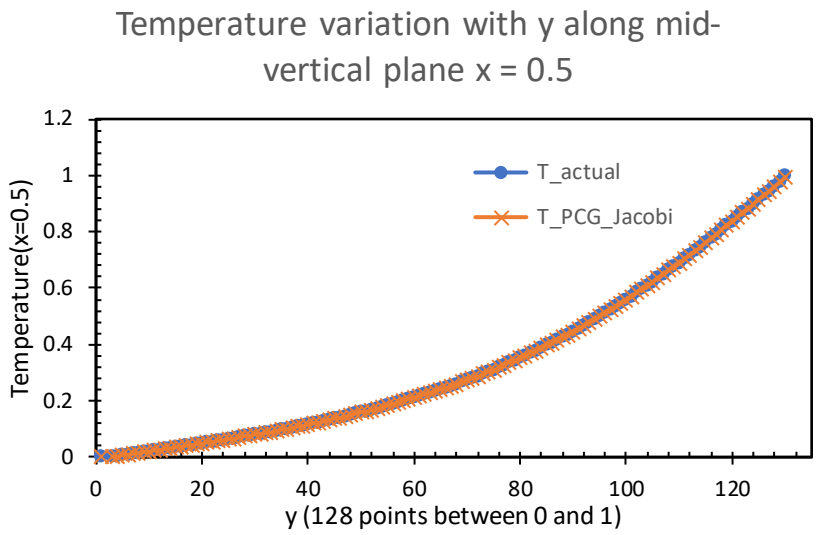
Temperature variation with y along mid-vertical plane x = 0.5

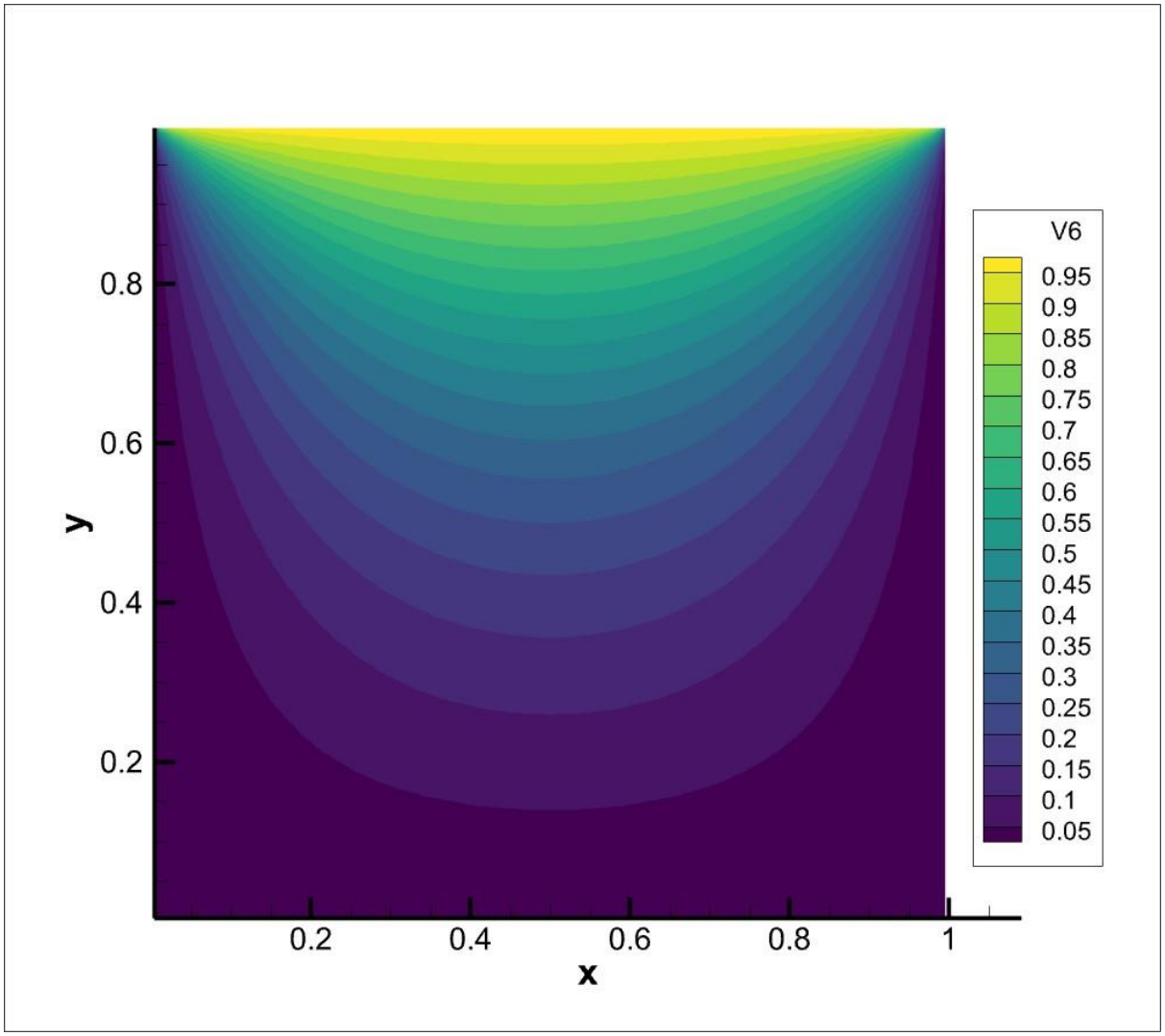




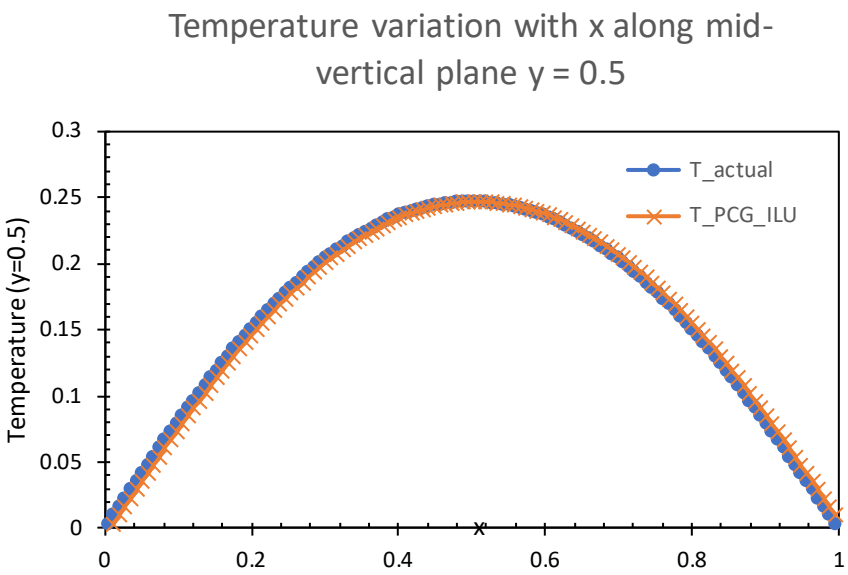
4.0 Jacobi- Pre Conditioner Conjugate Gradient Method

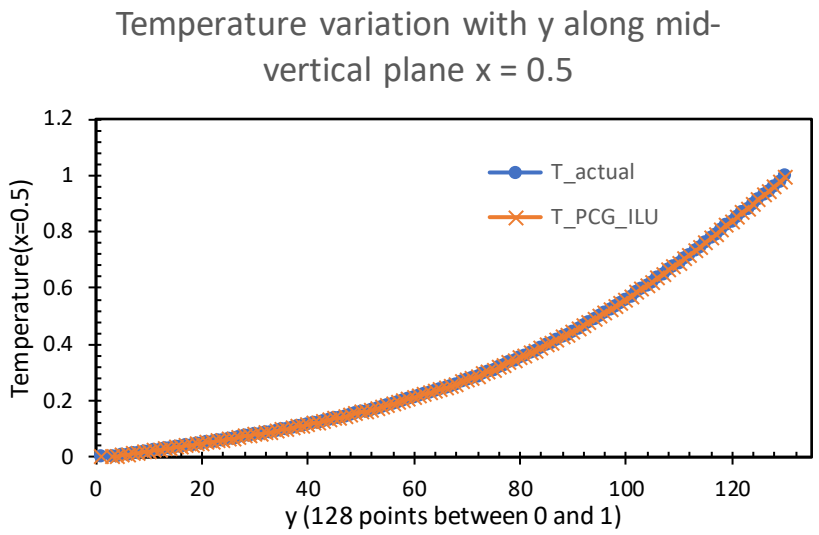


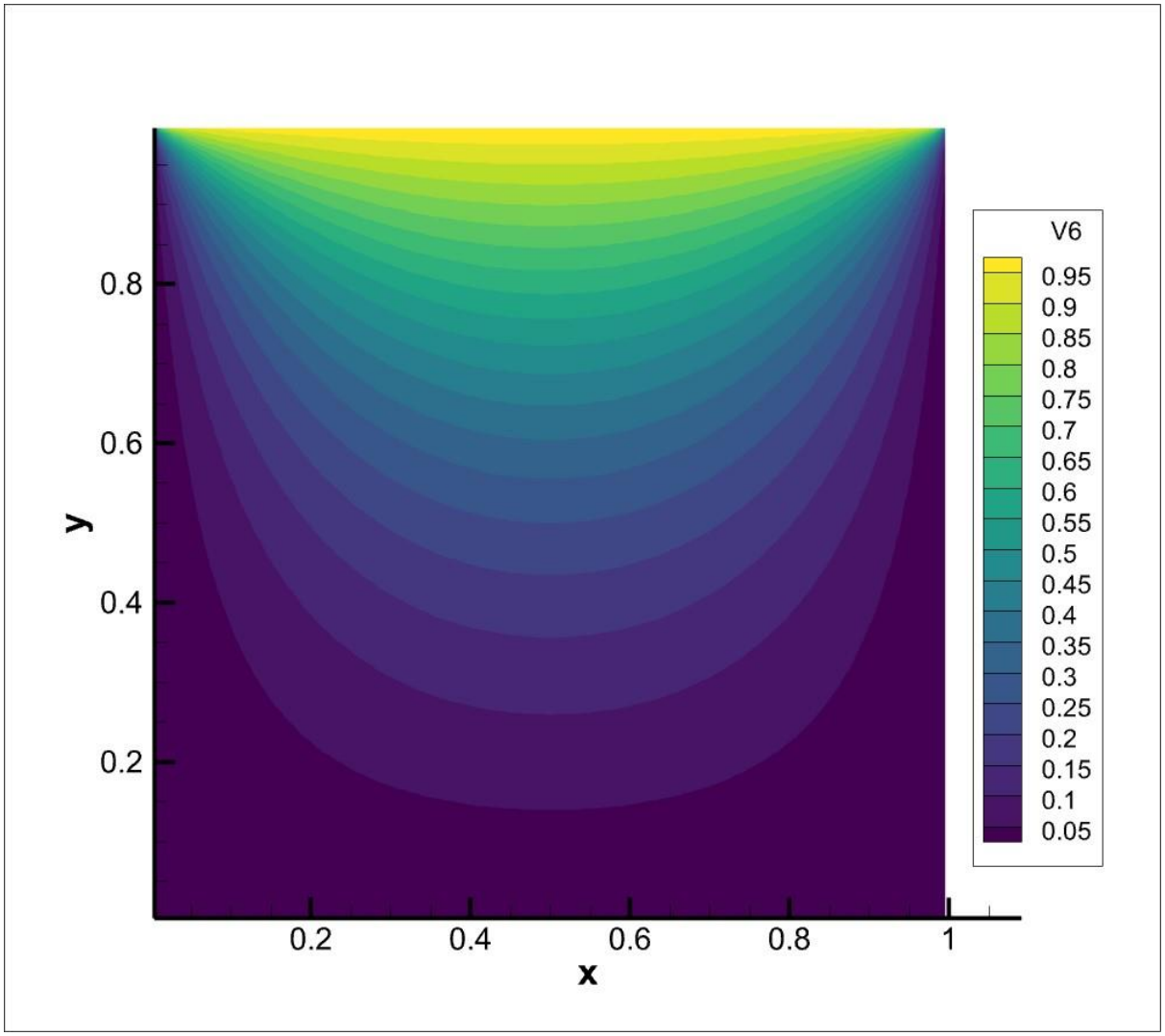



V6
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
y
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1
x

5.0 ILU- Pre Conditioner Conjugate Gradient Method






V6
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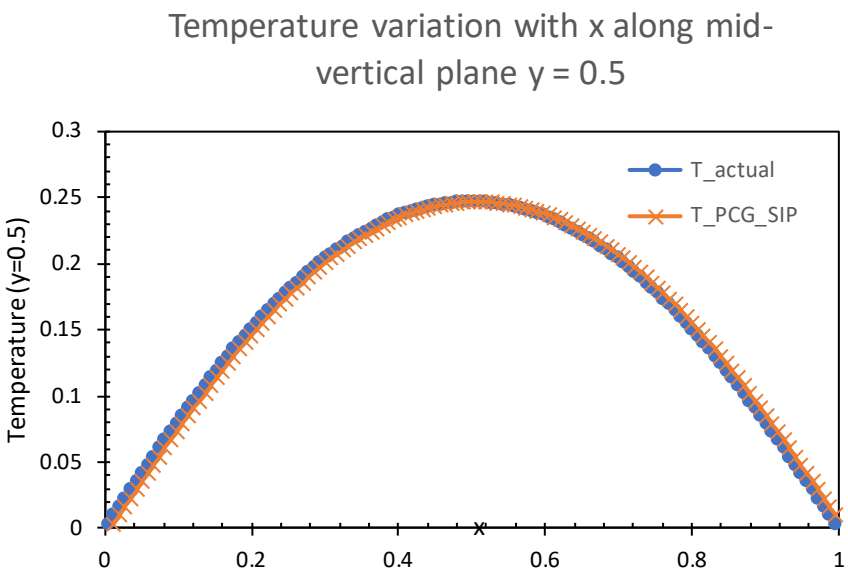
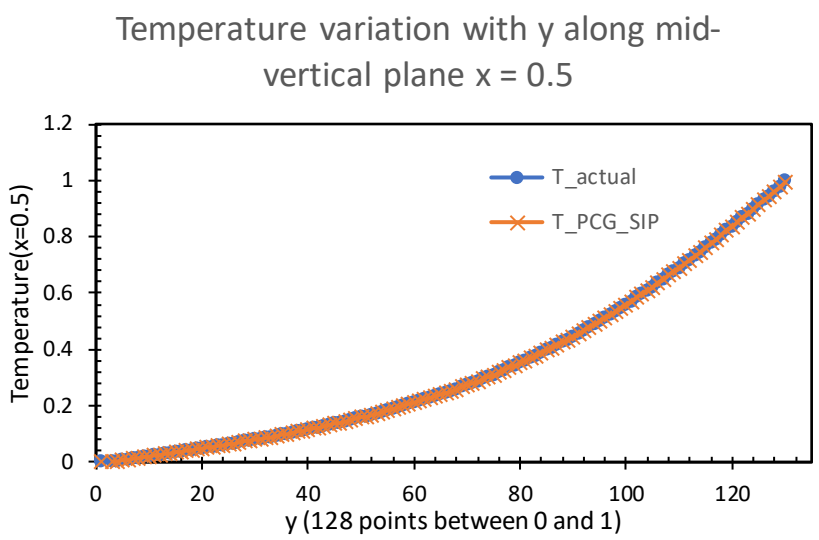
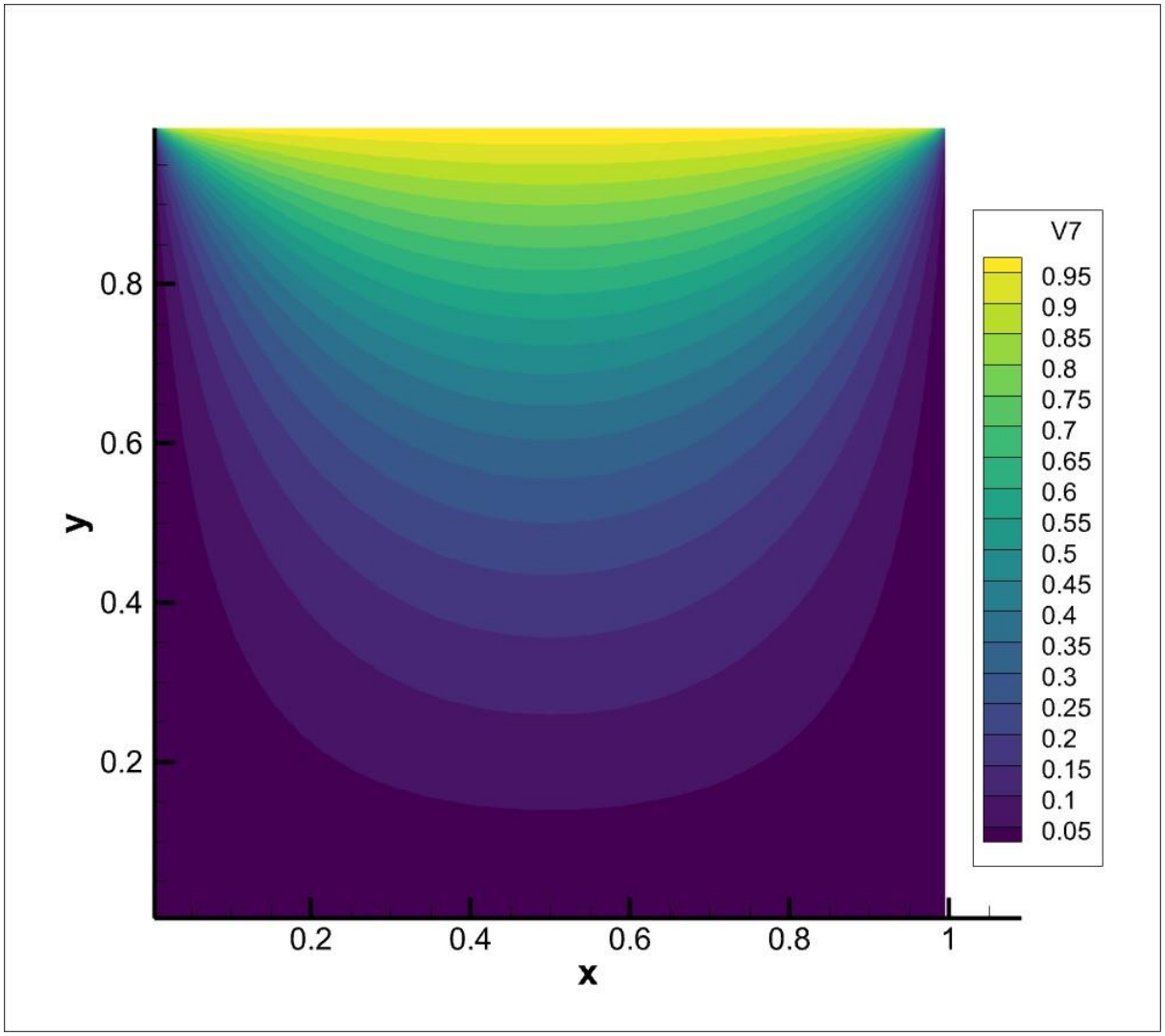
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
y
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1
x

6.0 SIP- Pre Conditioner Conjugate Gradient Method







7.0 L and U matrices coming from ILU and SIP factorization

ILU

Ni = 4 , Nj = 4 , N = 16

Point(n)	L_W	L_S	L_P	U_N	U_E
1	0	0	-6	-0.16667	-0.16667
2	0	1	-4.83333	-0.2069	-0.2069
3	0	1	-4.7931	-0.20863	-0.20863
4	0	1	-5.79137	0	-0.17267
5	1	0	-4.83333	-0.2069	-0.2069
6	1	1	-3.58621	-0.27885	-0.27885
7	1	1	-3.51252	-0.2847	-0.2847
8	1	1	-4.54263	0	-0.22014
9	1	0	-4.7931	-0.20863	-0.20863
10	1	1	-3.51252	-0.2847	-0.2847
11	1	1	-3.43061	-0.29149	-0.29149
12	1	1	-4.48837	0	-0.2228

13	1	0	-5.79137	-0.17267	0
14	1	1	-4.54263	-0.22014	0
15	1	1	-4.48837	-0.2228	0
16	1	1	-5.5544	0	0

SIP

Ni = 4 , Nj = 4 , N = 16

Point(n)	L_W	L_S	L_P	U_N	U_E
1	0	0	-6	-0.16667	-0.16667
2	0	1.090909	-4.90909	-0.2037	-0.22222
3	0	1.125	-4.89583	-0.20426	-0.22979
4	0	1.129808	-5.89904	0	-0.19152
5	1.090909	0	-4.90909	-0.22222	-0.2037
6	1.113402	1.113402	-3.73196	-0.29834	-0.29834
7	1.113744	1.175325	-3.6825	-0.30244	-0.31917
8	1	1.189885	-4.63849	0	-0.25652
9	1.125	0	-4.89583	-0.22979	-0.20426
10	1.175325	1.113744	-3.6825	-0.31917	-0.30244
11	1.178164	1.178164	-3.60427	-0.32688	-0.32688
12	1	1.195372	-4.5481	0	-0.26283
13	1.129808	0	-5.89904	-0.19152	0
14	1.189885	1	-4.63849	-0.25652	0
15	1.195372	1	-4.5481	-0.26283	0
16	1	1	-5.47434	0	0