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ME 543 Computational Fluid Dynamics

Computer Assignment – 3

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Stream-Vorticity Equations

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$u = \frac{\partial \psi}{\partial y}\,, \quad v = -\frac{\partial \psi}{\partial x}$$

1.1. Question and Data

Given:

Length – L = 1 units, Height – H = 1 units $\quad \epsilon < 10^{-6}$

(Boundary Conditions are as shown below)

Input Parameters:

M=101 (Number of points on Horizontal Side)

N=101 (Number of points on Vertical Side)

Δx = L/(M-1) = 0.01 $\qquad \left(\beta = \frac{(\Delta x)}{(\Delta y)} \right)$

Δy = H/(N-1) = 0.01

$Re_H = 100$, 400

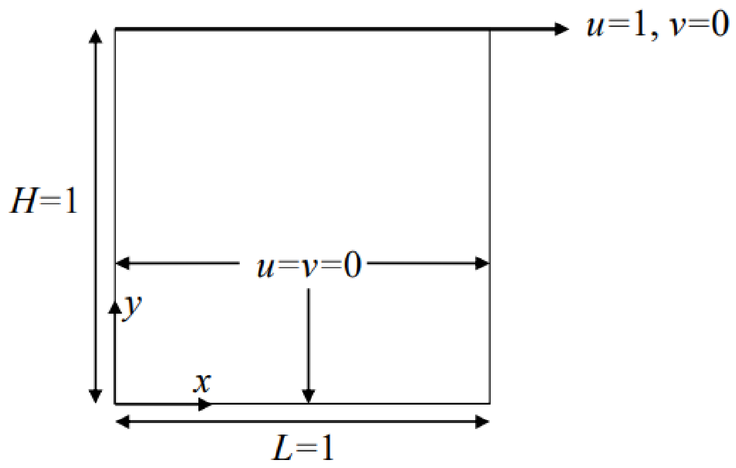


Figure 1: Flow inside a lid-driven cavity

1.2. Discretised Equation

Non-Dimensional Equation $(\beta = \frac{1}{\gamma})$

Streamline Equation:

$$\Psi_{i,j}^{k+1} = \frac{1}{2*(1+\beta^2)} \left([(\Delta x)^2 * \omega_{i,j}^{k+1}] + \beta^2 * \Psi_{i,j-1}^{k+1} + \Psi_{i-1,j}^{k+1} + \Psi_{i+1,j}^{k} + \beta^2 * \Psi_{i,j+1}^{k} \right)$$

Vorticity Equation:

$$\begin{aligned}\omega_{i,j}^{k+1} = \frac{1}{2*(1+\beta^2)} \Big[& \left\{1 - \left(\Psi_{i,j+1}^{k+1} - \Psi_{i,j-1}^{k+1}\right) * \left(\frac{\beta * Re}{4}\right)\right\} * \omega_{i+1,j}^{k} \\ & + \left\{1 + \left(\Psi_{i,j+1}^{k+1} - \Psi_{i,j-1}^{k+1}\right) * \left(\frac{\beta * Re}{4}\right)\right\} * \omega_{i-1,j}^{k+1} \\ & + \left\{1 + \left(\Psi_{i+1,j}^{k+1} - \Psi_{i-1,j}^{k+1}\right) * \left(\frac{Re}{4 * \beta}\right)\right\} * \beta^2 * \omega_{i,j+1}^{k} \\ & + \left\{1 - \left(\Psi_{i+1,j}^{k+1} - \Psi_{i-1,j}^{k+1}\right) * \left(\frac{Re}{4 * \beta}\right)\right\} * \beta^2 * \omega_{i,j-1}^{k+1} \Big]\end{aligned}$$

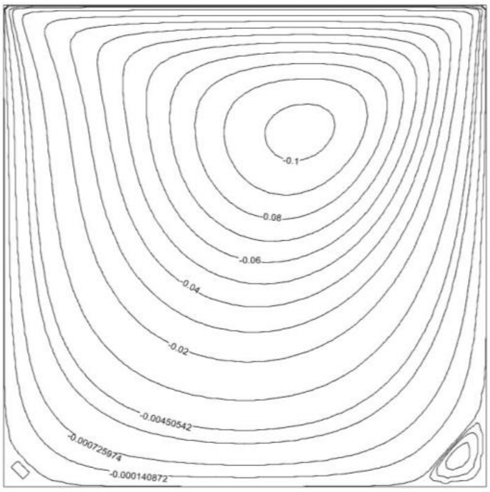
Velocity Equations:

$$u_{i,j} = \frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{2*\Delta y} \qquad v_{i,j} = -\frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2*\Delta x}$$

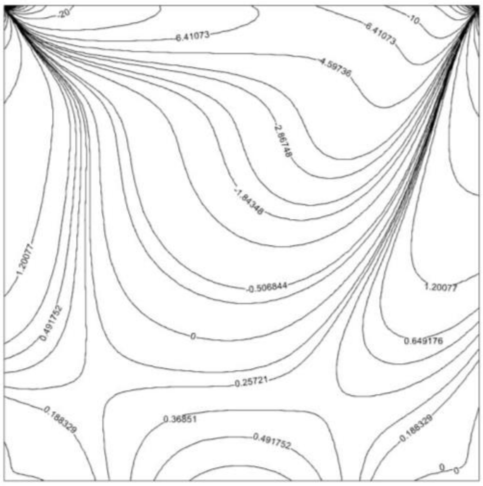
(The velocities are calculated at last after our psi (Streamlines) and omega (Vorticity) values Converge)

1.3. Results

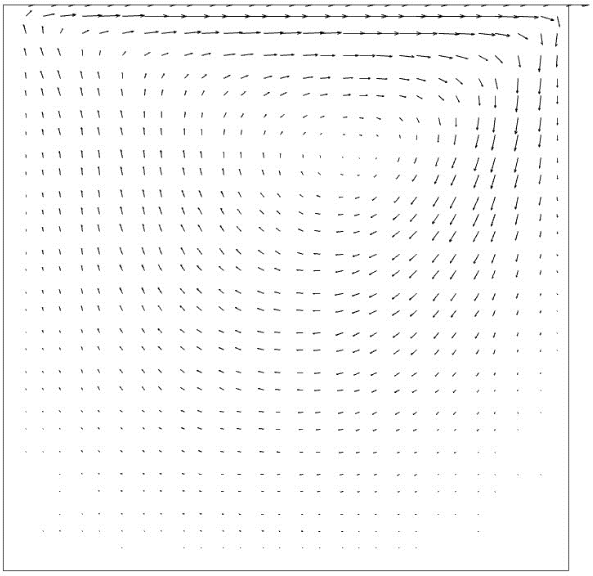
Reynolds Number (Re) = 100



Streamline



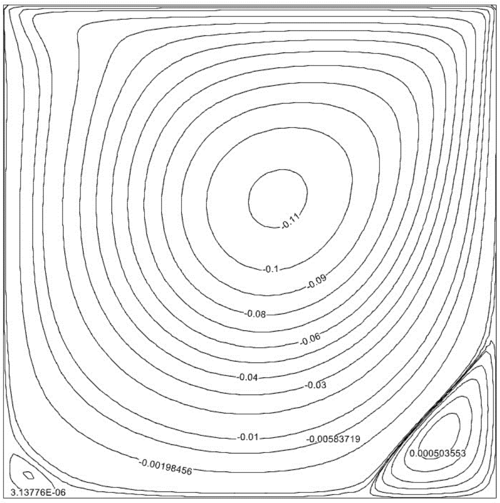
Vorticity



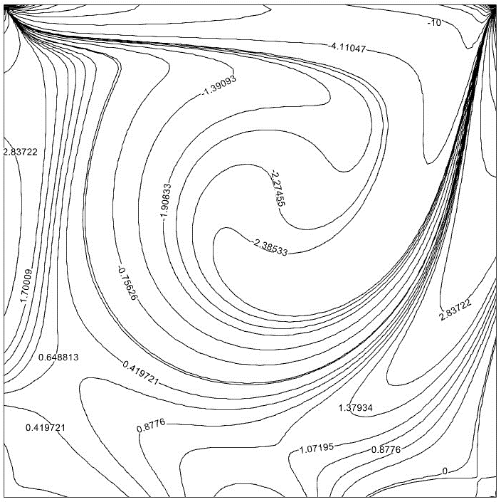
Velocity

Figure 2 : Re = 100 (Results)

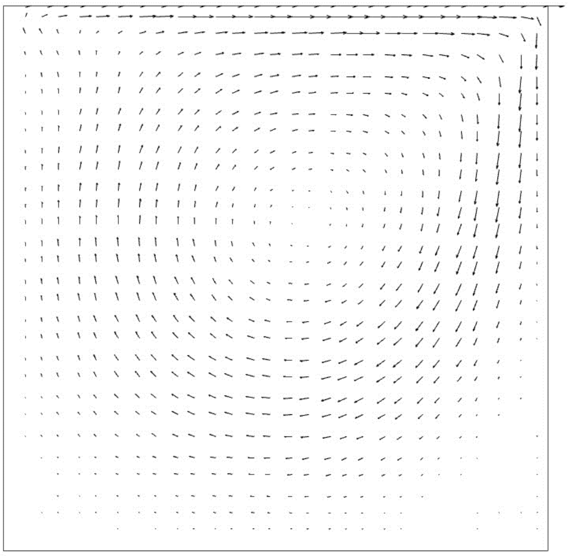
Reynolds Number (Re) = 400



Streamline



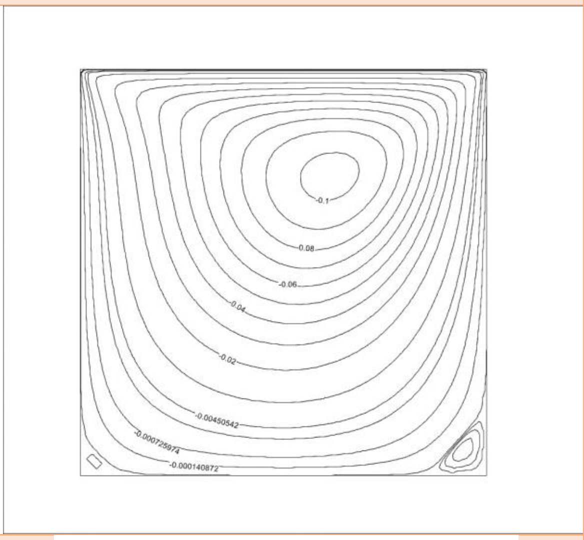
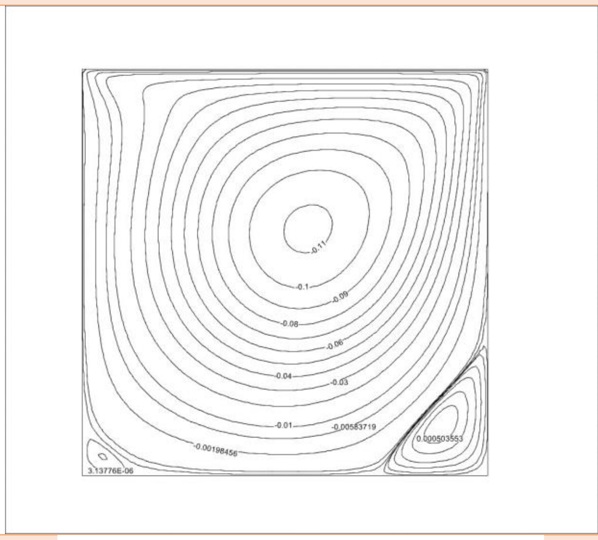
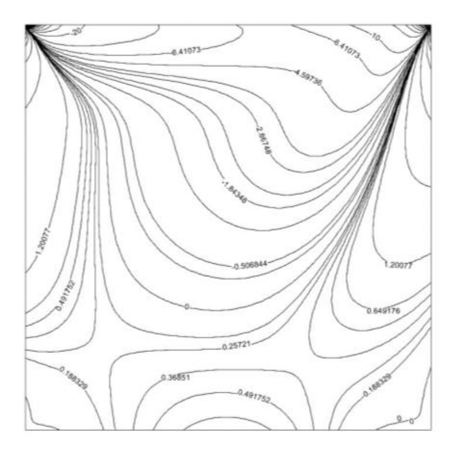
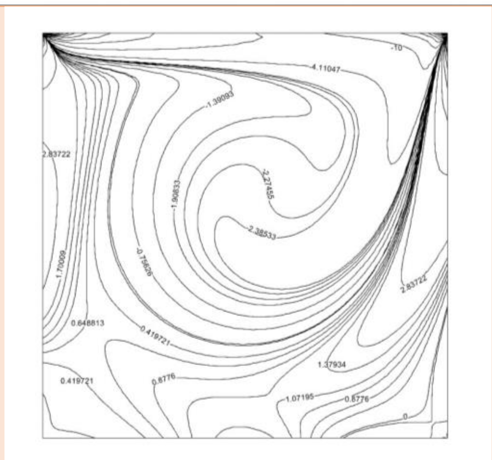
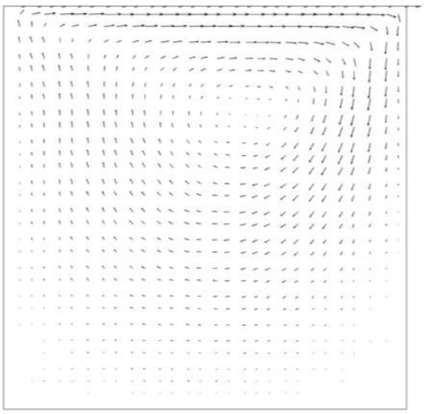
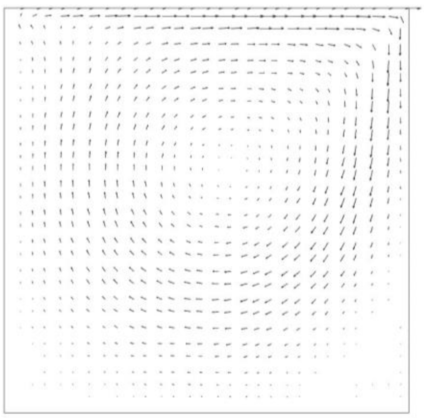
Vorticity



Velovity

Figure 3: Re = 400 (Results)

1.4. Comparison Table

Reynold's Number	Re = 100	Re 400
Kinematic Viscosity (γ)	0.01	0.0025
Streamlines		
Vorticity		
Velocity		
Number of Iterations	6536	7657
Computational Convergence Time (seconds)	52.641576	64.732577

1.5. Comparison of Experimental and Numerical Simulation Data

U velocity (Centreline)

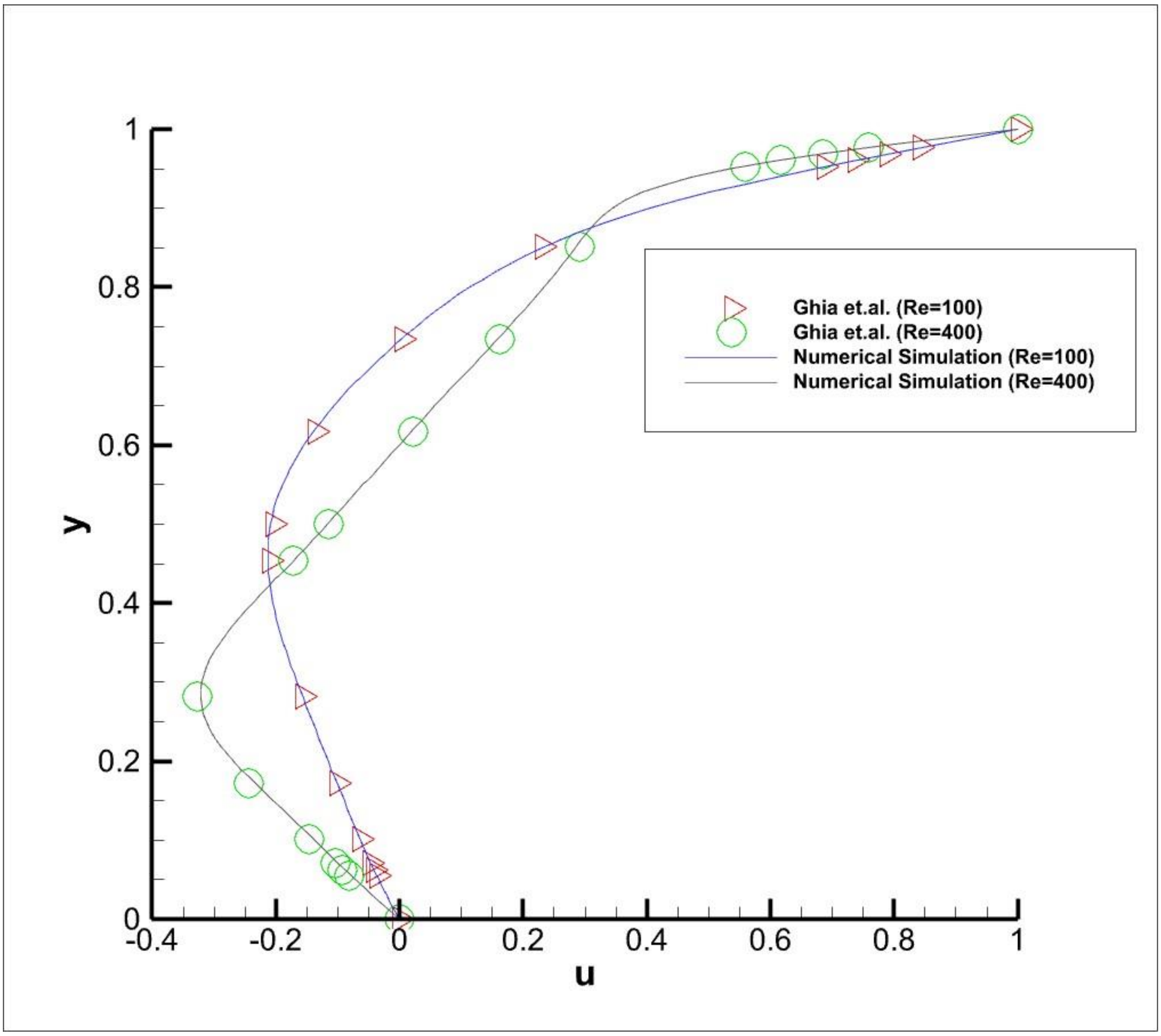


Figure 4: U velocity (Centreline)

V velocity (Centreline)

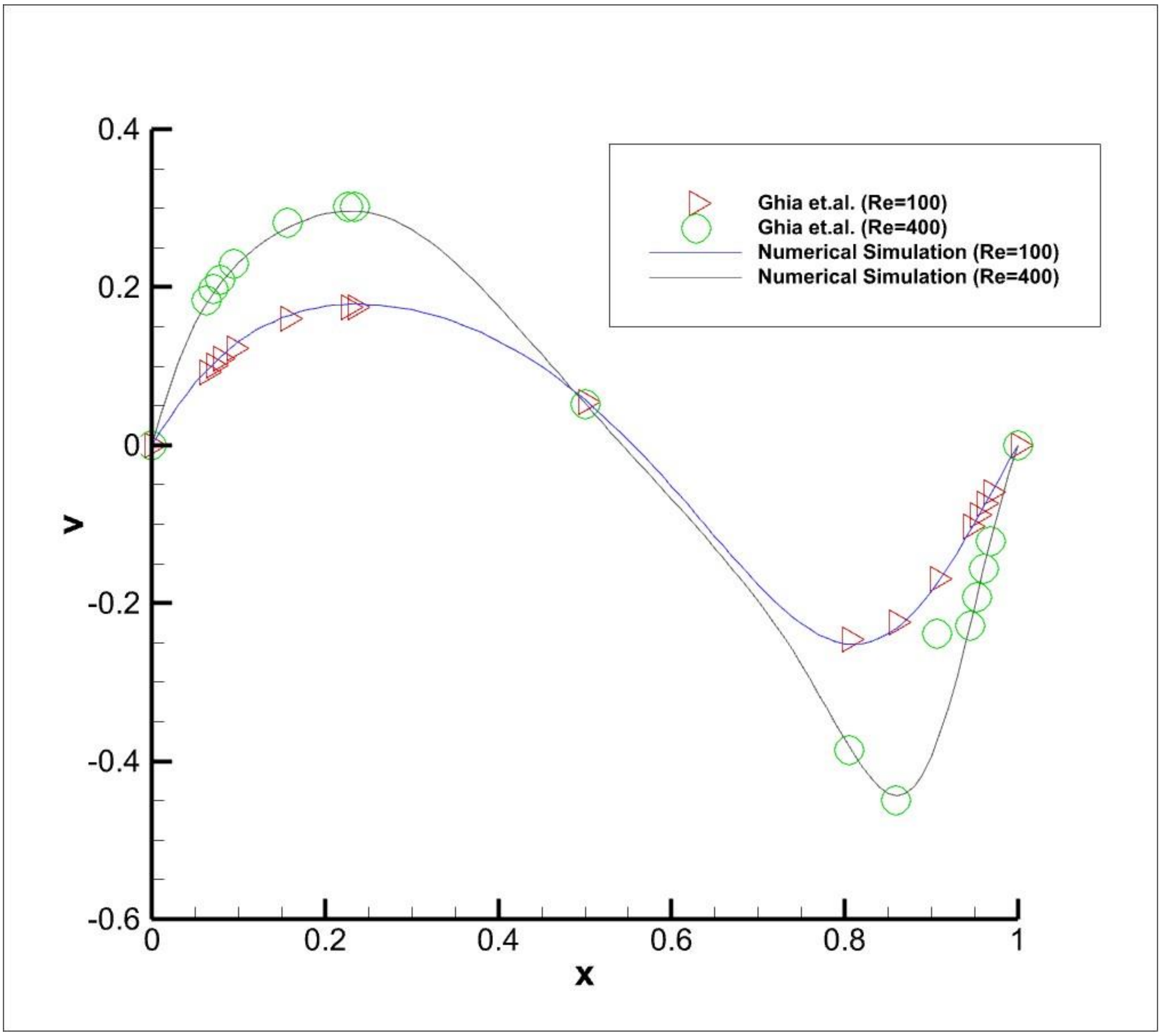


Figure 5: V velocity (Centreline)

1.6. Conclusion:

- The Numerical Data and the Experimental Data (Ghia et. Al.) are in Sync with each other.
- The Streamline Contour depict the magnitude (intensity) of velocity and shows how it is low near the boundaries, moderate in centre and maximum at the top surface.
- The Vorticity Contour depicts the rotational tensor which signifies the fluids shear strain and its variation in the flow leading to a vortex in centre.
- The Vortex is significant for high Reynolds number (low viscosity).
- The Streamlines are more symmetric for high Reynolds number (low viscosity) about centre.
- The Centreline Velocities shift show more variation away from average velocity for high Reynolds number
- The Vortex becomes more predominant for high Reynolds number.