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ME 543 Computational Fluid Dynamics

Computer Assignment – 3B

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Stream-Vorticity Equations

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$u = \frac{\partial \psi}{\partial y}\ , \quad v = -\frac{\partial \psi}{\partial x}$$

1.1. Question and Data

Given:

Length – L = 15 units, Height – H = 2 units $\quad \epsilon < 10^{-6}$

(Boundary Conditions are as shown below)

Input Parameters:

M=76 (Number of points on Horizontal Side)

N=31 (Number of points on Vertical Side)

$\Delta x = L/(M-1) = 0.2$ $\quad \left(\beta = \frac{(\Delta x)}{(\Delta y)} = 3 \right)$

$\Delta y = H/(N-1) = 0.0667$

$$Re_H = 100$$

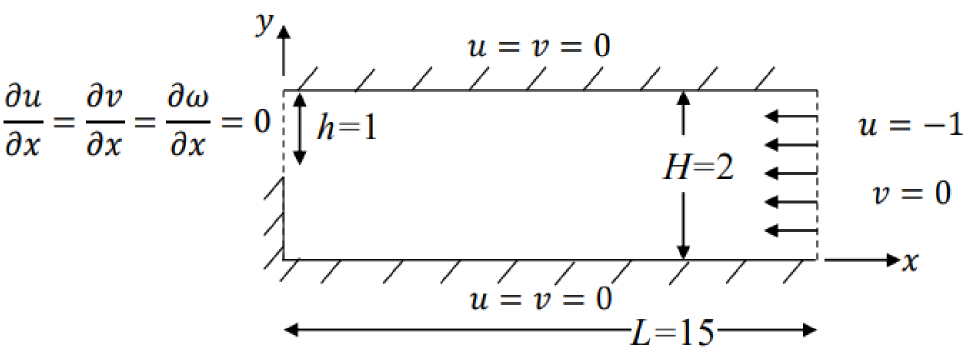

Figure: Flow through a sudden contraction

Figure 1: Flow through a sudden contraction

1.2. General Discretised Equation

Non-Dimensional Equation $(\beta = \frac{1}{\gamma})$

Streamline Equation:

$$\Psi_{i,j}^{k+1} = \frac{1}{2*(1+\beta^2)}\left([(\Delta x)^2 * \omega_{i,j}^{k+1}] + \beta^2 * \Psi_{i,j-1}^{k+1} + \Psi_{i-1,j}^{k+1} + \Psi_{i+1,j}^{k} + \beta^2 * \Psi_{i,j+1}^{k}\right)$$

Vorticity Equation:

$$\begin{aligned}\omega_{i,j}^{k+1} = \frac{1}{2*(1+\beta^2)}\Bigg[&\left\{1 - \left(\Psi_{i,j+1}^{k+1} - \Psi_{i,j-1}^{k+1}\right) * \left(\frac{\beta * Re}{4}\right)\right\} * \omega_{i+1,j}^{k} \\ + &\left\{1 + \left(\Psi_{i,j+1}^{k+1} - \Psi_{i,j-1}^{k+1}\right) * \left(\frac{\beta * Re}{4}\right)\right\} * \omega_{i-1,j}^{k+1} \\ + &\left\{1 + \left(\Psi_{i+1,j}^{k+1} - \Psi_{i-1,j}^{k+1}\right) * \left(\frac{Re}{4 * \beta}\right)\right\} * \beta^2 * \omega_{i,j+1}^{k} \\ + &\left\{1 - \left(\Psi_{i+1,j}^{k+1} - \Psi_{i-1,j}^{k+1}\right) * \left(\frac{Re}{4 * \beta}\right)\right\} * \beta^2 * \omega_{i,j-1}^{k+1} \Bigg]\end{aligned}$$

Velocity Equations:

$$u_{i,j} = \frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{2*\Delta y} \qquad v_{i,j} = -\frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2*\Delta x}$$

(The velocities are calculated at last after our psi (Streamlines) and omega (Vorticity) values Converge)

Flow Between two stationary Plates (Given Geometry)

Here The Vorticity values are high, so we need to use a underrelaxation factor (w=0.1) to limit the values until convergence

Under Relaxation Factor -> w=0.1

$$\begin{aligned}\omega_{i,j}^{k+1} = (1-w) * \omega_{i,j}^{k+1} + \frac{w}{2*(1+\beta^2)}\Bigg[&\left\{1 - \left(\Psi_{i,j+1}^{k+1} - \Psi_{i,j-1}^{k+1}\right) * \left(\frac{\beta * Re}{4}\right)\right\} * \omega_{i+1,j}^{k} \\ + &\left\{1 + \left(\Psi_{i,j+1}^{k+1} - \Psi_{i,j-1}^{k+1}\right) * \left(\frac{\beta * Re}{4}\right)\right\} * \omega_{i-1,j}^{k+1} \\ + &\left\{1 + \left(\Psi_{i+1,j}^{k+1} - \Psi_{i-1,j}^{k+1}\right) * \left(\frac{Re}{4 * \beta}\right)\right\} * \beta^2 * \omega_{i,j+1}^{k} \\ + &\left\{1 - \left(\Psi_{i+1,j}^{k+1} - \Psi_{i-1,j}^{k+1}\right) * \left(\frac{Re}{4 * \beta}\right)\right\} * \beta^2 * \omega_{i,j-1}^{k+1} \Bigg]\end{aligned}$$

Velocity and Streamline equation remain same and don't need underrelaxation factor

1.3. Boundary Condition

Boundary Condition		
i=1 to M (M=76)	U=-1	j=1 to N (N=31)

Top Boundary	1<=i<=M ; j=N	u=0;
		v=0;
		$\psi_{i,N}$ = U*H = -2;
		$\omega_{i,N} = -\frac{2}{\Delta y * \Delta y}(\psi_{i,N-1} - \psi_{i,N})$;

Bottom Boundary	1<=i<=M ; j=1	u=0;
		v=0;
		$\psi_{i,1}$ = 0;
		$\omega_{i,1} = -\frac{2}{\Delta y * \Delta y}(\psi_{i,2} - \psi_{i,1})$;

Right Boundary	1<=j<=N ; i=M	u=-1;
		v=0;
		$\psi_{M,1} = 0$;
		$\psi_{M,j} = \psi_{M,j-1} + u * \Delta y$; (j!=1);
		$\omega_{M,j} = -\frac{2}{\Delta x * \Delta x}(\psi_{M,j} - \psi_{M-1,j})$;

Upper Left Boundary	1<=j<=N/2 ; i=1	$u_{1,j} = u_{2,j}$; $v_{1,j} = v_{2,j}$; $\psi_{1,j} = \psi_{2,j}$; $\omega_{1,j} = \omega_{2,j}$;

Bottom Left Boundary	N/2<j<=N ; i=1	u=0;
		v=0;
		$\psi_{1,j} = 0$;
		$\omega_{M,j} = -\frac{2}{\Delta x * \Delta x}(\psi_{2,j} - \psi_{1,j})$;

1.4. Results

Reynolds Number (Re) = 100

Streamline

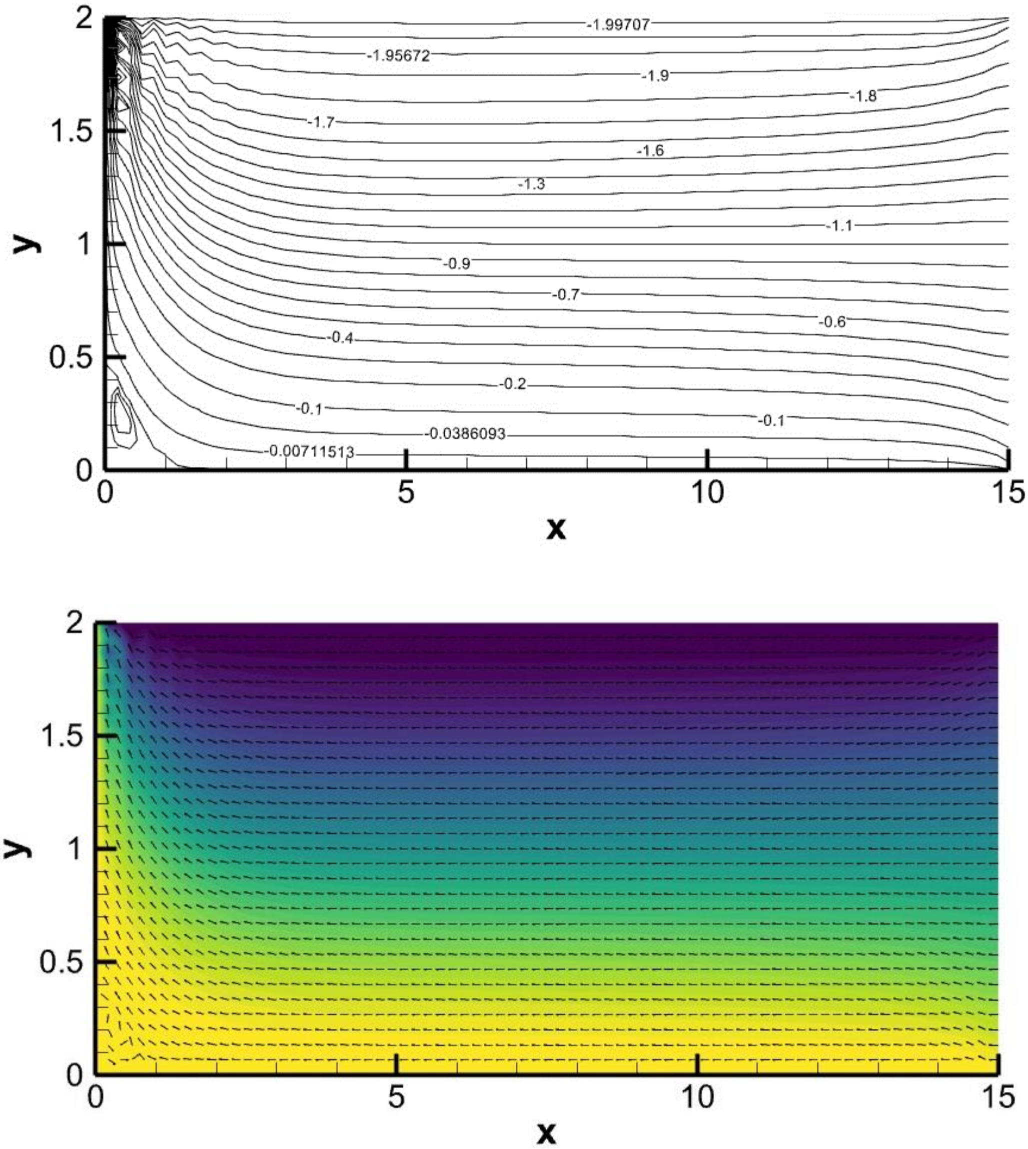

Figure 2 : Re = 100 -> Streamline Contour

Vorticity

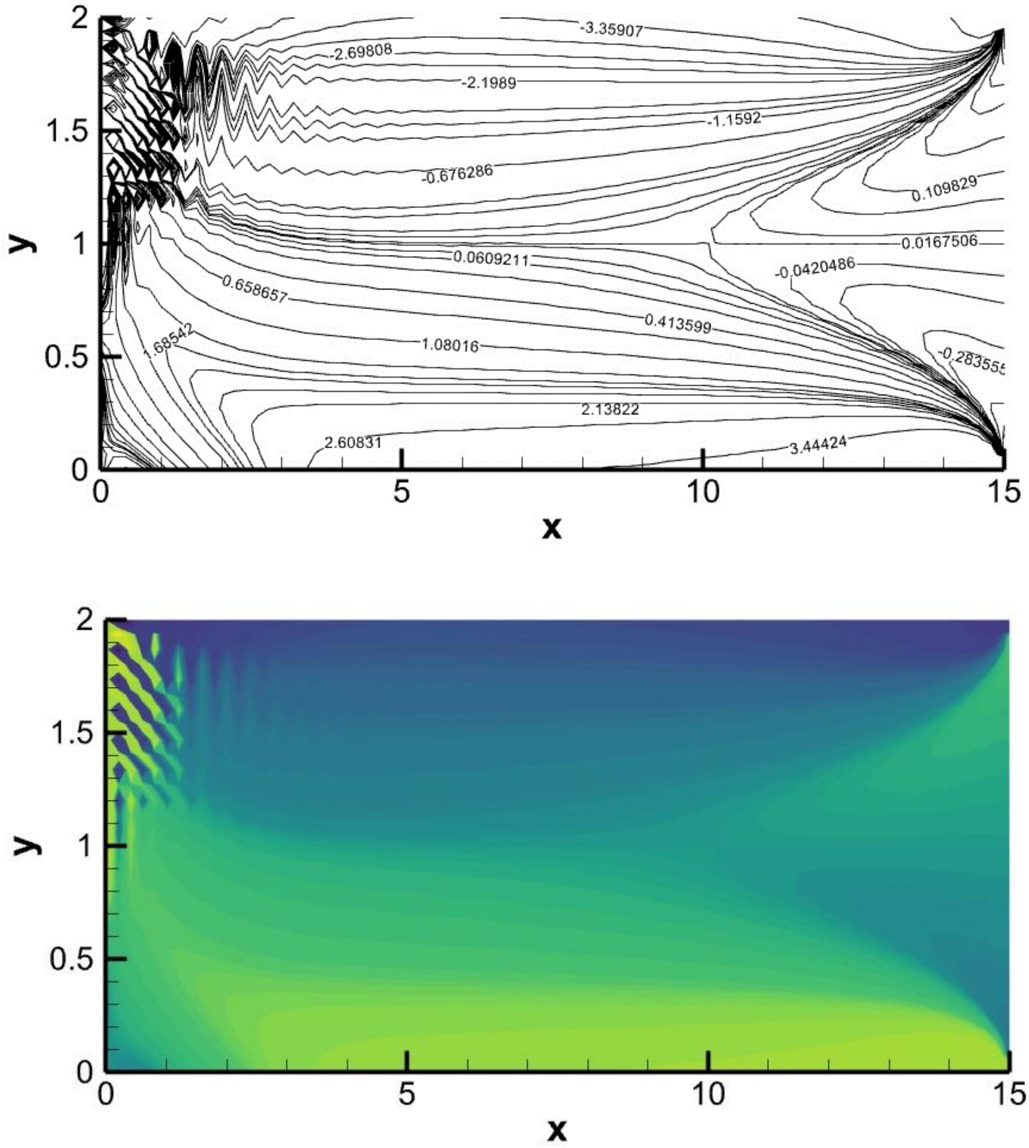

Figure 3: Re = 100 -> Vorticity Contour

Velocity Contour

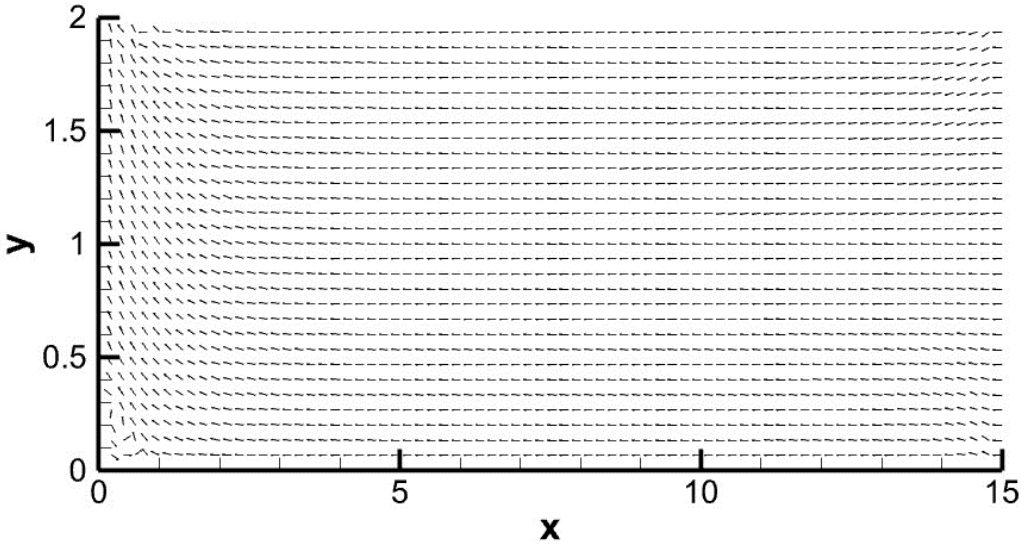

Figure 4: Re = 100 -> Velocity Contour

1.5. U-Velocity

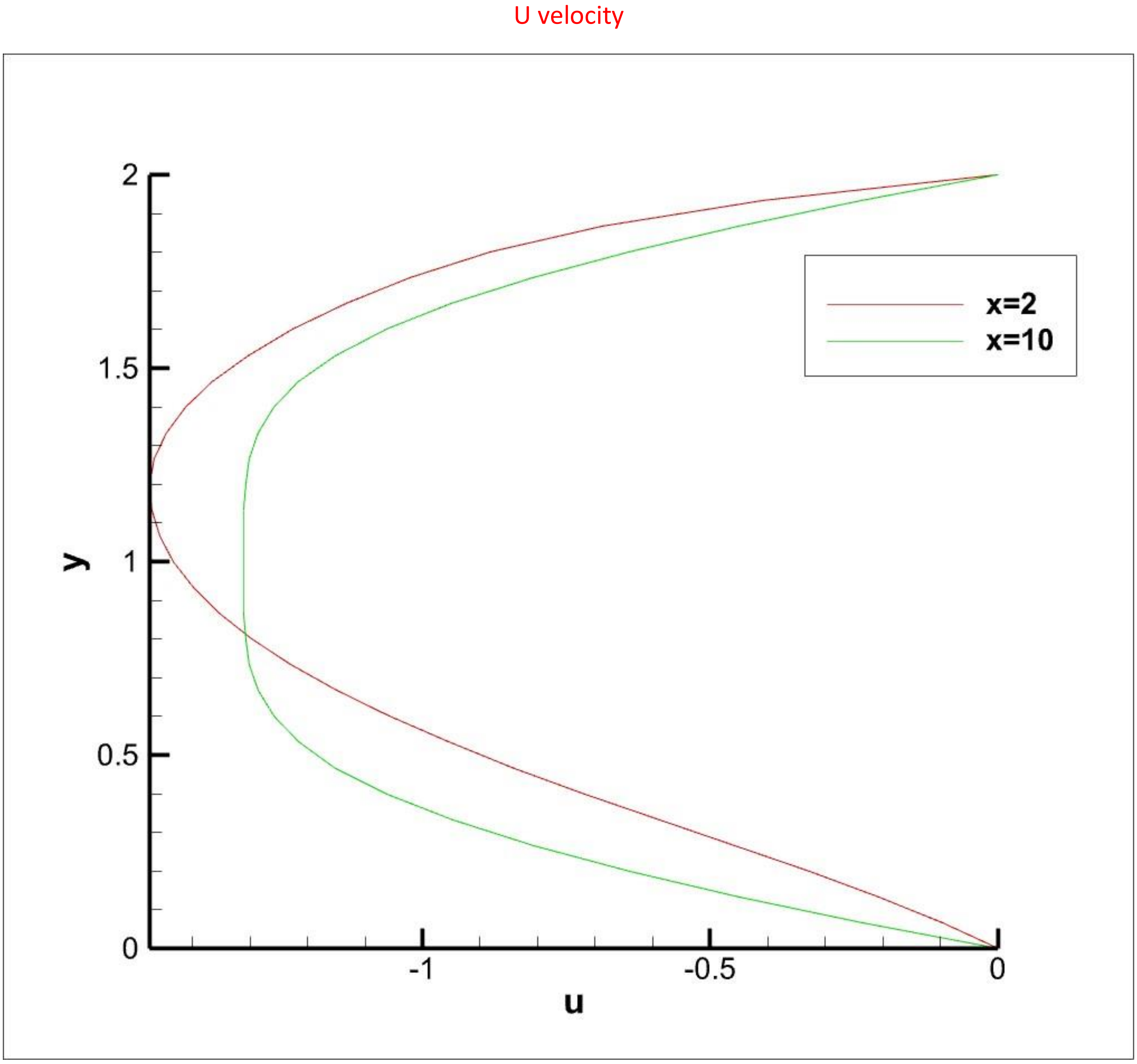

Figure 5: U velocity (x=2 , x=10)

1.6. Conclusion:

- The streamlines starting uniformly and spaced equidistant at right end all travel through half cross section and become dense at left end
- The Vorticity Contour depicts the rotational tensor which signifies the fluids shear strain and its negligible here due to straight flow and only small vortex at left bottom corner.
- The u velocity which is having Horizontal U profile at right end (Developing region) tends to become a blunt U profile as we move towards left and it is clearly seen from graph.