AMC 10 2024

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–	A
–	November 6, 2024
1	What is the value of $9901 \cdot 101 - 99 \cdot 10101?$ **(A)** 2 **(B)** 20 **(C)** 21 **(D)** 200 **(E)** 2020
2	A model used to estimate the time it will take to hike to the top of the mountain on a trail is of the form $T = aL + bG,$ where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimate it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet? **(A)** 240 **(B)** 246 **(C)** 252 **(D)** 258 **(E)** 264
3	What is the sum of the digits of the smallest prime that can be written as a sum of 5 distinct primes? **(A)** 5 **(B)** 7 **(C)** 9 **(D)** 10 **(E)** 11
4	The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum? **(A)** 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24
5	What is the least value of n such that $n!$ is a multiple of 2024? **(A)** 11 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 253
6	What is the minimum number of successive swaps of adjacent letters in the string ABCDEF that are needed to change the string to FEDCBA? (For example, 3 swaps are required to change ABC to CBA; one such sequence of swaps is ABC $\rightarrow$ BAC $\rightarrow$ BCA $\rightarrow$ CBA.) **(A)** 6 **(B)** 10 **(C)** 12 **(D)** 15 **(E)** 24
7	The product of three integers is 60. What is the least possible positive sum of the three integers?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 13

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8 Amy, Bomani, Charlie, and Daria work in a chocolate factory. On Monday Amy, Bomani, and Charlie started working at $1{:}00$ PM and were able to pack 4, 3, and 3 packages, respectively, every 3 minutes. At some later time, Daria joined the group, and Daria was able to pack 5 packages every 4 minutes. Together, they finished packing 450 packages at exactly $2{:}45$ PM. At what time did Daria join the group?

(A) $1{:}25$ PM **(B)** $1{:}35$ PM **(C)** $1{:}45$ PM **(D)** $1{:}55$ PM **(E)** $2{:}05$ PM

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9 In how many ways can 6 juniors and 6 seniors form 3 disjoint teams of 4 people so that each team has 2 juniors and 2 seniors?

(A) 720 **(B)** 1350 **(C)** 2700 **(D)** 3280 **(E)** 8100

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10 Consider the following operation. Given a positive integer n, if n is a multiple of 3, then you replace n by $\frac{n}{3}$. If n is not a multiple of 3, then you replace n by $n+10$. Then continue this process. For example, beginning with $n=4$, this procedure gives $4 \to 14 \to 24 \to 8 \to 18 \to 6 \to 2 \to 12 \to \cdots$. Suppose you start with $n=100$. What value results if you perform this operation exactly 100 times?

(A) 10 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 50

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11 How many ordered pairs of integers (m,n) satisfy $\sqrt{n^2-49}=m$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** Infinitely many

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12 Zelda played the *Adventures of Math* game on August 1 and scored 1700 points. She continued to play daily over the next 5 days. The bar chart below shows the daily change in her score compared to the day before. (For example, Zelda's score on August 2 was $1700+80=1780$ points.) What was Zelda's average score in points over the 6 days?

`https://cdn.artofproblemsolving.com/attachments/5/c/d246d9bf4002bfe23f859bd21605f882d8b7b`
`png`

(A) 1700 **(B)** 1702 **(C)** 1703 **(D)** 1713 **(E)** 1715

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13 Two transformations are said to *commute* if applying the first followed by the second gives the same result as applying the second followed by the first. Consider these four transformations of the coordinate plane:

- A translation 2 units to the right
- A 90°- rotation counterclockwise about the origin.
- A reflection across the x-axis, and
- A dilation centered at the origin with scale factor 2.

Of the 6 pairs of distinct transformations from this list, how many commute? **(A)** 1 **(B)** 2 **(C)** 3 **(D)

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14 One side of an equilateral triangle of height 24 lies on line ℓ. A circle of radius 12 is tangent to ℓ and is externally tangent to the triangle. The area of the region exterior to the triangle and the circle and bounded by the triangle, the circle, and line ℓ can be written as $a\sqrt{b} - c\pi$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is $a + b + c$?
(A) 72 **(B)** 73 **(C)** 74 **(D)** 75 **(E)** 76

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15 Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

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16 All of the rectangles in the figure below, which is drawn to scale, are similar to the enclosing rectangle. Each number represents the area of the rectangle. What is length AB?

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`png`

(A) $4 + 4\sqrt{5}$ **(B)** $10\sqrt{2}$ **(C)** $5 + 5\sqrt{5}$ **(D)** $10\sqrt[4]{8}$ **(E)** 20

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17 Two teams are in a best-two-out-of-three playoff: the teams will play at most 3 games, and the winner of the playoff is the first team to win 2 games. The first game is played on Team A's home field, and the remaining games are played on Team B's home field. Team A has a $\frac{2}{3}$ chance of winning at home, and its probability of winning when playing away from home is p. Outcomes of the games are independent. The probability that Team A wins the playoff is $\frac{1}{2}$. Then p can be written in the form $\frac{1}{2}(m - \sqrt{n})$, where m and n are positive integers. What is $m + n$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

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18 There are exactly K positive integers b with $5 \leq b \leq 2024$ such that the base-b integer 2024_b is divisible by 16 (where 16 is in base ten). What is the sum of the digits of K?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 20 **(E)** 21

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19 The first three terms of a geometric sequence are the integers $a,\ 720,$ and b, where $a < 720 < b$. What is the sum of the digits of the least possible value of b?

(A) 9 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 21

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20 Let S be a subset of $\{1, 2, 3, \ldots, 2024\}$ such that the following two conditions hold:
- If x and y are distinct elements of S, then $|x - y| > 2$
- If x and y are distinct odd elements of S, then $|x - y| > 6$.
What is the maximum possible number of elements in S?

(A) 436 **(B)** 506 **(C)** 608 **(D)** 654 **(E)** 675

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21 The numbers, in order, of each row and the numbers, in order, of each column of a 5×5 array of integers form an arithmetic progression of length 5. The numbers in positions $(5,5)$, $(2,4)$, $(4,3)$, and $(3,1)$ are $0, 48, 16,$ and 12, respectively. What number is in position $(1,2)$?

$$\begin{bmatrix} . & ? & . & . & . \\ . & . & . & 48 & . \\ 12 & . & . & . & . \\ . & . & 16 & . & . \\ . & . & . & . & 0 \end{bmatrix}$$

(A) 19 **(B)** 24 **(C)** 29 **(D)** 34 **(E)** 39

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22 Let $\mathcal{K}$ be the kite formed by joining two right triangles with legs 1 and $\sqrt{3}$ along a common hypotenuse. Eight copies of $\mathcal{K}$ are used to form the polygon shown below. What is the area of triangle ΔABC?

`https://cdn.artofproblemsolving.com/attachments/1/3/03abbd4df2932f4a1d16a34c2b9e15b683dec png`

(A) $2+3\sqrt{3}$ **(B)** $\frac{9}{2}\sqrt{3}$ **(C)** $\frac{10+8\sqrt{3}}{3}$ **(D)** 8 **(E)** $5\sqrt{3}$

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23 Integers a, b, and c satisfy $ab+c=100$, $bc+a=87$, and $ca+b=60$. What is $ab+bc+ca$?

(A) 212 **(B)** 247 **(C)** 258 **(D)** 276 **(E)** 284

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24 A bee is moving in three-dimensional space. A fair six-sided die with faces labeled A^+, A^-, B^+, B^-, C^+, and C^- is rolled. Suppose the bee occupies the point (a,b,c). If the die shows A^+, then the bee moves to the point $(a+1,b,c)$ and if the die shows A^-, then the bee moves to the point $(a-1,b,c)$. Analogous moves are made with the other four outcomes. Suppose the bee starts at the point $(0,0,0)$ and the die is rolled four times. What is the probability that the bee traverses four distinct edges of some unit cube? **(A)** $\frac{1}{54}$ **(B)** $\frac{7}{54}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{5}{18}$ **(E)** $\frac{2}{5}$

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25 The figure below shows a dotted grid 8 cells wide and 3 cells tall consisting of $1'' \times 1''$ squares. Carl places 1-inch toothpicks along some of the sides of the squares to create a closed loop that does not intersect itself. The numbers in the cells indicate the number of sides of that square that are to be covered by toothpicks, and any number of toothpicks are allowed if no number is written. In how many ways can Carl place the toothpicks?

1	1	1	1	1	1	1	1

(A) 130 **(B)** 144 **(C)** 146 **(D)** 162 **(E)** 196

– B

– November 12, 2024

1 In a long line of people, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

(A) 2021 **(B)** 2022 **(C)** 2023 **(D)** 2024 **(E)** 2025

2 What is $10! - 7! \cdot 6!$?

(A) -120 **(B)** 0 **(C)** 120 **(D)** 600 **(E)** 720

3 For how many integer values of x is $|2x| \leq 7\pi$?

(A) 16 **(B)** 17 **(C)** 19 **(D)** 20 **(E)** 21

4 Balls numbered $1, 2, 3, \ldots$ are deposited in 5 bins, labeled A, B, C, D, and E, using the following procedure. Ball 1 is deposited in bin A, and balls 2 and 3 are deposted in B. The next three balls are deposited in bin C, the next 4 in bin D, and so on, cycling back to bin A after balls are deposited in bin E. (For example, $22, 23, \ldots, 28$ are despoited in bin B at step 7 of this process.) In which bin is ball 2024 deposited?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

5 In the following expression, Melanie changed some of the plus signs to minus signs:

$$1 + 3 + 5 + 7 + \cdots + 97 + 99$$

When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

6 A rectangle has integer side lengths and an area of 2024. What is the least possible perimeter of the rectangle?

(A) 160 **(B)** 180 **(C)** 222 **(D)** 228 **(E)** 390

7 What is the remainder when $7^{2024} + 7^{2025} + 7^{2026}$ is divided by 19?

(A) 0 **(B)** 1 **(C)** 7 **(D)** 11 **(E)** 18

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8 Let N be the product of all the positive integer divisors of 42. What is the units digit of N?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

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9 Real numbers a, b and c have arithmetic mean 0. The arithmetic mean of a^2, b^2 and c^2 is 10. What is the arithmetic mean of ab, ac and bc?

(A) -5 **(B)** $-\frac{10}{3}$ **(C)** $-\frac{10}{9}$ **(D)** 0 **(E)** $\frac{10}{9}$

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10 Quadrilateral $ABCD$ is a parallelogram, and E is the midpoint of the side $\overline{AD}$. Let F be the intersection of lines EB and AC. What is the ratio of the area of quadrilateral $CDEF$ to the area of triangle CFB?

(A) $5:4$ **(B)** $4:3$ **(C)** $3:2$ **(D)** $5:3$ **(E)** $2:1$

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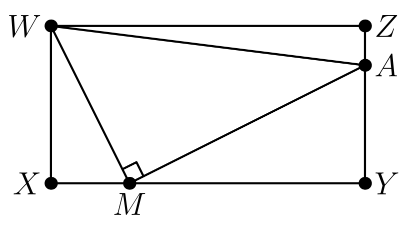
11 In the figure below $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies $\overline{XY}$, point A lies on $\overline{YZ}$, and $\angle WMA$ is a right angle. The areas of $\triangle WXM$ and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?



(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

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12 A group of 100 students from different countries meet at a mathematics competition. Each student speaks the same number of languages, and, for every pair of students A and B, student A speaks some language that student B does not speak, and student B speaks some language that student A does not speak. What is the least possible total number of languages spoken by all the students?

(A) 9 **(B)** 10 **(C)** 12 **(D)** 51 **(E)** 100

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13 Positive integers x and y satisfy the equation $\sqrt{x} + \sqrt{y} = \sqrt{1183}$. What is the minimum possible value of $x + y$?

(A) 585 **(B)** 595 **(C)** 623 **(D)** 700 **(E)** 791

14 A dartboard is the region B in the coordinate plane consisting of points (x, y) such that $|x|+|y| \leq 8$. A target T is the region where $(x^2 + y^2 - 25)^2 \leq 49$. A dart is thrown at a random point in B. The probability that the dart lands in T can be expressed as $\frac{m}{n}\pi$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 39 **(B)** 71 **(C)** 73 **(D)** 75 **(E)** 135

15 A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, 7, as well as x, y, z with $x \leq y \leq z$. The range of the list is 7, and the mean and median are both positive integers. How many ordered triples (x, y, z) are possible?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E) infinitely many**

16 Jerry likes to play with numbers. One day, he wrote all the integers from 1 to 2024 on the whiteboard. Then he repeatedly chose four numbers on the whiteboard, erased them, and replaced them with either their sum or their product. (For example, Jerry's first step might have been to erase $1, 2, 3$, and 5, and then write either 11, their sum, or 30, their product, on the whiteboard.) After repeatedly performing this operation, Jerry noticed that all the remaining numbers on the board were odd. What is the maximum possible number of integers on the board at that time?

(A) 1010 **(B)** 1011 **(C)** 1012 **(D)** 1013 **(E)** 1014

17 In a race among 5 snails, there is at most one tie, but that tie can involve any number of snails. For example, the result of the race might be that Dazzler is first; Abby, Cyrus, and Elroy are tied for second, and Bruna is fifth. How many different results of the race are possible?

(A) 180 **(B)** 361 **(C)** 420 **(D)** 431 **(E)** 720

18 How many different remainders can result when the 100th power of an integer is divided by 125?

(A) 1 **(B)** 2 **(C)** 5 **(D)** 25 **(E)** 125

19 In the following table, each question mark is to be replaced by "Possible" or "Not Possible" to indicate whether a nonvertical line with the given slope can contain the given number of lattice points (points both of whose coordinates are integers). How many of the 12 entries will be "Possible"?

	zero	**exactly one**	**exactly two**	**more than two**
zero slope	?	?	?	?
nonzero rational slope	?	?	?	?
irrational slope	?	?	?	?

(A) 4 **(B)** 5 **(C)** 6

20 Three different pairs of shoes are placed in a row so that no left shoe is next to a right shoe from a different pair. In how many ways can these six shoes be lined up?

(A) 60 **(B)** 72 **(C)** 90 **(D)** 108 **(E)** 120

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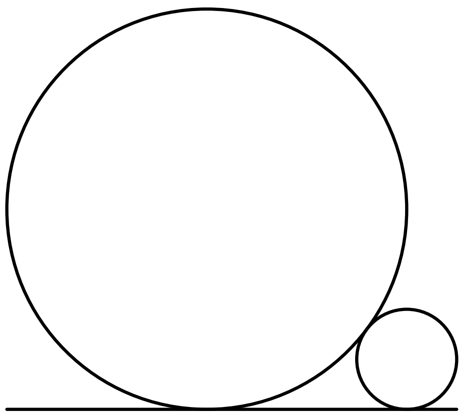
21 Two straight pipes (circular cylinders), with radii 1 and $\frac{1}{4}$, lie parallel and in contact on a flat floor. The figure below shows a head-on view. What is the sum of the possible radii of a third parallel pipe lying on the same floor and in contact with both?



(A) $\frac{1}{9}$ **(B)** 1 **(C)** $\frac{10}{9}$ **(D)** $\frac{11}{9}$ **(E)** $\frac{19}{9}$

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22 A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

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23 The Fibonacci numbers are defined by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. What is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}}?$$

(A) 318 **(B)** 319 **(C)** 320 **(D)** 321 **(E)** 322

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24 Let

$$P(m) = \frac{m}{2} + \frac{m^2}{4} + \frac{m^4}{8} + \frac{m^8}{8}.$$

How many of the values of $P(2022)$, $P(2023)$, $P(2024)$, and $P(2025)$ are integers?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

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25 Each of 27 bricks (right rectangular prisms) has dimensions $a \times b \times c$, where a, b, and c are pairwise relatively prime positive integers. These bricks are arranged to form a $3 \times 3 \times 3$ block, as shown on the left below. A 28[sup]th[/sup] brick with the same dimensions is introduced, and these bricks are reconfigured into a $2 \times 2 \times 7$ block, shown on the right. The new block is 1 unit taller, 1 unit wider, and 1 unit deeper than the old one. What is $a + b + c$?

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png

(A) 88 **(B)** 89 **(C)** 90 **(D)** 91 **(E)** 92
