AMC 10 2023

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–	A
–	November 8, 2023

1 Cities A and B are 45 miles apart. Alicia lives in A and Beth lives in B. Alicia bikes towards B at 18 miles per hour. Leaving at the same time, Beth bikes toward A at 12 miles per hour. How many miles from City A will they be when they meet? **(A)** 20 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27

2 The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

(A) $1\frac{4}{5}$ **(B)** 2 **(C)** $2\frac{2}{5}$ **(D)** 3 **(E)** $3\frac{3}{5}$

3 How many positive perfect squares less than 2023 are divisible by 5?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

4 A quadrilateral has all integer sides lengths, a perimeter of 26, and one side of length 4. What is the greatest possible length of one side of this quadrilateral?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

5 How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

6 An integer is assigned to each vertex of a cube. The value of an edge is defined to be the sum of the values of the two vertices it touches, and the value of a face is defined to be the sum of the values of the four edges surrounding it. The value of the cube is defined as the sum of the values of its six faces. Suppose the sum of the integers assigned to the vertices is 21. What is the value of the cube?

(A) 42 **(B)** 63 **(C)** 84 **(D)** 126 **(E)** 252

7 Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3?

(A) $\frac{2}{9}$ **(B)** $\frac{49}{216}$ **(C)** $\frac{25}{108}$ **(D)** $\frac{17}{72}$ **(E)** $\frac{13}{54}$

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8 Barb the baker creates a new temperature system for baking bread, Breadus, which is linearly based on Fahrenheit. Bread rises at 110 F°, which is 0 on the Breadus scale. Bread bakes at 350 F°, which is 100 on the Breadus scale. Bread is done when it's internal temperature is 200 F°. What is this temperature on the Breadus scale? **(A)** 33 **(B)** 34.5 **(C)** 36 **(D)** 37.5 **(E)** 39

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9 A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 will each digit appear an even number of times in the 8-digital display for that date?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

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10 Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently?
(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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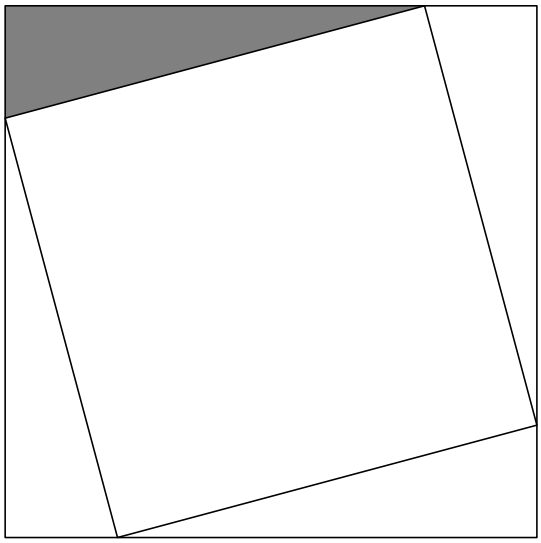
11 A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?



(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{3}$ **(D)** $\sqrt{3}-\sqrt{2}$ **(E)** $\sqrt{2}-1$

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12 How many three-digit positive integers N satisfy the following properties?

- The number N is divisible by 7.
- The number formed by reversing the digits of N is divisible by 5.

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

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13 Abdul and Chiang are standing 48 feet apart in a field. Bharat is standing in the same field as far from Abdul as possible so that the angle formed by his lines of sight to Abdul and Chiang measures 60°. What is the square of the distance (in feet) between Abdul and Bharat?

(A) 1728 **(B)** 2601 **(C)** 3072 **(D)** 4608 **(E)** 6912

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14 A number is chosen at random from among the first 100 positive integers, and a positive integer divisor of that number is then chosen at random. What is the probability that the chosen divisor is divisible by 11?

(A) $\frac{4}{100}$ **(B)** $\frac{9}{200}$ **(C)** $\frac{1}{20}$ **(D)** $\frac{11}{200}$ **(E)** $\frac{3}{50}$

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15 An even number of circles are nested, starting with a radius of 1 and increasing by 1 each time, all sharing a common point. The region between every other circle is shaded, starting with the region inside the circle of radius 2 but outside the circle of radius 1. An example showing 8 circles is displayed below. What is the least number of circles needed to make the total shaded area at least 2023π?

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16 In a table tennis tournament every participant played every other participant exactly once. Although there were twice as many right-handed players as left-handed players, the number of games won by left-handed players was 40% more than the number of games won by right-handed players. (There were no ties and no ambidextrous players.) What is the total number of games played?

(A) 15 **(B)** 36 **(C)** 45 **(D)** 48 **(E)** 66

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17 Let $ABCD$ be a rectangle with $AB = 30$ and $BC = 28$. Point P and Q lie on $\overline{BC}$ and $\overline{CD}$ respectively so that all sides of $\triangle ABP, \triangle PCQ$, and $\triangle QDA$ have integer lengths. What is the perimeter of $\triangle APQ$?

(A) 84 (B) 86 (C) 88 (D)90 (E)92

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18 A rhombic dodecahedron is a solid with 12 congruent rhombus faces. At every vertex, 3 or 4 edges meet, depending on the vertex. How many vertices have exactly 3 edges meet? **(A)** 5 **(B)** 6 **(C)**

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19 The line segment formed by $A(1,2)$ and $B(3,3)$ is rotated to the line segment formed by $A'(3,1)$ and $B'(4,3)$ about the point $P(r,s)$. What is $|r-s|$?

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) $\frac{2}{3}$ E) 1

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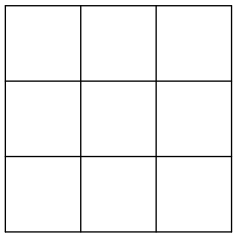
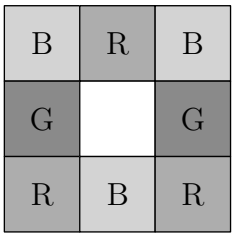
20 Each square in a 3×3 grid of squares is colored red, white, blue, or green so that every 2×2 square contains one square of each color. One such coloring is shown on the right below. How many different colorings are possible?





(A) 24 **(B)** 48 **(C)** 60 **(D)** 72 **(E)** 96

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21 Let $P(x)$ be the unique polynomial of minimal degree with the following properties:

$P(x)$ has leading coefficient 1, 1 is a root of $P(x)-1$, 2 is a root of $P(x-2)$, 3 is a root of $P(3x)$, 4 is a root of $4P(x)$

The roots of $P(x)$ are integers, with one exception. The root that is not an integer can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 41 **(B)** 43 **(C)** 45 **(D)** 47 **(E)** 49

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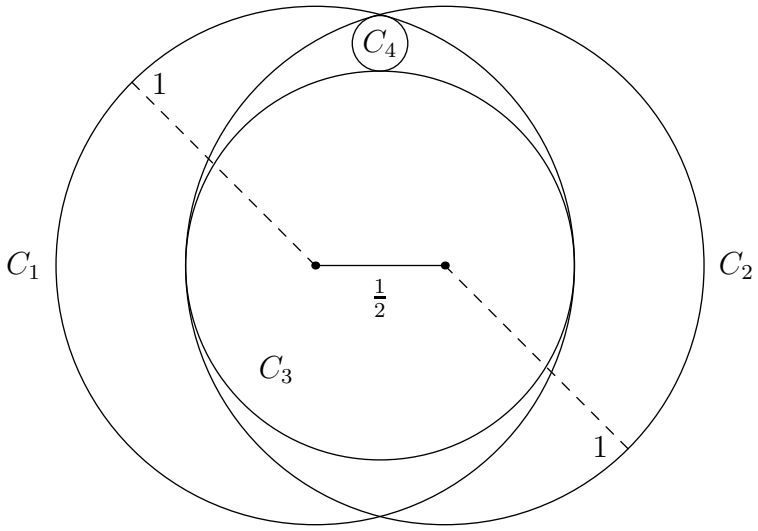
22 Circle C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and externally tangent to C_3. What is the radius of C_4?



(A) $\frac{1}{14}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{3}{28}$ **(E)** $\frac{1}{9}$

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23 Positive integer divisors a and b of n are called *complementary* if $ab = n$. Given that N has a pair of complementary divisors that differ by 20 and a pair of complementary divisors that differ by 23, find the sum of the digits of N.

(A) 11 **(B)** 13 **(C)** 15 **(D)** 17 **(E)** 19

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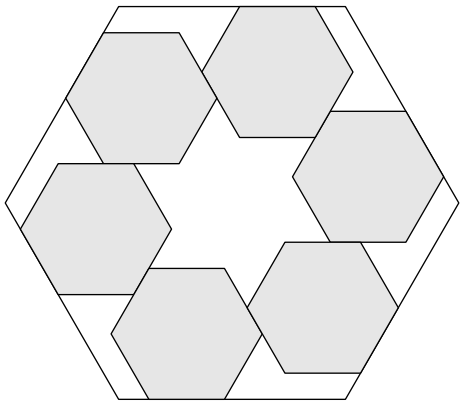
24 Six regular hexagonal blocks of side length 1 unit are arranged inside a regular hexagonal frame. Each block lies along an inside edge of the frame and is aligned with two other blocks, as shown in the figure below. The distance from any corner of the frame to the nearest vertex of a block is $\frac{3}{7}$ unit. What is the area of the region inside the frame not occupied by the blocks?



(A) $\frac{13\sqrt{3}}{3}$ **(B)** $\frac{216\sqrt{3}}{49}$ **(C)** $\frac{9\sqrt{3}}{2}$ **(D)** $\frac{14\sqrt{3}}{3}$ **(E)** $\frac{243\sqrt{3}}{49}$

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25 If A and B are vertices of a polyhedron, define the *distance* $d(A, B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A, B) = 1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A, B) = 2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q, R) > d(R, S)$?

(A) $\frac{7}{22}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{1}{2}$

– B

– November 14, 2023

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1 Mrs. Jones is pouring orange juice for her 4 kids into 4 identical glasses. She fills the first 3 full, but only has enough orange juice to fill one third of the last glass. What fraction of a glass of orange juice does she need to pour from the 3 full glasses into the last glass so that all glasses have an equal amount of orange juice? **(A)** $\frac{1}{12}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{2}{9}$

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2 Carlos went to a sports store to buy running shoes. Running shoes were on sale, with prices reduced by 20

A)46 B)50 C)48 D)47 E)49

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3 A $3-4-5$ right triangle is inscribed in circle A, and a $5-12-13$ right triangle is inscribed in circle B. What is the ratio of the area of circle A to the area of circle B?

(A) $\frac{9}{25}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{25}{169}$ **(E)** $\frac{4}{25}$

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4 Jackson's paintbrush makes a narrow strip that is 6.5 mm wide. Jackson has enough paint to make a strip of 25 meters. How much can he paint, in cm^2? **(A)** $162{,}500$ **(B)** 162.5 **(C)** $1{,}625$ **(D)** $1{,}6$

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5 Maddy and Lara see a list of numbers written on a blackboard. Maddy adds 3 to each number in the list and finds that the sum of her new numbers is 45. Lara multiplies each number in the list by 3 and finds that the sum of her new numbers is also 45. How many numbers are written on the blackboard?

(A) 10 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

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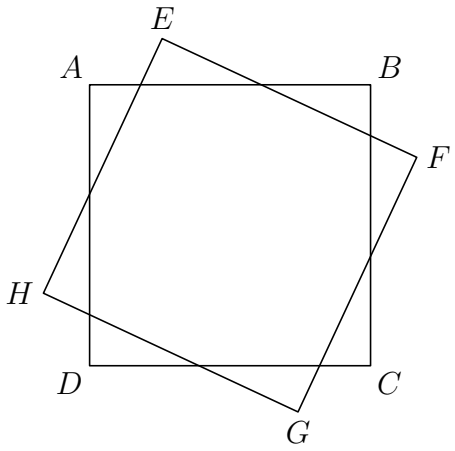
6 Let $L_1 = 1$, $L_2 = 3$, and $L_{n+2} = L_{n+1} + L_n$ for $n \geq 1$. How many terms in the sequence $L_1, L_2, L_3, \ldots, L_{2023}$ are even?

(A) 673 **(B)** 1011 **(C)** 675 **(D)** 1010 **(E)** 674

7 Square $ABCD$ is rotated 20° clockwise about its center to obtain square $EFGH$, as shown below. What is the degree measure of $\angle EAB$?



(A) 20° **(B)** 30° **(C)** 32° **(D)** 35° **(E)** 45°

8 What is the units digit of $2022^{2023} + 2023^{2022}$?

(A) 7 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 9

9 The numbers 16 and 25 are a pair of consecutive perfect squares whose difference is 9. How many pairs of consecutive positive perfect squares have a difference of less than or equal to 2023?

(A) 674 **(B)** 1011 **(C)** 1010 **(D)** 2019 **(E)** 2017

10 You are playing a game. A 2×1 rectangle covers two adjacent squares (oriented either horizontally or vertically) of a 3×3 grid of squares, but you are not told which two squares are covered. Your goal is to find at least one square that is covered by the rectangle. A "turn" consists of you guessing a square, after which you are told whether that square is covered by the hidden rectangle. What is the minimum number of turns you need to ensue that at least one of your guessed squares is covered by the rectangle?

(A) 3 **(B)** 5 **(C)** 4 **(D)** 8 **(E)** 6

11 Suzanne went to the bank and withdrew $\$800$. The teller gave her this amount using $\$20$ bills, $\$50$ bills, and $\$100$ bills, with at least one of each denomination. How many different collections of bills could Suzanne have received?

(A) 45 **(B)** 21 **(C)** 36 **(D)** 28 **(E)** 32

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12 When the roots of the polynomial

$$P(x) = \prod_{i=1}^{10} (x-i)^i$$

are removed from the real number line, what remains is the union of 11 disjoint open intervals. On how many of those intervals is $P(x)$ positive?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

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13 What is the area of the region in the coordinate plane defined by the inequality

$$||x|-1|+||y|-1| \leq 1?$$

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

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14 How many ordered pairs of integers (m,n) satisfy the equation $m^2+mn+n^2=m^2n^2$?

(A) 7 **(B)** 1 **(C)** 3 **(D)** 6 **(E)** 5

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15 What is the least positive integer m such that $m \cdot 2! \cdot 3! \cdot 4! \cdot 5! \cdots 16!$ is a perfect square?

(A) 30 **(B)** 30030 **(C)** 70 **(D)** 1430 **(E)** 1001

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16 Define an *upno* to be a positive integer of 2 or more digits where the digits are strictly increasing moving left to right. Similarly, define a *downno* to be a positive integer of 2 or more digits where the digits are strictly decreasing moving left to right. For instance, the number 258 is an *upno* and 8620 is a *downno*. Let U equal the total number of *upno*s and let D equal the total number of *downno*s. What is $|U-D|$?

(A) 512 **(B)** 10 **(C)** 0 **(D)** 9 **(E)** 511

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17 A rectangular box $\mathcal{P}$ has distinct edge lengths a, b, and c. The sum of the lengths of all 12 edges of $\mathcal{P}$ is 13, the sum of the areas of all 6 faces of $\mathcal{P}$ is $\frac{11}{2}$, and the volume of $\mathcal{P}$ is $\frac{1}{2}$. What is the length of the longest interior diagonal connecting two vertices of $\mathcal{P}$?

(A) 2 **(B)** $\frac{3}{8}$ **(C)** $\frac{9}{8}$ **(D)** $\frac{9}{4}$ **(E)** $\frac{3}{2}$

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18 Suppose a, b, and c are positive integers such that

$$\frac{a}{14}+\frac{b}{15}=\frac{c}{210}.$$

Which of the following statements are necessarily true?

I. If $\gcd(a, 14) = 1$ or $\gcd(b, 15) = 1$ or both, then $\gcd(c, 210) = 1$.
II. If $\gcd(c, 210) = 1$, then $\gcd(a, 14) = 1$ or $\gcd(b, 15) = 1$ or both.
III. $\gcd(c, 210) = 1$ if and only if $\gcd(a, 14) = \gcd(b, 15) = 1$.

(A) I, II, and III **(B)** I only **(C)** I and II only **(D)** III only **(E)** II and III only

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19 Sonya the frog chooses a point uniformly at random lying within the square $[0, 6] \times [0, 6]$ in the coordinate plane and hops to that point. She then randomly chooses a distance uniformly at random from $[0, 1]$ and a direction uniformly at random from north, south east, west. All he choices are independent. She now hops the distance in the chosen direction. What is the probability that she lands outside the square?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{10}$ **(E)** $\frac{1}{9}$

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20 Four congruent semicircles are drawn on the surface of a sphere with radius 2, as shown, creating a close curve that divides the surface into two congruent regions. The length of the curcve is $\pi\sqrt{n}$. What is n?

(A) 32 **(B)** 12 **(C)** 48 **(D)** 36 **(E)** 27

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21 Each of 2023 balls is placed in on of 3 bins. Which of the following is closest to the probability that each of the bins will contain an odd number of balls?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{4}$

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22 How many distinct values of x satisfy $\lfloor x \rfloor^2 – 3x + 2 = 0$ where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x?

(A) an infinite number **(B)** 4 **(C)** 2 **(D)** 3 **(E)** 0

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23 An arithmetic sequence has $n \geq 3$ terms, initial term a and common difference $d > 1$. Carl wrote down all the terms in this sequence correctly except for one term which was off by 1. The sum of the terms was 222. What was $a + d + n$

(A) 24 **(B)** 20 **(C)** 22 **(D)** 28 **(E)** 26

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24 What is the perimeter of the boundary of the region consisting of all points which can be expressed as $(2u – 3w, v + 4w)$ with $0 \leq u \leq 1$, $0 \leq v \leq 1$, and $0 \leq w \leq 1$?

(A) $10\sqrt{3}$ **(B)** 10 **(C)** 12 **(D)** 18 **(E)** 16

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25 A regular pentagon with area $\sqrt{5} + 1$ is printed on paper and cut out. The five vertices of the pentagon are folded into the center of the pentagon, creating a smaller pentagon. What is the area of the new pentagon?

(A) $4 – \sqrt{5}$ **(B)** $\sqrt{5} – 1$ **(C)** $8 – 3\sqrt{5}$ **(D)** $\frac{\sqrt{5}+1}{2}$ **(E)** $\frac{2+\sqrt{5}}{3}$

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