

## Jharkhand University of Technology, Ranchi

### Diploma 1st Semester Examination, 2023 (NEP)

Subject : Engineering Mathematics-I

Subject Code : AEC01

Time Allowed : 3 hours

Full Marks : 70

Answer any five questions in which Question No. 1 is compulsory and answer any four from the rest questions.

1. Answer the following Multiple Choice Type questions:

7×2=14

- (i) How many terms are there between 9, 12, 15, 18, ..., 87?
  - (a) 25
  - (b) 26
  - (c) 27
  - (d) None of these
  
- (ii) The value of  $\log_{1/N} N$  is equal to
  - (a) -1
  - (b) 1
  - (c) 0
  - (d) None of these
  
- (iii) The type of matrix which have a multiplicative inverse—
  - (a) row matrix
  - (b) column matrix
  - (c) square matrix
  - (d) None of these
  
- (iv) The system of equation  $x = y$  and  $x = -y$  has the number of solution—
  - (a) one solution
  - (b) two solutions
  - (c) many solutions
  - (d) no solution
  
- (v) Maximum value of  $\sin \theta \cdot \cos \theta$  is equal to
  - (a) 2
  - (b) 1
  - (c)  $\frac{1}{2}$
  - (d) None of these
  
- (vi)  ${}^n C_r$  is equal to
  - (a)  ${}^n C_{n-r}$
  - (b)  ${}^n C_{r-n}$
  - (c)  ${}^n C_{n-1}$
  - (d) None of these
  
- (vii) The slope and  $x$ -intercept of the line  $3x - y + k = 0$  are equal then the value of  $k$  is equal to
  - (a) 0
  - (b) -1
  - (c) 3
  - (d) -9

2. (a) Resolve  $\frac{x^3}{x^2+8x+12}$  into partial function.

7+7

(b) The  $p$ th and  $q$ th term of a GP are  $a$  and  $b$  respectively. Find its  $n$ th term.

AEC01

(2)

7+7

3. (a) Solve these simultaneous equations by Cramer's rule.

$$x + y + 2z = 4, 2x - y - z = 1 \text{ and } 3x - 2y - z = 3$$

- (b) Show that if  $n$  is any integer greater than 1 then  $6^{2n} - 35n - 1$  is divisible by 1225.

4. (a) In any triangle ABC prove that  $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$ .

7+7

$$(b) \text{ Prove that } 2 \tan^{-1} \sqrt{x} = \sec^{-1} \frac{1+x}{1-x}.$$

5. (a) Find the equation of straight line passing through the point  $(3, -2)$  and making an angle  $45^\circ$  with the line  $6x + 5y = 1$ .

7+7

$$(b) \text{ If } f(x) = \tan^{-1} x \text{ prove that } f(x) - f(y) = f\left(\frac{x-y}{1+xy}\right).$$

7+7

6. (a) Find the differential coefficient of  $\sin^{-1} x$  from the first principle.

$$(b) \text{ Find the inverse of the matrix } A = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}.$$

7+7

7. Solve any two of the following:

$$(a) \text{ Evaluate } \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$(b) \text{ Differentiate } y = \tan^{-1} \frac{2x}{1-x^2} \text{ with respect to } \sin^{-1} \frac{2x}{1+x^2}$$

$$(c) \text{ If } x^m y^n = (x+y)^{m+n} \text{ then evaluate } \frac{dy}{dx}$$

$$\begin{aligned} (3^2) &= m \cdot 2 \\ 9 &= m \cdot 2 \cdot \frac{a}{b} \cdot \frac{-6}{5} \\ 45 &= -12 \end{aligned}$$

$$(y - y_1) = m(x - x_1)$$

$$(y - 2) = \frac{6}{5}(x - 3)$$

$$5y - 10 = 6x + 18$$

$$6x + 5y - 28 = 0$$