Practice Set 12

Robotics & Automation
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Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

Given an arbitrary body twist V_b and transformation $T = T_{sb}$:

$$V_b = \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}, \qquad T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \tag{1}$$

Prove that $V_s = Ad_T V_b$. Remember that $x \times y = -y \times x$.

Problem 2

Write a function that computes the adjoint of a transformation matrix. Use that function to find the adjoint of transformation T:

$$T = \begin{bmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 & 0 & 5 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Problem 3

Numerically show that $Ad_T^{-1} = Ad_{T^{-1}}$ using T from the previous problem.

Problem 1

Given an arbitrary body twist and transformation $T = T_{sb}$:

$$V_b = \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}, \qquad T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \tag{3}$$

Prove that $V_s = Ad_T V_b$. Remember that $x \times y = -y \times x$.

Start by plugging in the terms:

$$V_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \begin{bmatrix} \omega_b \\ v_b \end{bmatrix} \tag{4}$$

From this we get that:

$$\omega_s = R\omega_b \tag{5}$$

$$v_s = [p]R\omega_b + Rv_b \tag{6}$$

We know that $\omega_s = R\omega_b$ from previous lectures; we are just rotating the angular velocity into frame $\{s\}$. For the second equation we can simplify:

$$v_s = [p]R\omega_b + Rv_b = p \times \omega_s + Rv_b = -[\omega_s]p + Rv_b = -[\omega_s]p + \dot{p} \tag{7}$$

As expected, this is the linear velocity of a point at $\{s\}$ expressed in $\{s\}$. Look back at your notes to confirm this result.

Problem 2

Write a function that computes the adjoint of a transformation matrix. Use that function to find the adjoint of transformation T:

$$T = \begin{bmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 & 0 & 5 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

```
function AdT = Adjoint(T)

    R = T(1:3, 1:3);
    p = T(1:3, 4);
    AdT = [R, zeros(3, 3); bracket(p)*R, R];

    function w_matrix = bracket(w)
        w_matrix = [0 -w(3) w(2);
            w(3) 0 -w(1);
            -w(2) w(1) 0];
    end

end
```

See the code above. Using the given matrix T we get:

$$\begin{bmatrix} -0.707 & -0.707 & 0 & 0 & 0 & 0 \\ 0.707 & -0.707 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -2.12 & 2.12 & -1 & -0.707 & -0.707 & 0 \\ -2.12 & -2.12 & -5 & 0.707 & -0.707 & 0 \\ 2.83 & -4.24 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Remember that the adjoint is a 6×6 matrix.

Problem 3

Numerically show that $Ad_T^{-1} = Ad_{T^{-1}}$ using T from the previous problem.

```
>> inv(Adjoint(T))

ans =

   -0.7071    0.7071    0.0000   -0.0000   -0.0000   -0.0000
   -0.7071   -0.7071    0.0000   -0.0000   -0.0000         0
         0         0    1.0000         0         0         0
   -2.1213   -2.1213    2.8284   -0.7071    0.7071         0
    2.1213   -2.1213   -4.2426   -0.7071   -0.7071   -0.0000
   -1.0000   -5.0000    0.0000   -0.0000   -0.0000    1.0000
```

```
>> Adjoint(inv(T))

ans =

   -0.7071    0.7071         0         0         0         0
   -0.7071   -0.7071         0         0         0         0
         0         0    1.0000         0         0         0
   -2.1213   -2.1213    2.8284   -0.7071    0.7071         0
    2.1213   -2.1213   -4.2426   -0.7071   -0.7071         0
   -1.0000   -5.0000         0         0         0    1.0000
```