Practice Set 22

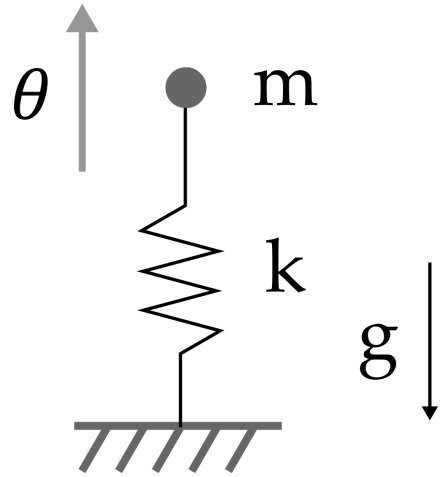
Robotics & Automation
Dylan Losey, Virginia Tech

Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

What are the dimensions of the Lagrangian? What are the SI units for the Lagrangian?

Problem 2



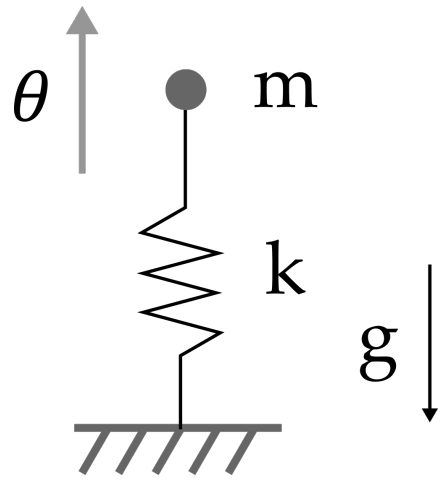
Apply the Euler-Lagrange equation to find the dynamics of the point mass shown above. The spring is at equilibrium at position θ_0, and the spring's potential energy is $\frac{1}{2}k(\theta - \theta_0)^2$.

Problem 1

What are the dimensions of the Lagrangian? What are the SI units for the Lagrangian?

The Lagrangian is a scalar. Remember that the Lagrangian is the kinetic energy minus the potential energy, $L = K - P$, and therefore the units of the Lagrangian must be units for energy. The SI unit here is $kg \cdot m^2/s^2$.

Problem 2



Apply the Euler-Lagrange equation to find the dynamics of the point mass shown above. The spring is at equilibrium at position θ_0, and the spring's potential energy is $\frac{1}{2}k(\theta - \theta_0)^2$.

Start by finding the kinetic and potential energy:

$$K(\theta, \dot{\theta}) = \frac{1}{2}m\dot{\theta}^2 \tag{1}$$

$$P(\theta) = mg\theta + \frac{1}{2}k(\theta - \theta_0)^2 \tag{2}$$

Subtract to get the Lagrangian:

$$L(\theta, \dot{\theta}) = \frac{1}{2}m\dot{\theta}^2 - mg\theta - \frac{1}{2}k(\theta - \theta_0)^2 \tag{3}$$

Then apply the Euler-Lagrange equation:

$$f = \frac{d}{dt}\frac{\partial L(\theta, \dot{\theta})}{\partial \dot{\theta}} - \frac{\partial L(\theta, \dot{\theta})}{\partial \theta} = m\ddot{\theta} - \big(-mg - k(\theta - \theta_0)\big) \tag{4}$$

$$f = m\ddot{\theta} + mg + k(\theta - \theta_0) \tag{5}$$