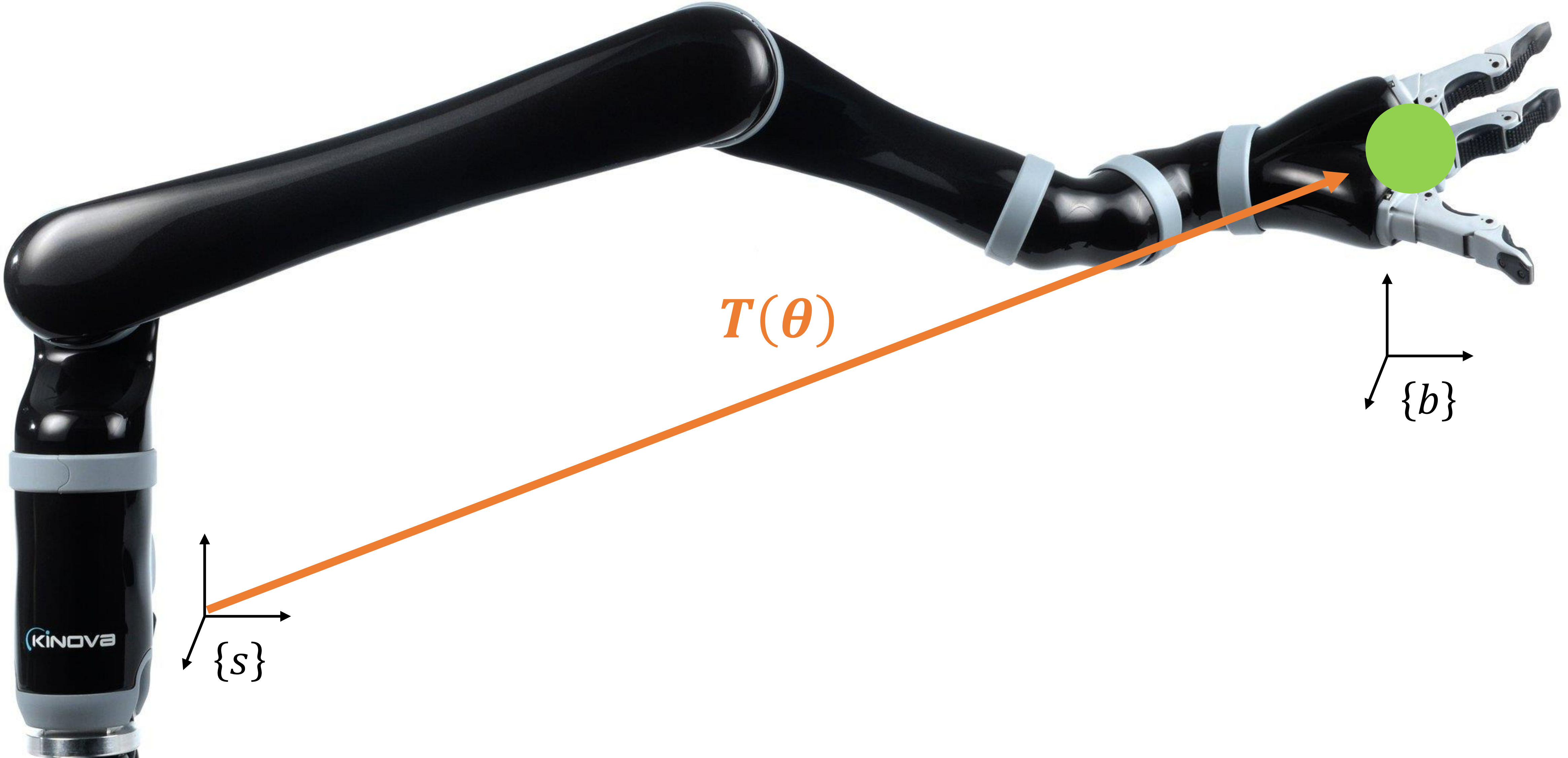
Inverse Kinematics

Reading: Modern Robotics 6.1

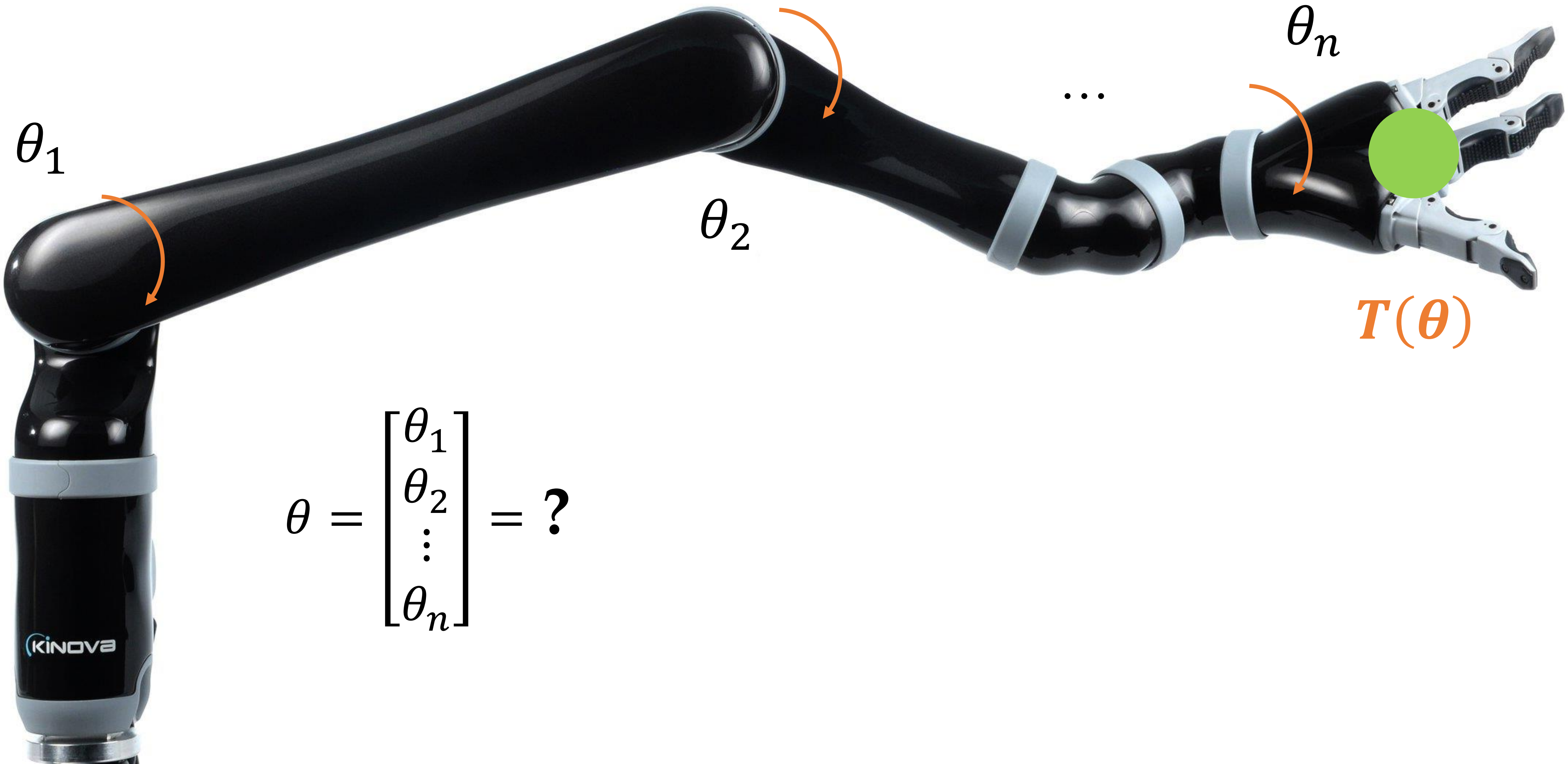
This Lecture

- What are inverse kinematics?
- How can we find the inverse kinematics?
- What makes inverse kinematics challenging?

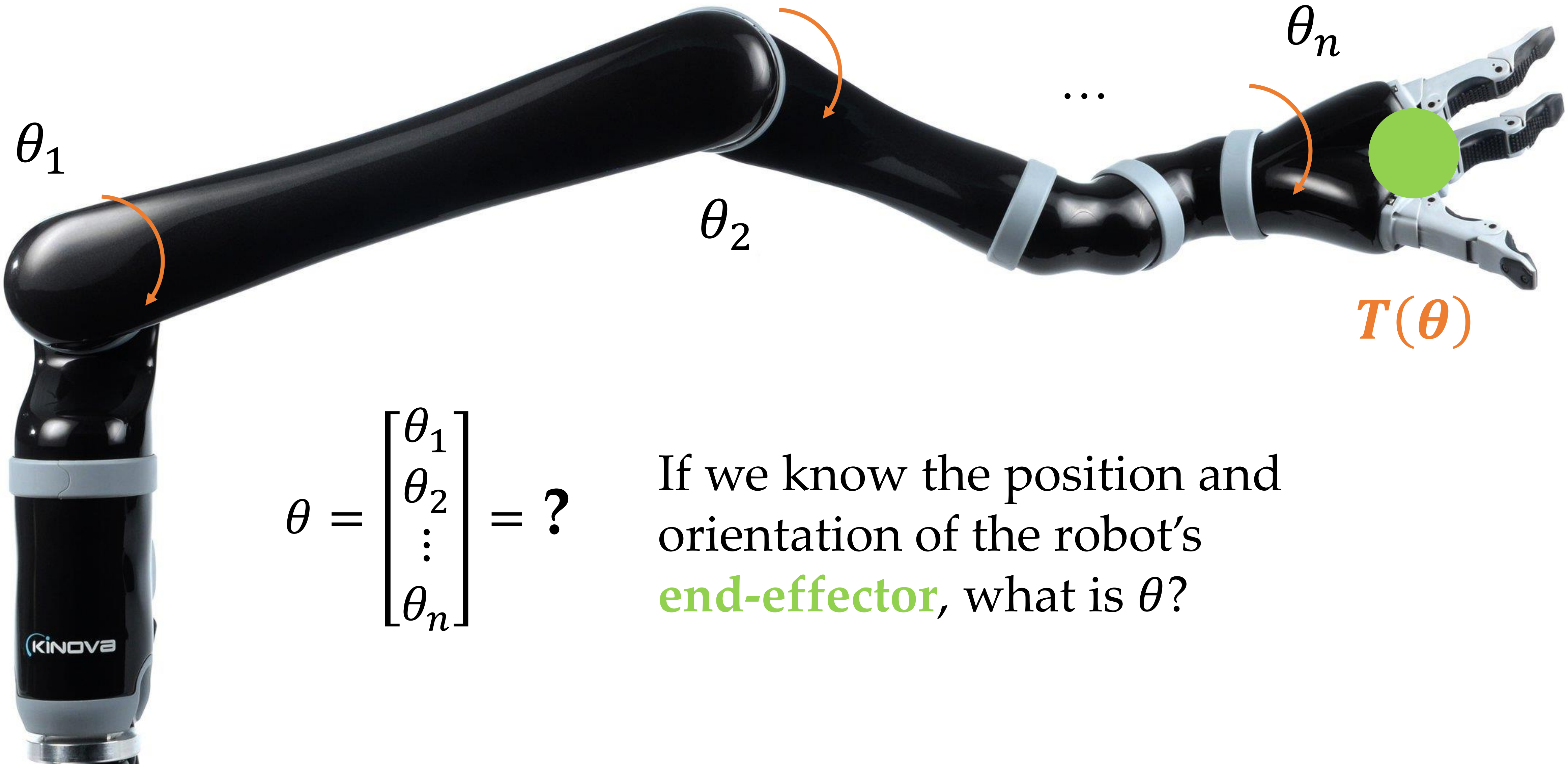

KINOVA



$T(\theta)$
$\{s\}$
$\{b\}$
KINOVA



$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} = \textbf{?}$$



$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} = \boldsymbol{?}$$

If we know the position and orientation of the robot's **end-effector**, what is θ?

Given a robot with:

- fixed frame $\{s\}$ at the base
- body frame $\{b\}$ at point of interest

inverse kinematics is the mapping from T_{sb} to joint values θ

Note: this is the ***opposite*** of forward kinematics!

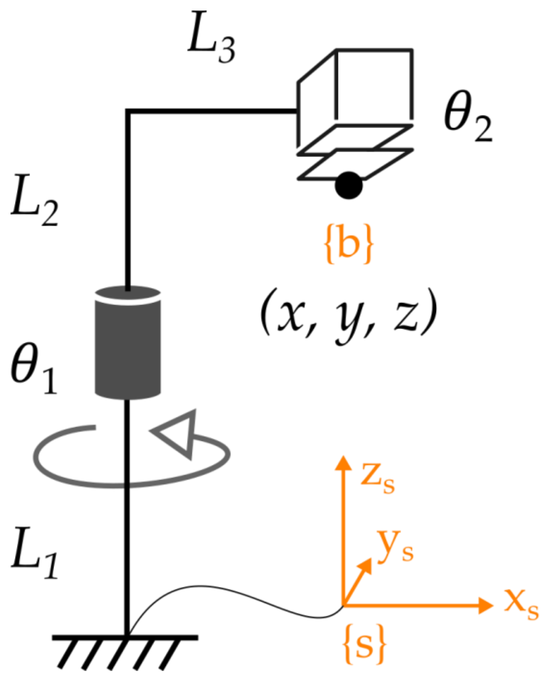
How do we **find** the **inverse kinematics**?

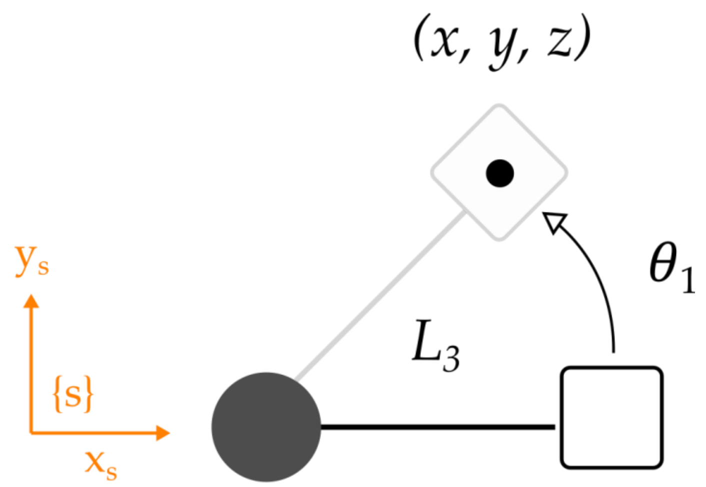
Analytical Solutions

The end-effector position is:

$$p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Solve for θ_1 and θ_2





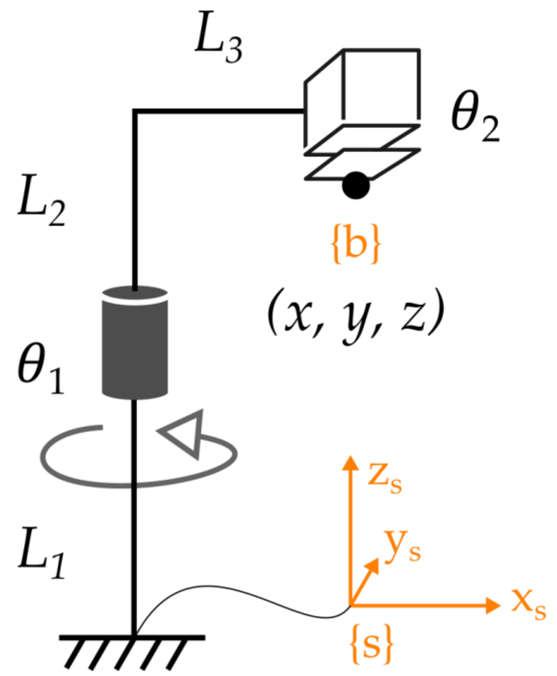
Analytical Solutions

The height of the end-effector is:

$$z = L_1 + L_2 - \theta_2$$

So if we are given z, then θ_2 is:

$$\boxed{\theta_2 = L_1 + L_2 - z}$$



Analytical Solutions

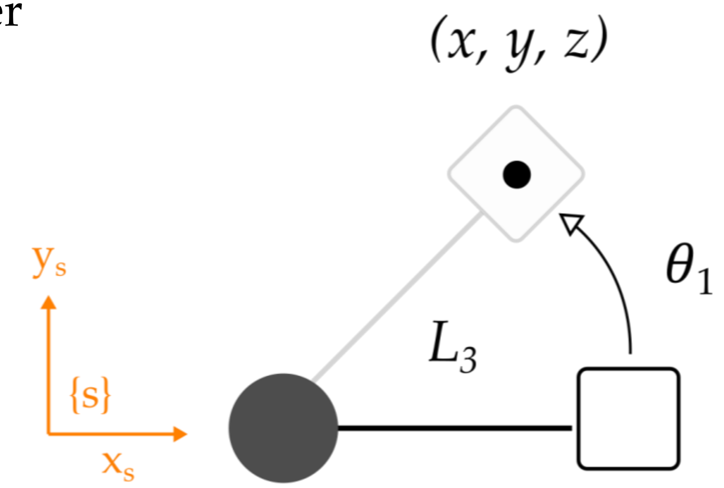
The angle of the end-effector is:

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$$

Gives wrong answer when $x < 0$

In robotics we improve this using:

$$\boxed{\theta_1 = \text{atan2}(y, x)}$$



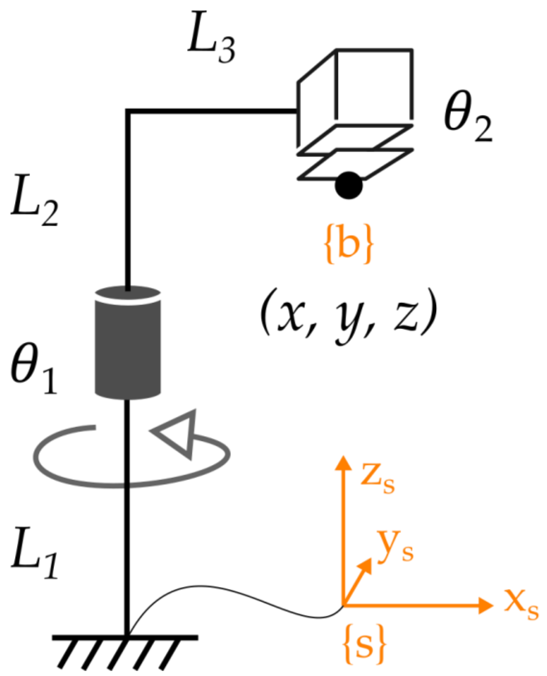
Analytical Solutions

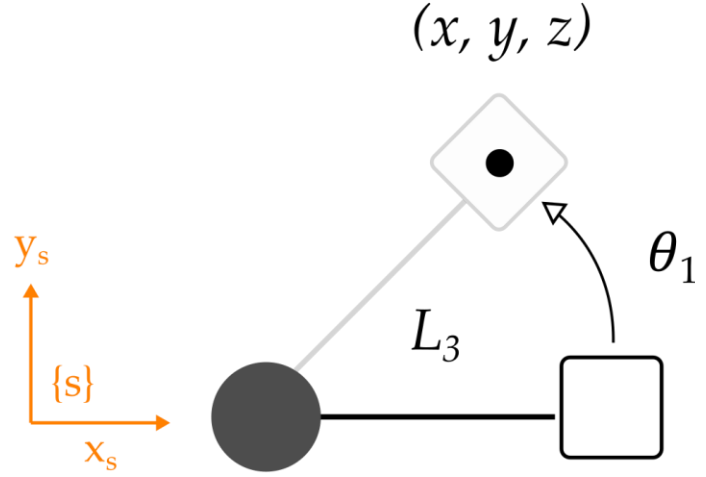
Given end-effector position (x, y, z), the joint position is:

$$\theta_1 = \text{atan2}(y, x)$$

$$\theta_2 = L_1 + L_2 - z$$

Inverse kinematics



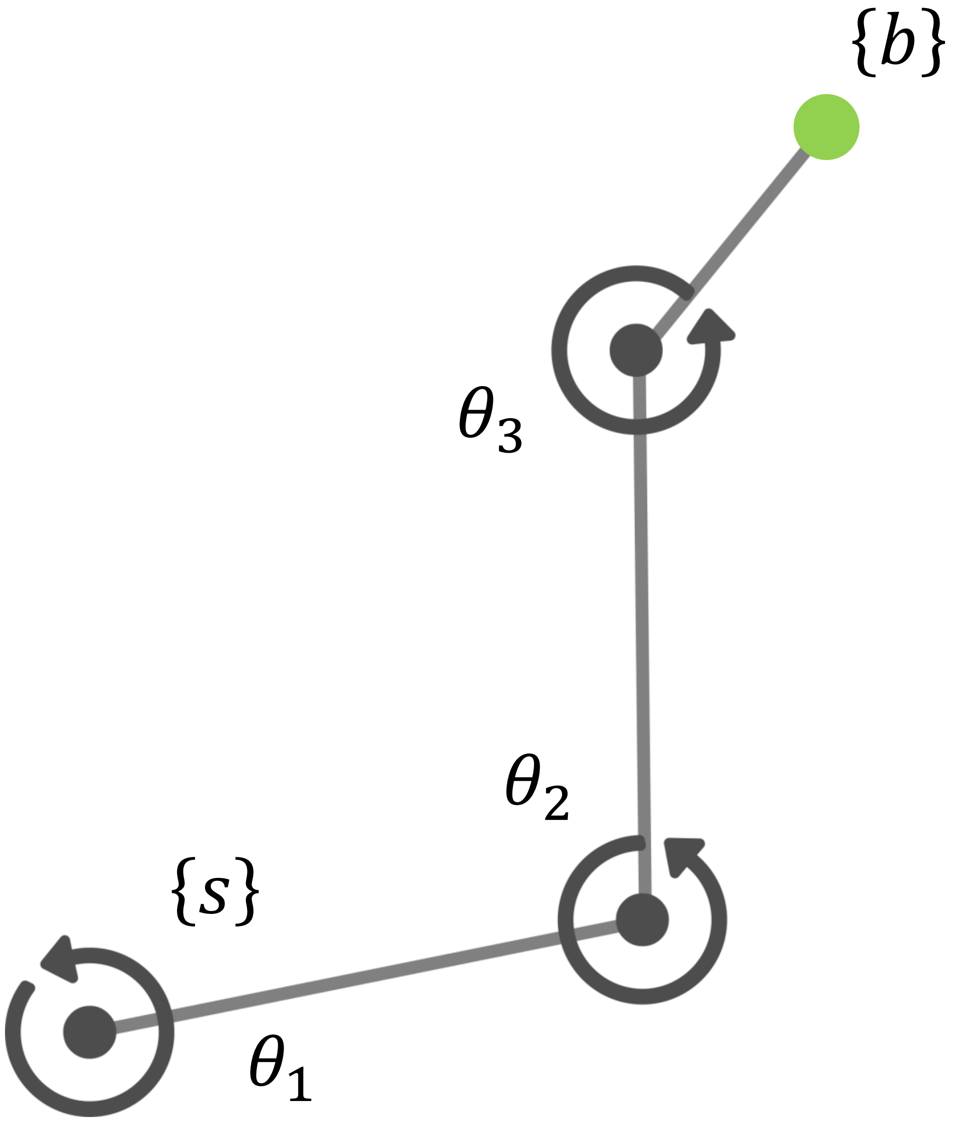


Inverse kinematics often have **more than one** solution

Given transformation matrix T_{sb}
this robot could have joint values:

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

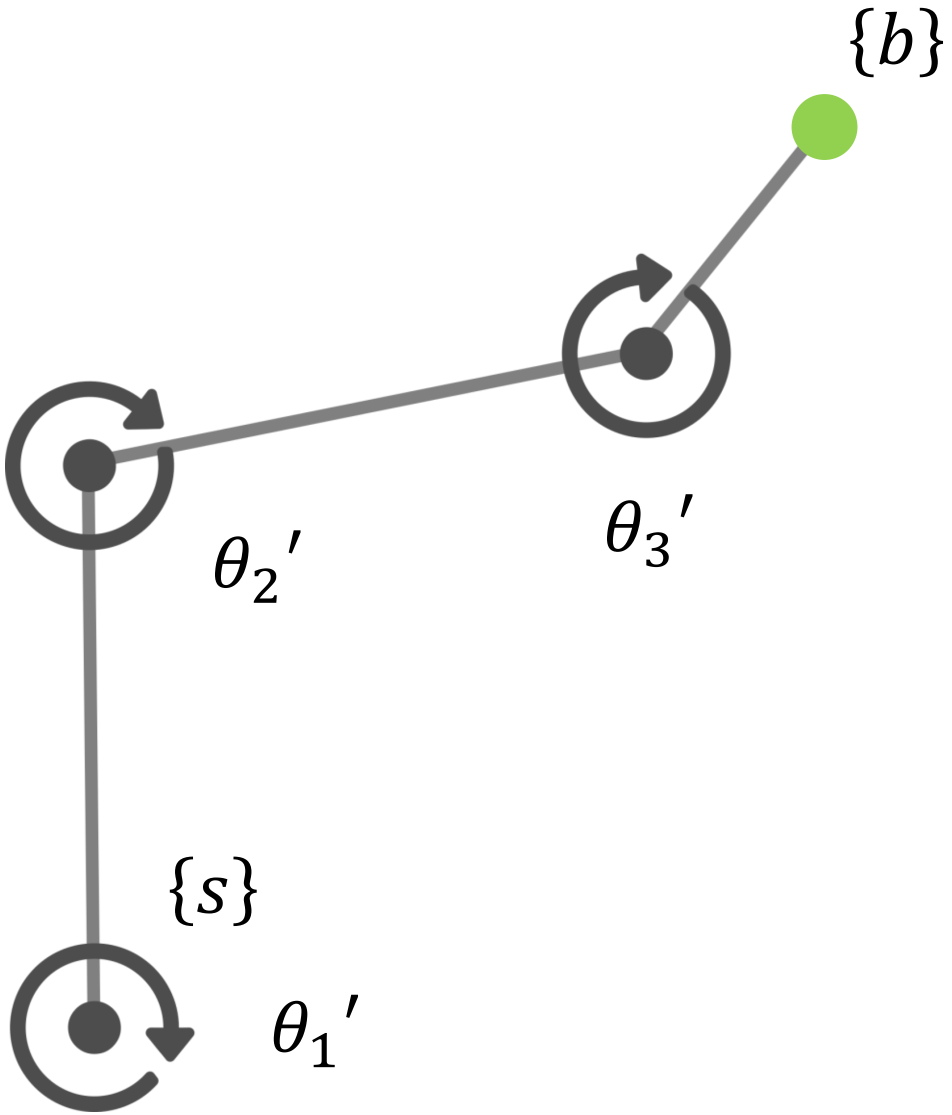
Call this joint position *elbow down*



Given transformation matrix T_{sb}
this robot could **instead** have:

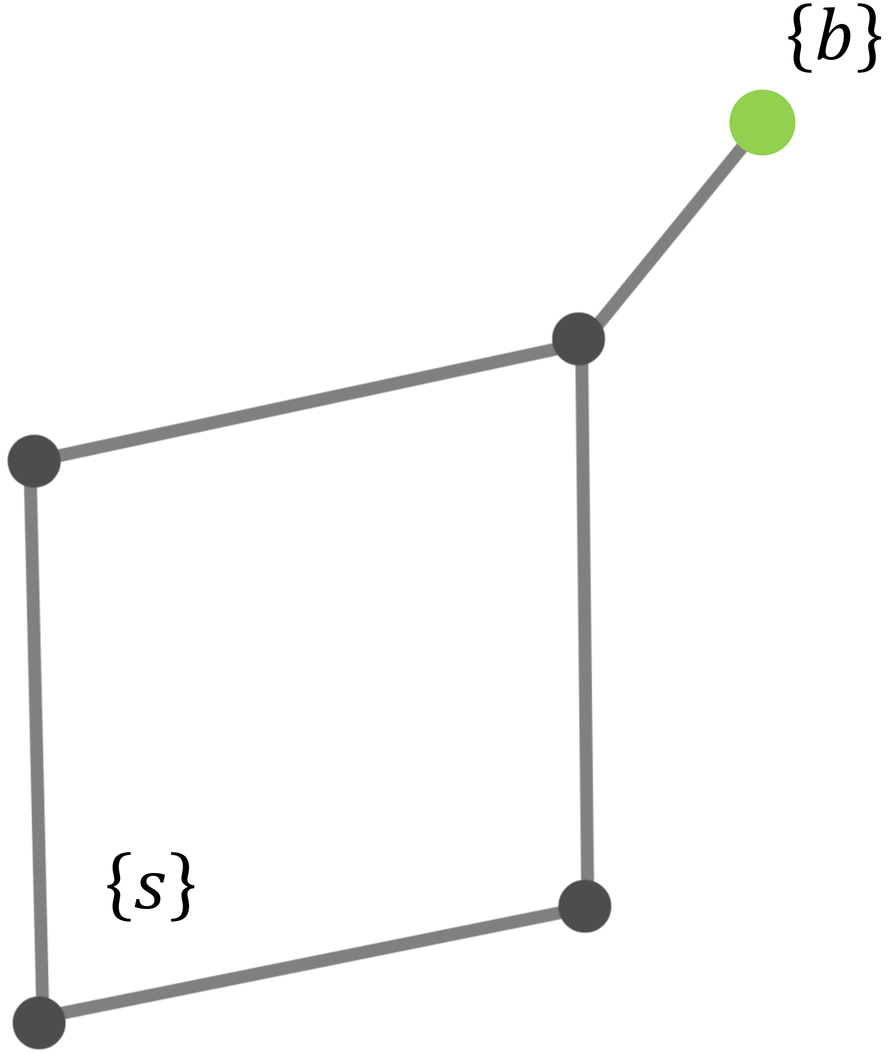
$$\theta' = \begin{bmatrix} \theta_1{}' \\ \theta_2{}' \\ \theta_3{}' \end{bmatrix}$$

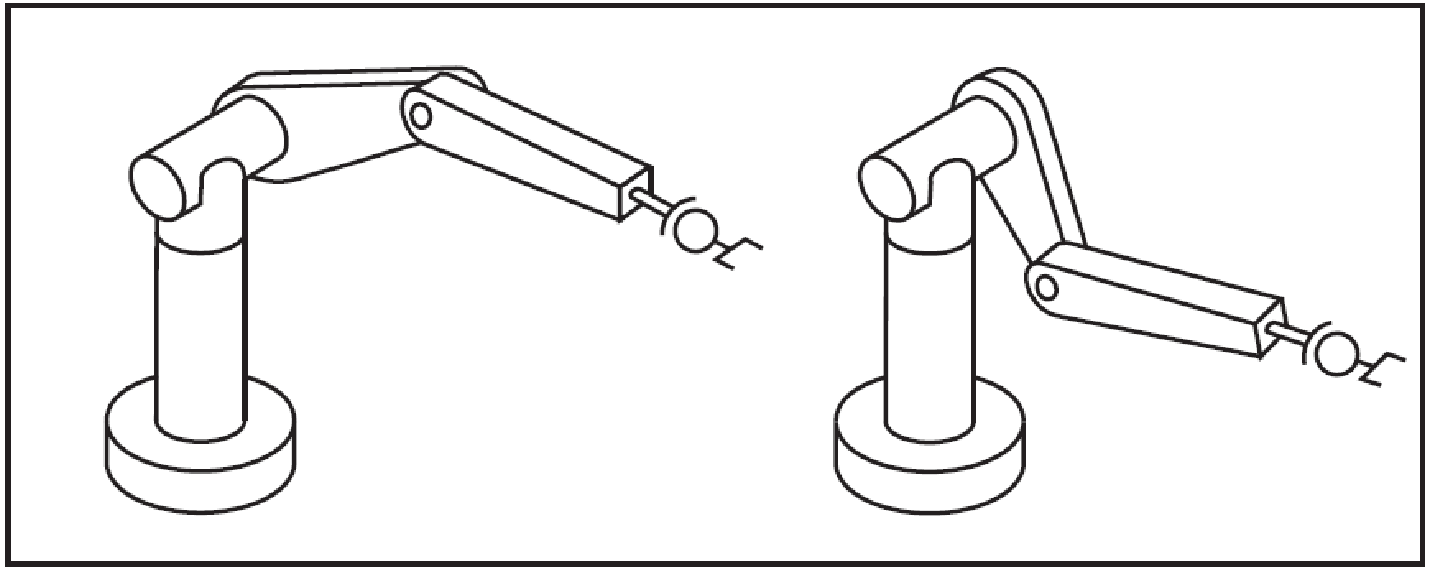
Call this joint position *elbow up*

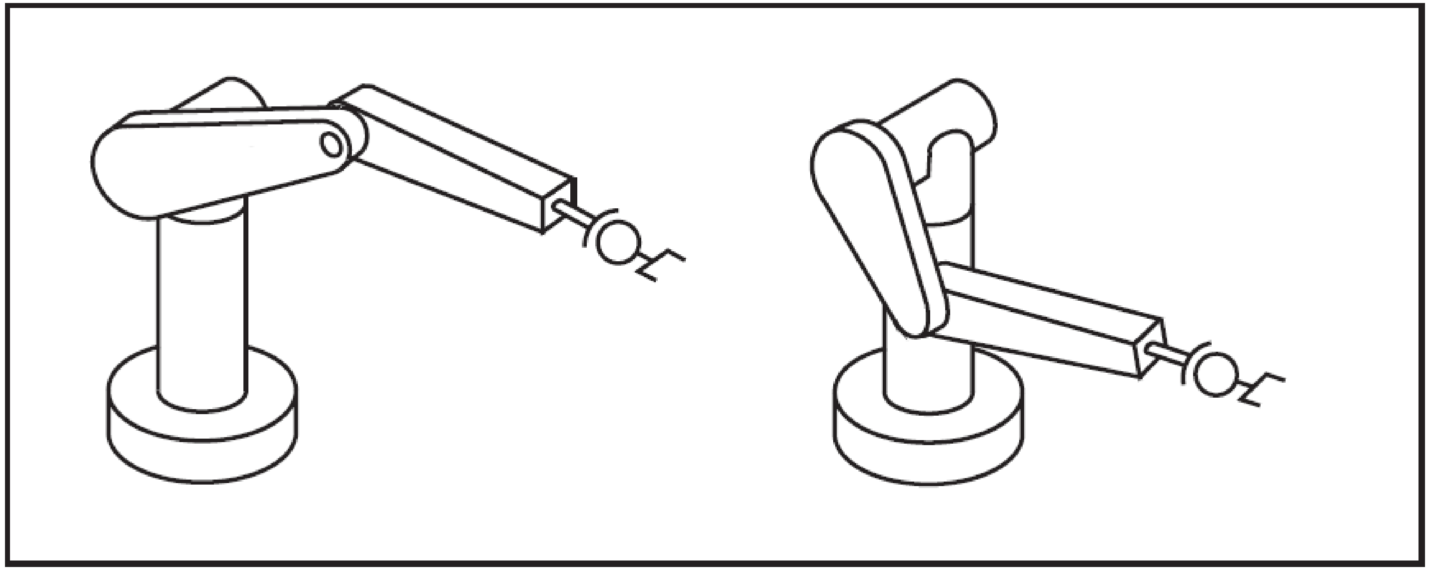


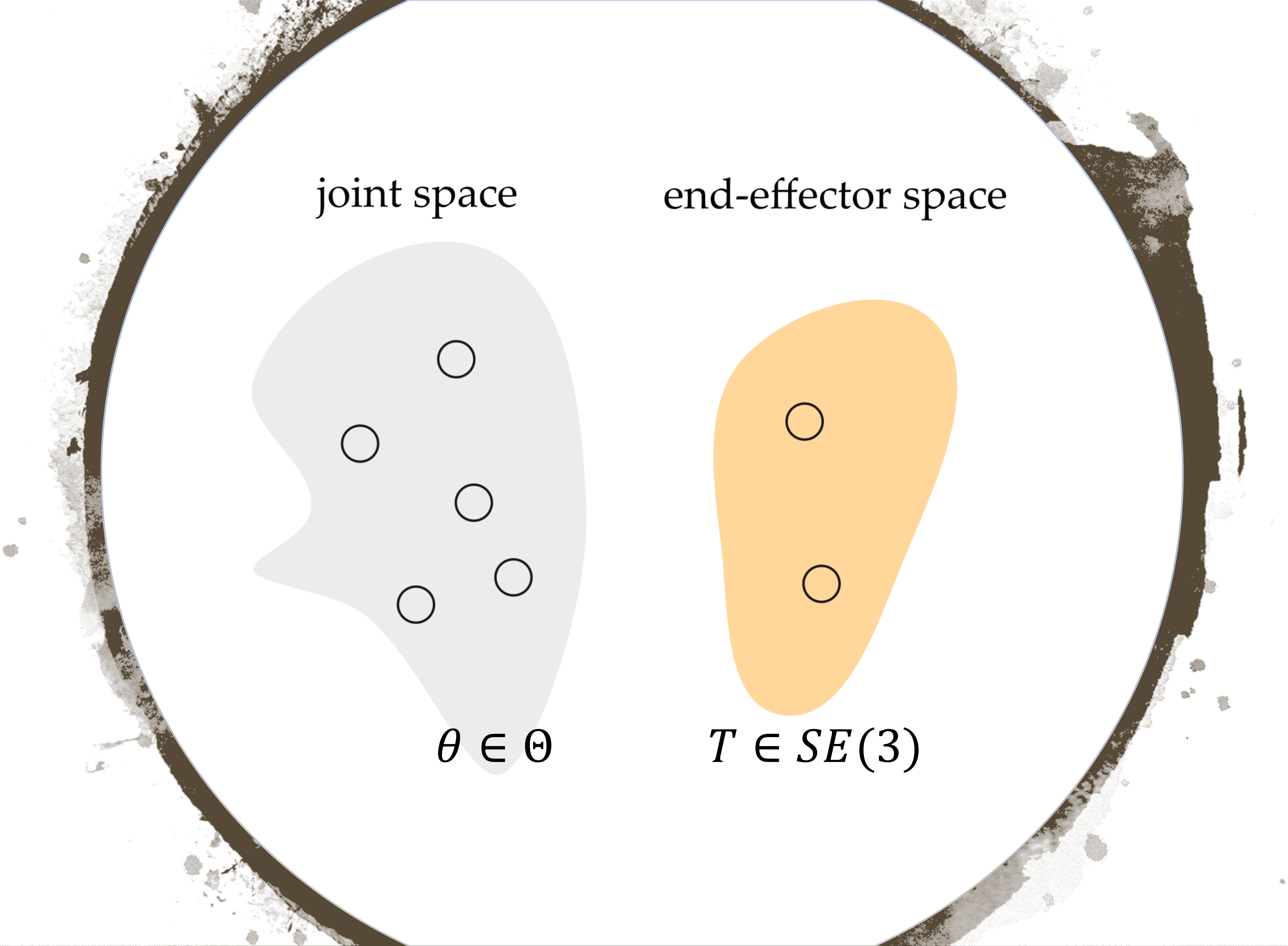
Both *elbow up* and *elbow down* are valid solutions for the inverse kinematics.

Given T_{sb}, we often find multiple choices for joint values θ

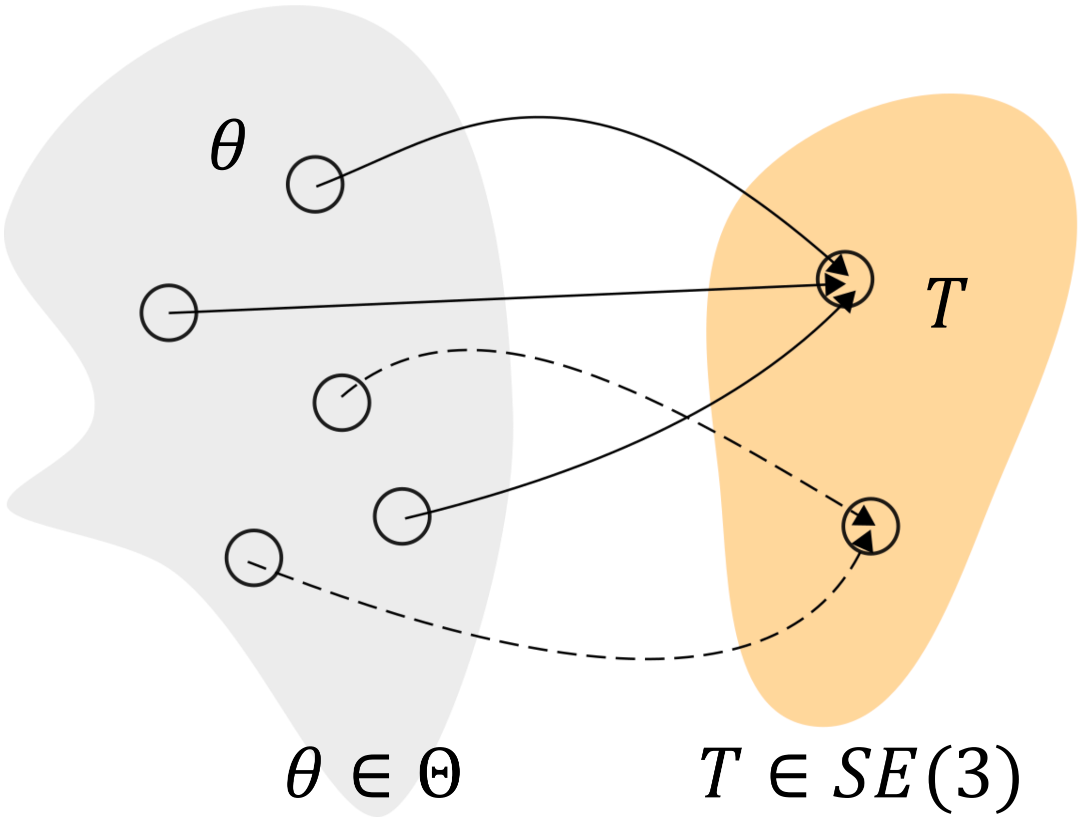




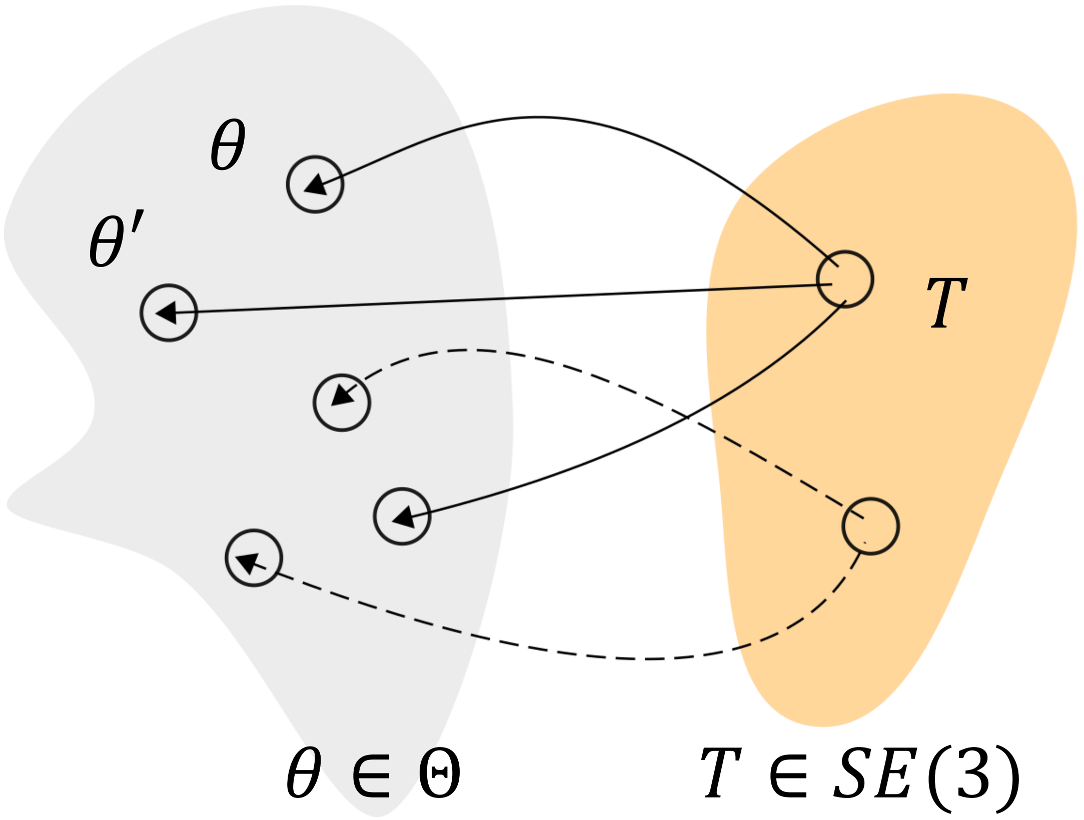



joint space
end-effector space
$\theta \in \Theta$
$T \in SE(3)$

Forward Kinematics: **one** solution



Inverse Kinematics: many solutions



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Next Lecture

- Analytical inverse kinematics work for simple robots. But what about complex, real-world robot arms?