Practice Set 16

Robotics & Automation
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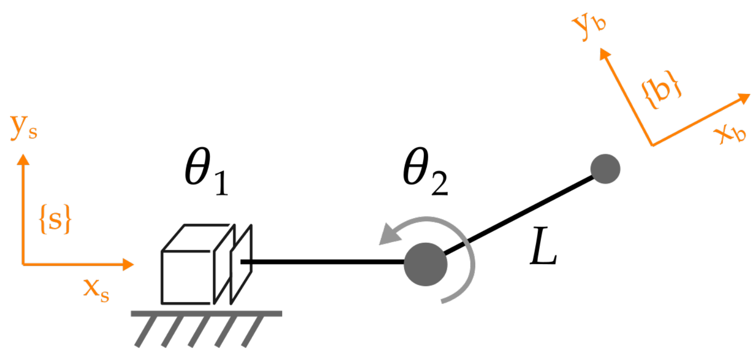
Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1



Consider the robot shown above. The task space of this robot is the x-y position of the end-effector. Draw at least two singular positions.

Problem 2

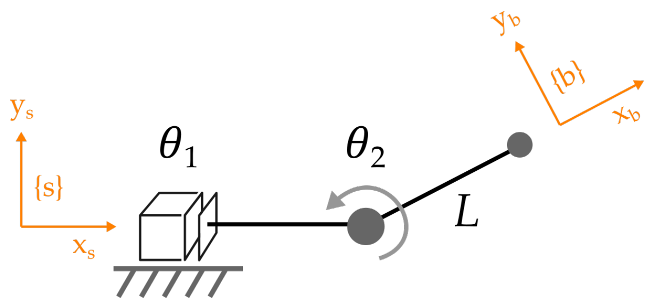
Let the robot have joint position $\theta_1 = 5$ and $\theta_2 = \pi/2$.

- Is this a singular configuration?
- In what direction(s) can the robot move?
- In what direction(s) can the robot not move?

Problem 3

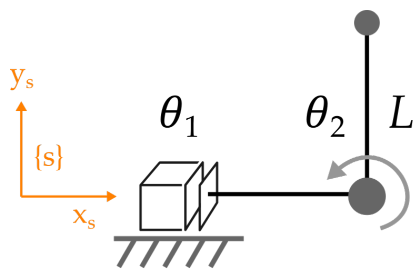
Find a joint position that maximizes the robot's manipulability.

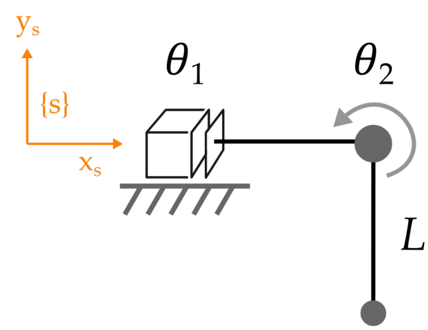
Problem 1



Consider the robot shown above. The task space of this robot is the x-y position of the end-effector. Draw at least two singular positions.

See the drawing below. No calculations were needed here — in these positions the robot can move along the x_s axis (left and right), but cannot move along the y_s axis (up and down).





Problem 2

Let the robot have joint position $\theta_1 = 5$ and $\theta_2 = \pi/2$.

- Is this a singular configuration?

To reinforce concepts from lecture let's get the geometric Jacobian.

$$J(\theta) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & -L\sin(\theta_2) \\ 0 & L\cos(\theta_2) \\ 0 & 0 \end{bmatrix} \tag{1}$$

We are given that the task space of this robot is the x-y position of the end-effector. For singularity analysis, we take the **only the rows of the Jacobian** that contribute to motion in x and motion in y:

$$J(\theta) = \begin{bmatrix} 1 & -L\sin(\theta_2) \\ 0 & L\cos(\theta_2) \end{bmatrix} \tag{2}$$

Substitute in the given value for θ and take the determinant:

$$J(\theta) = \begin{bmatrix} 1 & -L \\ 0 & 0 \end{bmatrix}, \qquad \det(J) = 0 \tag{3}$$

Because the determinant is zero, **this is a singular configuration**.

- In what direction(s) can the robot move?

The robot can achieve velocities in range(J). Here `colspace(J)` outputs: $[1, 0]^T$. Hence, the robot can move along the x_s axis.

- In what direction(s) can the robot not move?

The robot cannot move in null(J^T). Here `null(J')` outputs: $[0, 1]^T$. Hence, the robot cannot move along the y_s axis. Note that our results here match what we intuitively recognized in Problem 1.

Problem 3

Find a joint position that maximizes the robot's manipulability.

We want to find a joint position that maximizes $\sqrt{\det(JJ^T)}$. Plugging in:

$$J(\theta) = \begin{bmatrix} 1 & -L\sin(\theta_2) \\ 0 & L\cos(\theta_2) \end{bmatrix}, \quad \sqrt{\det(JJ^T)} = \sqrt{L^2\cos(\theta_2)^2} = L|\cos(\theta_2)| \tag{4}$$

We maximize the robot's manipulability when $\theta = 0 + n\pi$. Here the robot is farthest from a singularity when it is fully extended!