

Singularities



Reading: Modern Robotics 5.3 – 5.4



19:13:31 05/06/2015 UTC

This Lecture

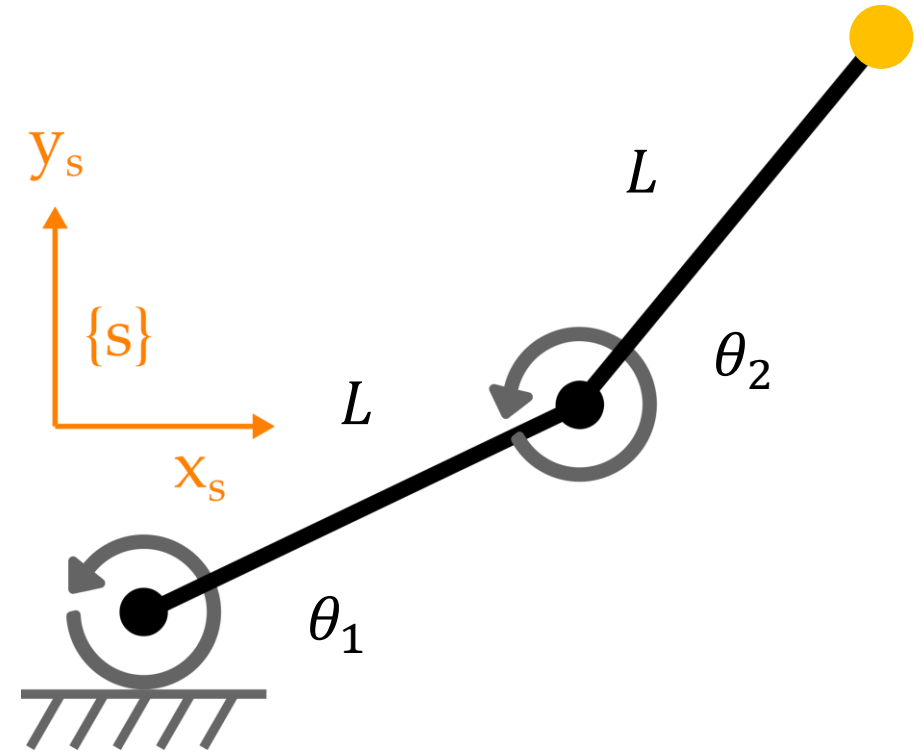


- What is a singularity?
- What directions can robots move in singularities?
- When are we close to or far from singularities?

Here the geometric **Jacobian** is:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

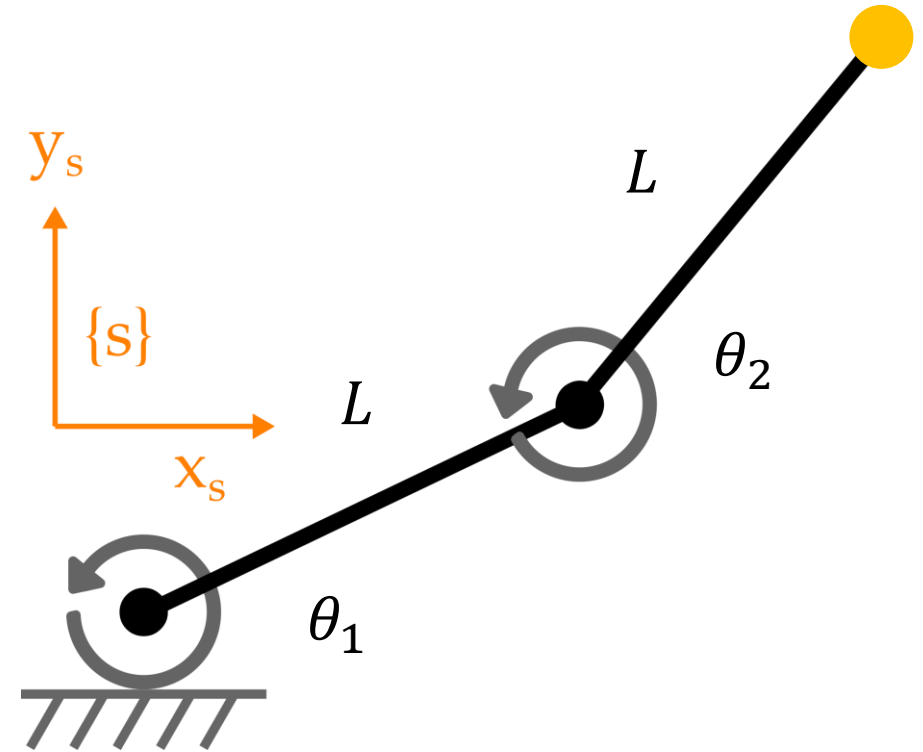
The robot's end-effector is moving in the $x - y$ plane. This is the *task space* for this 2-DoF robot.



Here the geometric **Jacobian** is:

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

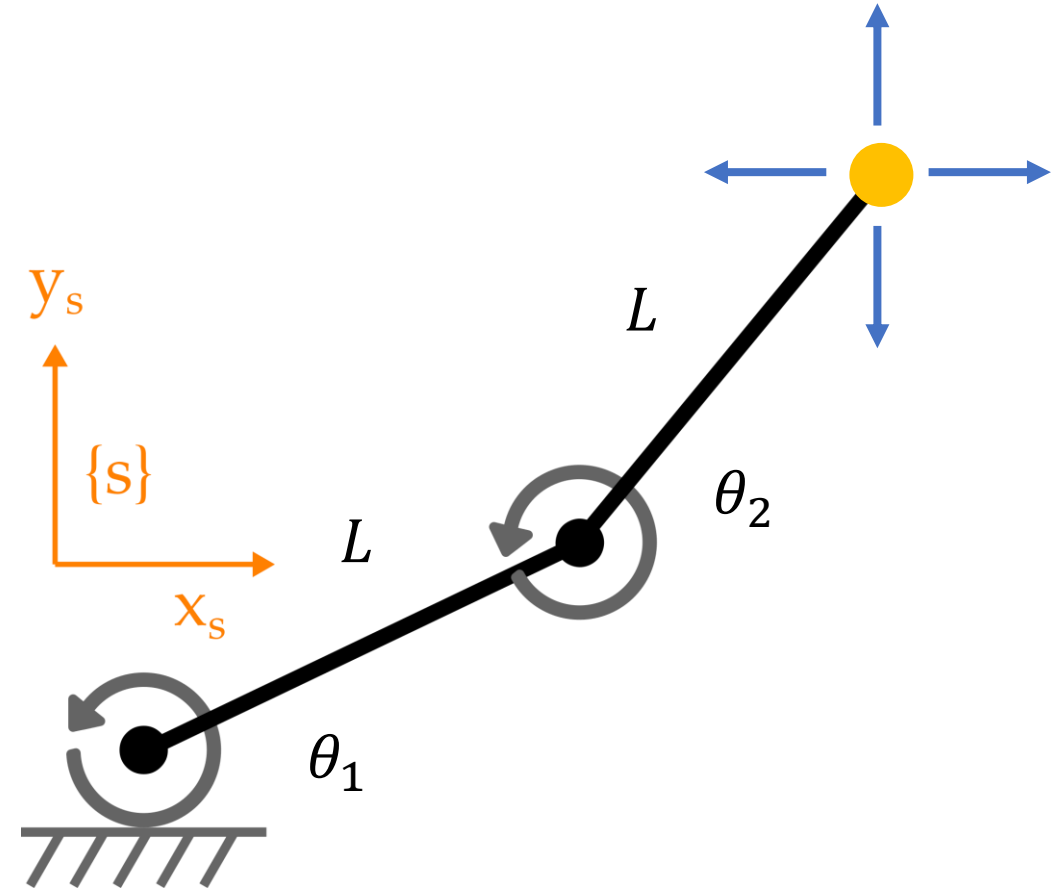
Let's restrict the Jacobian to rows that contribute motion in x and y (i.e., the robot's *task space*)



Here the geometric **Jacobian** is:

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

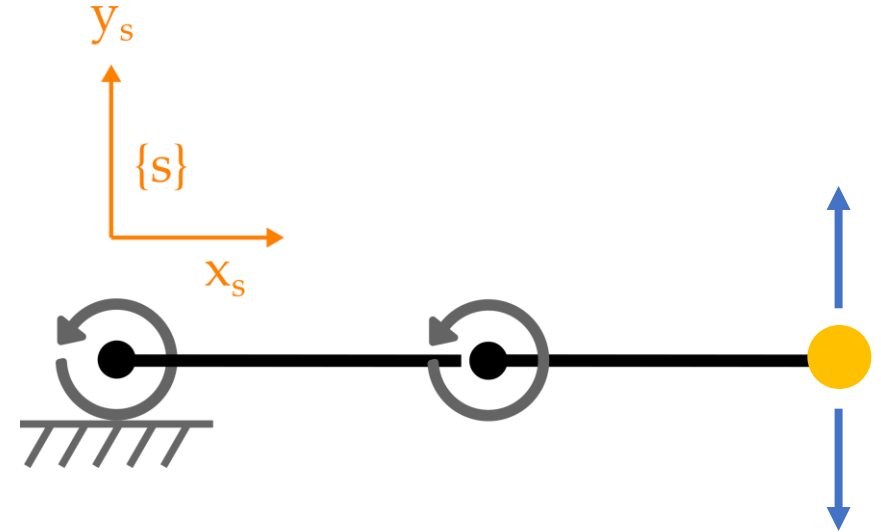
We want to be able to **move**
the end-effector **in all directions**
of our task space

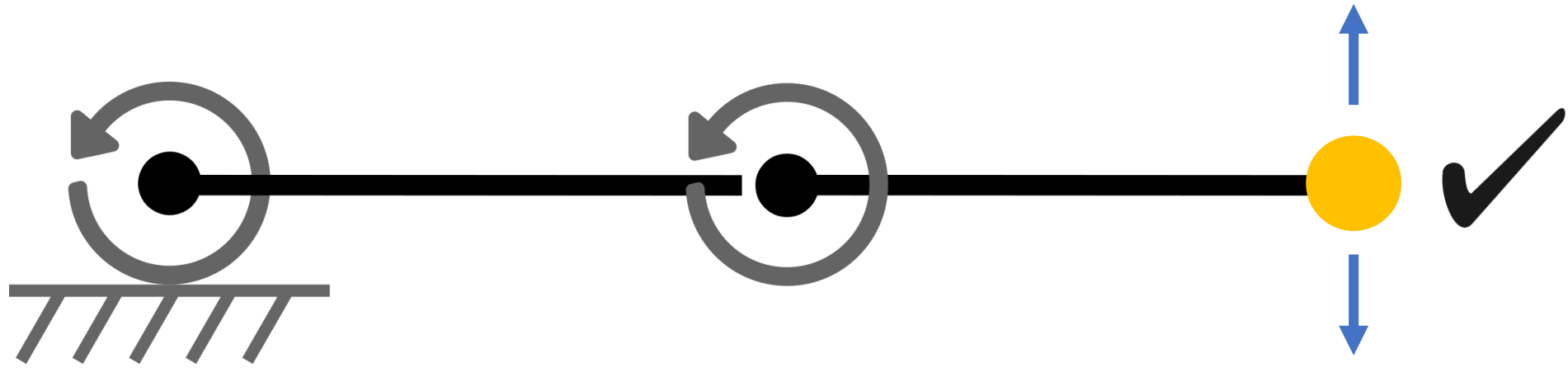


Here the geometric **Jacobian** is:

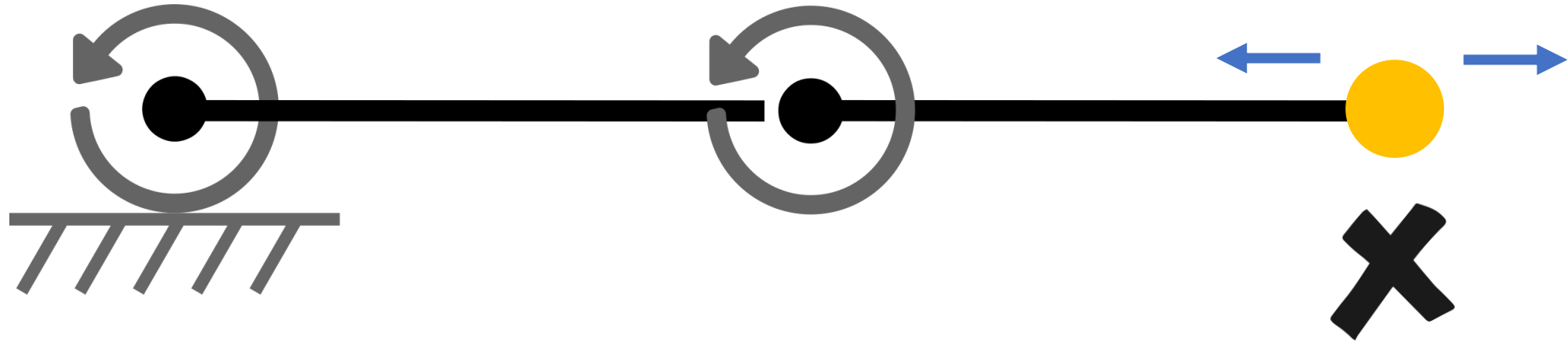
$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2L & L \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

But when $\theta = 0$, we **can't**
instantaneously move in x





When the arm is fully extended,
we easily move *up-down*...



...but can't immediately move *left-right*

A man with dark hair and glasses, wearing a grey cable-knit sweater, is leaning over a white robotic arm. He is holding a thin rod or wire with both hands, appearing to be adjusting or working on the base of the arm. The robotic arm is white with black joints and a black gripper. In the background, there is a cardboard box and some other equipment on a table. The overall scene is dimly lit, with a soft light source from the left.

This is an example
of a **singularity**

Singularity

A **singularity** is a joint position θ where the robot loses the ability to instantaneously move or rotate in one or more directions.

- Let n be the dimensions of the robot's task space.
- Let J be Jacobian, and let \mathbf{J} be the rows of the Jacobian associated with the task space
- Robot is in a singularity if the **rank of $\mathbf{J}(\theta)$** drops below n

Singularity

Singularities **do not depend** on the type of Jacobian:

$$\text{rank } \mathbf{J}_s(\theta) = \text{rank } \mathbf{J}_b(\theta) = \text{rank } \mathbf{J}(\theta)$$

Singularities are a result of kinematics,
not how we represent the Jacobian

Singularity

Use determinant to check if the **rank of $J(\theta)$** drops:

$$\det(J) = 0$$

If J is a square matrix

$$\det(JJ^T) = 0$$

If the robot is redundant
and J has more columns than rows

A man with dark hair and glasses, wearing a grey cable-knit sweater, is leaning over a white robotic arm. He is holding a thin rod or wire with both hands, appearing to be adjusting or working on the arm's base. The robotic arm is white with black accents and is positioned vertically. In the background, there is a cardboard box and some other equipment on a table. The overall scene is dimly lit, with a soft light source from the left.

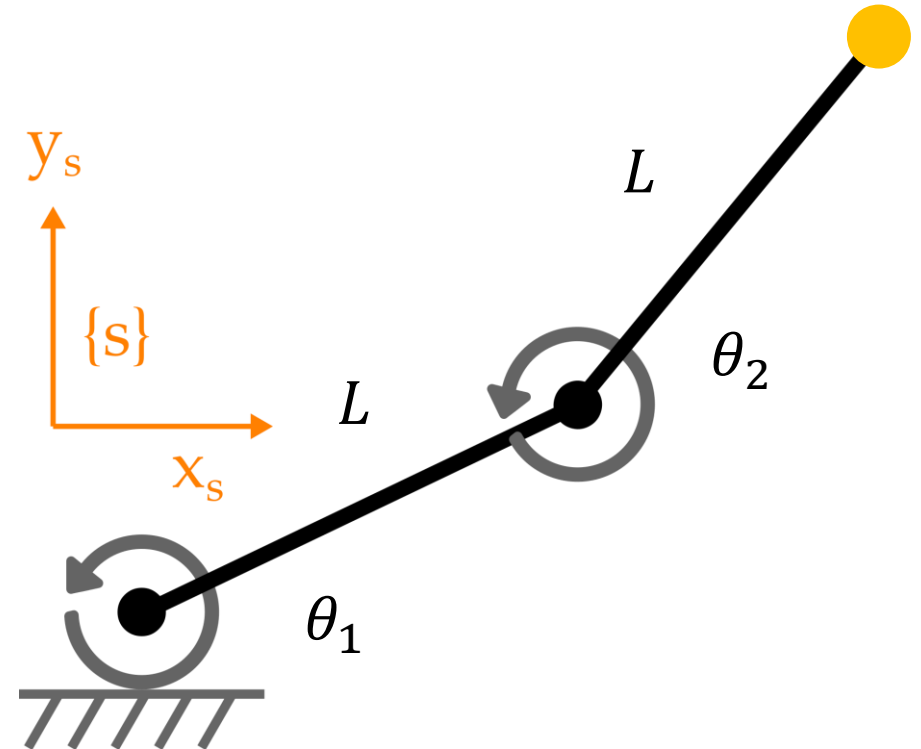
How do we find
singularities?

Singularity

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Task space

Jacobian J for task space

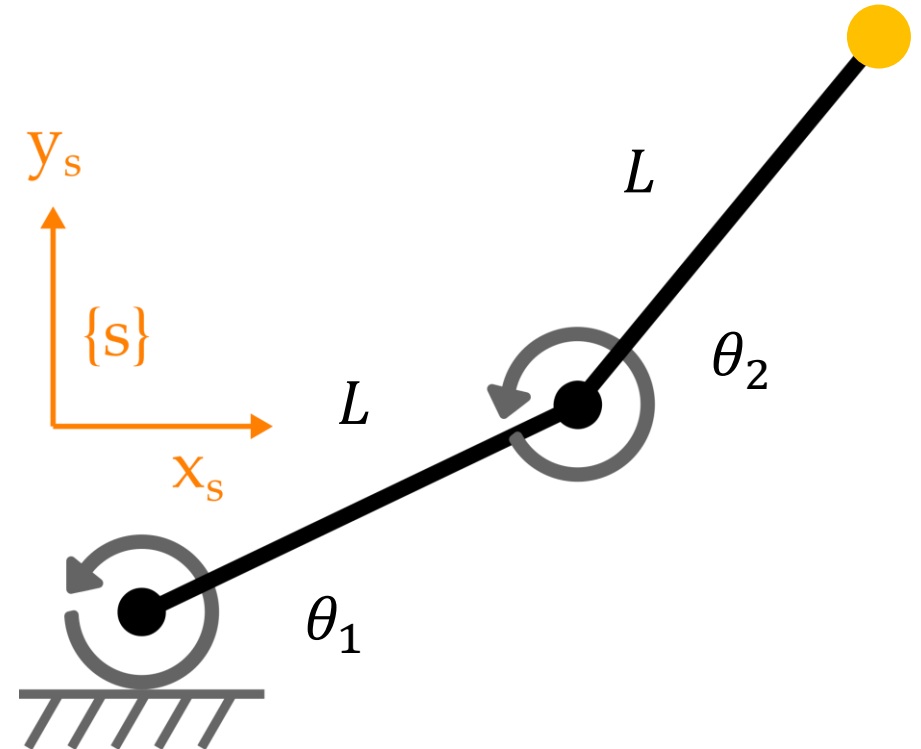


Singularity

$$J = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix}$$

$$\det(J) = L^2 \sin \theta_2$$

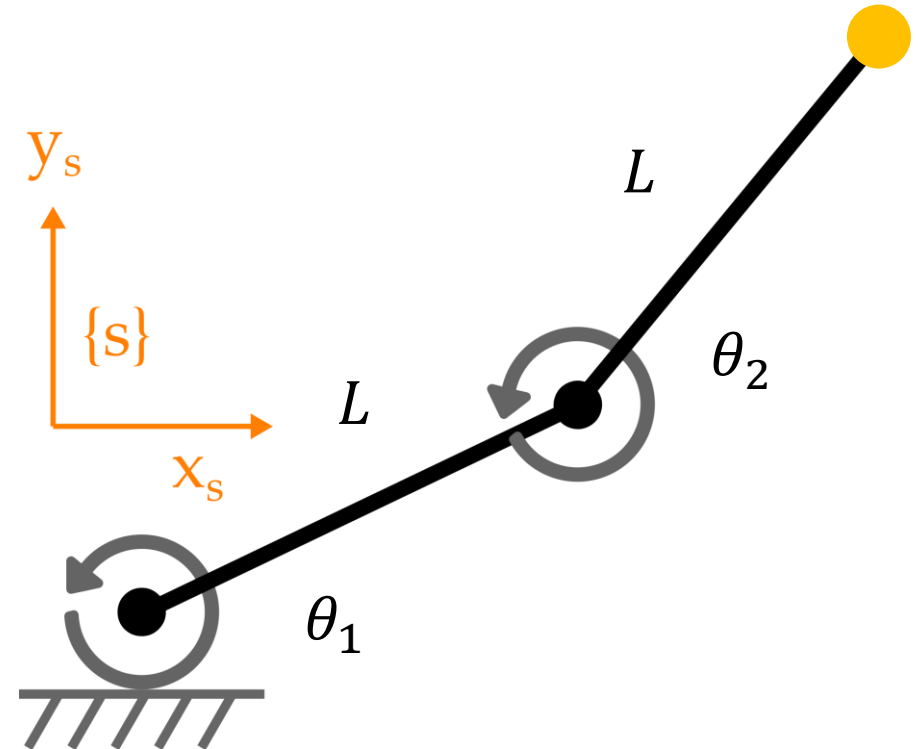
Solve for joint positions
where $\det(J) = 0$



Singularity

$$J = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix}$$

Singularity (Jacobian drops rank)
when $\theta_2 = 0$ or $\theta_2 = \pi$



A man with dark hair and glasses, wearing a grey cable-knit sweater, is leaning over a white robotic arm. He is holding a thin rod or wire with both hands, appearing to be adjusting or working on the base of the arm. The robotic arm is white with black joints and a black gripper. In the background, there are cardboard boxes and a small white device on a table. The scene is set in a workshop or laboratory environment.

In **what direction(s)**
can the robot move?

Singularity

$$\text{range } J + \text{null } J^T = \mathbb{R}^n$$

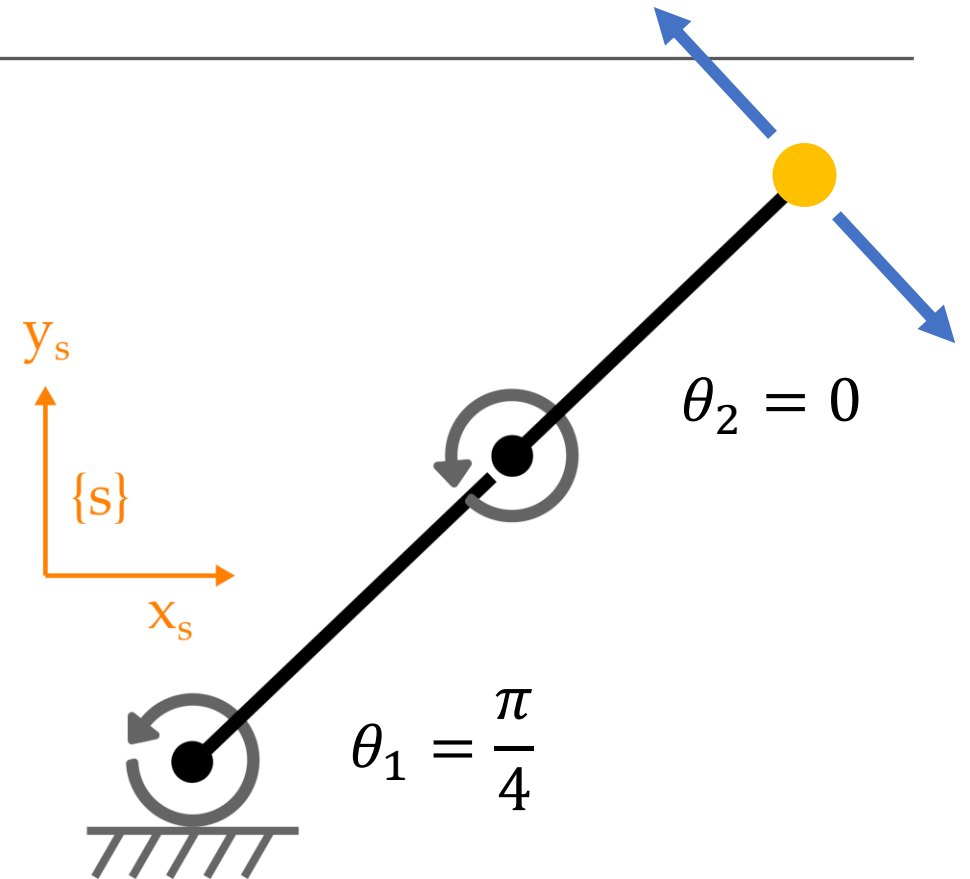
- \mathbb{R}^n are all velocities the task space
- We **can** achieve any velocity in $\text{range}(J)$
- We **cannot** achieve any velocity in $\text{null}(J^T)$

Singularity

$$J = \begin{bmatrix} -L\sqrt{2} & -L\sqrt{2}/2 \\ L\sqrt{2} & L\sqrt{2}/2 \end{bmatrix}$$

$$\text{range } J = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Use **colspace** or **orth** in Matlab

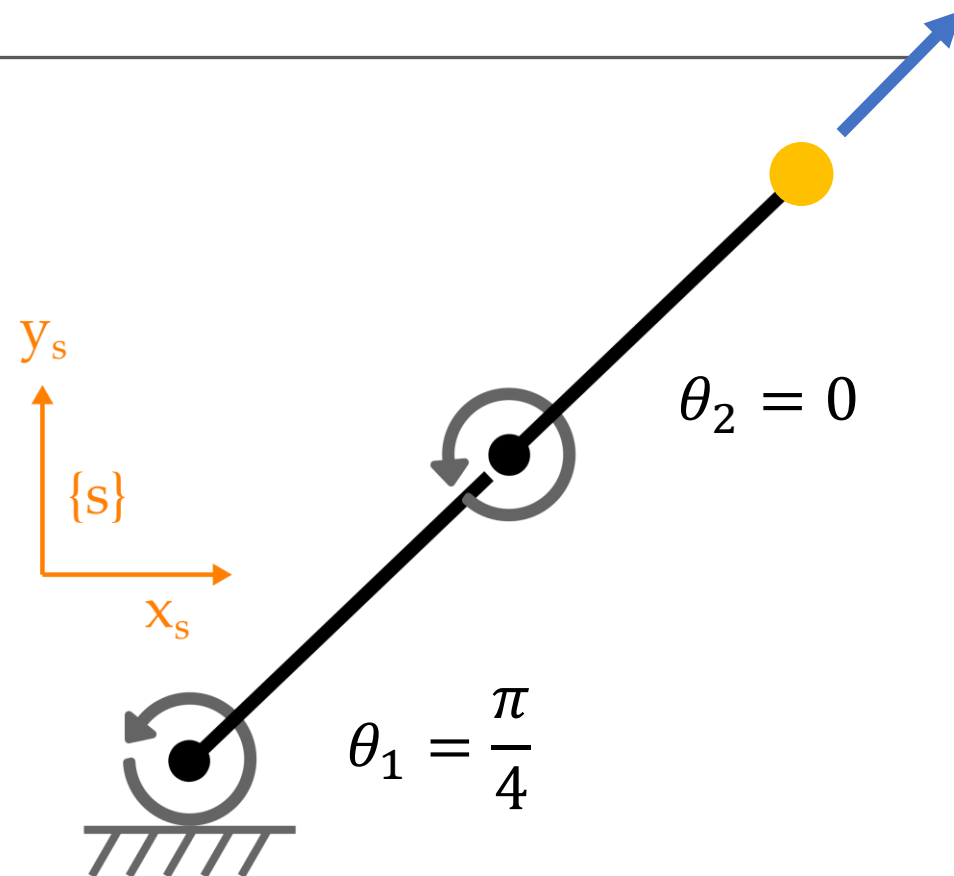


Singularity

$$J = \begin{bmatrix} -L\sqrt{2} & -L\sqrt{2}/2 \\ L\sqrt{2} & L\sqrt{2}/2 \end{bmatrix}$$

$$\text{null } J^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Use **null** in Matlab, 'r' argument



A man with dark hair and glasses, wearing a grey cable-knit sweater, is focused on adjusting a white robotic arm. He is holding a thin metal rod or wire with both hands. The robotic arm is white with black joints and a black gripper. In the background, there are cardboard boxes and a small white device on a wooden table. The scene is dimly lit, with a soft light source from the left.

How do we know we're
close to singularity?

Manipulability

Manipulability measures how easy it is for the end-effector to move in different directions

$$\sqrt{\det(\mathbf{J}\mathbf{J}^T)}$$

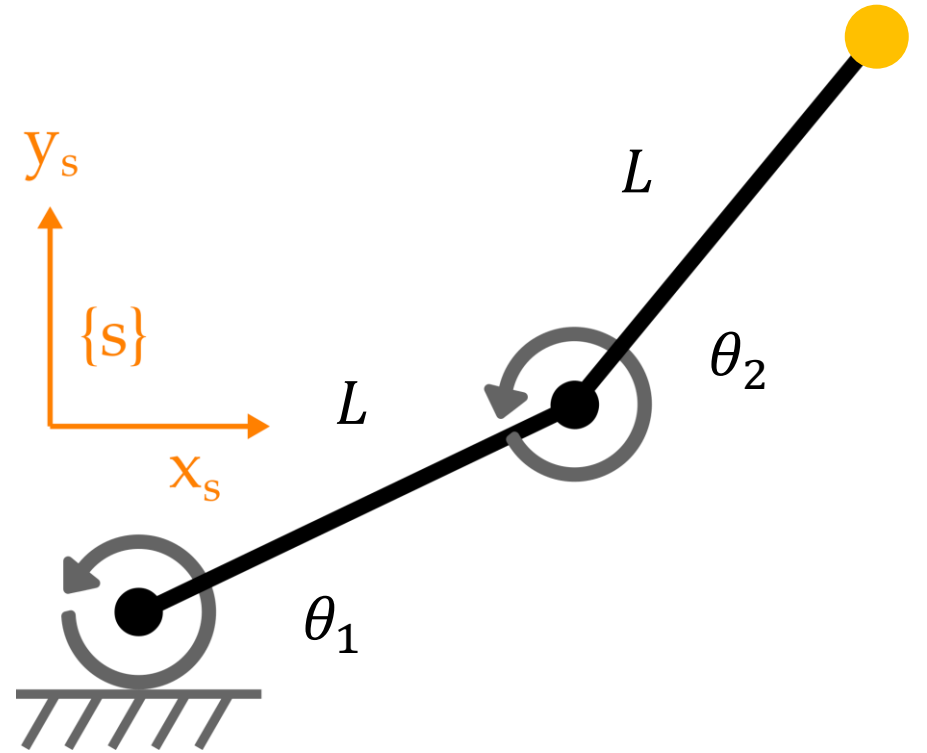
- When $\det \mathbf{J}\mathbf{J}^T \rightarrow 0$ we are approaching a singularity
- When $\det \mathbf{J}\mathbf{J}^T \rightarrow \infty$ we are far from singularities

Manipulability

$$J(\theta) = \begin{bmatrix} -Ls_1 - Ls_{12} & -Ls_{12} \\ Lc_1 + Lc_{12} & Lc_{12} \end{bmatrix}$$

$$\sqrt{\det(JJ^T)} = L^2 |\sin(\theta_2)|$$

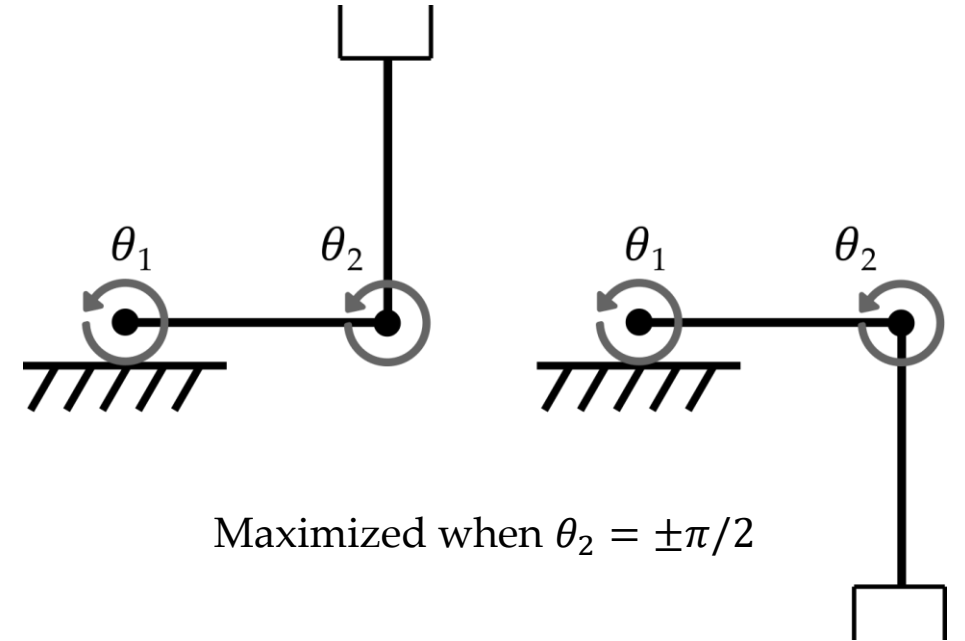
What joint positions
maximize manipulability?



Manipulability

$$J(\theta) = \begin{bmatrix} -Ls_1 - Ls_{12} & -Ls_{12} \\ Lc_1 + Lc_{12} & Lc_{12} \end{bmatrix}$$

$$\sqrt{\det(JJ^T)} = L^2 |\sin(\theta_2)|$$



This Lecture



- What is a singularity?
- What directions can robots move in singularities?
- When are we close to or far from singularities?

Next Lecture



- If we know where we want my robot to go, what joint positions get us there?