Practice Set 9

Robotics & Automation
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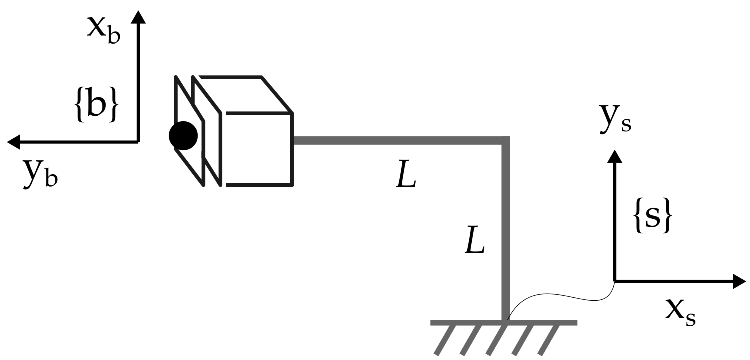
Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1



What is the screw S for the prismatic joint shown above?

Problem 2

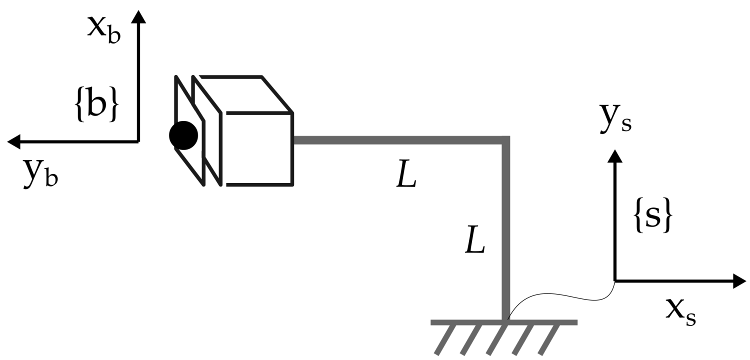
Write code that inputs screw S and joint position θ, and outputs transformation matrix:

$$T = e^{[S]\theta} \tag{1}$$

Problem 3

Using your code, see if you can find T_{sb} for the robot in Problem 1 if the prismatic joint translates by $\theta = +L/2$ units. Does you answer make sense?

Problem 1



What is the screw S for the prismatic joint shown above?

The prismatic joint is translating along the $-x_s$ axis. The screw is:

$$S = \begin{bmatrix} 0 \\ v_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

Problem 2

Write code that inputs screw S and joint position θ, and outputs transformation matrix:

$$T = e^{[S]\theta} \tag{3}$$

```
function T = screw2matrix(S, theta)
    T = expm(bracket(S) * theta);

    function bracket_S = bracket(S)
        bracket_S = [0 -S(3) S(2) S(4);
            S(3) 0 -S(1) S(5);
            -S(2) S(1) 0 S(6); 0 0 0 0];
    end
end
```

Notice the similarities between this code and what we wrote to convert an axis and angle to a rotation matrix.

Problem 3

Using your code, see if you can find T_{sb} for the robot in Problem 1 if the prismatic joint translates by $\theta = +L/2$ units. Does you answer make sense?

Start with the equation we found in lecture by solving the linear differential equation:

$$T_{sb}(\theta) = e^{[S]\theta} T_{sb}(0) \tag{4}$$

We can get $T_{sb}(0)$ by looking at the robot. We want the position and orientation of $\{b\}$ from the perspective of $\{s\}$, which is:

$$T_{sb}(0) = \begin{bmatrix} 0 & -1 & 0 & -L \\ 1 & 0 & 0 & L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Now we just leverage our code while leaving L as a symbolic variable. As a reminder, this is `syms L real` in Matlab. Multiplying out:

$$T_{sb}(L/2) = \begin{bmatrix} 0 & -1 & 0 & -3L/2 \\ 1 & 0 & 0 & L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

This answer is intuitive. The prismatic joint has moved $L/2$ units to the left, and thus $\{b\}$ is $L/2$ farther along the $-x_s$ axis. There is no change in rotation because the prismatic joint only translates.