Problem Set 3

Robotics & Automation
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Instructions. Please write legibly and do not attempt to fit your work into the smallest space possible. It is important to show all work, but basic arithmetic can be omitted. You are encouraged to use Matlab when possible to avoid hand calculations, but print and submit your commented code for non-trivial calculations. You can attach a pdf of your code to the homework, use live scripts or the publish feature in Matlab, or include a snapshot of your code. Do not submit .m files — we will not open or grade these files.

1 Implementing Forward Kinematics

1.1 (25 points)

Write a general forward kinematics function in Matlab: `T = fk(M, S, theta)`

- $M = T(0)$ is the forward kinematics when the robot is in its home position
- $S = [S_1, \ldots, S_n]$ is a matrix composed of n screw axes. The i-th column is S_i, the screw axis for the i-th joint
- $\theta = [\theta_1, \ldots, \theta_n]$ a vector of n joint values
- T is the transformation matrix from the fixed frame to the body frame

You can double check your code by testing it with our in-class examples.

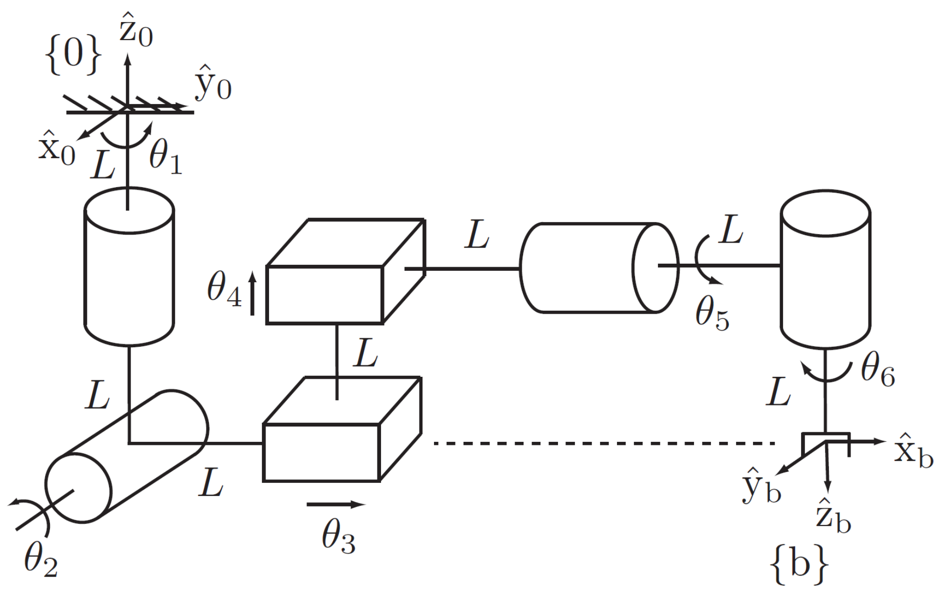
```
function T = fk(M, S, theta)

    T = eye(4);
    for idx = 1:length(theta)
        Si = S(:, idx);
        T = T * expm(bracket(Si) * theta(idx));
    end
    T = T * M;

    function S_matrix = bracket(S)
        S_matrix = [0 -S(3) S(2) S(4);
            S(3) 0 -S(1) S(5);
            -S(2) S(1) 0 S(6); 0 0 0 0];
    end
end
```

See the code above.

2 Finding Forward Kinematics



In this problem you will get the forward kinematics of the robot shown above. Frame $\{0\}$ is our fixed frame and we want the pose of frame $\{b\}$.

2.1 (20 points)

Find the home transformation matrix $M = T_{sb}(0)$ and write the screw axes $S_1, \ldots, S_6$.

We obtain M by writing the pose of $\{b\}$ relative to $\{0\}$ when $\theta = 0$.

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3L \\ 0 & 0 & -1 & -2L \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Next we need the screw axes $S_1, \ldots, S_6$. Remember to use the left equation for revolute joints and the right equation for prismatic joints:

$$S = \begin{bmatrix} \omega \\ -\omega \times q \end{bmatrix} \qquad S = \begin{bmatrix} 0 \\ v \end{bmatrix} \tag{2}$$

where ω is a unit vector in the direction of the joint axis, and q is any point along that axis, and v is a unit vector in the direction of positive translation.

$$\omega_1 = [0, 0, 1]^T, \quad q_1 = [0, 0, 0]^T, \quad S_1 = [0, 0, 1, 0, 0, 0]^T \tag{3}$$

$$\omega_2 = [1, 0, 0]^T, \quad q_2 = [0, 0, -2T]^T, \quad S_2 = [1, 0, 0, 0, -2L, 0]^T \tag{4}$$

$$v_3 = [0, 1, 0]^T, \quad S_3 = [0, 0, 0, 0, 1, 0]^T \tag{5}$$

$$v_4 = [0, 0, 1]^T, \quad S_4 = [0, 0, 0, 0, 0, 1]^T \tag{6}$$

$$\omega_5 = [0, 1, 0]^T, \quad q_5 = [0, 0, -L]^T, \quad S_5 = [0, 1, 0, L, 0, 0]^T \tag{7}$$

$$\omega_6 = [0, 0, -1]^T, \quad q_6 = [0, 3L, 0]^T, \quad S_6 = [0, 0, -1, -3L, 0, 0]^T \tag{8}$$

2.2 (5 points)

Use your forward kinematics function from Problem 1 to find T_{0b} when:

Case 1: $L = 1$, $\theta = [\pi/2, 0, 2, -0.5, 2\pi/3, -\pi/4]^T$

Case 2: $L = 2$, $\theta = [-\pi/2, -\pi/4, 0, 1, \pi/3, \pi/2]^T$

Your answers should be two transformation matrices with numerical values.

Plug the answers from Problem 2.1 into the Matlab function `fk`, and use the given values for θ. The end-effector's pose is:

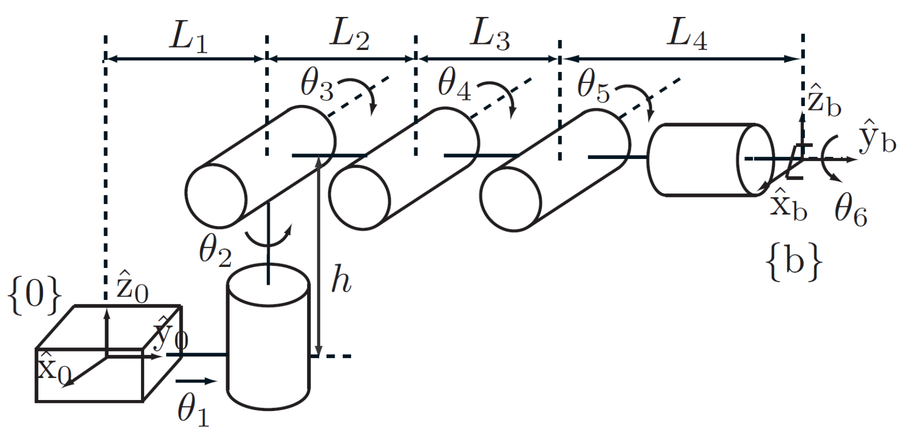
Case 1: $L = 1$, $\theta = [\pi/2, 0, 2, -0.5, 2\pi/3, -\pi/4]^T$

$$T_{0b} = \begin{bmatrix} -0.7071 & -0.7071 & 0 & -5 \\ 0.3536 & -0.3536 & -0.866 & -0.866 \\ 0.6124 & -0.6124 & 0.5 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Case 2: $L = 2$, $\theta = [-\pi/2, -\pi/4, 0, 1, \pi/3, \pi/2]^T$

$$T_{0b} = \begin{bmatrix} -0.6124 & -0.7071 & -0.3536 & 5.6569 \\ -0.5 & 0 & 0.866 & 1.7321 \\ -0.6124 & 0.7071 & -0.3536 & -6.8284 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

3 Finding More Forward Kinematics



In this problem you will get the forward kinematics of the robot shown above. Frame $\{0\}$ is our fixed frame and we want the pose of frame $\{b\}$.

3.1 (20 points)

Find the home transformation matrix $M = T_{sb}(0)$ and write the screw axes $S_1, \dots, S_6$.

We obtain M by writing the pose of $\{b\}$ relative to $\{0\}$ when $\theta = 0$.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1 + L_2 + L_3 + L_4 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

Next we find the screw axes. Using the equations for prismatic and revolute joints:

$$S_1 = [0, 0, 0, 0, 1, 0]^T \tag{12}$$

$$S_2 = [0, 0, 1, L_1, 0, 0]^T \tag{13}$$

$$S_3 = [-1, 0, 0, 0, -h, L_1]^T \tag{14}$$

$$S_4 = [-1, 0, 0, 0, -h, L_1 + L_2]^T \tag{15}$$

$$S_5 = [-1, 0, 0, 0, -h, L_1 + L_2 + L_3]^T \tag{16}$$

$$S_6 = [0, 1, 0, -h, 0, 0]^T \tag{17}$$

3.2 (5 points)

Use your forward kinematics function from Problem 1 to find T_{0b} when:

- $h = 2, \quad L_1 = L_2 = L_3 = L_4 = 1, \quad \theta = [1, \pi/4, -\pi/4, 0, \pi/4, \pi/2]^T$

Your answer should be a transformation matrix with numerical values.

Plug the answers from Problem 3.1 into the Matlab function `fk`, and use the given values for θ. The end-effector's pose is:

$$T_{0b} = \begin{bmatrix} 0 & -0.7071 & 0.7071 & -1.7071 \\ 0 & 0.7071 & 0.7071 & 3.7071 \\ -1 & 0 & 0 & 3.4142 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

4 Arbitrary Point of Interest

Although we generally use forward kinematics to get the pose of the end-effector, sometimes there are other points we are interested in. For instance, perhaps we want to check that the elbow of our robot arm is not going to collide with an object. Here you will get the forward kinematics of the marked frame $\{b\}$ with respect to fixed frame $\{0\}$.

4.1 (5 points)

If you change θ_5 and θ_6 for this robot while leaving all other joints fixed, does T_{0b} change?

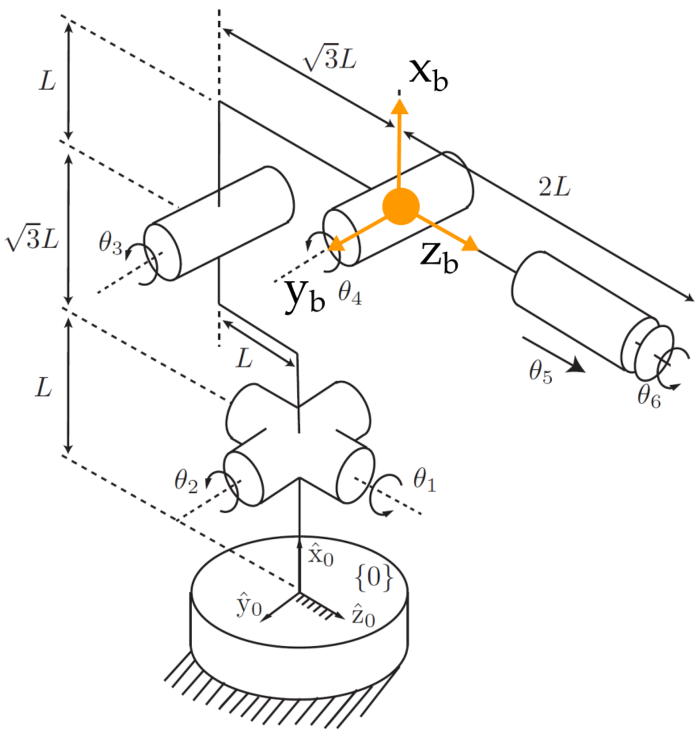
No. For any serial robot arm, joints after the point of interest do not affect its pose.

4.2 (20 points)

Find the forward kinematics T_{0b} for the following cases:

Case 1: $L = 1, \quad \theta = [\pi/4, \pi/4, 0, \pi/2, 2, -\pi/4]^T$

Case 2: $L = 1, \quad \theta = [-\pi/2, 0, \pi/6, -\pi/3, 1, \pi/2]^T$



We obtain M by writing the pose of $\{b\}$ relative to $\{0\}$ when $\theta = 0$.

$$M = \begin{bmatrix} 1 & 0 & 0 & 2L + \sqrt{3}L \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L + \sqrt{3}L \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

Next we need the screw axes *that affect the pose of* $\{b\}$. We can ignore S_5 and S_6 since these joints come after $\{b\}$. Using the equations for prismatic and revolute joints:

$$S_1 = [0, 0, 1, 0, -L, 0]^T \tag{20}$$

$$S_2 = [0, 1, 0, 0, 0, L]^T \tag{21}$$

$$S_3 = [0, 1, 0, L, 0, L + \sqrt{3}L]^T \tag{22}$$

$$S_4 = [0, 1, 0, L - \sqrt{3}L, 0, 2L + \sqrt{3}L]^T \tag{23}$$

Now we plug S and θ into the Matlab function `fk`. Because we only care about the first four joints, we can ignore θ_5 and θ_6. The pose of the point of interest is:

Case 1: $\quad L = 1, \quad \theta = [\pi/4, \pi/4, 0, \pi/2]^T$

$$T_{0b} = \begin{bmatrix} -0.5 & -0.7071 & 0.5 & 2.7321 \\ -0.5 & 0.7071 & 0.5 & 1.7321 \\ -0.7071 & 0 & -0.7071 & -1.4142 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Case 2: $\quad L = 1, \quad \theta = [-\pi/2, 0, \pi/6, -\pi/3]^T$

$$T_{0b} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -0.866 & 0 & 0.5 & -3.4641 \\ 0.5 & 0 & 0.866 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$