Practice Set 2

Robotics & Automation
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Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

Is matrix X a rotation matrix? Prove why or why not.

$$X = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \tag{1}$$

Problem 2

Is matrix X a rotation matrix? Prove why or why not.

$$X = \begin{bmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Problem 3

Let R_1 and R_2 be two rotation matrices. Prove that $R_3 = R_1 R_2$ is also a rotation matrix. As a hint: $\det(AB) = \det(A) \cdot \det(B)$

Problem 4

Let R_1 and R_2 be two rotation matrices. The commutative property states that $R_1 R_2 = R_2 R_1$. Are rotation matrices commutative? Try finding a counterexample.

Problem 1

Is matrix X a rotation matrix? Prove why or why not.

$$X = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \tag{3}$$

For X to be a rotation matrix we must have $R^T R = I$ and $\det(R) = +1$. This means that each column (and row) must be a unit vector. But we can easily see that the third column is *not* a unit vector. Hence X is **not** a rotation matrix.

Problem 2

Is matrix X a rotation matrix? Prove why or why not.

$$X = \begin{bmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

For X to be a rotation matrix we must have $R^T R = I$ and $\det(R) = +1$. Here we satisfy both properties, and so X **is** a rotation matrix.

Problem 3

Let R_1 and R_2 be two rotation matrices. Prove that $R_3 = R_1 R_2$ is also a rotation matrix. As a hint: $\det(AB) = \det(A) \cdot \det(B)$

Let's start by checking if R_3 is an orthonormal matrix:

$$R_3^T R_3 = (R_1 R_2)^T (R_1 R_2) = R_2^T R_1^T R_1 R_2 \tag{5}$$

But we know that $R_1^T R_1 = I$, so this simplifies to:

$$R_3^T R_3 = R_2^T R_2 \tag{6}$$

Since R_2 is a rotation matrix we also have that $R_2^T R_2 = I$.

$$R_3^T R_3 = I \tag{7}$$

Next we need to confirm that $\det(R_3) = +1$. Using the hint:

$$\det(R_3) = \det(R_1) \cdot \det(R_2) = 1 \cdot 1 = 1 \tag{8}$$

We conclude that $R_3^T R_3 = I$ and $\det(R_3) = 1$. **Whenever you multiply two rotation matrices, the result is also a rotation matrix.**

Problem 4

Let R_1 and R_2 be two rotation matrices. The commutative property states that $R_1 R_2 = R_2 R_1$. Are rotation matrices commutative? Try finding a counterexample.

Here we want two rotation matrices where $R_1 R_2 \neq R_2 R_1$. Make sure that the R_1 and R_2 you choose are actually rotation matrices! Here is one option:

$$R_1 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_1^T R_1 = I, \quad \det(R_1) = +1 \tag{9}$$

$$R_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad R_2^T R_2 = I, \quad \det(R_2) = +1 \tag{10}$$

Now that we have verified that these choices are rotation matrices, we plug into the commutative definition:

$$R_1 R_2 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}, \quad R_2 R_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{11}$$

Notice that $R_1 R_2 \neq R_2 R_1$. **In general, rotation matrices are not commutative.**