Practice Set 15

Robotics & Automation
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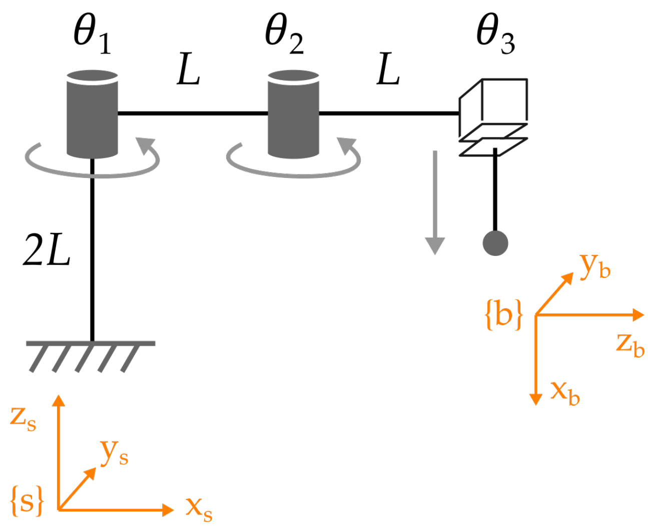
Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1



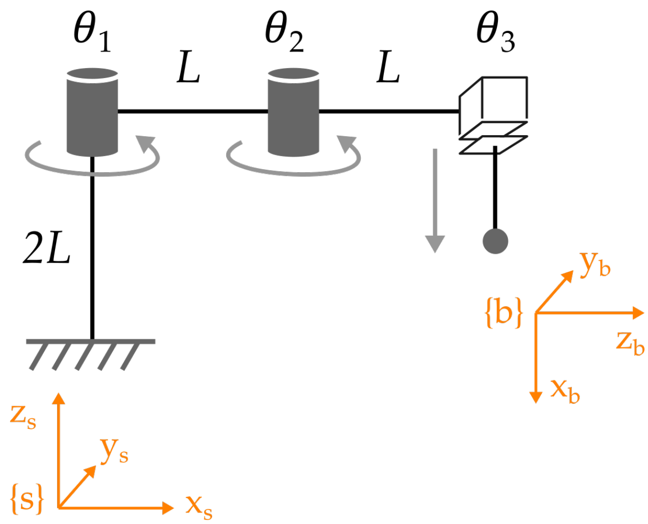
Find the geometric Jacobian for the robot shown above. Leave L and θ as symbolic variables.

Problem 2

Break down the Jacobian matrix to answer the following questions:

- Can this robot have angular velocity in the x_s axis?
- Does $\dot{\theta}_3$ contribute to the angular velocity?
- If we actuate joints one and two, can we apply a linear velocity in the z_s axis?
- Which joint has a larger effect on angular velocity — joint one or joint two?

Problem 1



Find the geometric Jacobian for the robot shown above. Leave L and θ as symbolic variables.

We found the space Jacobian for this robot in a previous lecture:

$$J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & L\sin(\theta_1) & 0 \\ 0 & -L\cos(\theta_1) & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{1}$$

Your next step is to get the robot's forward kinematics.

$$T_{sb}(\theta) = \begin{bmatrix} 0 & -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & L\cos(\theta_1) + L\cos(\theta_1+\theta_2) \\ 0 & \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & L\sin(\theta_1) + L\sin(\theta_1+\theta_2) \\ -1 & 0 & 0 & L-\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The final step is to convert the space Jacobian to the geometric Jacobian:

$$J(\theta) = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} Ad_{T^{-1}} J_s(\theta) \tag{3}$$

See the examples from lecture. You should obtain:

$$J(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ -L\sin(\theta_1) - L\sin(\theta_1+\theta_2) & -L\sin(\theta_1+\theta_2) & 0 \\ L\cos(\theta_1) + L\cos(\theta_1+\theta_2) & L\cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{4}$$

Problem 2

Break down the Jacobian matrix to answer the following questions:

For the following questions it is useful to think about the components of the Jacobian, and how they contribute to the velocity of the end-effector:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ -L\sin(\theta_1) - L\sin(\theta_1+\theta_2) & -L\sin(\theta_1+\theta_2) & 0 \\ L\cos(\theta_1) + L\cos(\theta_1+\theta_2) & L\cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{5}$$

- Can this robot have angular velocity in the x_s axis?

  No. The first row captures the angular velocity in the x_s axis, and this row is all zeros. So $\omega_x = 0$ always.

- Does $\dot{\theta}_3$ contribute to the angular velocity?

  No. The first three rows capture the angular velocity, and when we multiply these rows we find that $\omega_x = 0$, $\omega_y = 0$, $\omega_z = \dot{\theta}_1 + \dot{\theta}_2$.

- If we actuate joints one and two, can we apply a linear velocity in the z_s axis?

  No. $\dot{p}_z = -\dot{\theta}_3$. Moving the first two joints does not move the robot in z_s.

- Which joint has a larger effect on angular velocity — joint one or joint two?

  It is a tie. We have that $\omega_z = \dot{\theta}_1 + \dot{\theta}_2$. Both joints have equal effects on the angular velocity of the end-effector.