

Statics

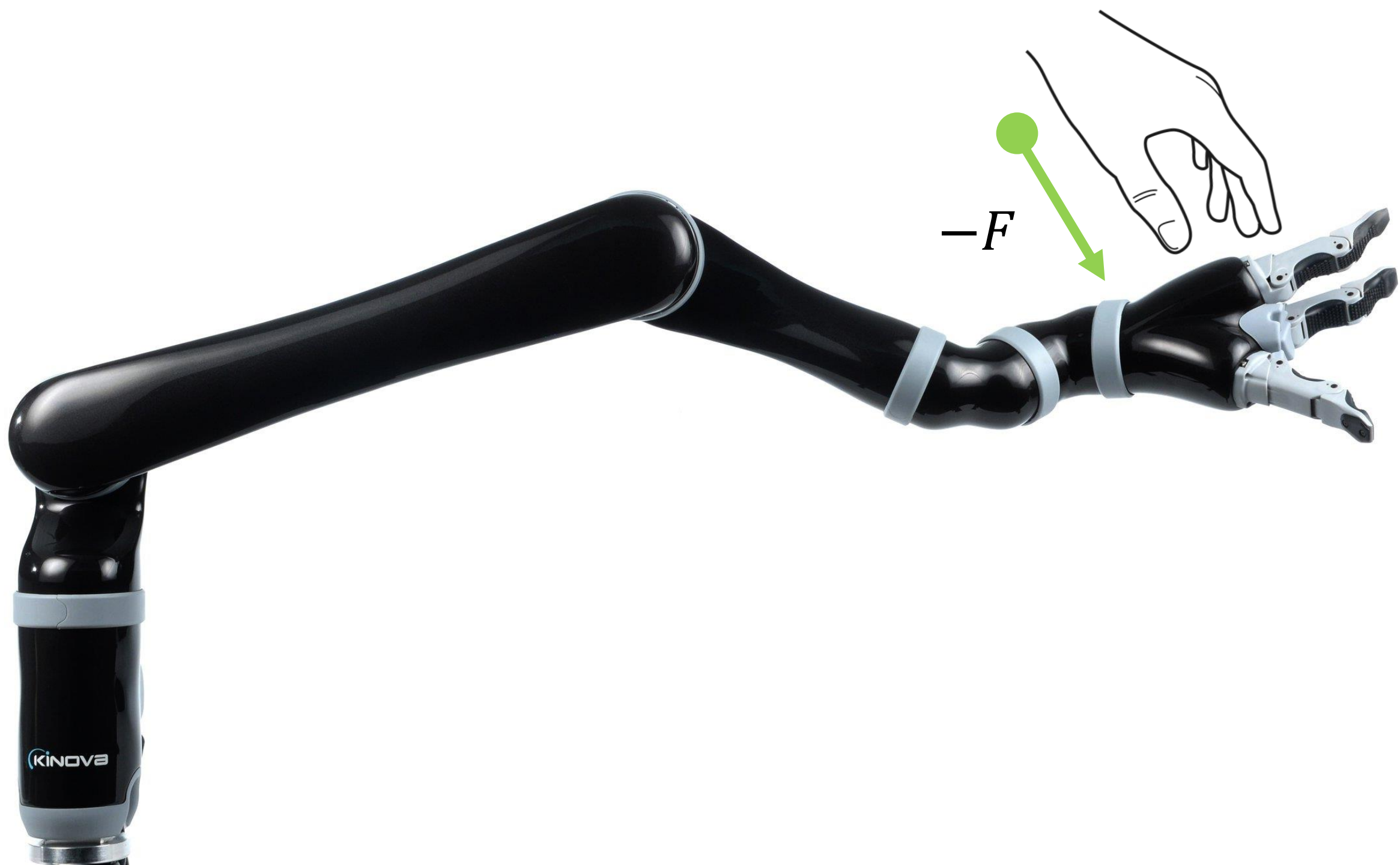
Reading: Modern Robotics 5.2

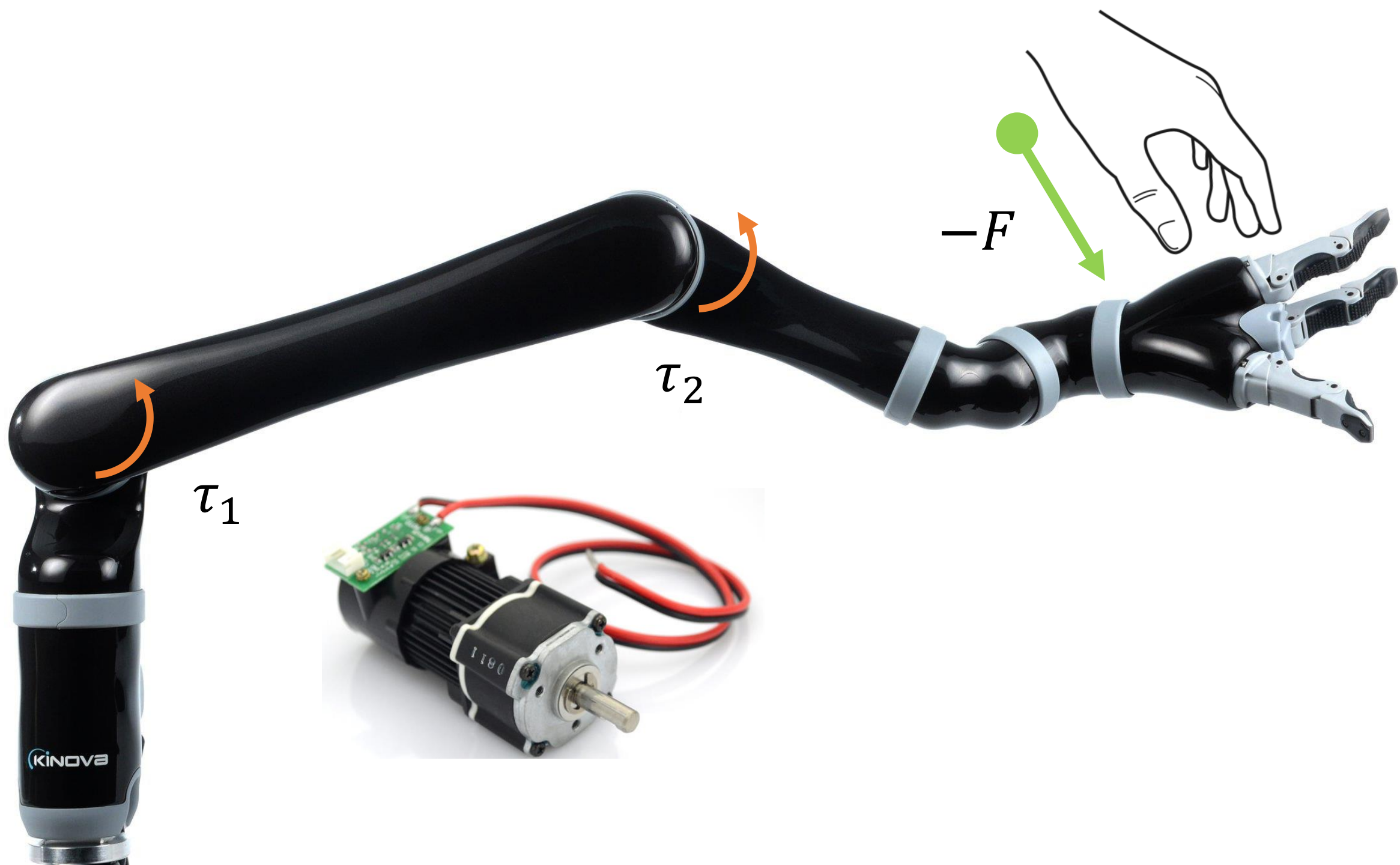


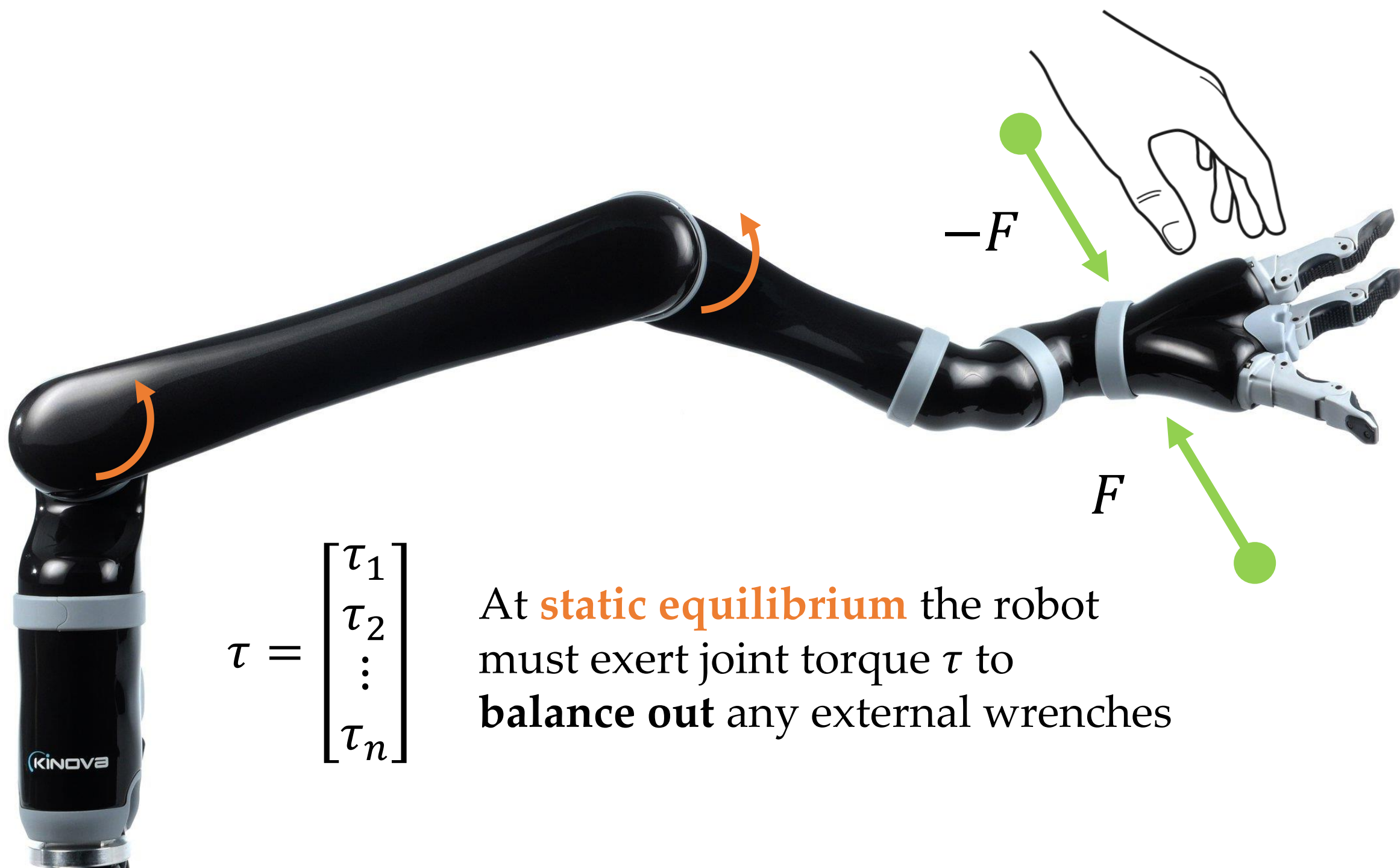
This Lecture



- What joint torques keep the robot in place?
- How do singularities affect statics?







$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$$

At **static equilibrium** the robot must exert joint torque τ to **balance out** any external wrenches

The background of the slide is a photograph of an industrial manufacturing environment. Several large, orange robotic arms, likely KUKA brand as indicated by the logo, are visible. They are positioned around a central area where a white, skeletal structure of a car body is being assembled. The scene is dimly lit, with the primary light source coming from the overhead industrial lights, creating a professional and technical atmosphere. The text is overlaid on this image, with the words 'joint torques' and 'wrenches' highlighted in yellow and orange respectively to draw attention to the key concepts of the question.

How do we convert between
joint torques and
end-effector **wrenches**?

Equilibrium

At static equilibrium, the power in the joints is equal to the power at the end-effector (taking the limit as velocity goes towards zero):

$$\dot{\theta}^T \tau = V^T F$$

Power in joint space

Power in end-effector space

Equilibrium

At static equilibrium, the power in the joints is equal to the power at the end-effector (taking the limit as velocity goes towards zero):

$$\dot{\theta}^T \tau = \underline{V^T F}$$

Use the Jacobian:

$$V = J(\theta)\dot{\theta}$$

Equilibrium

At static equilibrium, the power in the joints is equal to the power at the end-effector (taking the limit as velocity goes towards zero):

$$\dot{\theta}^T (\tau = J(\theta)^T F)$$

$$\tau = J(\theta)^T F$$

Equilibrium

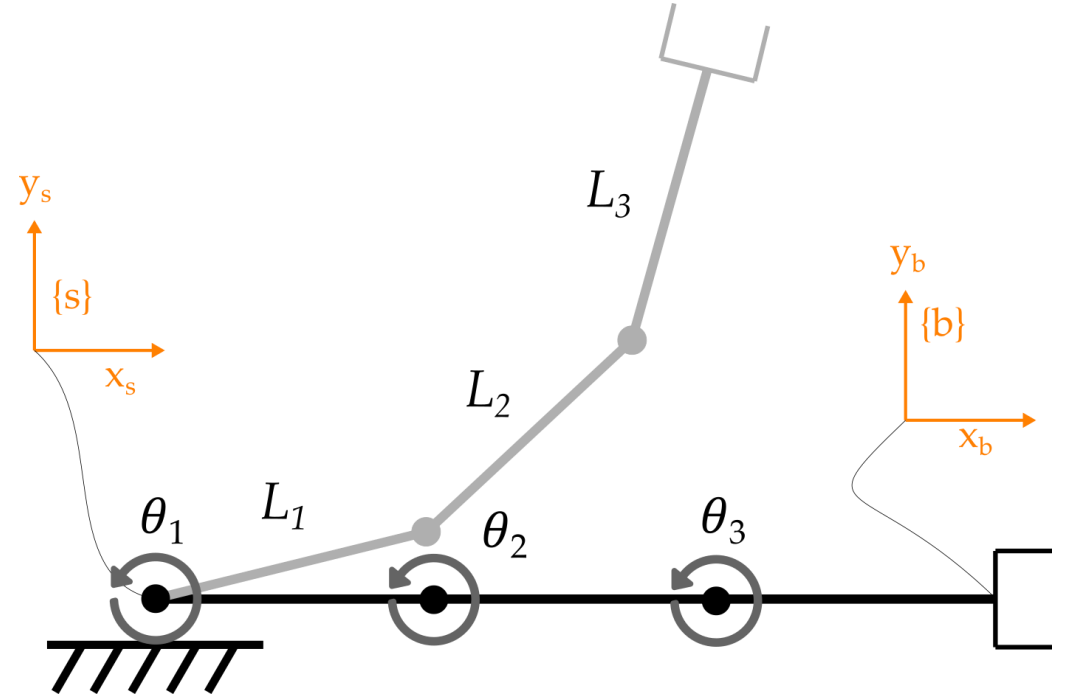
$$\tau = J(\theta)^T F$$

- Given wrench F , find the joint torques τ that apply that wrench
- Works for any Jacobian if F in **same frame** (i.e., if $F = F_s$ is in spatial frame, then use space Jacobian, and if $F = F_b$ is in body frame, then use body Jacobian)

Example

To apply F at the end-effector, what joint torque do we need?

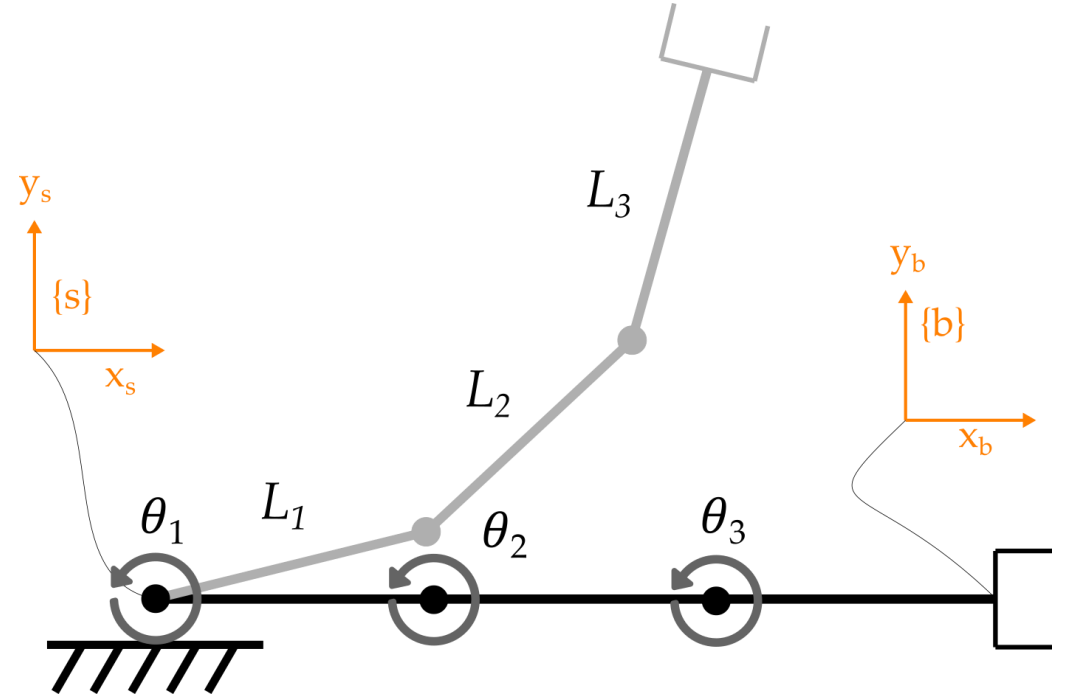
$$J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & L_1 s_1 & L_1 s_1 + L_2 s_{12} \\ 0 & -L_1 c_1 & -L_1 c_1 - L_2 c_{12} \\ 0 & 0 & 0 \end{bmatrix}$$



Example

To apply F at the end-effector, what joint torque do we need?

$$\underbrace{\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}}_{\tau} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & L_1 s_1 & L_1 s_1 + L_2 s_{12} \\ 0 & -L_1 c_1 & -L_1 c_1 - L_2 c_{12} \\ 0 & 0 & 0 \end{bmatrix}^T}_{J_s(\theta)^T} \underbrace{\begin{bmatrix} m_x \\ m_y \\ m_z \\ f_x \\ f_y \\ f_z \end{bmatrix}}_{F_s}$$

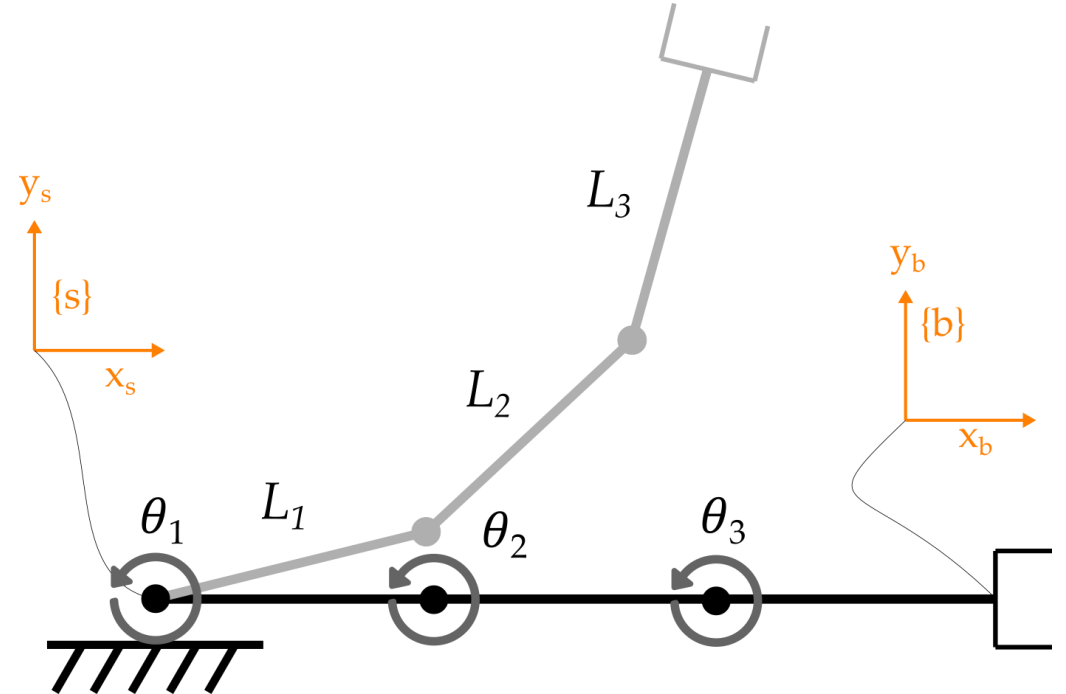


Example

To apply F at the end-effector, what joint torque do we need?

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} m_z \\ m_z + f_x L_1 s_1 - f_y L_1 c_1 \\ m_z + f_x (L_1 s_1 + L_2 s_{12}) - f_y (L_1 c_1 + L_2 c_{12}) \end{bmatrix}$$

No f_z here. No matter what f_z is applied, it does not affect the joint torque



A photograph of an automotive assembly line. Several orange robotic arms are positioned around a silver car chassis. The background shows more of the factory floor with various equipment and structures. The text "How do statics change at singularities?" is overlaid on the left side of the image. The word "singularities" is highlighted in orange, matching the color of the robots. A horizontal white line is positioned below the text.

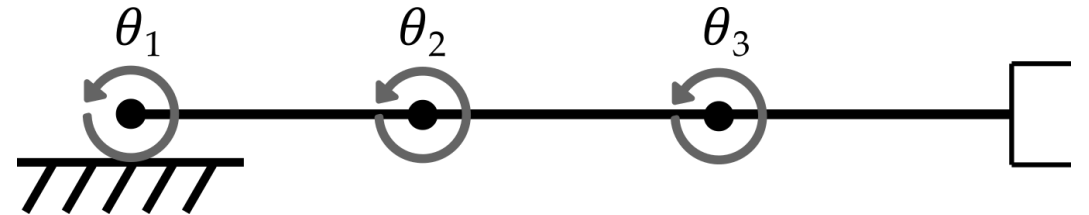
How do statics
change at
singularities?

Example

This robot is at a **singularity** when $\theta = 0$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} m_z \\ m_z + \mathbf{f}_x \mathbf{L}_1 \mathbf{s}_1 - f_y L_1 c_1 \\ m_z + \mathbf{f}_x (\mathbf{L}_1 \mathbf{s}_1 + \mathbf{L}_2 \mathbf{s}_{12}) - f_y (L_1 c_1 + L_2 c_{12}) \end{bmatrix}$$

Here $\sin(\theta_1) = \sin(\theta_1 + \theta_2) = 0$

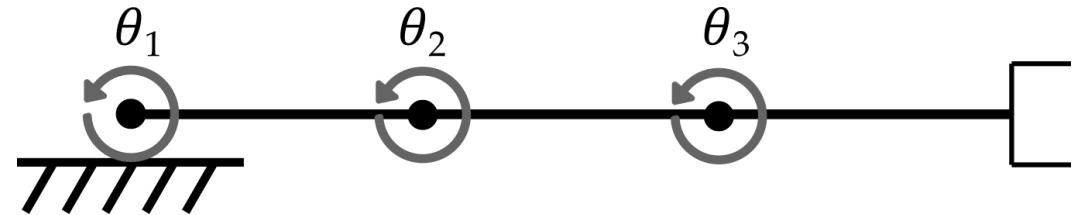


Example

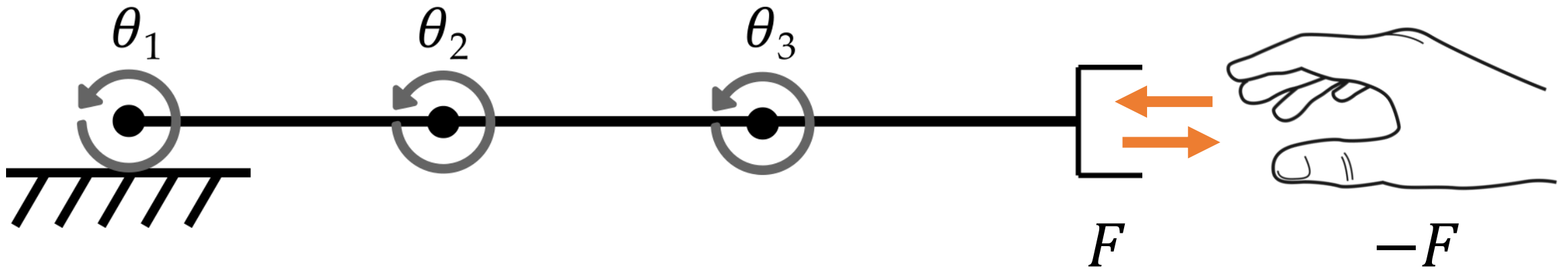
This robot is at a **singularity**
when $\theta = 0$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} m_z \\ m_z - f_y L_1 c_1 \\ m_z - f_y (L_1 c_1 + L_2 c_{12}) \end{bmatrix}$$

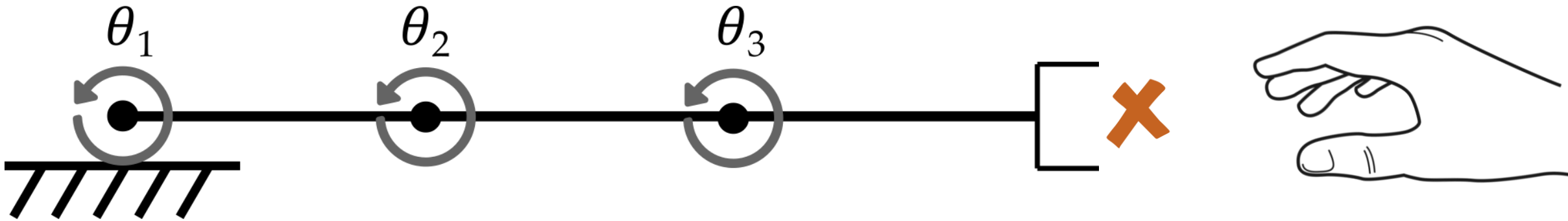
No f_x now. No matter what f_x is applied,
it does not affect the joint torque



The robot **passively resists** wrenches
in the direction of the singularity



But the robot **cannot apply** wrenches
in the direction of the singularity



Takeaways

Wrenches are like twists, but deal with **moment and force** instead of **angular and linear velocity**.

$$V = J(\theta)\dot{\theta}$$

At singularity, cannot move
along null J^T

$$\tau = J(\theta)^T F$$

At singularity, cannot control
wrenches along null J^T

This Lecture



- What joint torques keep the robot in place?
- How do singularities affect statics?

Next Lecture



- We've dealt with statics...
...what about dynamics?