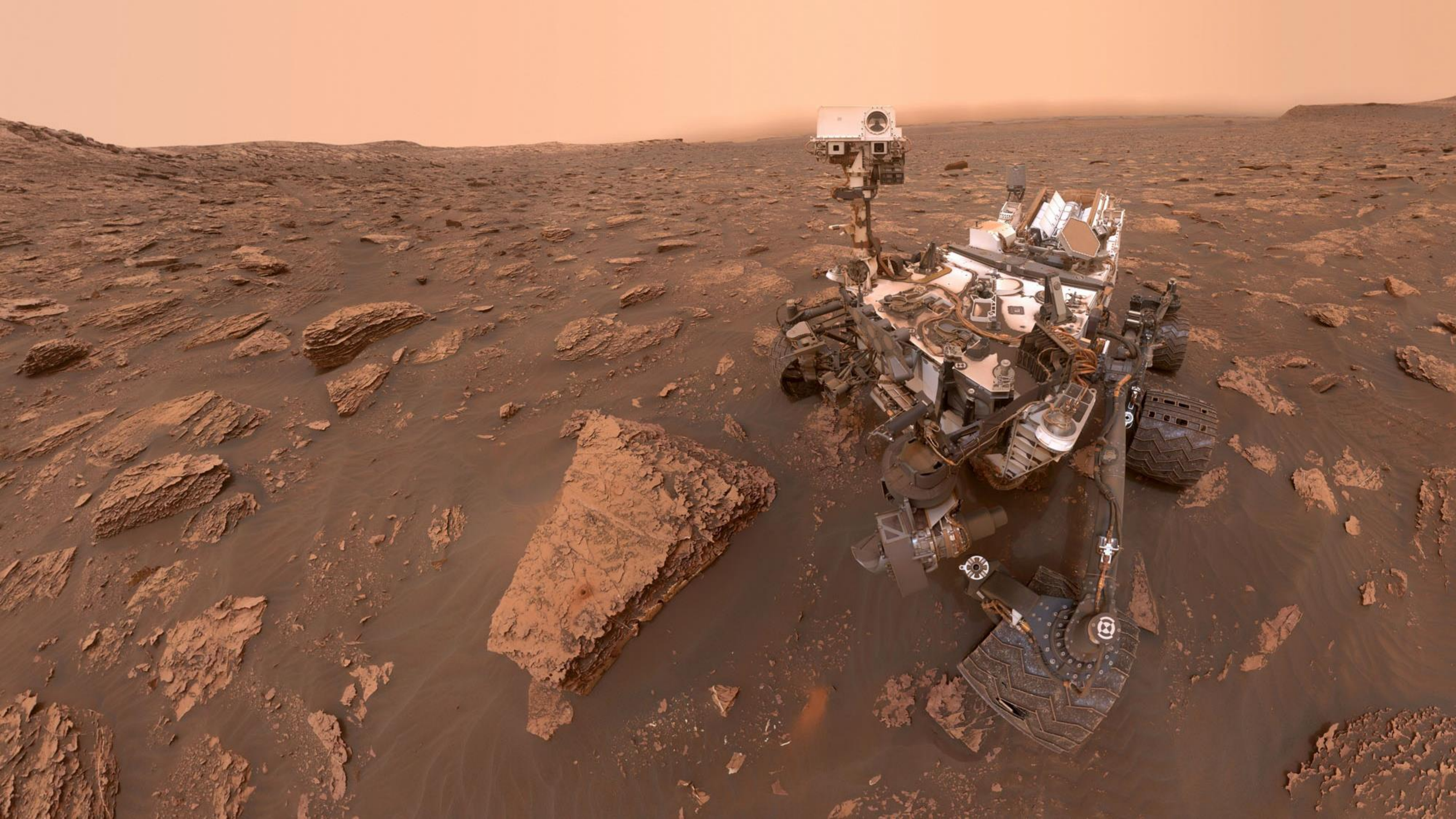
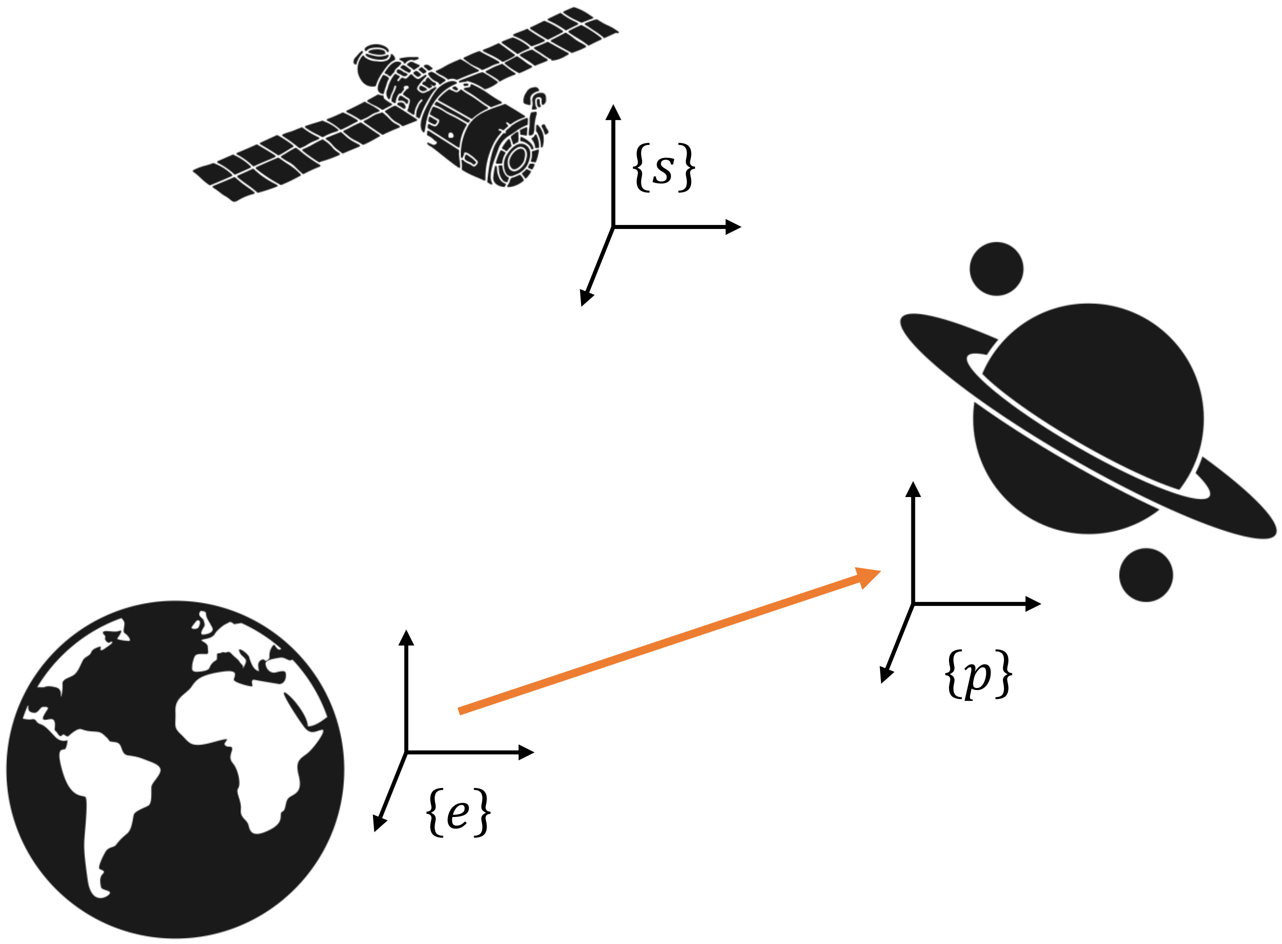
Changing Frames

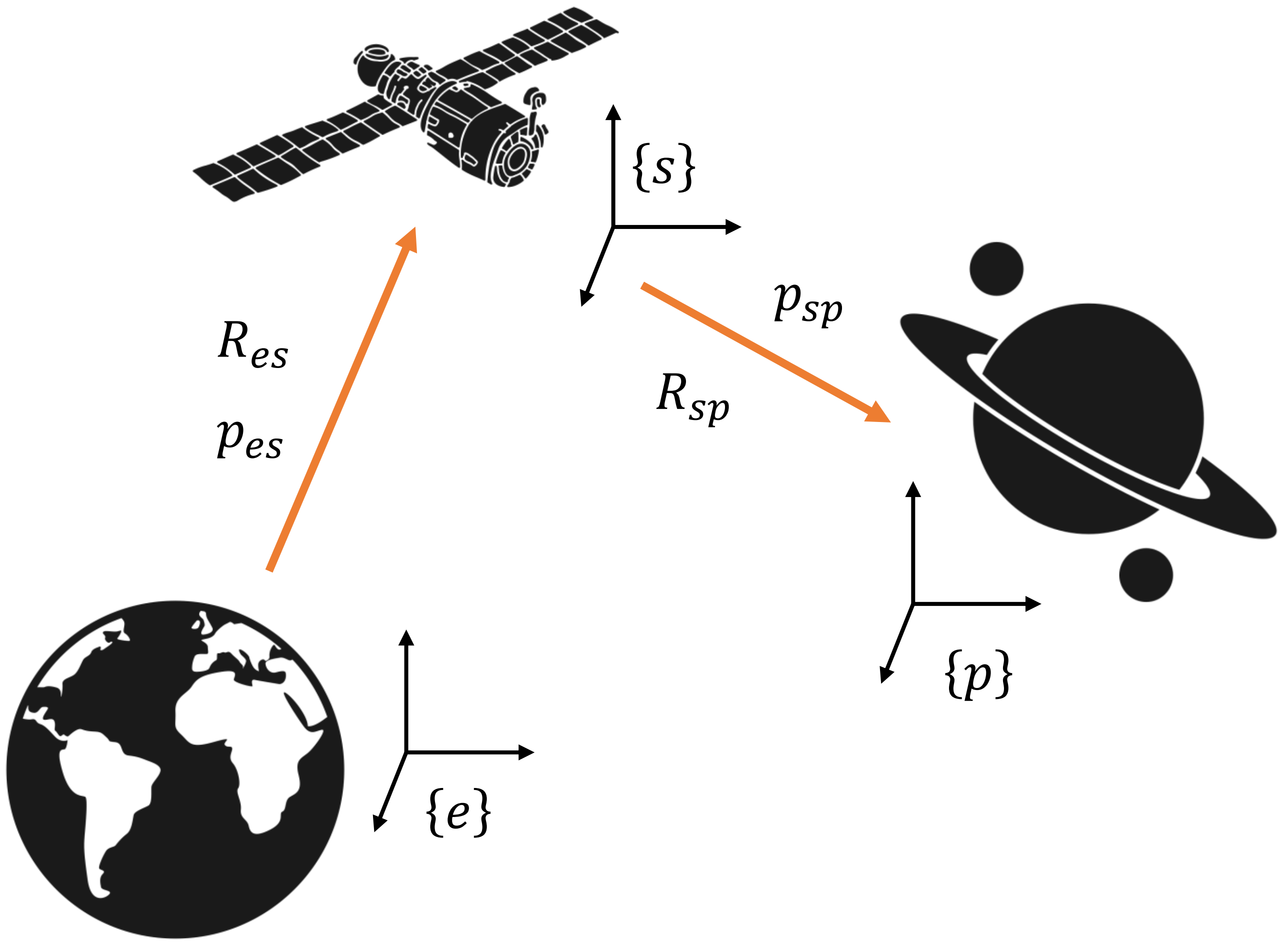
Reading: Modern Robotics 3.3.1



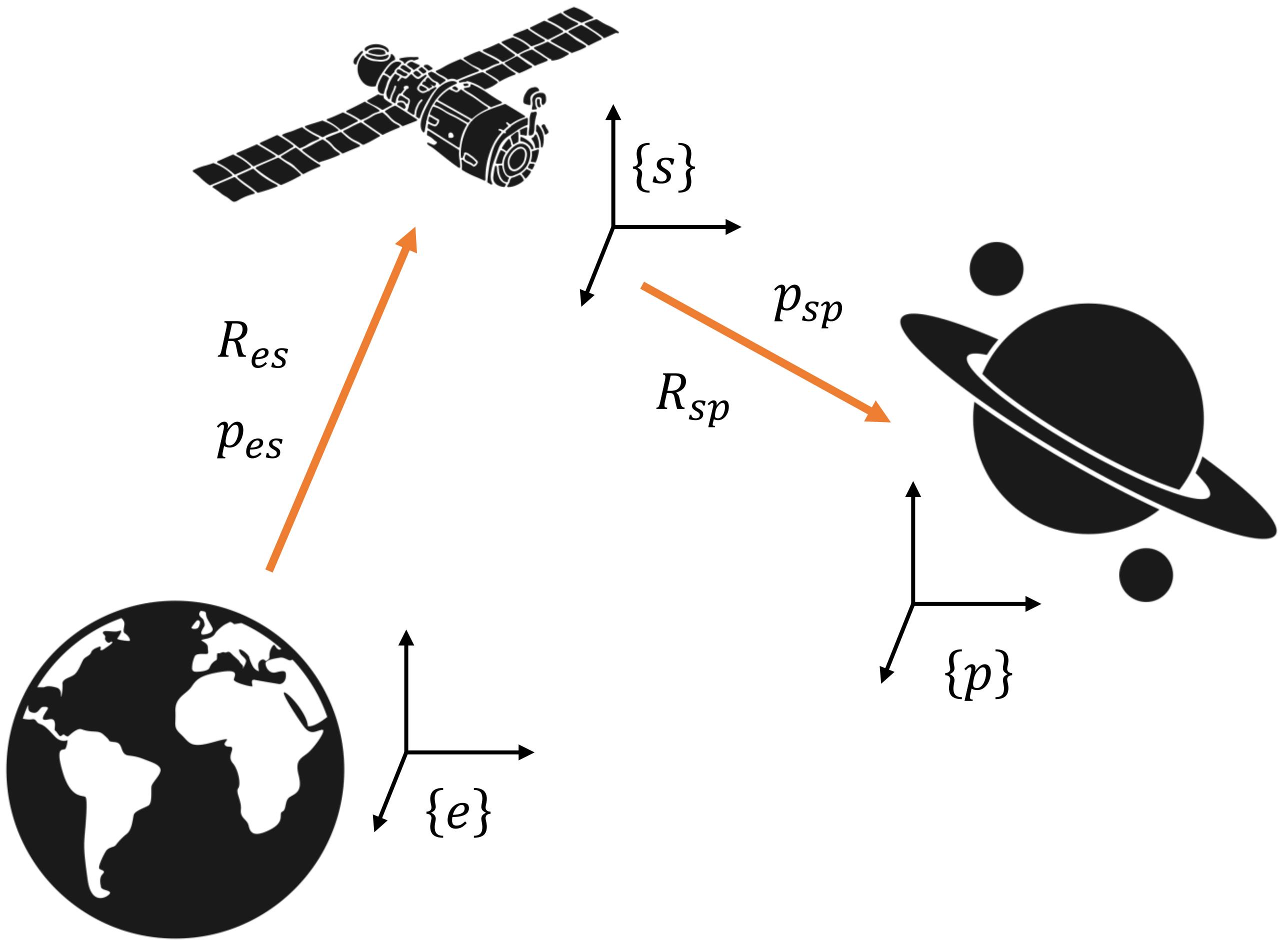
This Lecture

- How do we use transformation matrices?
- Which matrices should we multiply to get a desired transformation?


{s}
{p}
{e}

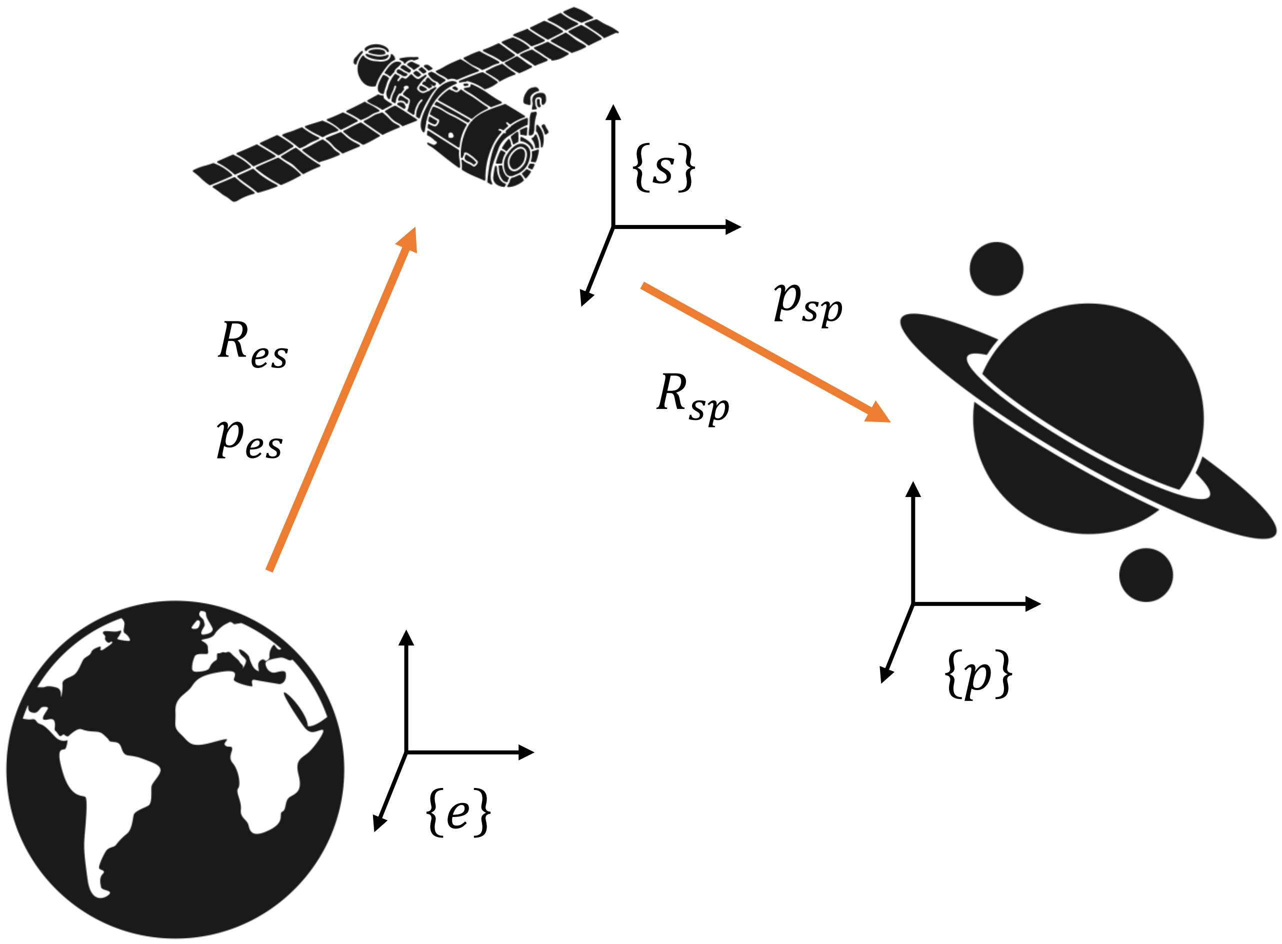


$\{s\}$
p_{sp}
R_{es}
R_{sp}
p_{es}
$\{p\}$
$\{e\}$



What is the orientation of $\{p\}$ with respect to the earth $\{e\}$?

What is the position of $\{p\}$ relative to the earth $\{e\}$?

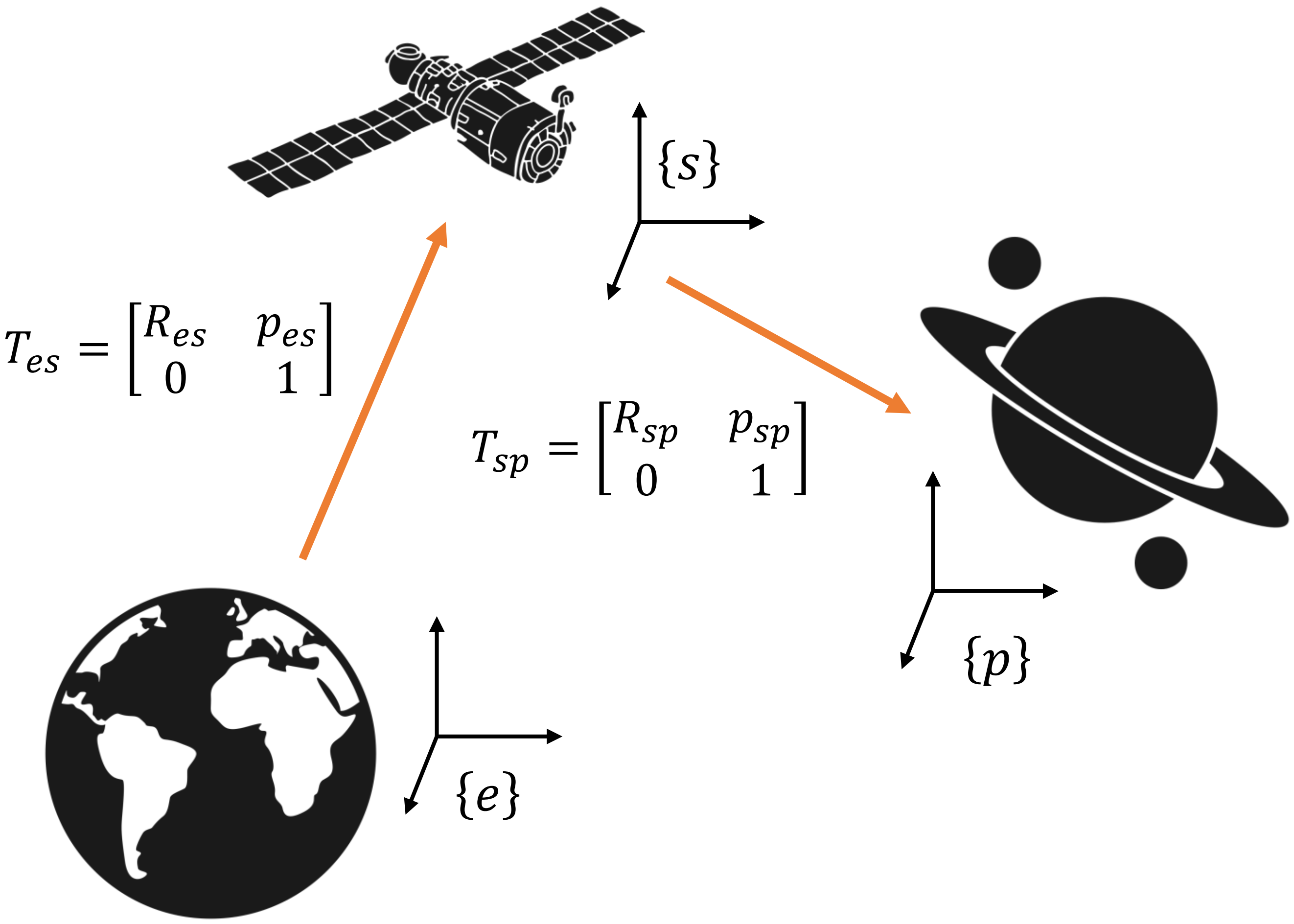


What is the orientation of $\{p\}$ with respect to the earth $\{e\}$?

$$R_{ep} = R_{es} R_{sp}$$

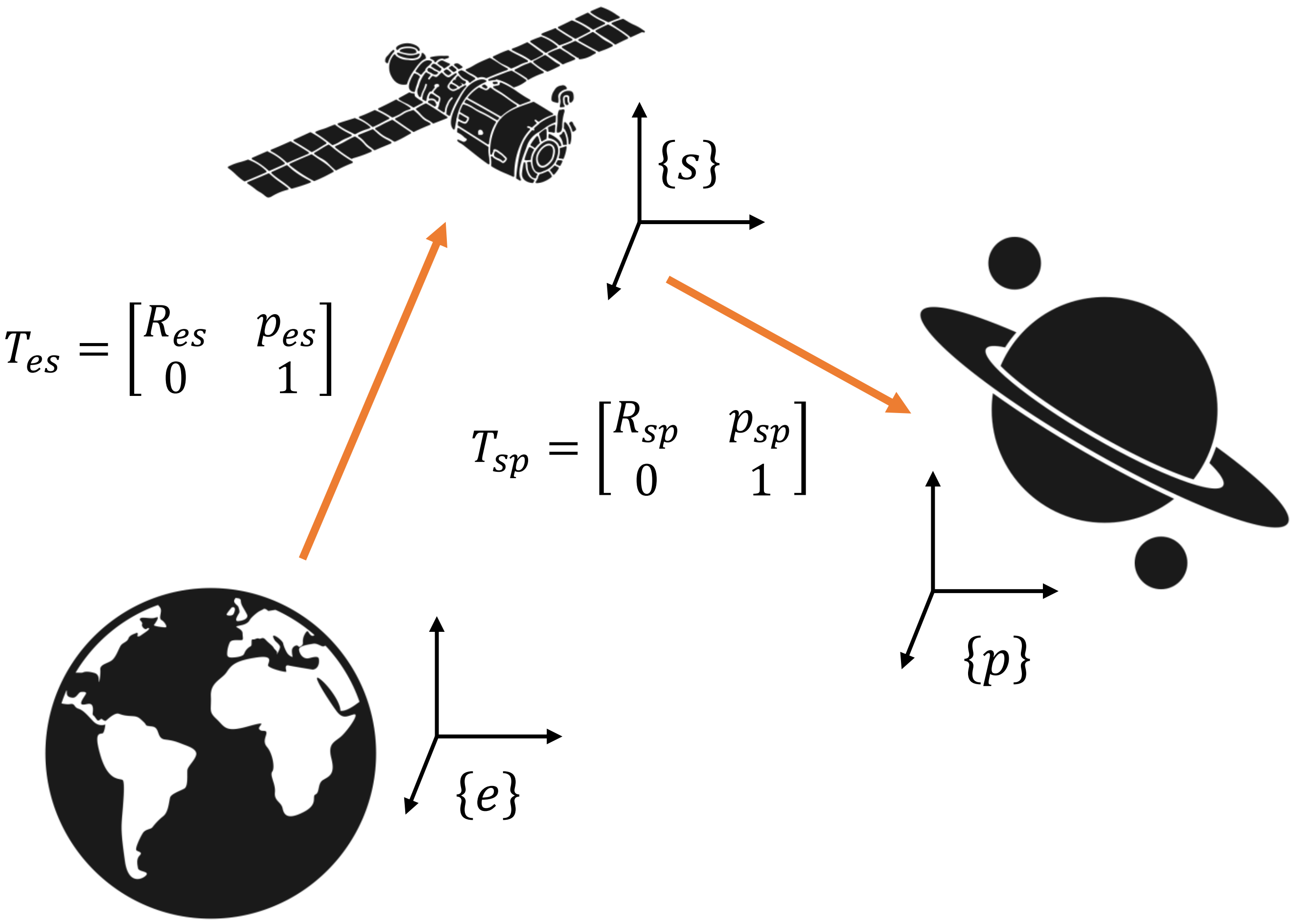
What is the position of $\{p\}$ relative to the earth $\{e\}$?

$$p_{ep} = R_{es} p_{sp} + p_{es}$$



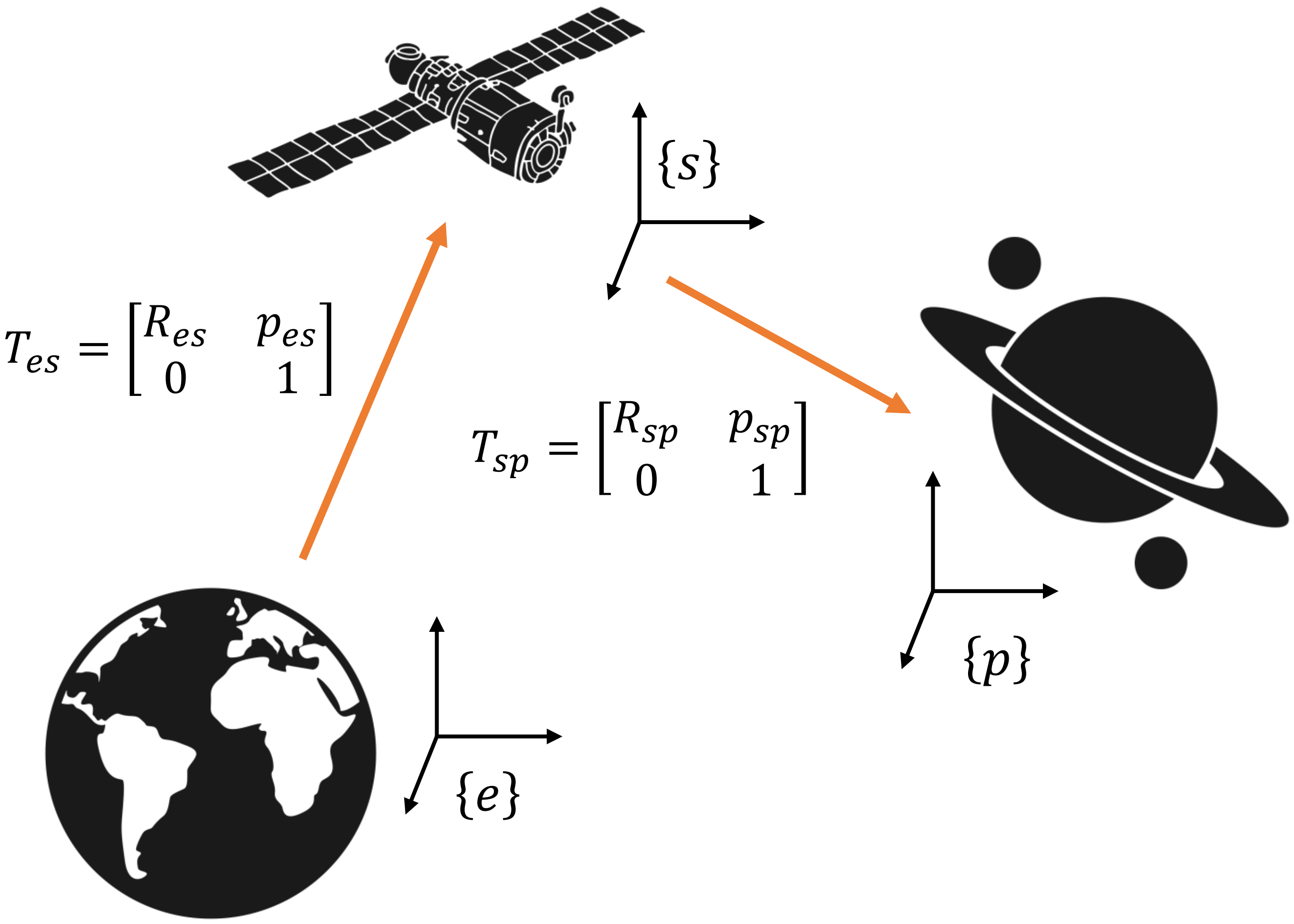
What is the **pose** of $\{p\}$ with respect to the earth $\{e\}$?

Notation: T_{ab} is the position and orientation of frame $\{b\}$ expressed in frame $\{a\}$



What is the **pose** of $\{p\}$ with respect to the earth $\{e\}$?

$$T_{ep} = T_{es} T_{sp}$$



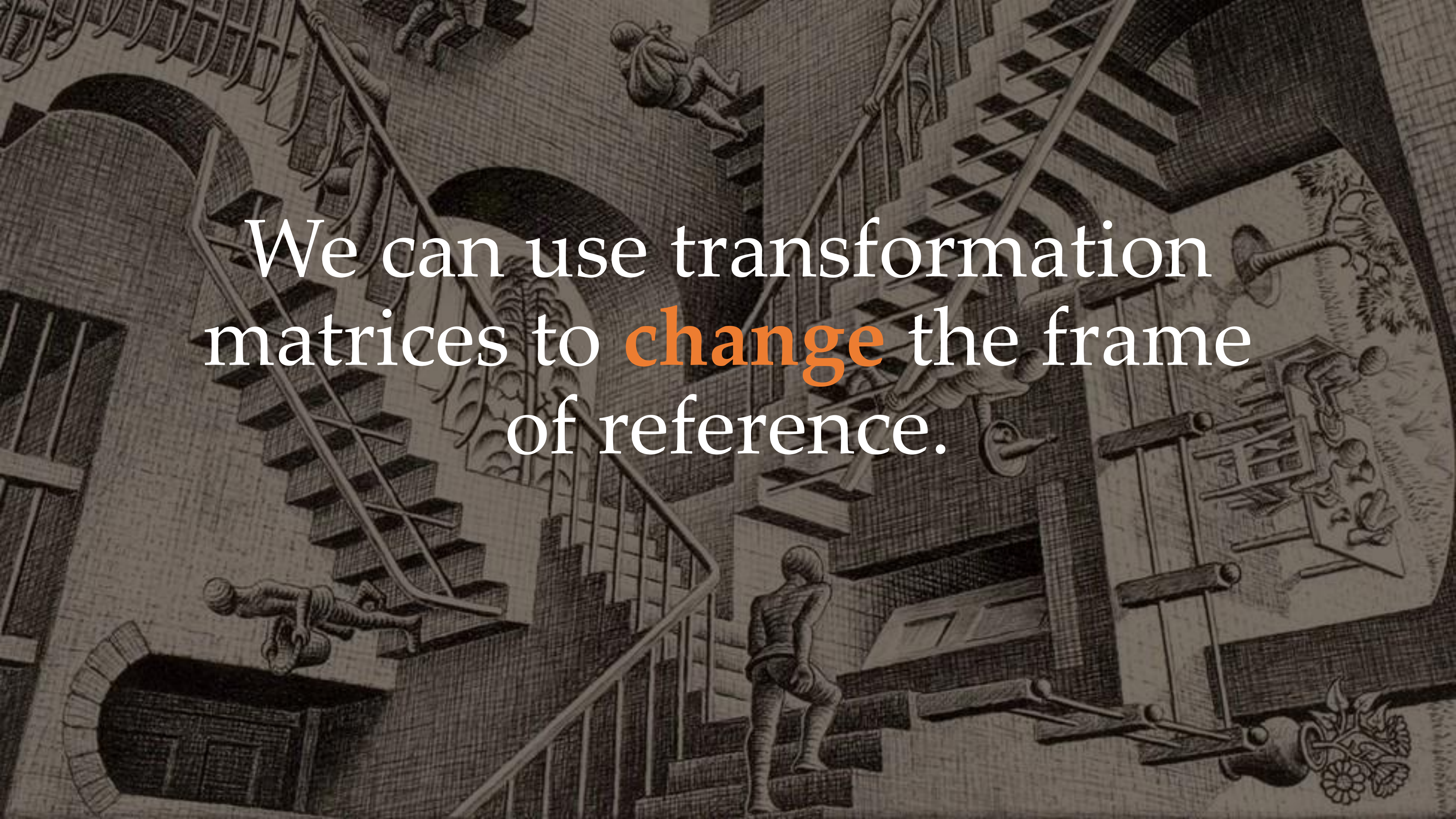
What is the **pose** of $\{p\}$ with respect to the earth $\{e\}$?

$$T_{ep} = T_{es}T_{sp}$$

$$= \begin{bmatrix} R_{es} & p_{es} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R_{sp} & p_{sp} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_{es}R_{sp} & R_{es}p_{sp} + p_{es} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_{ep} & p_{ep} \\ 0 & 1 \end{bmatrix}$$



We can use transformation matrices to **change** the frame of reference.

Changing Frames

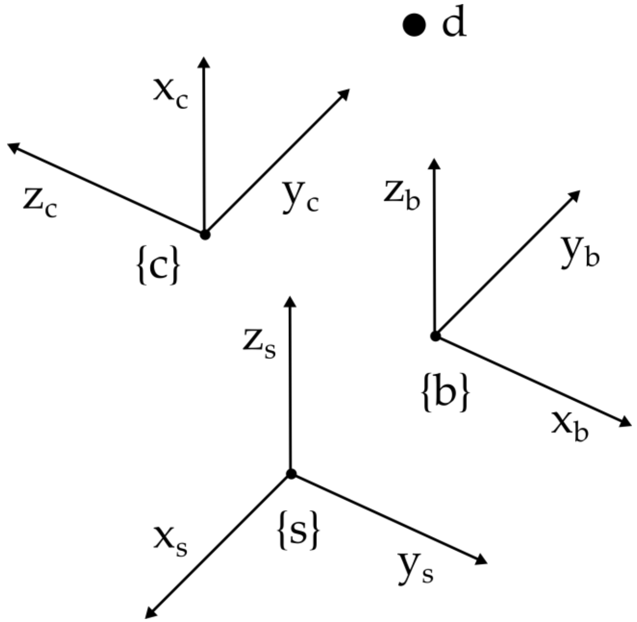
When we multiply transformation matrices, if the subscripts *cancel* then we **change** the frame of reference

$$T_{ep} = T_{es}T_{sp}$$

$$T_{ad} = T_{ab}T_{bc}T_{cd}$$

$$p_{ac} = T_{ab}p_{bc}$$

Example

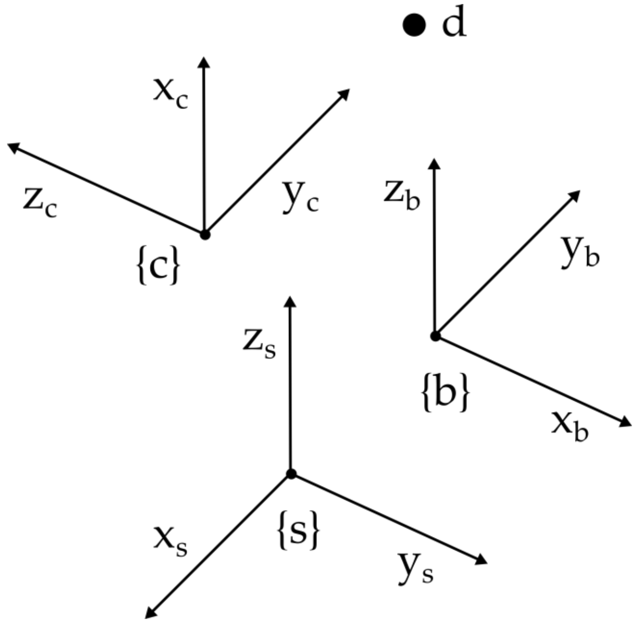


We want to find the position of d relative to frame {s} given:

$$\boldsymbol{T_{sb}} = \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T_{bc}} = \begin{bmatrix} R_{bc} & p_{bc} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{p_{cd}} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

Example



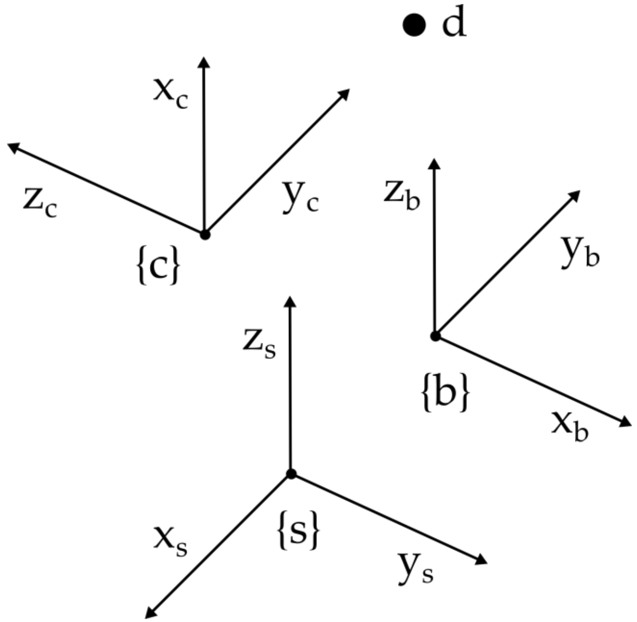
We want to find the position of d relative to frame {s} given:

$$p_{sd} = T_{sb}T_{bc}p_{cd}$$

$$p_{sd} = \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix} \boxed{\begin{bmatrix} R_{bc} & p_{bc} \\ 0 & 1 \end{bmatrix} p_{cd}}$$

4×4 matrix times a 3×1 vector?

Example



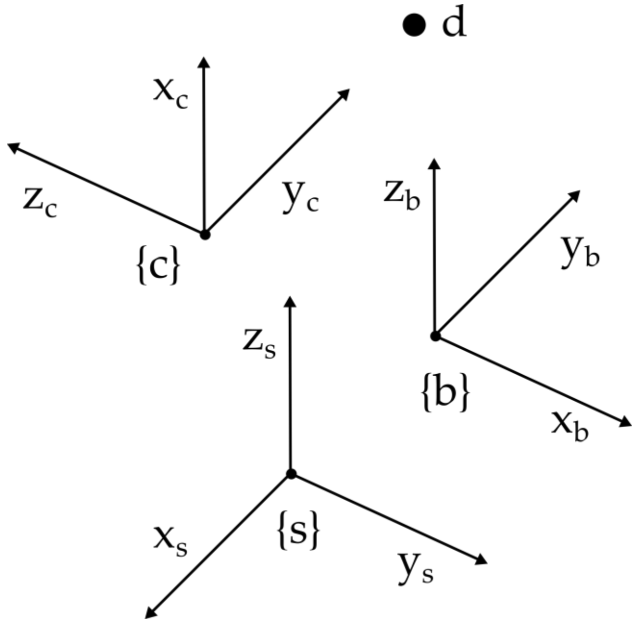
We want to find the position of d relative to frame {s} given:

$$p_{sd} = T_{sb} T_{bc} p_{cd}$$

$$p_{sd} = \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix} \boxed{\begin{bmatrix} R_{bc} & p_{bc} \\ 0 & 1 \end{bmatrix} p_{cd}}$$

When we multiply a position vector by a transformation, we **append a '1'** to make it a 4-dimensional vector.

Example

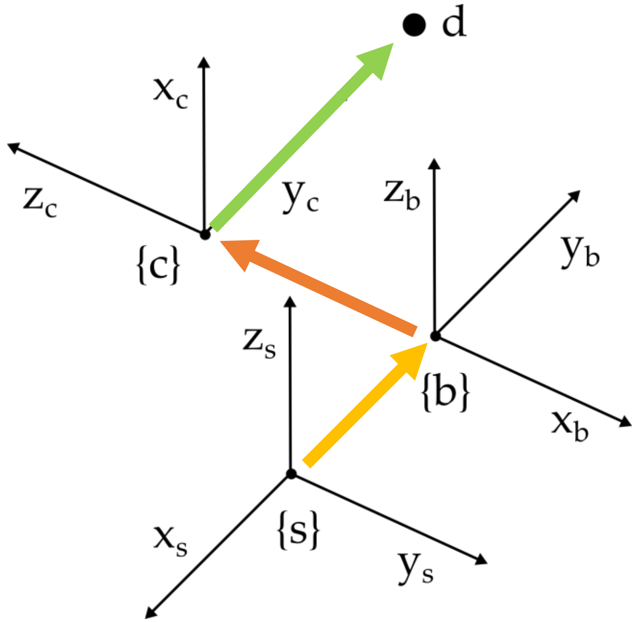


We want to find the position of d relative to frame {s} given:

$$p_{sd} = T_{sb}T_{bc}p_{cd}$$

$$p_{sd} = \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R_{bc} & p_{bc} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p_{cd} \\ 1 \end{bmatrix}$$

Example



We want to find the position of d relative to frame {s} given:

$$p_{sb} = T_{sb}T_{bc}p_{cd}$$

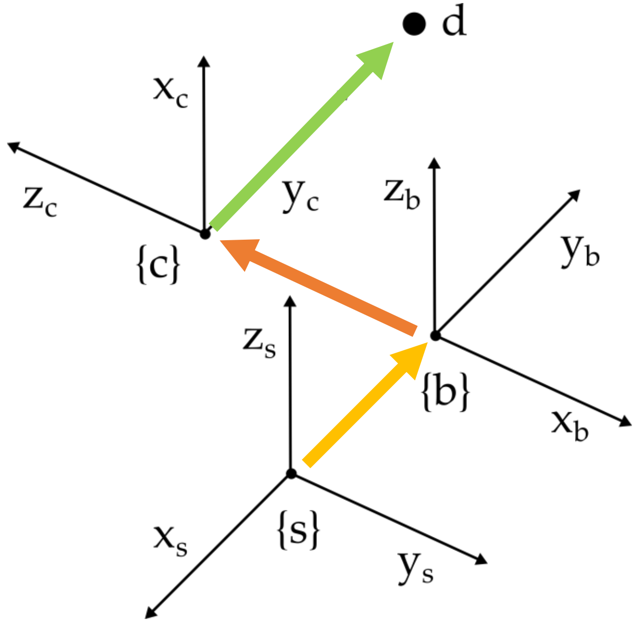
$$p_{sd} = \begin{bmatrix} R_{sb}R_{bc}p_{cd} + R_{sb}p_{bc} + p_{sb} \\ 1 \end{bmatrix}$$

{s} to {b} in frame {s}

{b} to {c} in frame {s}

{c} to d in frame {s}

Example



We want to find the position of d relative to frame {s} given:

$$p_{sb} = T_{sb} T_{bc} p_{cd}$$

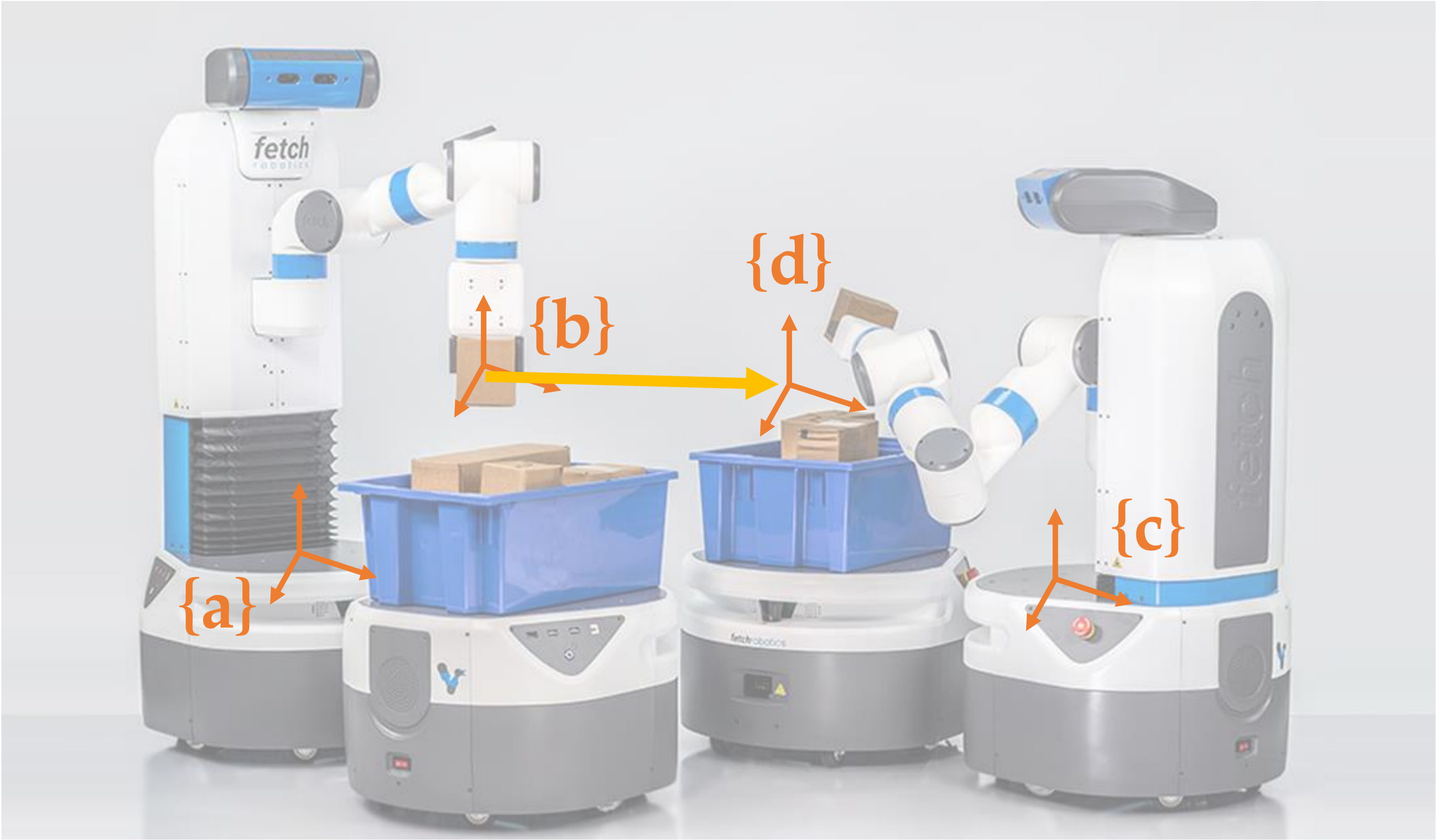
$$p_{sd} = \begin{bmatrix} \begin{bmatrix} -5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -5 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

We can find a **desired transformation** matrix from given transformations

We want to find T_{bd}

We are given:

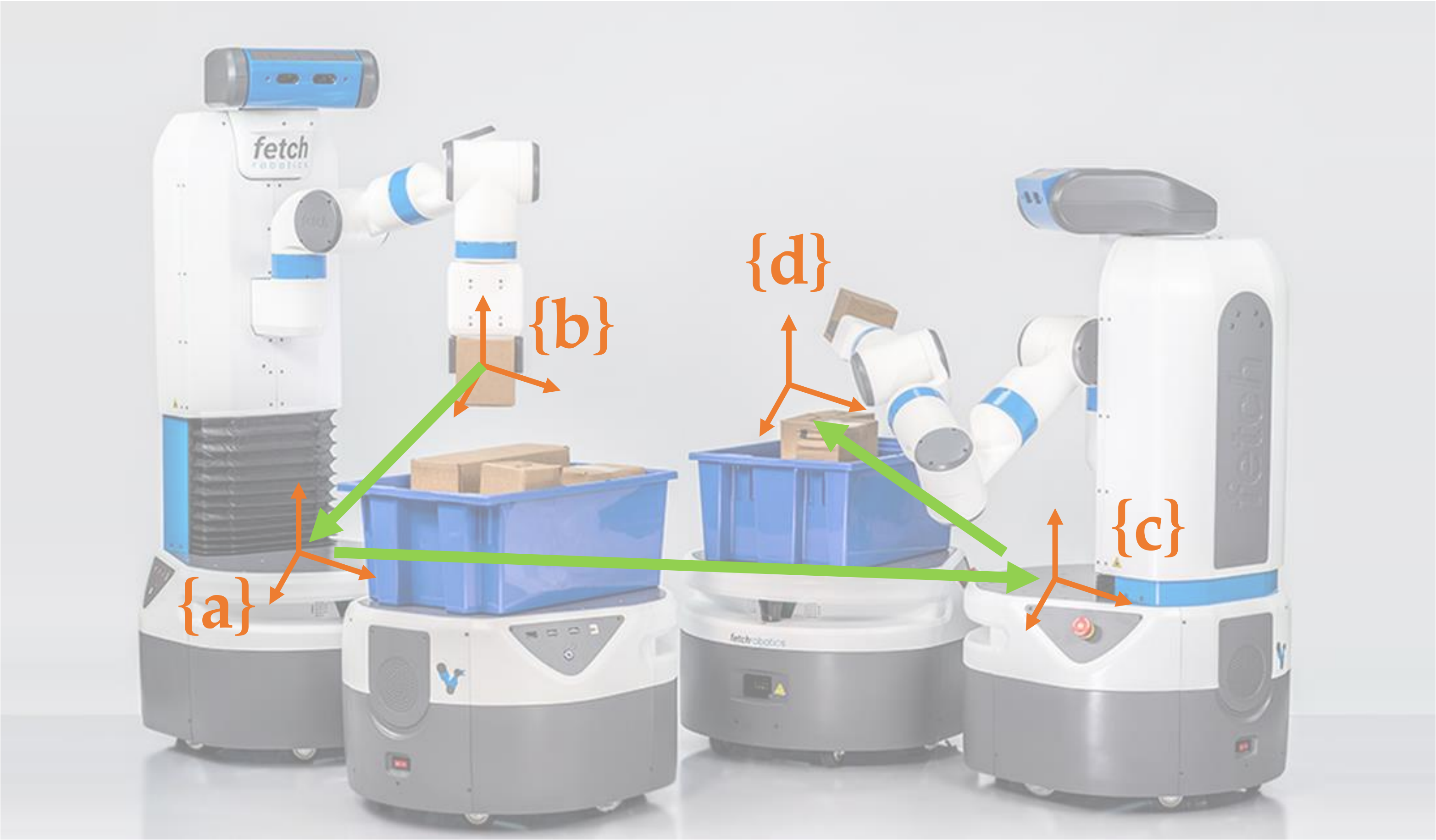
- Base to end-effector (T_{ab})
- Base to base (T_{ac})
- Base to box (T_{cd})



We want to find T_{bd}

Draw a path from $\{b\}$ to $\{d\}$ using the transformations.

$$T_{bd} = T_{ba} T_{ac} T_{cd}$$

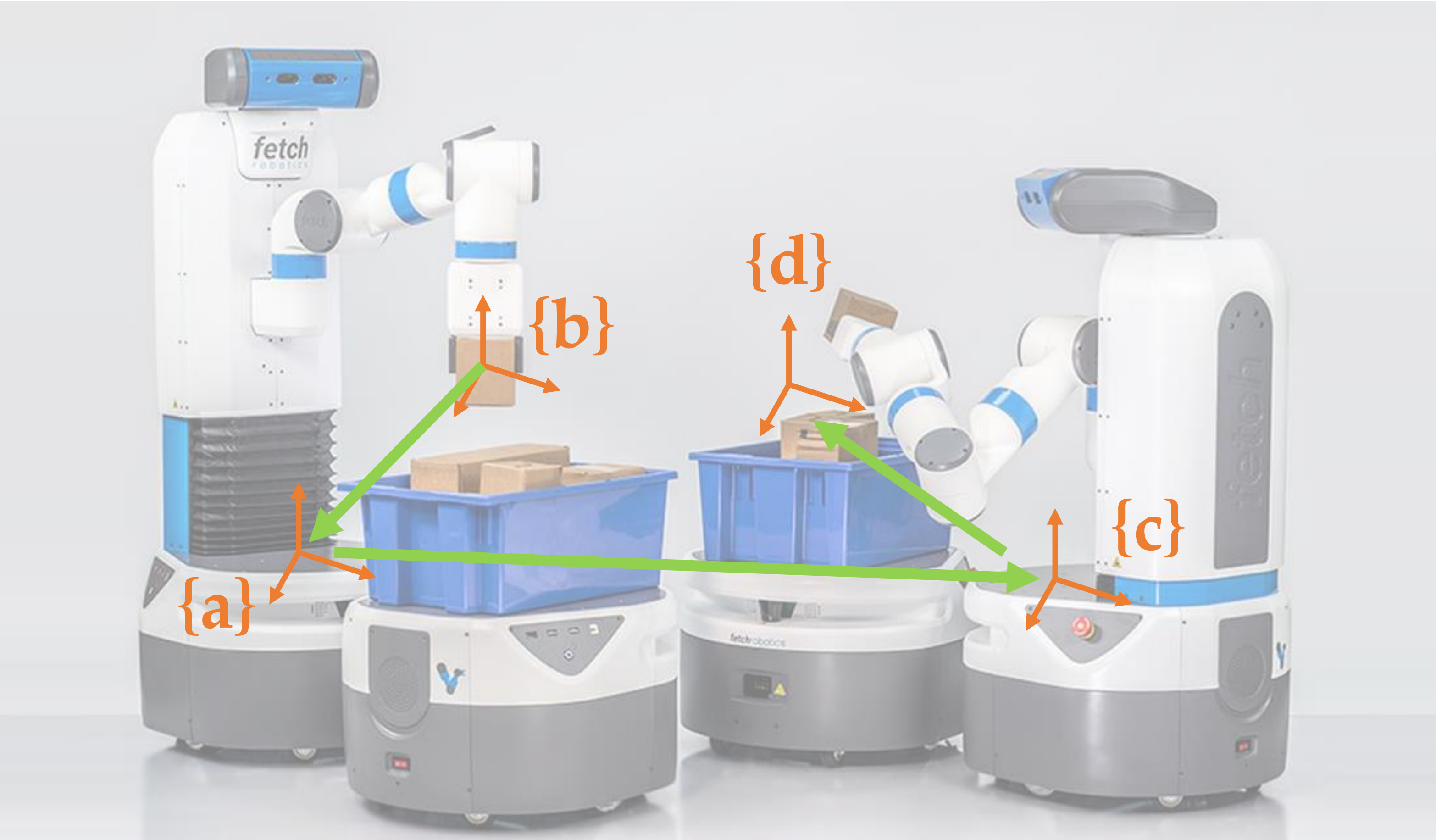


We want to find T_{bd}

Draw a path from $\{b\}$ to $\{d\}$ using the transformations.

$$T_{bd} = \underline{T_{ba}} T_{ac} T_{cd}$$

Use the ***inverse*** *property:* $T_{ba} = T_{ab}{}^{-1}$

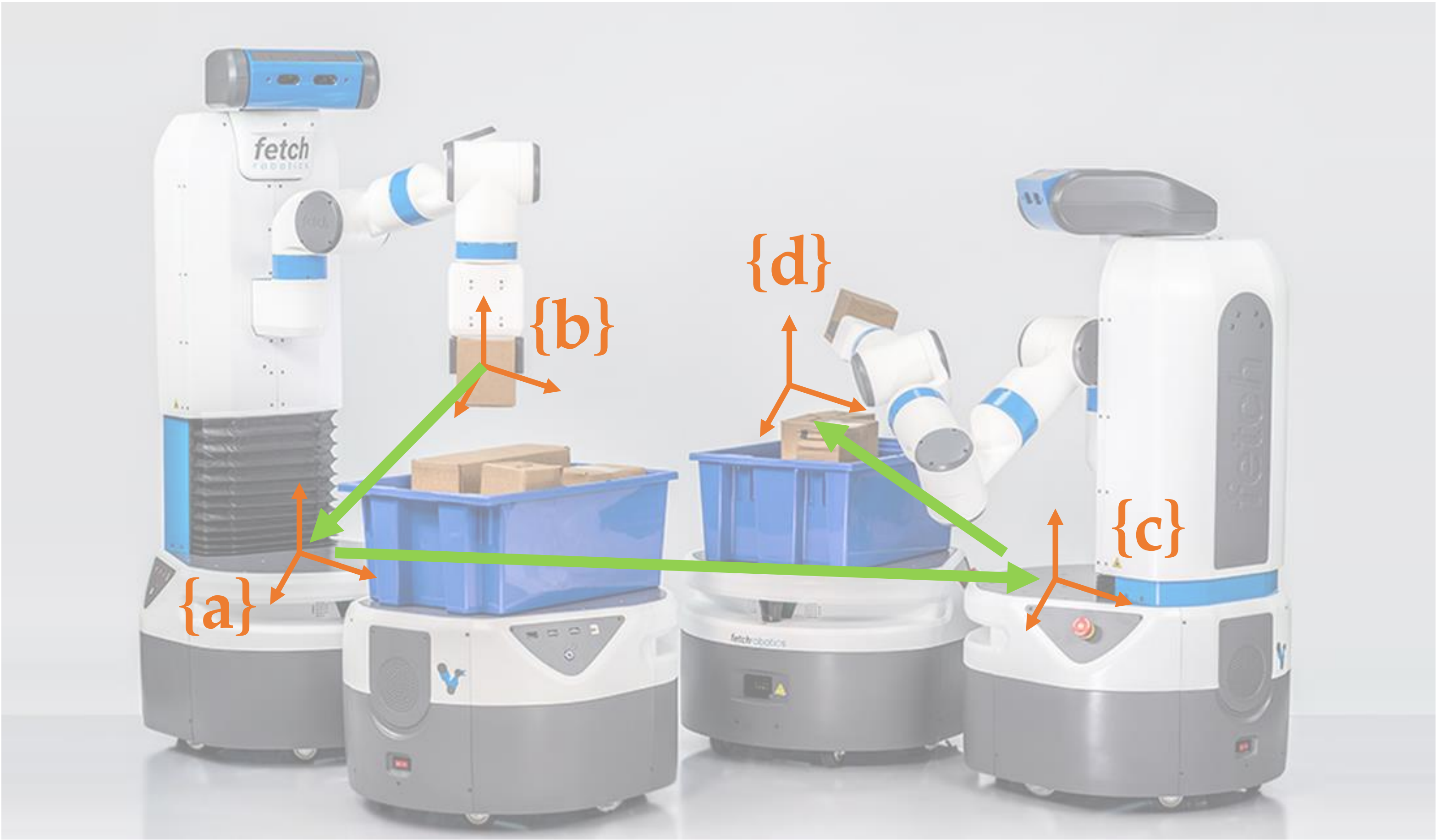


We want to find T_{bd}

We are given:

- Base to end-effector (T_{ab})
- Base to base (T_{ac})
- Base to box (T_{cd})

$$T_{bd} = T_{ab}^{-1} T_{ac} T_{cd}$$



This Lecture

- How do we use transformation matrices?
- Which matrices should we multiply to get a desired transformation?

Next Lecture

- How can we use transformation matrices to move objects?