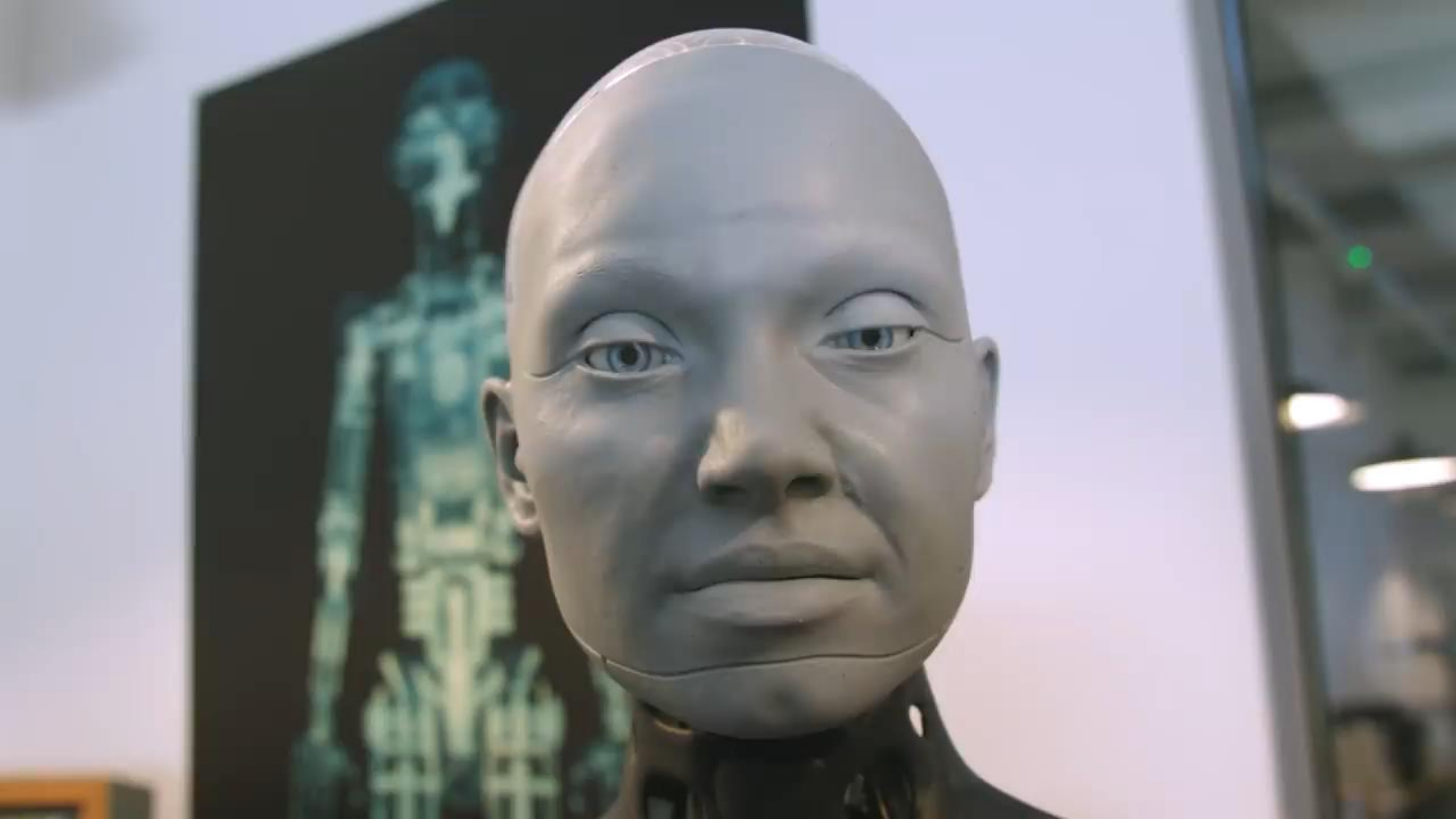
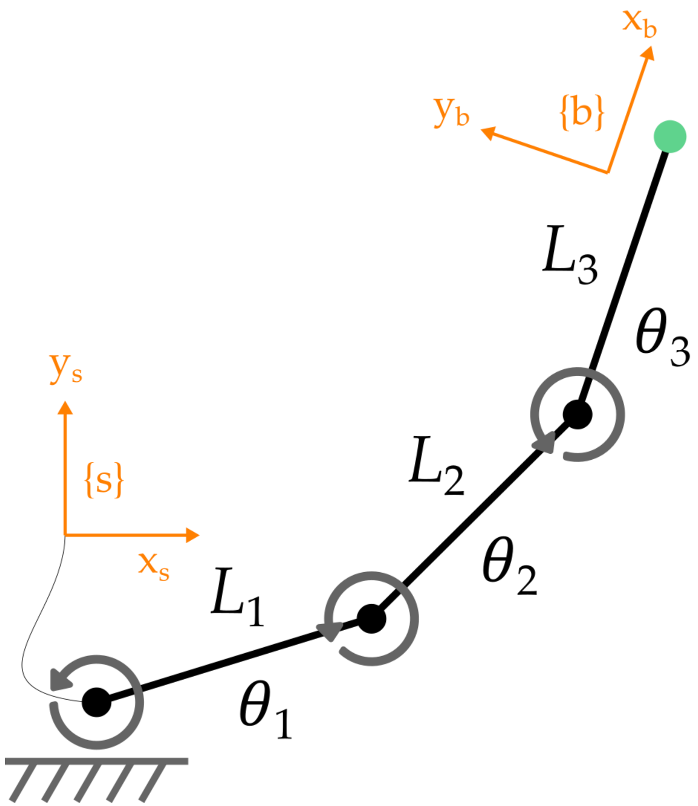
Interpreting the Jacobian

Reading: Modern Robotics 5.1.3 – 5.1.4



This Lecture

- What do the terms of the Jacobian intuitively mean?
- What can we learn about our robot from the Jacobian?



Last lecture we found the geometric **Jacobian**:

$$J(\boldsymbol{\theta}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ -L_1 s_1 - L_2 s_{12} - L_3 s_{123} & -L_2 s_{12} - L_3 s_{123} & -L_3 s_{123} \\ L_1 c_1 + L_2 c_{12} + L_3 c_{123} & L_2 c_{12} + L_3 c_{123} & L_3 c_{123} \\ 0 & 0 & 0 \end{bmatrix}$$

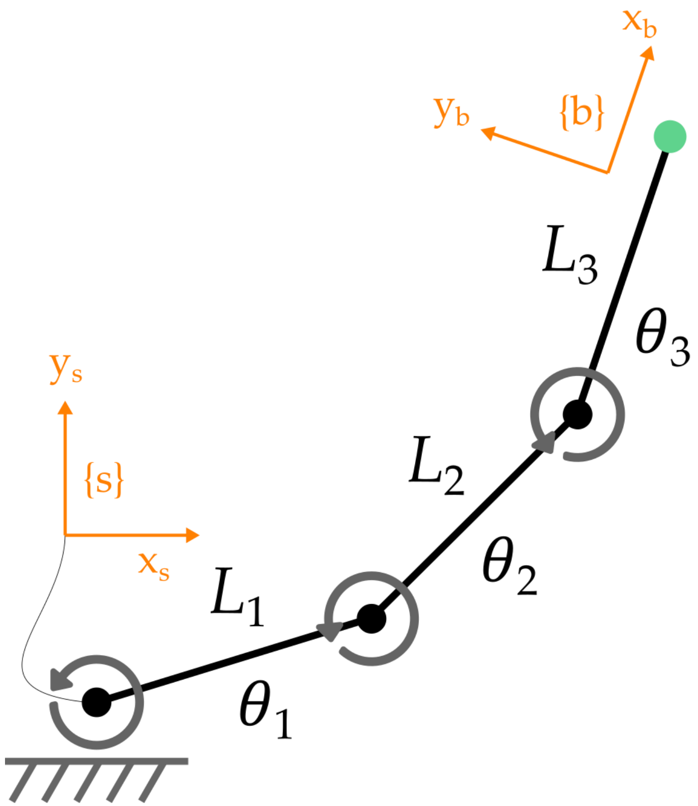

What does this
matrix mean?

$$\begin{bmatrix} \omega_s \\ \dot{p} \end{bmatrix} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}$$

top three rows capture angular velocity

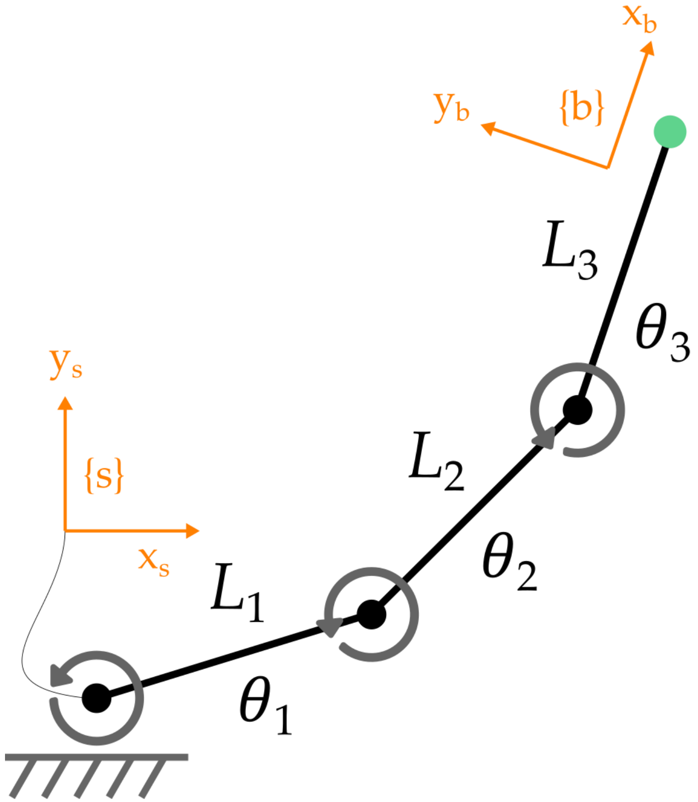
$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ -L_1 s_1 - L_2 s_{12} - L_3 s_{123} & -L_2 s_{12} - L_3 s_{123} & -L_3 s_{123} \\ L_1 c_1 + L_2 c_{12} + L_3 c_{123} & L_2 c_{12} + L_3 c_{123} & L_3 c_{123} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

bottom three rows capture linear velocity



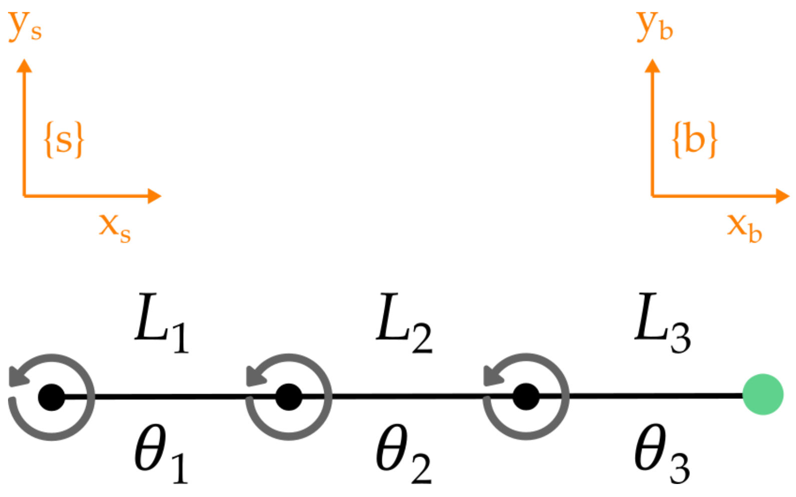
$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \underline{\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}}$$

No matter what $\dot{\theta}$ is, we cannot have angular velocity about x_s or y_s



$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} (-L_1 s_1 - L_2 s_{12} - L_3 s_{123})\dot{\theta}_1 + (-L_2 s_{12} - L_3 s_{123})\dot{\theta}_2 + (-L_3 s_{123})\dot{\theta}_3 \\ (L_1 c_1 + L_2 c_{12} + L_3 c_{123})\dot{\theta}_1 + (L_2 c_{12} + L_3 c_{123})\dot{\theta}_2 + (L_3 c_{123})\dot{\theta}_3 \\ 0 \end{bmatrix}$$

No matter what $\dot{\theta}$ is, we cannot have linear velocity in z_s



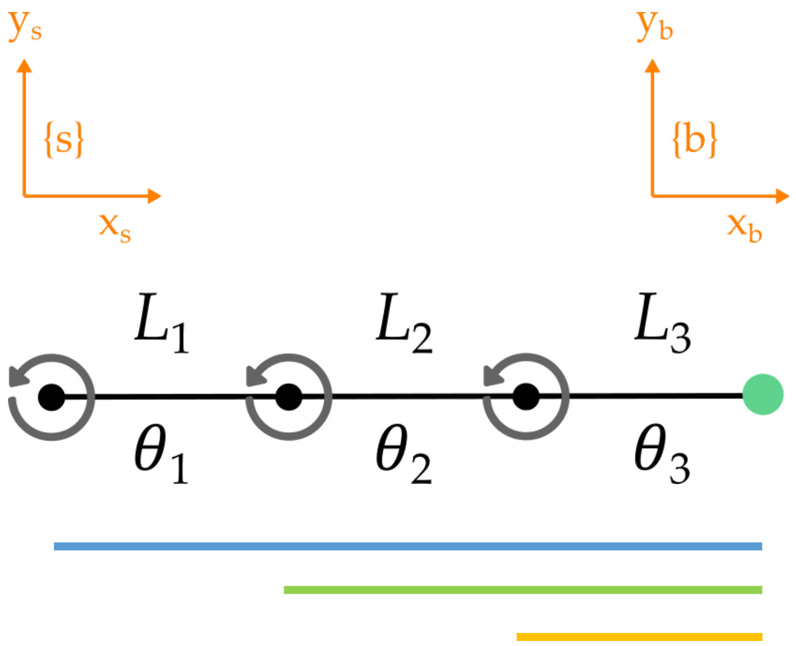
Evaluate. At joint position

$$\theta_1 = 0, \qquad \theta_2 = 0, \qquad \theta_3 = 0$$

The linear velocity of the end-effector is:

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} 0 \\ (L_1 + L_2 + L_3)\dot{\theta}_1 + (L_2 + L_3)\dot{\theta}_2 + (L_3)\dot{\theta}_3 \\ 0 \end{bmatrix}$$

Here we cannot move in x_s or z_s

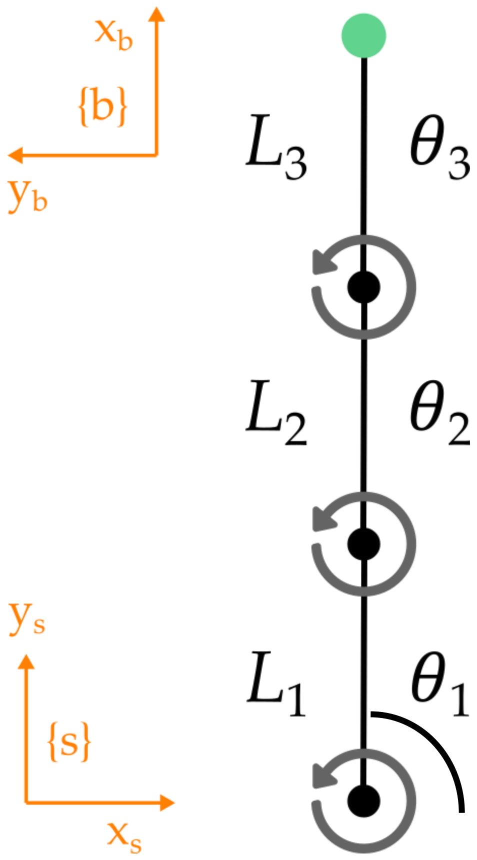


Evaluate. At joint position

$$\theta_1 = 0, \qquad \theta_2 = 0, \qquad \theta_3 = 0$$

The linear velocity of the end-effector is:

$$\dot{p}_y = \underbrace{(L_1 + L_2 + L_3)}_{\text{moment arm}}\dot{\theta}_1 + \underbrace{(L_2 + L_3)}_{\text{moment arm}}\dot{\theta}_2 + \underbrace{(L_3)}_{\text{moment arm}}\dot{\theta}_3$$



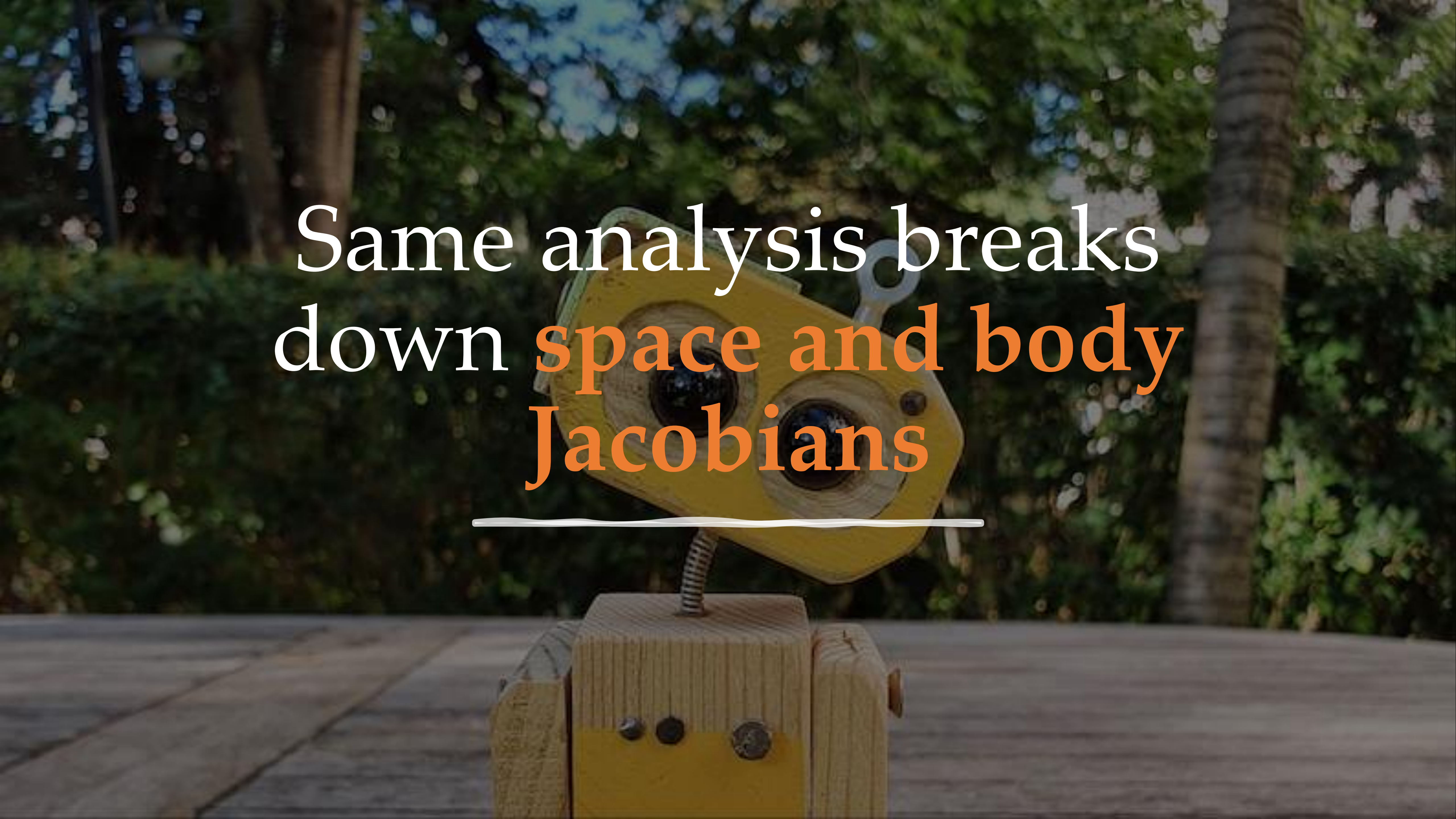
Evaluate. At joint position

$$\theta_1 = \pi/2, \qquad \theta_2 = 0, \qquad \theta_3 = 0$$

The linear velocity of the end-effector is:

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} -(L_1 + L_2 + L_3)\dot{\theta}_1 - (L_2 + L_3)\dot{\theta}_2 - (L_3)\dot{\theta}_3 \\ 0 \\ 0 \end{bmatrix}$$

Remember that the Jacobian often **depends** on joint position



Same analysis breaks down **space and body Jacobians**

This Lecture

- What do the terms of the Jacobian intuitively mean?
- What can we learn about our robot from the Jacobian?

Next Lecture

- How does the Jacobian change our ability to move the end-effector?