Body & Geometric Jacobian

Reading: Modern Robotics 5.1.2


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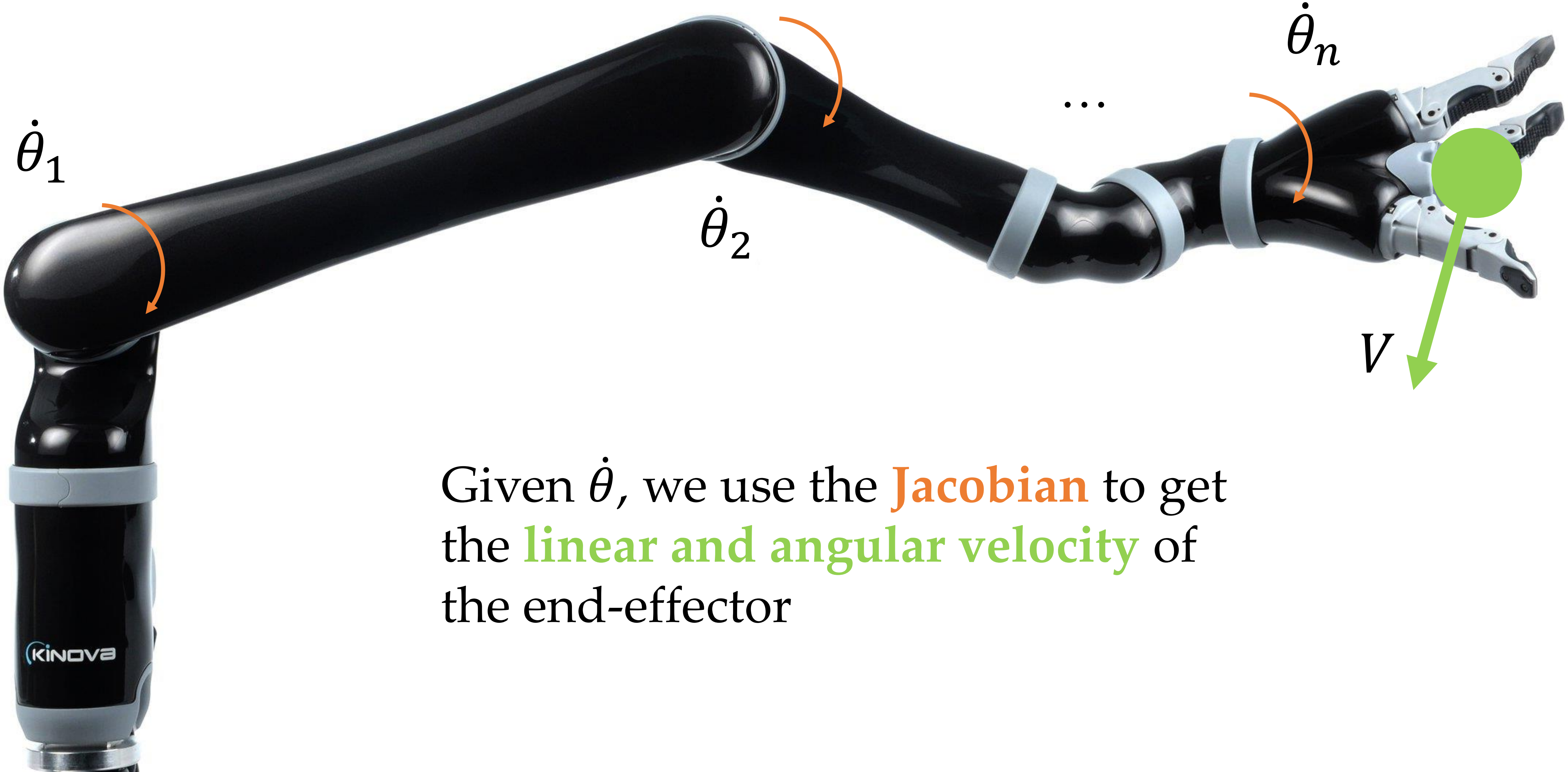
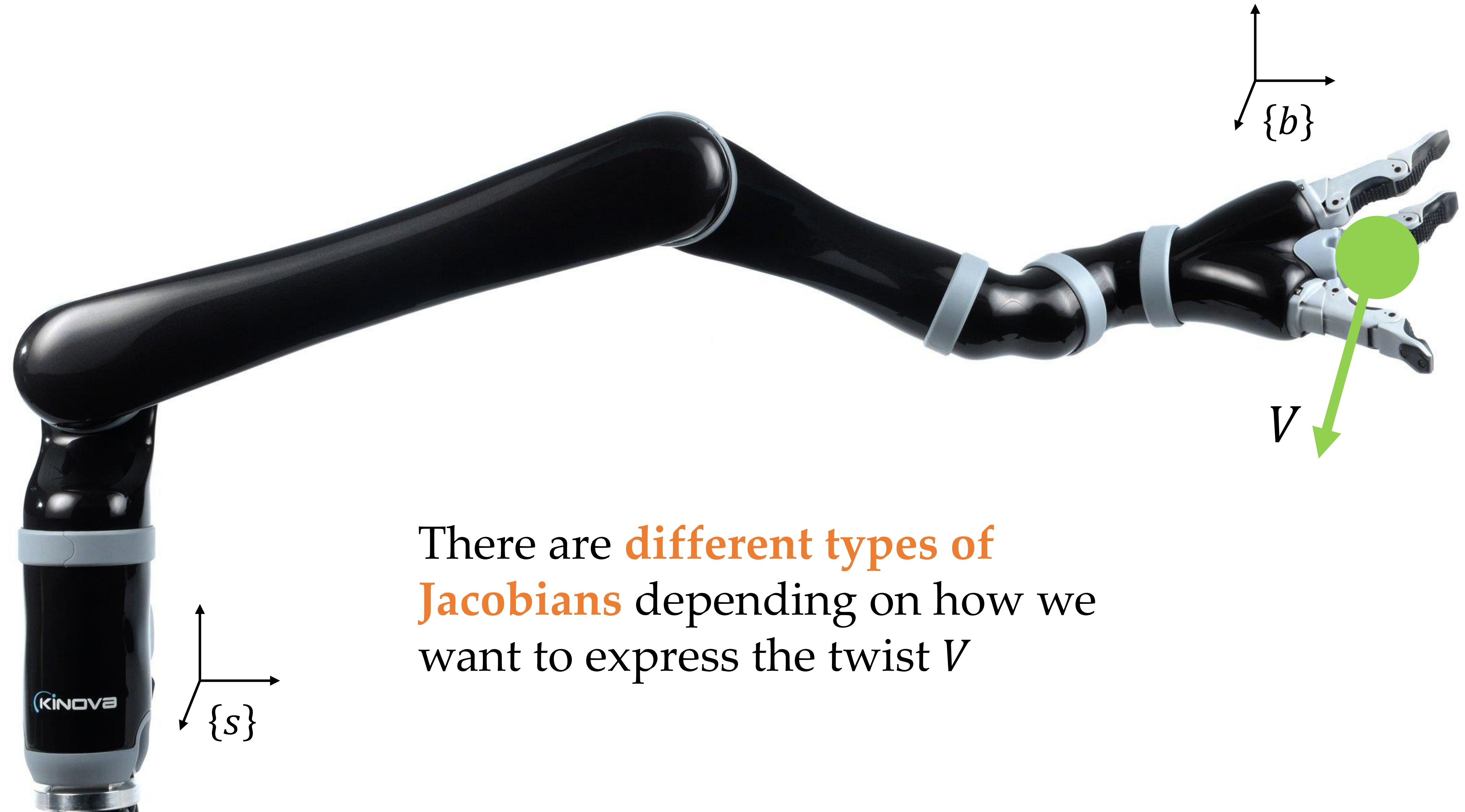
TECHCRUNCH

This Lecture

- What are the different types of Jacobians?
- How do we convert from one Jacobian to another?



Given $\dot{\theta}$, we use the **Jacobian** to get the **linear and angular velocity** of the end-effector



There are **different types of Jacobians** depending on how we want to express the twist V

Body Jacobian

Body Jacobian relates **joint velocity** to **body twist**:

$$V_b = \boldsymbol{J_b}(\boldsymbol{\theta})\dot{\theta}$$

we can find the body Jacobian
using the **space Jacobian**

Body Jacobian

Body Jacobian relates **joint velocity** to **body twist**:

$$V_b = \boldsymbol{J_b}(\boldsymbol{\theta})\dot{\theta}$$

$$\boxed{V_s} = \mathrm{Ad}_{T_{sb}} \boxed{V_b}$$

$J_s(\theta)\dot{\theta}$ $\quad$ $J_b(\theta)\dot{\theta}$

Body Jacobian

Body Jacobian relates **joint velocity** to **body twist**:

$$V_b = \boldsymbol{J_b}(\boldsymbol{\theta})\dot{\theta}$$

$$\boldsymbol{J_b}(\boldsymbol{\theta}) = \underline{\mathrm{Ad}_{T_{sb}^{-1}}}\, J_s(\theta)$$

remember that $T_{sb} = T(\theta)$ is
your **forward kinematics**

There is one more type of **Jacobian** you will often use

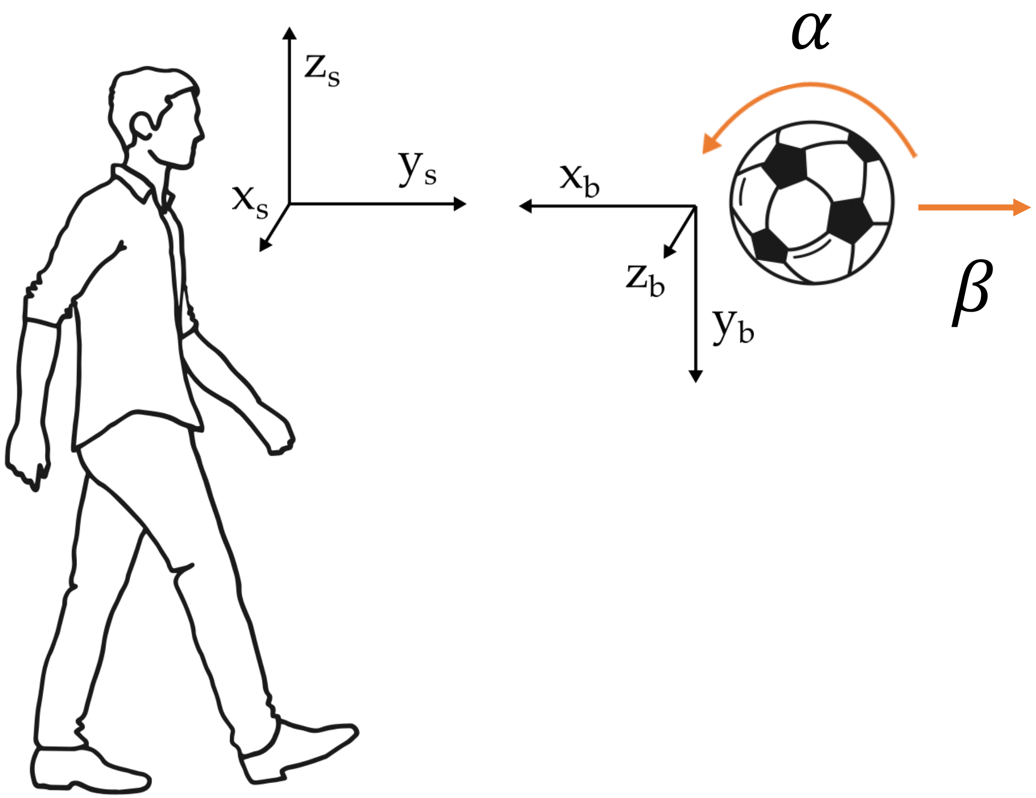
Geometric Jacobian

Taking a step back, remember that:

$$V_b = \begin{bmatrix} \omega_b \\ v_b \end{bmatrix} = \begin{bmatrix} R^T \omega_s \\ R^T \dot{p} \end{bmatrix}$$

- ω_s is angular velocity from $\{s\}$ perspective
- $\dot{p} = \dot{p}_{sb}$ is linear velocity of a **point at $\{b\}$ from $\{s\}$ perspective**

Geometric Jacobian



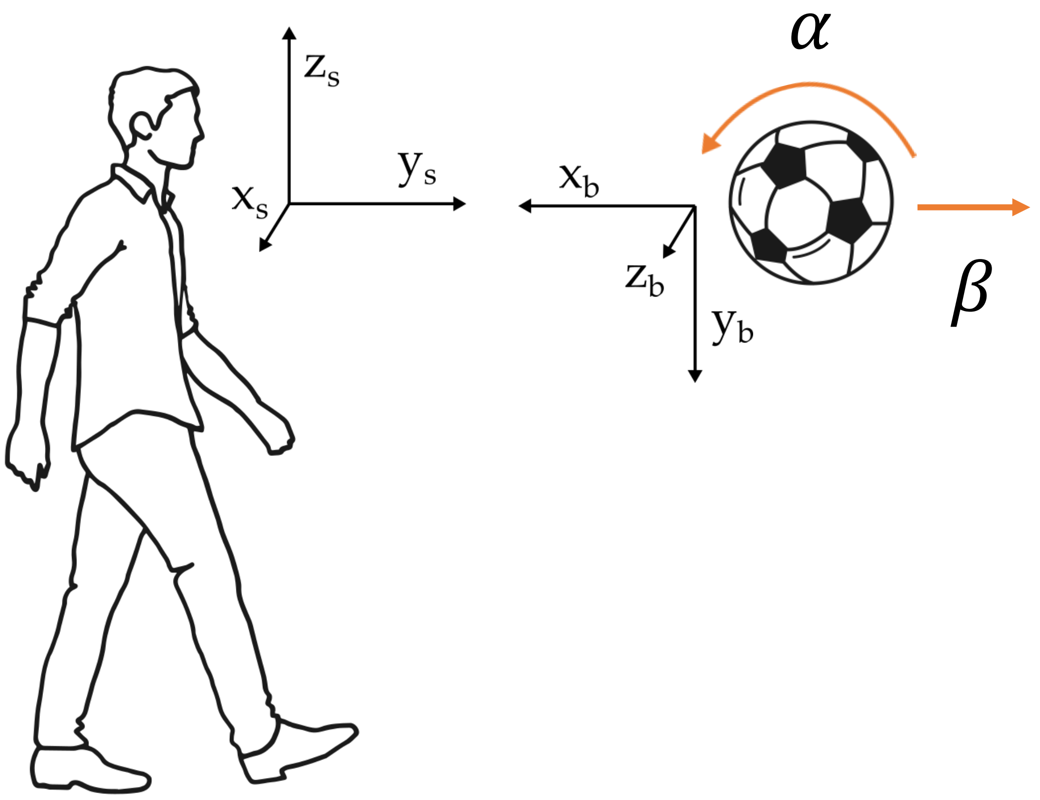
ω_s is angular velocity from $\{s\}$ perspective

What is ω_s here?

$\dot{p}$ is linear velocity of a point at $\{b\}$ from $\{s\}$ perspective

What is $\dot{p}$ here?

Geometric Jacobian



ω_s is angular velocity from $\{s\}$ perspective

$$\omega_s = \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$$

$\dot{p}$ is linear velocity of a point at $\{b\}$ from $\{s\}$ perspective

$$\dot{p} = \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix}$$

Geometric Jacobian

Relates **joint velocity** to linear and angular velocity of a **point at {b} expressed in {s}**

$$\begin{bmatrix} \omega_s \\ \dot{p} \end{bmatrix} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}$$

we can find the geometric Jacobian
using the **body Jacobian**

Geometric Jacobian

Relates **joint velocity** to linear and angular velocity of a **point at {b} expressed in {s}**

$$\begin{bmatrix} \omega_s \\ \dot{p} \end{bmatrix} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}$$

$$V_b = \begin{bmatrix} R^T \omega_s \\ R^T \dot{p} \end{bmatrix}, \qquad \begin{bmatrix} \omega_s \\ \dot{p} \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} R^T \omega_s \\ R^T \dot{p} \end{bmatrix}$$

Geometric Jacobian

Relates **joint velocity** to linear and angular velocity of a **point at {b} expressed in {s}**

$$\begin{bmatrix} \omega_s \\ \dot{p} \end{bmatrix} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}$$

$$\boldsymbol{J}(\boldsymbol{\theta}) = \underline{\begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}} J_b(\theta)$$

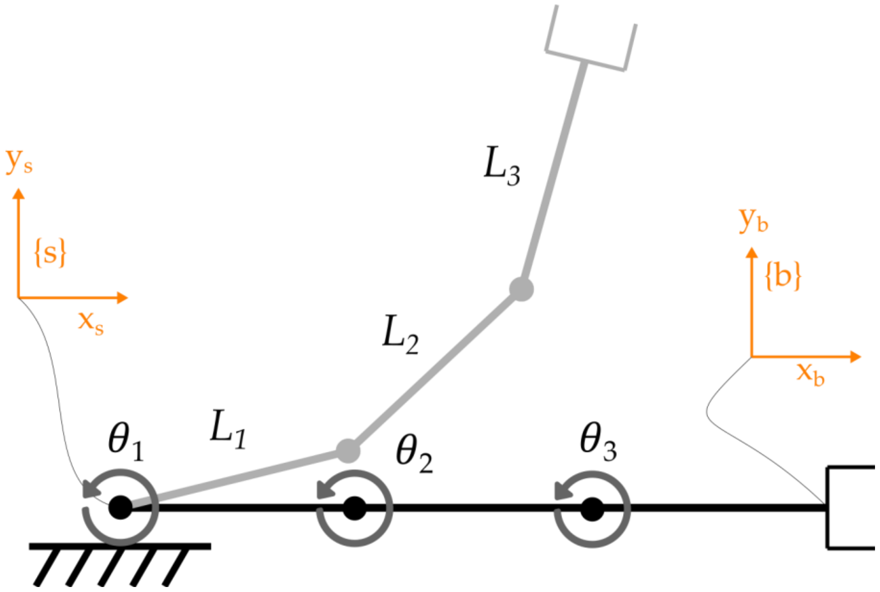
remember that R is part of
your **forward kinematics** T

Body and geometric Jacobian example

Example

Three-DoF robot arm.

Given joint values θ, what is the **body Jacobian**?
What is the **geometric Jacobian**?

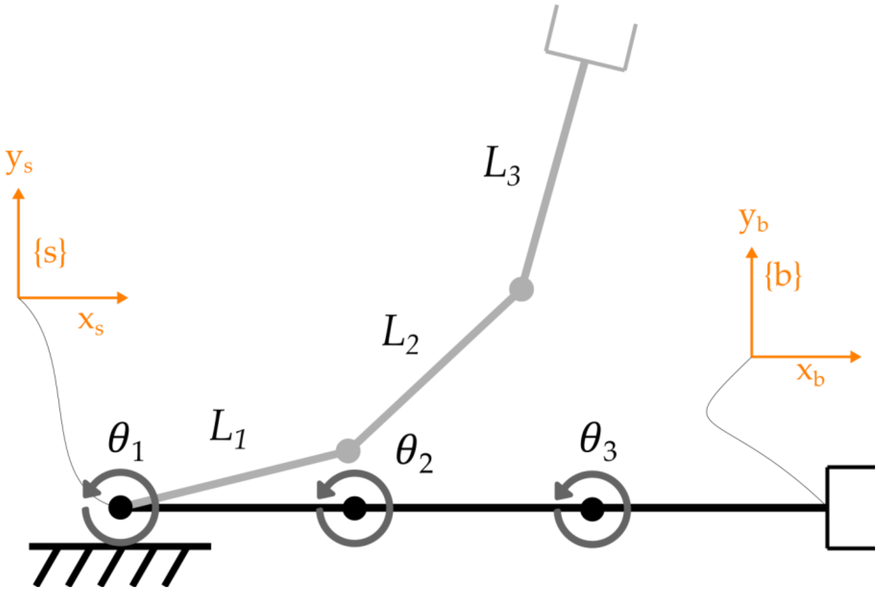


Example

Three-DoF robot arm.

Given joint values θ, what is the **body Jacobian**?
What is the **geometric Jacobian**?

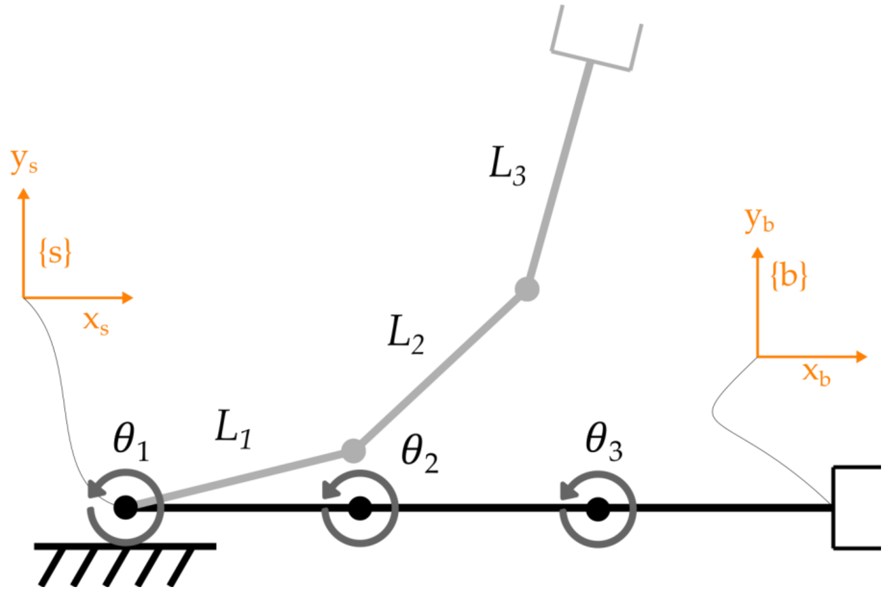
$$\boldsymbol{J_s}(\boldsymbol{\theta}) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3]$$



Example

Step 1. S_i is the screw for the i-th joint when the robot is in home position

$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -L_1 \\ 0 \end{bmatrix}, \qquad S_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -L_1 - L_2 \\ 0 \end{bmatrix}$$



Example

Step 2. Use adjoints to get each column of the space Jacobian

$$\boldsymbol{J_s}(\boldsymbol{\theta}) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3]$$

$$\boldsymbol{J_s}(\boldsymbol{\theta}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & L_1 s_1 & L_1 s_1 + L_2 s_{12} \\ 0 & -L_1 c_1 & -L_1 c_1 - L_2 c_{12} \\ 0 & 0 & 0 \end{bmatrix}$$

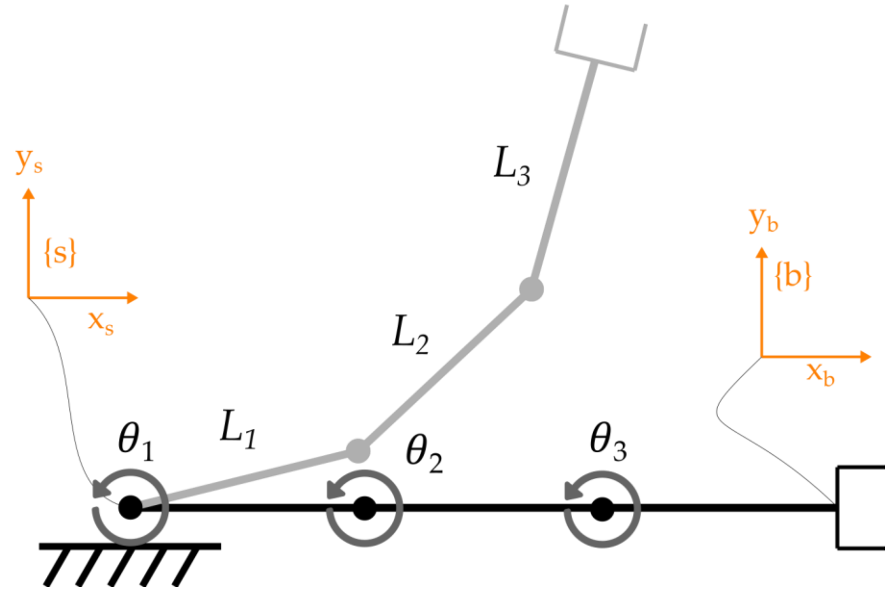
Example

Step 3. Convert space Jacobian to body Jacobian

$$\boldsymbol{J_b}(\boldsymbol{\theta}) = \underline{\mathrm{Ad}_{T_{sb}^{-1}}}\, J_s(\theta)$$

need forward kinematics

$$T_{sb} = \begin{bmatrix} c_{123} & -s_{123} & 0 & L_1c_1 + L_2c_{12} + L_3c_{123} \\ s_{123} & c_{123} & 0 & L_1s_1 + L_2s_{12} + L_3s_{123} \\ 0 & 0 & 1 & 0 \\ & 0 & & 1 \end{bmatrix}$$

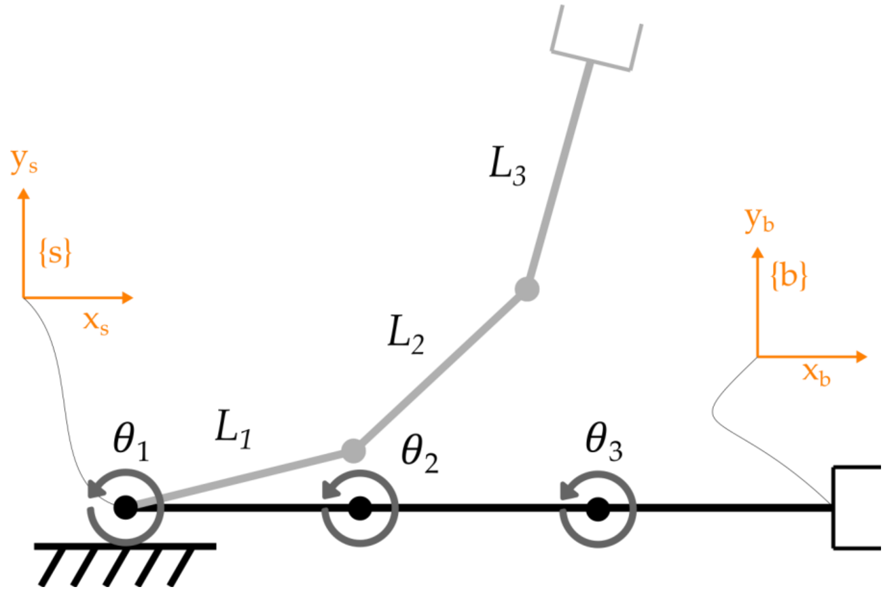


Example

Step 3. Convert space Jacobian to body Jacobian

$$\boldsymbol{J_b}(\boldsymbol{\theta}) = \mathrm{Ad}_{T_{sb}^{-1}} J_s(\theta)$$

$$\boldsymbol{J_b}(\boldsymbol{\theta}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ L_2 s_3 + L_1 s_{23} & L_2 s_3 & 0 \\ L_3 + L_2 c_3 + L_1 c_{23} & L_3 + L_2 c_3 & L_3 \\ 0 & 0 & 0 \end{bmatrix}$$



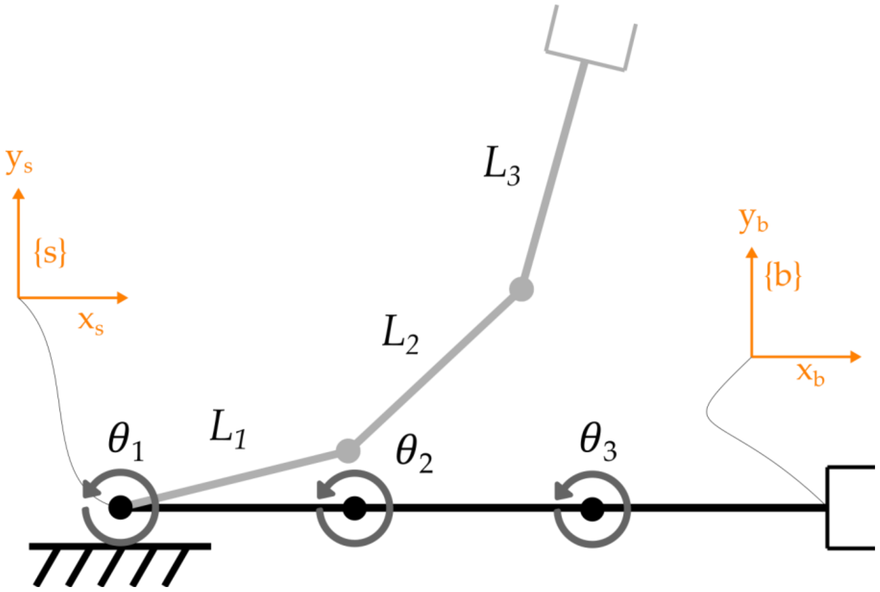
Example

Step 4. Convert body to geometric Jacobian

$$\boldsymbol{J}(\boldsymbol{\theta}) = \underline{\begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}} J_b(\theta)$$

forward kinematics $T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$

$$R = \begin{bmatrix} c_{123} & -s_{123} & 0 \\ s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

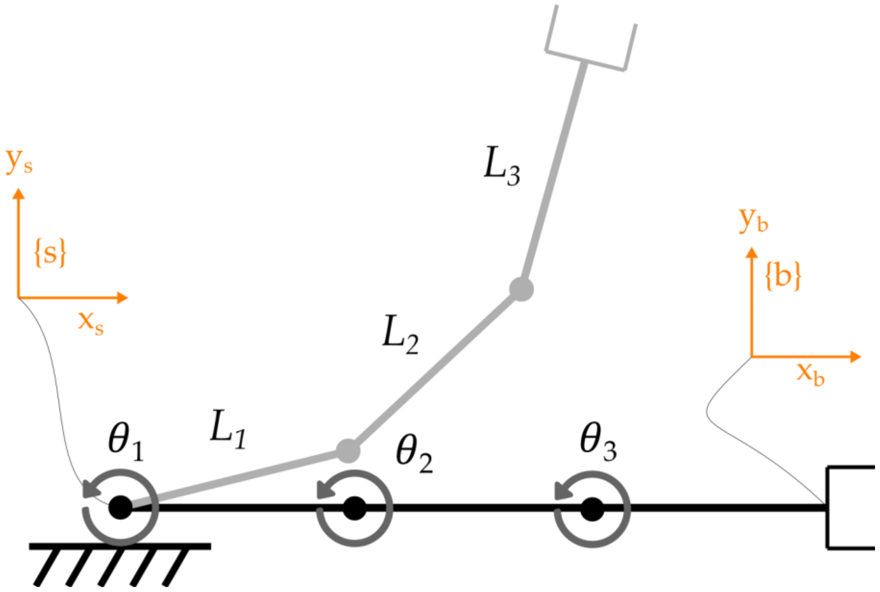


Example

Step 4. Convert body to geometric Jacobian

$$\boldsymbol{J}(\boldsymbol{\theta}) = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} J_b(\theta)$$

$$\boldsymbol{J}(\boldsymbol{\theta}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ -L_1 s_1 - L_2 s_{12} - L_3 s_{123} & -L_2 s_{12} - L_3 s_{123} & -L_3 s_{123} \\ L_1 c_1 + L_2 c_{12} + L_3 c_{123} & L_2 c_{12} + L_3 c_{123} & L_3 c_{123} \\ 0 & 0 & 0 \end{bmatrix}$$



This Lecture

- What are the different types of Jacobians?
- How do we convert from one Jacobian to another?

Next Lecture

- How should we interpret Jacobians?