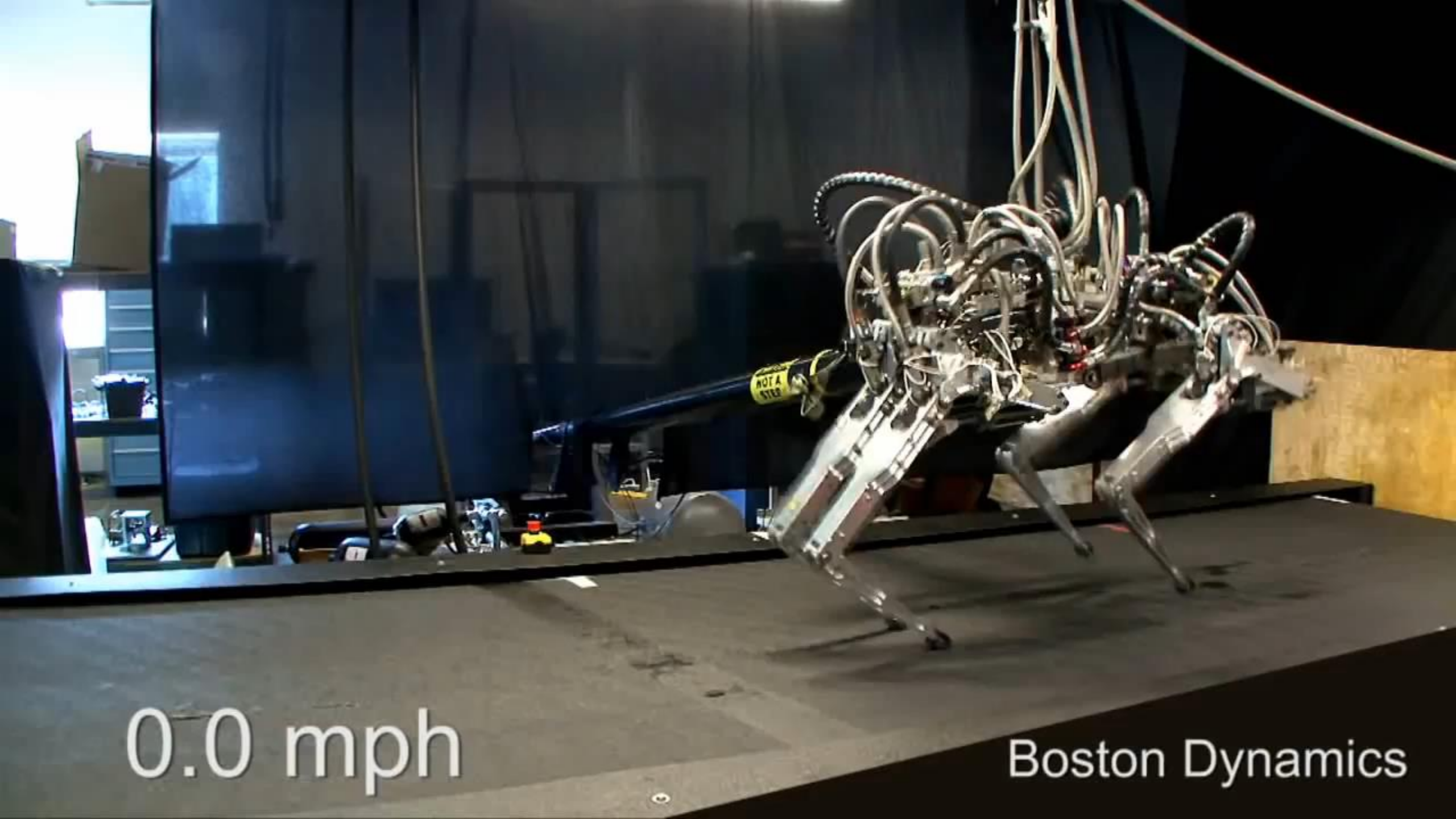


# Introducing the Jacobian



Reading: Modern Robotics 5.0 + 3.3.2



0.0 mph

Boston Dynamics

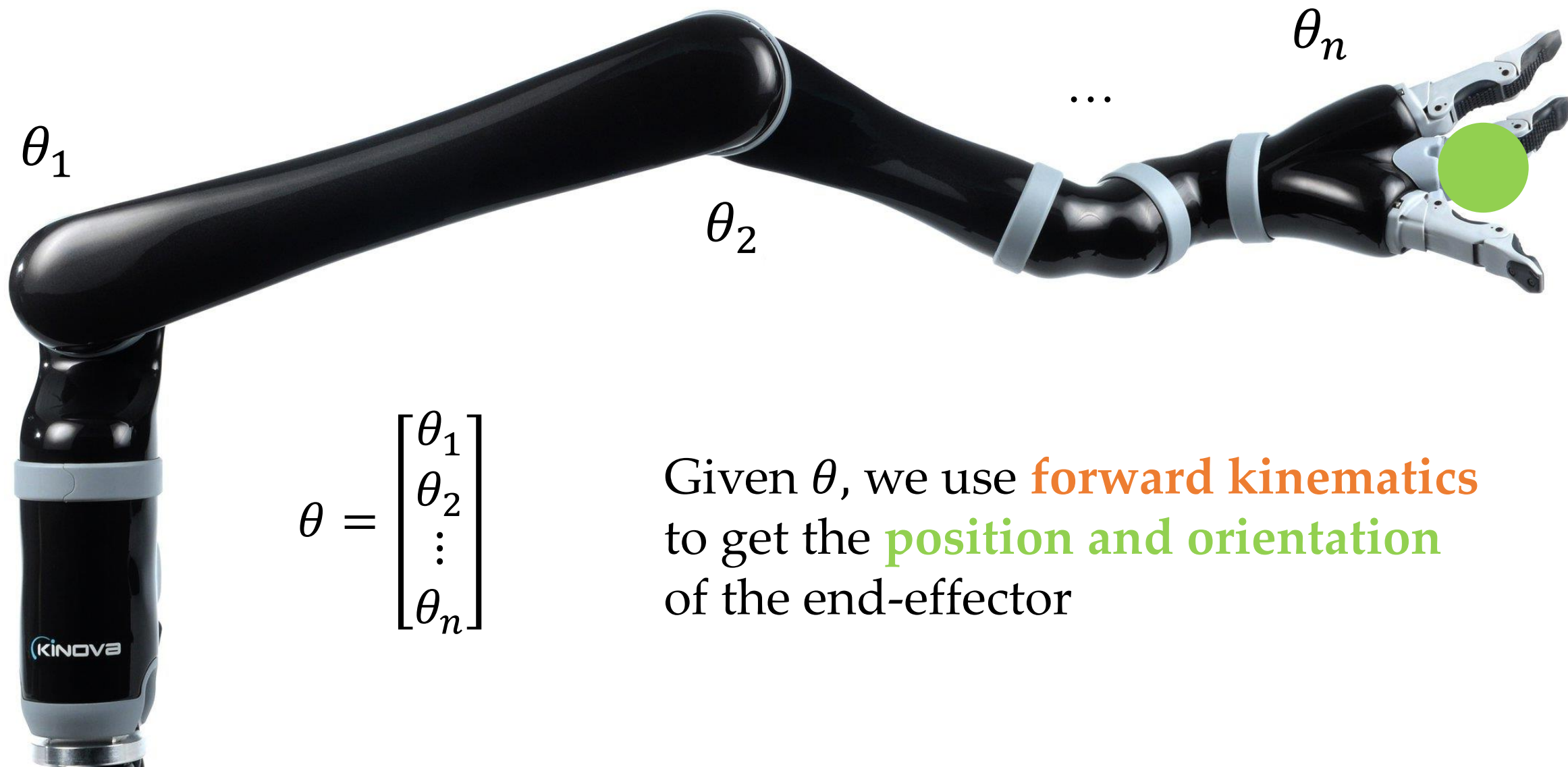
# This Lecture



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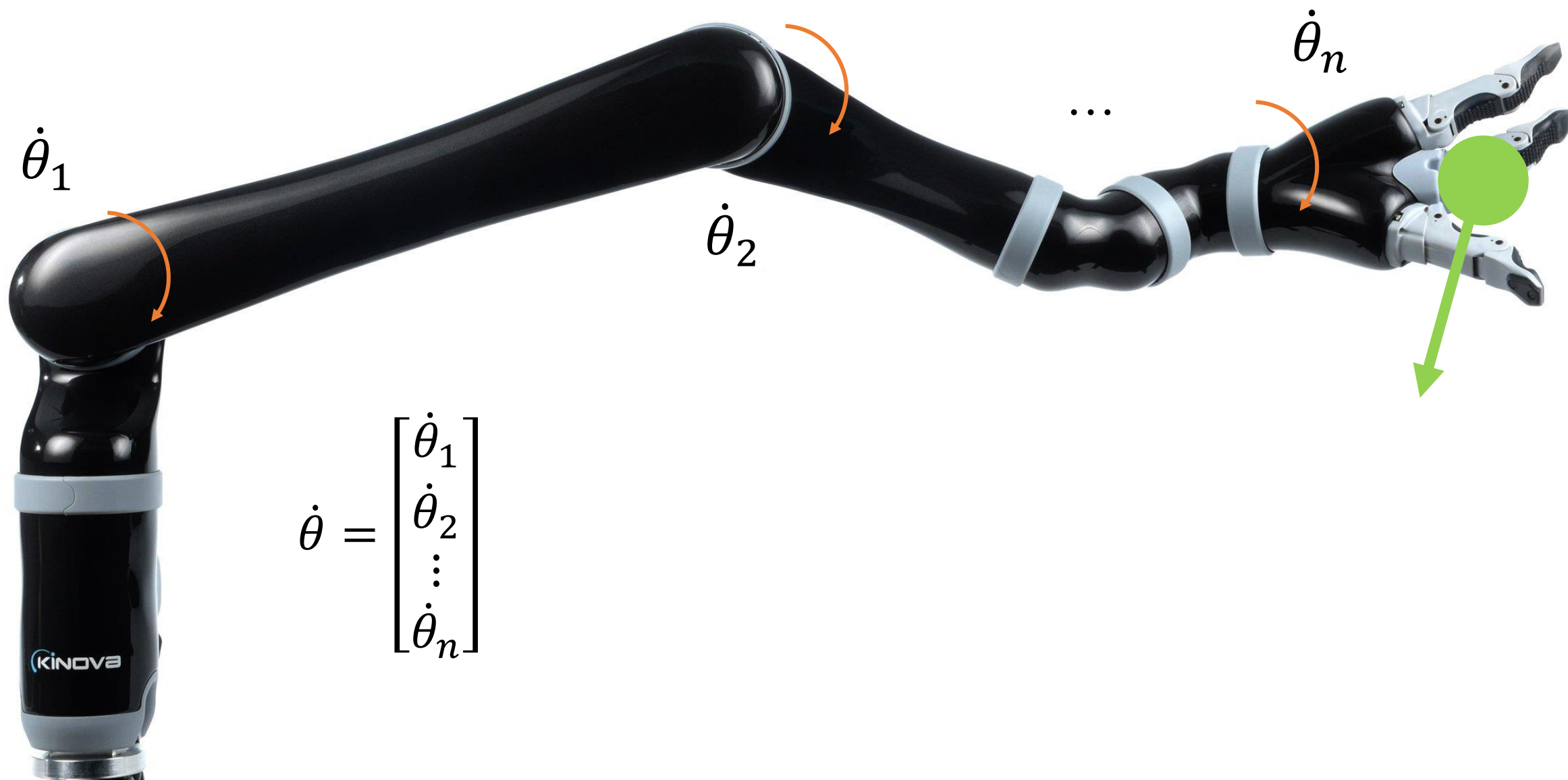
- What is the velocity of our robot arm?
- What is a robot Jacobian?
- How do we relate velocities in different frames?





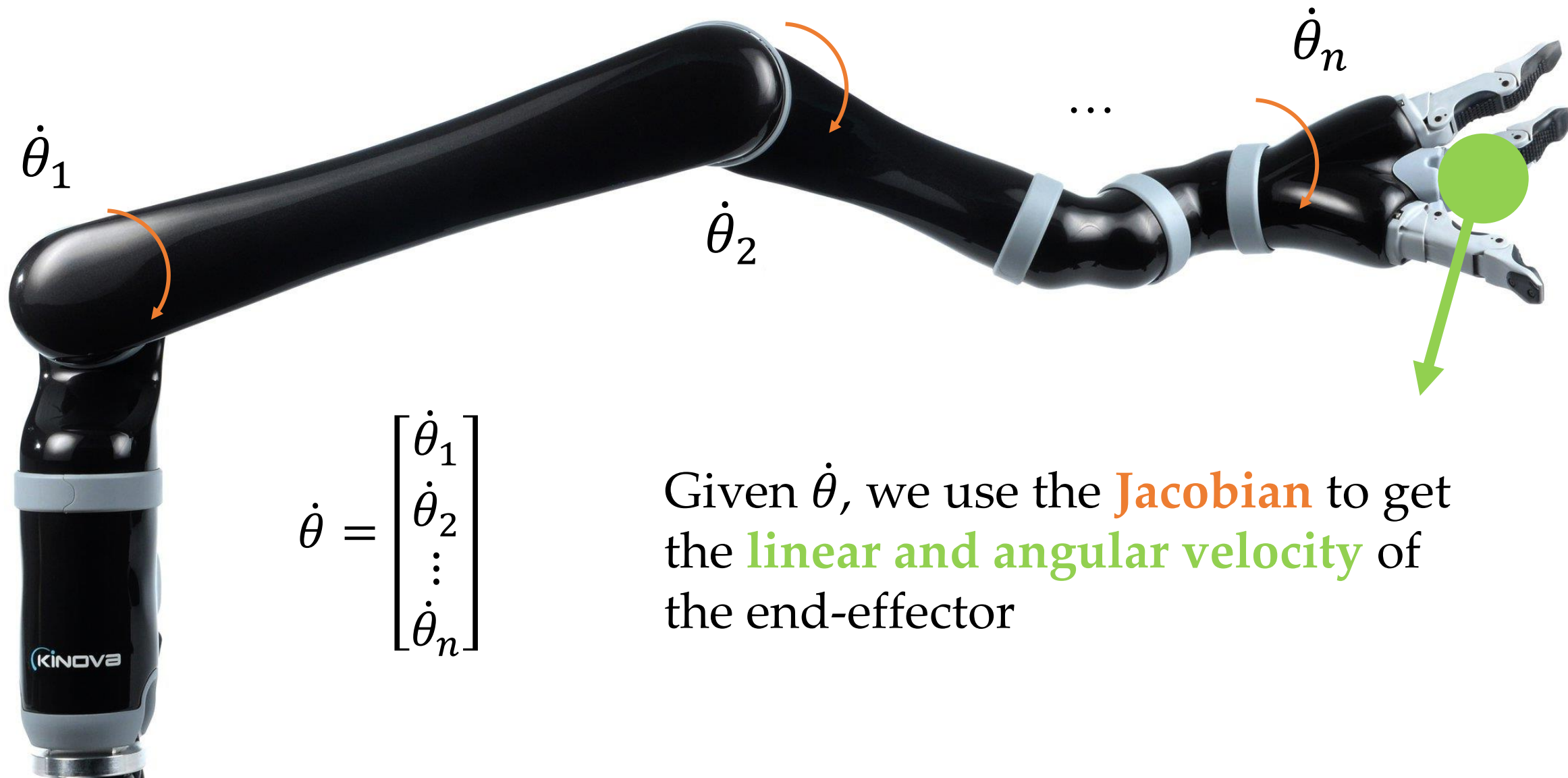
$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

Given  $\theta$ , we use **forward kinematics** to get the **position and orientation** of the end-effector



$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$





$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

Given  $\dot{\theta}$ , we use the **Jacobian** to get the **linear and angular velocity** of the end-effector

The background features three wireframe car models in a dark space with a grid pattern. The car in the upper left has red sensor waves emanating from its front. The car in the lower right has yellow sensor waves. A third car is partially visible in the upper right. The text 'What is the Jacobian?' is centered, with 'Jacobian' in orange and 'What is the' and '?' in white. A white horizontal line is positioned below the text.

What is the **Jacobian**?

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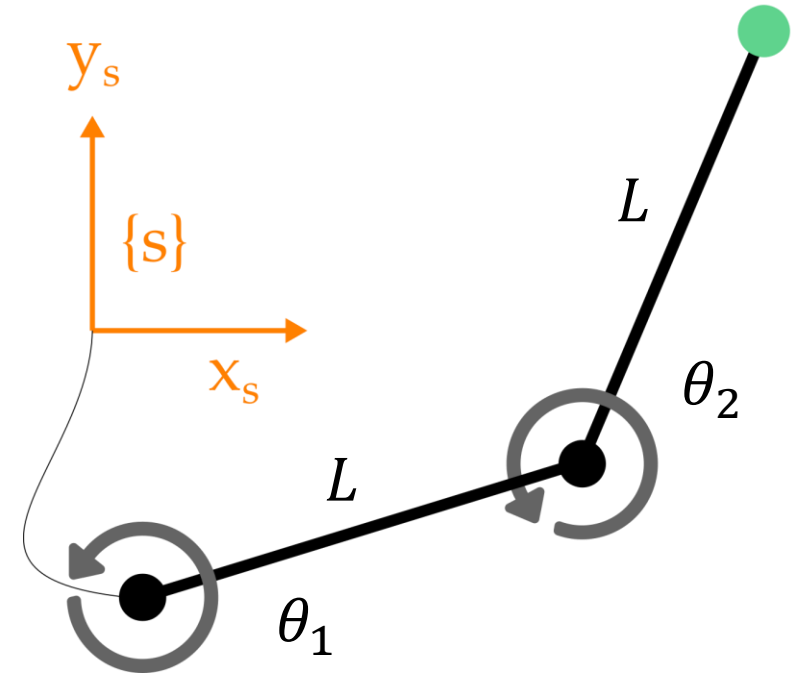
# Example

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Here the forward kinematics are:

$$\underline{p(t) = \begin{bmatrix} L \cos \theta_1(t) + L \cos(\theta_1(t) + \theta_2(t)) \\ L \sin \theta_1(t) + L \sin(\theta_1(t) + \theta_2(t)) \end{bmatrix} = f(\theta(t))}$$

position of end-effector in  $\{s\}$

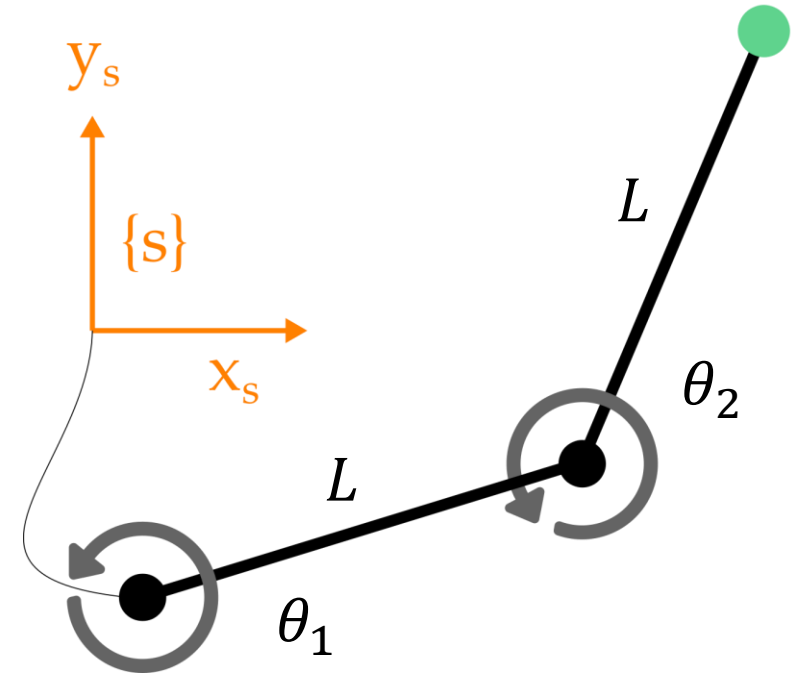


# Example

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The velocity of the end-effector is:

$$\dot{p}(t) = \frac{df}{dt} = \frac{\partial f(\theta)}{\partial \theta} \frac{d\theta(t)}{dt}$$

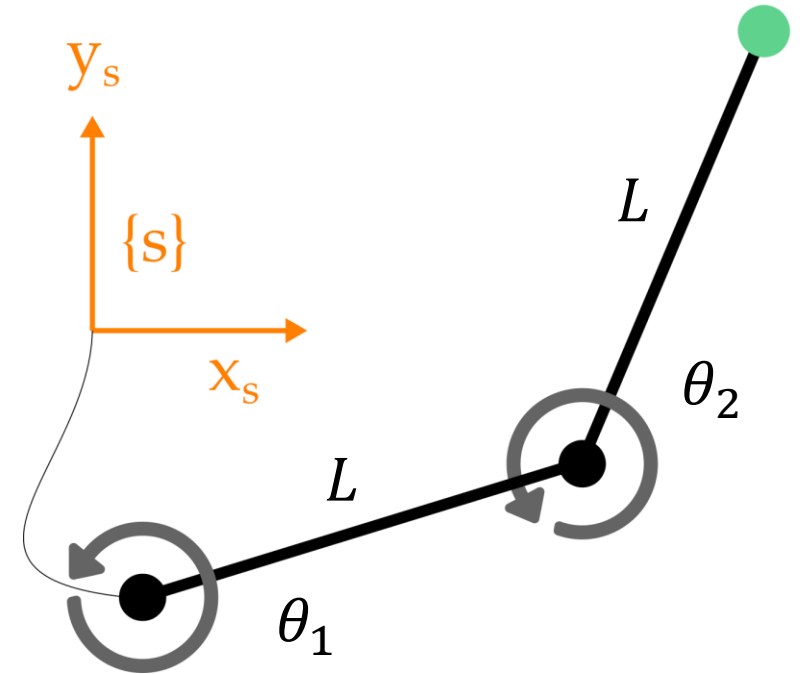


# Example

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The velocity of the end-effector is:

$$\dot{p} = \frac{\partial f(\theta)}{\partial \theta} \dot{\theta}$$



# Example

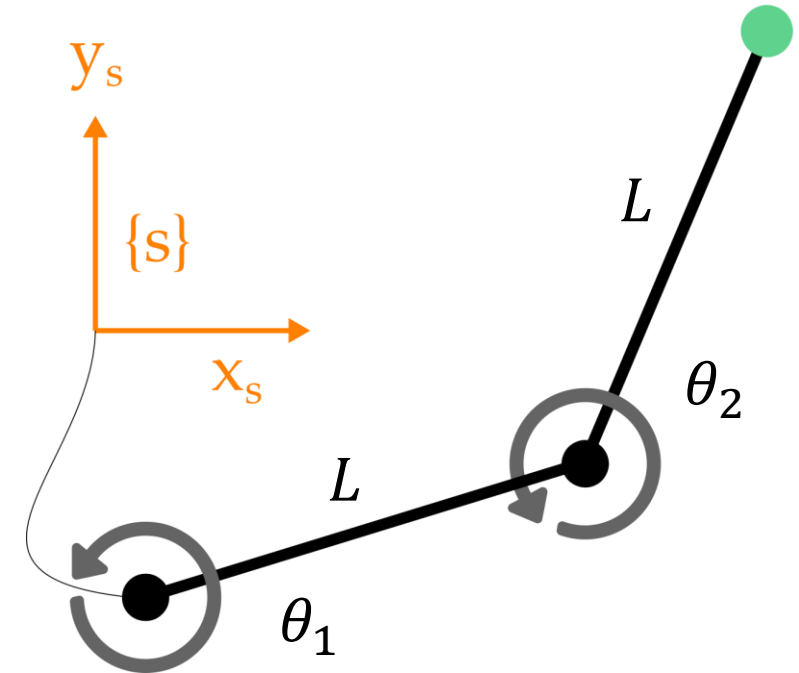
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The velocity of the end-effector is:

$$\dot{p} = \frac{\partial f(\theta)}{\partial \theta} \dot{\theta}$$

$$\dot{p} = J(\theta) \dot{\theta}$$

We refer to  $J(\theta)$  as the **Jacobian**



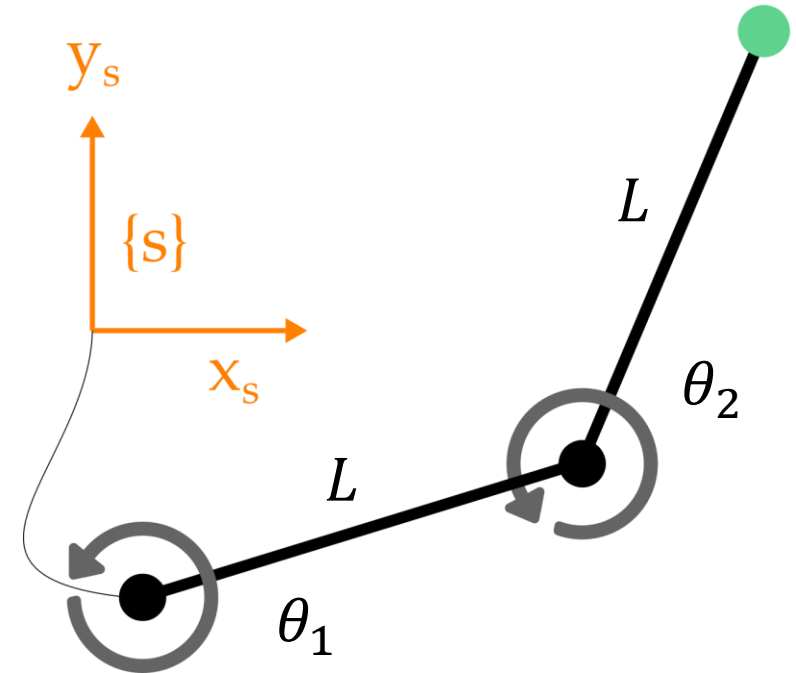
# Example

---

The velocity of the end-effector is:

$$\dot{p} = J(\theta)\dot{\theta}$$

$$p(t) = \begin{bmatrix} L \cos \theta_1(t) + L \cos(\theta_1(t) + \theta_2(t)) \\ L \sin \theta_1(t) + L \sin(\theta_1(t) + \theta_2(t)) \end{bmatrix}$$





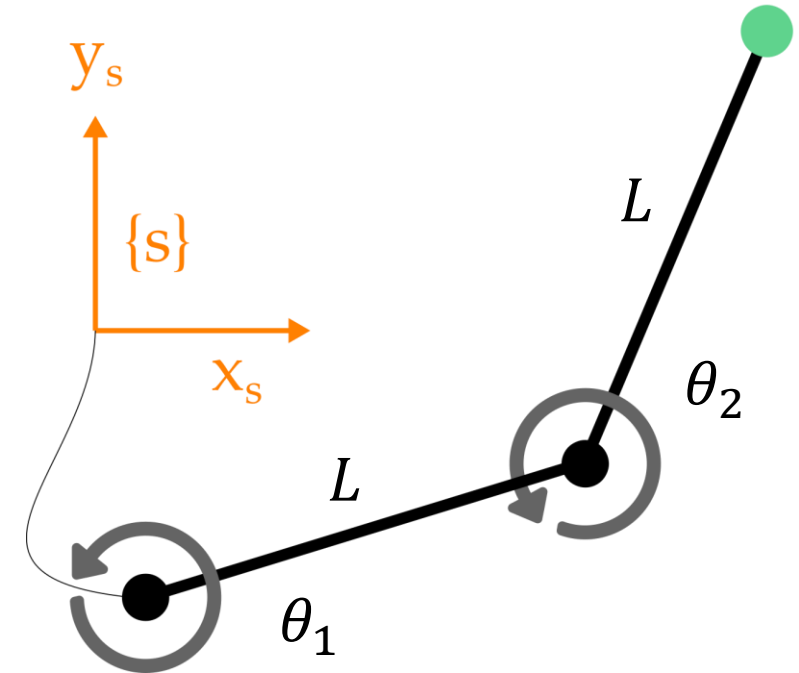
# Example

---

The velocity of the end-effector is:

$$\dot{p} = J(\theta)\dot{\theta}$$

$$\dot{p} = \begin{bmatrix} -L\dot{\theta}_1 \sin \theta_1 - L(\dot{\theta}_1 + \dot{\theta}_2) \sin(\theta_1 + \theta_2) \\ L\dot{\theta}_1 \cos \theta_1 + L(\dot{\theta}_1 + \dot{\theta}_2) \cos(\theta_1 + \theta_2) \end{bmatrix}$$



# Example

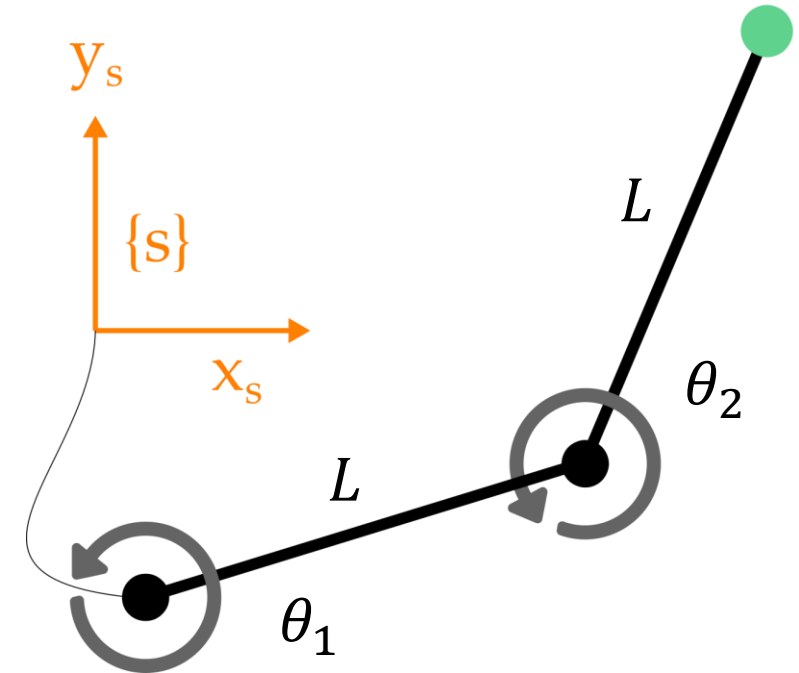
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The velocity of the end-effector is:

$$\dot{p} = J(\theta)\dot{\theta}$$

$$\dot{p} = \begin{bmatrix} -L \sin \theta_1 - L \sin(\theta_1 + \theta_2) & -L \sin(\theta_1 + \theta_2) \\ L \cos \theta_1 + L \cos(\theta_1 + \theta_2) & L \cos(\theta_1 + \theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

this  $2 \times 2$  matrix is this robot's **Jacobian**



A wireframe model of a car is shown in a dark, space-like environment with a grid of stars. The car is depicted with several concentric, glowing orange and yellow lines around it, suggesting motion or a field of influence. The text is overlaid on this background.

The **Jacobian** is a matrix  
that maps **joint velocity** to  
**end-effector** velocity

---

# Adjoint

To find the Jacobian, we need to a tool for **relating velocities** in different frames

$$V_s = \left[ \quad \quad \right] V_b$$

---

Given body twist, how do we get  
spatial twist (and vice versa)?

# Adjoint

To find the Jacobian, we need to a tool for **relating velocities** in different frames

$$\begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix} \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}$$

---

This  $6 \times 6$  matrix is the **adjoint**.  
Obtained by rearranging the  
equations for  $V_s$  and  $V_b$



# Adjoint

**Definition.** For transformation matrix  $T$  the adjoint is:

$$\text{Ad}_T = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}$$

$$(\text{Ad}_T)^{-1} = \text{Ad}_{T^{-1}}$$

---

*Useful property of adjoints*

# Adjoint

**Application.** For transformation matrix  $T_{sb}$  we have that:

$$V_s = \mathbf{Ad}_{T_{sb}} V_b$$

$$V_b = \mathbf{Ad}_{T_{sb}^{-1}} V_s$$

# This Lecture



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- What is the velocity of our robot arm?
- What is a robot Jacobian?
- How do we relate velocities in different frames?

# Next Lecture



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- How do we get the Jacobian of a robot arm?