Screws

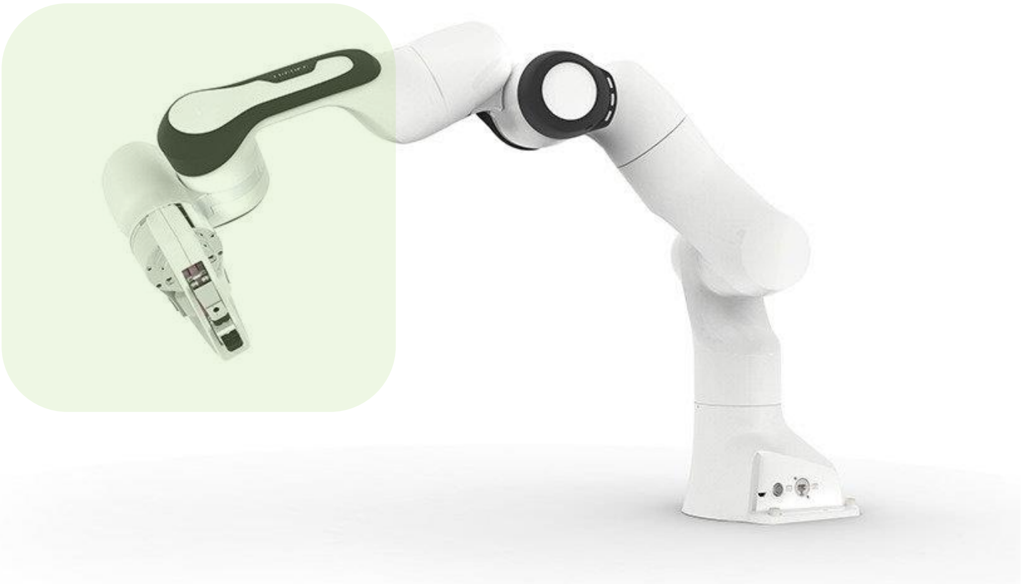
Reading: Modern Robotics 3.3.2 – 3.3.3

This Lecture

- What twists are associated with prismatic and revolute joints?
- What are screws?
- How can we use screws to find the pose of a moving joint?


hello robot

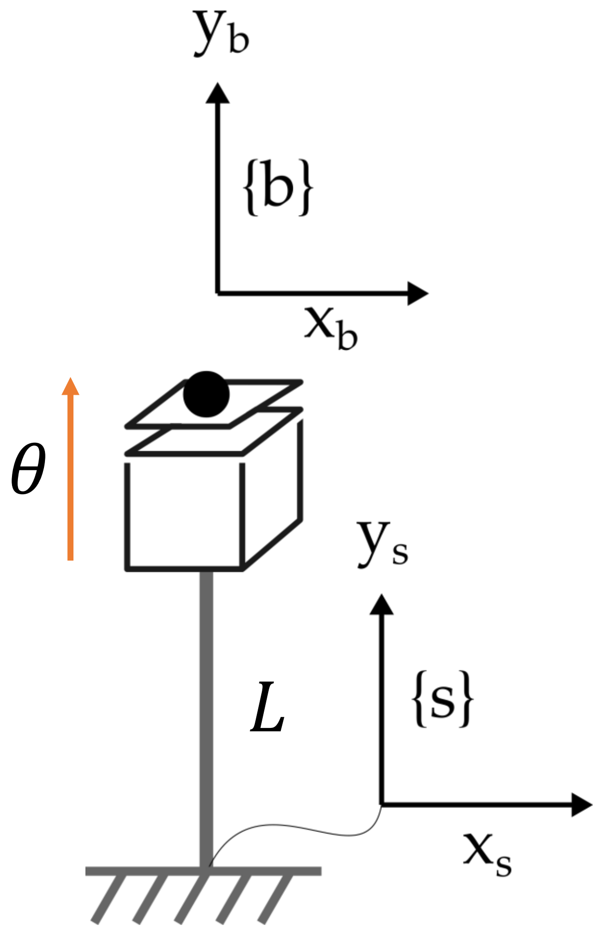
hello robot



Prismatic Joints

Pure translation. 1-DoF joint that enables the link to translate but not rotate

$$V_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \begin{bmatrix} \omega_s \\ -\omega_s \times p + \dot{p} \end{bmatrix}$$



Prismatic Joints

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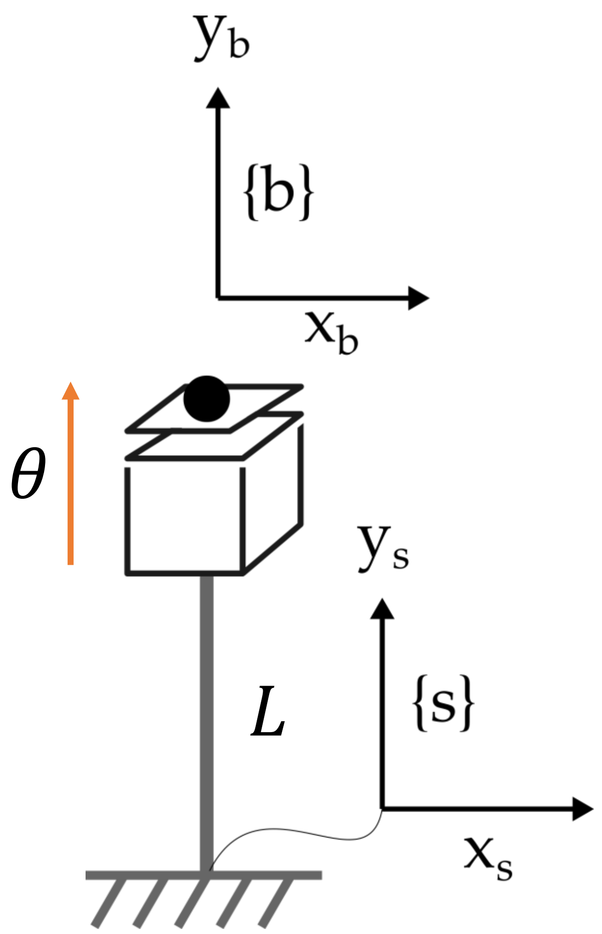
$$V_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \underbrace{\begin{bmatrix} \omega_s \\ -\omega_s \times p + \dot{p} \end{bmatrix}}_{\omega_s = 0}$$

$$\boxed{V_s = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{p} \end{bmatrix}}$$



Revolute Joints

Pure rotation. 1-DoF joint that enables the link to rotate but not translate

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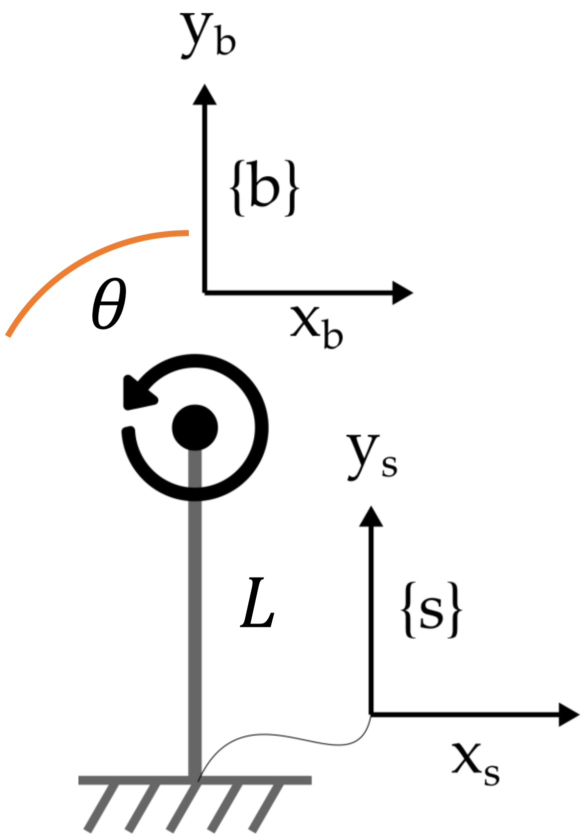

Revolute Joints

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$\dot{p} = 0$

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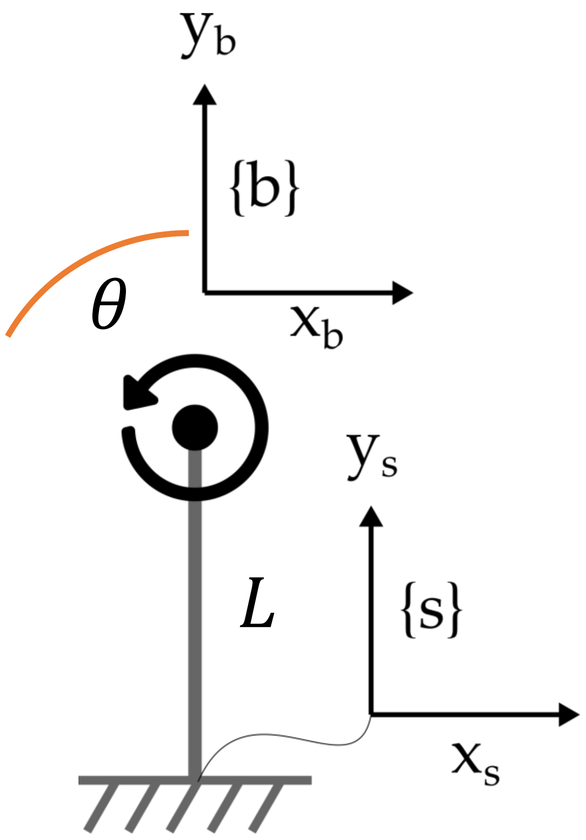


Screws

A screw is a **normalized twist**:

$$S = \begin{bmatrix} 0 \\ v_s \end{bmatrix} \qquad S = \begin{bmatrix} \omega_s \\ -\omega_s \times p \end{bmatrix}$$

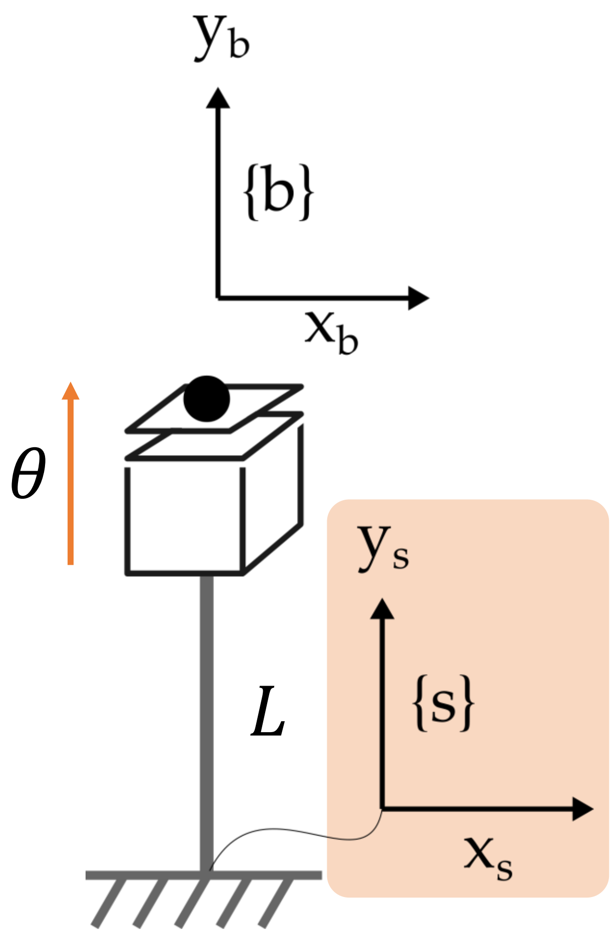
prismatic **revolute**

Prismatic: $\omega_s = 0$ and v_s is a unit vector
Revolute: ω_s is a unit vector and $v_s = -\omega_s \times p$

Prismatic Joints

Pure translation. 1-DoF joint that enables the link to translate but not rotate

$$S = \begin{bmatrix} 0 \\ v_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

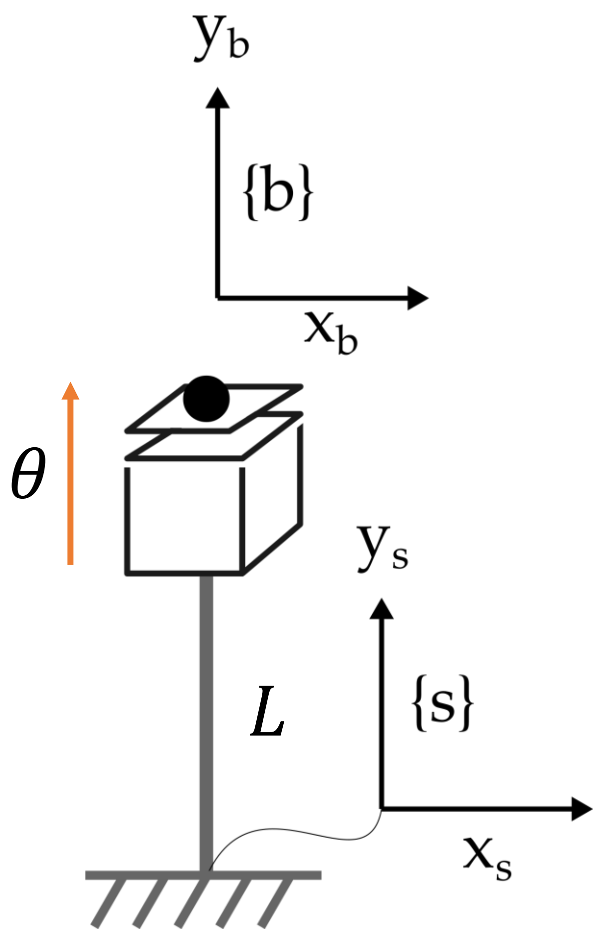


Prismatic Joints

Pure translation. 1-DoF joint that enables the link to translate but not rotate

$$S = \begin{bmatrix} 0 \\ v_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

unit vector

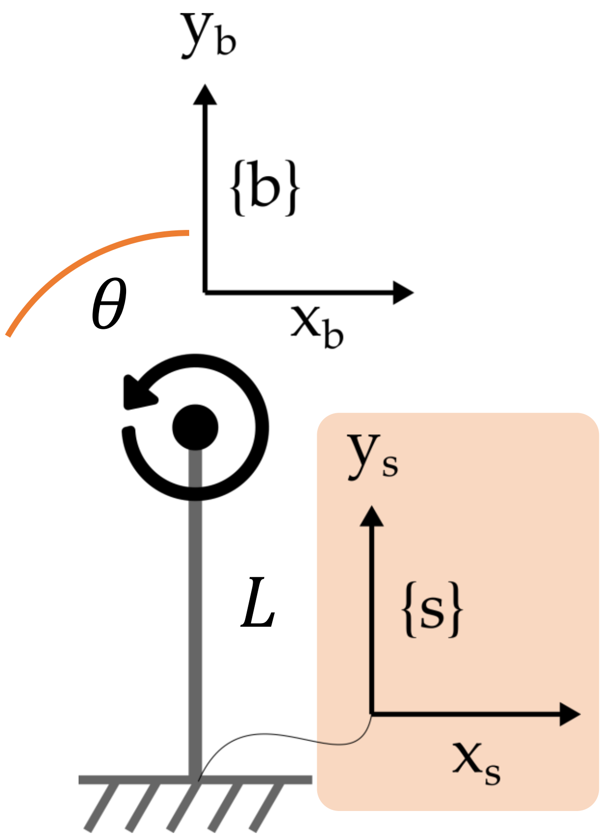


Revolute Joints

Pure rotation. 1-DoF joint that enables the link to rotate but not translate

$$S = \begin{bmatrix} \omega_s \\ -\omega_s \times p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ L \\ 0 \\ 0 \end{bmatrix}$$

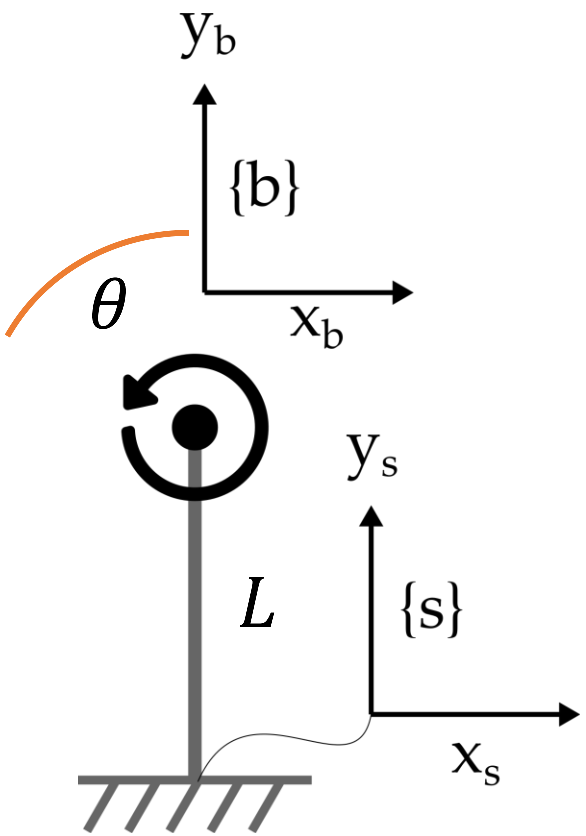
$$p = p_{sb} = \begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix}$$



Revolute Joints

Pure rotation. 1-DoF joint that enables the link to rotate but not translate

$$S = \begin{bmatrix} \omega_s \\ -\omega_s \times p \end{bmatrix} = \begin{array}{c} \textit{unit vector} \\ \begin{bmatrix} \boxed{\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}} \\ L \\ 0 \\ 0 \end{bmatrix} \end{array}$$



We can use screws to capture the **motion** caused by a joint.

Joint Motion

We use $[S]$ notation the same as $[V]$ to write the screw as a 4×4 matrix

$$\dot{T} = [S]T$$

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$$\dot{T} = [S]T$$

$$\boxed{T(\theta) = e^{[S]\theta}T(0)}$$

$T(0)$ is the initial transformation from $\{s\}$ to $\{b\}$

Joint Motion

We use $[S]$ notation the same as $[V]$ to write the screw as a 4×4 matrix

$$\dot{T} = [S]T$$

$$\boxed{T(\theta) = e^{[S]\theta}\,T(0)}$$

$e^{[S]\theta}$ is a transformation matrix. *See expm in matlab*.
This captures the motion in the **fixed frame**.

Joint Motion

We use $[S]$ notation the same as $[V]$ to write the screw as a 4×4 matrix

$$\dot{T} = [S]T$$

$$\boxed{T(\theta) = e^{[S]\theta}T(0)}$$

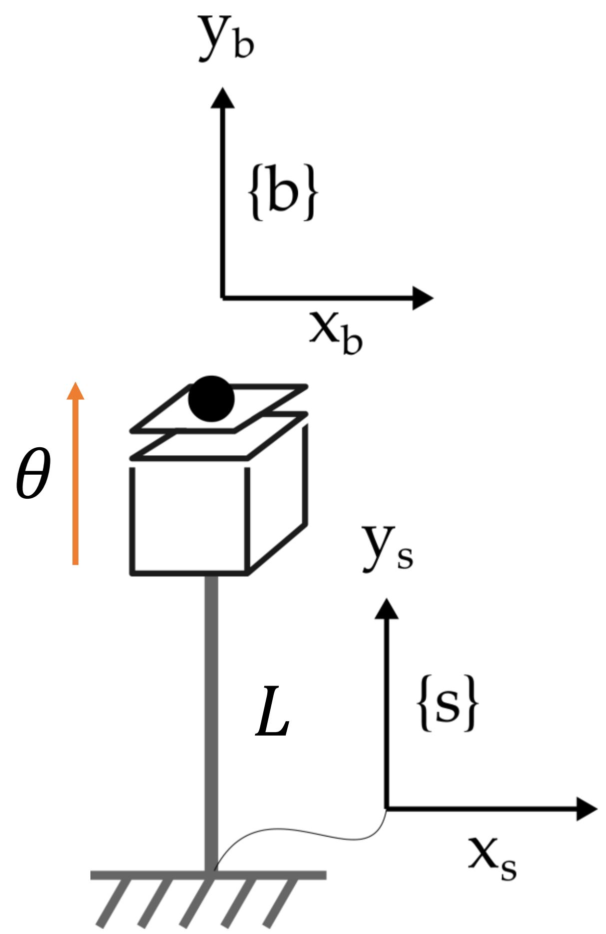
$T(\theta)$ is the new transformation from $\{s\}$ to $\{b\}$ after **translating the prismatic joint by θ units** or **rotating the revolute joint by θ units**

Let’s see some examples!

Prismatic Joints

By looking at the drawing, we found:

$$S = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad T(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L \\ 0 & 0 & 1 & 0 \\ & 0 & & 1 \end{bmatrix}$$



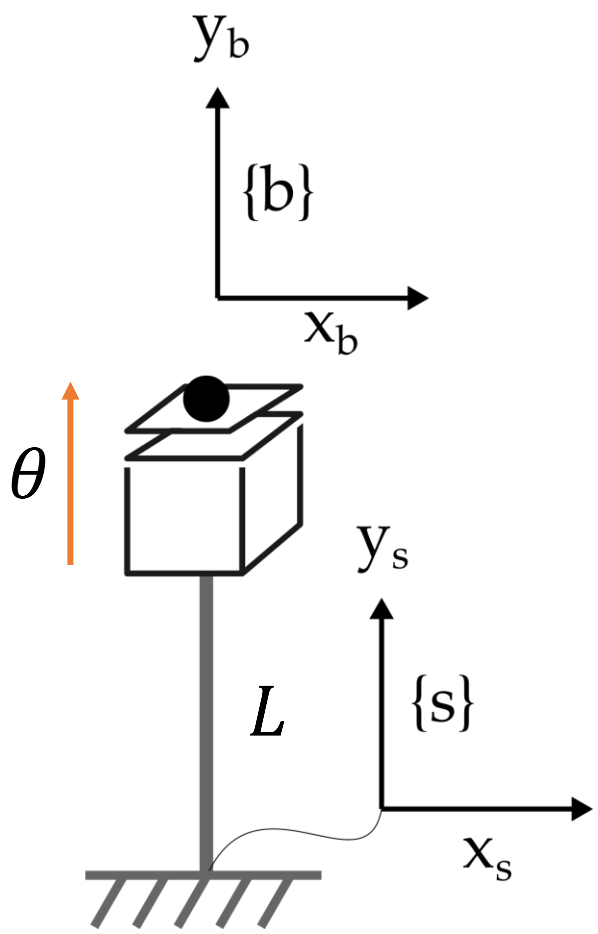
Prismatic Joints

Converting screws to joint motion:

$$T(\theta) = e^{[S]\theta}T(0)$$

$$T(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L+\theta \\ 0 & 0 & 1 & 0 \\ & 0 & & 1 \end{bmatrix}$$

{b} translates along y_s axis



Revolute Joints

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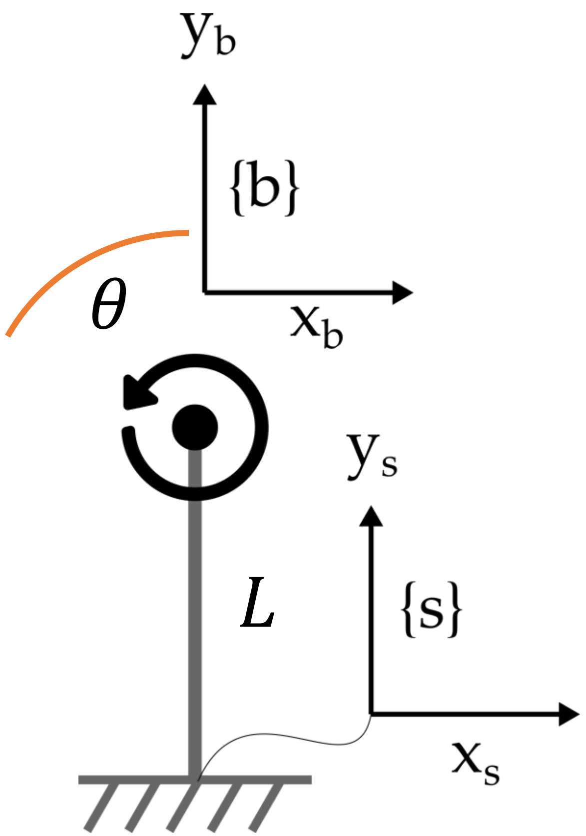


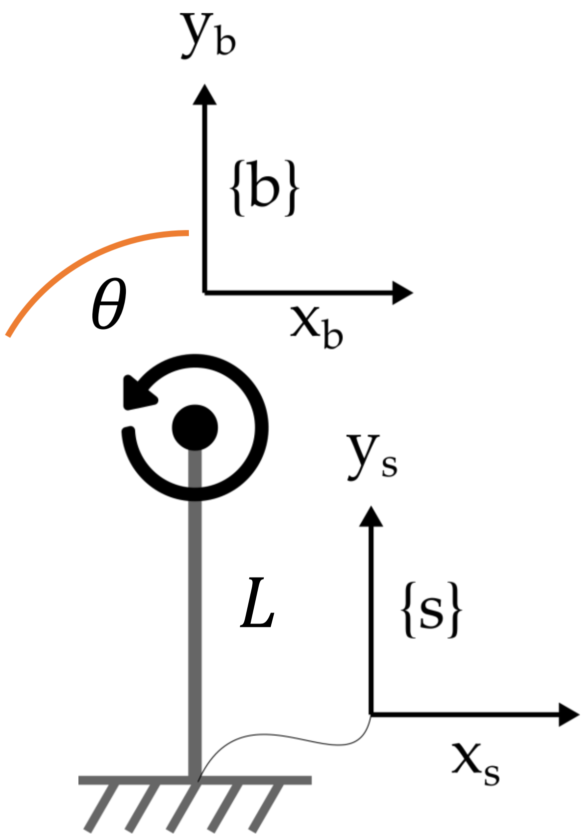
Revolute Joints

Converting screws to joint motion:

$$T(\theta) = e^{[S]\theta}T(0)$$

$$T(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & L \\ 0 & 0 & 1 & 0 \\ & 0 & & 1 \end{bmatrix}$$

{b} rotates around z_s axis



This Lecture

- What twists are associated with prismatic and revolute joints?
- What are screws?
- How can we use screws to find the pose of a moving joint?

Next Lecture

- How can we extend this to find the pose of a robot arm?