Practice Set 4

Robotics & Automation
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Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

Given two vectors x and y, the cross product is:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \qquad x \times y = \begin{bmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{bmatrix} \tag{1}$$

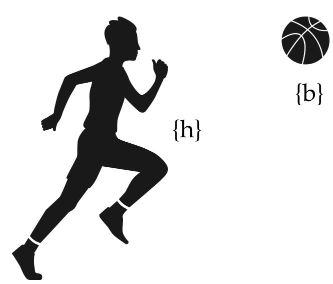
Prove that $[x]y$ is equal to $x \times y$.

Problem 2

Write a function that computes the rotation matrix R from an axis ω and an angle θ. Then use that function to find the following rotation matrices:

- Let $\omega = [1, 0, 0]^T$ and leave θ as a symbolic variable
- Let $\omega = [0, 0, 1]^T$ and let $\theta = 5$
- Let $\omega = [0, 0, 5]^T$ and let $\theta = 1$. Compare this to your previous answer; can you explain why they are the same?

Problem 3



You're spinning a basketball. The angular velocity of the spinning ball is:

$$\omega_h = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} \text{ rad/s} \tag{2}$$

Initially $R_{hb} = I$. If this basketball spins for 3 seconds, what is the new rotation R_{hb}?

Problem 1

Given two vectors x and y, the cross product is:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad x \times y = \begin{bmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{bmatrix} \tag{3}$$

Prove that $[x]y$ is equal to $x \times y$.

Use the definition of a screw symmetric matrix:

$$[x]y = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -x_3y_2 + x_2y_3 \\ x_3y_1 - x_1y_3 \\ -x_2y_1 + x_1y_2 \end{bmatrix} \tag{4}$$

Problem 2

Write a function that computes the rotation matrix R from an axis ω and an angle θ. Then use that function to find the following rotation matrices:

```
function R = axisangle(omega, theta)
    R = expm(skew(omega) * theta);

    function bracket_x = skew(x)
        bracket_x = [0 -x(3) x(2);
            x(3) 0 -x(1); -x(2) x(1) 0];
    end
end
```

- Let $\omega = [1, 0, 0]^T$ and leave θ as a symbolic variable

  To make θ a symbolic variable in Matlab, use:
  `syms theta real`
  The `real` specifies that θ is a real number. Our code outputs:

  $$R = e^{[\omega]\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{5}$$

  You should recognize that this is a rotation about the x-axis.

- Let $\omega = [0, 0, 1]^T$ and let $\theta = 5$

  $$R = e^{[\omega]\theta} = \begin{bmatrix} 0.284 & 0.959 & 0 \\ -0.959 & 0.284 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

- Let $\omega = [0, 0, 5]^T$ and let $\theta = 1$. Compare this to your previous answer; can you explain why they are the same?

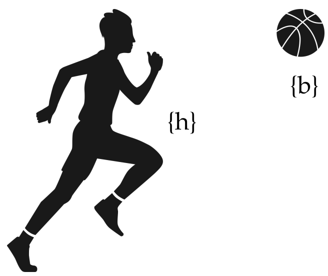
$$R = e^{[\omega]\theta} = \begin{bmatrix} 0.284 & 0.959 & 0 \\ -0.959 & 0.284 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Let's define $\omega_1 = [0, 0, 1]^T$ and $\omega_2 = [0, 0, 5]^T$. Because $\omega_2 = 5\omega_1$, we have that $[\omega_2] = [\omega_1] \cdot 5$. More generally, let ω be a vector and define $\hat{\omega}$ as the corresponding unit vector:

$$e^{[\omega]\theta} = e^{[\hat{\omega}]\cdot\|\omega\|\theta} \tag{8}$$

In practice, this means that you do not need to normalize the axis vector ω. However, if the axis is not normalized, just remember that the magnitude of that vector contributes to how much you rotate.

Problem 3



You're spinning a basketball. The angular velocity of the spinning ball is:

$$\omega_h = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} \text{ rad/s} \tag{9}$$

Initially $R_{hb} = I$. If this basketball spins for 3 seconds, what is the new rotation R_{hb}?

This question is asking us to find the rotation associated with spinning at angular velocity ω_h for 3 seconds. We can directly use our axis-angle formula:

$$R_{hb} = e^{[\omega_h]\theta} \tag{10}$$

where $\theta = 3$. Plugging in:

$$R_{hb} = \begin{bmatrix} 0.96 & 0 & 0.28 \\ 0 & 1 & 0 \\ -0.28 & 0 & 0.96 \end{bmatrix} \tag{11}$$