Problem Set 1

Robotics & Automation
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Instructions. Please write legibly and do not attempt to fit your work into the smallest space possible. It is important to show all work, but basic arithmetic can be omitted. You are encouraged to use Matlab when possible to avoid hand calculations, but print and submit your commented code for non-trivial calculations. You can attach a pdf of your code to the homework, use live scripts or the publish feature in Matlab, or include a snapshot of your code. Do not submit .m files — we will not open or grade these files.

1 Linear Algebra Review

When working with robots — from self-driving cars to assistive arms — we often leverage sets of vectors to describe where our robot is in space, how it moves, and how it reacts to forces. When we use these sets of vectors, one important concept is their *linear independence* or *linear dependence*. A set of vectors $\{v_1, v_2, \ldots, v_n\}$ is linearly independent if the coefficients $c_1, c_2, \ldots, c_n$ must all be zero in order for $c_1 v_1 + c_2 v_2 + \ldots + c_n v_n$ to equal zero.

1.1 (5 points)

Prove that the following vectors are linearly independent:

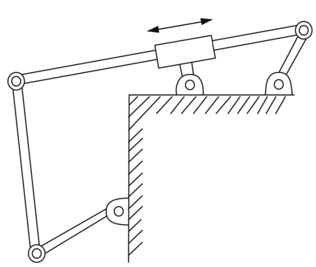
$$v_1 = [1, 0, 0]^T, \quad v_2 = [1, 0, 1]^T, \quad v_3 = [0, 1, 1]^T \tag{1}$$

1.2 (5 points)

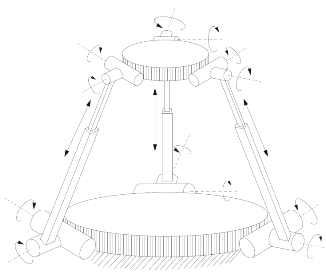
Prove that the following vectors are linearly dependent:

$$v_1 = [1, 1, 0]^T, \quad v_2 = [1, 2, 1]^T, \quad v_3 = [0, 1, 1]^T \tag{2}$$

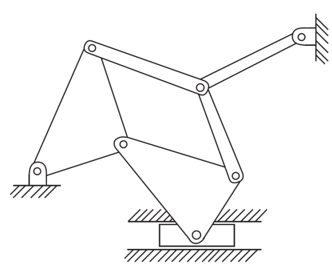
2 Degrees-of-Freedom



(a)



(b)



(c)

Imagine that you are developing a new robot. Three very different mechanism designs have been proposed. For each design, you want to determine how many degrees-of-freedom the robot has. In (a) one single link is translating through the indicated slider. In (b) each of the legs are composed of a universal joint at the base, a prismatic joint, and a universal joint joint at the platform.

2.1 (5 points)

How many degrees of freedom does robot (a) have?

2.2 (10 points)

How many degrees of freedom does robot (b) have?

2.3 (5 points)

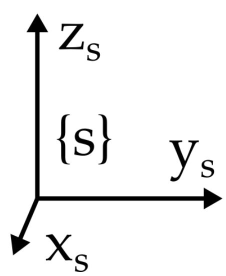
How many degrees of freedom does robot (c) have?

2.4 (5 points)

Once you have chosen your design you will need to use actuators to move the robot. How many actuators do you need to control robot (a)?

3 Properties of Rotation Matrices

3.1 (5 points)



Using $\{s\}$, draw coordinate frame $\{b\}$ if

$$R_{sb} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{3}$$

Point p has position $p_b = [0, 0, -2]^T$ in frame $\{b\}$. Draw this point in coordinate frame $\{s\}$. What is its position with respect to $\{s\}$?

3.2 (5 points)

If R is a rotation matrix, we require that $\det(R) = +1$. Why is this? Your answer should include a drawing of a coordinate frame X where $\det(X) = -1$.

3.3 (5 points)

Given vector x and rotation matrices R_1, R_2, and R_3, prove that $y = R_1 R_2 R_3 x$ has the same magnitude as x. **Hint –** define magnitude (i.e., the length) of vector x as $\|x\| = \sqrt{x^T x}$

3.4 (5 points)

Given two rotation matrices R_1 and R_2 in 2D space, prove that these rotation matrices are always commutative. When we rotate in a plane, the rotation matrix is:

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{4}$$

4 Implementing Rotation Matrices

4.1 (5 points)

Program the following functions. Here $Rot(x, \theta) = \texttt{rotx}(\theta)$ means we rotate around the x-axis by θ radians.

- `R = rotx(`θ`)`
- `R = roty(`θ`)`
- `R = rotz(`θ`)`

4.2 (5 points)

Multiply the following rotation matrices:

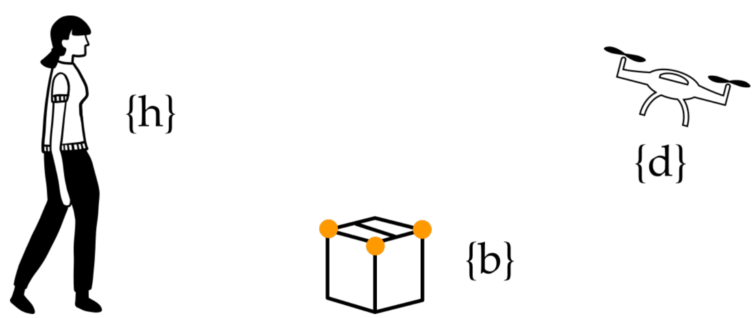
$$R = Rot(z, \pi/4)Rot(y, -\pi/3)Rot(z, \pi/2) \tag{5}$$

Prove that your answer R is also a rotation matrix.

4.3 (5 points)

Using R from the previous part, show that $R^{-1} = R^T$.

5 Using Rotation Matrices



A drone is picking up and delivering packages. In order to locate these packages the drone has an attached imaging system, which can detect the corners of a box. From the drone's perspective the orientation of the box is:

$$R_{db} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} \tag{6}$$

A person is also looking at the box. From their perspective the orientation of the box is:

$$R_{hb} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

5.1 (10 points)

What is the orientation of the person relative to the drone?

5.2 (5 points)

What is the orientation of the drone relative to the person?

5.3 (15 points)

From the human's perspective the position of the box is $p_h = \overrightarrow{hb}_h = [1, 2, 0]^T$. From the drone's perspective the position of the box is $p_d = \overrightarrow{db}_d = [2, 0, -2]^T$.

What is the position of the drone in frame $\{h\}$?