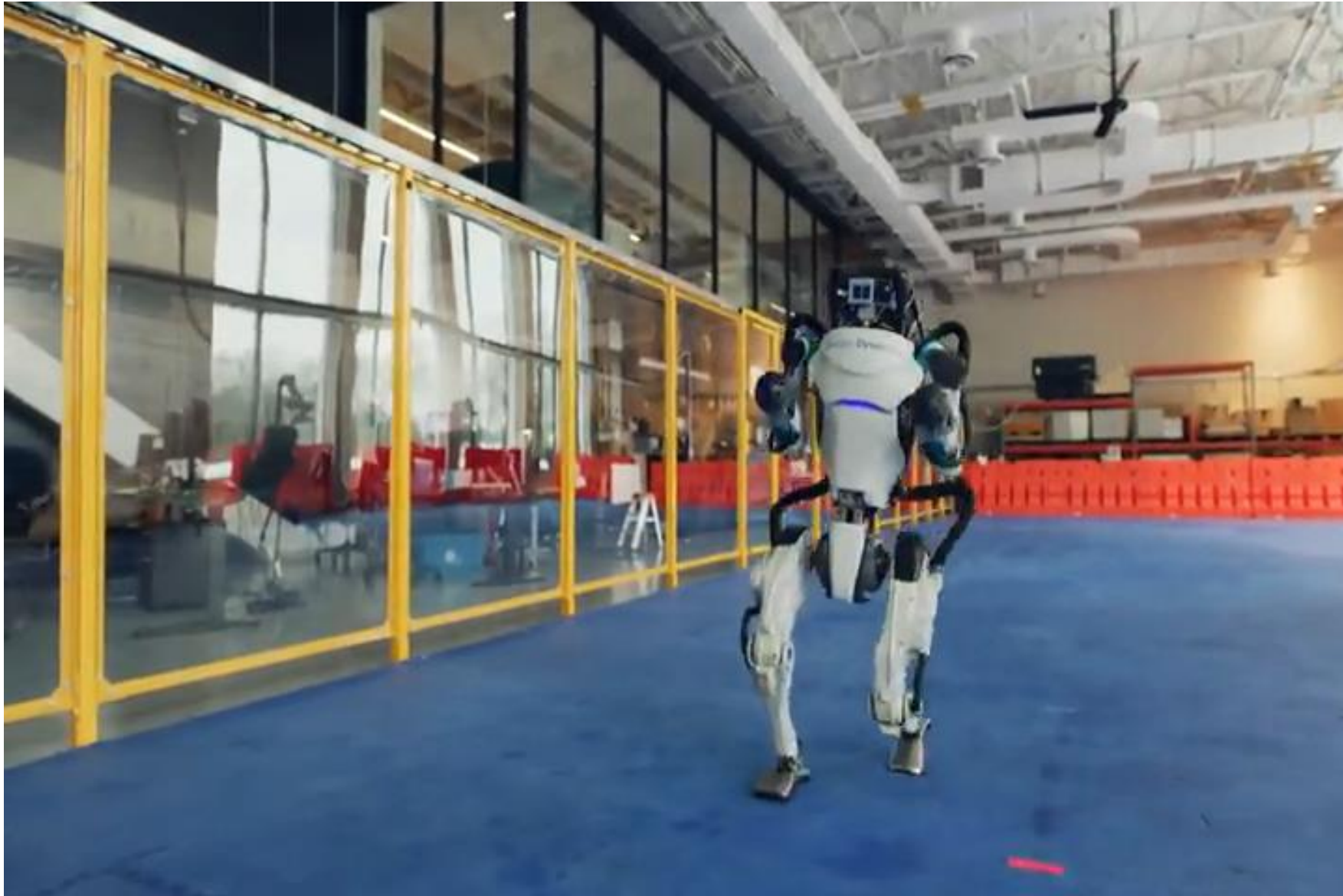


# Euler-Lagrange



Reading: Robot Modeling and Control 7.1.1



# This Lecture

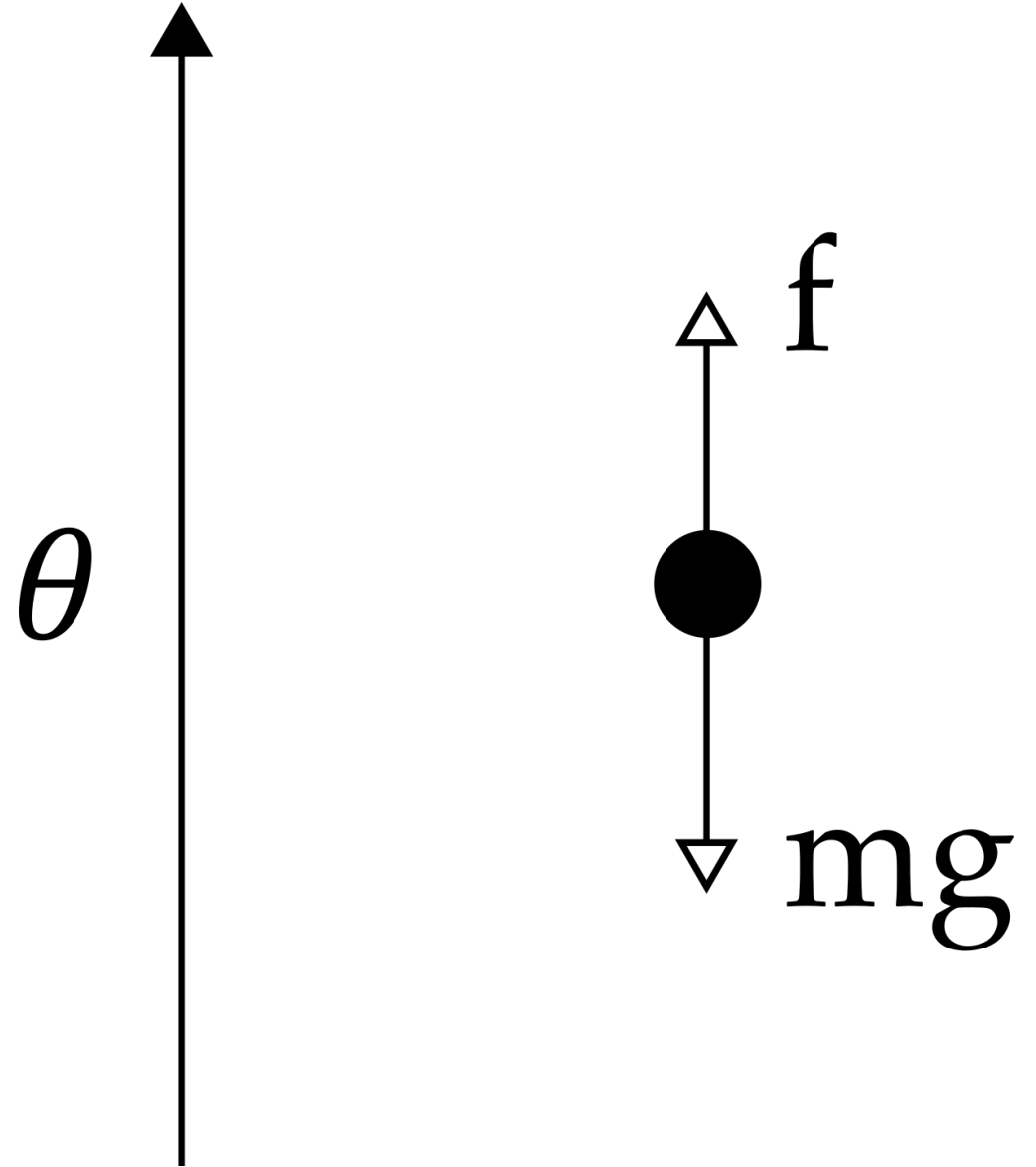


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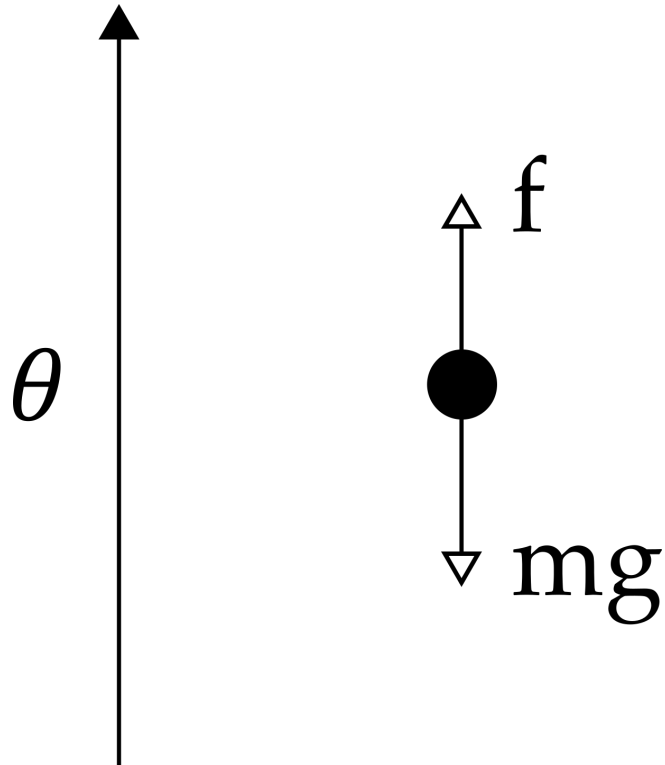
- How do we use kinetic and potential energy to get dynamics?
- What is the Euler-Lagrange equation?

# Point Mass

Let's start with a 1-DoF example



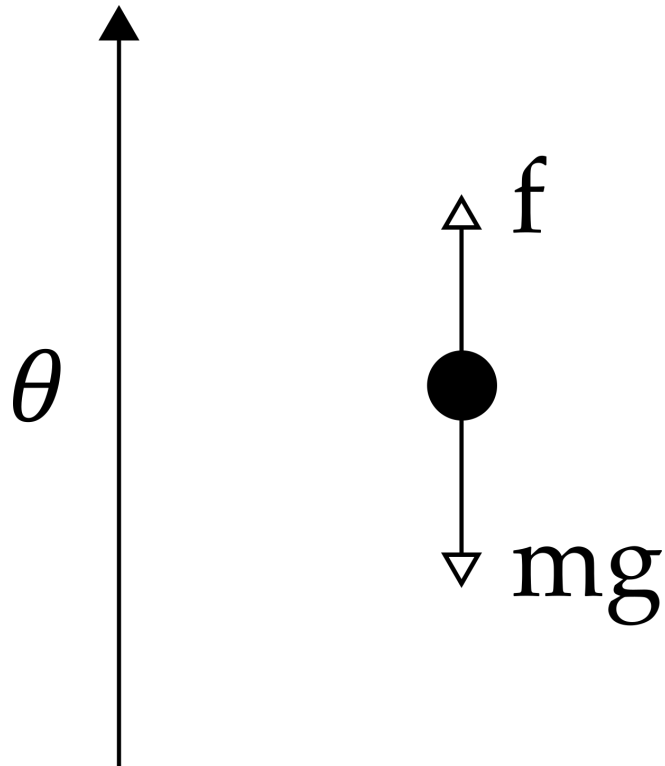
# Point Mass



What is the **equation of motion** for the point mass?

- Particle with mass  $m$
- Moves up and down with position  $\theta$
- Force  $f$  pushing up, gravity pulling down

# Point Mass




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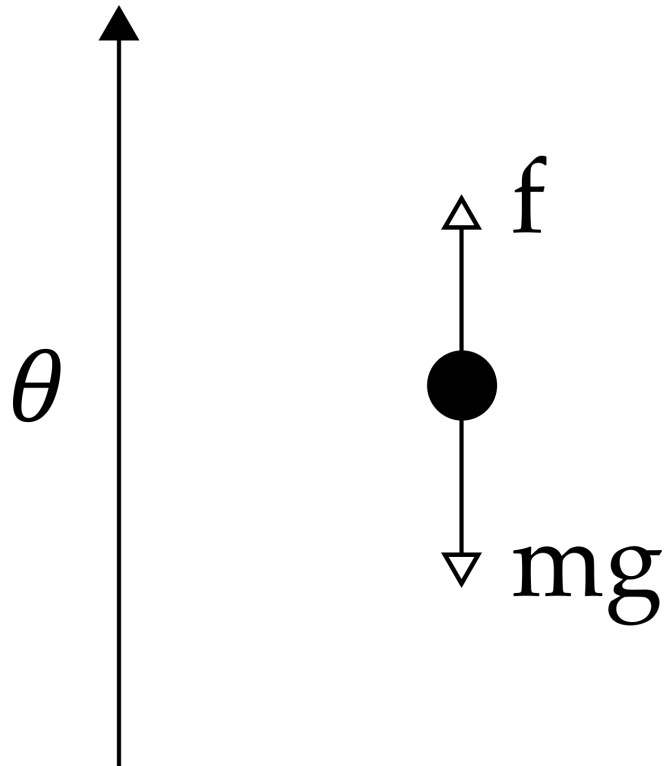
$$m\ddot{\theta} = f - mg$$

*Newton's second law*

A humanoid robot is captured in mid-air, performing a jump over a large log on a workshop floor. The robot has a white and grey body with blue accents on its joints. The background features a workshop environment with yellow structural beams, a blue door with a 'NOT AN EXIT' sign, a black tool chest, and white storage cabinets. The text 'Let's use kinetic and potential energy to get the same result' is overlaid on the image, with 'kinetic' in yellow and 'potential energy' in orange.

Let's use **kinetic** and **potential energy** to get the same result

# Lagrangian

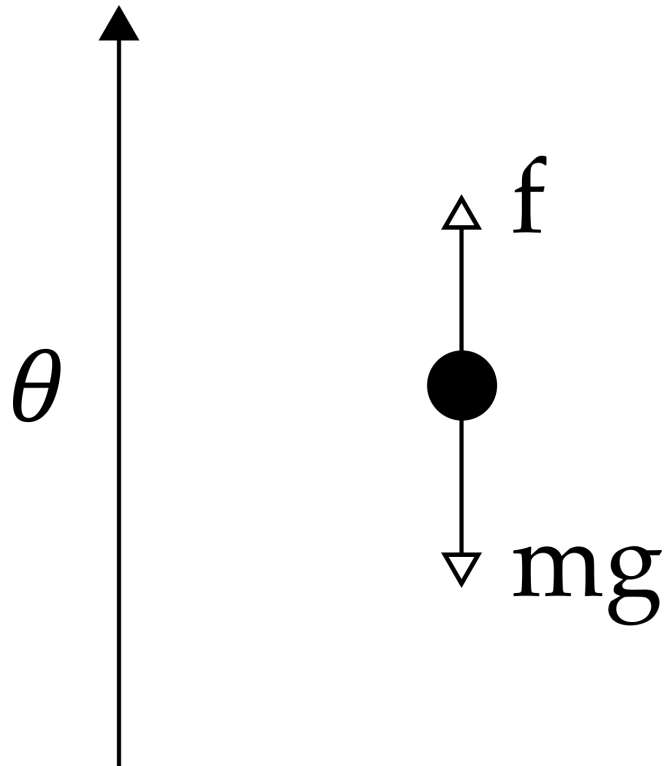


**Lagrangian**  $L$  is the difference between kinetic and potential energy

$$L(\theta, \dot{\theta}) = \underbrace{K(\theta, \dot{\theta})}_{\text{Kinetic energy}} - \underbrace{P(\theta)}_{\text{Potential energy}}$$



# Lagrangian



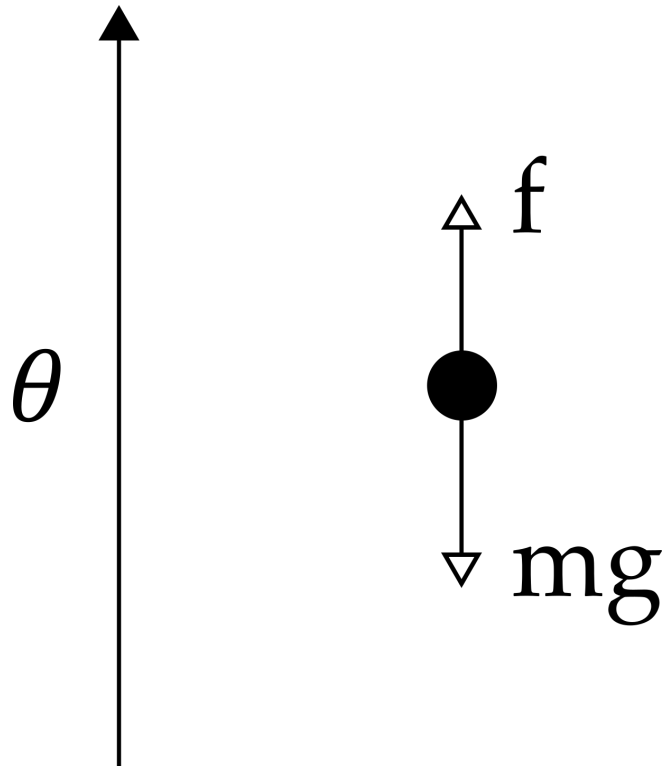
**Lagrangian**  $L$  is the difference between kinetic and potential energy

$$L(\theta, \dot{\theta}) = \underbrace{K(\theta, \dot{\theta})}_{\text{Kinetic energy}} - \underbrace{P(\theta)}_{\text{Potential energy}}$$

Kinetic energy:  $K(\theta, \dot{\theta}) = \frac{1}{2}m\dot{\theta}^2$

Potential energy:  $P(\theta) = mg\theta$

# Lagrangian



**Lagrangian**  $L$  is the difference between kinetic and potential energy

$$L(\theta, \dot{\theta}) = \underbrace{\frac{1}{2}m\dot{\theta}^2}_{\text{Kinetic energy}} - \underbrace{mg\theta}_{\text{Potential energy}}$$

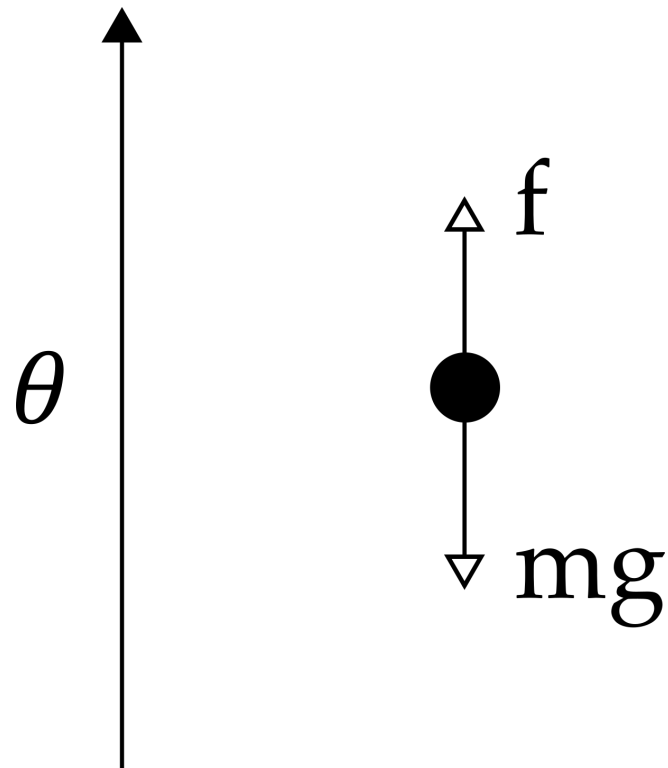
# Euler-Lagrange Equation

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$$f = \frac{d}{dt} \frac{\partial L(\theta, \dot{\theta})}{\partial \dot{\theta}} - \frac{\partial L(\theta, \dot{\theta})}{\partial \theta}$$

Converts Lagrangian  $L$  to dynamic **equations of motion**

# Euler-Lagrange



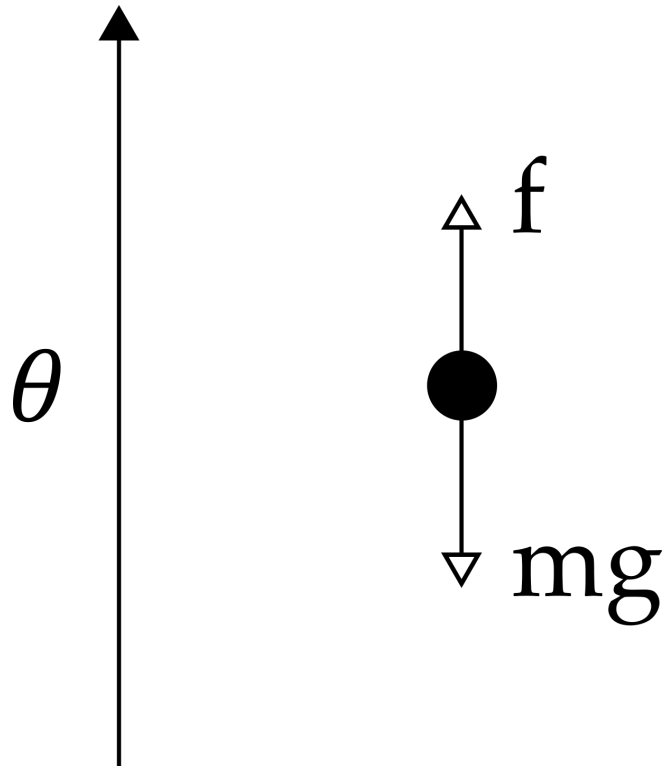
Apply **Euler-Lagrange Equation** to get dynamics.

$$L(\theta, \dot{\theta}) = \frac{1}{2}m\dot{\theta}^2 - mg\theta$$

$$f = \frac{d}{dt} \underbrace{\frac{\partial L(\theta, \dot{\theta})}{\partial \dot{\theta}}}_{\text{1. partial derivative wrt } \dot{\theta}} - \underbrace{\frac{\partial L(\theta, \dot{\theta})}{\partial \theta}}_{\text{2. derivative of result wrt time } t}$$

3. partial derivative wrt  $\theta$

# Euler-Lagrange

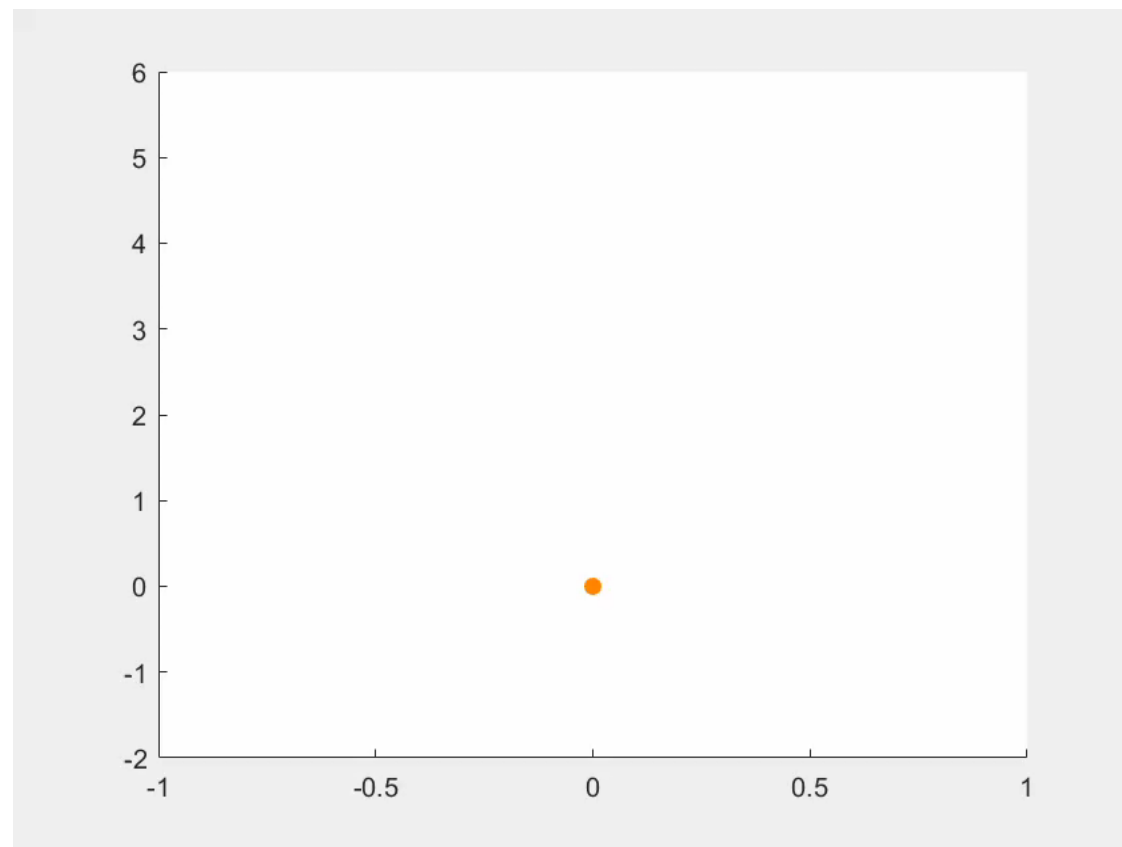
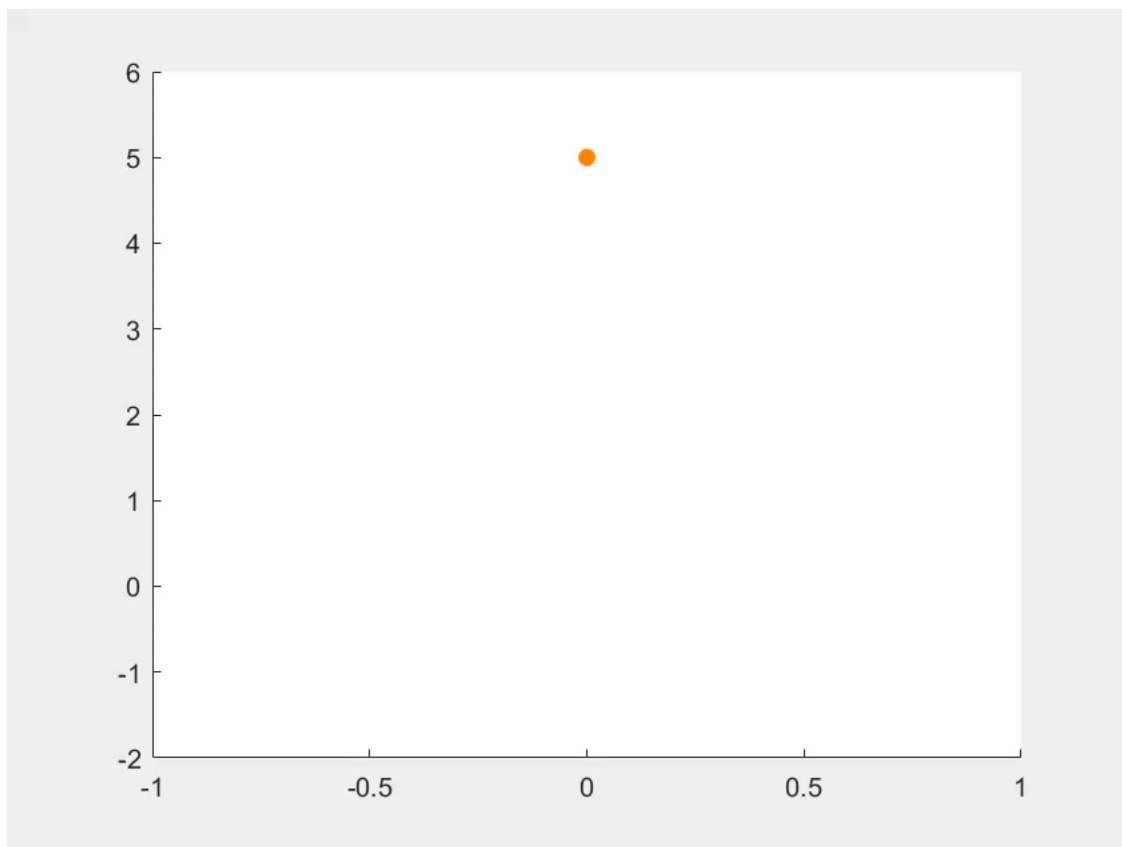


Apply **Euler-Lagrange Equation** to get dynamics.

$$L(\theta, \dot{\theta}) = \frac{1}{2}m\dot{\theta}^2 - mg\theta$$

$$f = m\ddot{\theta} + mg$$





A yellow and black Boston Dynamics robot arm is shown reaching for a white cup on a wooden table. The arm is extended upwards and to the left, with the gripper positioned just above the cup. The robot's body is yellow, and its legs are black. The background is a plain, light gray.

How do we extend this  
approach to **robot arms**?

# Takeaways

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$$L(\theta, \dot{\theta}) = K(\theta, \dot{\theta}) - P(\theta)$$

total kinetic (and potential) energy summed across every joint

$$\tau_i = \frac{d}{dt} \frac{\partial L(\theta, \dot{\theta})}{\partial \dot{\theta}_i} - \frac{\partial L(\theta, \dot{\theta})}{\partial \theta_i}$$

torque at  $i$ -th joint    velocity of  $i$ -th joint    position of  $i$ -th joint



# This Lecture



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- How do we use kinetic and potential energy to get dynamics?
- What is the Euler-Lagrange equation?

# Next Lecture



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- How do we find the kinetic and potential energy for a robot arm?