Practice Set 29

Robotics & Automation
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Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

Consider a "bug" setting where you are programming a drone to fly around 2D obstacles. The drone knows where the goal is, but it does not know about the obstacles *a priori*.

Form teams of two. One partner proposes a bug algorithm, and the other partner tries to design an environment that leads to worst-case performance. Switch roles and repeat.

Problem 2

Consider an environment with no obstacles. The potential energy is:

$$U(\theta) = \frac{1}{2}\beta\|g - \theta\|^2 \tag{1}$$

where g is the goal and $\beta > 0$ is a scaling parameter. Using gradient descent with learning rate α, find the condition for $\alpha \cdot \beta$ under which the motion planner converges to the goal.

Hint: Think about the inequality $\|\theta^{t+1} - g\| < \|\theta^t - g\|$

Problem 2

Consider an environment with no obstacles. The potential energy is:

$$U(\theta) = \frac{1}{2}\beta\|g - \theta\|^2 \tag{2}$$

where g is the goal and β is a scaling parameter. Using gradient descent with learning rate a, find the condition for $\alpha \cdot \beta$ under which the motion planner converges to the goal.

Hint: Think about the inequality $\|\theta^{t+1} - g\| < \|\theta^t - g\|$

Start by taking the gradient of the potential energy:

$$\nabla U = -\beta(g - \theta) \tag{3}$$

Use gradient descent to find the next joint position:

$$\theta^{t+1} = \theta^t - \alpha\nabla U(\theta^t) \tag{4}$$

$$\theta^{t+1} = \theta^t + \alpha\beta(g - \theta^t) \tag{5}$$

For the system to converge to g, the robot's position must get closer to the goal, i.e., $\|\theta^{t+1} - g\| < \|\theta^t - g\|$. Expanding these terms:

$$\|\theta^{t+1} - g\| = \|\theta^t + \alpha\beta(g - \theta^t) - g\| \tag{6}$$

$$\|\theta^{t+1} - g\| = \|(1 - \alpha\beta)(\theta^t - g)\| \tag{7}$$

$$\|\theta^{t+1} - g\| = |1 - \alpha\beta| \cdot \|\theta^t - g\| \tag{8}$$

We now have the stability condition:

$$|1 - \alpha\beta| \cdot \|\theta^t - g\| < \|\theta^t - g\| \tag{9}$$

which is satisfied so long as:

$$0 < \alpha \cdot \beta < 2 \tag{10}$$

In summary, having a learning rate that is too high can cause the potential field approach to fail to reach the goal, even when there are no obstacles in the environment.