Robotics & Automation

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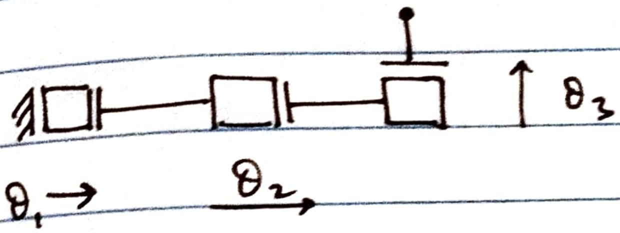
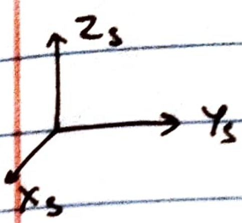
Problem Set - 5

1



1.1



3 prismatic joints

End-effector position $p = [x \quad y \quad z]^T$

∴ Working on inverse kinematics

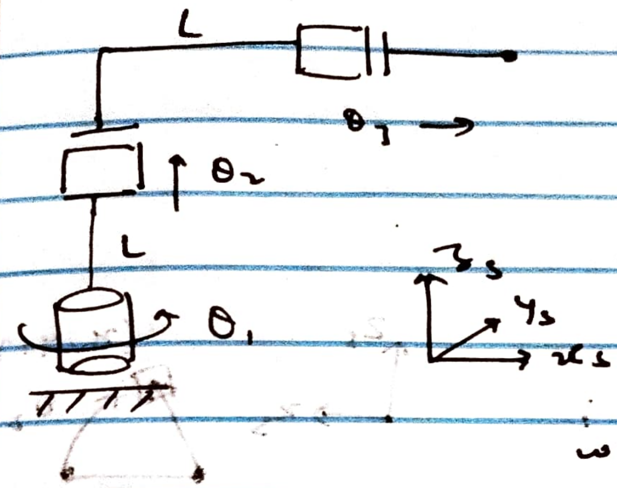
→ $y = (\theta_1 + \theta_2)$ [Translation joints θ_1 & θ_2]

→ $z = \theta_3$

→ $x = 0$

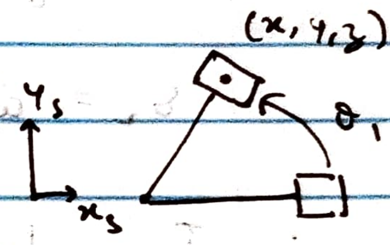
∴ $$\boxed{\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} y - \theta_2 \\ y - \theta_1 \\ z \end{bmatrix}}$$

1.21



wrong answer if $x < 0$

• **Revolute joint θ_1 :**



$\Rightarrow \theta_1 = \tan^{-1}(y/x) = \text{atan2}(y, x)$

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& $z = (L + \theta_2)$

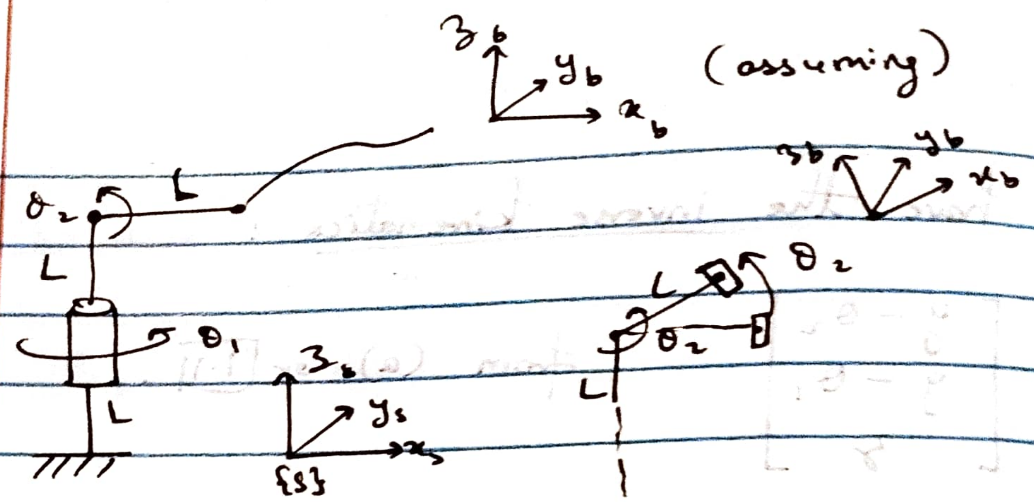
$\Rightarrow x = (L + \theta_3)$

$\therefore \quad \theta_2 = (z - L) \quad \& \quad \theta_3 = (x - L)$

$\therefore$ **Inverse kinematics :**

$$\boxed{\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} \text{atan2}(y, x) \\ (z - L) \\ (x - L) \end{bmatrix}}$$

1.3



- For revolute joint θ_1 :-

The angle of the end-effector is :-

$$\theta_1 = \operatorname{atan}(y, x)$$

- Now, for joint θ_2 :-

$$z = 2L + L\sin(\theta_2)$$

α, $$x = L\cos\theta_2$$

$$\Rightarrow \theta_2 = \sin^{-1}\left(\frac{z - 2L}{L}\right)$$

$$\theta_2 = \cos^{-1}\left(\frac{x}{L}\right)$$

∴ Inverse kinematics

$$\theta = \begin{bmatrix} \operatorname{atan}(y, x) \\ \sin^{-1}\left(\frac{z - 2L}{L}\right) \end{bmatrix}$$

or

$$\theta = \begin{bmatrix} \operatorname{atan}(y, x) \\ \cos^{-1}\left(\frac{x}{L}\right) \end{bmatrix}$$

$\boxed{1.4}$ We have the inverse kinematics :

$$\theta = \begin{bmatrix} y - \theta_2 \\ y - \theta_1 \\ z \end{bmatrix} \quad \text{from (a) or } \boxed{1.1}$$

We can see that both θ_1 and θ_2 are dependent on each other for a solution.

In an ideal position, where $x = y = z = 0$

We can achieve the desired end-effector position $p = [0 \quad 2L \quad 0]^T$ when

$\theta_2 = 0$, $\theta_1 = -2L$ $\theta_3 = 0$ (or any value as it does not affect the kinematics)

But for other solutions where $y \neq 0$, we can see it affects both θ_1 & θ_2 joint positions and they are dependent on each other.

Infinite solutions are possible, example :-

Thus, let's say for $y = L$; $\theta_2 = L$ & $\theta_1 = \cancel{-}L$ & $\theta_3 = 0$

$\therefore$ There exists infinite solutions for this position as the inverse kinematics are in dependence with y & each other (θ_1 & θ_2)