Problem Set 6

Robotics & Automation
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Instructions. Please write legibly and do not attempt to fit your work into the smallest space possible. It is important to show all work, but basic arithmetic can be omitted. You are encouraged to use Matlab when possible to avoid hand calculations, but print and submit your commented code for non-trivial calculations. You can attach a pdf of your code to the homework, use live scripts or the publish feature in Matlab, or include a snapshot of your code. Do not submit .m files — we will not open or grade these files.

For this assignment we are asking you to also submit **videos** of your simulations. Follow the instructions to **label** these videos based on the problem number, and then submit them all within a **single zipped folder**.

1 Wrenches

1.1 (10 points)

Imagine an arbitrary wrench F and coordinate frames $\{s\}$ and $\{b\}$. If $f_s \neq 0$ and $m_s \neq 0$, under what conditions is $\|F_b\| = \|F_s\|$? This question does not refer to the figure below.

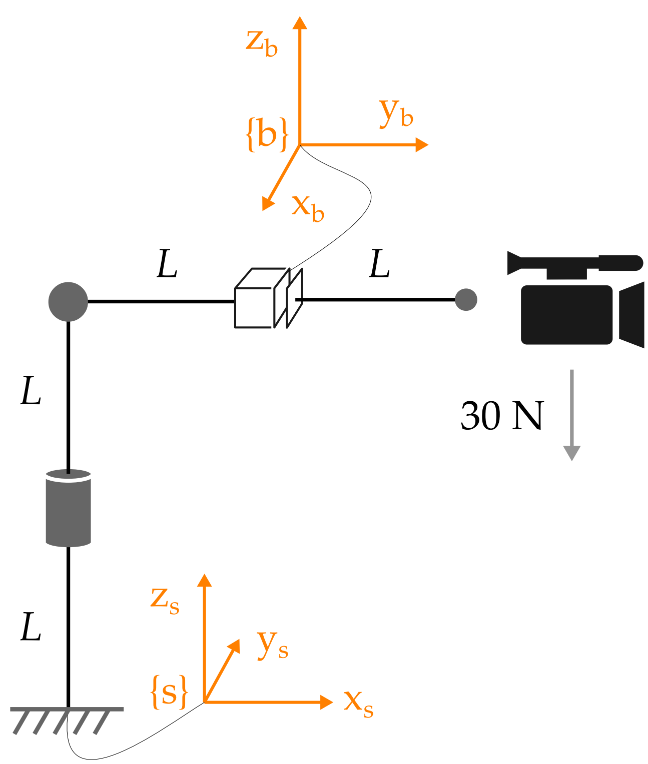


Figure 1: A robot arm holding a camera that weights 30 N.

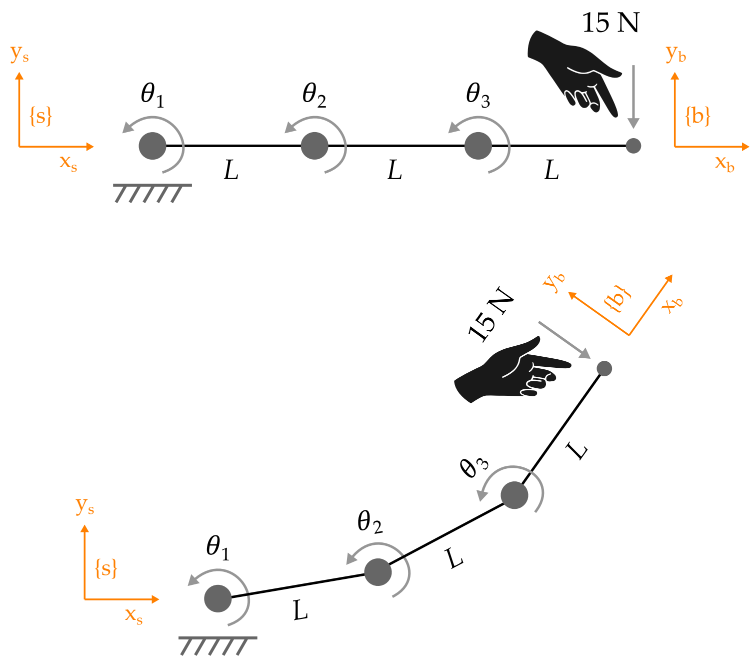
1.2 (5 points)

Consider the robot in Figure 1. Find the wrench applied by the camera in frame $\{b\}$.

1.3 (5 points)

Consider the robot in Figure 1. Find the wrench applied by the camera in frame $\{s\}$.

2 Statics



A human is pushing on the end-effector of the 3-DoF robot shown above. (Top Drawing) The human applies a 15 N force along the negative y_b axis. (Bottom Drawing) The human keeps this force aligned with the negative y_b axis as the robot moves.

2.1 (5 points)

What wrench does the robot need to apply at the end-effector to maintain static equilibrium?

2.2 (5 points)

Case 1. Let $L = 1$ and let $\theta = [0, \pi/4, \pi/4]^T$.
Find the joint torques needed to balance out the force applied by the human.

2.3 (5 points)

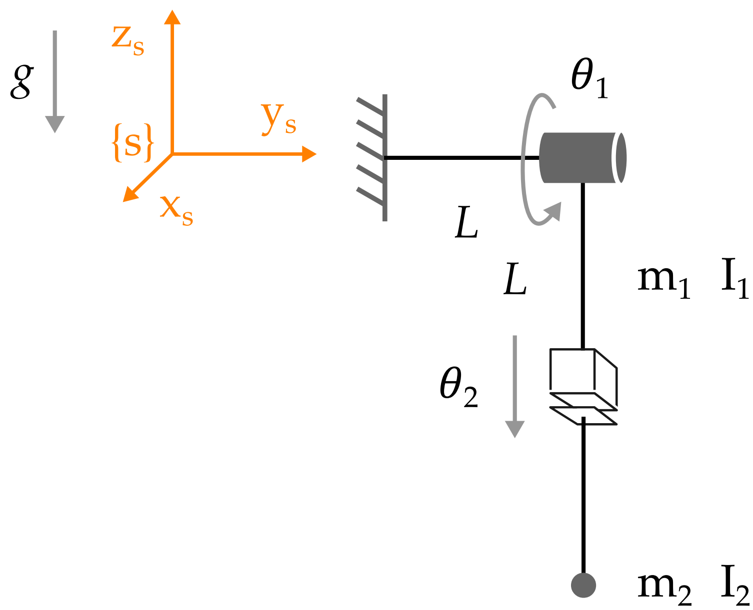
Case 2. Let $L = 1$ and let $\theta = [0, \pi/8, 0]^T$.
Find the joint torques needed to balance out the force applied by the human.

2.4 (15 points)

Let $\|\tau\|$ be the magnitude (i.e., the length) of the joint torque vector.

- Find a joint position θ that **maximizes** $\|\tau\|$
- Find a joint position θ that **minimizes** $\|\tau\|$

3 Dynamics: Revolute-Prismatic



3.1 (20 points)

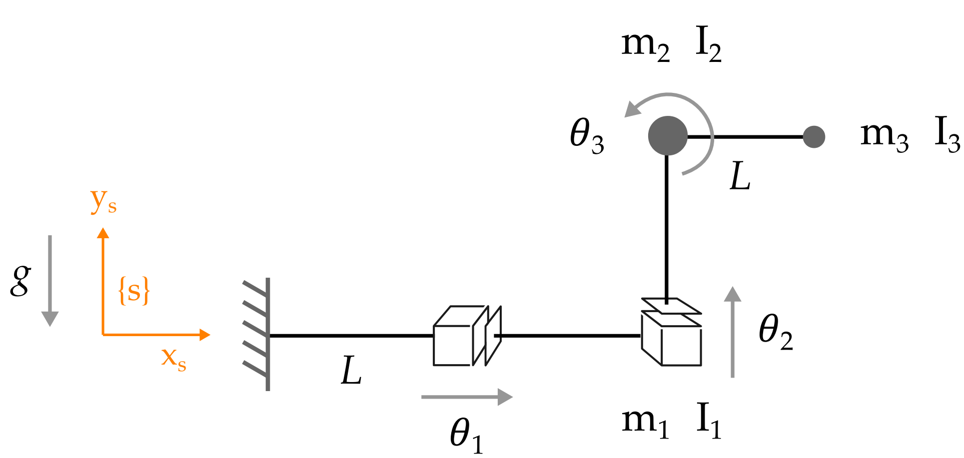
Find the dynamics for the robot shown above. Your answer should be of the form:

$$\tau = M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) \tag{1}$$

List the mass matrix, the Coriolis matrix, and the gravity vector. The center of mass for m_1 is located **halfway** between the revolute joint and the prismatic joint. The center of mass for m_2 is at the robot's end-effector. Inertia matrices I_1 and I_2 are:

$$I_1 = \begin{bmatrix} I_{x1} & 0 & 0 \\ 0 & I_{y1} & 0 \\ 0 & 0 & I_{z1} \end{bmatrix}, \quad I_2 = \begin{bmatrix} I_{x2} & 0 & 0 \\ 0 & I_{y2} & 0 \\ 0 & 0 & I_{z2} \end{bmatrix}$$

4 Dynamics: Prismatic-Prismatic-Revolute



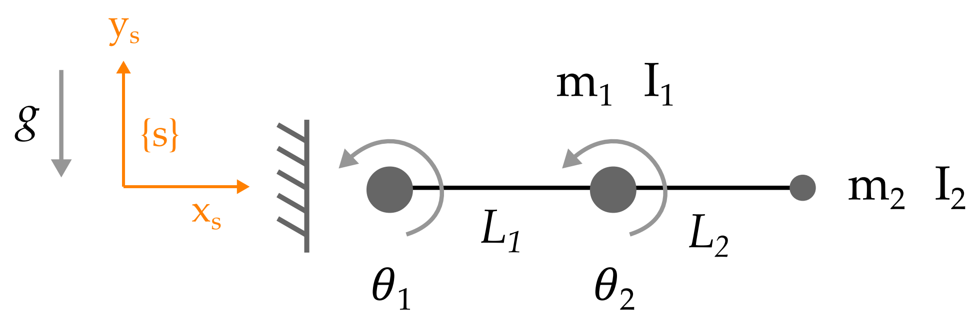
4.1 (20 points)

Find the dynamics for the robot shown above. Your answer should be of the form:

$$\tau = M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) \tag{2}$$

List the mass matrix, the Coriolis matrix, and the gravity vector. Each center of mass is at the end of the link.

5 Simulation



Here you will simulate the dynamics of the planar robot shown above. **We found the dynamics of this robot in lecture**. Start by **downloading** the Matlab file `make_simulation.m` that was provided with this assignment.

5.1 (10 points)

Combine `make_simulation.m` with the dynamics we derived. You will need to get the (x, y) position of each link to plot the robot. Use the given simulation parameters and frame rates; all videos should be 10 seconds in length. Turn in the following MP4 videos:

- Make a simulation where $\tau = [0, 0]^T$ and the robot has no friction. Title this video **Problem5_1.mp4**
- Make a simulation where $\tau = [0, 0]^T$ and the robot has viscous friction $B = I$. Title this video **Problem5_2.mp4**
- Make a simulation where $\tau = [20, 5]^T$ and the robot has viscous friction $B = I$. Title this video **Problem5_3.mp4**

Although not required, I encourage you to play with the parameters (such as mass, inertia, friction, and τ), and see how these parameters affect the simulation.