Rotation Matrices

Reading: Modern Robotics 3.1 – 3.2


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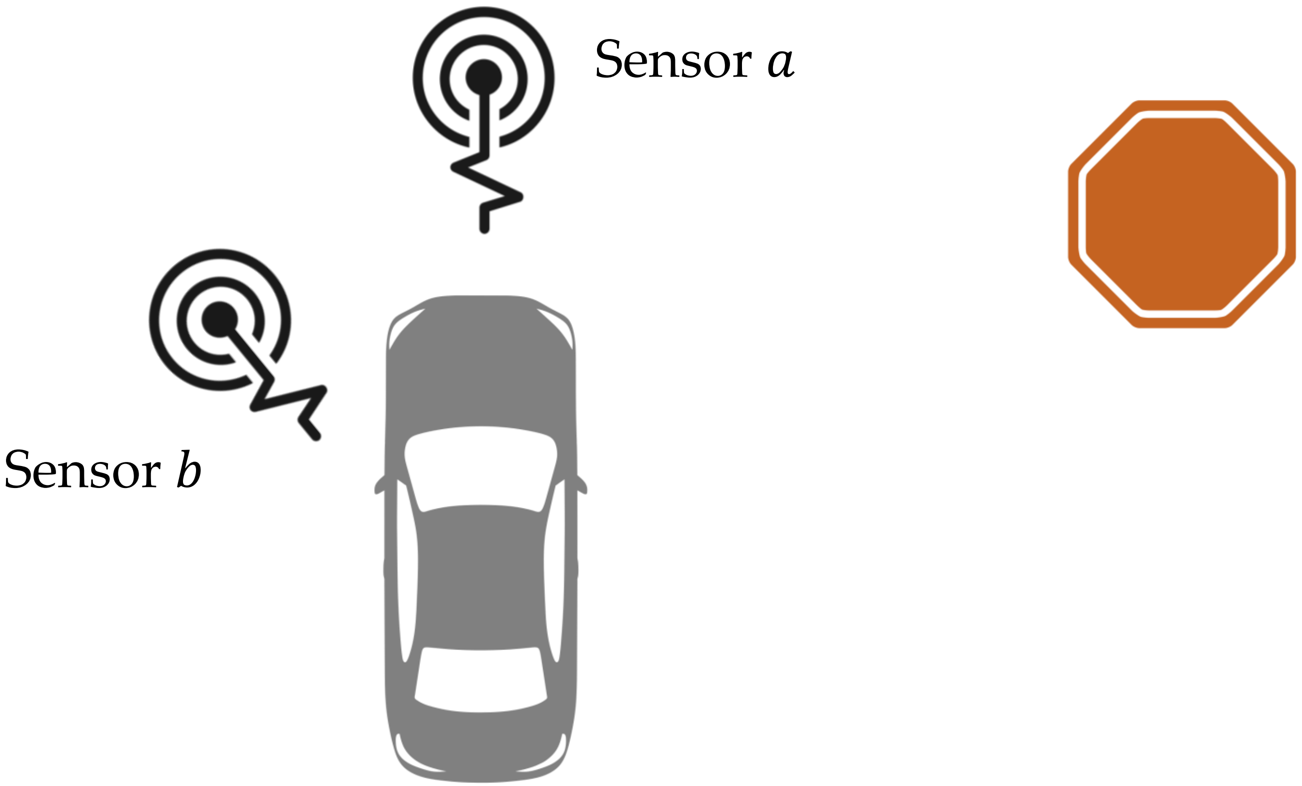
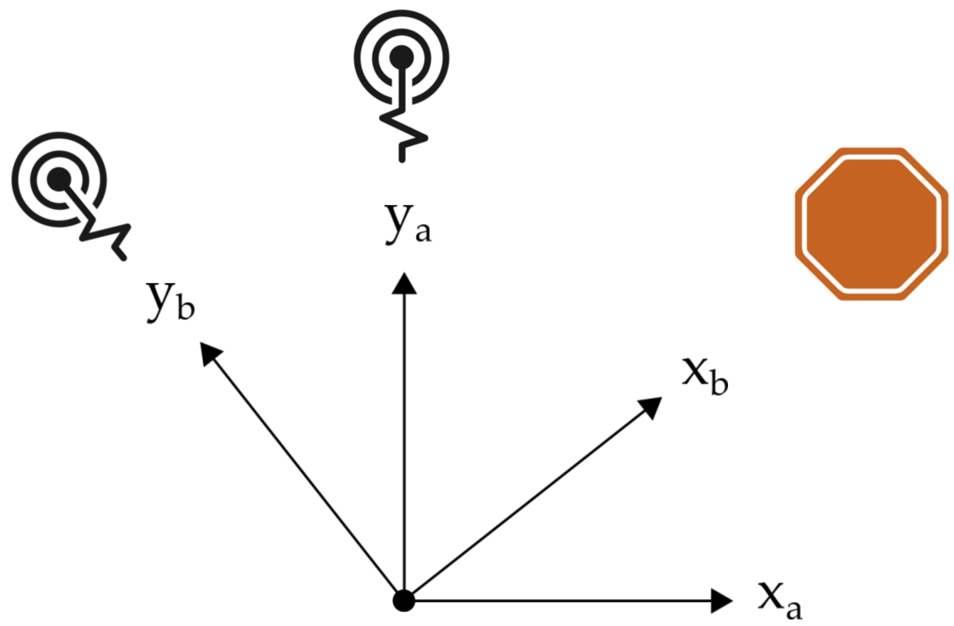
This Lecture

- Why do we use rotation matrices?



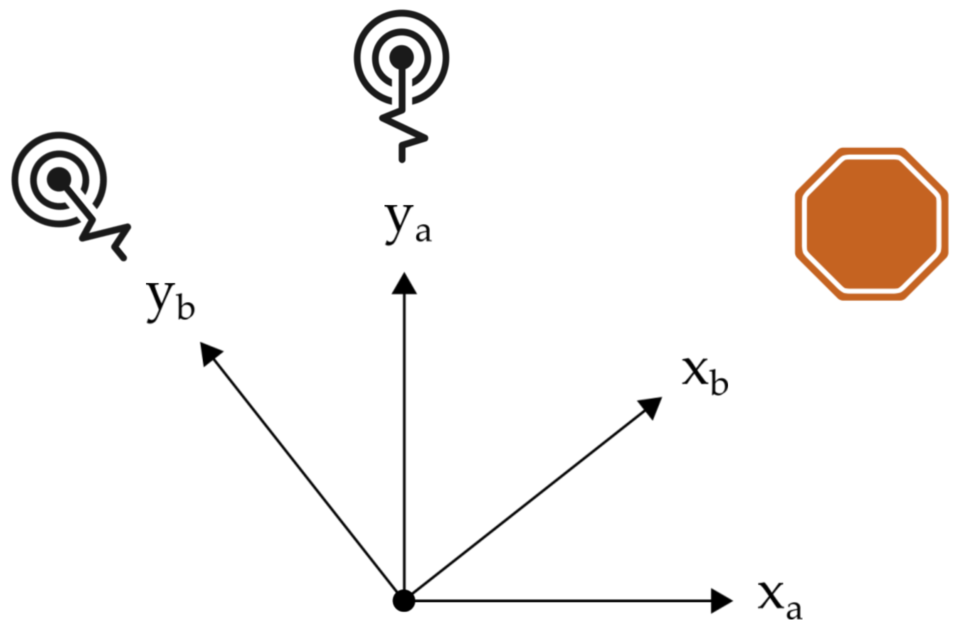
Sensor *a*
Sensor *b*

Given the stop sign in frame $\{b\}$, what is the position in frame $\{a\}$?



Given the stop sign in frame $\{b\}$, what is the position in frame $\{a\}$?

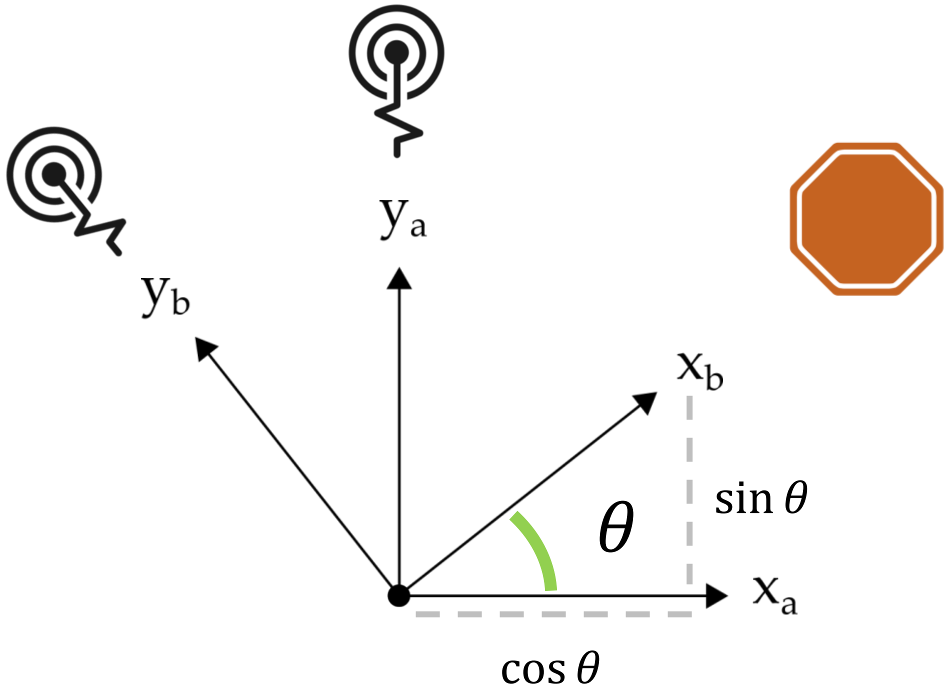
$$p_a = \boldsymbol{R_{ab}} p_b$$



Given the stop sign in frame $\{b\}$, what is the position in frame $\{a\}$?

$$p_a = \boldsymbol{R_{ab}} p_b$$

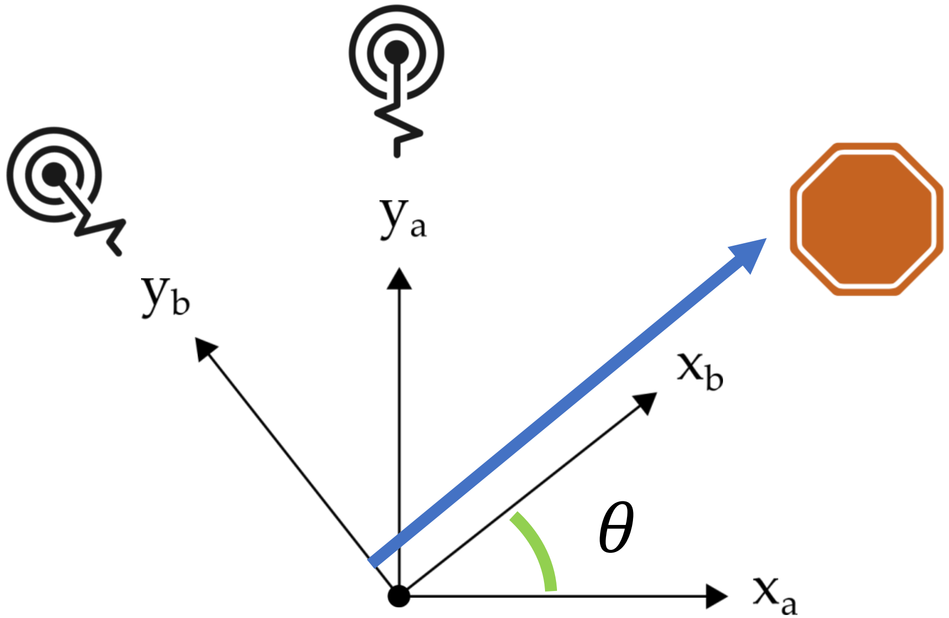
$$R_{ab} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Given the stop sign in frame $\{b\}$, what is the position in frame $\{a\}$?

$$p_a = \boldsymbol{R_{ab}} p_b$$

$$R_{ab} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad p_b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

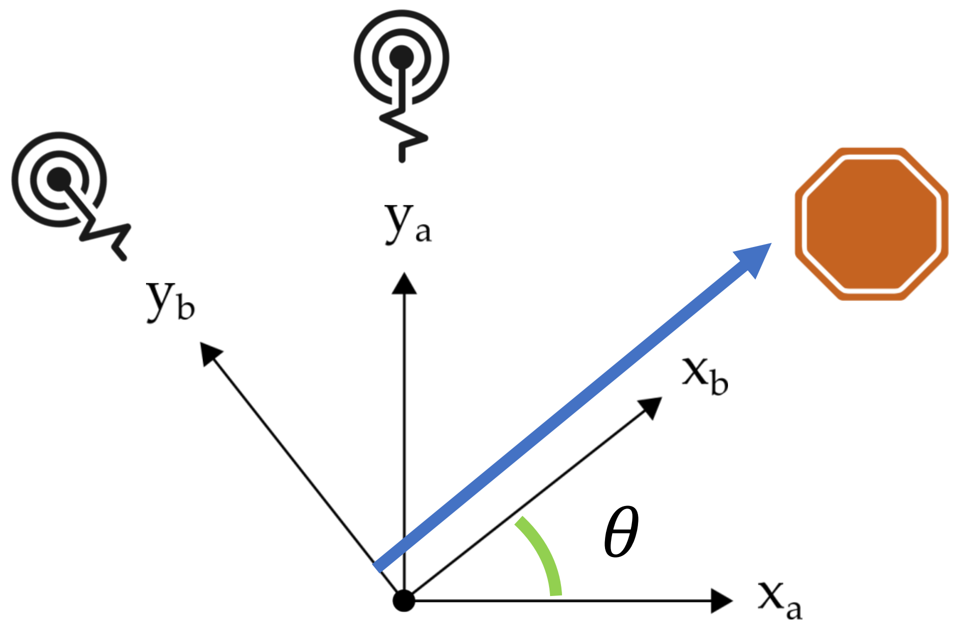


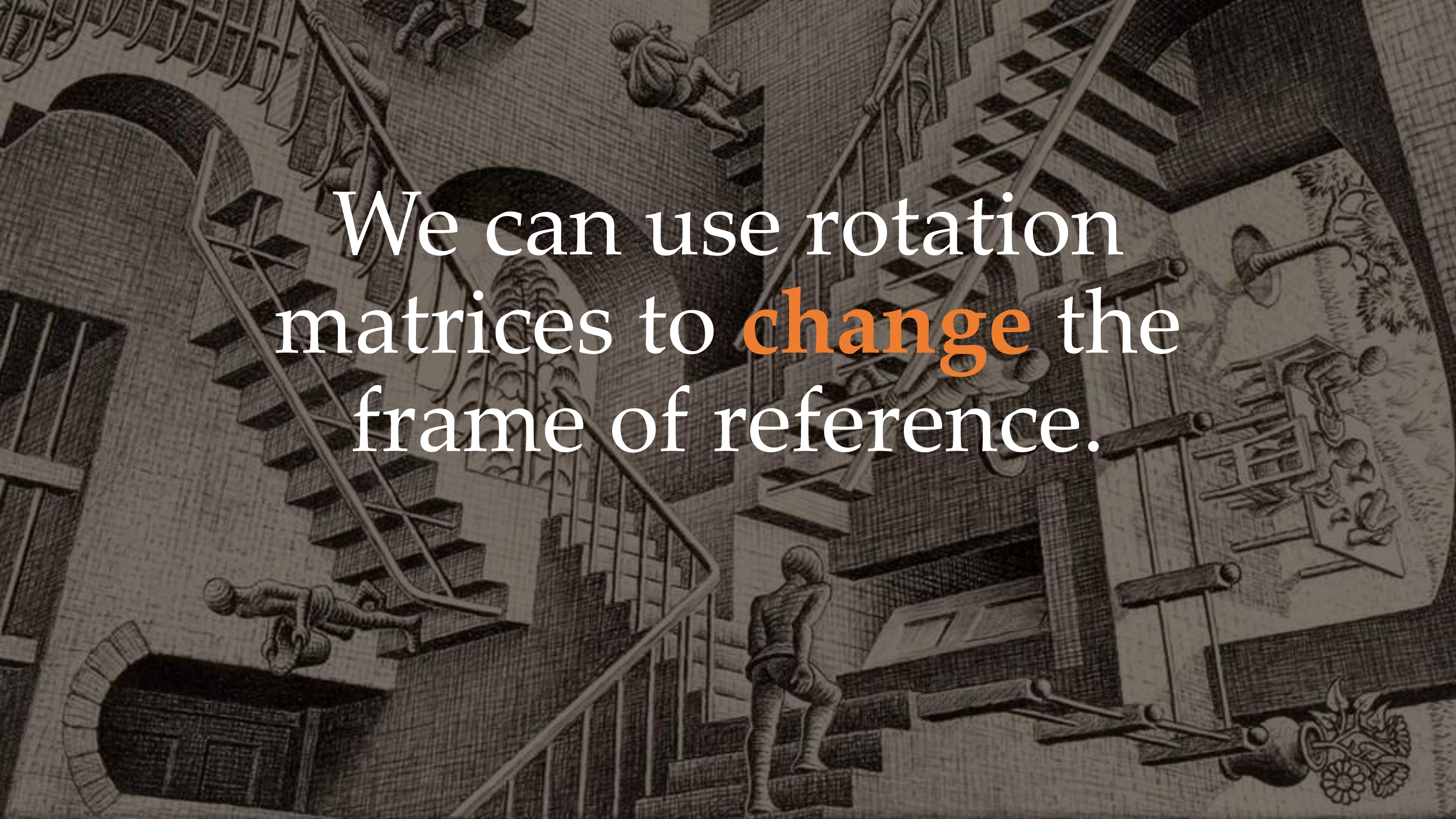
Given the stop sign in frame $\{b\}$,
what is the position in frame $\{a\}$?

$$p_a = \boldsymbol{R_{ab}} p_b$$

$$R_{ab} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad p_b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$p_a = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$





We can use rotation matrices to **change** the frame of reference.

Changing Perspectives

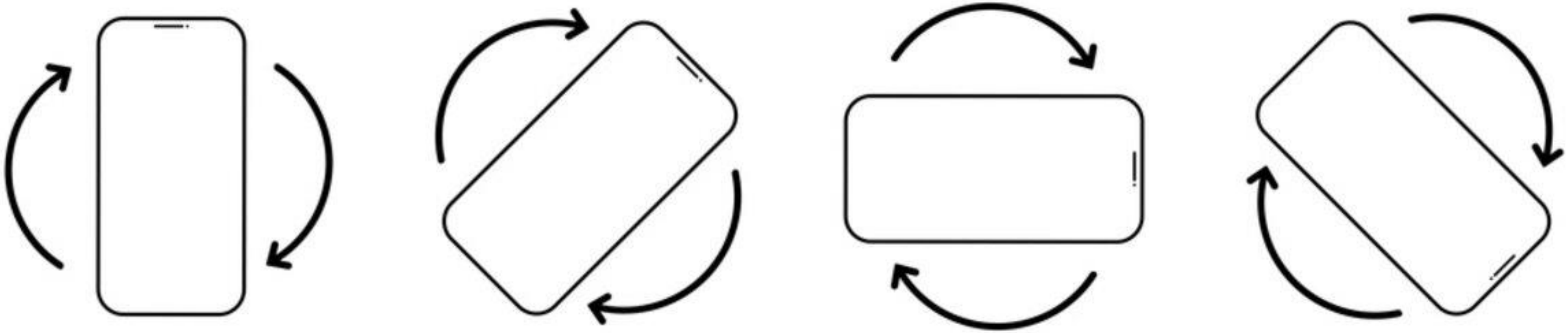
When we multiply rotation matrices, if the subscripts *cancel* then we **change** the frame of reference (i.e., the perspective)

$$p_a = R_{ab} p_b$$

$$R_{ac} = R_{ab} R_{bc}$$

$$p_a = R_{ab} R_{bc} p_c$$

We can also use rotation matrices to **rotate** a vector.



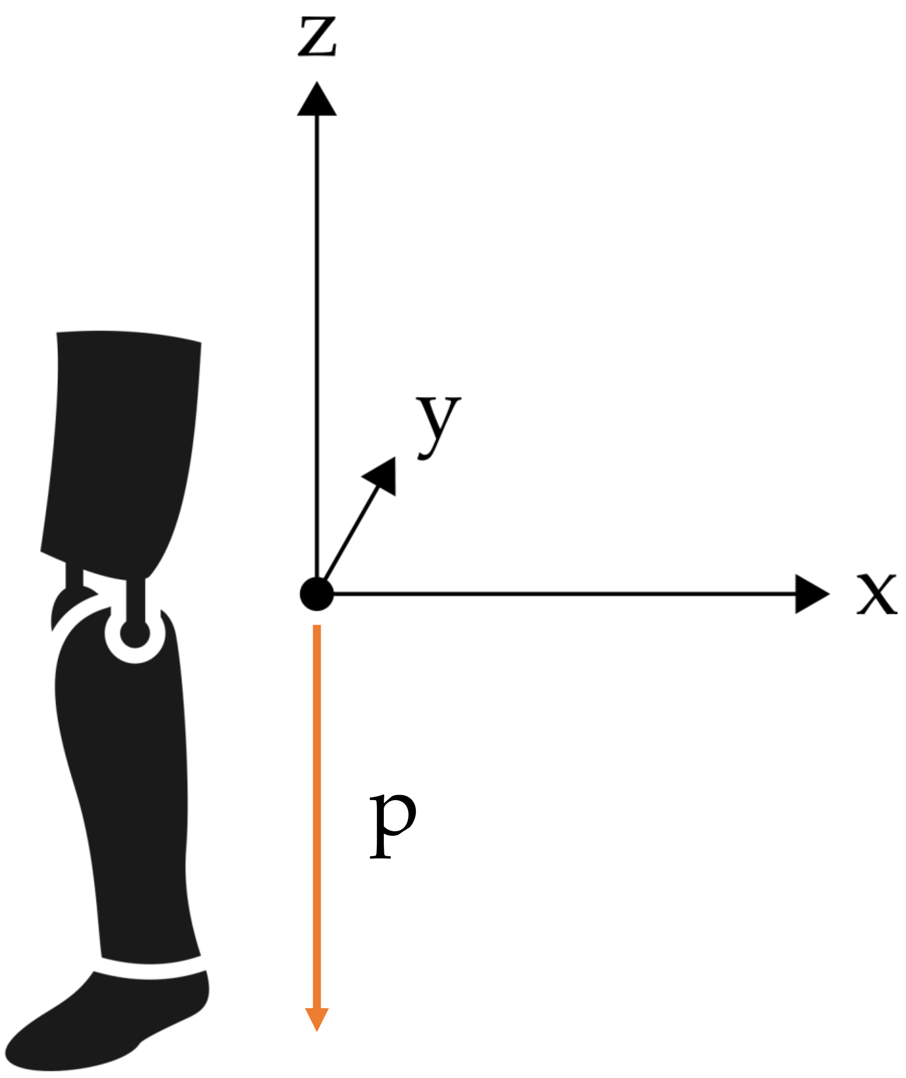
Rotating Objects

We can use rotation matrices as a mathematical operator to **rotate** an object in a single, fixed coordinate frame

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad Rot(y,\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

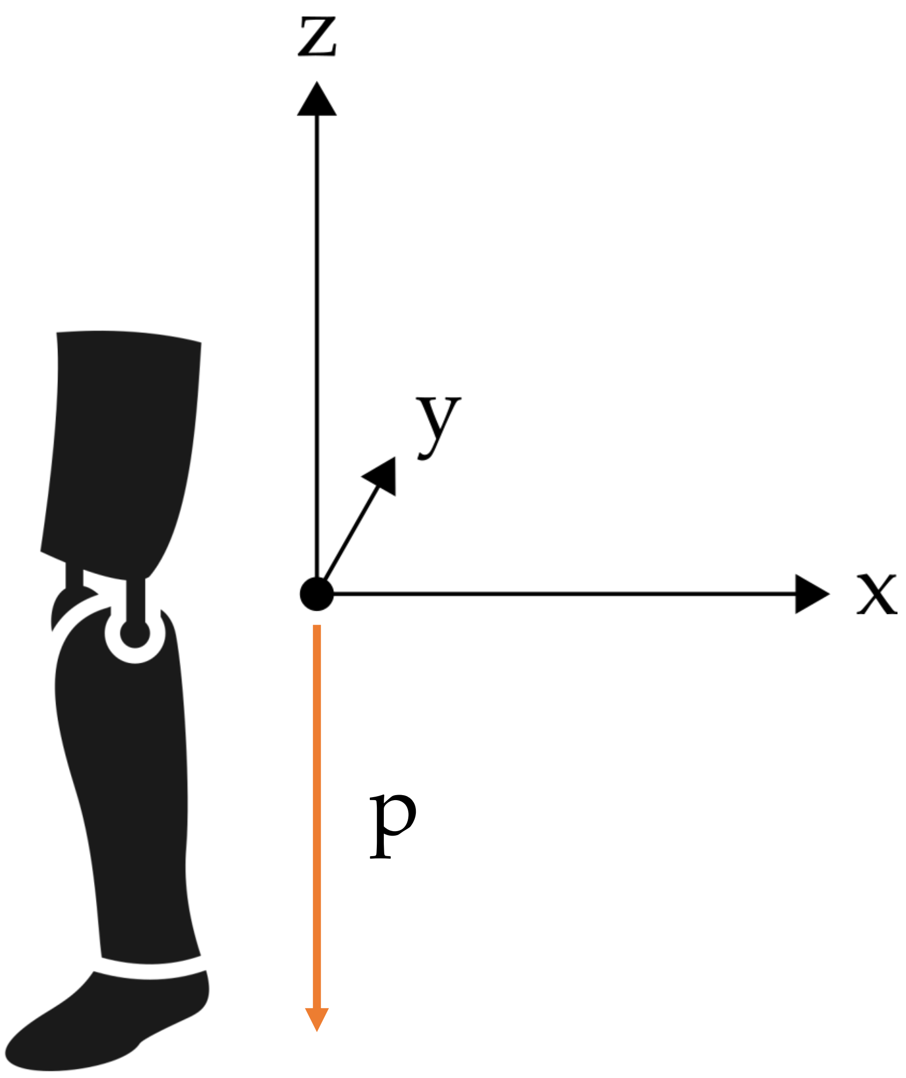
$$Rot(z,\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If we **rotate the base** of this prosthetic leg about y, where does the foot end up?



If we **rotate the base** of this prosthetic leg about y, where does the foot end up?

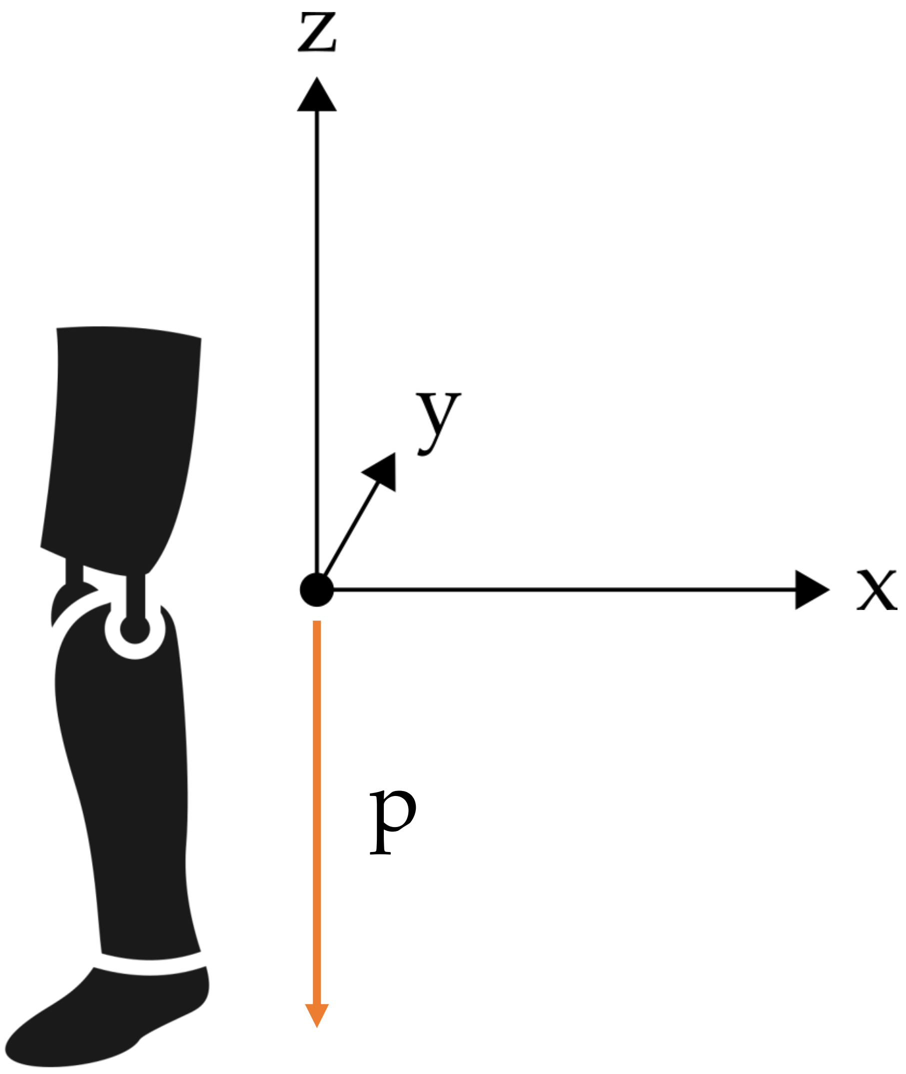
$$p' = Rp$$



If we **rotate the base** of this prosthetic leg about y, where does the foot end up?

$$p' = Rp$$

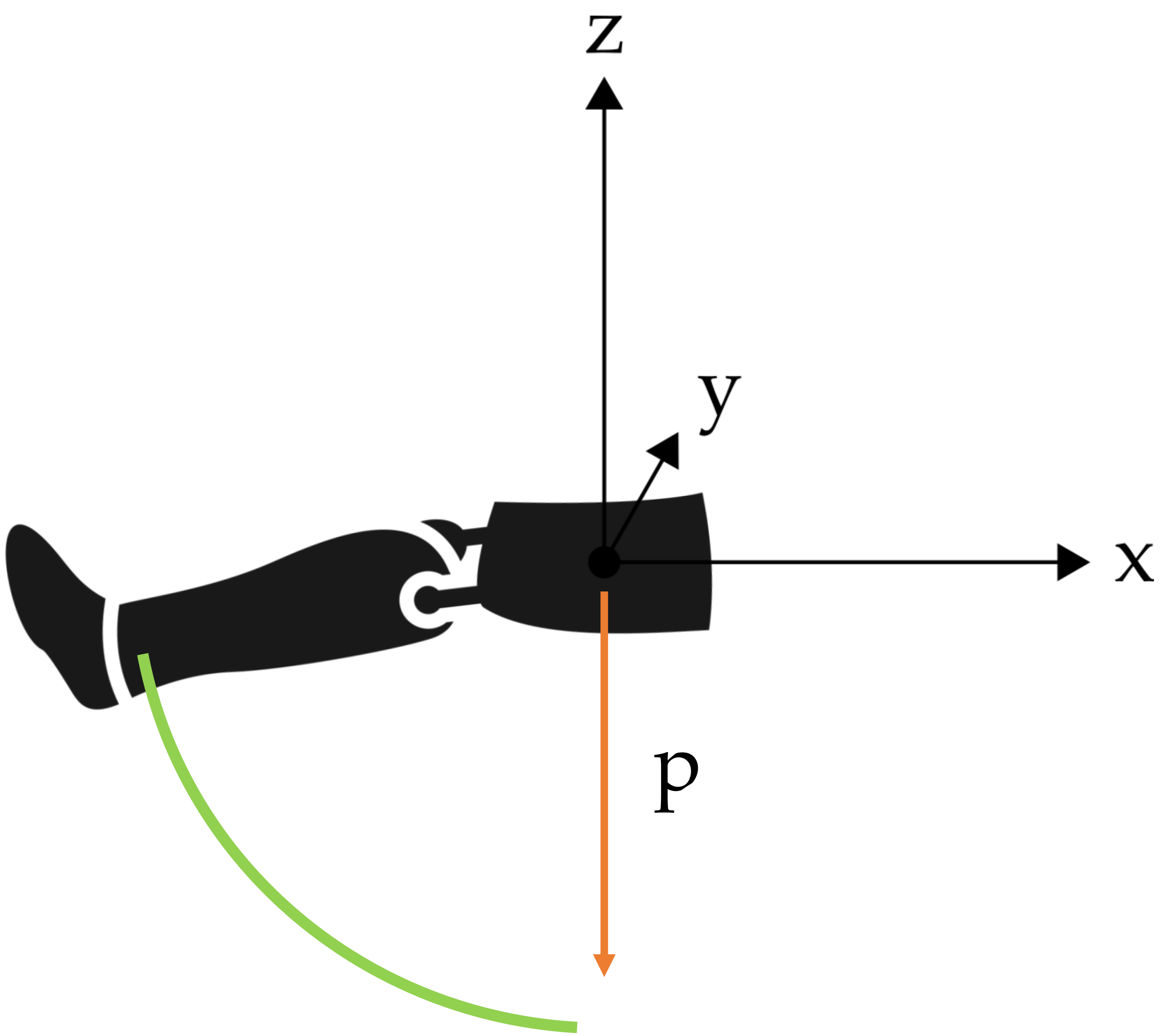
$$p' = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$



If we **rotate the base** of this prosthetic leg about y, where does the foot end up?

$$p' = Rp$$

$$p' = \begin{bmatrix} -2\sin\theta \\ 0 \\ 2\cos\theta \end{bmatrix}$$



Takeaways

Multiplying by rotation matrices can:

1. Change our frame of reference
2. Rotate a vector or frame

Next Lecture

- How do we describe angular velocity?
- What are some other ways to represent rotation?