Practice Set 21

Robotics & Automation
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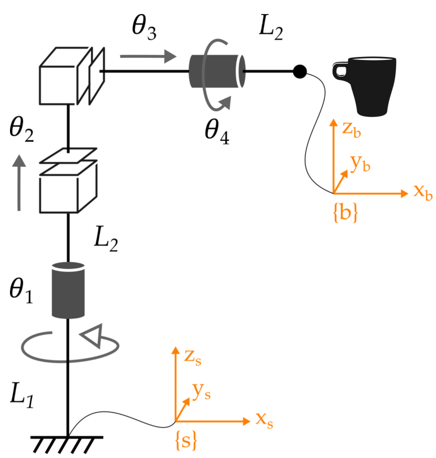
Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

Given wrench F, let F_s be the wrench in the fixed frame and let F_b be that wrench written in the body frame. Prove that the joint torques are equal for either choice of frame:

$$\tau = J_s(\theta)^T F_s = J_b(\theta)^T F_b \tag{1}$$

Problem 2



This robot is holding a mug that weights 10 newtons. The robot's forward kinematics are:

$$T(\theta) = \begin{bmatrix} c_1 & -c_4 s_1 & s_1 s_4 & (L_2+\theta_3)c_1 \\ s_1 & c_1 c_4 & -c_1 s_4 & (L_2+\theta_3)s_1 \\ 0 & s_4 & c_4 & L_1+L_2+\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and the robot's space Jacobian is (you found this in a previous homework assignment):

$$J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 & c_1 \\ 0 & 0 & 0 & s_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & c_1 & -(L_1+L_2+\theta_2)s_1 \\ 0 & 0 & s_1 & (L_1+L_2+\theta_2)c_1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

- What joint torques will hold the coffee in place if $\theta = [0, 0, 0, 0]^T$?
- What joint torques will hold the coffee in place if $\theta = [\pi/2, 4, 5, 0]^T$?

Problem 1

Given wrench F, let F_s be the wrench in the fixed frame and let F_b be that wrench written in the body frame. Prove that the joint torques are equal for either choice of frame:

$$\tau = J_s(\theta)^T F_s = J_b(\theta)^T F_b \tag{4}$$

Solve this problem using the relationships we have derived between (a) J_s and J_b and (b) F_s and F_b. You have the following equations in your notes:

$$J_s = \mathrm{Ad}_{T_{sb}} J_b \tag{5}$$

$$F_s = (\mathrm{Ad}_{T_{bs}})^T F_b \tag{6}$$

Now just plug these into $J_s(\theta)^T F_s$ and simplify:

$$\tau = J_s^T F_s = (\mathrm{Ad}_{T_{sb}} J_b)^T (\mathrm{Ad}_{T_{bs}})^T F_b \tag{7}$$

$$\tau = J_s^T F_s = J_b^T (\mathrm{Ad}_{T_{sb}})^T (\mathrm{Ad}_{T_{bs}})^T F_b \tag{8}$$

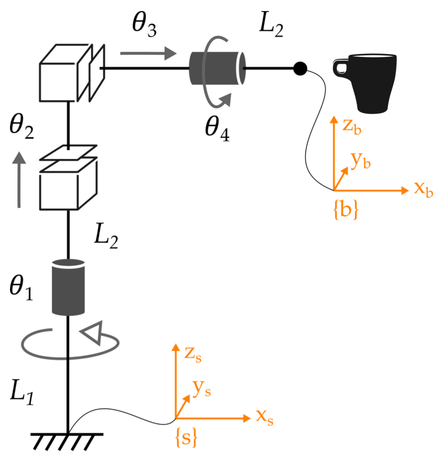
$$\tau = J_s^T F_s = J_b^T (\mathrm{Ad}_{T_{bs}} \mathrm{Ad}_{T_{sb}})^T F_b \tag{9}$$

One property of adjoints is $\mathrm{Ad}_{T^{-1}} = (\mathrm{Ad}_T)^{-1}$. Applying this, we get that:

$$\mathrm{Ad}_{T_{bs}} \mathrm{Ad}_{T_{sb}} = \mathrm{Ad}_{T_{sb}^{-1}} \mathrm{Ad}_{T_{sb}} = (\mathrm{Ad}_{T_{sb}})^{-1} \mathrm{Ad}_{T_{sb}} = I \tag{10}$$

So we can conclude that $\tau = J_s^T F_s = J_b^T F_b$. Physically, this means that we do not need to apply different joint torques τ just because we changed our perspective.

Problem 2



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and the robot's space Jacobian is (you found this in a previous homework assignment):

$$J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 & c_1 \\ 0 & 0 & 0 & s_1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & c_1 & -(L_1 + L_2 + \theta_2)s_1 \\ 0 & 0 & s_1 & (L_1 + L_2 + \theta_2)c_1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{12}$$

- What joint torques will hold the coffee in place if $\theta = [0, 0, 0, 0]^T$?
- What joint torques will hold the coffee in place if $\theta = [\pi/2, 4, 5, 0]^T$?

The cup weighs 10 N and this force is applied along the negative z_b axis. We want to apply an **equal and opposite force** to keep the cup from falling and maintain static equilibrium. Hence, the wrench we want to apply at the end-effector is:

$$F_b = \begin{bmatrix} m_b \\ f_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ +10 \end{bmatrix} \tag{13}$$

We are given J_s and T_{sb}. I'm going to solve this by first converting F_b to F_s:

$$F_s = (\mathrm{Ad}_{T_{bs}})^T F_b \tag{14}$$

and then calculating the joint torques for static equilibrium:

$$\tau = J_s(\theta)^T F_s = J_s(\theta)^T (\mathrm{Ad}_{T_{bs}})^T F_b \tag{15}$$

Evaluate this equation in Matlab using the given terms. Plugging in the home joint position $\theta = [0, 0, 0, 0]^T$ while leaving L_1 and L_2 as symbolic variables:

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

The first prismatic joint must apply a positive force of 10 N to hold up the cup. Let's see how it changes when the robot is in joint position $\theta = [\pi/2, 4, 5, 0]^T$:

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

No change! We use the same prismatic joint to support the weight of the coffee mug.