Practice Set 20

Robotics & Automation
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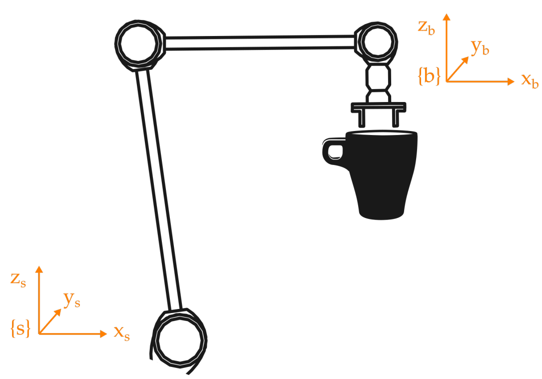
Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

Let x and y be two vectors in $\mathbb{R}^3$. Prove that $-x \times y = [x]^T y$. **Note:** We used this step when changing the frames for moments.

Problem 2



The robot shown above is holding a coffee cup. The coffee weights 15 newtons, and the transformation matrix from base $\{s\}$ to end-effector $\{b\}$ is:

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where both L_1 and L_2 are measured in meters.

- What is the applied wrench in frame $\{b\}$?
- What is the applied wrench in frame $\{s\}$?

Problem 1

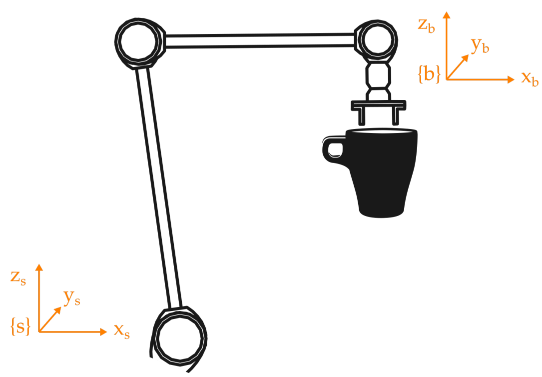
Let x and y be two vectors in $\mathbb{R}^3$. Prove that $-x \times y = [x]^T y$. **Note:** We used this step when changing the frames for moments.

By definition we know that $x \times y = [x]y$. This means that $-x \times y = -[x]y$. Compare the skew-symmetric matrix $[x]$ to $-[x]$ as shown below:

$$[x] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}, \qquad -[x] = \begin{bmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{bmatrix} \tag{2}$$

You should see that $-[x] = [x]^T$. Hence, we have $-x \times y = [x]^T y$.

Problem 2



The robot shown above is holding a coffee cup. The coffee weights 15 newtons, and the transformation matrix from base $\{s\}$ to end-effector $\{b\}$ is:

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where both L_1 and L_2 are measured in meters.

- What is the applied wrench in frame $\{b\}$?
- What is the applied wrench in frame $\{s\}$?

Start with F_b. The cup weighs 15 N, and this force is applied along the negative z_b axis. The cup is at (or directly below) frame $\{b\}$, so there is no moment:

$$F_b = \begin{bmatrix} m_b \\ f_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -15 \end{bmatrix} \tag{4}$$

To find F_s we use the relationship:

$$F_s = (\mathrm{Ad}_{T_{bs}})^T F_b \tag{5}$$

We are given T_{sb} and we know that we can switch the frame of reference by leveraging: $T_{bs} = T_{sb}^{-1}$. Substituting this in, we get:

$$F_s = (\mathrm{Ad}_{T_{sb}^{-1}})^T F_b \tag{6}$$

Now just plug in T_{sb} and F_b to find the answer:

$$F_s = \begin{bmatrix} m_s \\ f_s \end{bmatrix} = \begin{bmatrix} 0 \\ 15L_1 \\ 0 \\ 0 \\ 0 \\ -15 \end{bmatrix} \tag{7}$$

See if this answer agrees with your intuition. As we increase the length of L_1 (i.e., the horizontal offset between $\{s\}$ and $\{b\}$), we increase the moment about the y_s axis. Does this answer make sense?