Practice Set 26

Robotics & Automation
Dylan Losey, Virginia Tech

Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

For each of the characteristic equations shown below, decide if the system is stable.

- $s^2 + 5s = 0$
- $6s^2 + s - 15 = 0$
- $2s^3 + 4s^2 + 2s + 10 = 0$

Problem 2

Given the dynamics below, find a controller that results in closed-loop stability:

$$f(t) + d(t) = m\ddot{\theta}(t) - k\theta(t) \tag{1}$$

Assume you can observe $\theta(t)$ in real-time.

Problem 3

For the same dynamics as the previous problem, determine if a PI controller is stable.

Problem 1

For each of the characteristic equations shown below, decide if the system is stable.

- $s^2 + 5s = 0$
- $6s^2 + s - 15 = 0$
- $2s^3 + 4s^2 + 2s + 10 = 0$

For stability all poles of the characteristic equation must have negative real values. You can solve for the poles, or apply the Routh–Hurwitz criterion.

Equation 1. $s^2 + 5s = 0$. Poles are $s = 0$ and $s = -5$. This is **not stable** because of the pole $s = 0$. Technically this system is **marginally stable** and will oscillate forever.

Equation 2. $6s^2 + s - 15 = 0$. Poles are $s = -5/3$ and $s = 3/2$. This is **not stable**.

Equation 3. $2s^3 + 4s^2 + 2s + 10 = 0$. Poles are $s = -2.4$ and $s = 0.22 \pm 1.4i$. This is **not stable**. With the Routh–Hurwitz criterion, although all terms are positive, we have that $a_2 a_1 = 2 < a_0 = 5$.

Problem 2

Given the dynamics below, find a controller that results in closed-loop stability:

$$f(t) + d(t) = m\ddot{\theta}(t) - k\theta(t) \tag{2}$$

Assume you can observe $\theta(t)$ in real-time.

Convert the given dynamics into the Laplace domain to get the plant $G(s)$:

$$G(s) = \frac{1}{ms^2 - k} \tag{3}$$

The characteristic equation is:

$$1 + C(s)G(s) = 0 \tag{4}$$

Note that $H(s) = 1$ since we can directly observe θ. We are assuming a standard feedback loop; I will provide additional information (or a block diagram) if the loop has been altered. Now plugging in $G(s)$:

$$ms^2 - k + C(s) = 0 \tag{5}$$

According to the Routh–Hurwitz criterion, we need a positive term for $a_1 s$ and a positive term for a_0. Currently we have neither, so try a **PD controller**:

$$C(s) = k_d s + k_p, \qquad ms^2 + k_d s + (k_p - k) = 0 \tag{6}$$

This system is stable for any choice of $k_d > 0$ and $k_p > k$.

Problem 3

For the same dynamics as the previous problem, determine if a PI controller is stable.

Starting with the characteristic equation derived above:

$$ms^2 - k + C(s) = 0 \tag{7}$$

For a PI controller let $C(s) = k_p + k_i/s$. Plugging in:

$$ms^2 + (k_p - k) + \frac{k_i}{s} = 0 \tag{8}$$

$$ms^3 + (k_p - k)s + k_i = 0 \tag{9}$$

No matter what choices we make for k_p and k_i, the PI controller is **not stable**. Using Routh–Hurwitz $a_2 = 0$, and thus not all poles have negative real parts.