Position and Rotation

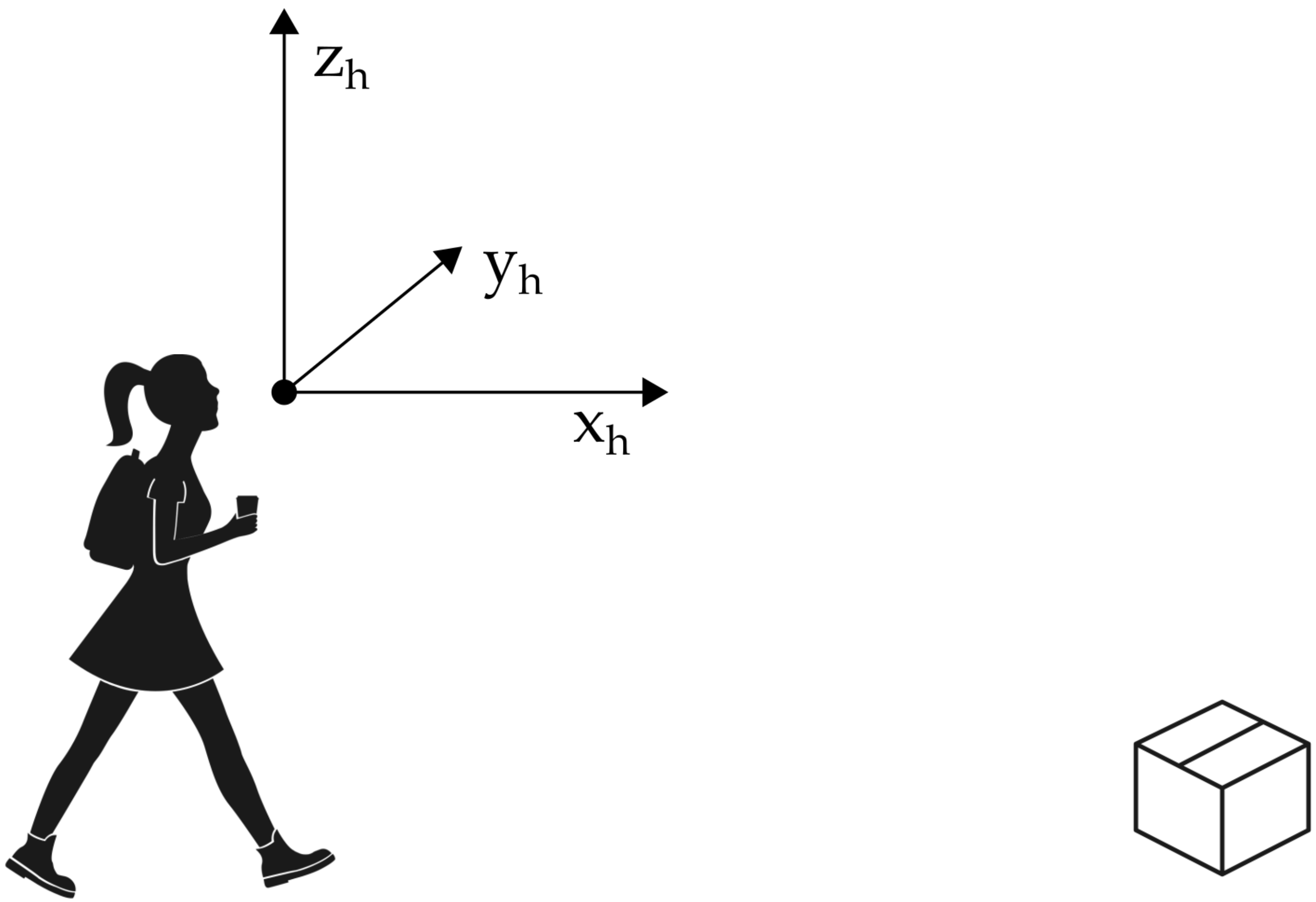
Reading: Modern Robotics 3.1 – 3.2


Domino
SUGAR

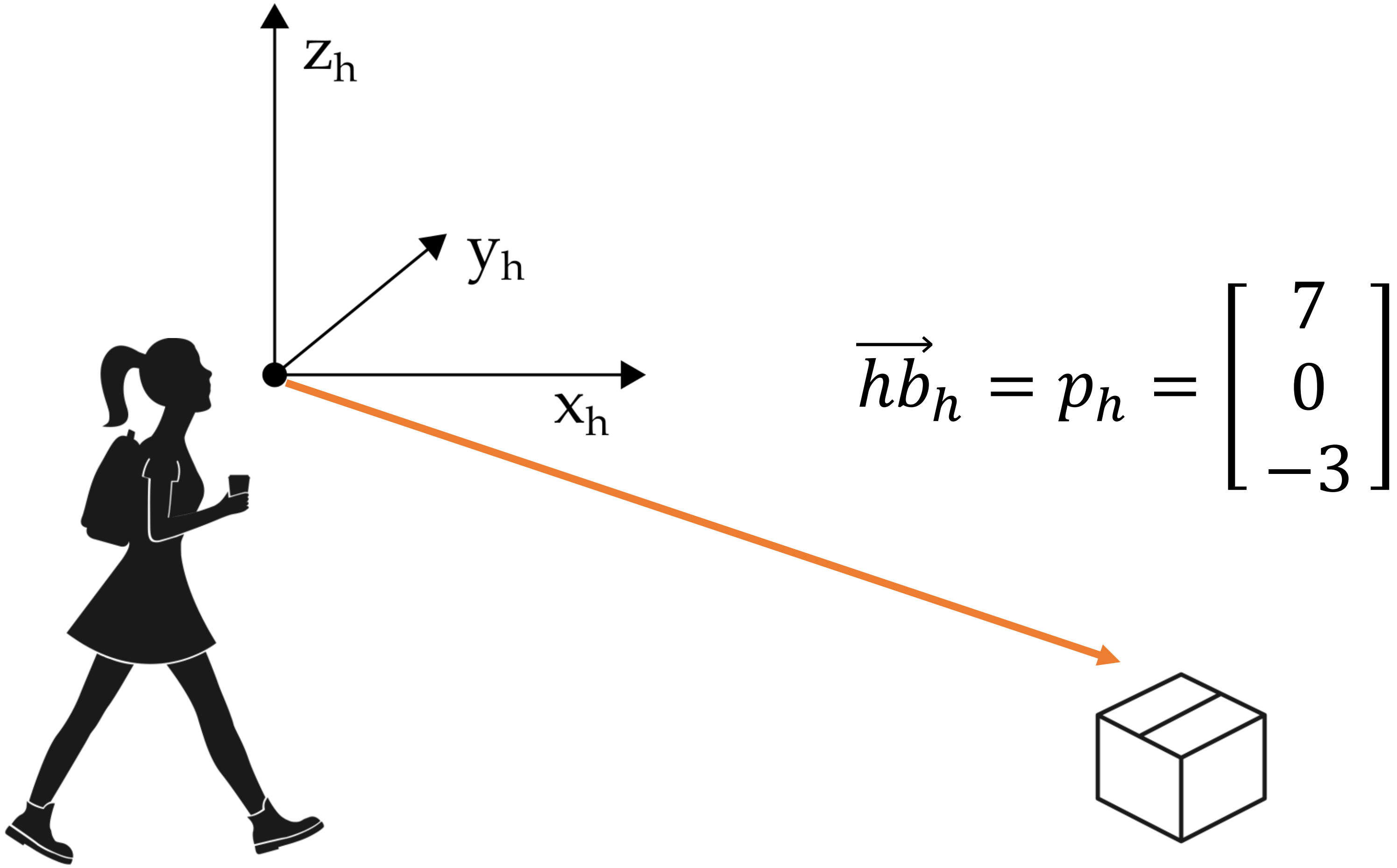
This Lecture

- How do we represent position?
- How do we represent rotation?
- What is a rotation matrix?

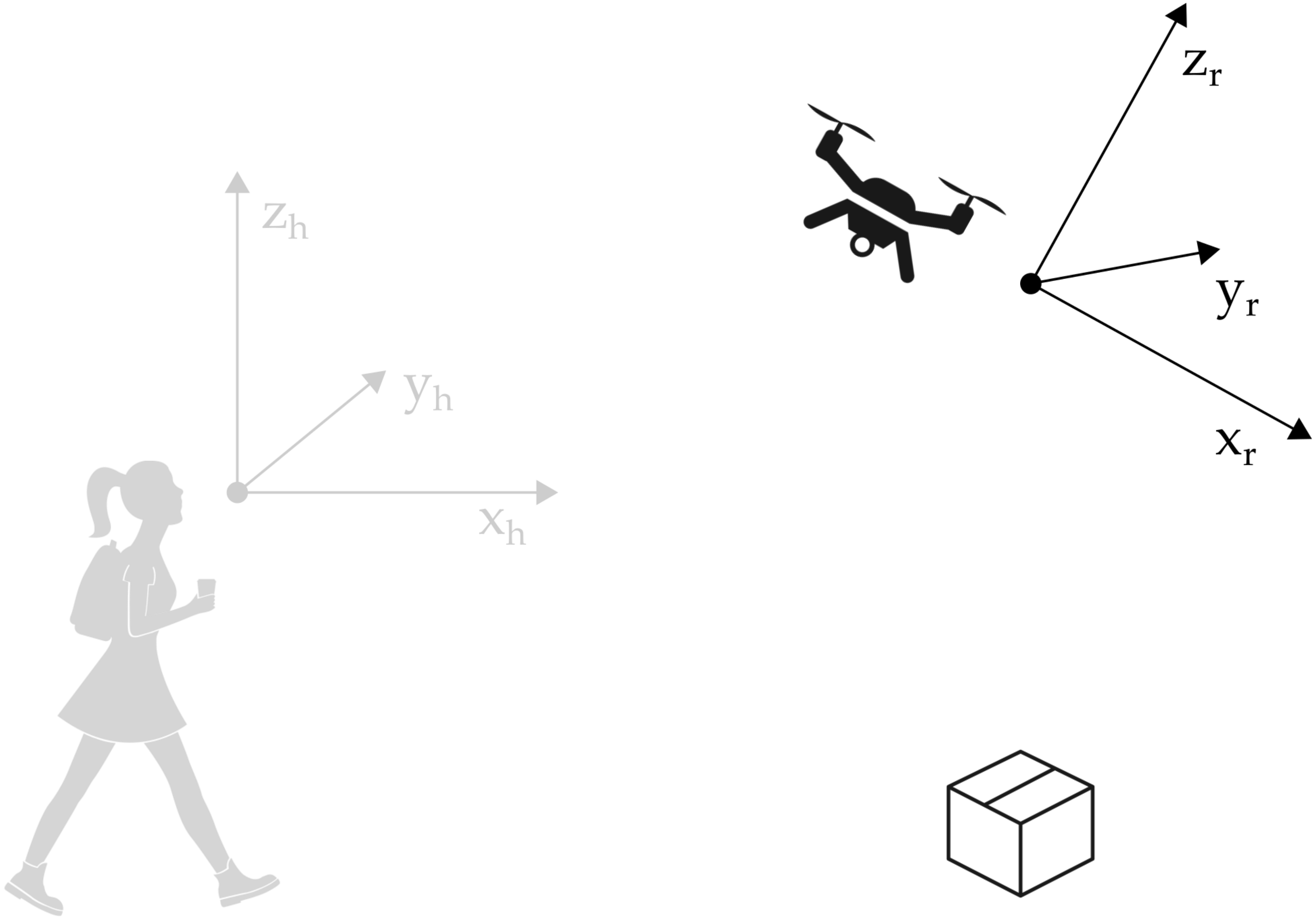
How do we represent **position**?



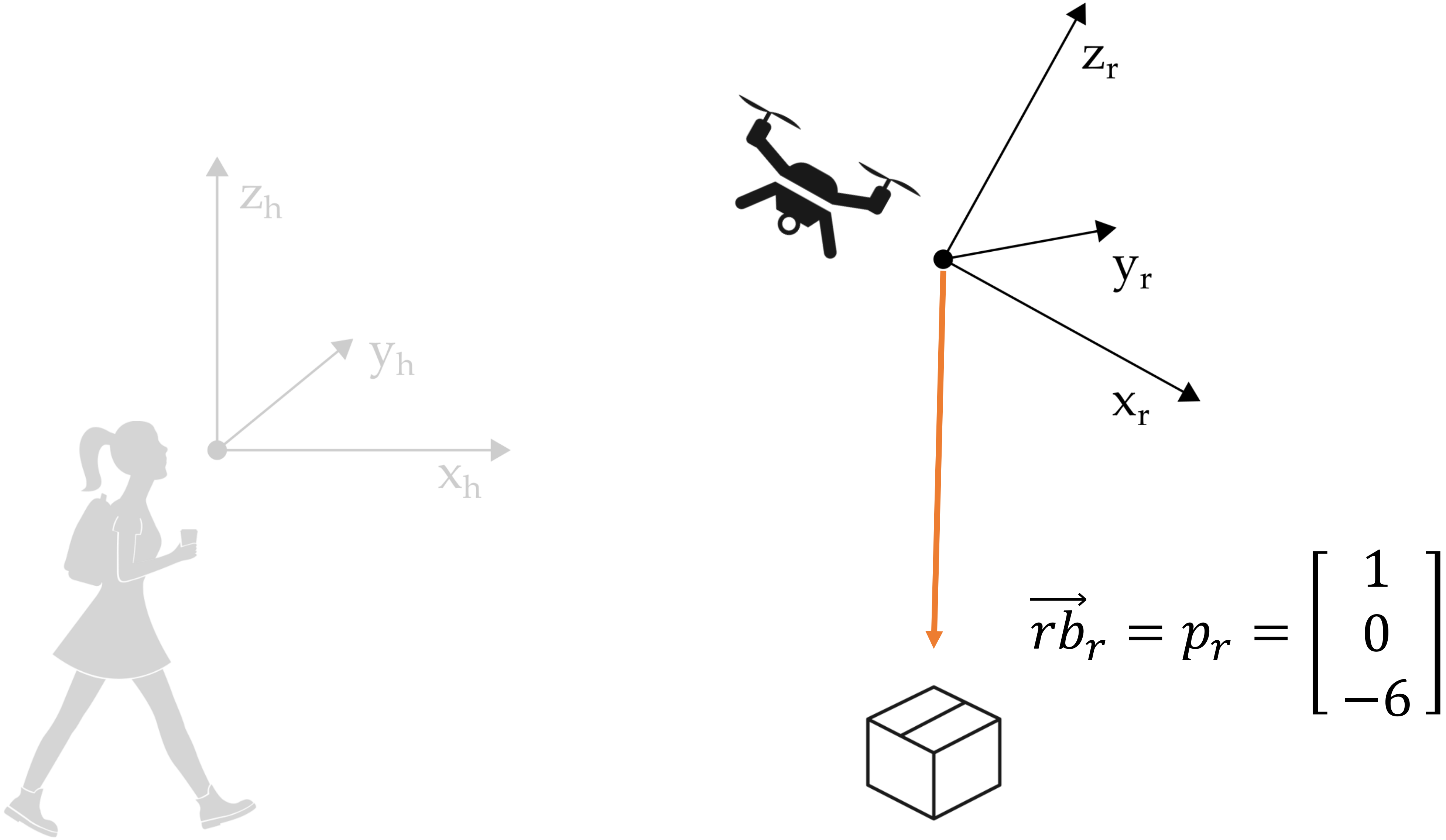
z_h
y_h
x_h



$$\overrightarrow{hb}_h = p_h = \begin{bmatrix} 7 \\ 0 \\ -3 \end{bmatrix}$$



z_h
y_h
x_h
z_r
y_r
x_r

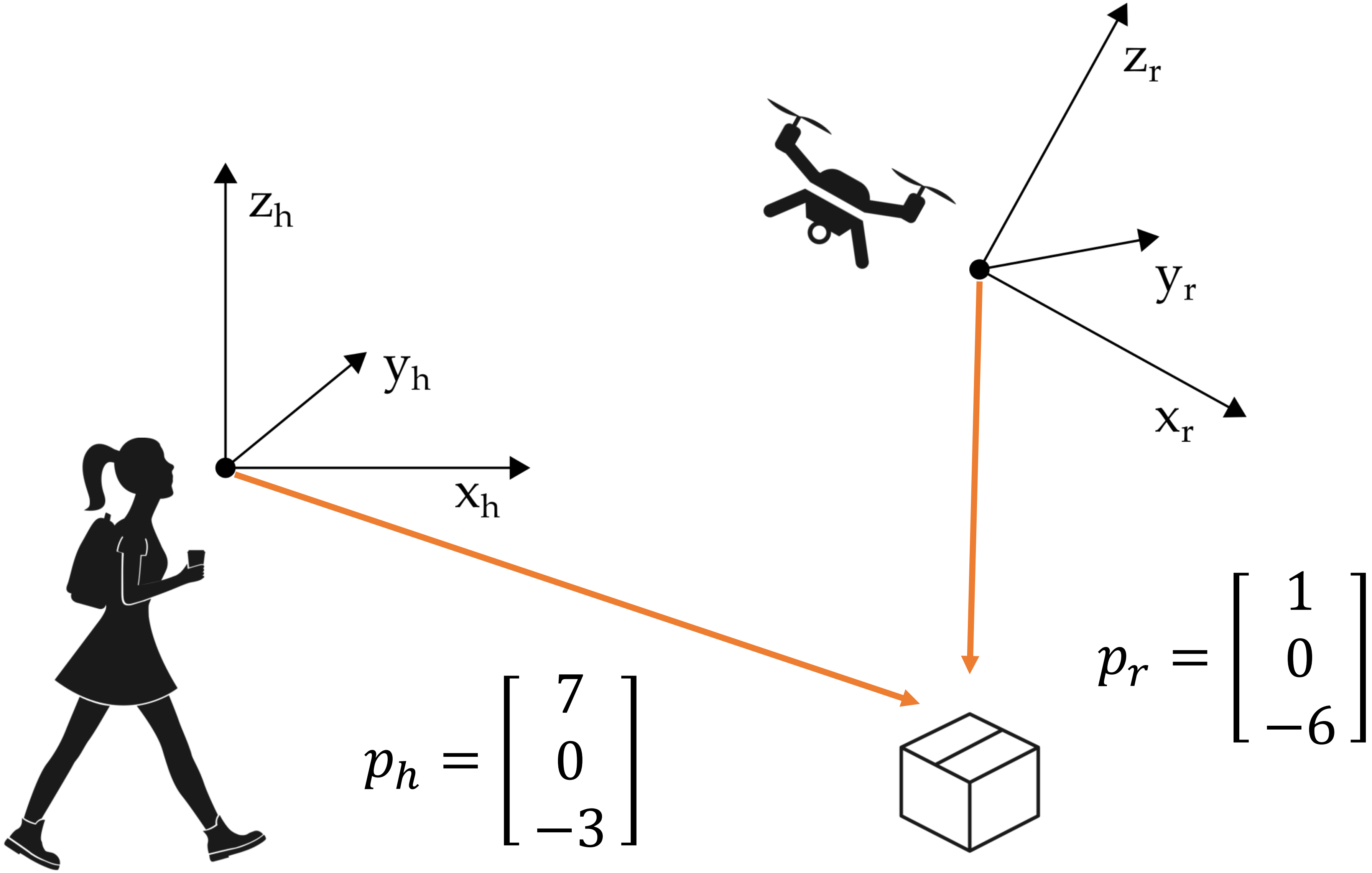

z_h
y_h
x_h
z_r
y_r
x_r
$\overrightarrow{rb}_r = p_r = \begin{bmatrix} 1 \\ 0 \\ -6 \end{bmatrix}$

Position

We represent position as a **vector**

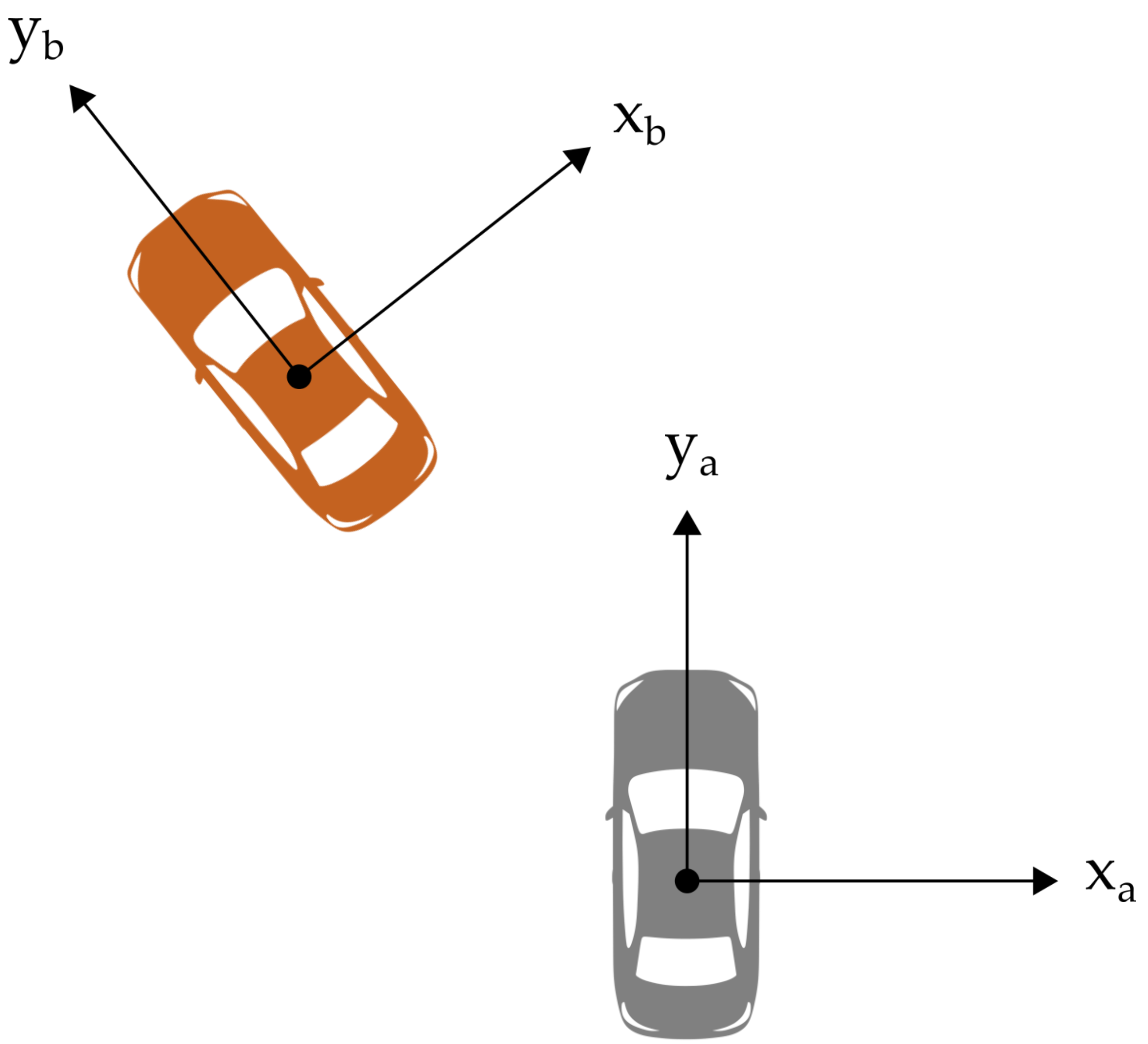
The position of an object depends on the frame of reference

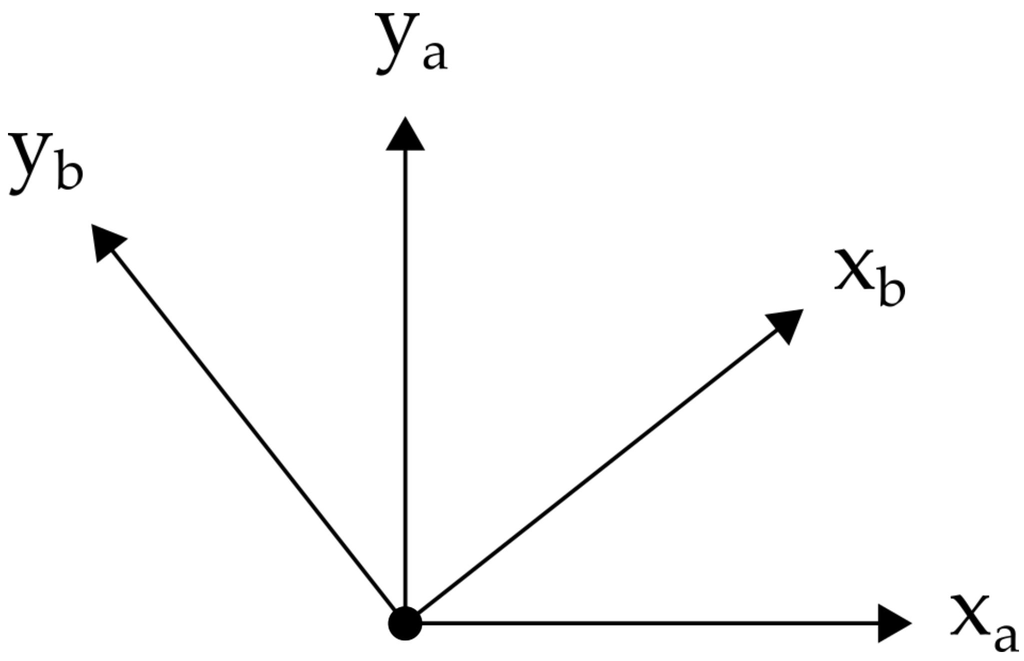
$\boldsymbol{p_a}$ is a vector written in coordinate frame $\boldsymbol{a}$



z_h
y_h
x_h
z_r
y_r
x_r
$p_h = \begin{bmatrix} 7 \\ 0 \\ -3 \end{bmatrix}$
$p_r = \begin{bmatrix} 1 \\ 0 \\ -6 \end{bmatrix}$

How do we represent **rotation**?

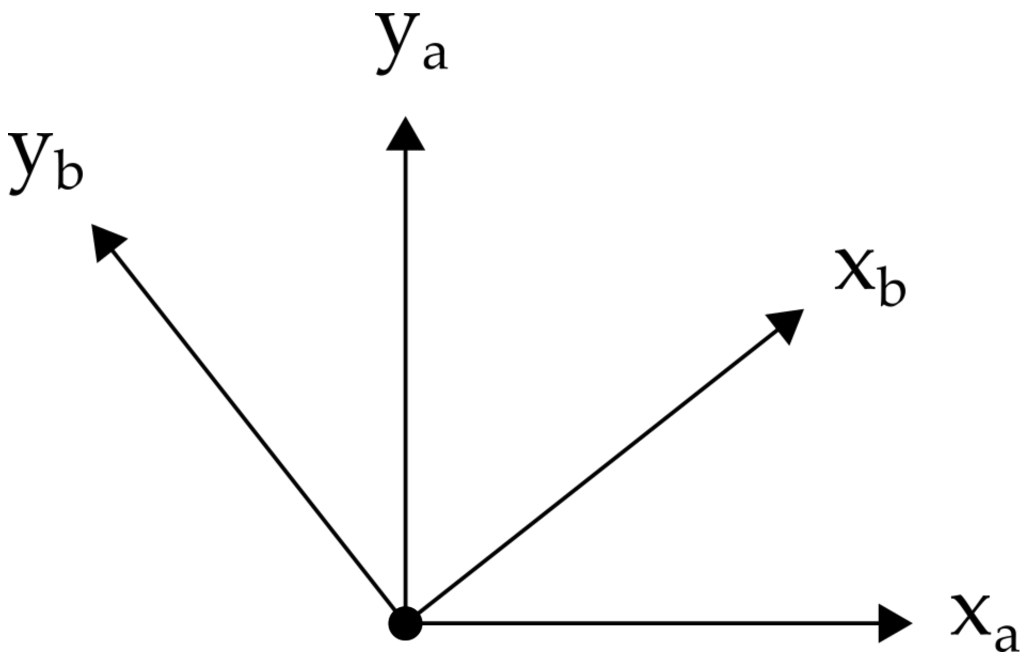

y_b
x_b
y_a
x_a



y_a
y_b
x_b
x_a

We represent rotation as a **matrix**

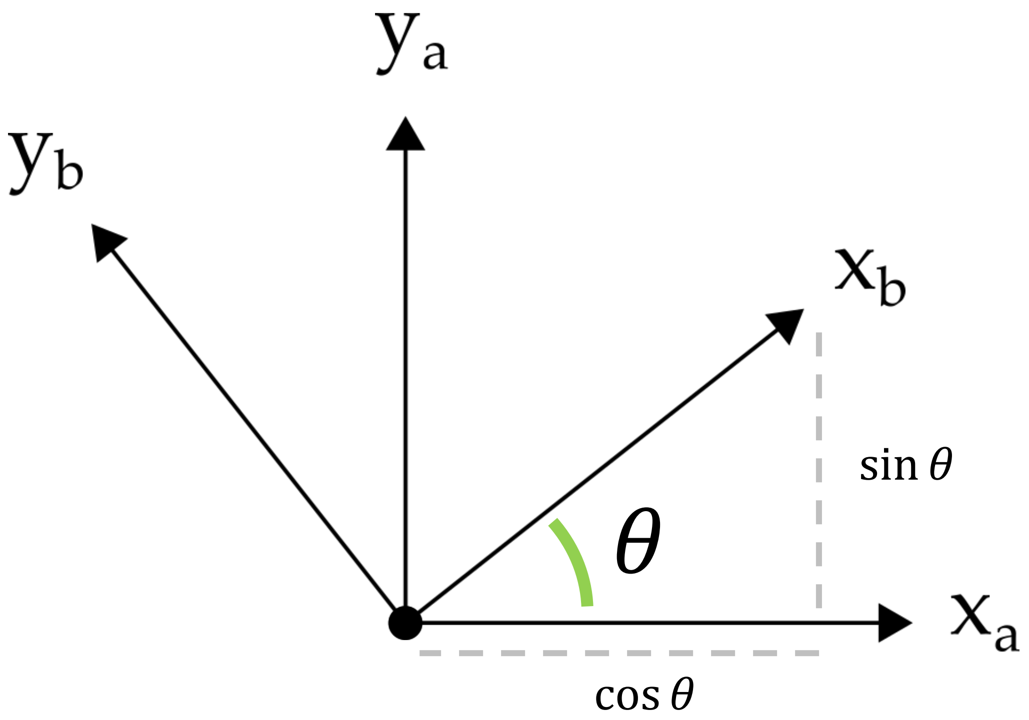
$$R_{ab} = [\ \underbrace{x_b \text{ in } \{a\}}_{\text{Column 1}},\ \underbrace{y_b \text{ in } \{a\}}_{\text{Column 2}}\ ]$$



We represent rotation as a **matrix**

$$R_{ab} = [\ x_b \text{ in } \{a\},\ y_b \text{ in } \{a\}\ ]$$

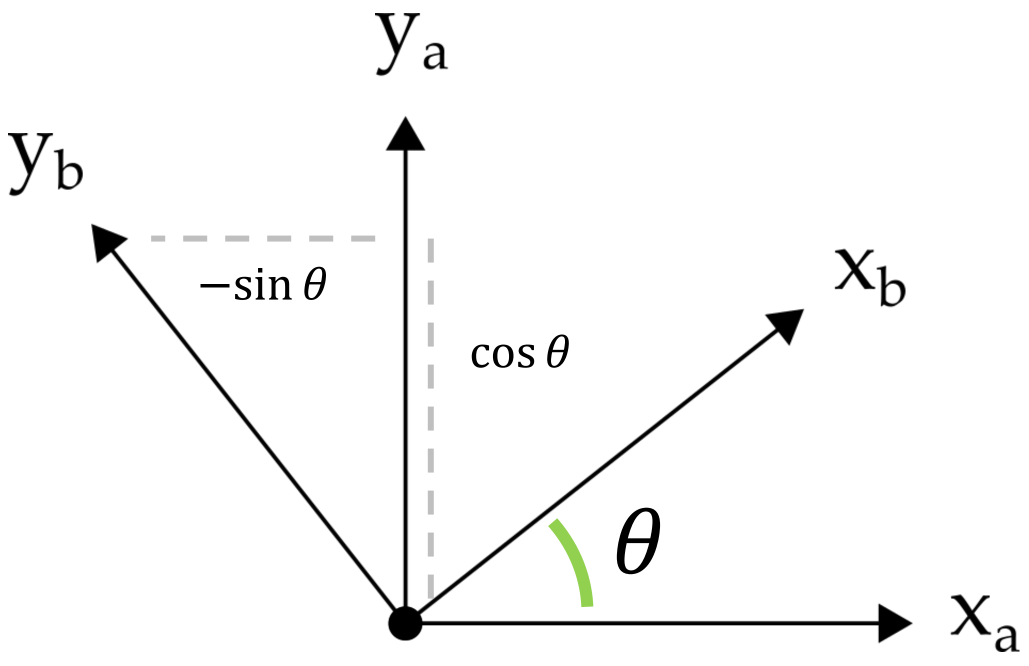
$$R_{ab} = \begin{bmatrix} \cos\theta & \\ \sin\theta & \end{bmatrix}$$



We represent rotation as a **matrix**

$$R_{ab} = [\ x_b \text{ in } \{a\},\ y_b \text{ in } \{a\}\ ]$$

$$R_{ab} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Rotation

We represent rotation as a **matrix**

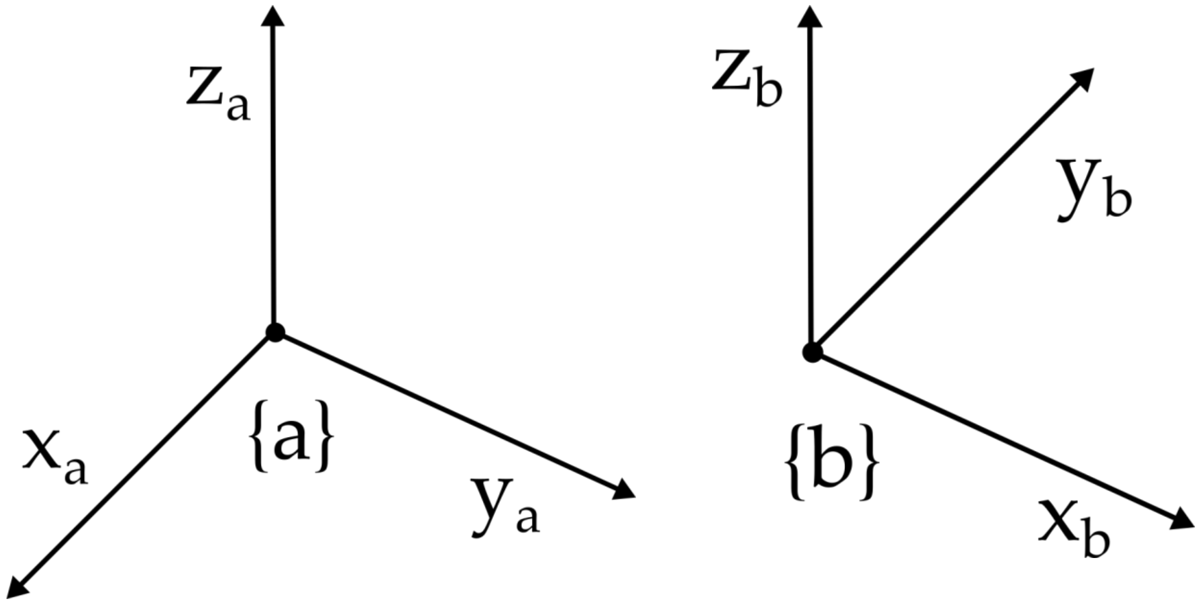
$\boldsymbol{R_{ab}}$ is the orientation of $\boldsymbol{b}$ with respect to coordinate frame $\boldsymbol{a}$

In our three-dimensional world, the rotation matrix is:

$$R_{ab} = [\ \underbrace{x_b \text{ in } \{a\}}_{\textit{Column 1}},\ \underbrace{y_b \text{ in } \{a\}}_{\textit{Column 2}},\ \underbrace{z_b \text{ in } \{a\}}_{\textit{Column 3}}\ ]$$

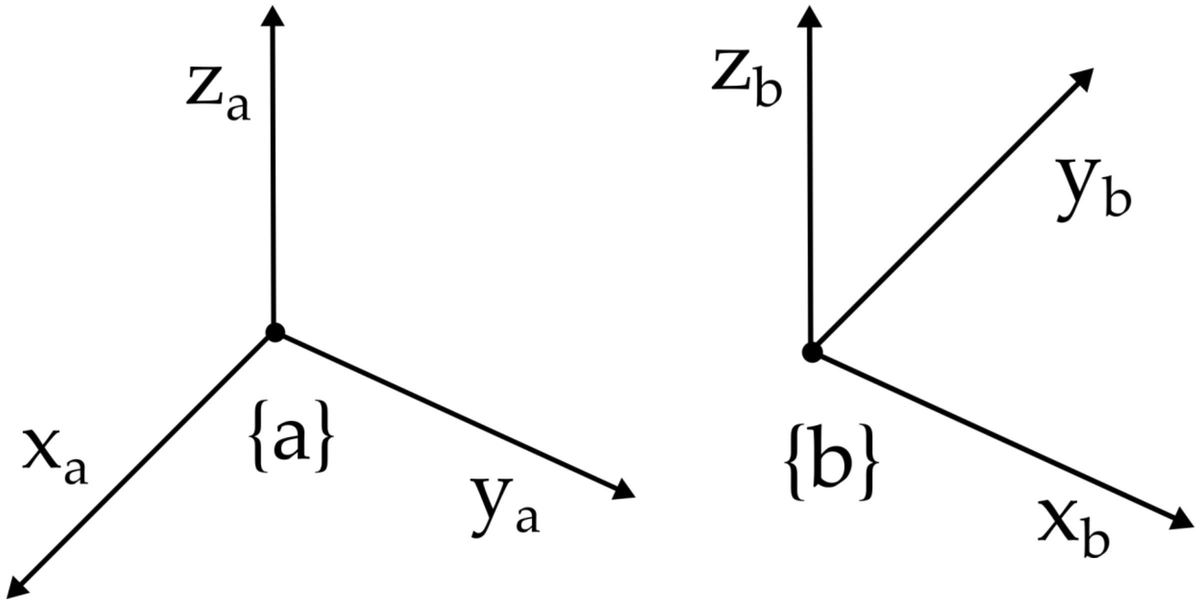
Practice

$$R_{ab} = [\ \underbrace{x_b \text{ in } \{a\}}_{\textit{Column 1}},\ \underbrace{y_b \text{ in } \{a\}}_{\textit{Column 2}},\ \underbrace{z_b \text{ in } \{a\}}_{\textit{Column 3}}\ ]$$



Practice

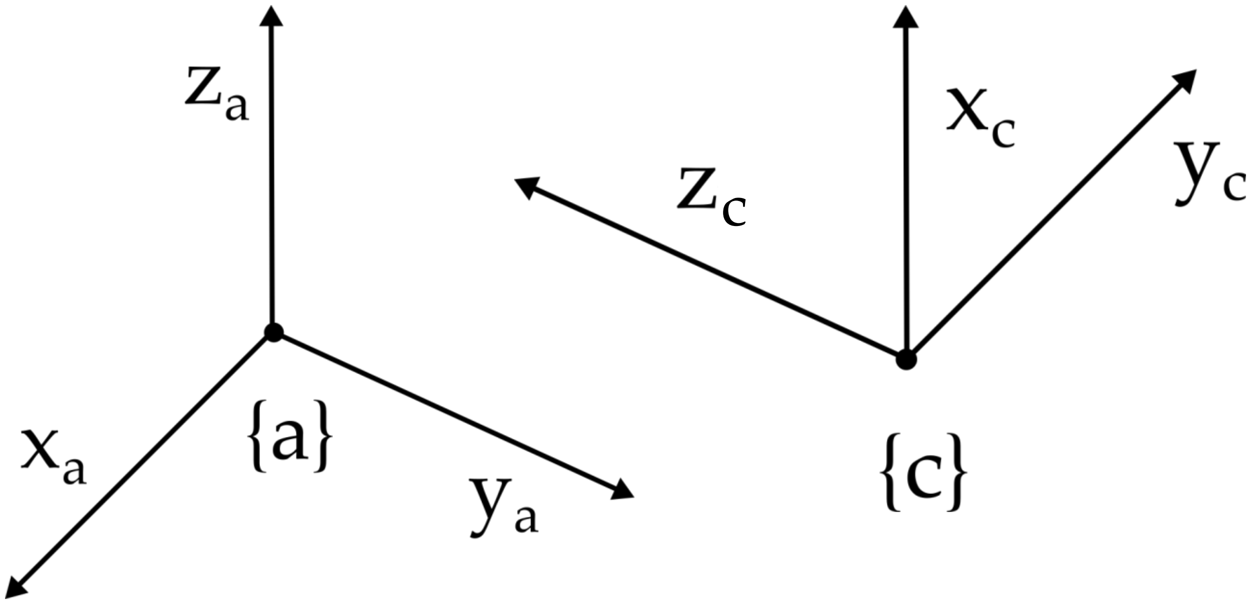
$$R_{ab} = [\ \underbrace{x_b \text{ in } \{a\}}_{\textit{Column 1}},\ \underbrace{y_b \text{ in } \{a\}}_{\textit{Column 2}},\ \underbrace{z_b \text{ in } \{a\}}_{\textit{Column 3}}\ ]$$



$$R_{ab} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Practice

$$R_{ac} = [\ \underbrace{x_c \text{ in } \{a\}}_{\textit{Column 1}},\ \underbrace{y_c \text{ in } \{a\}}_{\textit{Column 2}},\ \underbrace{z_c \text{ in } \{a\}}_{\textit{Column 3}}\ ]$$



$$R_{ac} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

What is a **rotation matrix**?

Properties

Matrix R is a rotation matrix if and only if:

$$\boldsymbol{R^T R = I}$$
$$\mathbf{det}(\boldsymbol{R}) = \mathbf{+1}$$

Properties

Matrix R is a rotation matrix if and only if:

$$R^T R = I$$
$$\det(R) = +1$$

Inverse of a rotation matrix is its transpose:

$$R^T R = I$$

$$R^T R(R^{-1}) = I(R^{-1})$$

$$\boldsymbol{R^T = R^{-1}}$$

Properties

Matrix R is a rotation matrix if and only if:

$$R^T R = I$$
$$\det(R) = +1$$

Transpose switches the frame of reference:

$$\boldsymbol{R_{ab}}^{\boldsymbol{T}} = \boldsymbol{R_{ba}}$$

Properties

Matrix R is a rotation matrix if and only if:

$$R^T R = I$$
$$\det(R) = +1$$

Product is a rotation matrix:

Let R_1 and R_2 be two rotation matrices and let $\boldsymbol{R_3 = R_1 R_2}$. You can prove that:

$$R_3^T R_3 = I$$
$$\det(R_3) = +1$$

This Lecture

- How do we represent position?
- How do we represent rotation?
- What is a rotation matrix?

Next Lecture

- Why do we use rotation matrices?