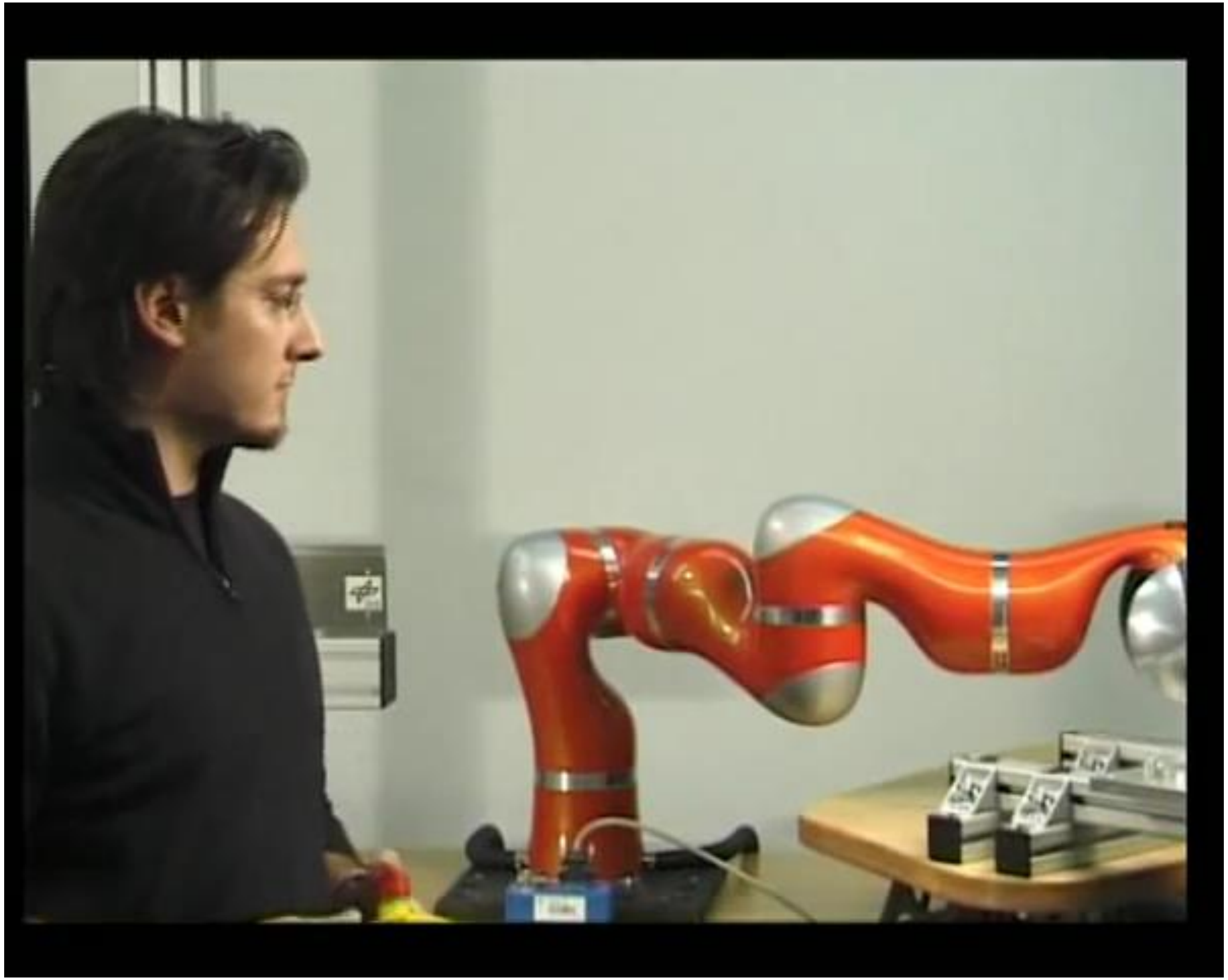
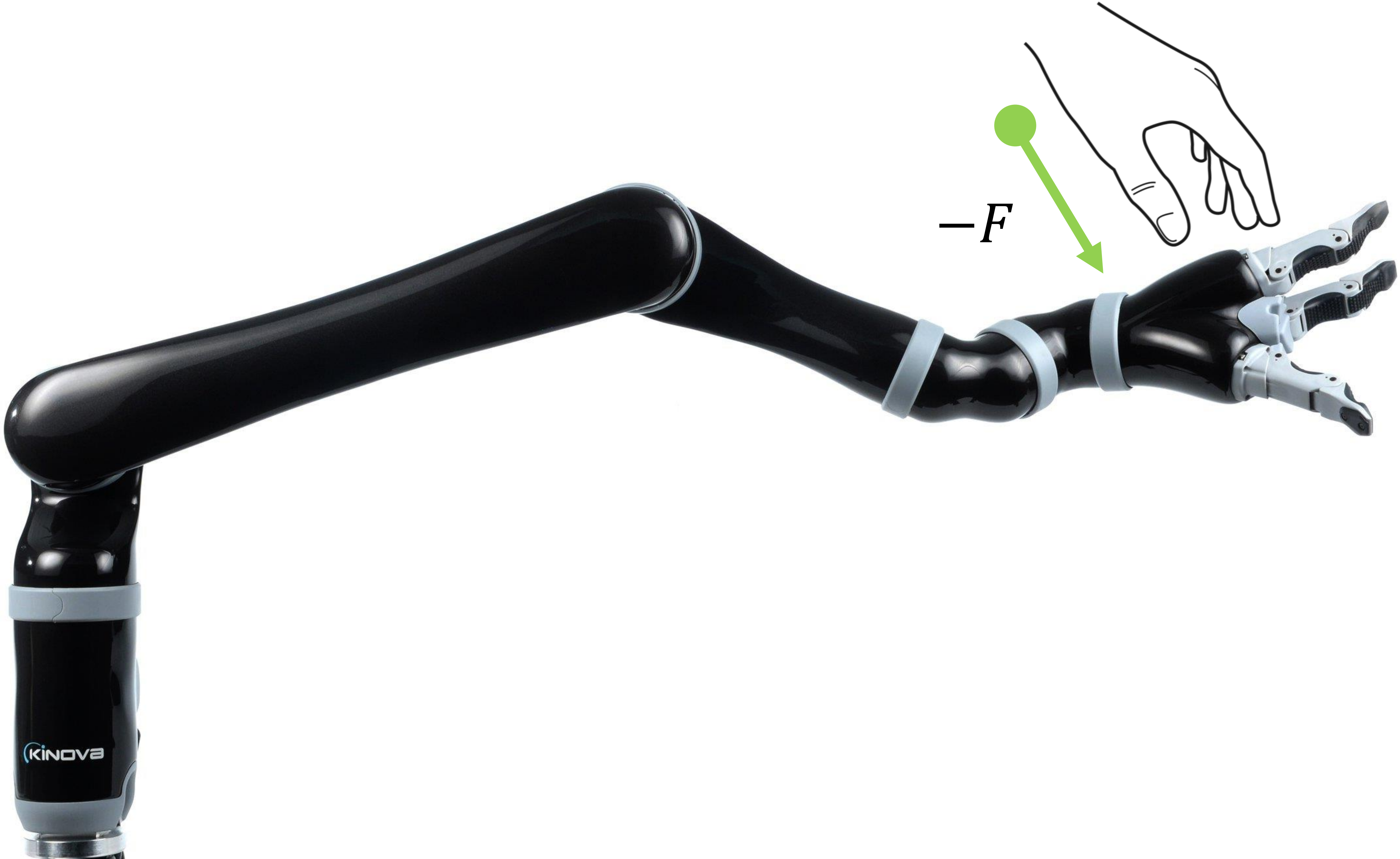
Statics

Reading: Modern Robotics 5.2

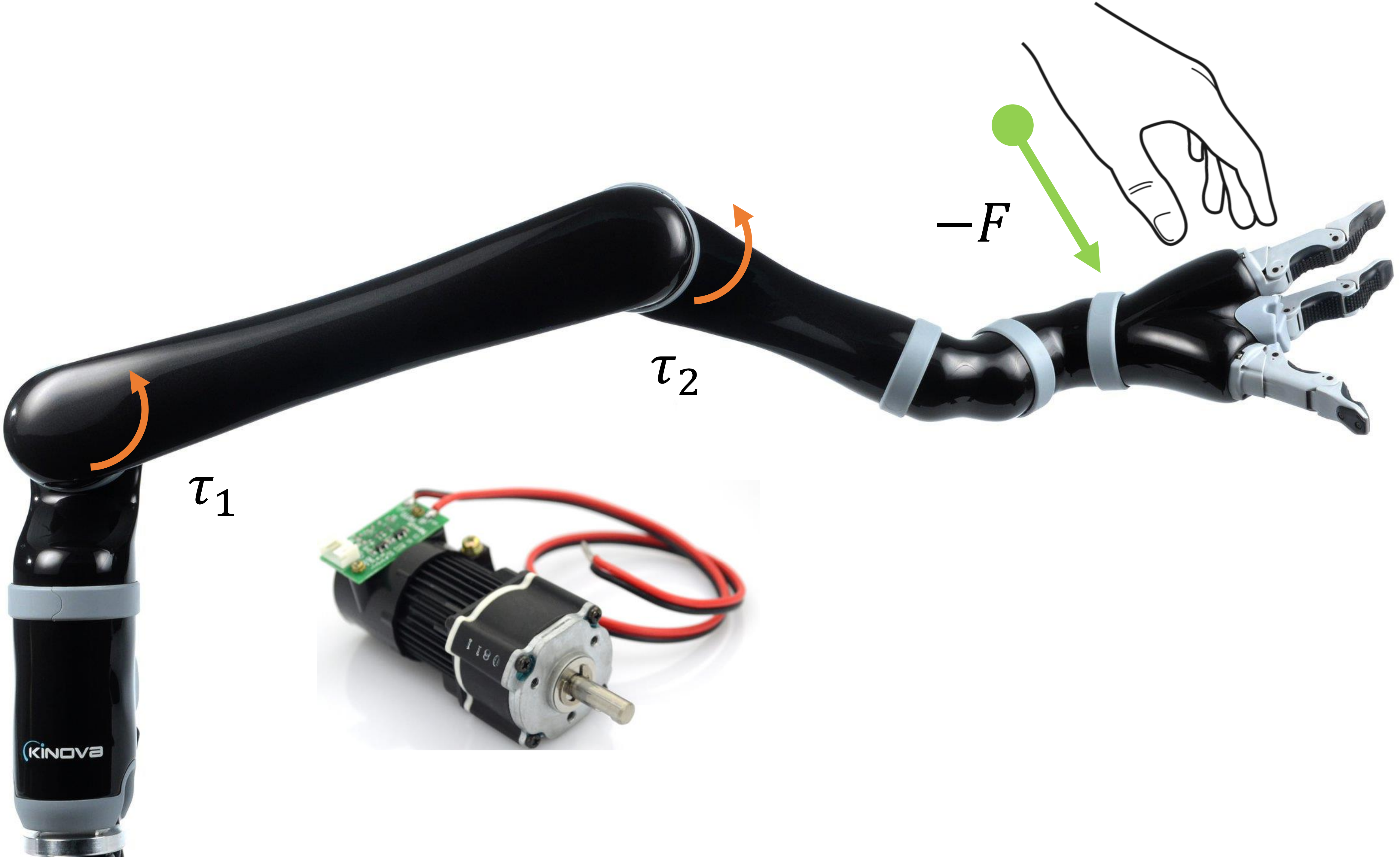


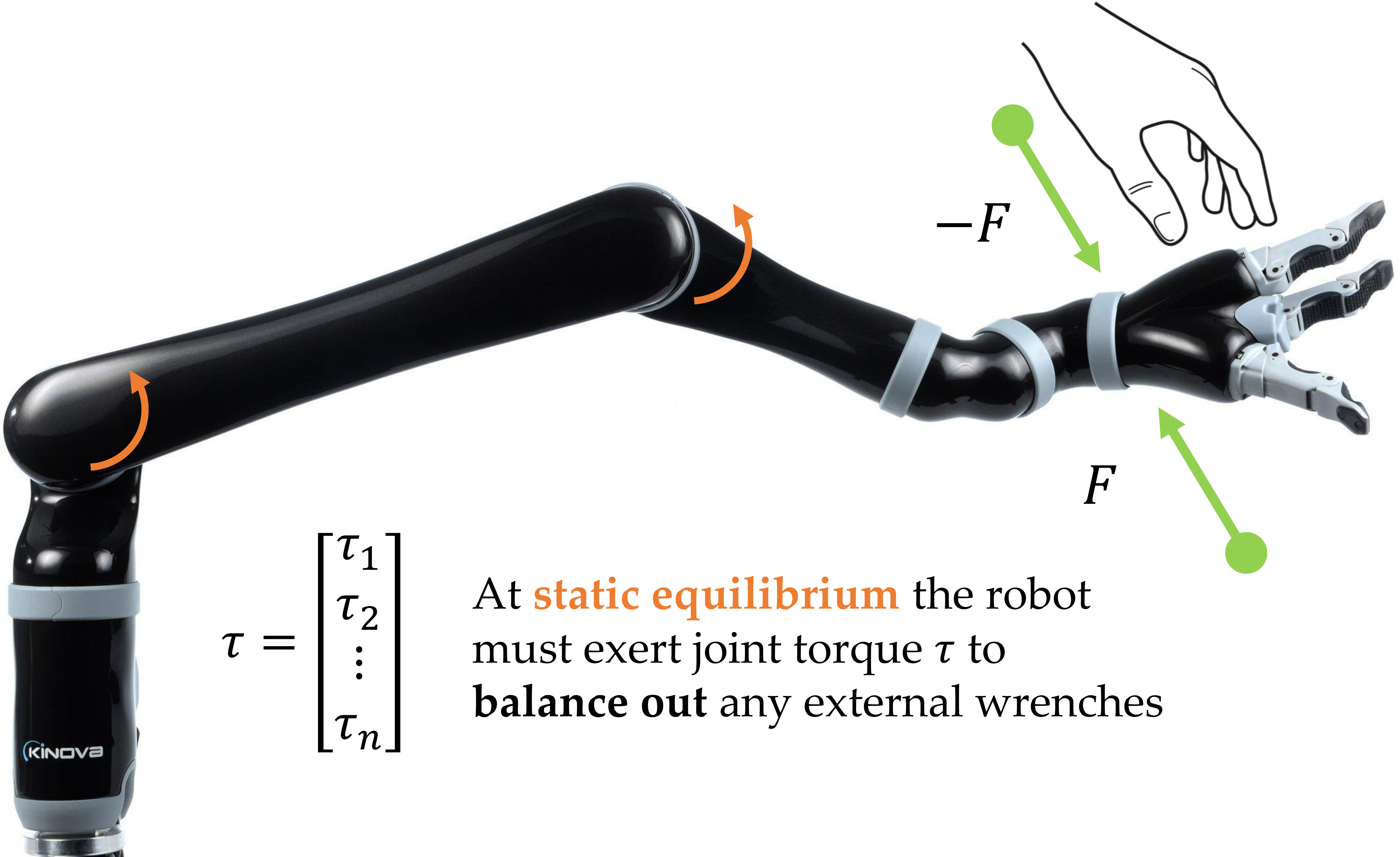
This Lecture

- What joint torques keep the robot in place?
- How do singularities affect statics?



$-F$
KINOVA


$-F$
τ_2
τ_1
KINOVA



$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$$

At **static equilibrium** the robot must exert joint torque τ to **balance out** any external wrenches

How do we convert between **joint torques** and end-effector **wrenches**?

Equilibrium

At static equilibrium, the power in the joints is equal to the power at the end-effector (taking the limit as velocity goes towards zero):

$$\underbrace{\dot{\theta}^T \tau}_{\textit{Power in joint space}} = \underbrace{V^T F}_{\textit{Power in end-effector space}}$$

Equilibrium

At static equilibrium, the power in the joints is equal to the power at the end-effector (taking the limit as velocity goes towards zero):

$$\dot{\theta}^T \tau = V^T F$$

Use the Jacobian:

$$V = J(\theta)\dot{\theta}$$

Equilibrium

At static equilibrium, the power in the joints is equal to the power at the end-effector (taking the limit as velocity goes towards zero):

$$\dot{\theta}^T(\tau = J(\theta)^T F)$$

$$\boxed{\tau = J(\theta)^T F}$$

Equilibrium

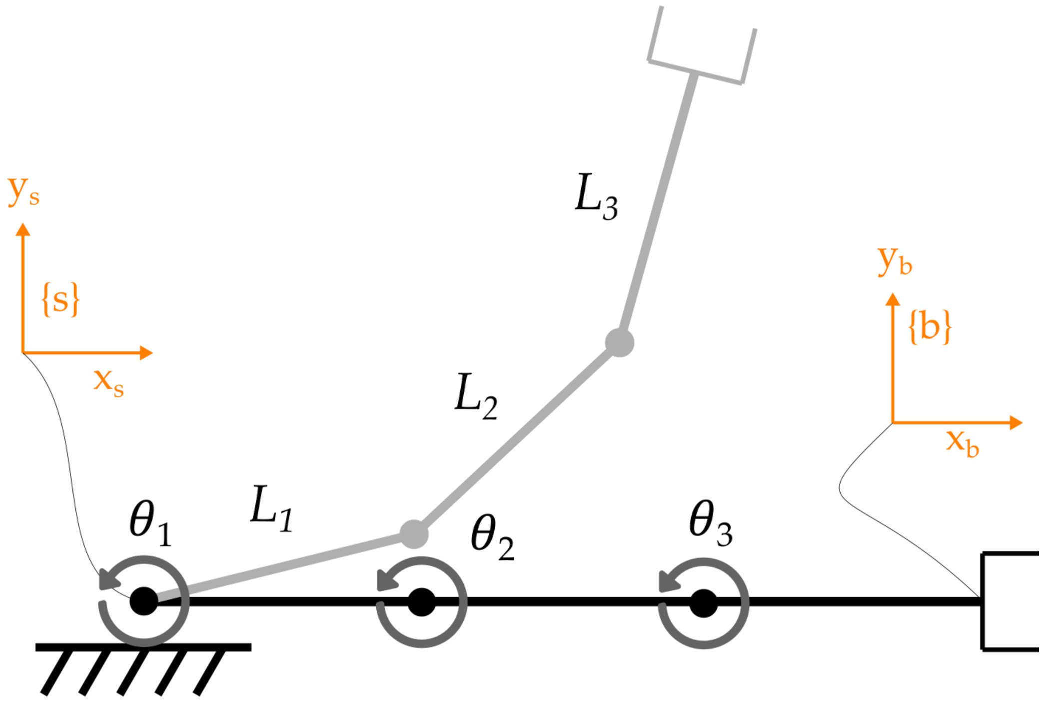
$$\tau = J(\theta)^T F$$

- Given wrench F, find the joint torques τ that apply that wrench
- Works for any Jacobian if F in **same frame** (i.e., if $F = F_s$ is in spatial frame, then use space Jacobian, and if $F = F_b$ is in body frame, then use body Jacobian)

Example

To apply F at the end-effector, what joint torque do we need?

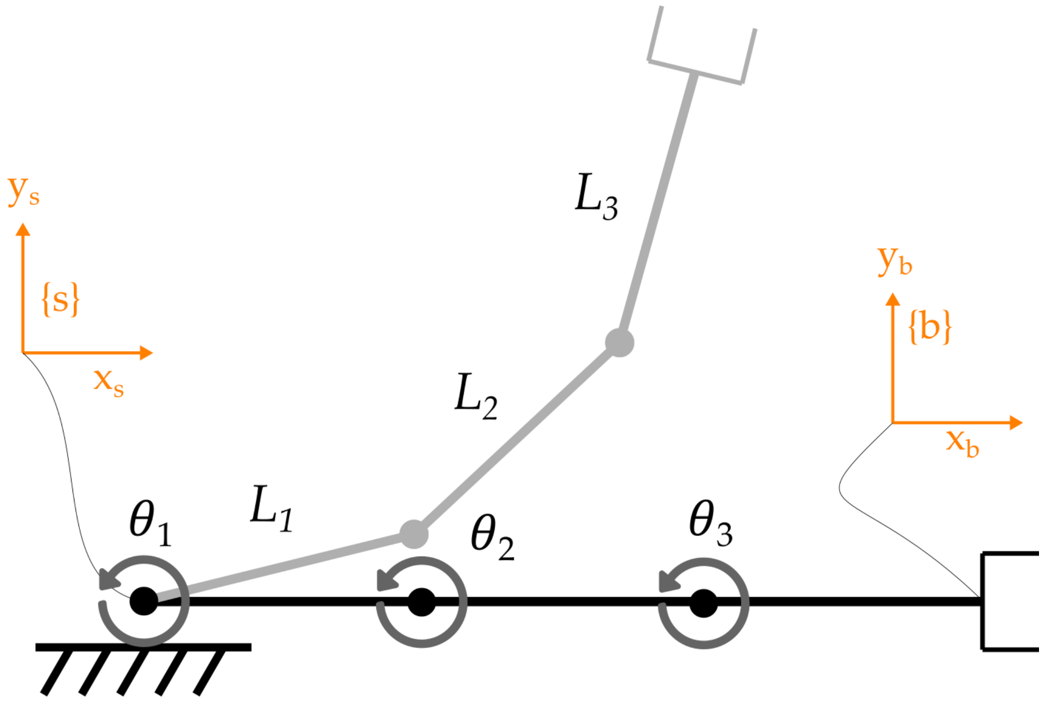
$$J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & L_1 s_1 & L_1 s_1 + L_2 s_{12} \\ 0 & -L_1 c_1 & -L_1 c_1 - L_2 c_{12} \\ 0 & 0 & 0 \end{bmatrix}$$



Example

To apply F at the end-effector, what joint torque do we need?

$$\underbrace{\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}}_{\tau} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & L_1 s_1 & L_1 s_1 + L_2 s_{12} \\ 0 & -L_1 c_1 & -L_1 c_1 - L_2 c_{12} \\ 0 & 0 & 0 \end{bmatrix}^T}_{J_s(\theta)^T} \underbrace{\begin{bmatrix} m_x \\ m_y \\ m_z \\ f_x \\ f_y \\ f_z \end{bmatrix}}_{F_s}$$

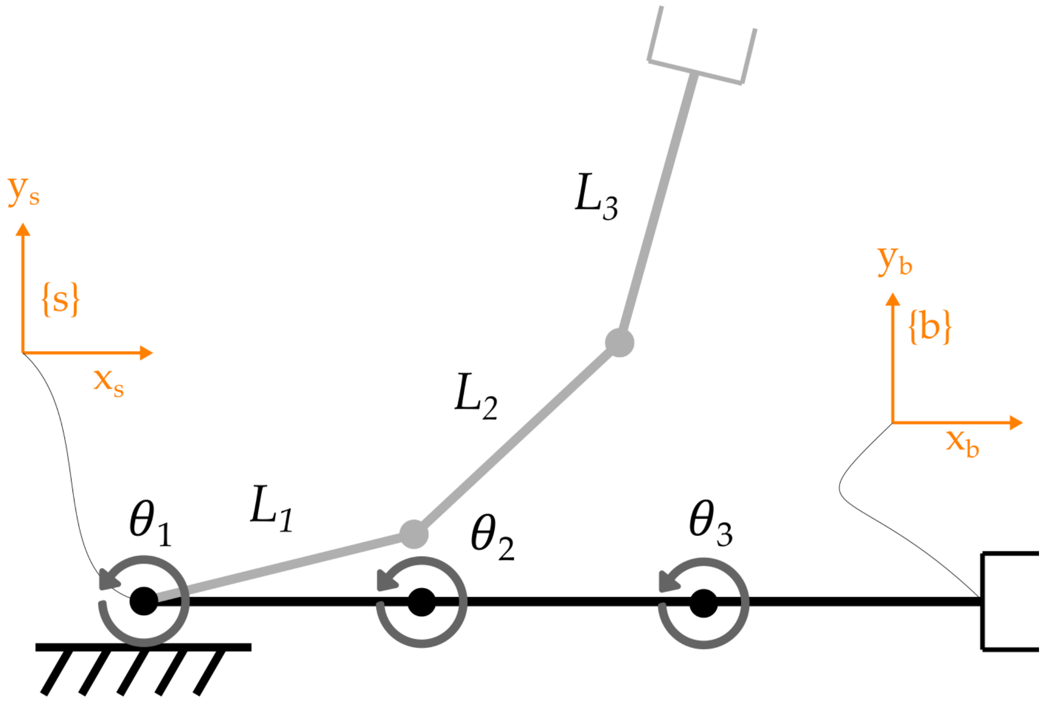


Example

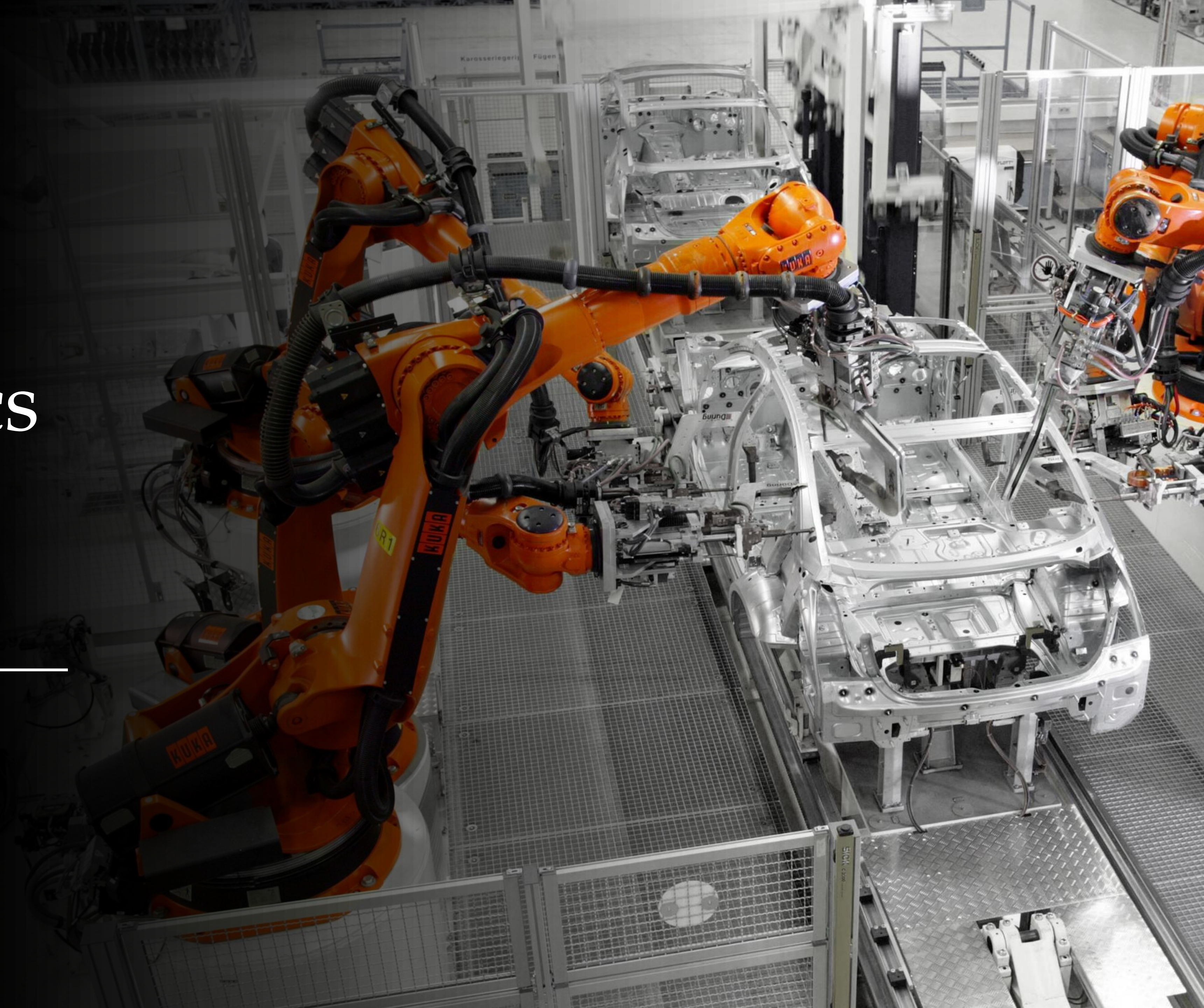
To apply F at the end-effector, what joint torque do we need?

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} m_z \\ m_z + f_x L_1 s_1 - f_y L_1 c_1 \\ m_z + f_x(L_1 s_1 + L_2 s_{12}) - f_y(L_1 c_1 + L_2 c_{12}) \end{bmatrix}$$

No f_z here. No matter what f_z is applied, it does not affect the joint torque



How do statics change at **singularities**?

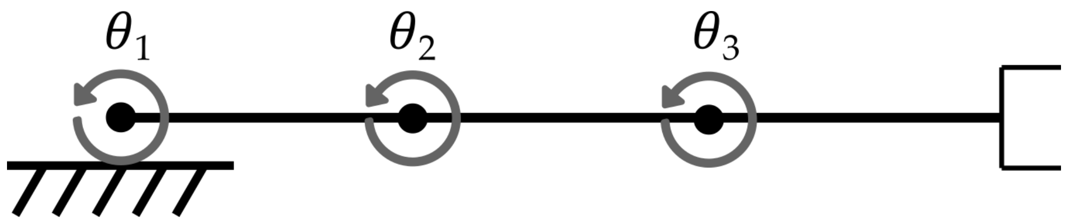


Example

This robot is at a **singularity**
when $\theta = 0$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} m_z \\ m_z + \boldsymbol{f_x L_1 s_1} - f_y L_1 c_1 \\ m_z + \boldsymbol{f_x (L_1 s_1 + L_2 s_{12})} - f_y (L_1 c_1 + L_2 c_{12}) \end{bmatrix}$$

Here $\sin(\theta_1) = \sin(\theta_1 + \theta_2) = 0$

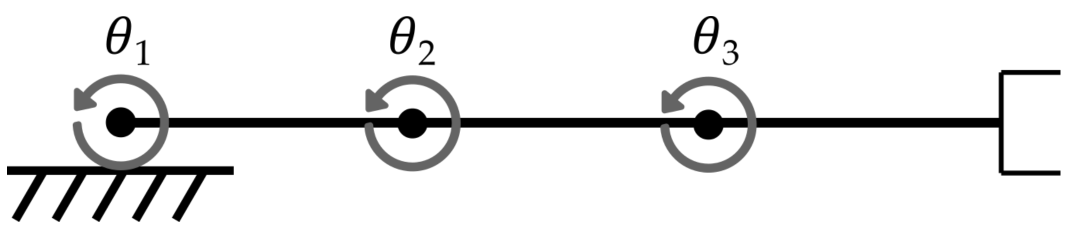


Example

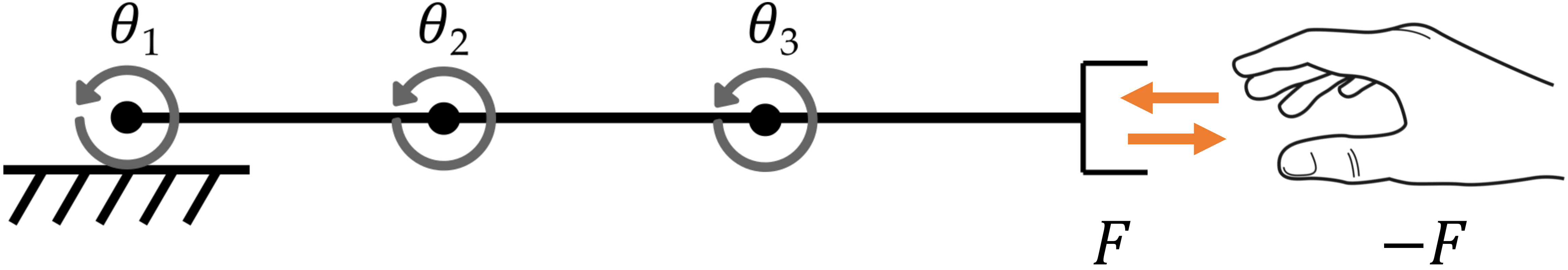
This robot is at a **singularity** when $\theta = 0$

$$\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix} = \begin{bmatrix} m_z \\ m_z - f_y L_1 c_1 \\ m_z - f_y(L_1 c_1 + L_2 c_{12}) \end{bmatrix}$$

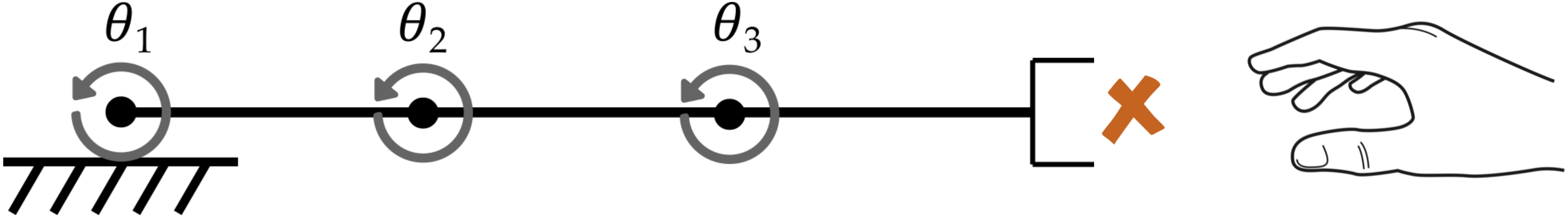
No f_x now. No matter what f_x is applied, it does not affect the joint torque



The robot **passively resists** wrenches in the direction of the singularity



But the robot **cannot apply** wrenches in the direction of the singularity



Takeaways

Wrenches are like twists, but deal with **moment and force** instead of **angular and linear velocity**.

$$V = J(\theta)\dot{\theta}$$

At singularity, cannot move along null J^T

$$\tau = J(\theta)^T F$$

At singularity, cannot control wrenches along null J^T

This Lecture

- What joint torques keep the robot in place?
- How do singularities affect statics?

Next Lecture

- We've dealt with statics…

  …what about dynamics?