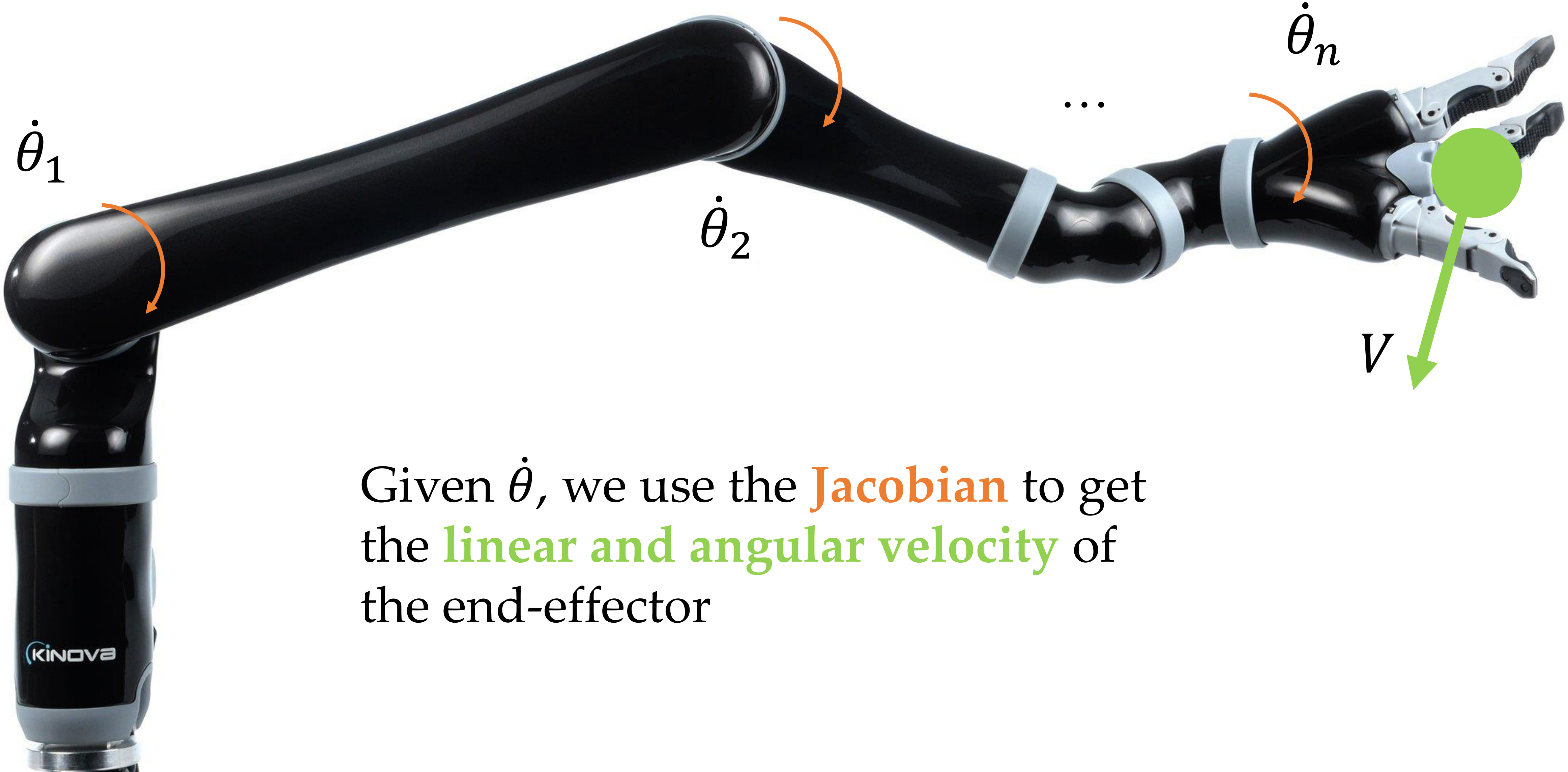
Space Jacobian

Reading: Modern Robotics 5.1.1

This Lecture

- How do we find the Jacobian of a robot arm?



Given $\dot{\theta}$, we use the **Jacobian** to get the **linear and angular velocity** of the end-effector

Deriving the Jacobian

We know that the **spatial twist** of the robot's end-effector is:

$$[V_s] = \dot{T}T^{-1}$$

$$T = e^{[S_1]\theta_1} e^{[S_2]\theta_2} M$$

for simplicity let's say the robot has two joints

Deriving the Jacobian

$$T = e^{[S_1]\theta_1} e^{[S_2]\theta_2} M$$

$$T^{-1} = M^{-1} e^{-[S_2]\theta_2} e^{-[S_1]\theta_1}$$

remember to flip order of matrices when taking inverse

$$\dot{T} = [S_1]\dot{\theta}_1 e^{[S_1]\theta_1} e^{[S_2]\theta_2} M + e^{[S_1]\theta_1} [S_2]\dot{\theta}_2 e^{[S_2]\theta_2} M$$

chain rule where θ is a function of time

Deriving the Jacobian

Plugging in and applying the properties of **adjoint** operators:

$$[V_s] = \dot{T}T^{-1}$$

$$\boxed{V_s = S_1\dot{\theta}_1 + Ad_{e^{[S_1]\theta_1}}S_2\dot{\theta}_2}$$

Deriving the Jacobian

Plugging in and applying the properties of **adjoint** operators:

$$[V_s] = \dot{T}T^{-1}$$

$$V_s = \underline{[S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2]}\dot{\theta}$$

this matrix is the robot's
space Jacobian

V_s
$\{s\}$

Space Jacobian

Space Jacobian relates **joint velocity** to **spatial twist**:

$$V_s = J_s(\theta)\dot{\theta}$$

$$J_s(\theta) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3 \cdots]$$

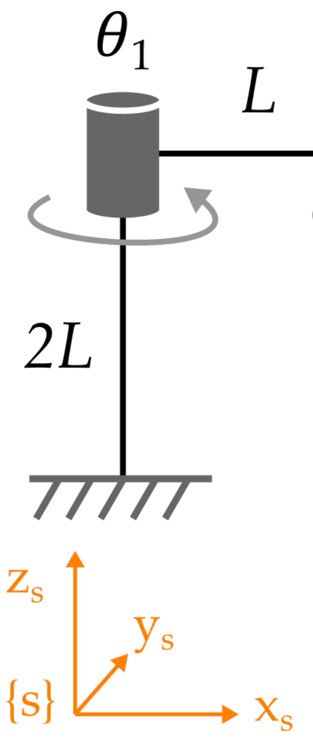
if robot has n joints, this matrix has n columns
each column is $\mathrm{Ad}_{e^{[S_1]\theta_1} \cdots e^{[S_{i-1}]\theta_{i-1}}} S_i$

Space Jacobian Example

Example

Three-DoF robot arm.

Given joint values θ and joint velocity $\dot{\theta}$, what is the spatial twist of the end-effector?



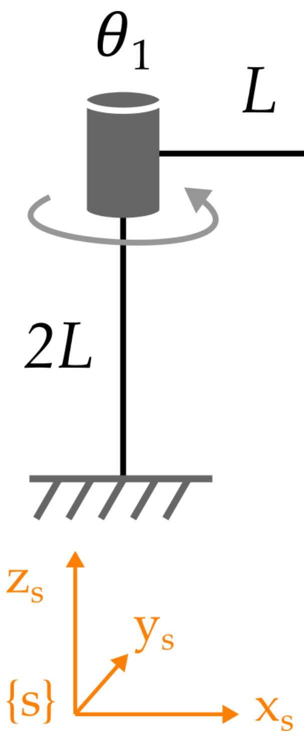
Example

Three-DoF robot arm.

Given joint values θ and joint velocity $\dot{\theta}$, what is the **spatial twist** of the end-effector?

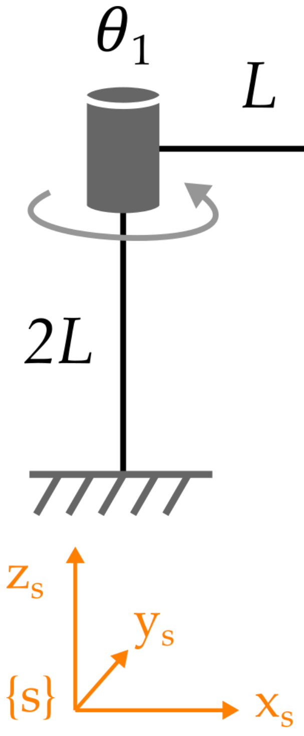
$$V_s = \boldsymbol{J_s}(\boldsymbol{\theta})\dot{\theta}$$

$$\boldsymbol{J_s}(\boldsymbol{\theta}) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad Ad_{e^{[S_1]\theta_1}e^{[S_2]\theta_2}} S_3]$$



Example

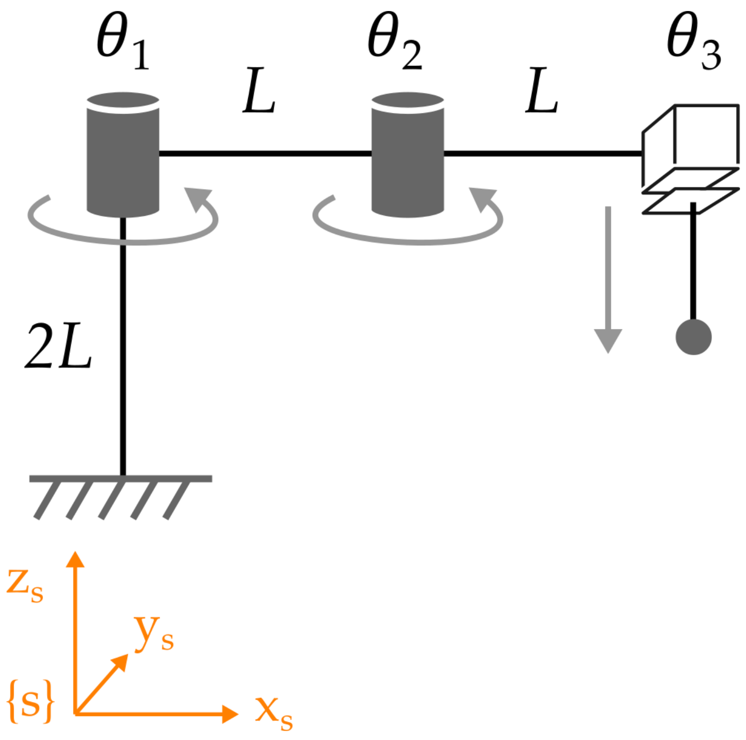
Step 1. S_i is the screw for the i-th joint when the robot is in home position



Example

Step 1. S_i is the screw for the i-th joint when the robot is in home position

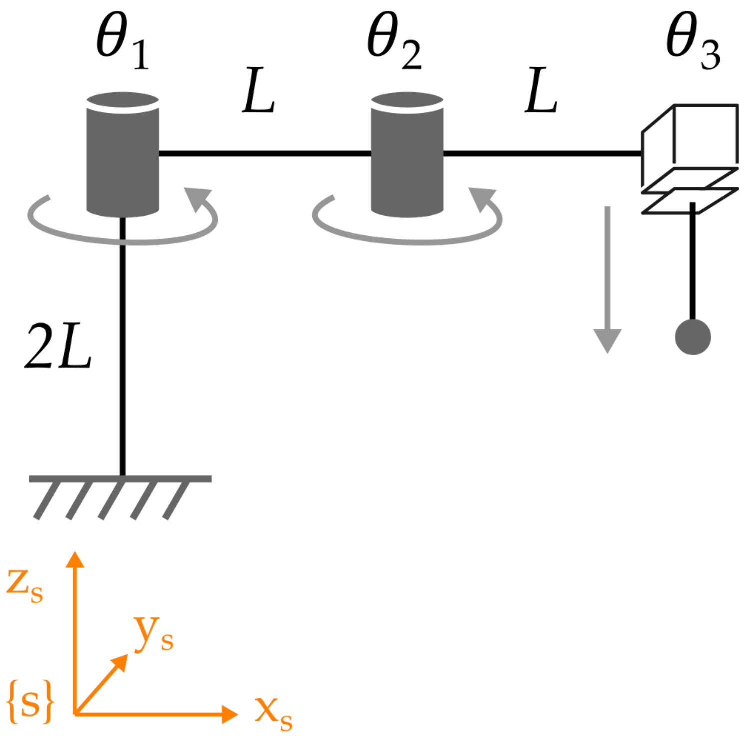
$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -L \\ 0 \end{bmatrix}, \qquad S_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$



Example

Step 2. Use adjoints to get each column of Jacobian

$$\boldsymbol{J_s}(\boldsymbol{\theta}) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3]$$

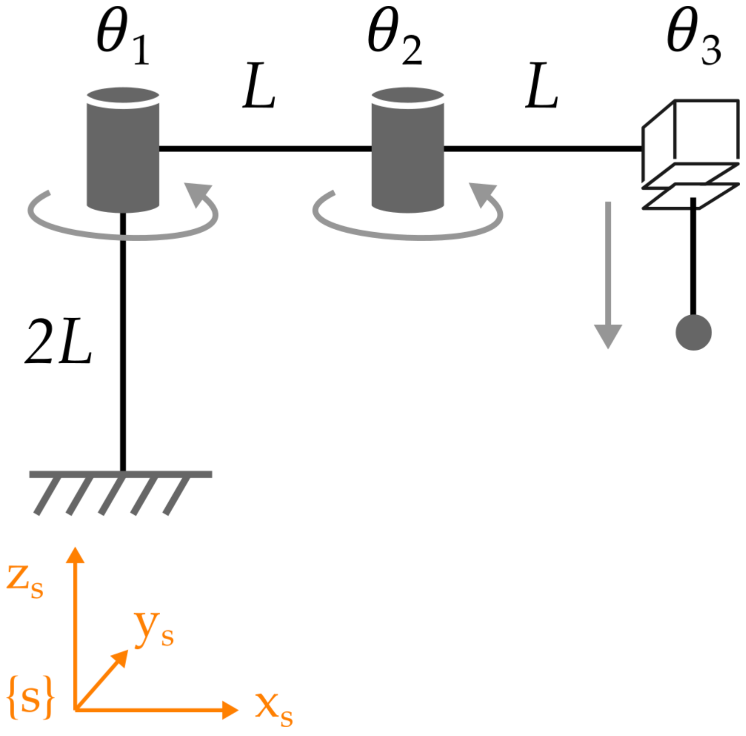


Example

Step 2. Use adjoints to get each column of Jacobian

$$\boldsymbol{J_s}(\boldsymbol{\theta}) = [S_1 \quad \underline{Ad_{e^{[S_1]\theta_1}} S_2} \quad Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3]$$

$$Ad_{e^{[S_1]\theta_1}} = \begin{bmatrix} c_1 & -s_1 & 0 & & 0 & \\ s_1 & c_1 & 0 & & & \\ 0 & 0 & 1 & & & \\ & & & c_1 & -s_1 & 0 \\ & 0 & & s_1 & c_1 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

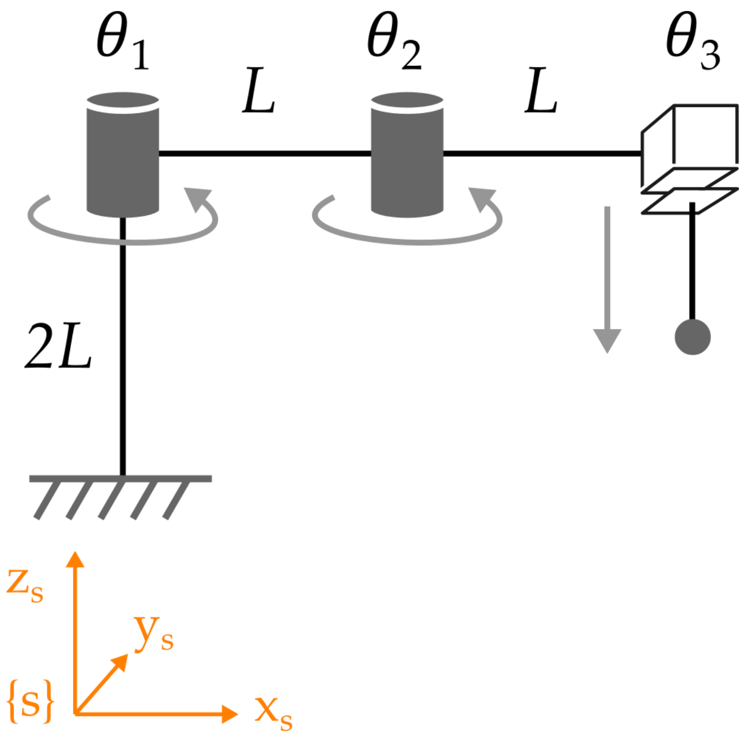


Example

Step 2. Use adjoints to get each column of Jacobian

$$J_s(\theta) = [S_1 \quad \underline{Ad_{e^{[S_1]\theta_1}} S_2} \quad Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3]$$

$$\begin{bmatrix} c_1 & -s_1 & 0 & & & \\ s_1 & c_1 & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ & & & c_1 & -s_1 & 0 \\ & 0 & & s_1 & c_1 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -L \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ Ls_1 \\ -Lc_1 \\ 0 \end{bmatrix}$$

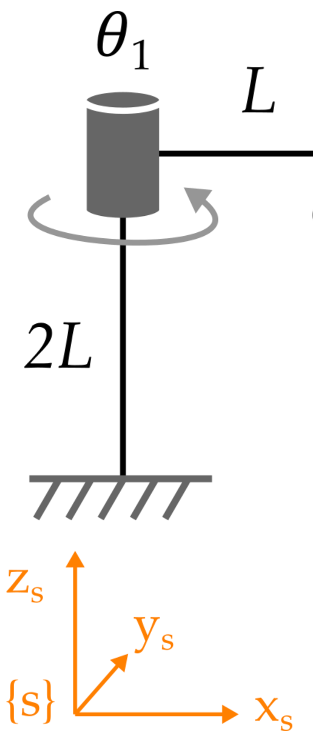


Example

Step 2. Use adjoints to get each column of Jacobian

$$\boldsymbol{J_s}(\boldsymbol{\theta}) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad \underline{Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3}]$$

Hint: Make sure to multiply $e^{[S_1]\theta_1} e^{[S_2]\theta_2}$ first, then take the adjoint of the result

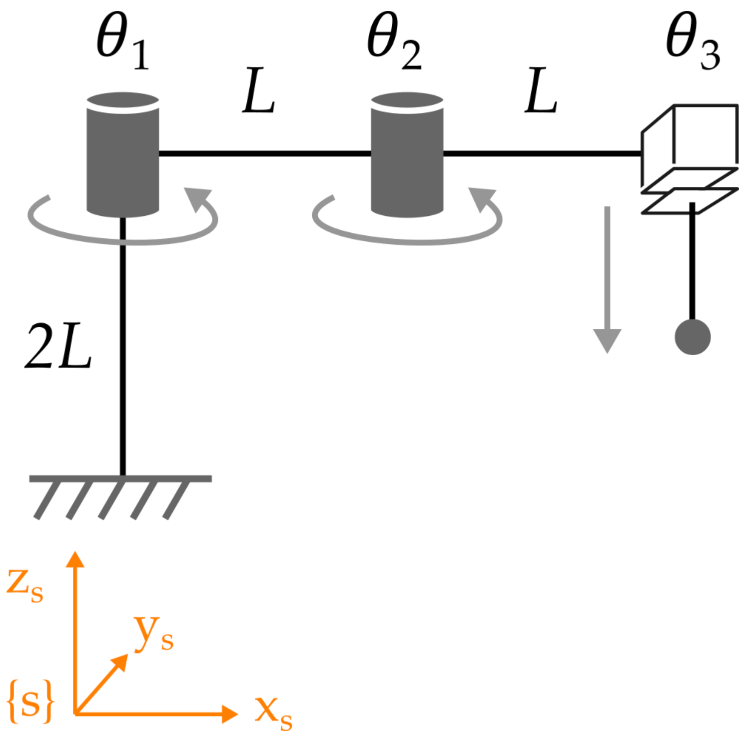


Example

Step 2. Use adjoints to get each column of Jacobian

$$J_s(\theta) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad \underline{Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3}]$$

$$e^{[S_1]\theta_1} e^{[S_2]\theta_2} = \begin{bmatrix} c_{12} & -s_{12} & 0 & L(c_1 - c_{12}) \\ s_{12} & c_{12} & 0 & L(s_1 - s_{12}) \\ 0 & 0 & 1 & 0 \\ & 0 & & 1 \end{bmatrix}$$

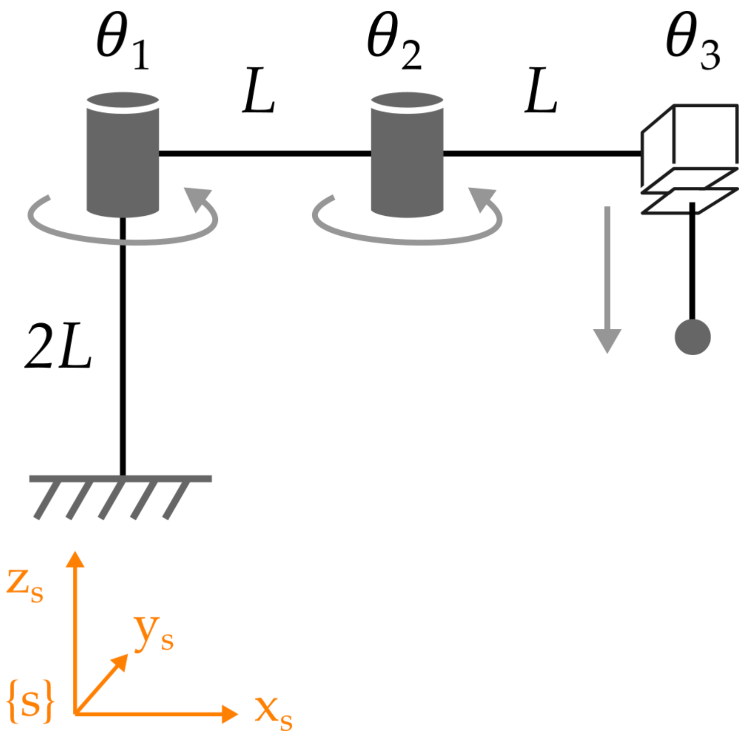


Example

Step 2. Use adjoints to get each column of Jacobian

$$\boldsymbol{J_s}(\boldsymbol{\theta}) = [S_1 \quad Ad_{e^{[S_1]\theta_1}} S_2 \quad \underline{Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} S_3}]$$

$$Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}} = \begin{bmatrix} c_{12} & -s_{12} & 0 & & & \\ s_{12} & c_{12} & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ & & & c_{12} & -s_{12} & 0 \\ & [p]R & & s_{12} & c_{12} & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$



Example

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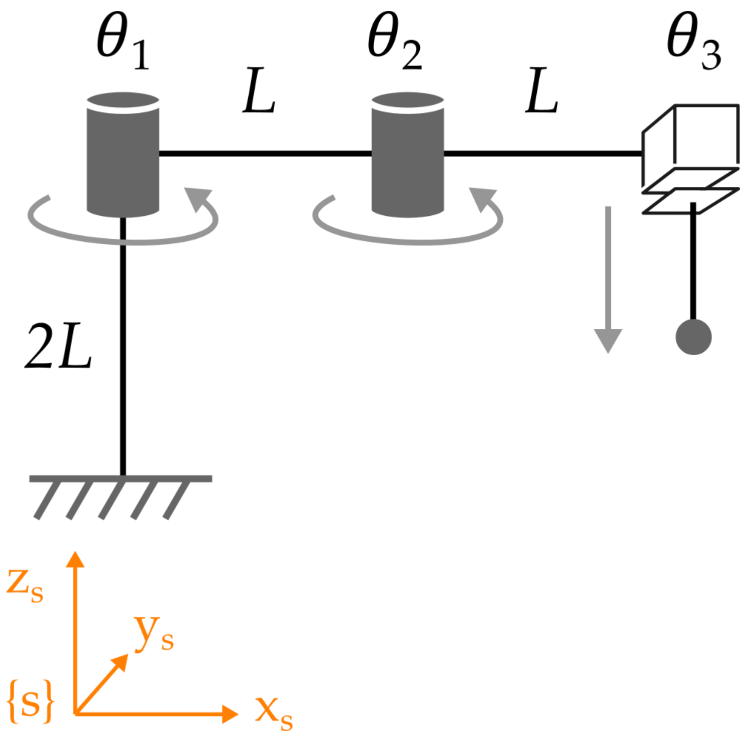
$$\begin{bmatrix} c_{12} & -s_{12} & 0 & & & \\ s_{12} & c_{12} & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ & & & c_{12} & -s_{12} & 0 \\ & [p]R & & s_{12} & c_{12} & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

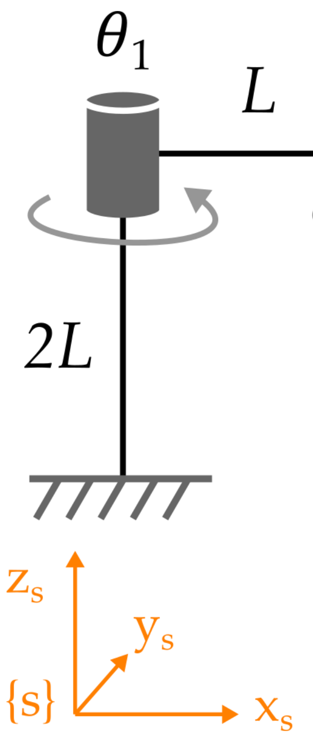


Example

Three-DoF robot arm.

Given joint values θ and joint velocity $\dot{\theta}$, what is the spatial twist of the end-effector?

$$V_s = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & L\sin\theta_1 & 0 \\ 0 & -L\cos\theta_1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{\boldsymbol{J_s(\theta)}} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$



This Lecture

- How do we find the Jacobian of a robot arm?

Next Lecture

- We found the Jacobian for the spatial twist…

  what if we want the body twist?
- What are some other useful Jacobians?