Problem Set 4

Robotics & Automation
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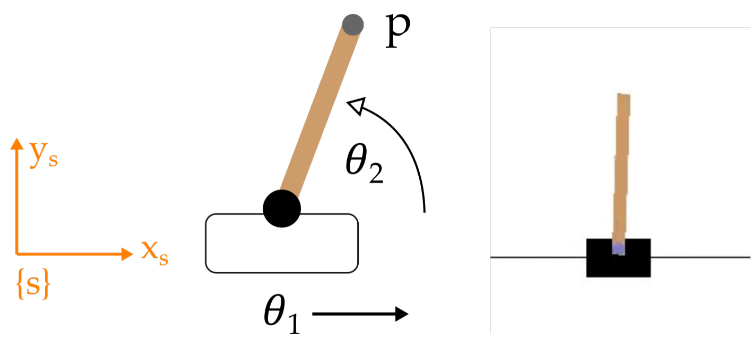
Instructions. Please write legibly and do not attempt to fit your work into the smallest space possible. It is important to show all work, but basic arithmetic can be omitted. You are encouraged to use Matlab when possible to avoid hand calculations, but print and submit your commented code for non-trivial calculations. You can attach a pdf of your code to the homework, use live scripts or the publish feature in Matlab, or include a snapshot of your code. Do not submit .m files — we will not open or grade these files.

1 Properties of Jacobians

1.1 (5 points)



The robot shown above has two joints: a cart, which can slide left and right along a track, and a pole (i.e., an inverted pendulum), which this cart is trying to balance. The pendulum has length L, and the vertical distance from $\{s\}$ to the base of the inverted pendulum is h. Solve for the Jacobian J such that $\dot{p} = J(\theta)\dot{\theta}$. Here p is the position of the end of the pole.

1.2 (5 points)

Consider a serial robot arm with three joints. Starting with the spatial twist $[V_s] = \dot{T}T^{-1}$, derive the spatial Jacobian:

$$V_s = J_s(\theta)\dot{\theta}, \quad J_s(\theta) = \begin{bmatrix} S_1 & \mathrm{Ad}_{e^{[S_1]\theta_1}} S_2 & \mathrm{Ad}_{e^{[S_1]\theta_1}e^{[S_2]\theta_2}} S_3 \end{bmatrix} \tag{1}$$

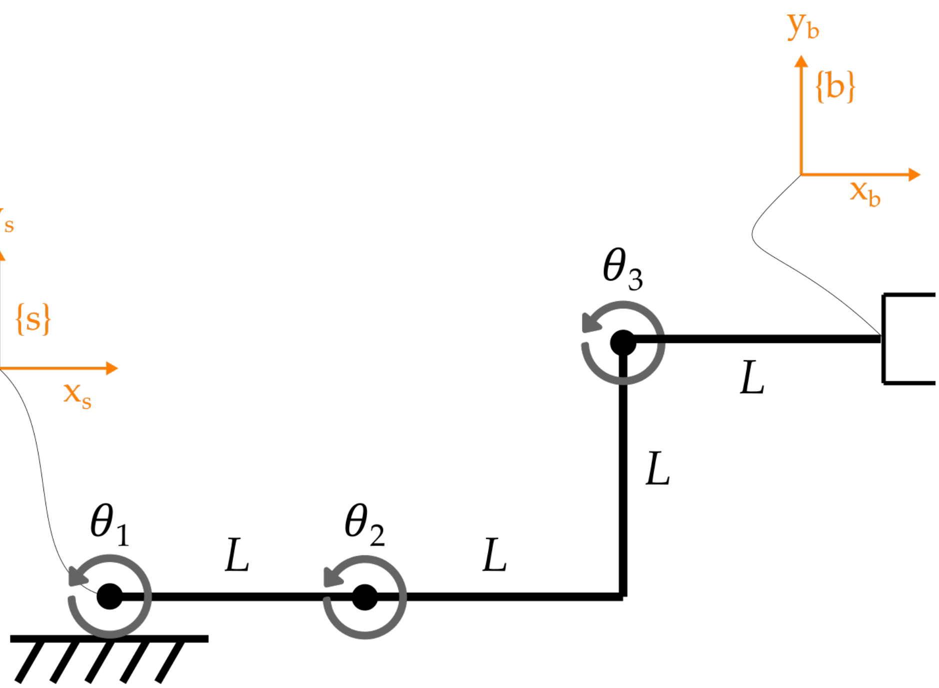
Hint. Let T be a transformation and S be a screw. By definition, $T[S]T^{-1} = [\mathrm{Ad}_T S]$.

1.3 (5 points)

Let J be a Jacobian (either space, body, or geometric) such that $V = J(\theta)\dot{\theta}$. Here V is a six-dimensional vector and the serial robot arm has n joints. Under what conditions is J invertible, i.e., when can you find J^{-1}?

2 Jacobian: Planar Robot

In this problem you will get the Jacobian for the planar robot shown below. Your answers should be in terms of the variables L, θ_1, θ_2, and θ_3.



2.1 (5 points)

Find the forward kinematics for the planar robot. Write out the home position M, the screw axes S_1, S_2, S_3, and the transformation matrix $T(\theta)$.

2.2 (5 points)

Find the Space Jacobian $J_s(\theta)$.

2.3 (5 points)

Find the Body Jacobian $J_b(\theta)$.

2.4 (5 points)

Find the Geometric Jacobian $J(\theta)$.

3 Jacobian: $3D$ Robot

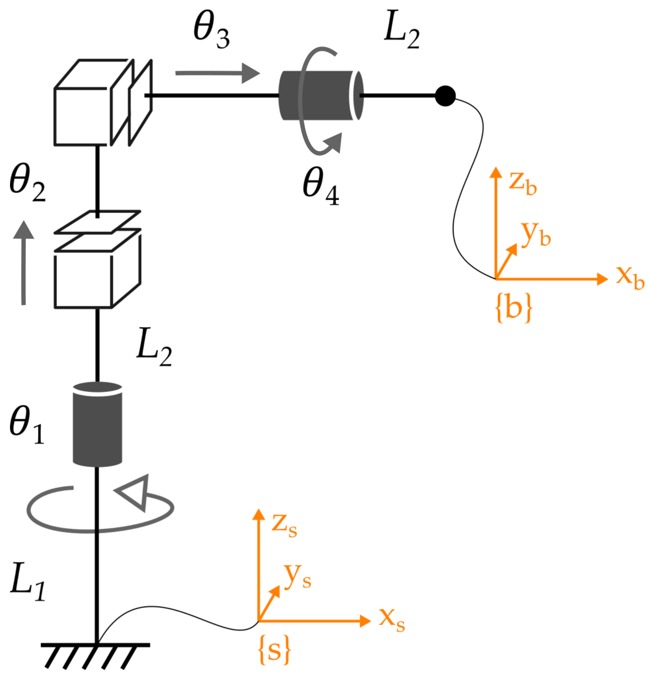
In this problem you will get the Jacobian for the robot moving in $3D$ space shown below. Your answers should be in terms of the variables L_1, L_2, θ_1, θ_2, θ_3, and θ_4.

3.1 (5 points)

Find the forward kinematics for the $3D$ robot. Write out the home position M, the screw axes S_1, S_2, S_3, S_4, and the transformation matrix $T(\theta)$.

3.2 (5 points)

Find the Space Jacobian $J_s(\theta)$.



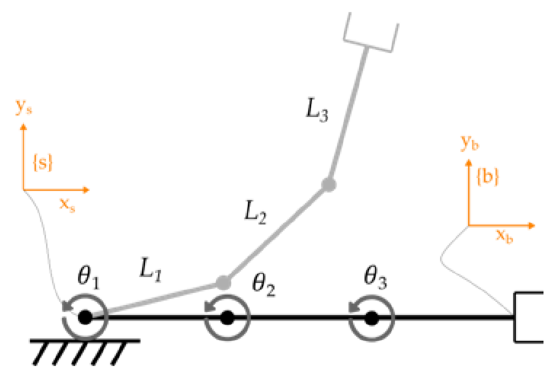
3.3 (5 points)

Find the Body Jacobian $J_b(\theta)$.

3.4 (5 points)

Find the Geometric Jacobian $J(\theta)$.

4 Interpreting the Jacobian



This problem deals with the planar robot shown above.

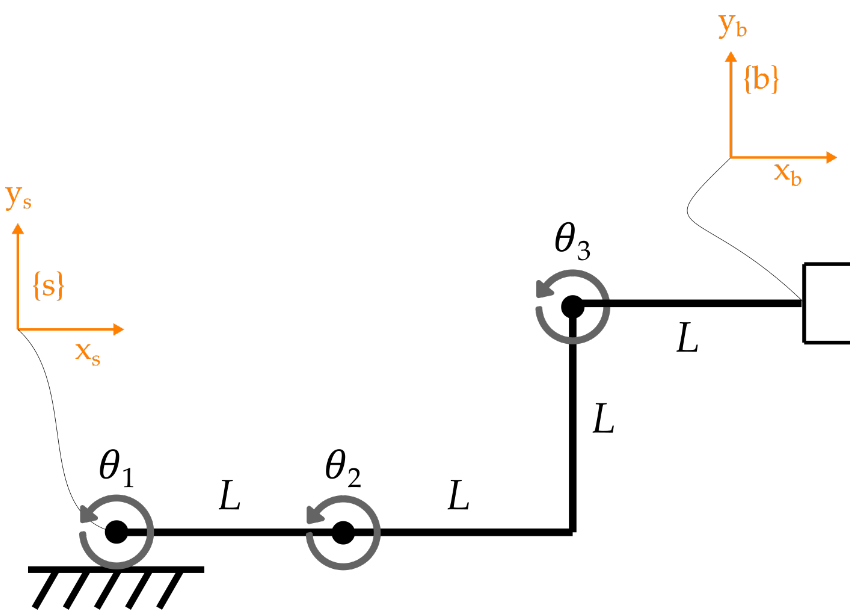
4.1 (10 points)

Imagine that you can only actuate one joint right now. In other words, either $\dot{\theta}_1 = 1$ rad/s, or $\dot{\theta}_2 = 1$ rad/s, or $\dot{\theta}_3 = 1$ rad/s. Let $p = p_{sb}$ be the position of the robot's end-effector, and assume that $L_1 = L_2 = L_3$.

- Which joint should you actuate to maximize $|\dot{p}_x|$ if $\theta = [-\pi/8, \pi/4, \pi/8]^T$?

- Which joint should you actuate to maximize $|\dot{p}_x|$ if $\theta = [3\pi/4, -\pi/4, 0]^T$?
- Which joint should you actuate to maximize $|\dot{p}_y|$ if $\theta = [\pi/2, -\pi/8, -\pi/2]^T$?

5 Singularities: Planar Robot



In this problem you will explore singularities and manipulability of the planar robot shown above. You previously found the Jacobian for this robot. The task space of this robot is the (x, y) position of the end-effector and the angle of the end-effector around the z axis.

5.1 (5 points)

Identify the joint position(s) where the robot is at a singular configuration.

5.2 (5 points)

Draw the robot in two different singular configurations.

5.3 (5 points)

Let $\theta = [\pi/4, -\pi/4, \pi/4]^T$.

- Is this a singular configuration?
- In what direction(s) can the robot move?
- In what direction(s) can the robot not move?

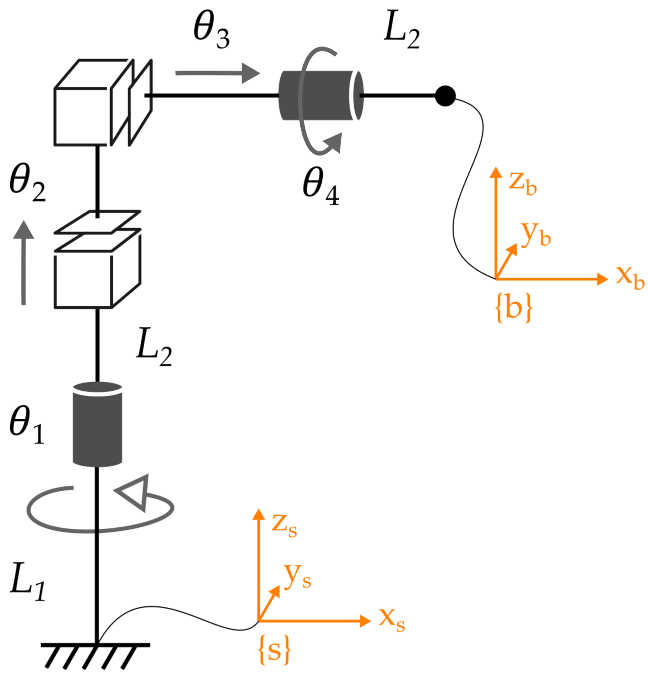
5.4 (5 points)

Find the joint position(s) that maximize the robot's manipulability.

5.5 (5 points)

Modify the planar robot's design to remove **all singular configuration(s)**. You are allowed to: add joints, change the geometry of the links, and/or change the type of joints. Prove that $\det(JJ^T) \neq 0$ for all θ with your modified robot.

6 Singularities: $3D$ Robot



In this problem you will explore singularities and manipulability of the planar robot shown above. You previously found the Jacobian for this robot.

6.1 (5 points)

Identify the joint position(s) where the robot is at a singular configuration. Alternatively, prove that none exist. **Hint:** A singularity occurs when the rank of this robot's Jacobian is less than its maximum rank.

6.2 (5 points)

Find the joint position(s) that maximize the robot's manipulability. **Hint:** There are multiple metrics for manipulability. For this problem treat manipulability as the robot's overall movement speed. In other words, at what joint positions can we maximize $V^T V$?