Practice Set 7

Robotics & Automation
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Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

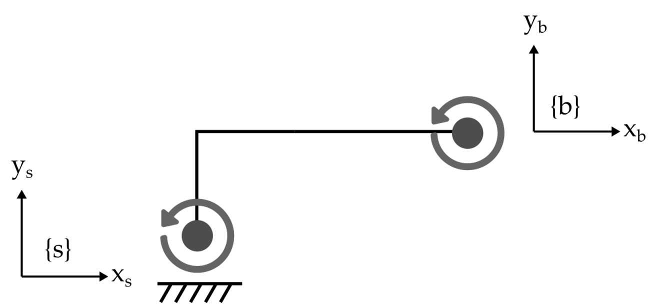
Problem 1

Consider the following sequence of rotations:

1. Rotate by θ_1 about the fixed x axis.
2. Then rotate by θ_2 about the body z axis.
3. Then rotate by θ_3 about the body x axis.
4. Then rotate by θ_4 about the fixed z axis.

Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplication).

Problem 2



Consider the robot arm shown above. This robot has two revolute joints: one at the base, next to frame $\{s\}$, and the other at the end-effector, next to frame $\{b\}$. The current transformation from the base to the end-effector is:

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

See if you can figure out what T_{sb} becomes if:

- We rotate the joint at $\{s\}$ by $\pi/2$ radians around z_s.
- We rotate the joint at $\{b\}$ by $\pi/2$ radians around z_b.

Problem 1

Consider the following sequence of rotations:

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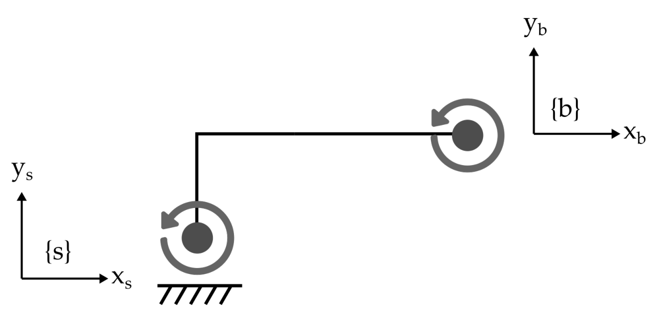
Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplication).

> To determine the cumulative effect of these rotations we begin with the first rotation $R(x, \theta_1)$ and pre- or post-multiply to obtain our final answer. Remember that if the motion is in the body frame (i.e., the current frame) we post-multiply. If the motion is in the fixed frame (i.e., the world frame) we pre-multiply.
>
> $$R = R(z, \theta_4)R(x, \theta_1)R(z, \theta_2)R(x, \theta_3) \tag{2}$$

Problem 2



Consider the robot arm shown above. This robot has two revolute joints: one at the base, next to frame $\{s\}$, and the other at the end-effector, next to frame $\{b\}$. The current transformation from the base to the end-effector is:

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

See if you can figure out what T_{sb} becomes if:

- We rotate the joint at $\{s\}$ by $\pi/2$ radians around z_s.

> This is a fixed-frame motion. We pre-multiply by a rotation around z_s:
>
> $$T'_{sb} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$
>
> In practice, we have rotated the entire arm by 90 degrees. The robot arm is now pointing up along the y_s axis. **This concept will come up again later**, so it may be worthwhile to draw the arm and visualize how it is moving.

- We rotate the joint at $\{b\}$ by $\pi/2$ radians around z_b.

This is a body-frame motion. We post-multiply by a rotation around z_b:

$$T'_{sb} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Here we are just rotating the wrist of the robot. The arm does not move at all (i.e., the end-effector is in the same position), but we have turned frame $\{b\}$ by 90 degrees.