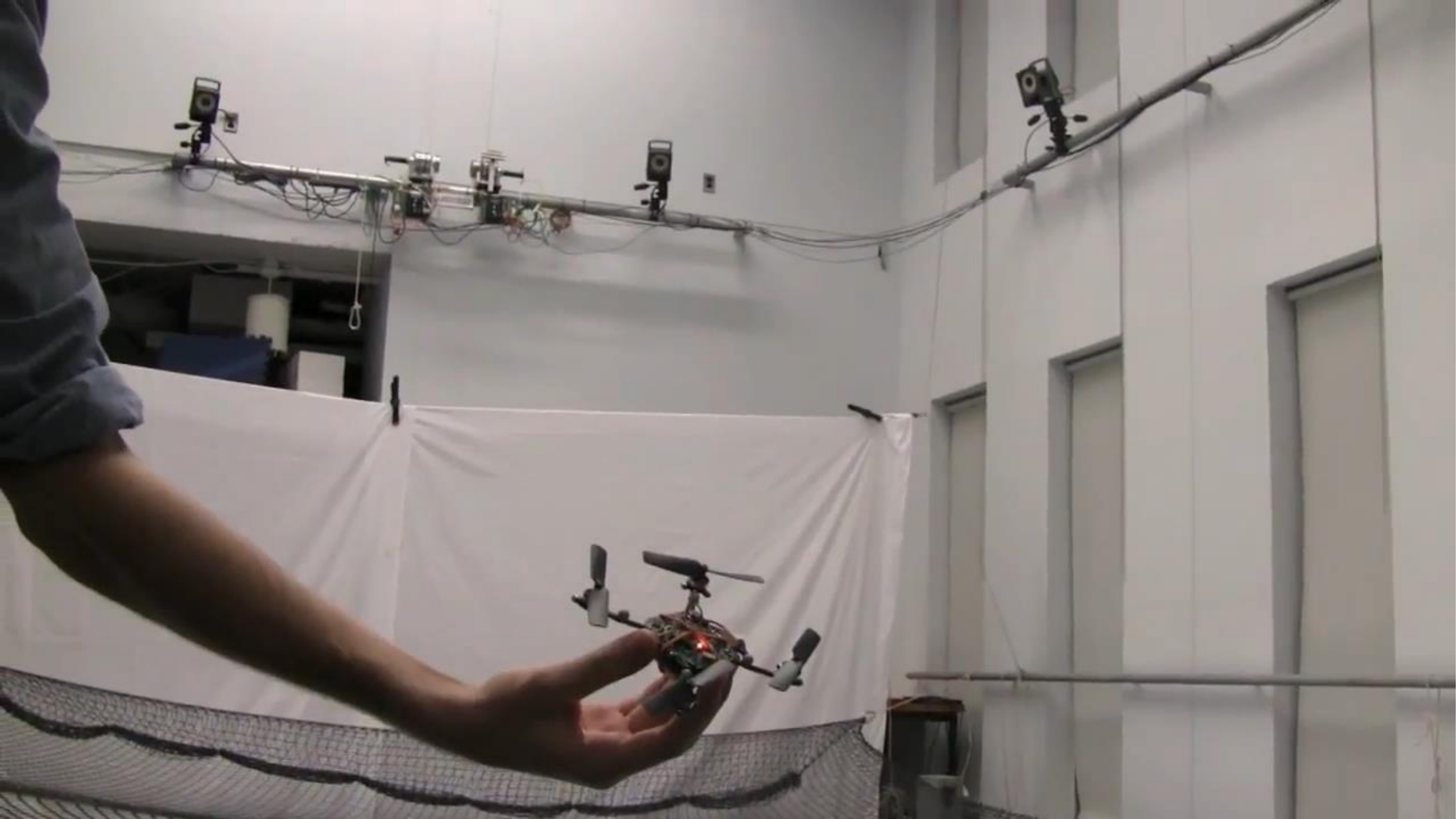
Fixed vs. Body Frame Motion

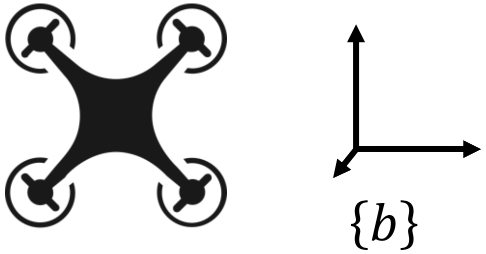
Reading: Modern Robotics 3.3.1



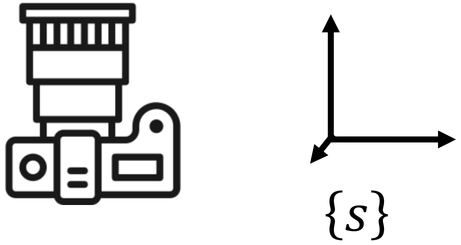
This Lecture

- How can we use transformation matrices to move objects?
- What is the difference between body frame and fixed frame motion?

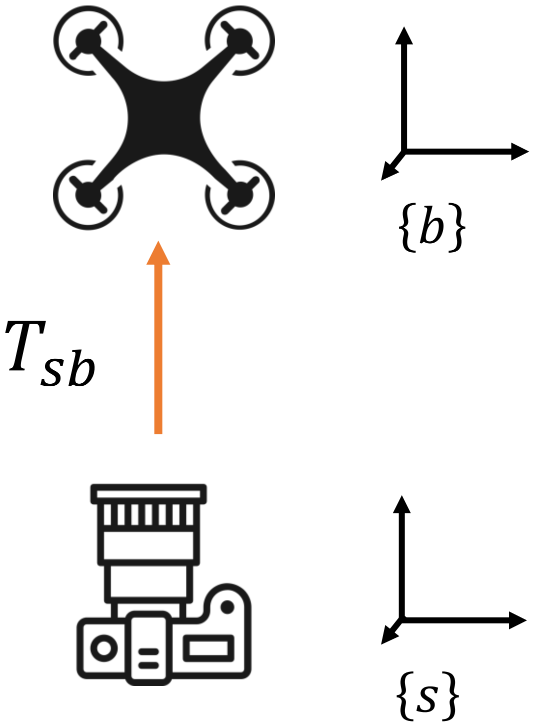
We can multiply transformation matrices to **move** a frame.



Body frame: this frame **moves** with the object



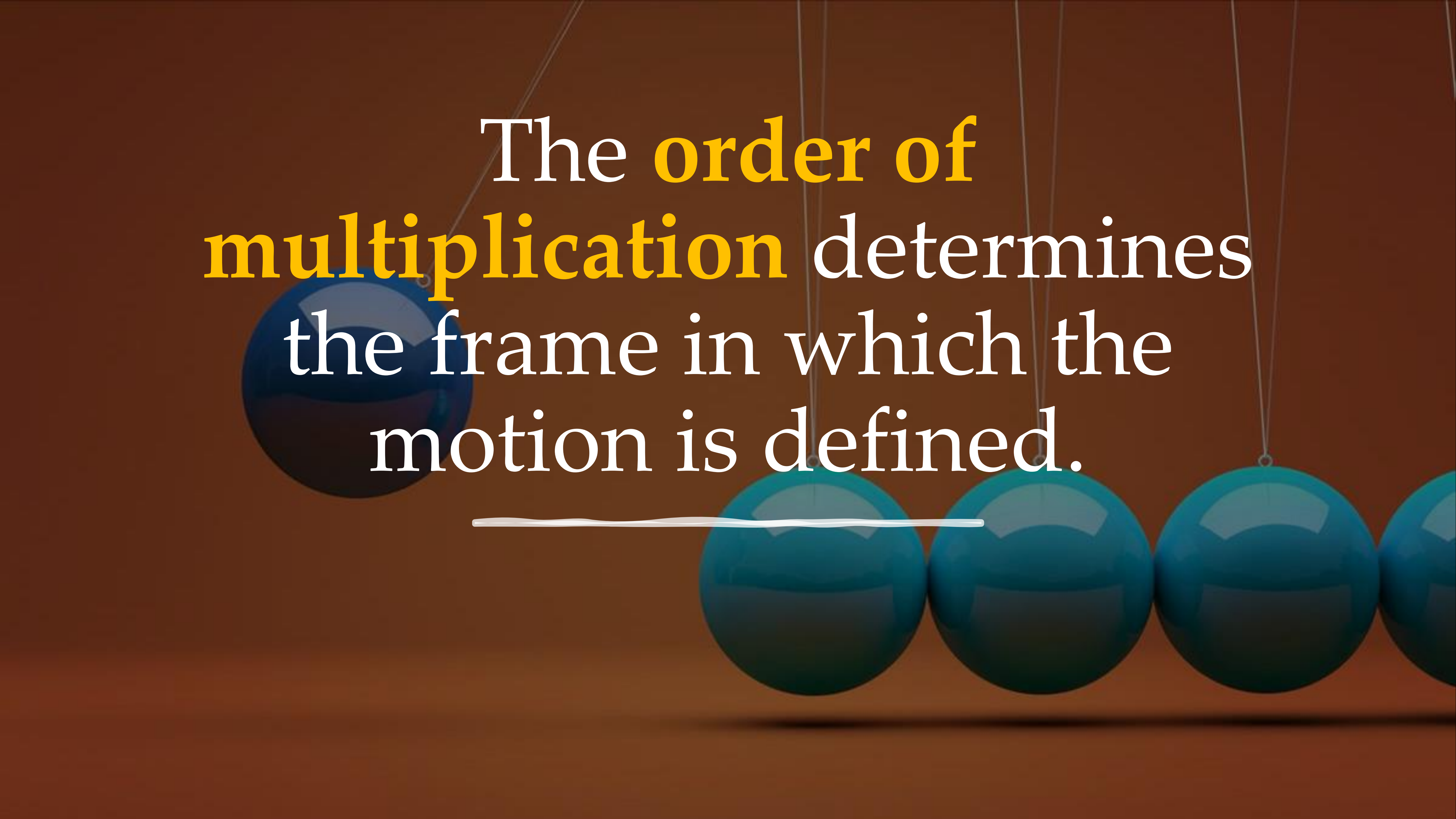
Fixed frame: this frame is always **stationary**



Let T be a rigid body motion.

$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

We want to **multiply** T and T_{sb} to find the pose of the quadrotor after this motion.


The order of multiplication determines the frame in which the motion is defined.

Body Frame

Post-multiply.

We perform a motion with respect to the **body frame $\{b\}$**

$$T_{sb'} = T_{sb}T$$

Body Frame

Post-multiply.

We perform a motion with respect to the **body frame $\{b\}$**

$$T_{sb'} = \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_{sb}R & p_{sb} + R_{sb}p \\ 0 & 1 \end{bmatrix}$$

Rotate from $\{s\}$ to $\{b\}$,
and then perform rotation R in frame $\{b\}$

Body Frame

Post-multiply.

We perform a motion with respect to the **body frame $\{b\}$**

$$T_{sb'} = \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_{sb}R & p_{sb} + R_{sb}p \\ 0 & 1 \end{bmatrix}$$

p is a vector in frame $\{b\}$ and
we use R_{sb} to write p in frame $\{s\}$

Fixed Frame

Pre-multiply.

We perform a motion with respect to the **fixed frame $\{s\}$**

$$T_{sb'} = T T_{sb}$$

Fixed Frame

Pre-multiply.

We perform a motion with respect to the **fixed frame $\{s\}$**

$$T_{sb'} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} RR_{sb} & Rp_{sb} + p \\ 0 & 1 \end{bmatrix}$$

Rotate the world by R
defined in the fixed frame $\{s\}$

Fixed Frame

Pre-multiply.

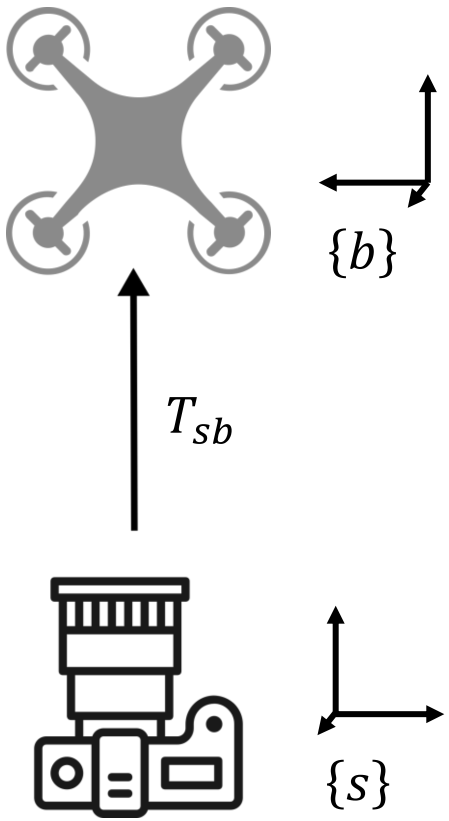
We perform a motion with respect to the **fixed frame $\{s\}$**

$$T_{sb'} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{sb} & p_{sb} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} RR_{sb} & Rp_{sb} + \boxed{p} \\ 0 & 1 \end{bmatrix}$$

Add displacement p
where p is a vector in frame $\{s\}$

Spot the Difference



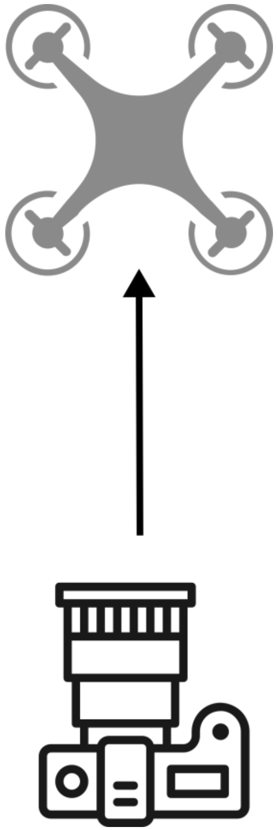
We want to find the **new position** of the quadrotor after motion T

$$R_{sb} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad p_{sb} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Which z-axis are we rotating around?
What y-axis are we translating in?

Spot the Difference



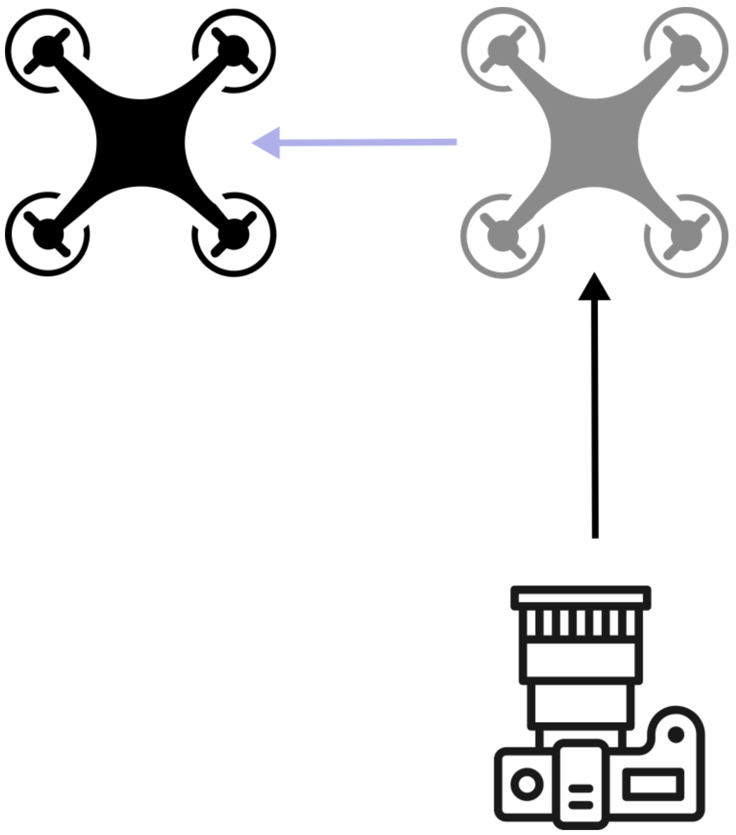
We want to find the **new position** of the quadrotor after motion T

Body Frame

$$p' = p_{sb} + R_{sb}p$$

$$p' = \underline{\begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Spot the Difference



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$$p' = p_{sb} + R_{sb}p$$

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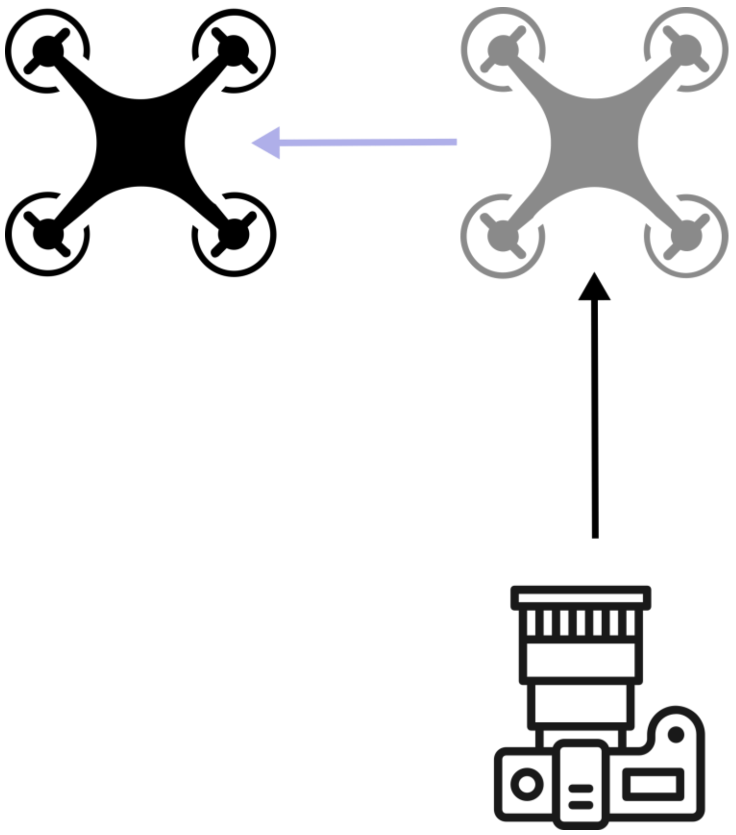

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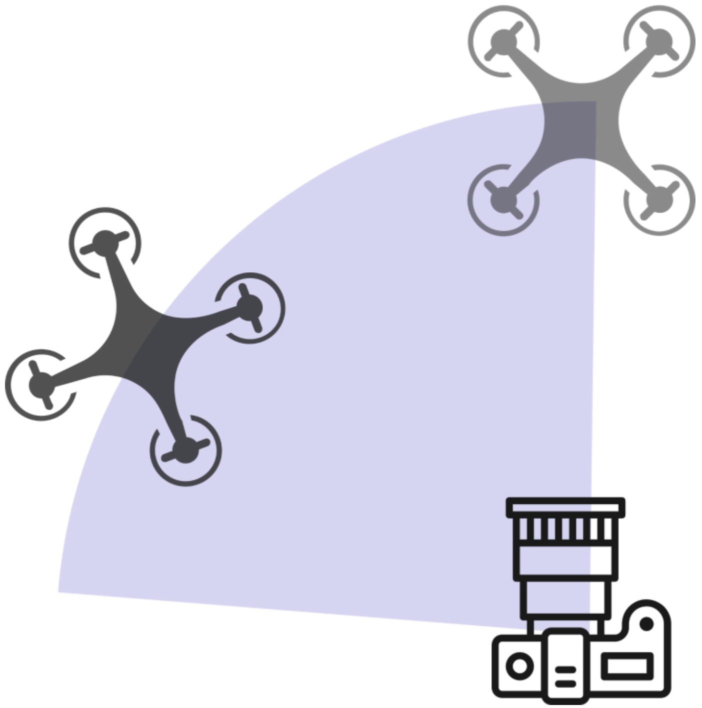
Body Frame

$$p' = p_{sb} + R_{sb}p$$

$$p' = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$p' = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$$

Spot the Difference



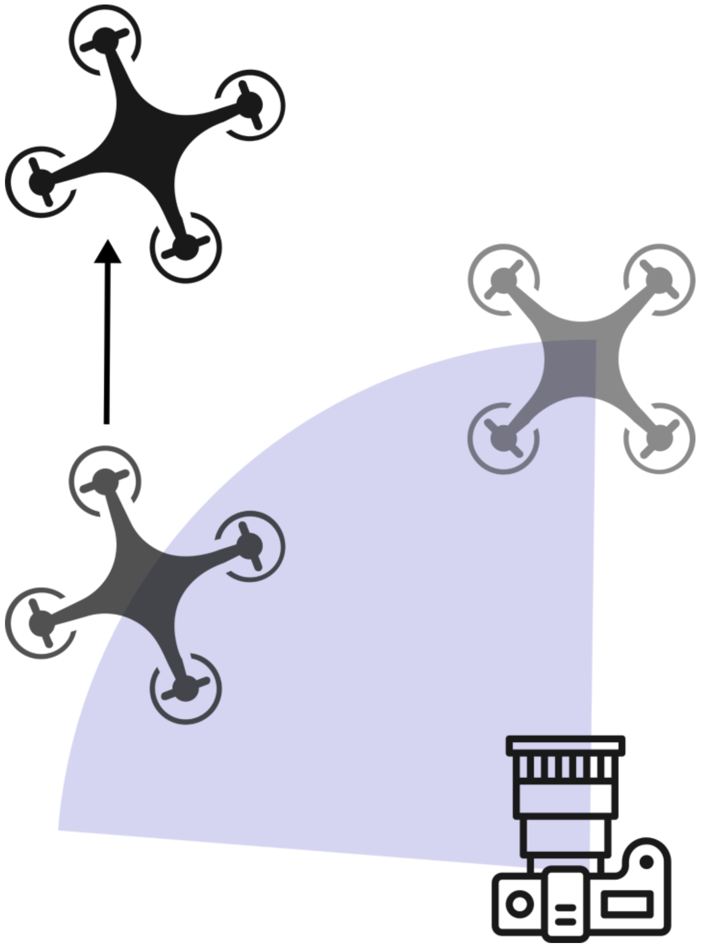
We want to find the **new position** of the quadrotor after motion T

Fixed Frame

$$p' = Rp_{sb} + p$$

$$p' = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Spot the Difference



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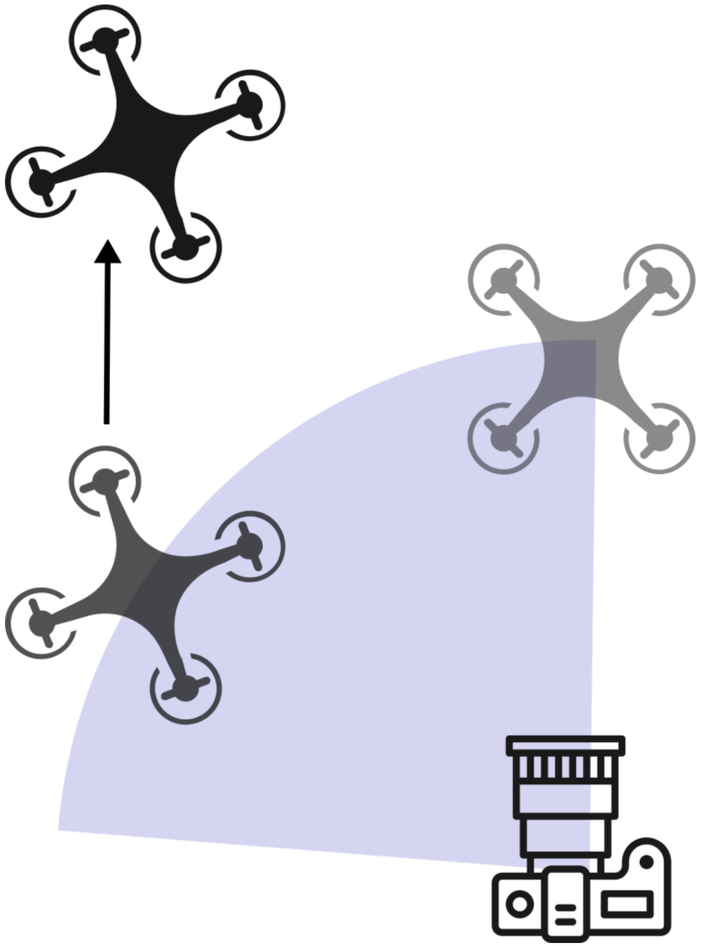
Spot the Difference



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$$p' = Rp_{sb} + p$$

$$p' = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$p' = \begin{bmatrix} -5\sin\theta \\ 1 + 5\cos\theta \\ 0 \end{bmatrix}$$

Everything we just discussed for *transformations* also applies to **rotation matrices**

This Lecture

- How can we use transformation matrices to move objects?
- What is the difference between body frame and fixed frame motion?

Next Lecture

- How do we capture the linear and angular velocity of an object?