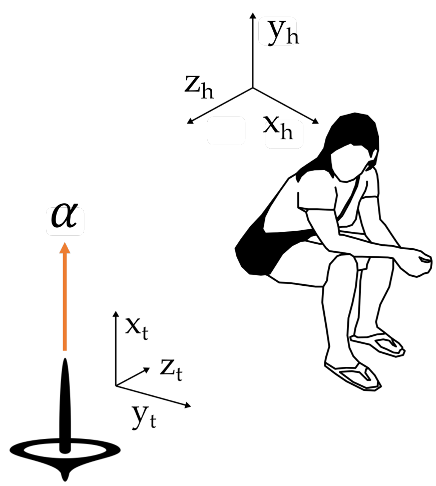
Problem Set 2

Robotics & Automation
Dylan Losey, Virginia Tech

Instructions. Please write legibly and do not attempt to fit your work into the smallest space possible. It is important to show all work, but basic arithmetic can be omitted. You are encouraged to use Matlab when possible to avoid hand calculations, but print and submit your commented code for non-trivial calculations. You can attach a pdf of your code to the homework, use live scripts or the publish feature in Matlab, or include a snapshot of your code. Do not submit .m files — we will not open or grade these files.

1 Angular Velocity

1.1 (5 points)



A top is spinning at α radians per second around the x_t axis. Using the drawing, find ω_t and ω_h. Numerically show that $\omega_h = R_{ht}\omega_t$

1.2 (5 points)

Explain why ZXX Euler angles do not exist.

1.3 (5 points)

Given the rotation matrix:

$$R = \begin{bmatrix} 0.3642 & -0.7799 & 0.5090 \\ 0.7017 & -0.1296 & -0.7006 \\ 0.6124 & 0.6124 & 0.5 \end{bmatrix} \tag{1}$$

Find the axis-angle representation of R.

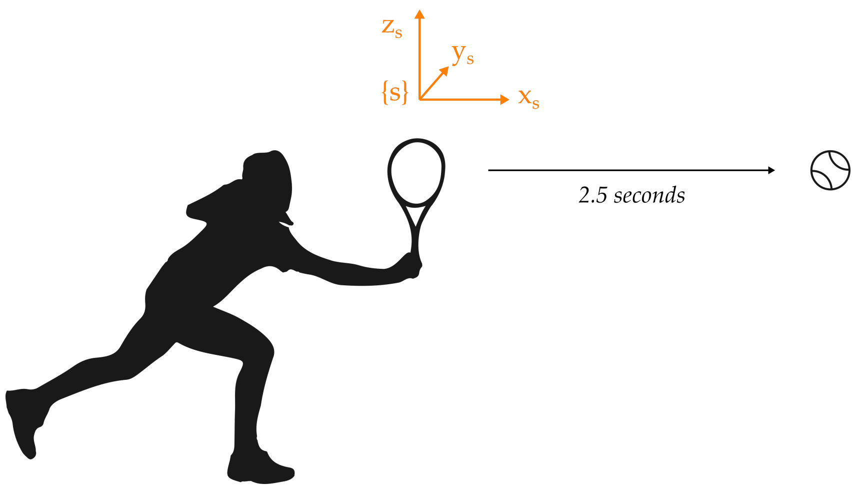
1.4 (5 points)

Write a function that computes the rotation matrix R from an axis ω and an angle θ. Then use that function to find the following rotation matrix:

$$\hat{\omega} = \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{bmatrix}, \quad \theta = \pi/4 \tag{2}$$

Prove that your result is a rotation matrix.

2 Combining Position and Rotation



You just hit a tennis ball. At time $t = 0$, the position and orientation of the tennis ball in your coordinate frame $\{s\}$ are:

$$p(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad R(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The ball is spinning around the $+z_s$ axis at α radians per second, and is flying forward along the $+x_s$ axis at β meters per second.

2.1 (5 points)

What is the position of the tennis ball after 2.5 seconds?

2.2 (5 points)

What is the orientation of the tennis ball after 2.5 seconds?

3 Properties of Transformation Matrices

3.1 (5 points)

Given two transformation matrices T_1 and T_2 where:

$$T_1 = \begin{bmatrix} R_1 & p_1 \\ 0 & 1 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} R_2 & p_2 \\ 0 & 1 \end{bmatrix} \tag{4}$$

Find an instance of T_1 and T_2 where the transformation matrices are commutative. In your answer R_1 and R_2 **cannot** be identity, and p_1 and p_2 **cannot** be zero. You also cannot choose $T_1 = T_2$. Alternatively, prove that no such special case exists.

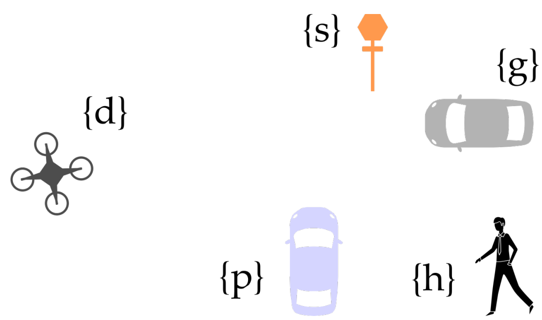
3.2 (5 points)

Write a function that inputs a 4×4 matrix X. This function should output `True` if X is a transformation matrix and `False` if X is not a transformation matrix. Demonstrate that your function works with the following matrices:

$$X_1 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & -5 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & -1.7 \\ 0 & 0 & 1 & -11 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad X_2 = \begin{bmatrix} 0.5 & -1/\sqrt{2} & 0.5 & 0 \\ 0.5 & 1/\sqrt{2} & 0.5 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} -1/\sqrt{2} & -0.5 & 0.5 & -1.1 \\ 1/\sqrt{2} & -0.5 & -0.5 & 4.5 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 1.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 Using Transformation Matrices



Consider the figure above. Your goal is to find the pose of the human expressed in the drone's coordinate frame. You know:

- the pose of the stop sign with respect to the drone
- the pose of the stop sign with respect to the gray car
- the pose of the purple car with respect to the human
- the pose of the purple car with respect to the gray car

4.1 (5 points)

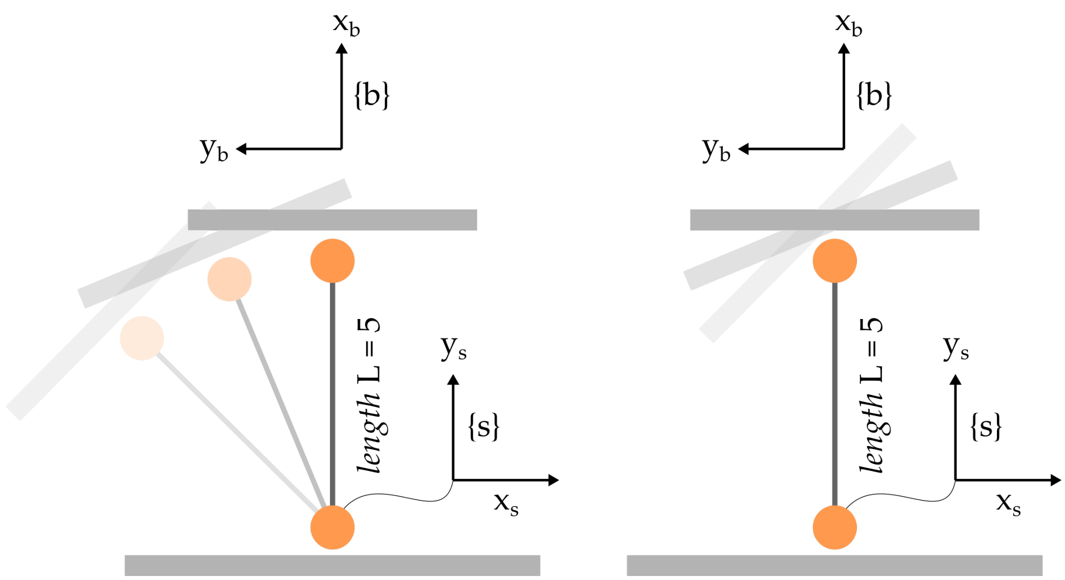
Sketch a system diagram. Draw arrows to mark the known transformation matrices, and label each arrow.

4.2 (5 points)

Using your diagram, determine the equation for the transformation matrix T_{dh}

5 Fixed and Body Frame Motion

We have a planar robot with two revolute joints. See the figure below: on the left we move the revolute joint at $\{s\}$, and on the right we move the revolute joint at $\{b\}$. For the following parts we will ask your to perform a *sequence* of motions. Solve for the transformation matrix T_{sb} after each motion.



5.1 (5 points)

First rotate the joint at $\{s\}$ by $\pi/4$.

5.2 (5 points)

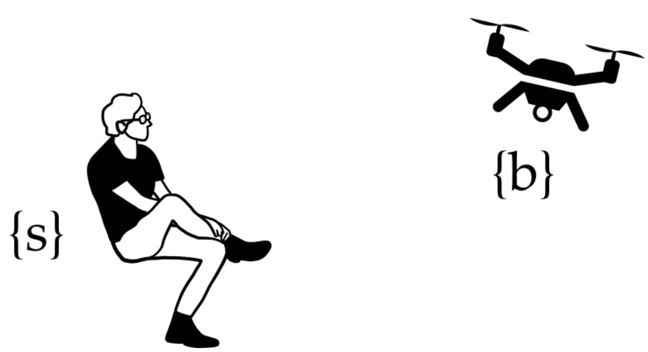
Then rotate the joint at $\{b\}$ by $-\pi/4$

5.3 (5 points)

Finally rotate the joint at $\{b\}$ by $\pi/2$

6 Properties of Twists

6.1 (5 points)



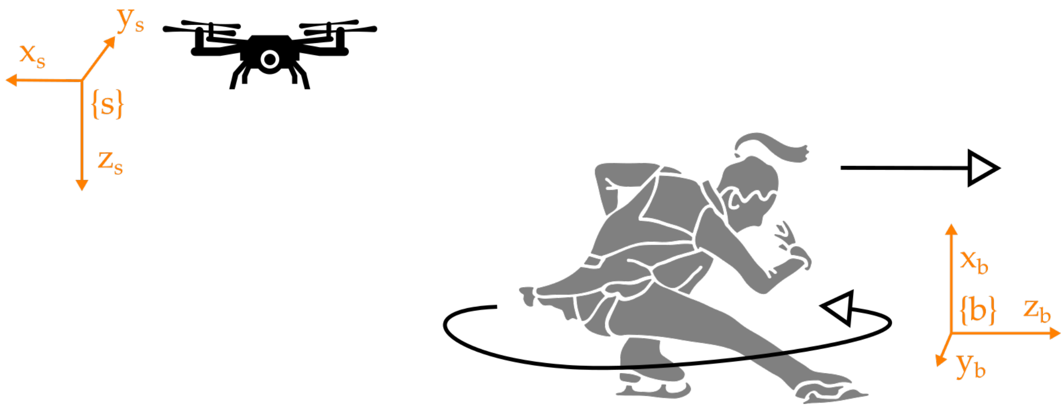
Consider a seated person watching a drone. Let V_s be a spatial twist and let V_b be a body twist. The transformation between the person and drone is currently:

$$T_{sb} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \tag{5}$$

- If $\omega_b \neq 0$ and $p \neq 0$, find sufficient condition(s) for $V_s = V_b$
- If $\omega_s \neq 0$ and $p \neq 0$, find the condition(s) that must be met for $\|V_s\| = \|V_b\|$. Here $\|\cdot\|$ denotes the magnitude (or length) of a vector.

6.2 (5 points)

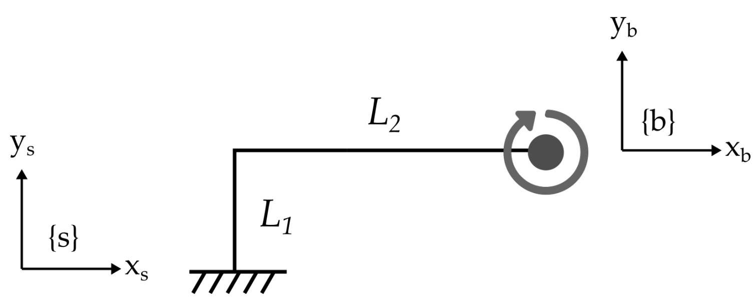
You have a video drone that is trying to track the motion of a figure skater. The skater is rotating around the x_b axis at α radians per second, and is moving forward at β meters per second. The drone is c_1 meters above the figure skater and c_2 meters to the left of the skater.



- What is the body twist?
- What is the spatial twist?

7 Properties of Screws

7.1 (5 points)



Consider the revolute joint shown above. This joint is rotating at α radians per second.

- What is the spatial twist V_s?
- What is the screw S?
- Show that your screw S is the "normalized" twist V_s

7.2 (5 points)

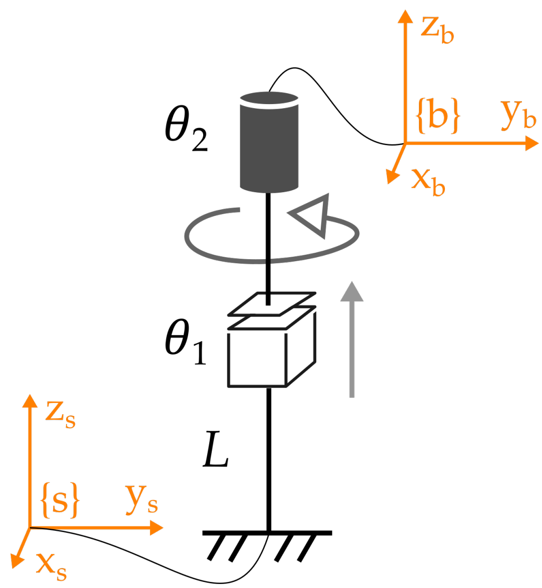
Given the screw S for a prismatic joint:

$$S = \begin{bmatrix} 0 \\ v_s \end{bmatrix} \tag{6}$$

Find the transformation matrix $T = e^{[S]\theta}$. Your answer should be an expression for T that works for any choice of v_s.

8 Using Screws

Answer the following questions based on the robot shown below.



8.1 (5 points)

Let S_1 be the screw for the prismatic joint (with joint position θ_1) and let S_2 be the screw for the revolute joint (with joint position θ_2). Write S_1 and S_2.

8.2 (5 points)

Convert the screws to joint motion to find T_{sb} as a function of θ.

8.3 (5 points)

If $L = 1$, $\theta_1 = 5$, and $\theta_2 = \pi/2$, what is T_{sb}?