# Practice Set 25

**Robotics & Automation**
Dylan Losey, Virginia Tech

Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

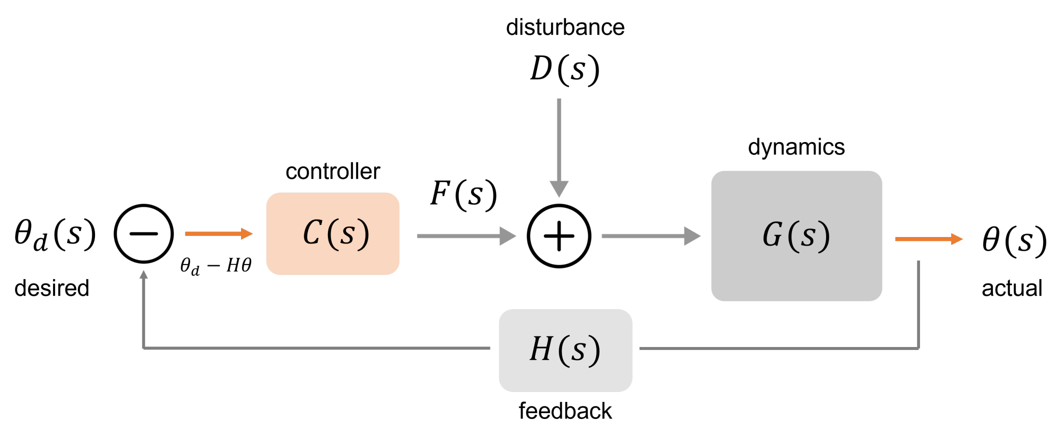
## Problem 1

Write the plant $G(s)$ for a revolute joint with dynamics:

$$\tau(t) + d(t) = I\ddot{\theta}(t) + k\theta(t) \quad (1)$$

Here $I$ is the joint's inertia and $G(s) = \frac{\theta(s)}{\tau(s)+D(s)}$

## Problem 2




For the block diagram shown above:

- Find the closed-loop transfer function $\theta(s)/\theta_d(s)$. To find this set $D(s) = 0$.
- Find the disturbance transfer function $\theta(s)/D(s)$. To find this set $\theta_d(s) = 0$.

## Problem 1

Write the plant $G(s)$ for a revolute joint with dynamics:

$$\tau(t) + d(t) = I\ddot{\theta}(t) + k\theta(t) \tag{2}$$

Here $I$ is the joint's inertia and $G(s) = \frac{\theta(s)}{\tau(s)+D(s)}$

First convert the time domain dynamics to the Laplace domain:

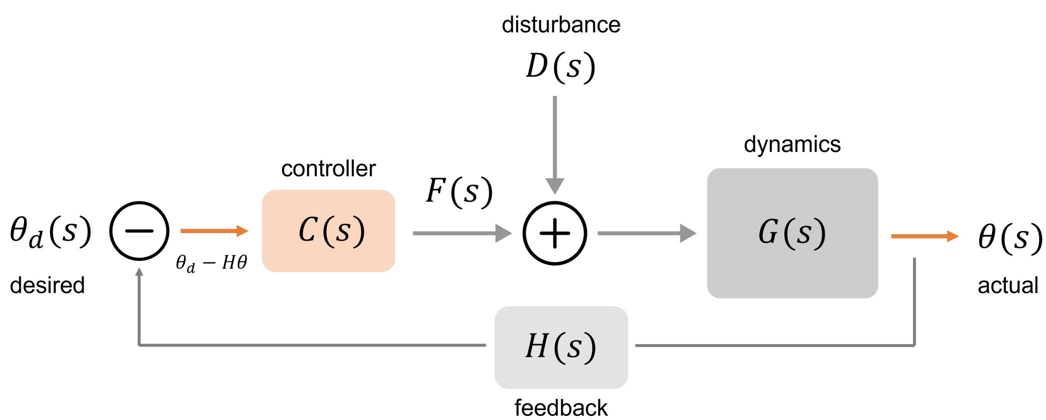
$$\tau(s) + D(s) = Is^s\theta(s) + k\theta(s) \tag{3}$$

Then rearrange to solve for $G(s)$:

$$G(s) = \frac{\theta(s)}{\tau(s) + D(s)} = \frac{1}{Is^2 + k} \tag{4}$$

Note that when we multiply $G(s)$ by the sum of the torques applied to the joint we output its position $\theta(s)$.

## Problem 2




For the block diagram shown above:

- Find the closed-loop transfer function $\theta(s)/\theta_d(s)$. To find this set $D(s) = 0$.
- Find the disturbance transfer function $\theta(s)/D(s)$. To find this set $\theta_d(s) = 0$.

From the block diagram above, we have two equations:

$$F(s) = C(s)\big(\theta_d(s) - H(s)\theta(s)\big) \tag{5}$$

$$\theta(s) = G(s)\big(F(s) + D(s)\big) \tag{6}$$

Combine these equations and isolate $\theta(s)$:

$$\theta(s) = G(s)C(s)\big(\theta_d(s) - H(s)\theta(s)\big) + G(s)D(s) \tag{7}$$

$$\theta(s) = C(s)G(s)\theta_d(s) - C(s)G(s)H(s)\theta(s) + G(s)D(s) \tag{8}$$

$$\theta(s)\big(1 + C(s)G(s)H(s)\big) = C(s)G(s)\theta_d(s) + G(s)D(s) \tag{9}$$

$$\theta(s) = \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}\theta_d(s) + \frac{G(s)}{1 + C(s)G(s)H(s)}D(s) \tag{10}$$

The first term is the closed-loop transfer function and the second term is the disturbance transfer function. Intuitively, these terms provide input-output relationships. Note that each element above is a scalar, so the order of multiplication is not important (e.g., $C(s)G(s) = G(s)C(s)$).