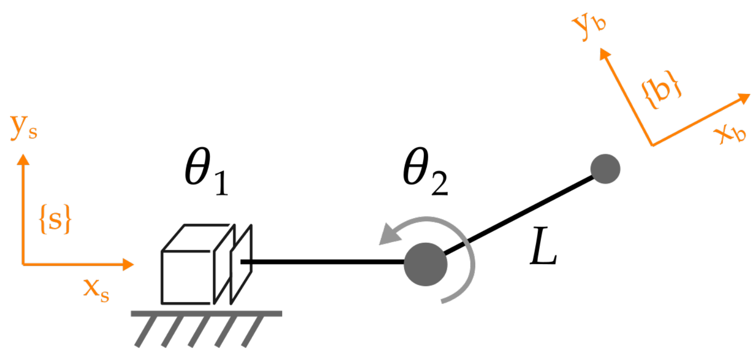
Practice Set 17

Robotics & Automation
Dylan Losey, Virginia Tech

Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1



Find the analytical inverse kinematics for this robot.

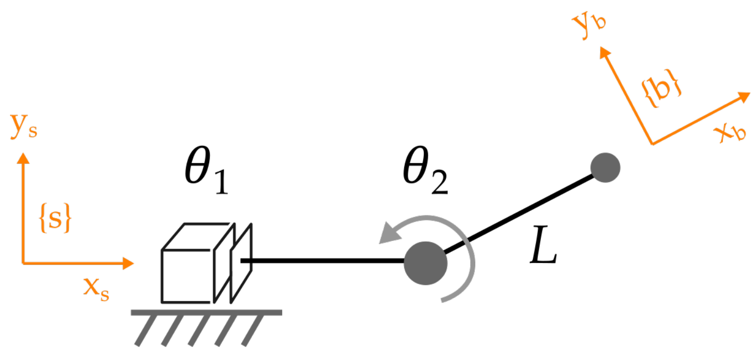
Problem 2

Let the robot's end-effector be in position:

$$p_{sb} = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

List the joint position(s) that achieve this end-effector position.

Problem 1



Find the analytical inverse kinematics for this robot.

Start by writing the x and y coordinates of the end-effector in terms of θ:

$$x = \theta_1 + L\cos(\theta_2) \tag{2}$$

$$y = L\sin(\theta_2) \tag{3}$$

Now solve this system of equations for θ_1 and θ_2:

$$\theta_2 = \arcsin(y/L) \tag{4}$$

$$\theta_1 = x - L\cos(\theta_2) \tag{5}$$

You might notice that out solution is not perfect. The $\arcsin(\cdot)$ function only provides outputs between $-\pi/2$ and $\pi/2$.

To highlight this issue let's test our solution when $\theta_1 = L$ and $\theta_2 = \pi$. Here $x = 0$ and $y = 0$. Ideally, we should recover $\theta_1 = L$, $\theta_2 = \pi$. But solving our inverse kinematics we obtain $\theta_1 = -L$, $\theta_2 = 0$. This solution places the end-effector at the correct (x, y) position. But it does not provide the joint values of the actual robot arm! This occurs because multiple joint positions map to the same end-effector position.

Problem 2

Let the robot's end-effector be in position:

$$p_{sb} = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

List the joint position(s) that achieve this end-effector position.

There are two different joint positions that put the end-effector at p_{sb}:

$$\theta_1 = 0, \quad \theta_2 = 0 \tag{7}$$

$$\theta_1 = 2L, \quad \theta_2 = \pi \tag{8}$$

Keep in mind that inverse kinematics often have more than one solution.