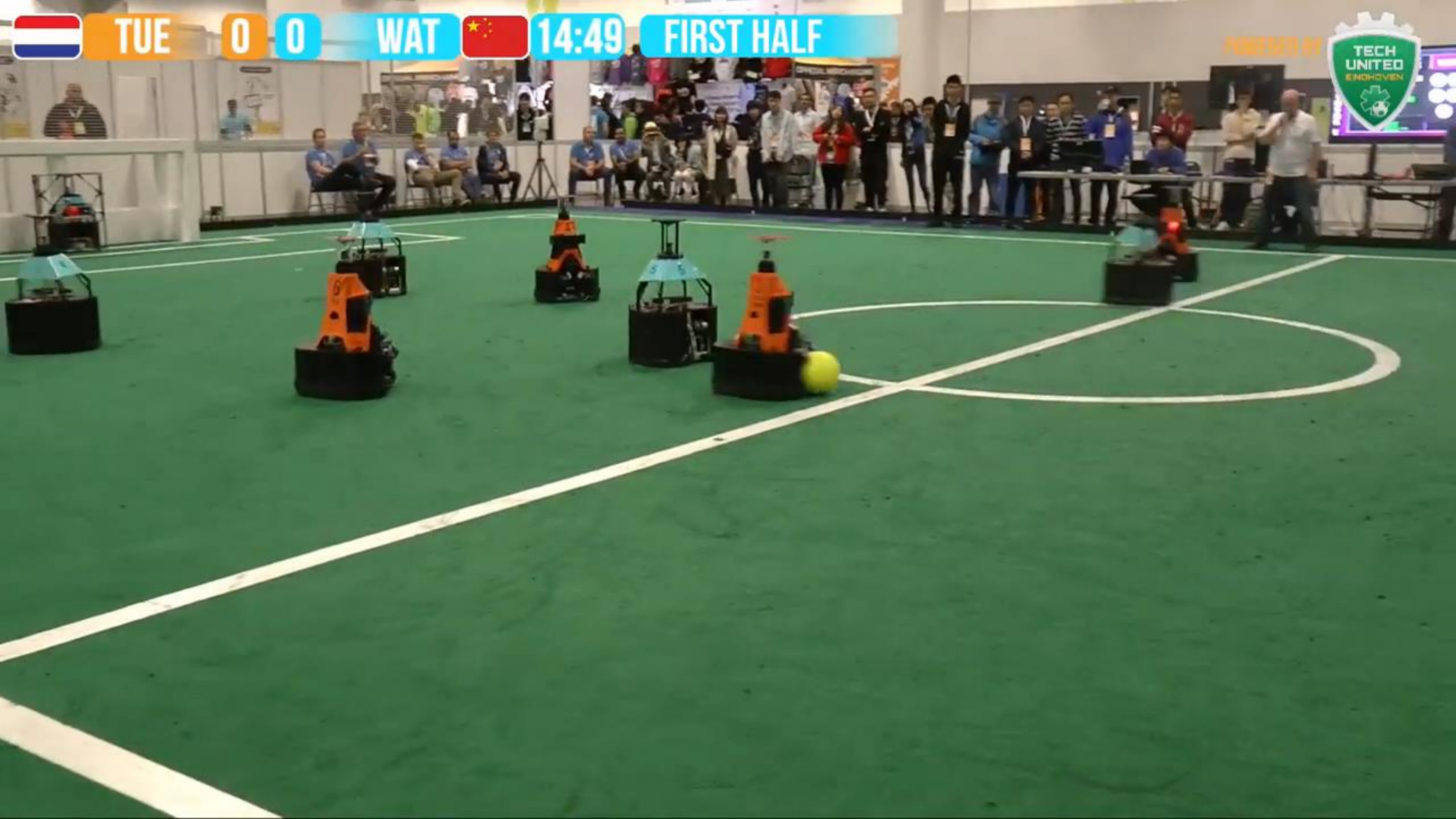
Twists

Reading: Modern Robotics 3.3.2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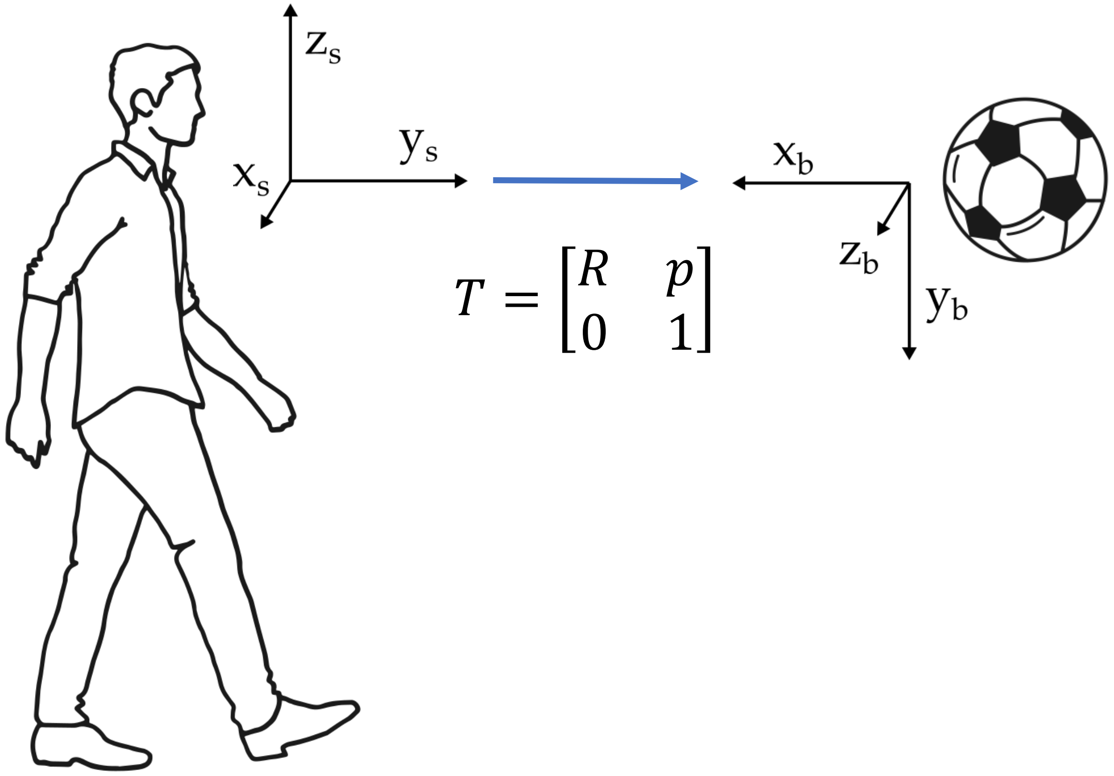
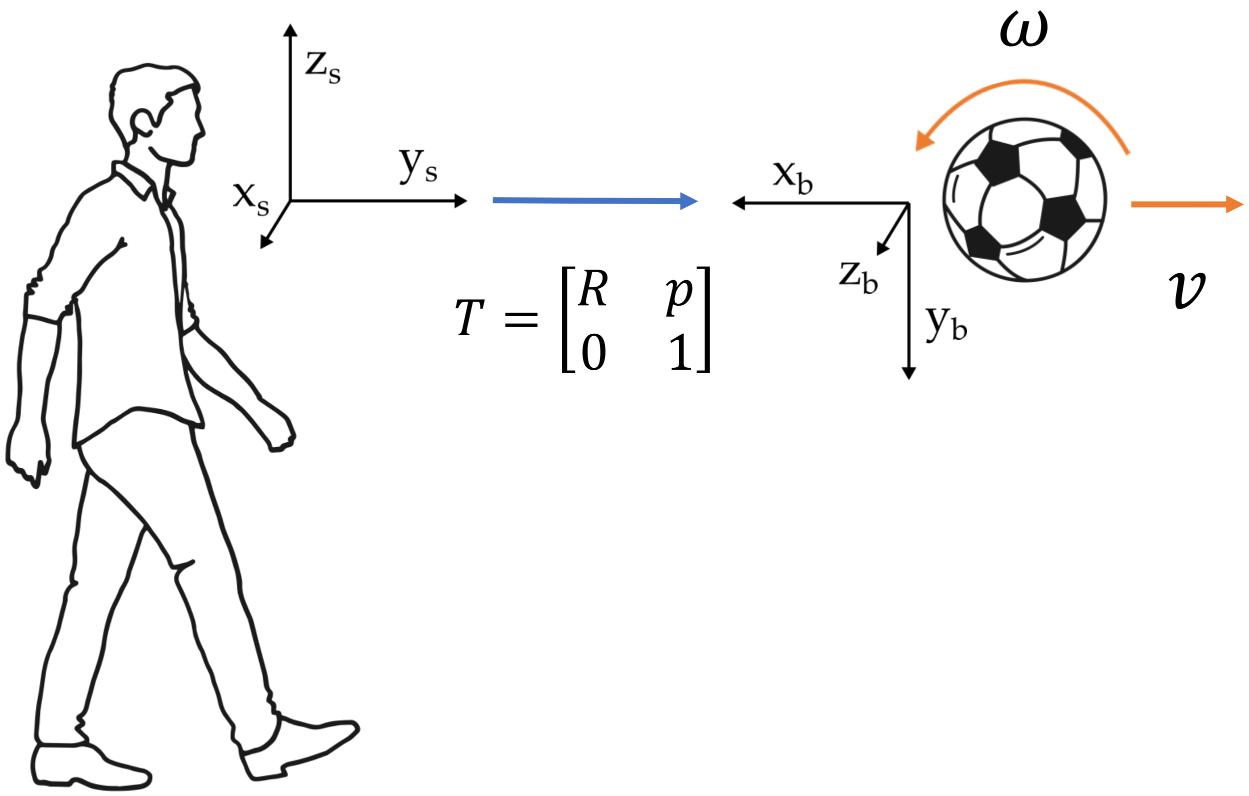
This Lecture

- How do we represent linear and angular velocity?
- How are twists related to transformation matrices?
- What are the two types of twists?



Transformation T captures where $\{b\}$ is **right now**.

But if the ball is moving, how do we write its velocity?



How can we go from position and rotation to **linear and angular velocity**?

Twists

A twist is a 6-dimensional **vector**:

$$V = \begin{bmatrix} \omega \\ v \end{bmatrix}$$

Where $\omega \in \mathbb{R}^3$ is the **angular velocity** and $v \in \mathbb{R}^3$ is the **linear velocity**

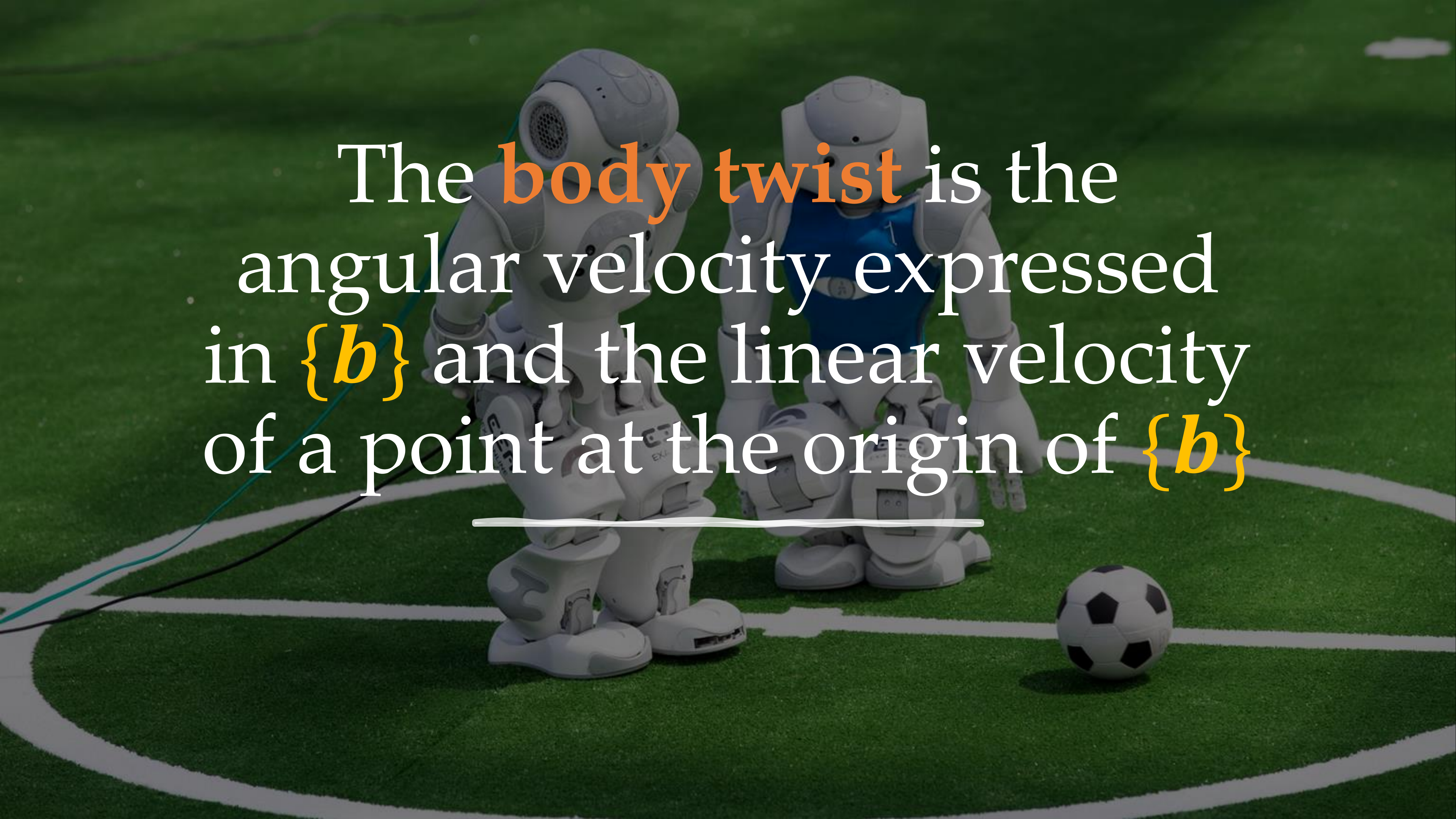
Twists

We often write twists as a 4×4 **matrix**:

$$[V] = \begin{bmatrix} [\omega] & v \\ 0 & 0 \end{bmatrix}$$

Remember that $[\omega]$ is skew-symmetric matrix

$$[\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$



The **body twist** is the angular velocity expressed in $\{b\}$ and the linear velocity of a point at the origin of $\{b\}$

Body Twist

The formula for the body twist is $[V_b] = T^{-1}\dot{T}$

Let's see why:

$$[V_b] = T^{-1}\dot{T} = \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{R} & \dot{p} \\ 0 & 0 \end{bmatrix}$$

$$[V_b] = T^{-1}\dot{T} = \begin{bmatrix} R^T \dot{R} & R^T \dot{p} \\ 0 & 0 \end{bmatrix}$$

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$$[V_b] = T^{-1}\dot{T} = \begin{bmatrix} \underline{R^T \dot{R}} & R^T \dot{p} \\ 0 & 0 \end{bmatrix}$$

From our lecture on angular velocity, we know that $R^T\dot{R} = [\omega_b]$

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$$[V_b] = T^{-1}\dot{T} = \begin{bmatrix} [\omega_b] & R^T \dot{p} \\ 0 & 0 \end{bmatrix}$$

$\dot{p}$ is the linear velocity of $\{b\}$ expressed in $\{s\}$, and $R^T = R_{bs}$ rotates this vector into frame $\{b\}$

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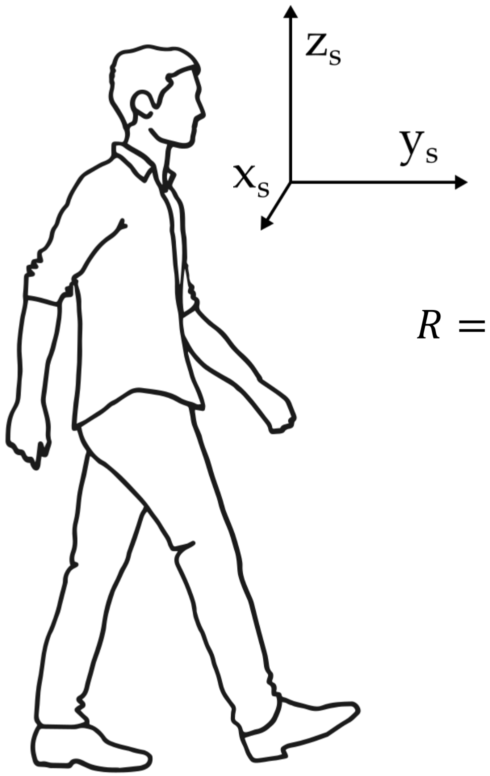
$$[V_b] = T^{-1}\dot{T} = \underline{\begin{bmatrix} [\omega_b] & v_b \\ 0 & 0 \end{bmatrix}}$$

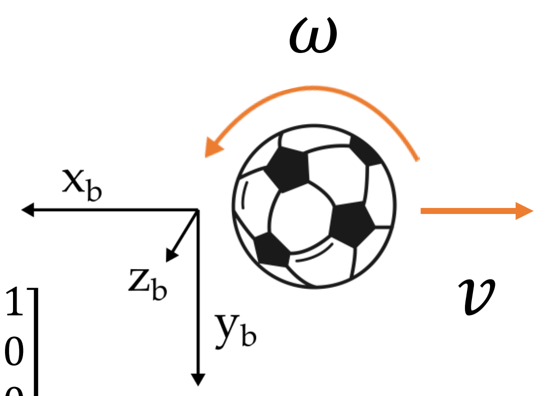
$[\omega_b] = R^T \dot{R}$
angular velocity
expressed in $\{b\}$

$v_b = R^T \dot{p}$
linear velocity of a point at $\{b\}$
expressed in $\{b\}$

Body Twist



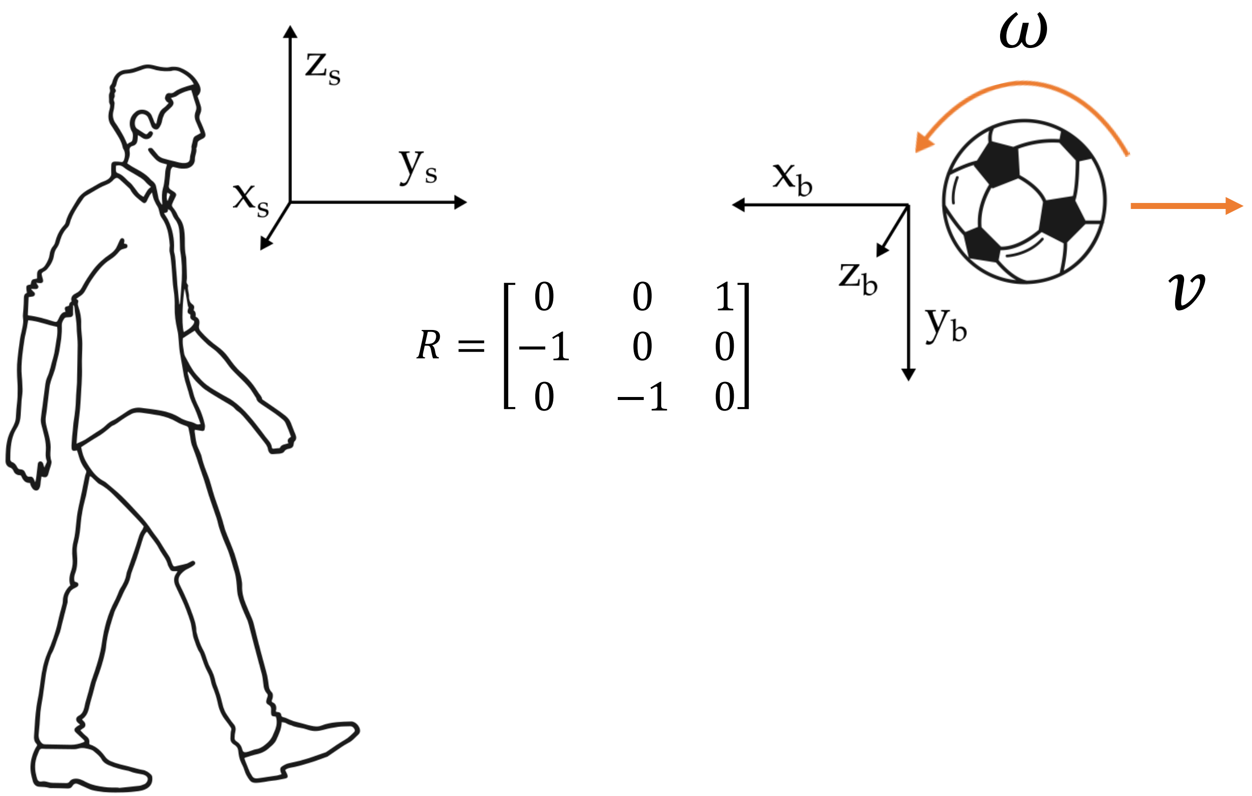
$$R = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$



Ball rotates around z_b axis at α rad/s, and moves right at β m/s.

What is the **body twist**?

Body Twist



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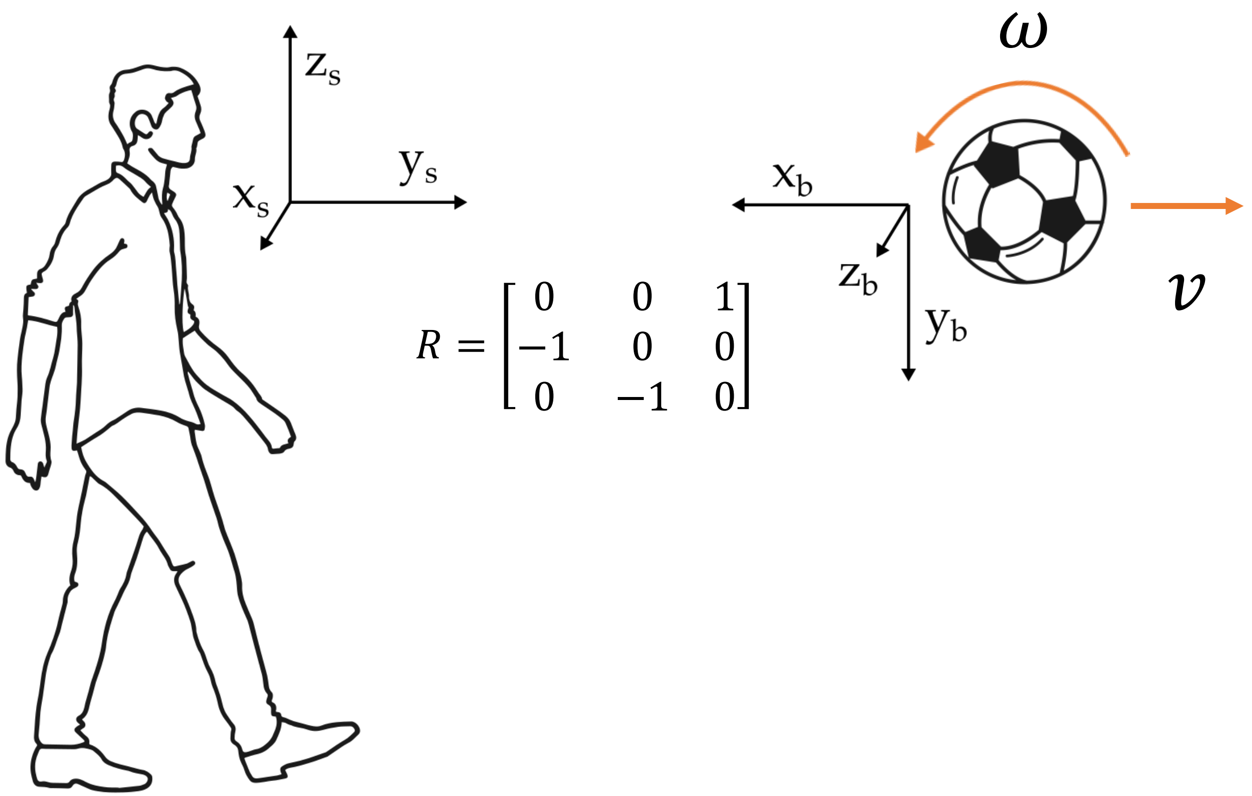
Ball rotates around z_b axis at α rad/s, and moves right at β m/s.

What is the **body twist**?

$$\omega_b = \begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$$

Body Twist



$$R = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

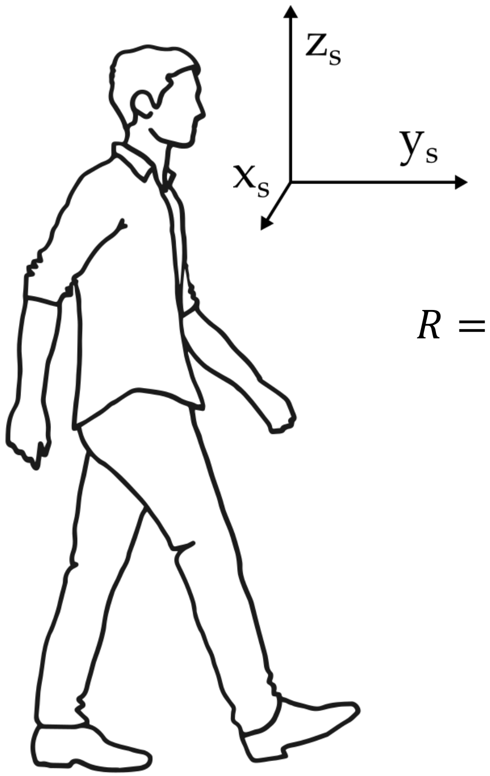
Ball rotates around z_b axis at α rad/s, and moves right at β m/s.

What is the **body twist**?

$$v_b = R^T \dot{p}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix} = \begin{bmatrix} -\beta \\ 0 \\ 0 \end{bmatrix}$$

Body Twist



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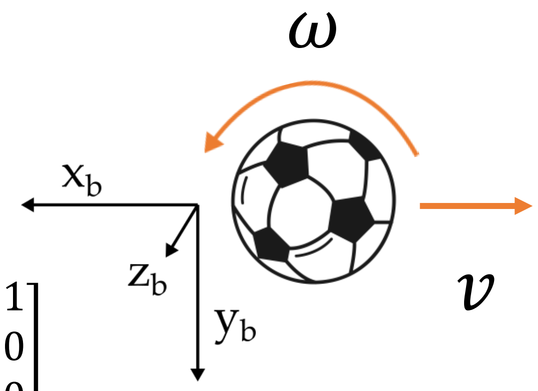

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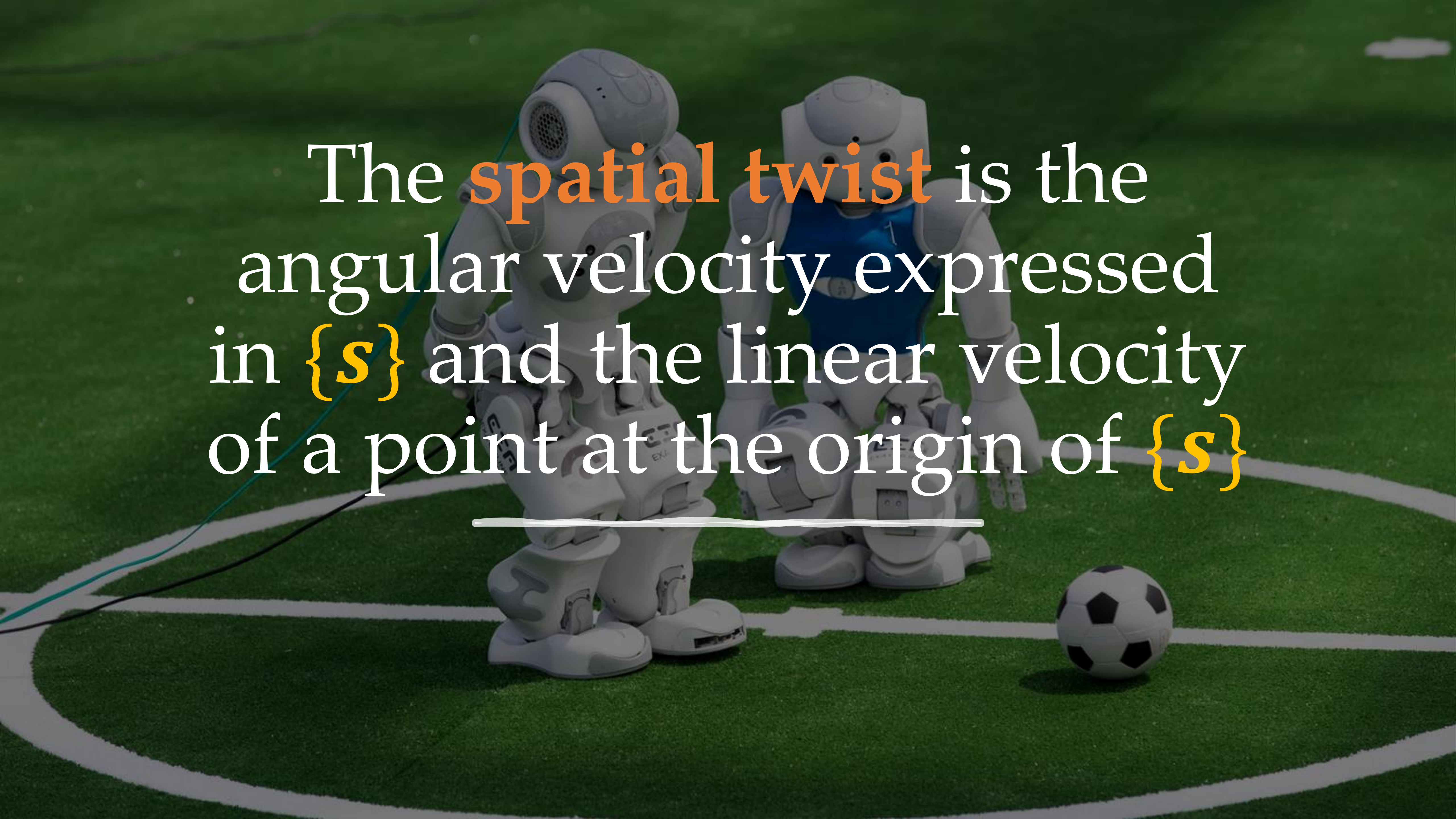
$$V_b = \begin{bmatrix} 0 \\ 0 \\ \alpha \\ -\beta \\ 0 \\ 0 \end{bmatrix}$$



The **spatial twist** is the angular velocity expressed in $\{s\}$ and the linear velocity of a point at the origin of $\{s\}$

Spatial Twist

The formula for the spatial twist is $[V_s] = \dot{T}T^{-1}$

Let's see why:

$$[V_s] = \dot{T}T^{-1} = \begin{bmatrix} \dot{R} & \dot{p} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix}$$

$$[V_s] = \dot{T}T^{-1} = \begin{bmatrix} \dot{R}R^T & -\dot{R}R^T p + \dot{p} \\ 0 & 0 \end{bmatrix}$$

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From our lecture on angular velocity, we know that $\dot{R}R^T = [\omega_s]$

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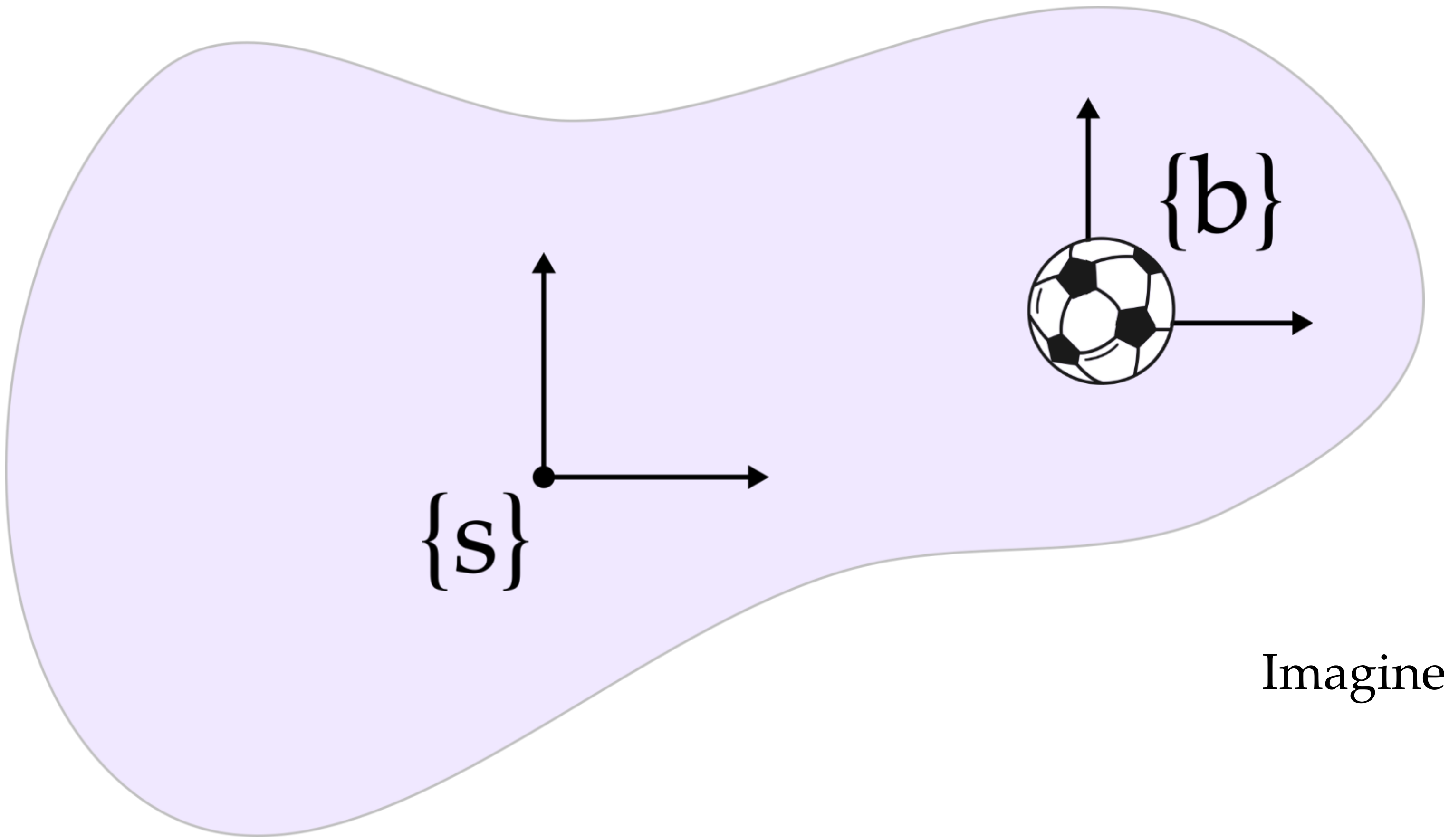
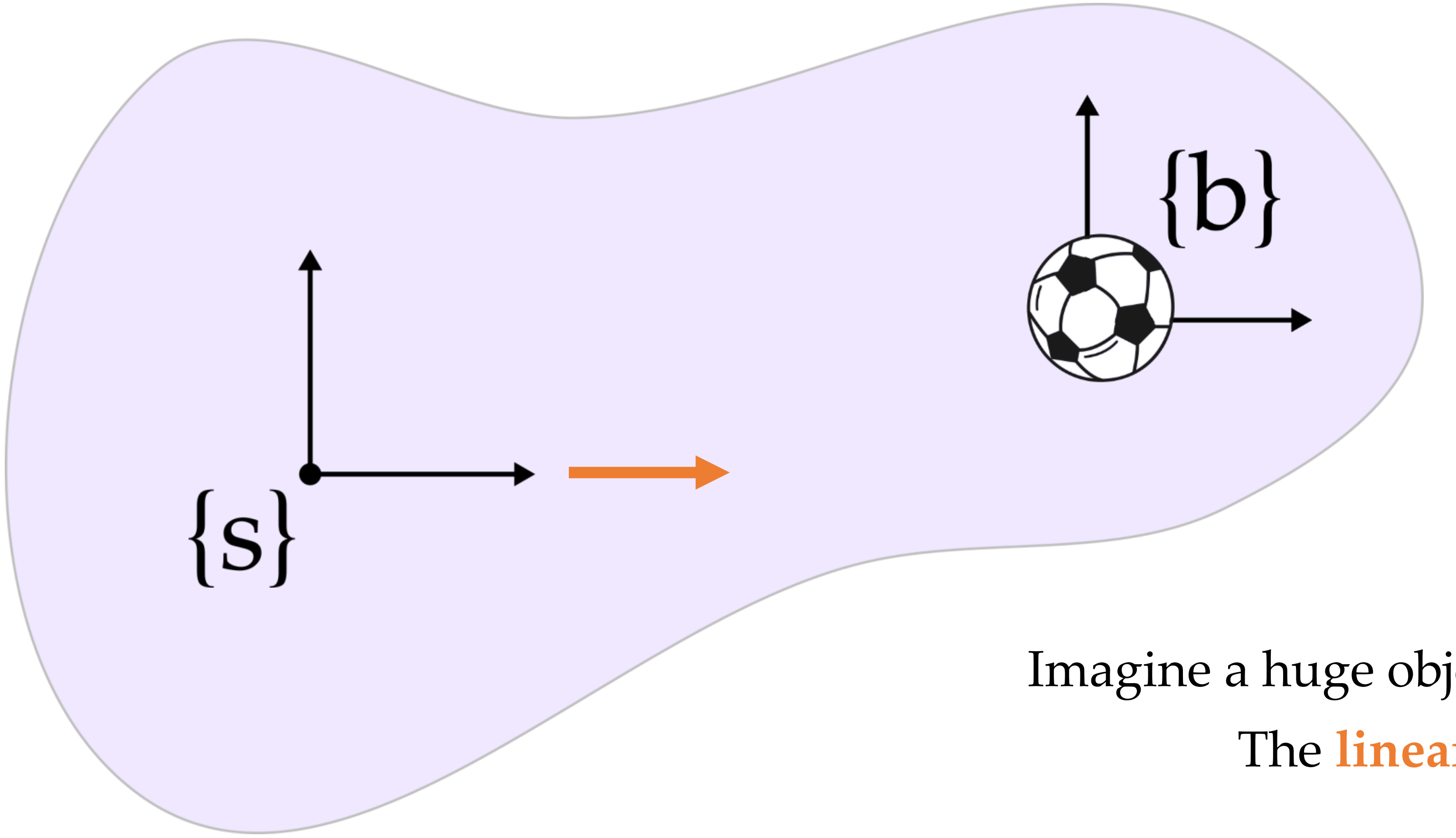
$$[V_s] = \dot{T}T^{-1} = \begin{bmatrix} [\omega_s] & \boxed{\omega_s \times (-p) + \dot{p}} \\ 0 & 0 \end{bmatrix}$$



Imagine a huge object attached to $\{b\}$

The **linear velocity** at $\{s\}$ is:

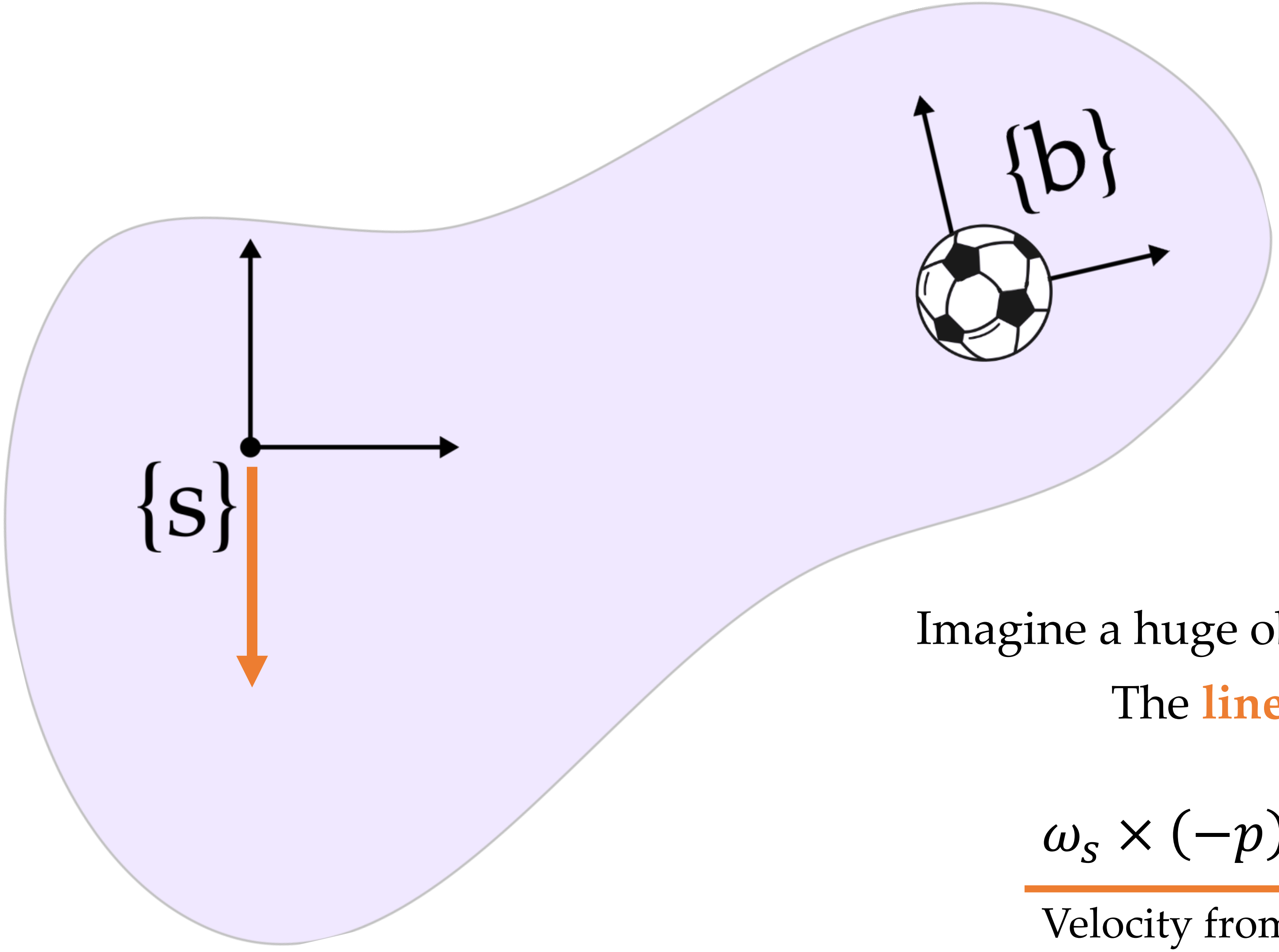


Imagine a huge object attached to $\{b\}$

The **linear velocity** at $\{s\}$ is:

$$\underline{\dot{p}}$$

Velocity from translation of $\{b\}$



Imagine a huge object attached to $\{b\}$

The **linear velocity** at $\{s\}$ is:

$$\underbrace{\omega_s \times (-p)} + \dot{p}$$

Velocity from rotation of $\{b\}$
Note that $(-p)$ is a vector from {b} to {s}

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$$[\mathcal{V}_s] = \dot{T}T^{-1} = \begin{bmatrix} [\omega_s] & \omega_s \times (-p) + \dot{p} \\ 0 & 0 \end{bmatrix}$$

$[\omega_s] = \dot{R}R^T$
angular velocity
expressed in $\{s\}$

$v_s = \omega_s \times (-p) + \dot{p}$
linear velocity of a point at $\{s\}$
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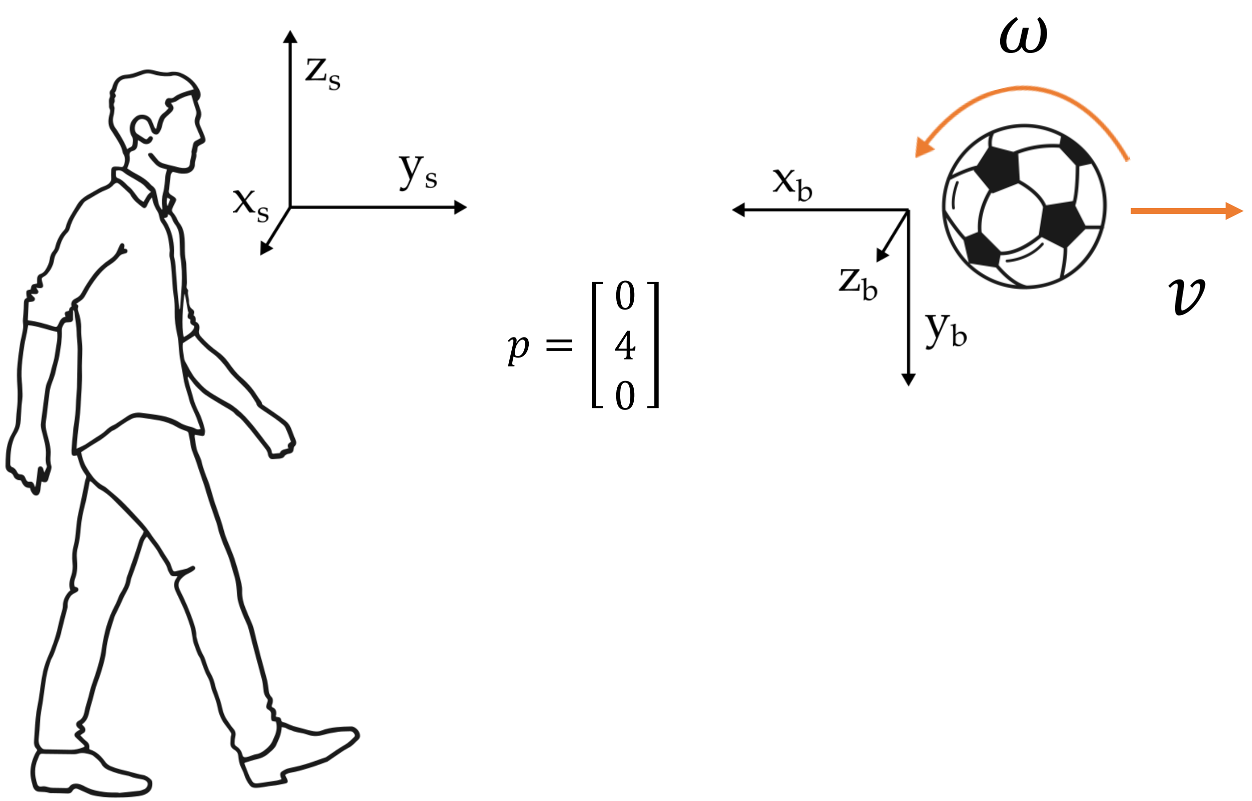
Spatial Twist

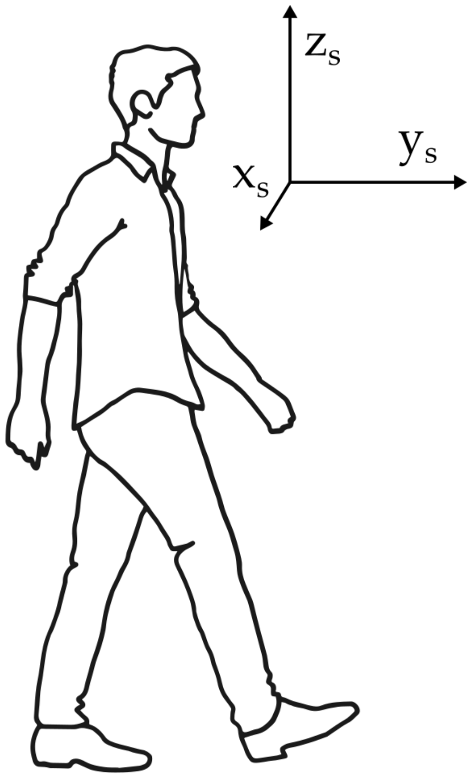


$$p = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

Ball rotates around z_b axis at α rad/s, and moves right at β m/s.

What is the **spatial twist**?

Spatial Twist



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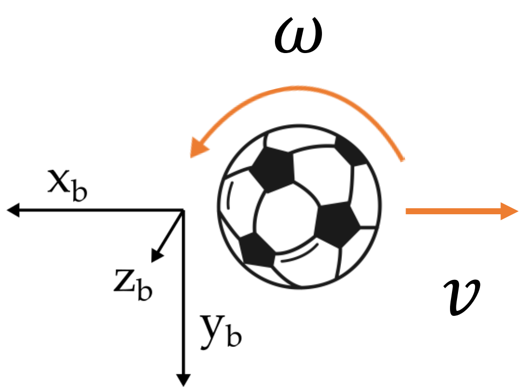

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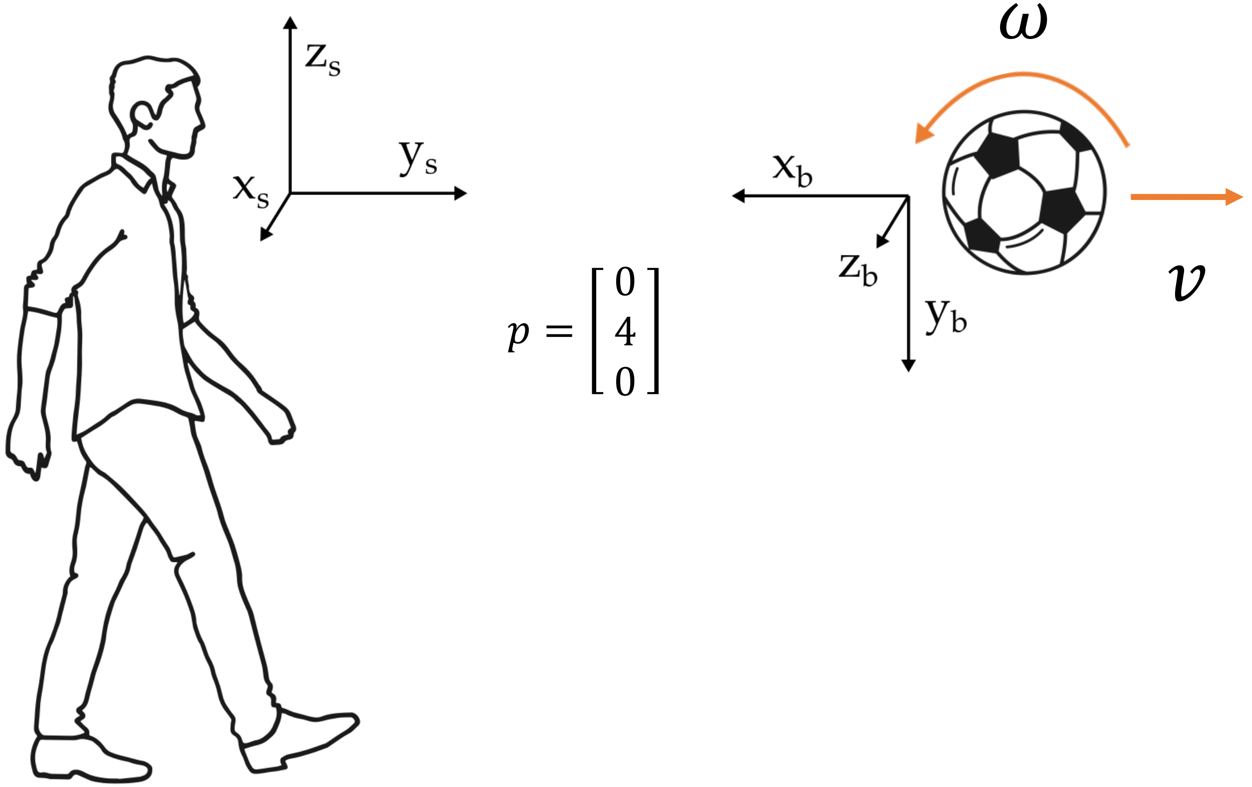
$$\omega_s = \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$$

Spatial Twist



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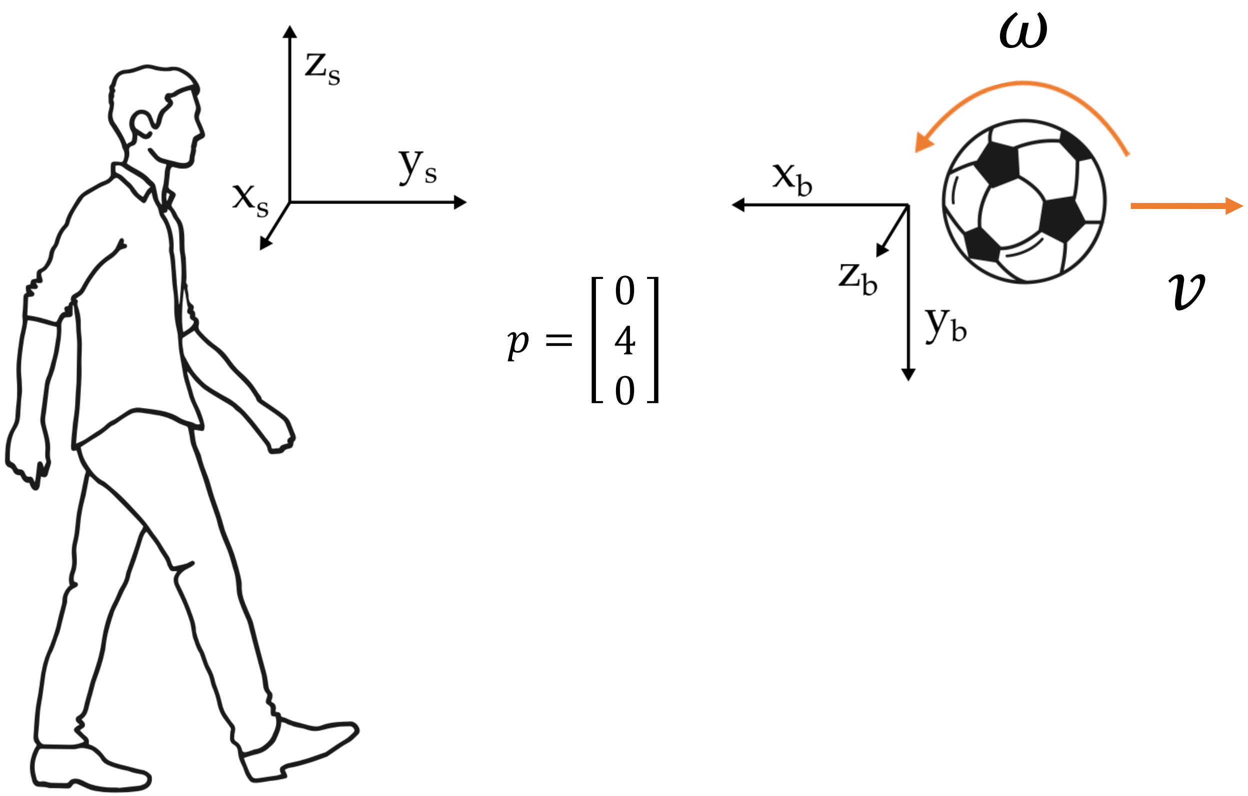
What is the **spatial twist**?

$$v_s = \omega_s \times (-p) + \dot{p}$$

$$= \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \beta \\ -4\alpha \end{bmatrix}$$

Spatial Twist



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$$V_s = \begin{bmatrix} \alpha \\ 0 \\ 0 \\ 0 \\ \beta \\ -4\alpha \end{bmatrix}$$

This Lecture

- How do we represent linear and angular velocity?
- How are twists related to transformation matrices?
- What are the two types of twists?

Next Lecture

- How can we use twists to describe the motion of robot joints?