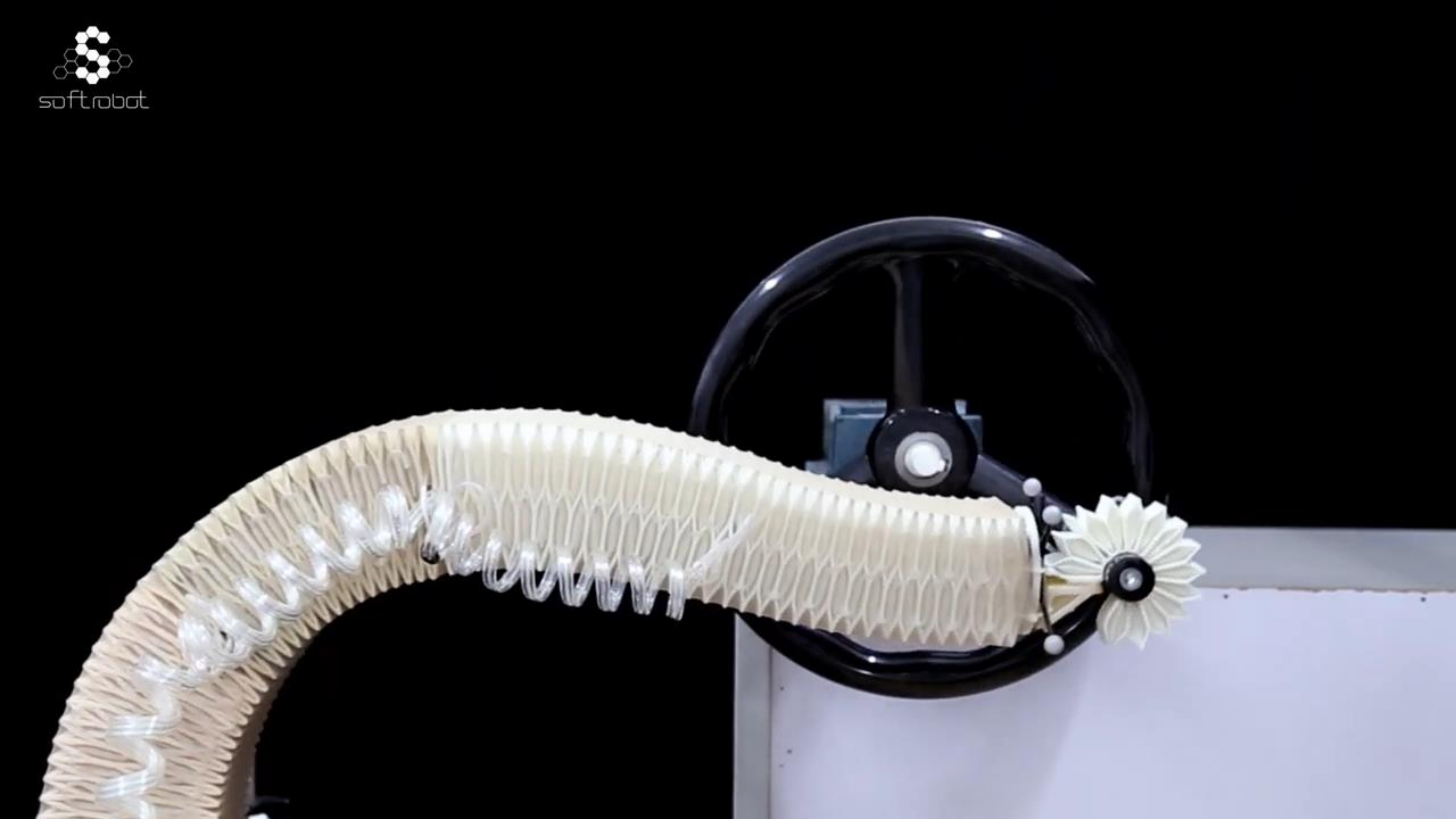
Forward Kinematics: More Examples

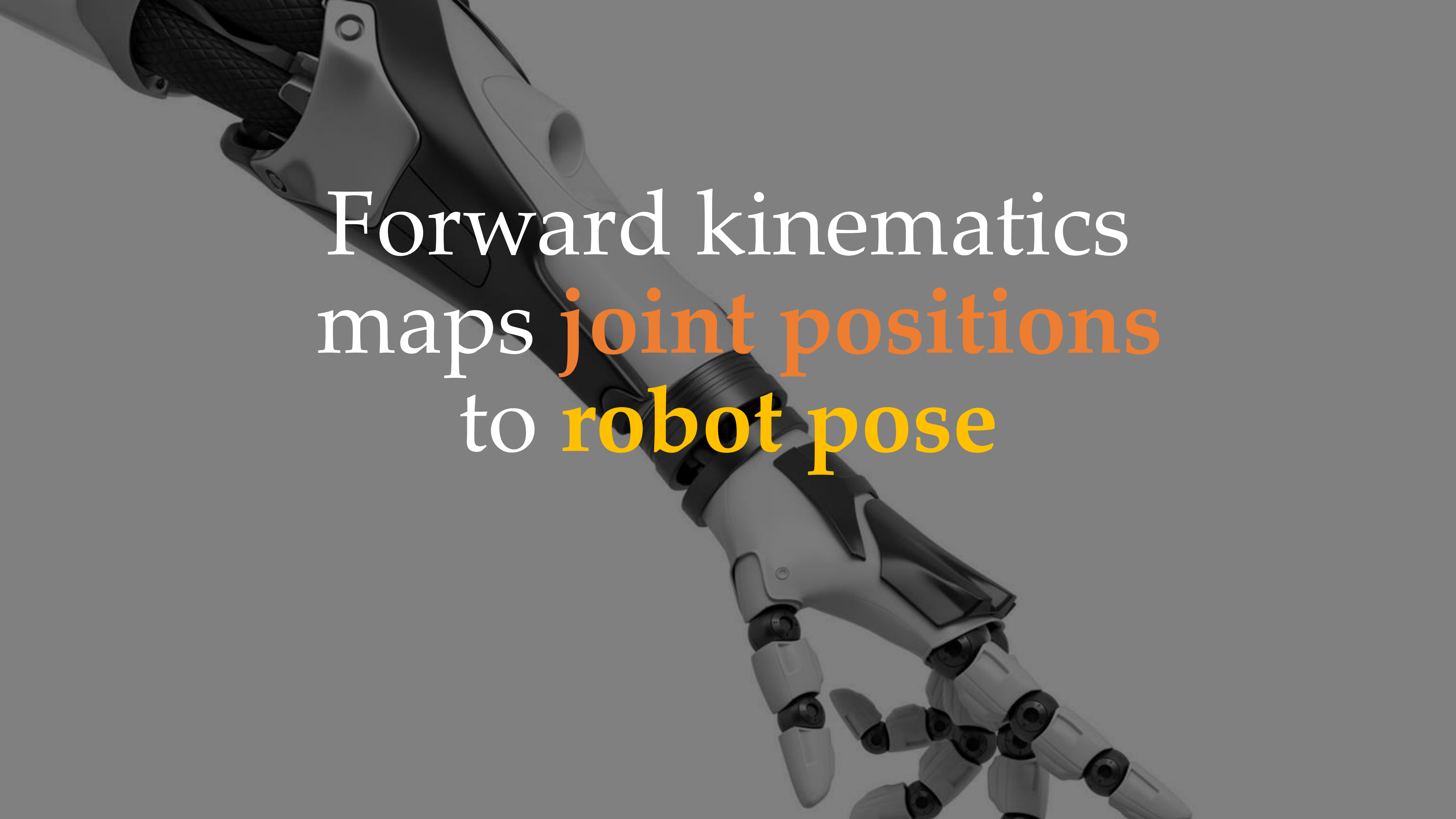
Reading: Modern Robotics 4.1



This Lecture

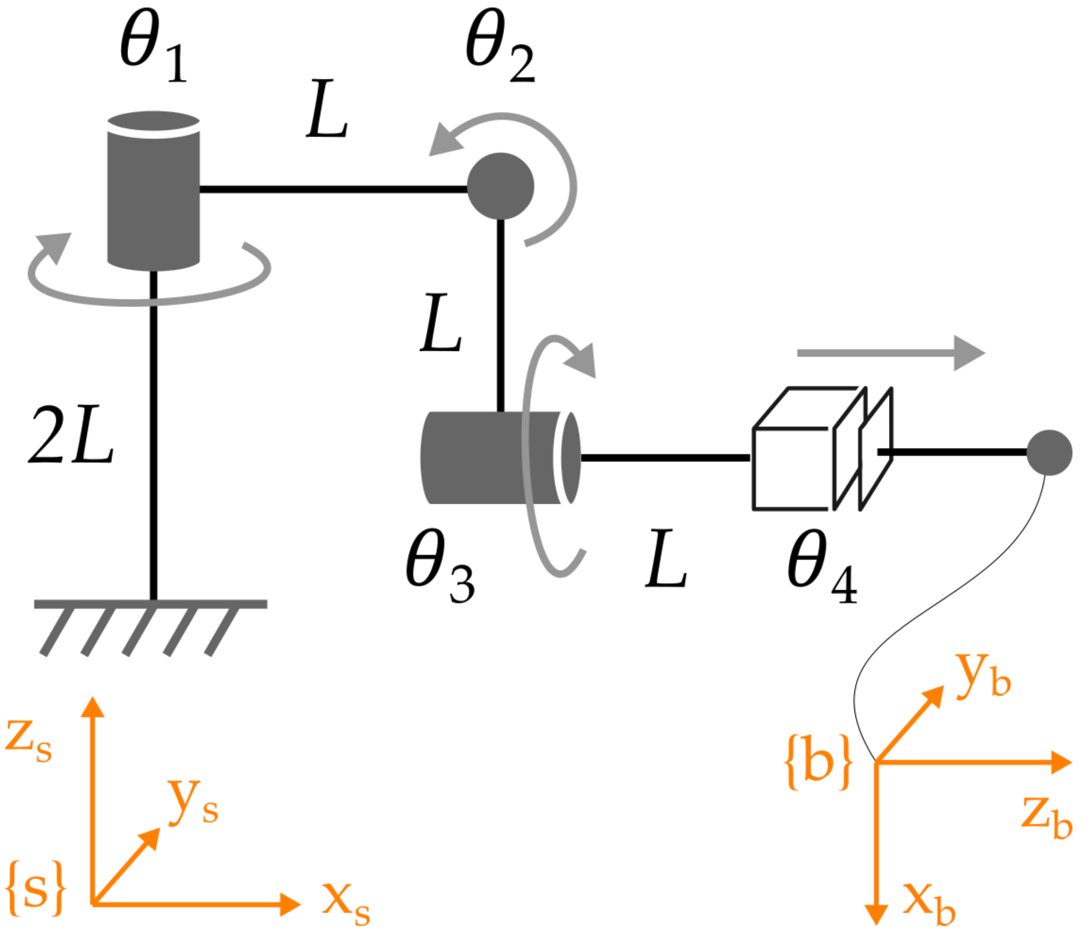
- How do I apply the product of exponentials formula?
- Practice forward kinematics with one final example

Forward kinematics maps joint positions to robot pose



Four-DoF robot arm.

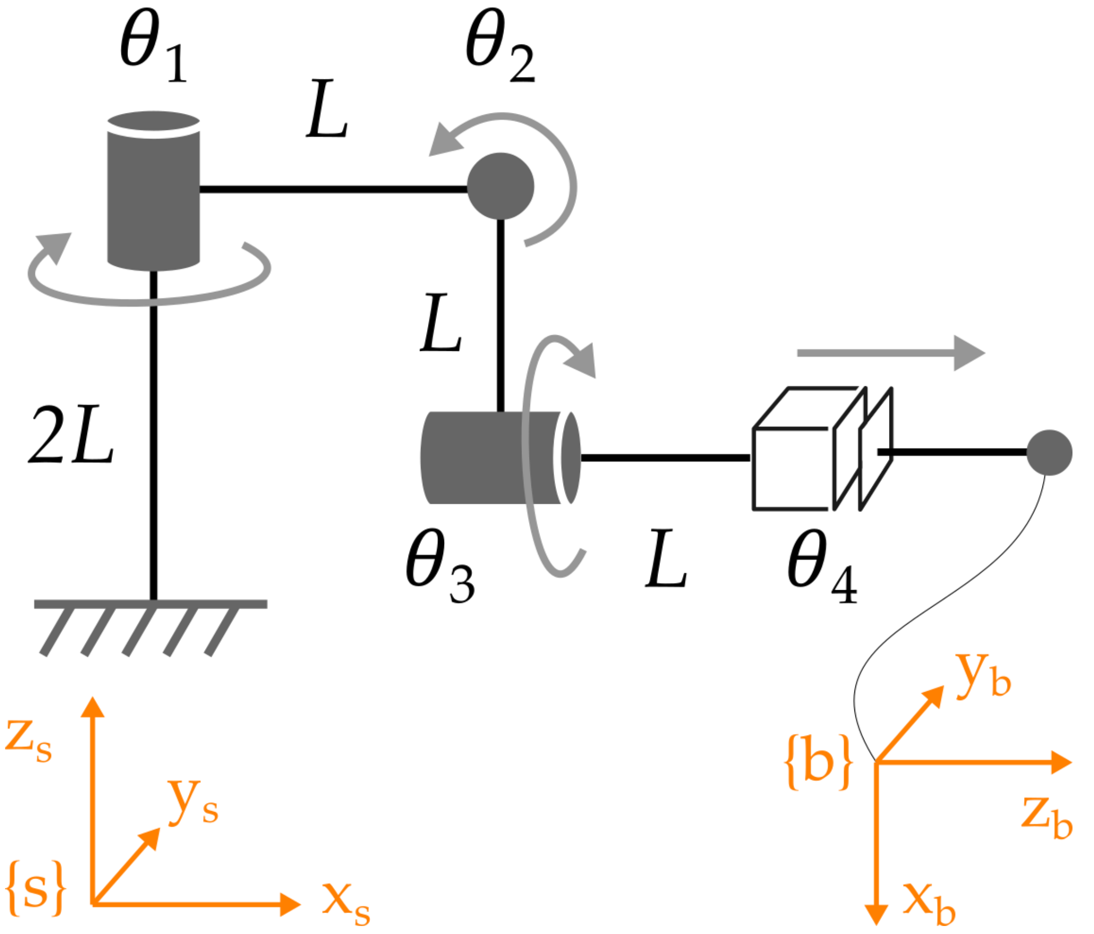
Given joint values θ, what is the **pose** of the end-effector?



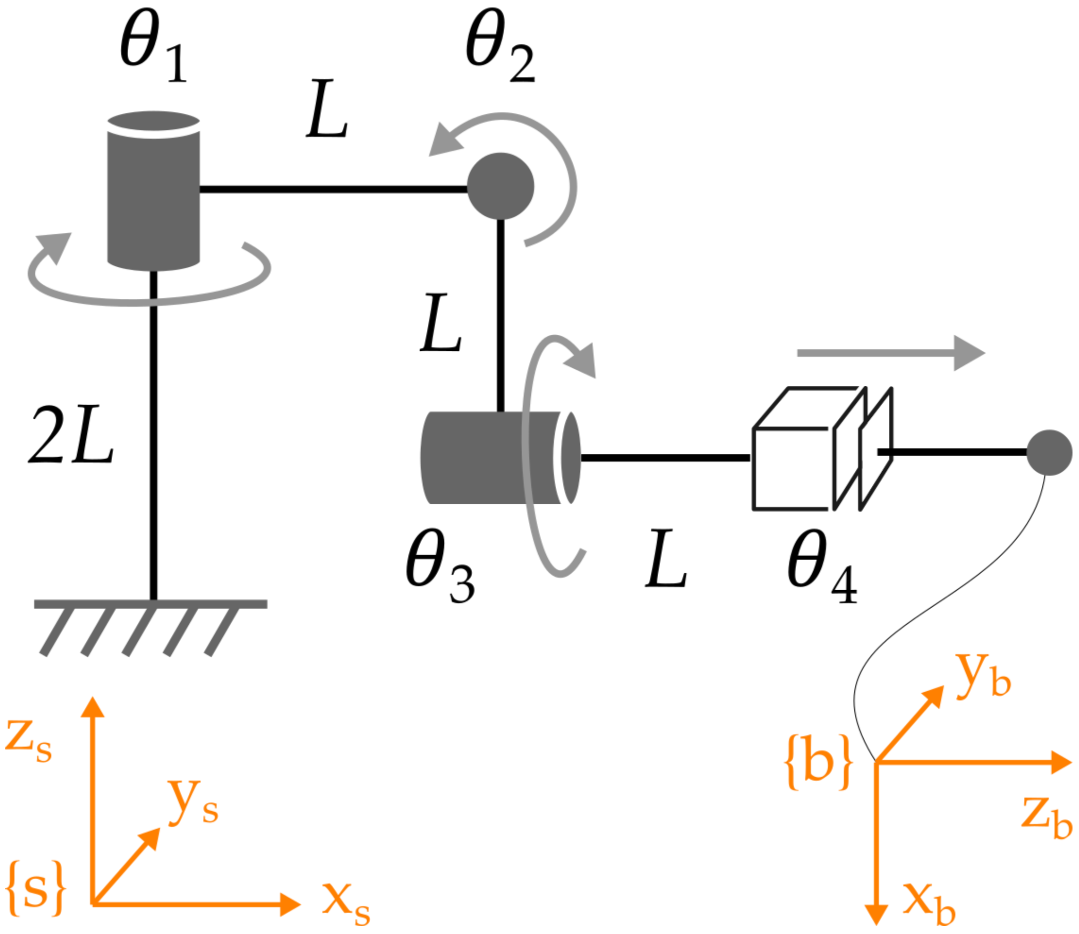
Four-DoF robot arm.

Given joint values θ, what is the **pose** of the end-effector?

$$T(\theta) = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4} M$$



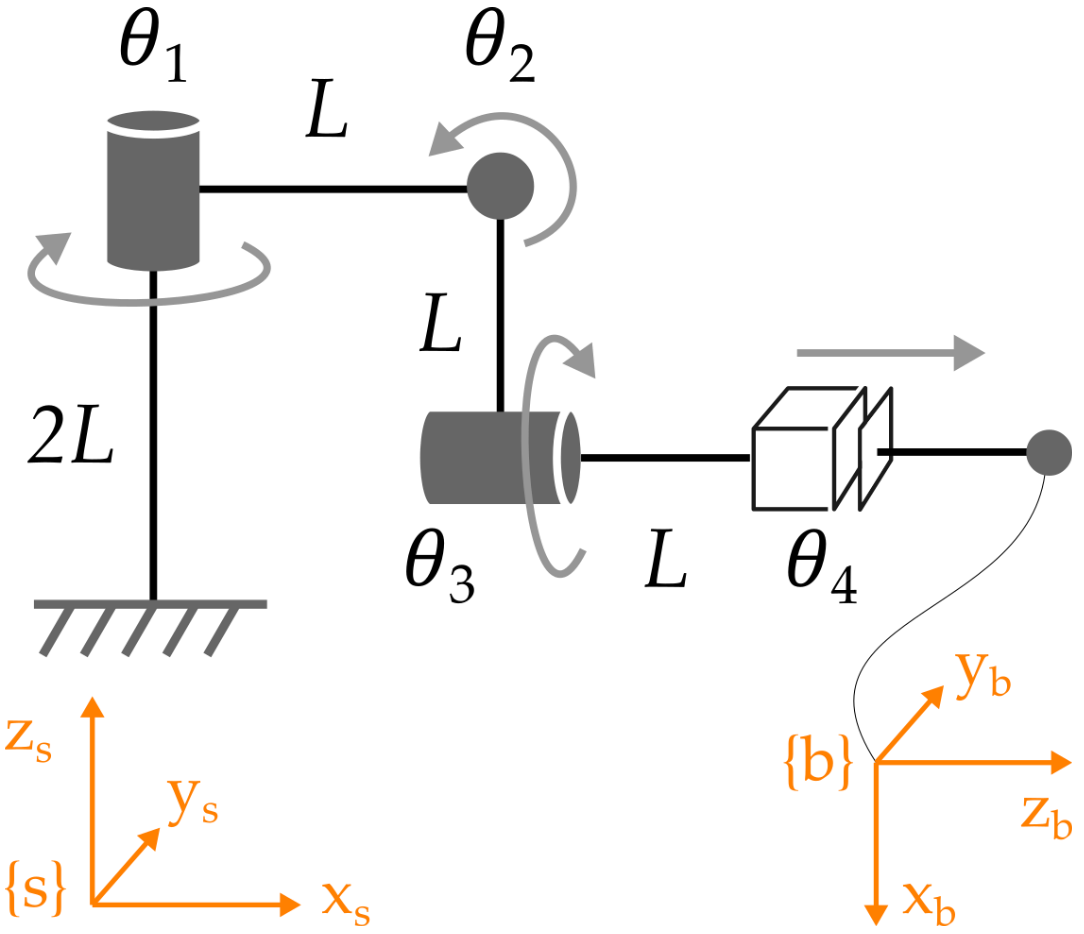
Step 1. $M = T_{sb}$ when the robot is in home position



Step 1. $M = T_{sb}$ when the robot is in home position

$$M = \begin{bmatrix} 0 & 0 & 1 & 2L \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & L \\ & 0 & & 1 \end{bmatrix}$$

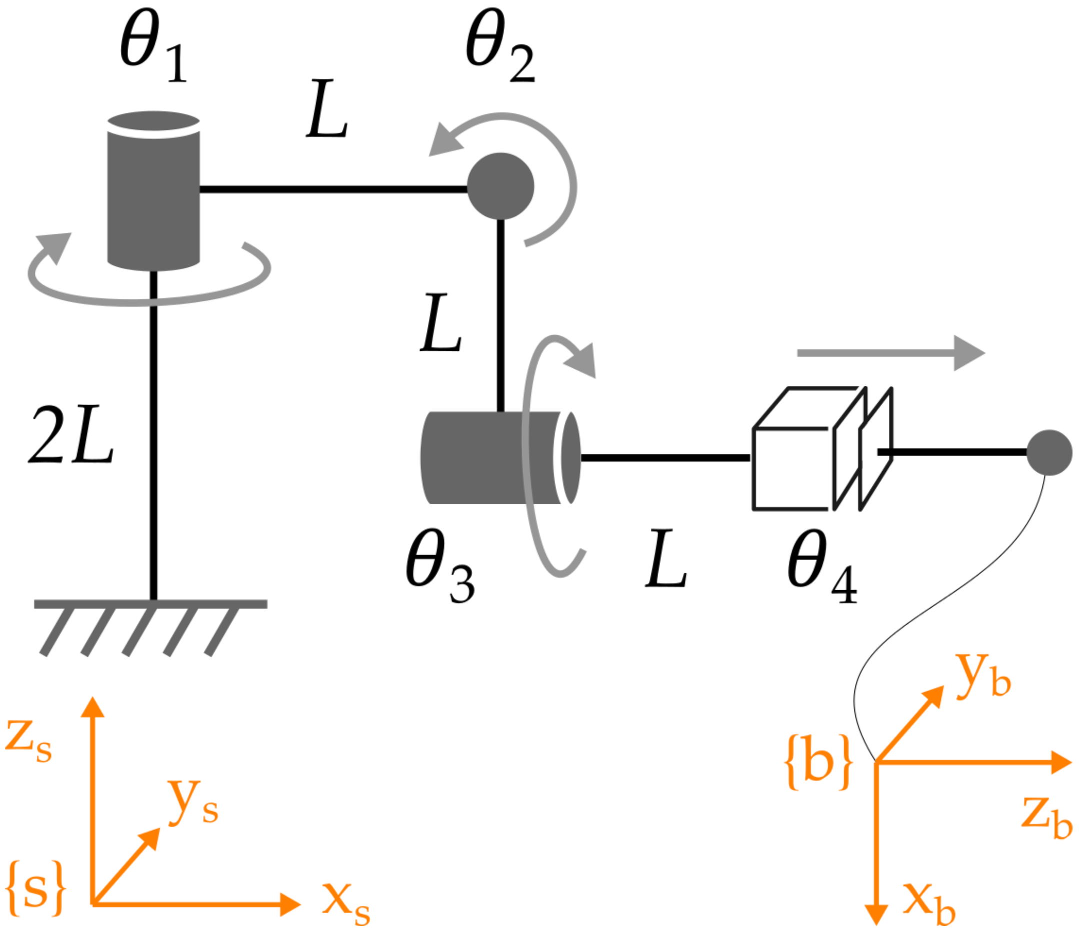
x_b aligned with $-z_s$
z_b aligned with x_s



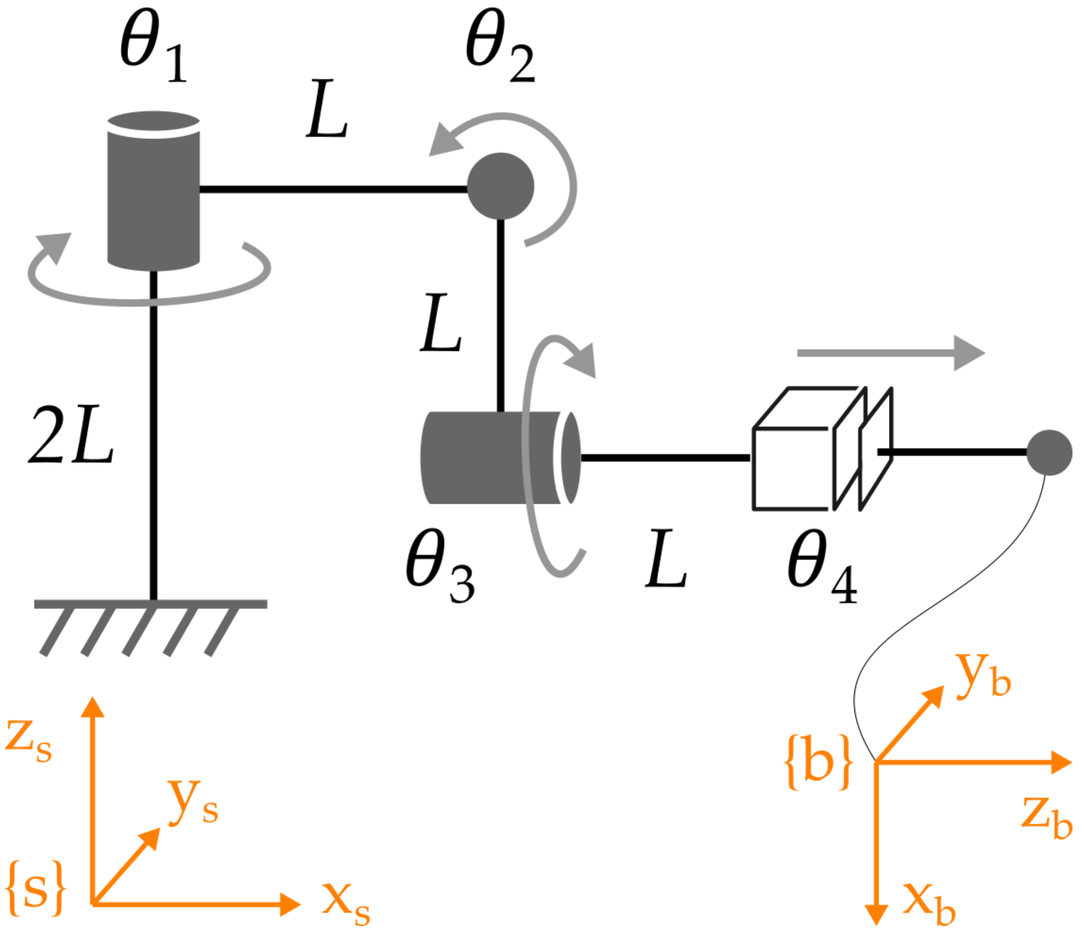
Step 1. $M = T_{sb}$ when the robot is in home position

$$M = \begin{bmatrix} 0 & 0 & 1 & 2L \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & L \\ \multicolumn{3}{c}{0} & 1 \end{bmatrix}$$

$\{b\}$ is $2L$ units along x_s
and L along z_s



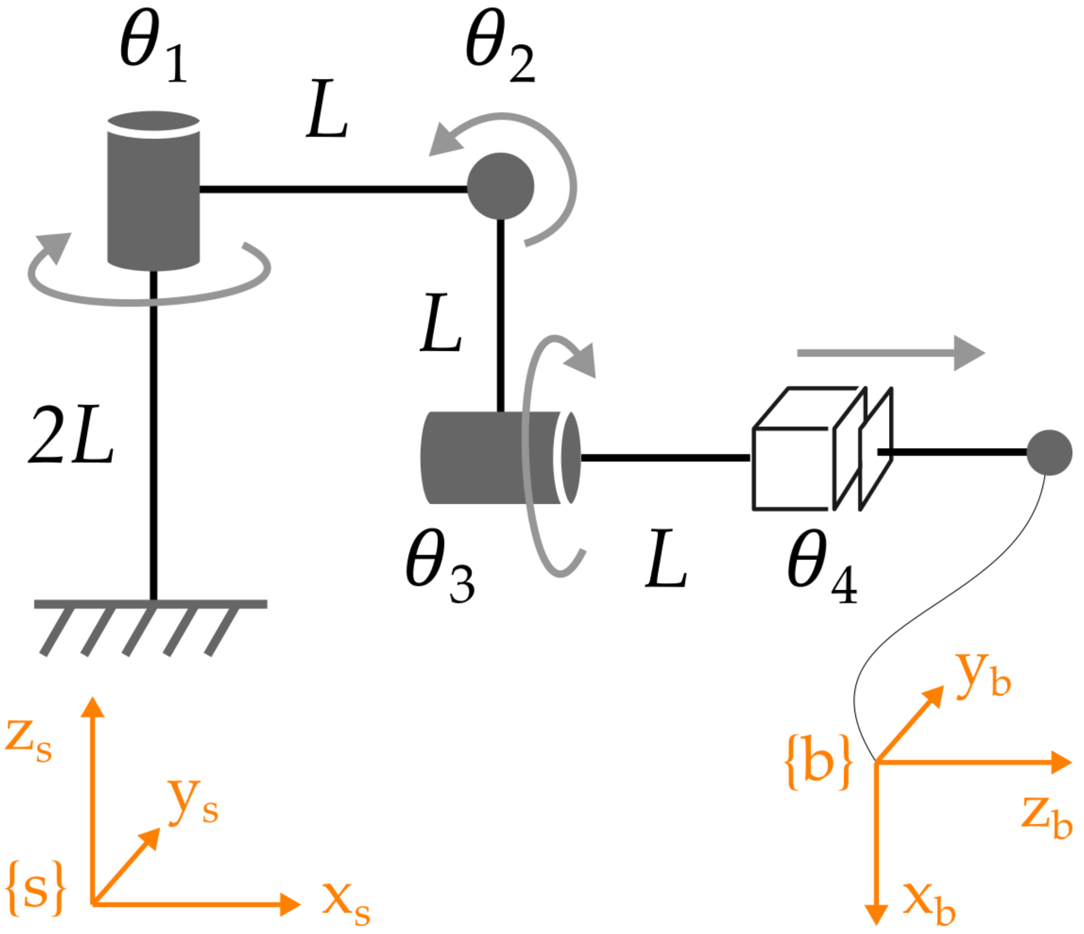
Step 2. S_i is the screw for the i-th joint when the robot is in home position



Step 2. S_i is the screw for the i-th joint when the robot is in home position

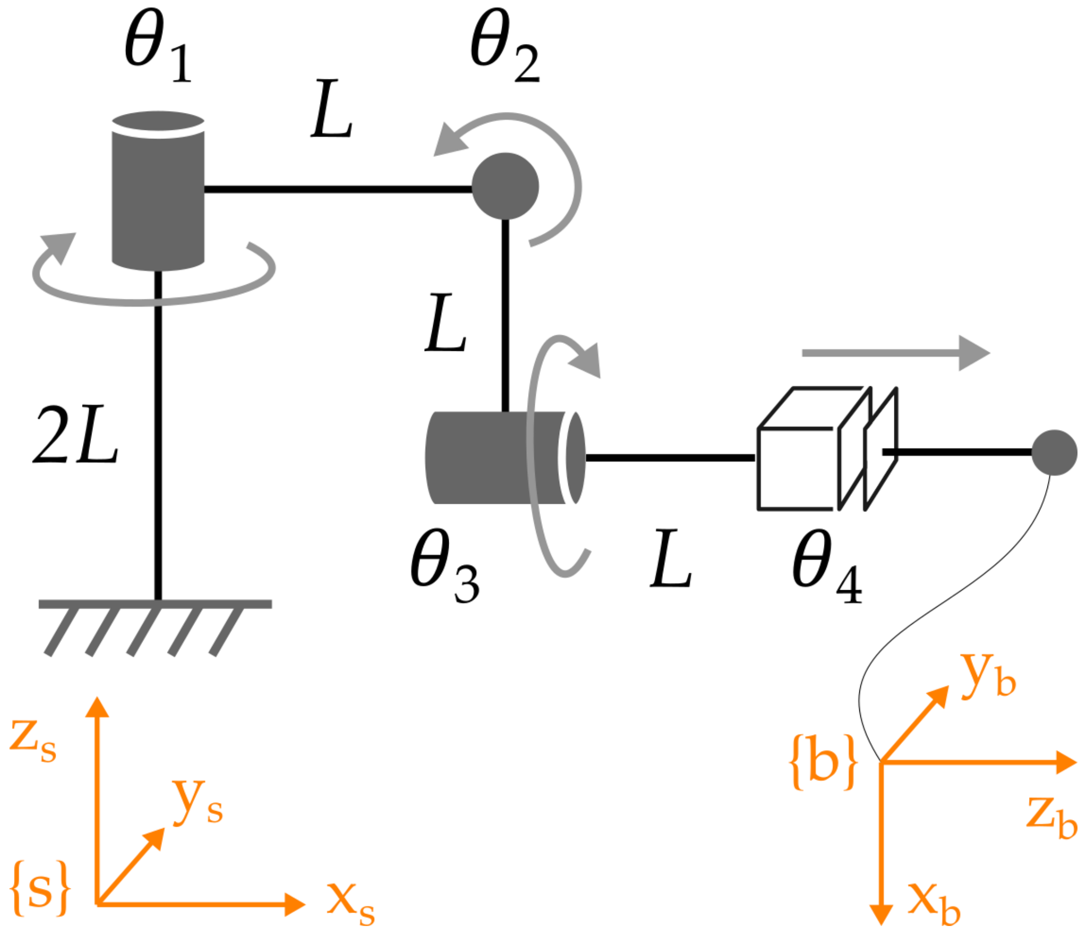
$$S = \begin{bmatrix} \omega_s \\ -\omega_s \times q \end{bmatrix}$$

- ω_s is unit vector in the direction of the axis of ***positive*** rotation
- q is vector from $\{s\}$ to the ***joint axis***



Step 2. S_i is the screw for the i-th joint when the robot is in home position

Hint: Remember to check for positive or negative rotation

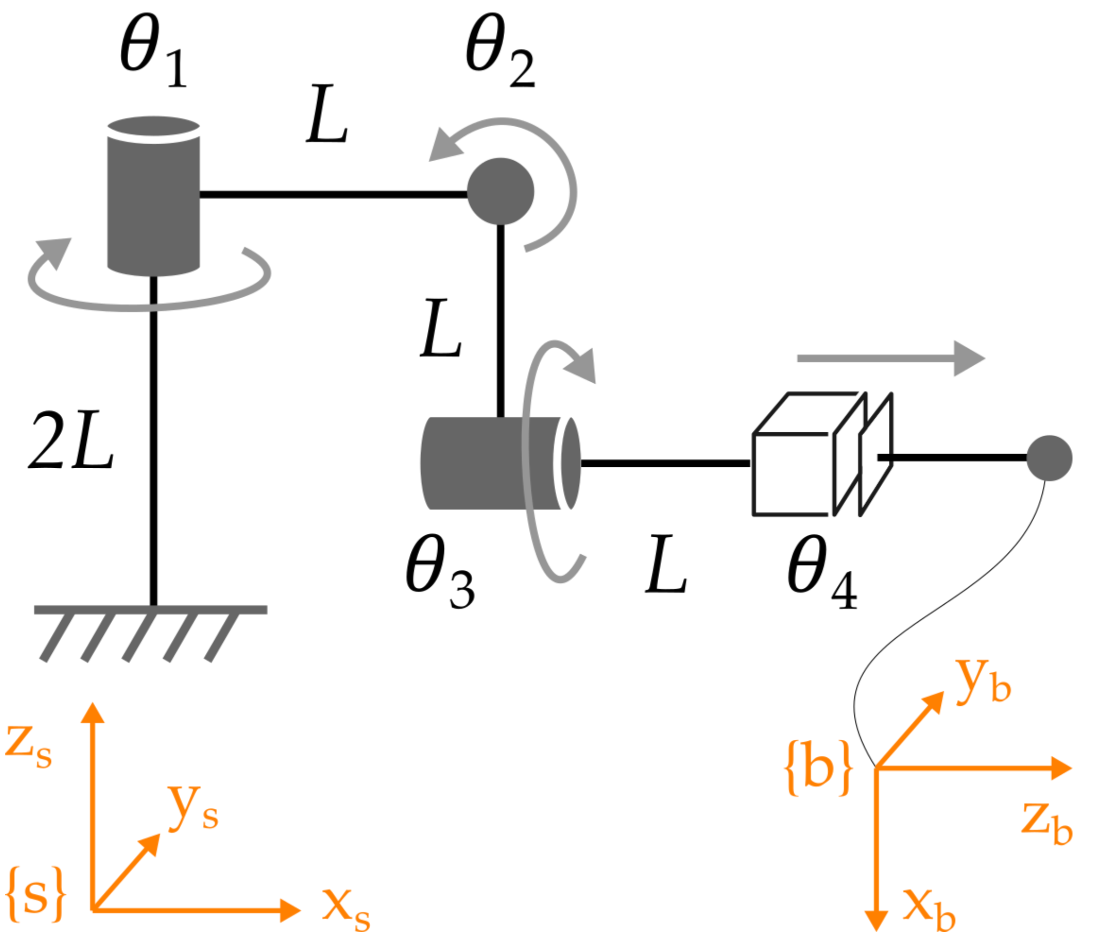


Step 2. S_i is the screw for the i-th joint when the robot is in home position

$$S = \begin{bmatrix} \omega_s \\ -\omega_s \times q \end{bmatrix}$$

$$\omega_{s1} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad S_1 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

1st joint is rotating around $-z_s$ axis



Step 2. S_i is the screw for the i-th joint when the robot is in home position

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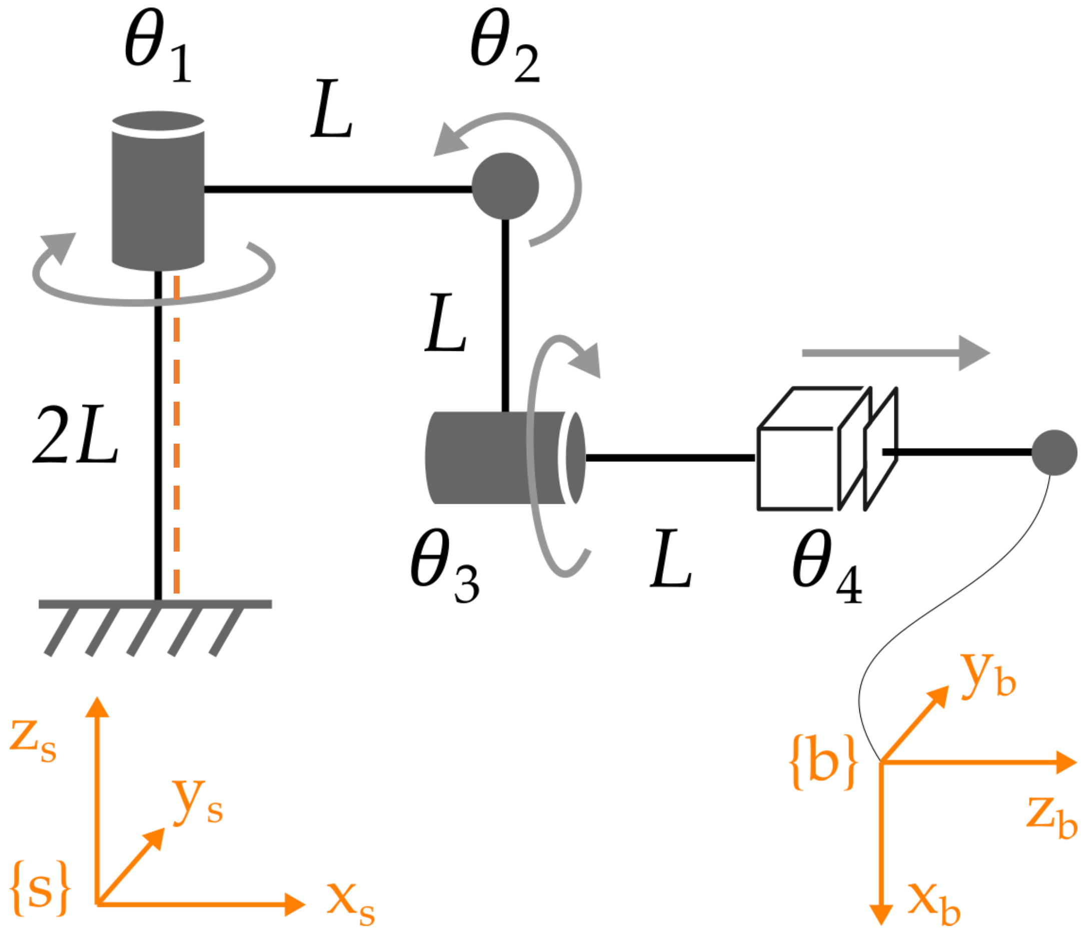
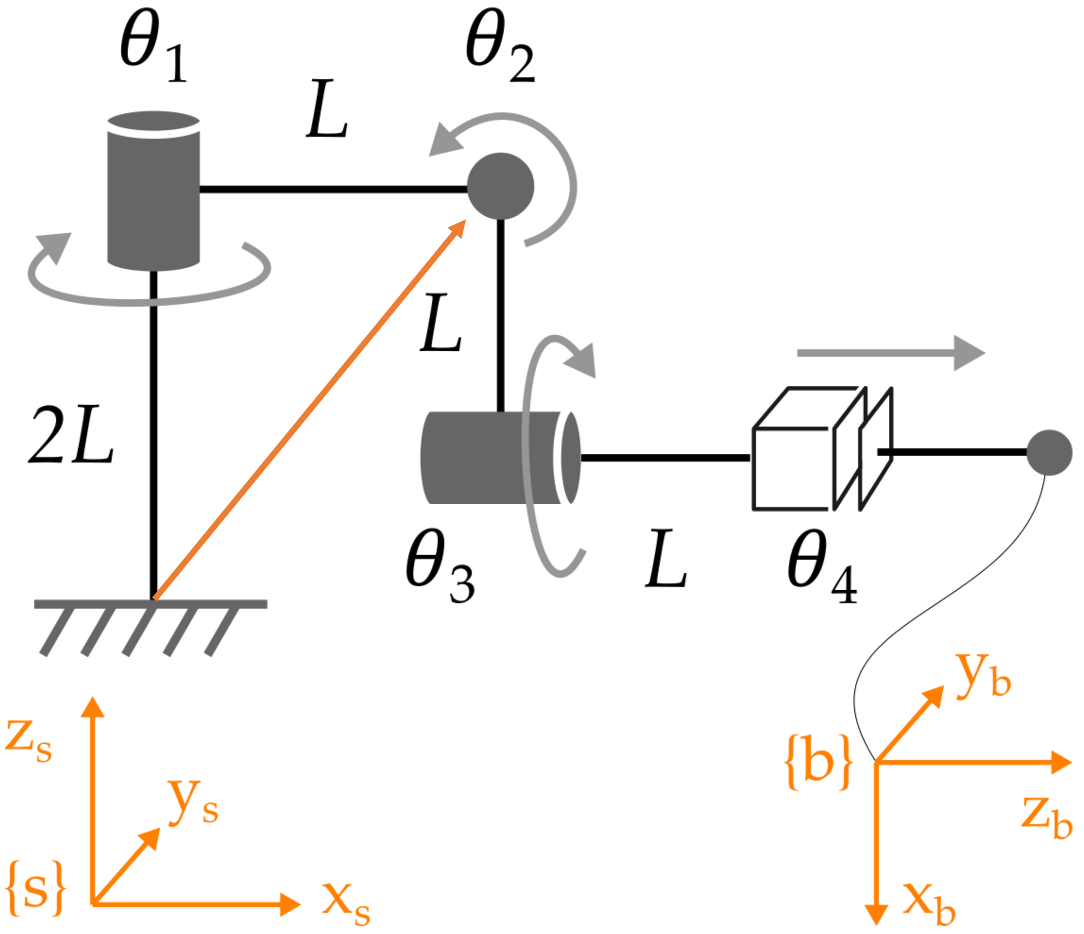
1st joint axis passes through $\{s\}$



Step 2. S_i is the screw for the i-th joint when the robot is in home position

$$S = \begin{bmatrix} \omega_s \\ -\omega_s \times q \end{bmatrix}$$

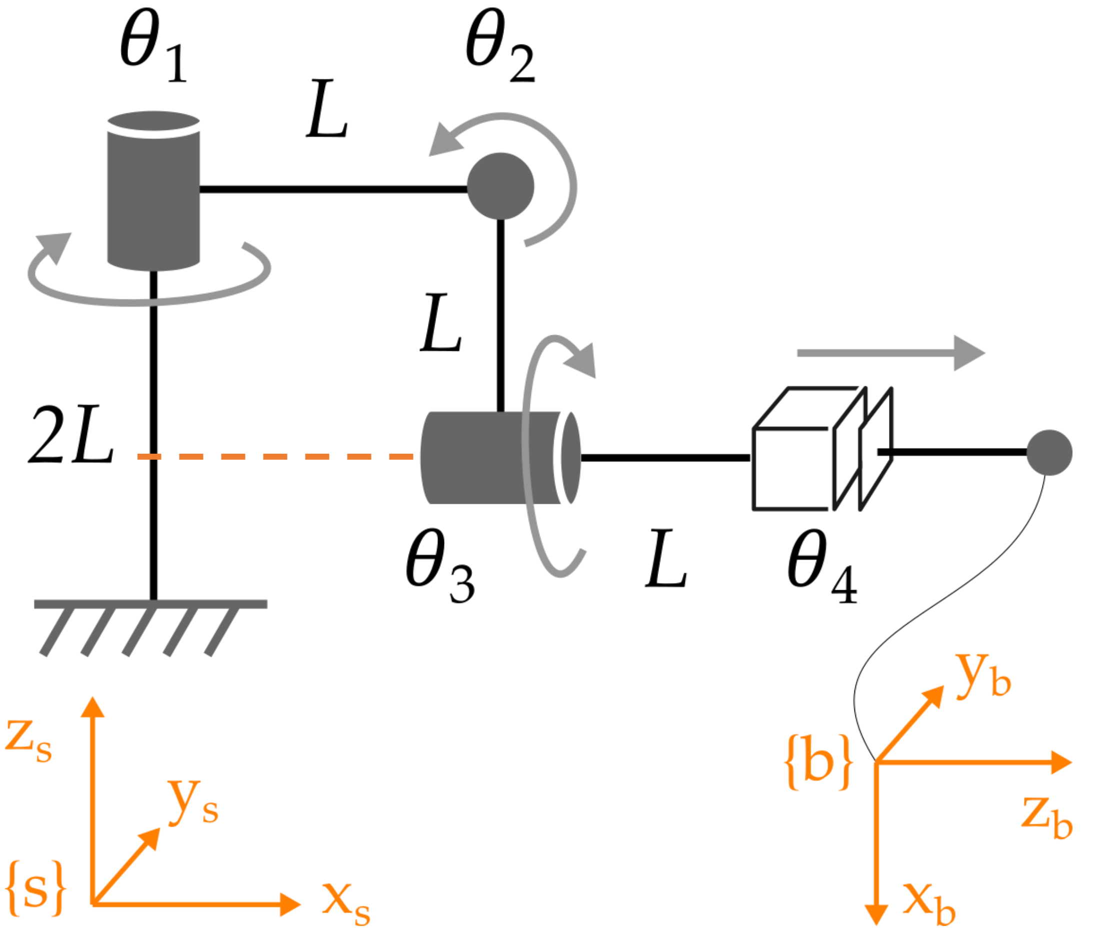
$$\omega_{s2} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad q_2 = \begin{bmatrix} L \\ 0 \\ 2L \end{bmatrix} \quad S_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 2L \\ 0 \\ -L \end{bmatrix}$$



Step 2. S_i is the screw for the i-th joint when the robot is in home position

$$S = \begin{bmatrix} \omega_s \\ -\omega_s \times q \end{bmatrix}$$

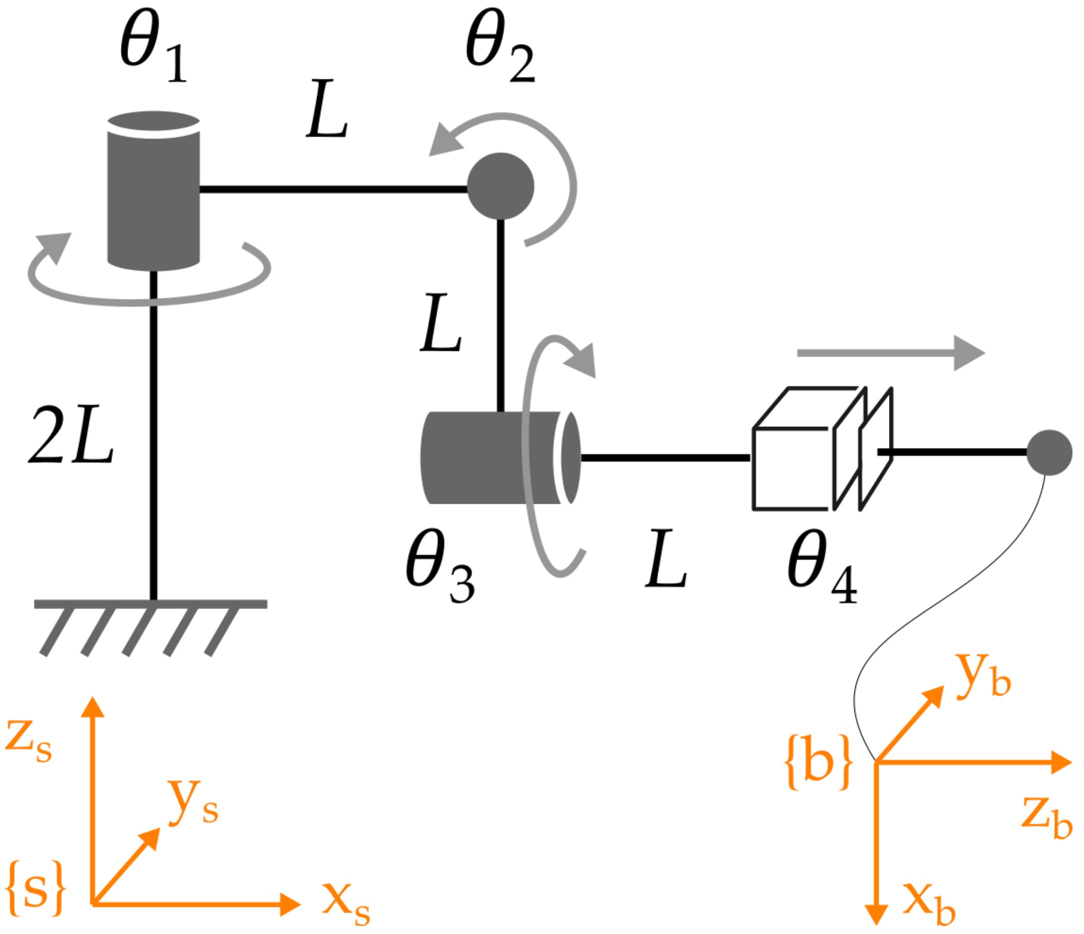
$$\omega_{s3} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad q_3 = \begin{bmatrix} 0 \\ 0 \\ L \end{bmatrix} \quad S_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ -L \\ 0 \end{bmatrix}$$



Step 2. S_i is the screw for the i-th joint when the robot is in home position

$$S = \begin{bmatrix} 0 \\ v_s \end{bmatrix}$$

- v_s is unit vector in the direction of positive translation

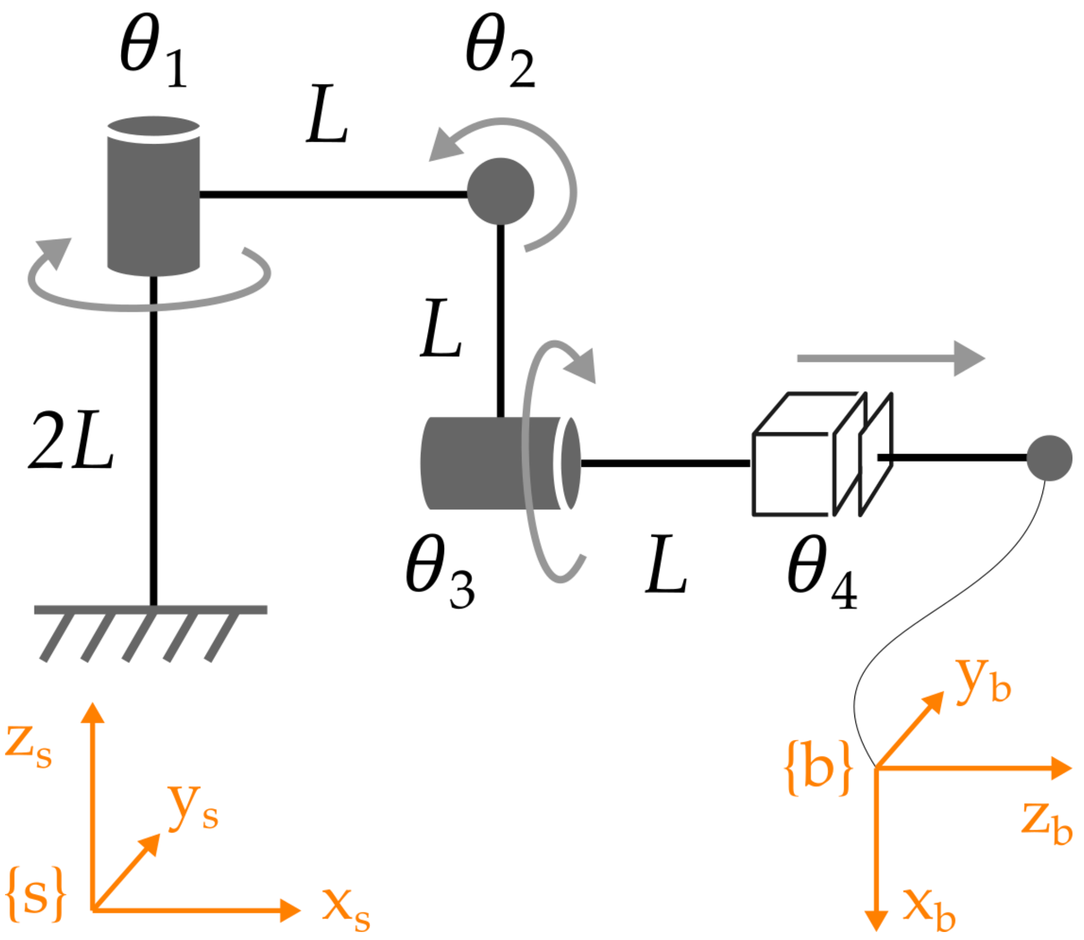


Step 2. S_i is the screw for the i-th joint when the robot is in home position

$$S = \begin{bmatrix} 0 \\ v_s \end{bmatrix}$$

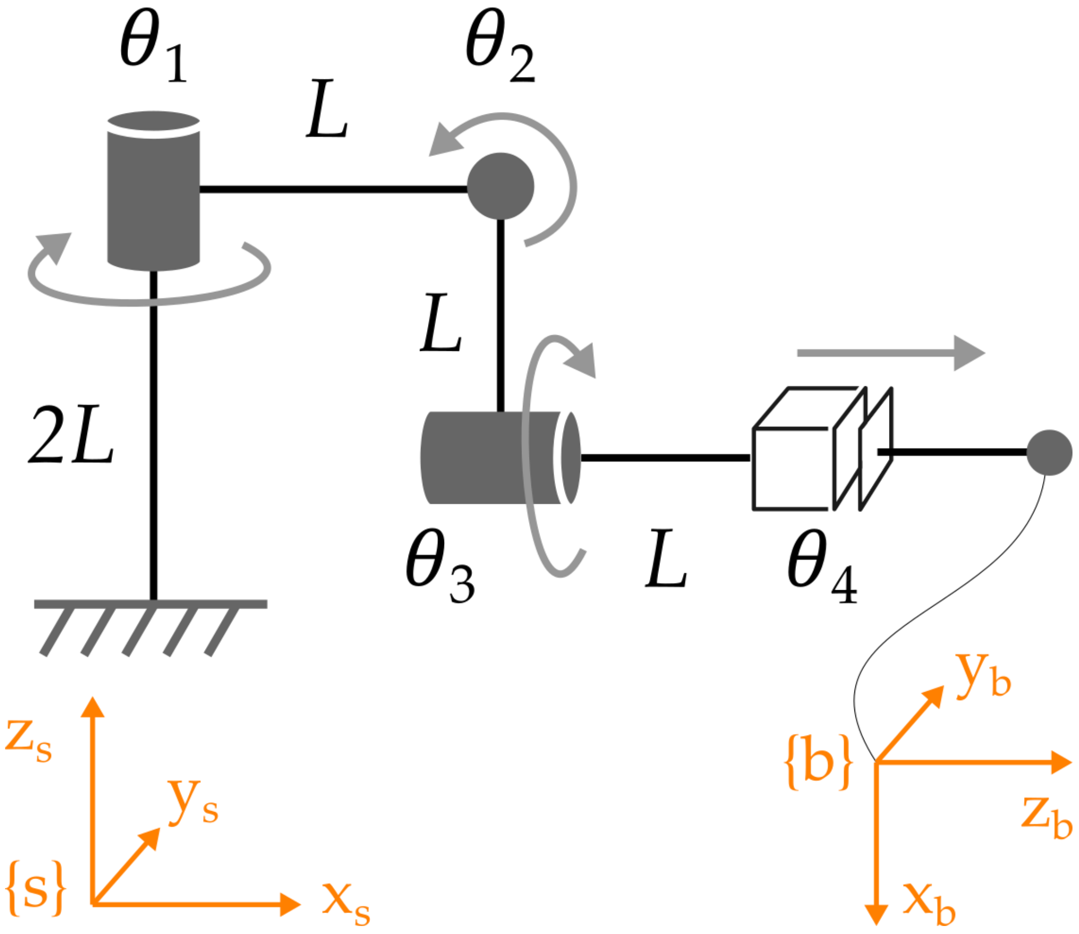
$$v_{s4} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad S_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Positive translation along x_s



Step 3. Use our formula to get $T(\theta)$

$$T(\theta) = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4} M$$



```matlab
function T = fk(theta1, theta2, theta3, theta4, L)

M = [0 0 1 2*L; 0 1 0 0; -1 0 0 L; 0 0 0 1];
S1 = [0; 0; -1; 0; 0; 0];
S2 = [0; -1; 0; 2*L; 0; -L];
S3 = [-1; 0; 0; 0; -L; 0];
S4 = [0; 0; 0; 1; 0; 0];
T = expm(bracket(S1)*theta1) * ...
    expm(bracket(S2)*theta2) * ...
    expm(bracket(S3)*theta3) * ...
    expm(bracket(S4)*theta4) * M;

    function S_matrix = bracket(S)
        S_matrix = [0 -S(3) S(2) S(4);
                    S(3) 0 -S(1) S(5);
                    -S(2) S(1) 0 S(6); 0 0 0 0];
    end
end
```

This Lecture

- How do I apply the product of exponentials formula?
- Practice forward kinematics with one final example

Next Lecture

- How do we find the velocity of our robot arm?