Practice Set 5

Robotics & Automation
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Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

Problem 1

Is matrix X a transformation matrix?

$$X = \begin{bmatrix} 0 & 0 & -1 & -10 \\ 0 & -1 & 0 & 2.5 \\ 0 & 0 & 0 & -0.1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

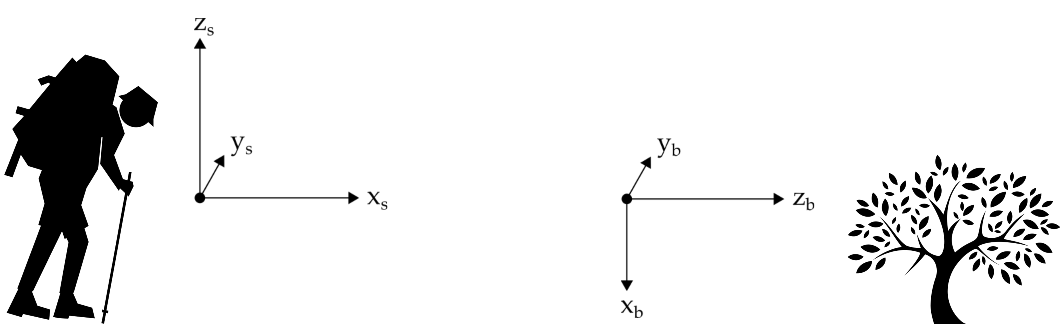
Problem 2

Let T be a transformation matrix. When is the transpose of T also a transformation matrix?

Problem 3

Let T_1 and T_2 be two transformation matrices. Are these transformations commutative?

Problem 4



A bush is 5 units in from of the hiker. The pose of $\{b\}$ relative to $\{s\}$ is:

$$T_{sb} = \begin{bmatrix} 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Try to find T_{bs} directly from the drawing, and then try to find T_{bs} using T_{sb}. Do your two answers agree?

Problem 1

Is matrix X a transformation matrix?

$$X = \begin{bmatrix} 0 & 0 & -1 & -10 \\ 0 & -1 & 0 & 2.5 \\ 0 & 0 & 0 & -0.1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

No, X is not a transformation matrix. Remember that:

$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \tag{4}$$

X is not a valid transformation because (a) R in X is not a rotation matrix and (b) the first element in the bottom row is 1 when it should always be zero.

Problem 2

Let T be a transformation matrix. When is the transpose of T also a transformation matrix?

We are looking for cases when T^T is a valid transformation matrix. Start with the definition of a transformation:

$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \qquad T^T = \begin{bmatrix} R^T & 0 \\ p^T & 1 \end{bmatrix} \tag{5}$$

For T^T to be a transformation we need $p = 0$.

You might remember that $R^T = R^{-1}$ is a rotation matrix. But that is not the case with transformation matrices! The transpose of a transformation matrix *is not* the same as the inverse of a transformation matrix.

Problem 3

Let T_1 and T_2 be two transformation matrices. Are these transformations commutative?

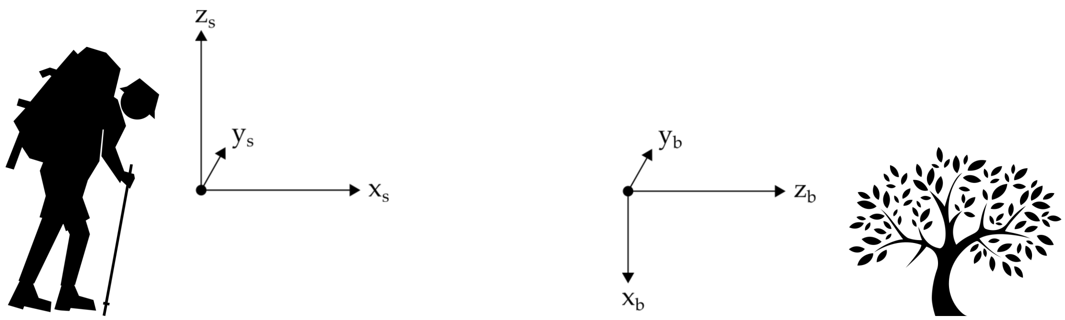
No, transformations matrices are not typically commutative.

$$T_1 = \begin{bmatrix} R_1 & p_1 \\ 0 & 1 \end{bmatrix} \qquad T_2 = \begin{bmatrix} R_2 & p_2 \\ 0 & 1 \end{bmatrix} \tag{6}$$

Multiplying these out, we get that:

$$T_1T_2 = \begin{bmatrix} R_1R_2 & R_1p_2 + p_1 \\ 0 & 1 \end{bmatrix} \qquad T_2T_1 = \begin{bmatrix} R_2R_1 & R_2p_1 + p_2 \\ 0 & 1 \end{bmatrix} \tag{7}$$

We previously proved that R_1R_2 is not necessarily equal to R_2R_1. Hence, we see that $T_1T_2 \neq T_2T_1$ at least for some choices of T_1 and T_2.



Problem 4

A bush is 5 units in from of the hiker. The pose of $\{b\}$ relative to $\{s\}$ is:

$$T_{sb} = \begin{bmatrix} 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

Try to find T_{bs} directly from the drawing, and then try to find T_{bs} using T_{sb}. Do your two answers agree?

Looking at the drawing and remembering our definitions of rotation matrices, you should be able to figure out that:

$$T_{bs} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Notice that the hiker is 5 units behind the tree along the z_b axis. Now let's use the properties of transformation matrices:

$$T_{bs} = T_{sb}^{-1} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

Yes! Taking the inverse of a transformation matrix changes the frame of reference.