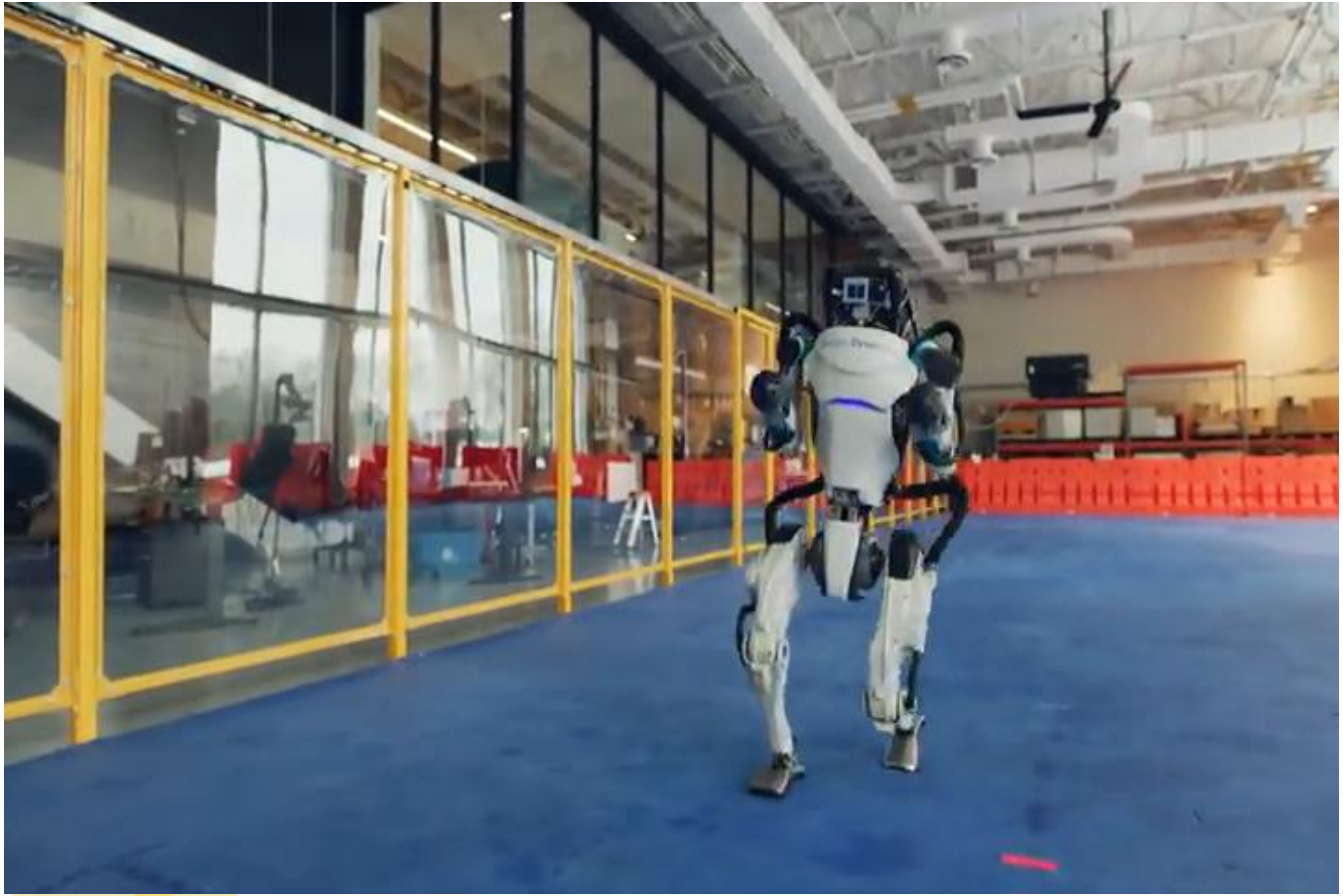
Euler-Lagrange

Reading: Robot Modeling and Control 7.1.1

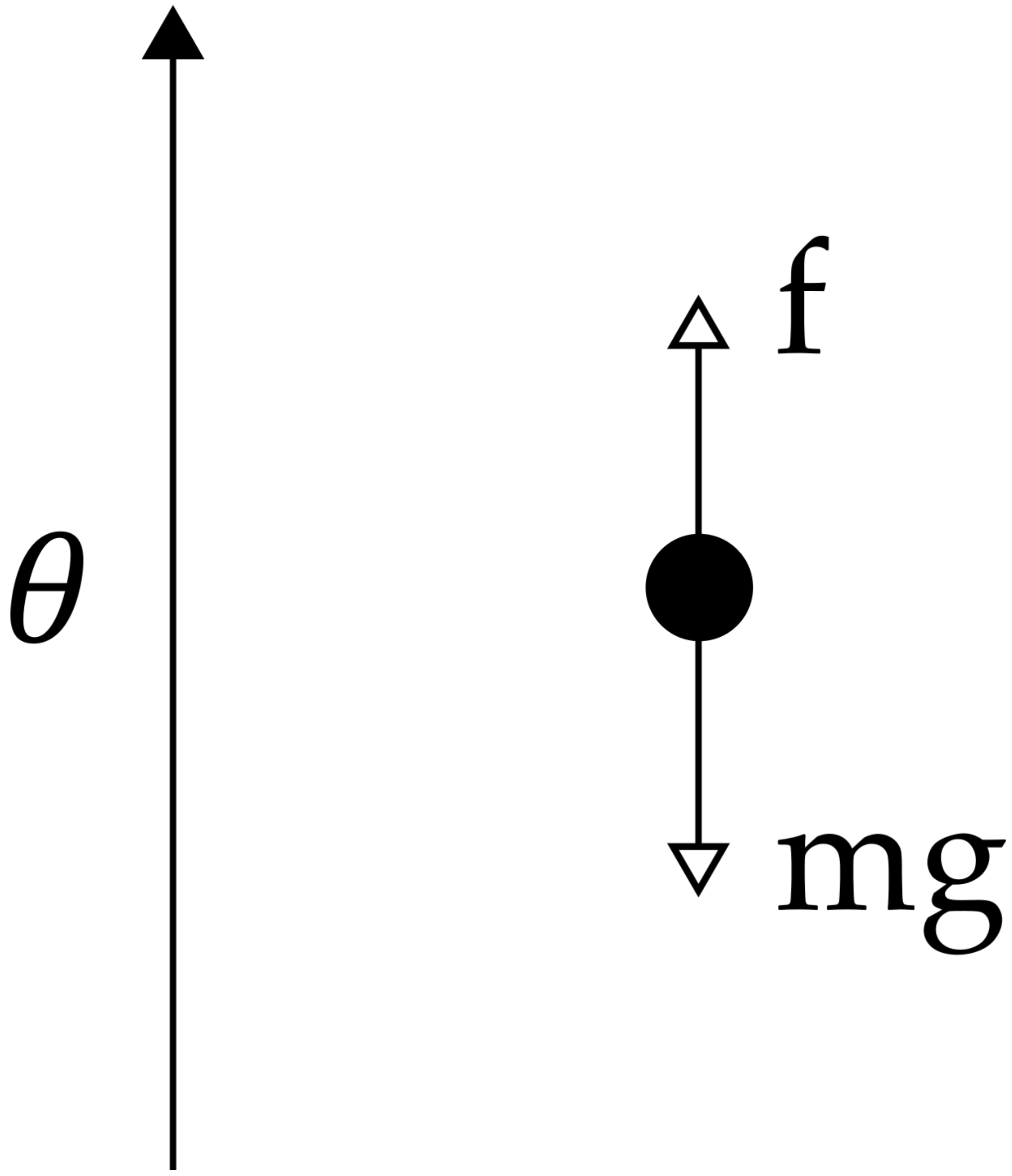


This Lecture

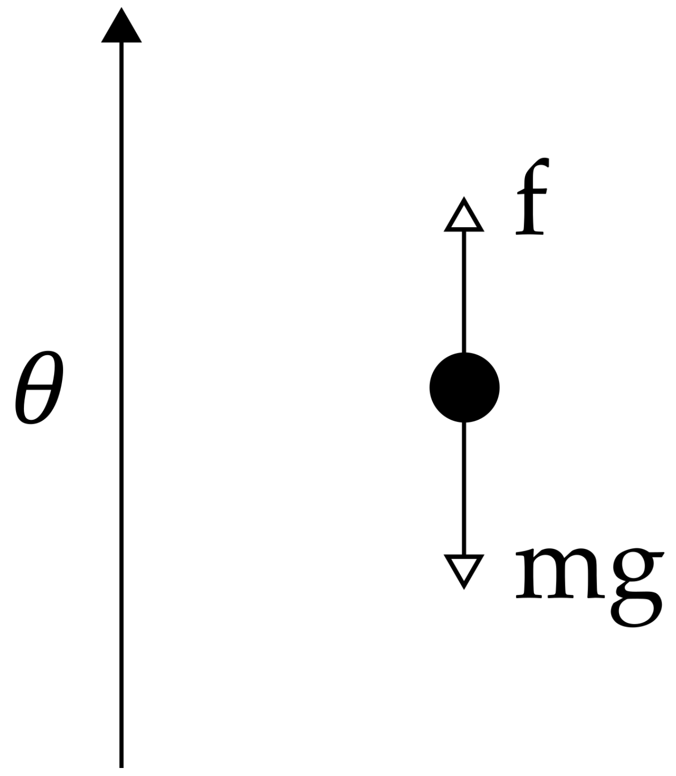
- How do we use kinetic and potential energy to get dynamics?
- What is the Euler-Lagrange equation?

Point Mass

Let’s start with a 1-DoF example



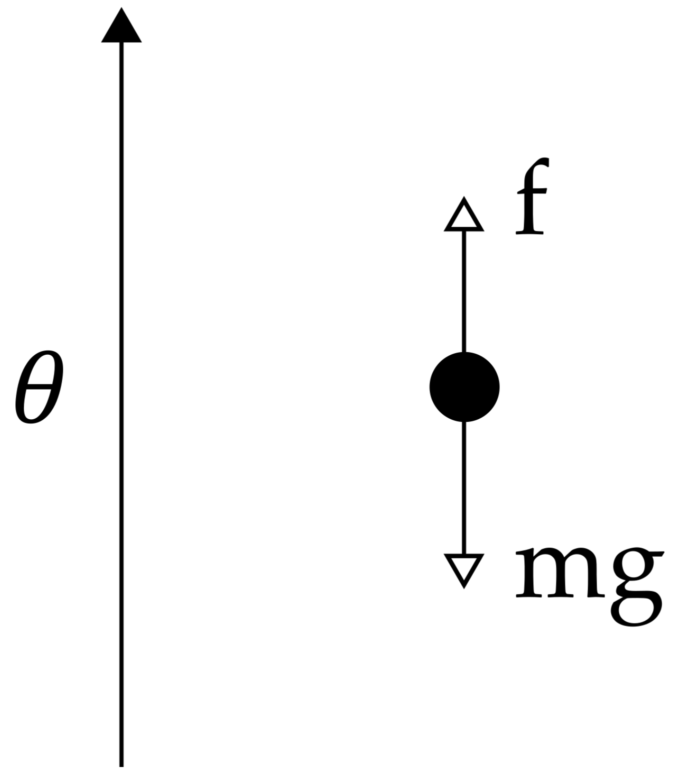
Point Mass



What is the **equation of motion** for the point mass?

- Particle with mass m
- Moves up and down with position θ
- Force f pushing up, gravity pulling down

Point Mass



What is the **equation of motion** for the point mass?

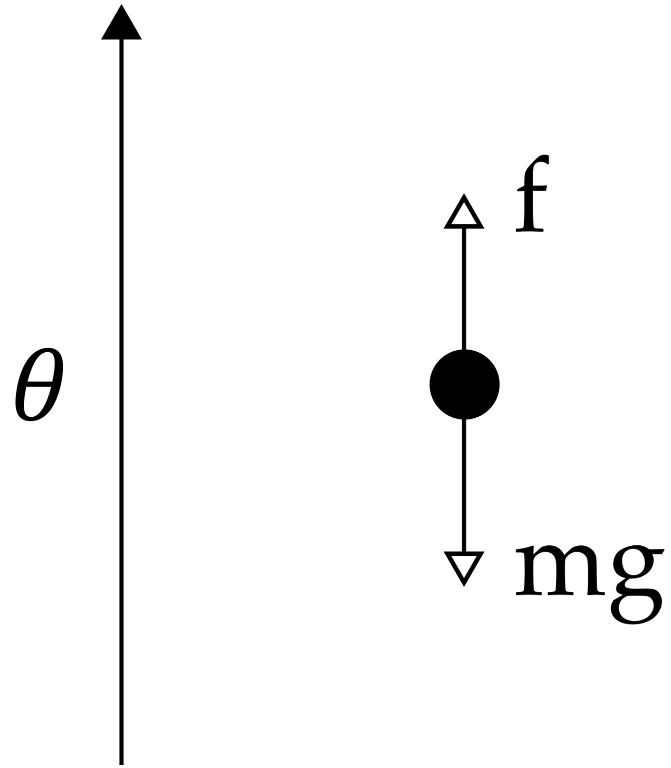
- Particle with mass m
- Moves up and down with position θ
- Force f pushing up, gravity pulling down

$$m\ddot{\theta} = f - mg$$

Newton's second law

Let's use **kinetic** and **potential energy** to get the same result

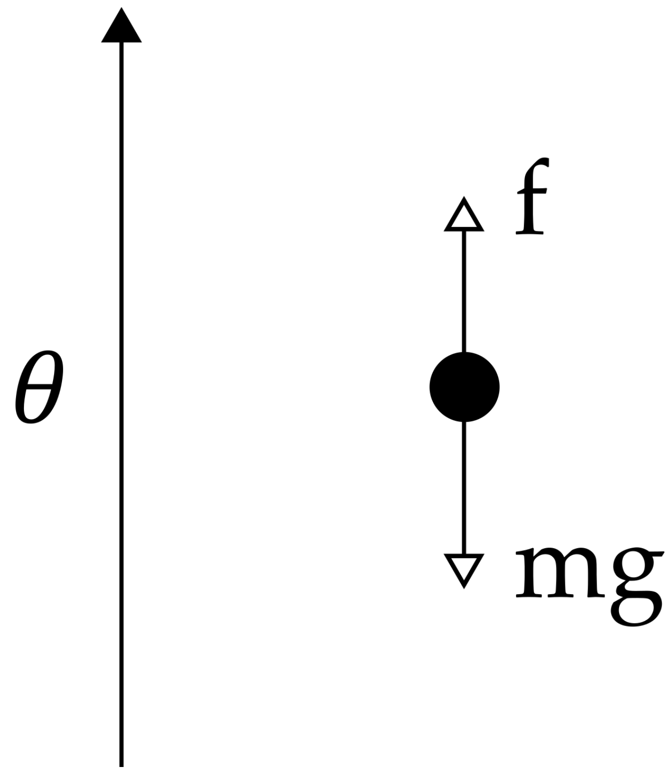
Lagrangian



Lagrangian L is the difference between kinetic and potential energy

$$L(\theta, \dot{\theta}) = \underbrace{K(\theta, \dot{\theta})}_{\textit{Kinetic energy}} - \underbrace{P(\theta)}_{\textit{Potential energy}}$$

Lagrangian



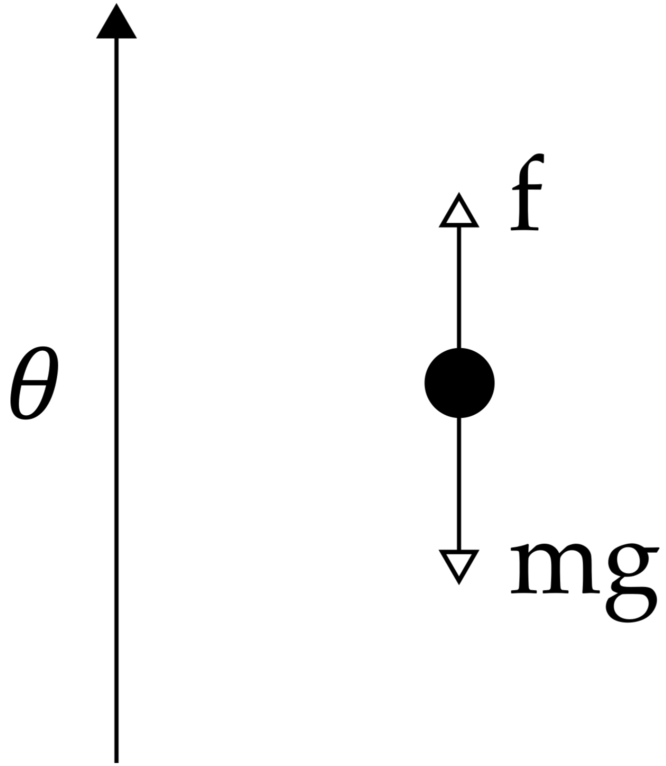
Lagrangian L is the difference between kinetic and potential energy

$$L(\theta, \dot{\theta}) = \underbrace{K(\theta, \dot{\theta})}_{\textit{Kinetic energy}} - \underbrace{P(\theta)}_{\textit{Potential energy}}$$

Kinetic energy: $K(\theta, \dot{\theta}) = \frac{1}{2} m \dot{\theta}^2$

Potential energy:$P(\theta) = mg\theta$

Lagrangian



Lagrangian L is the difference between kinetic and potential energy

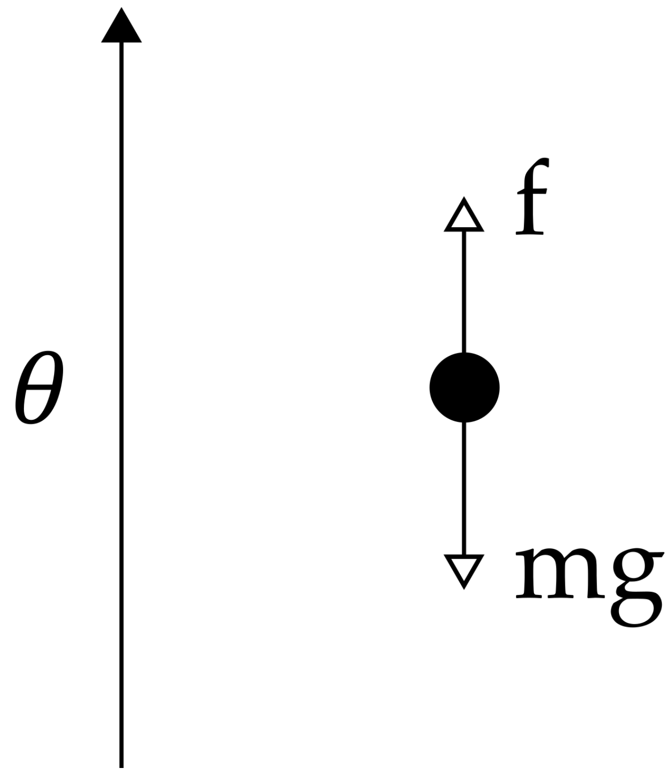
$$L(\theta, \dot{\theta}) = \underbrace{\frac{1}{2} m \dot{\theta}^2}_{\textit{Kinetic energy}} - \underbrace{mg\theta}_{\textit{Potential energy}}$$

Euler-Lagrange Equation

$$f = \frac{d}{dt}\frac{\partial L(\theta,\dot{\theta})}{\partial \dot{\theta}} - \frac{\partial L(\theta,\dot{\theta})}{\partial \theta}$$

Converts Lagrangian L to dynamic **equations of motion**

Euler-Lagrange



Apply **Euler-Lagrange Equation** to get dynamics.

$$L(\theta, \dot{\theta}) = \frac{1}{2} m\dot{\theta}^2 - mg\theta$$

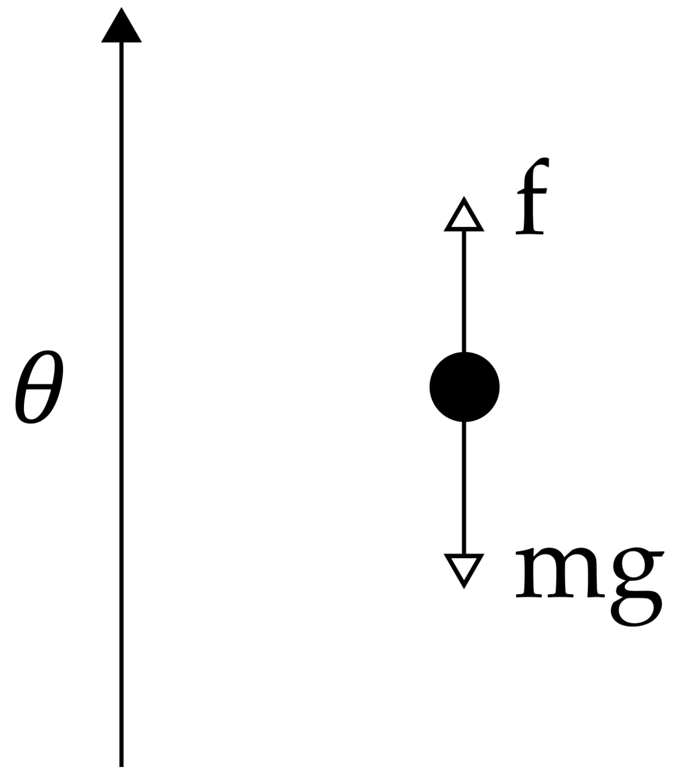
$$f = \frac{d}{dt}\frac{\partial L(\theta, \dot{\theta})}{\partial \dot{\theta}} - \frac{\partial L(\theta, \dot{\theta})}{\partial \theta}$$

1. partial derivative wrt $\dot{\theta}$

2. derivative of result wrt time t

3. partial derivative wrt θ

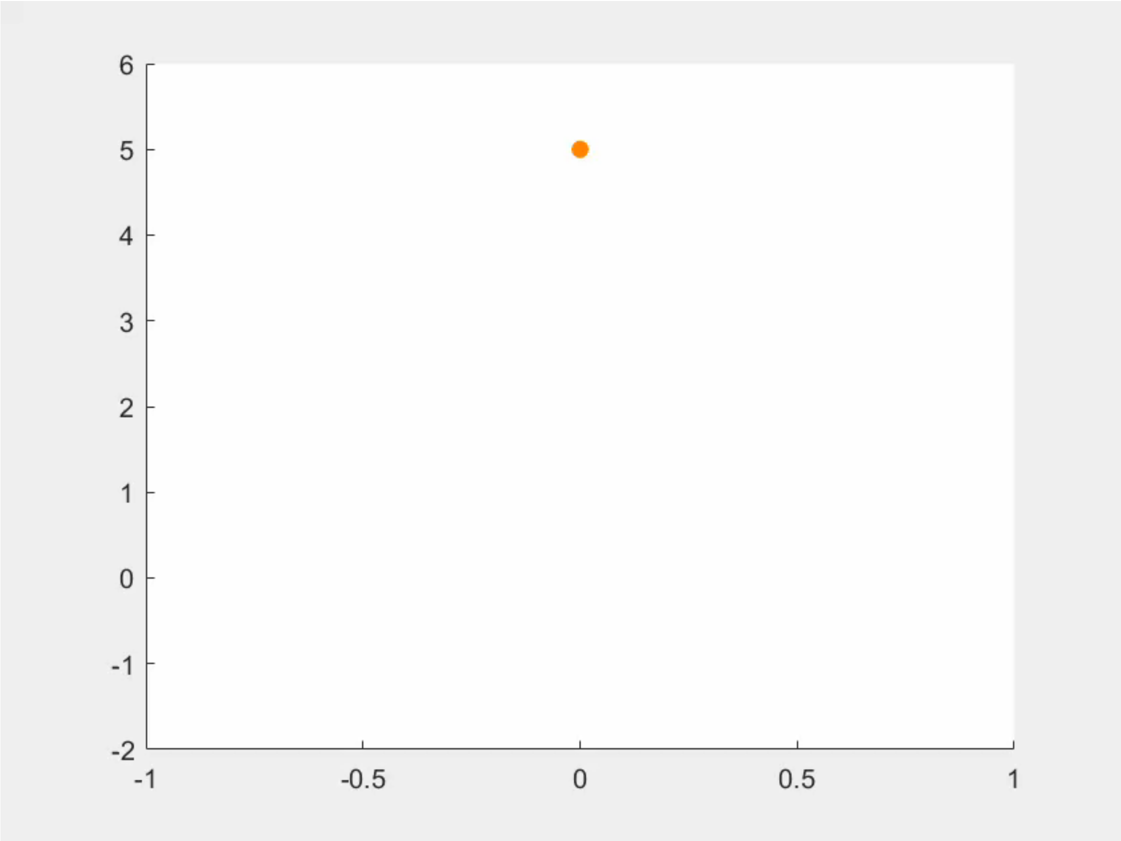
Euler-Lagrange



Apply **Euler-Lagrange Equation** to get dynamics.

$$L(\theta, \dot{\theta}) = \frac{1}{2} m\dot{\theta}^2 - mg\theta$$

$$f = m\ddot{\theta} + mg$$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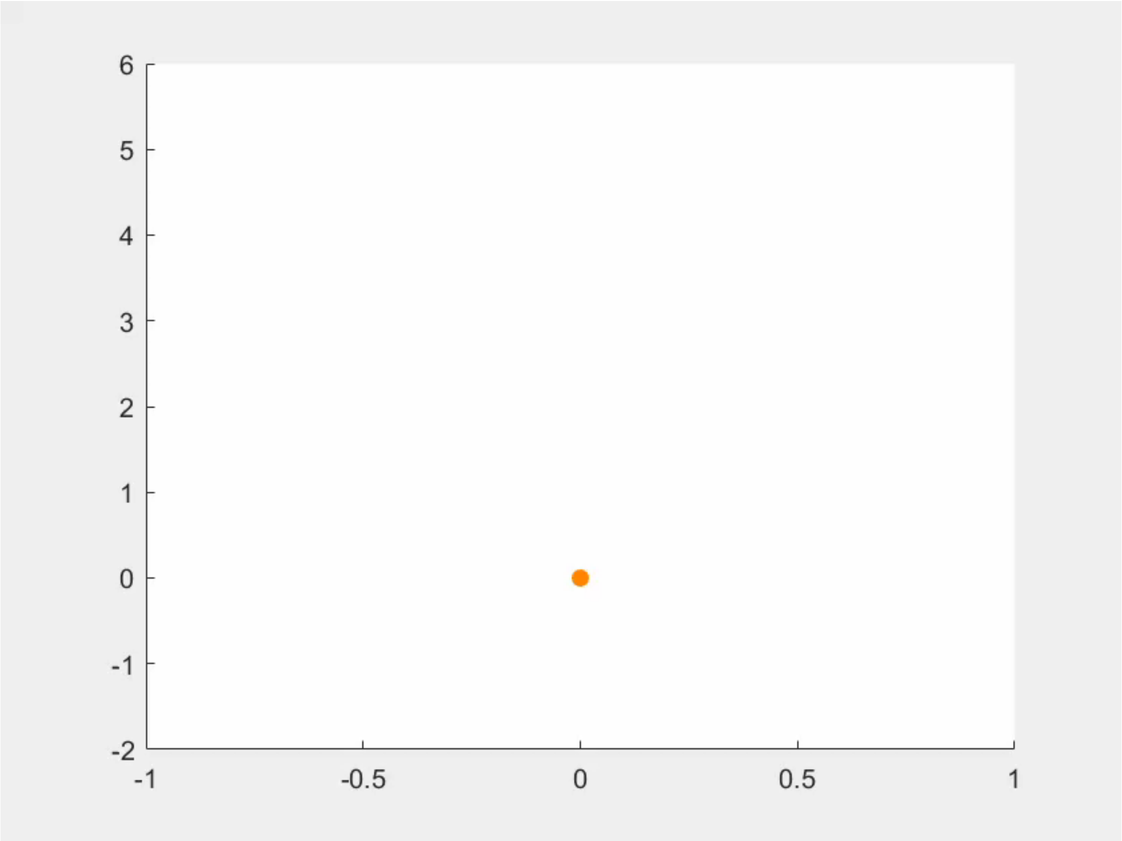
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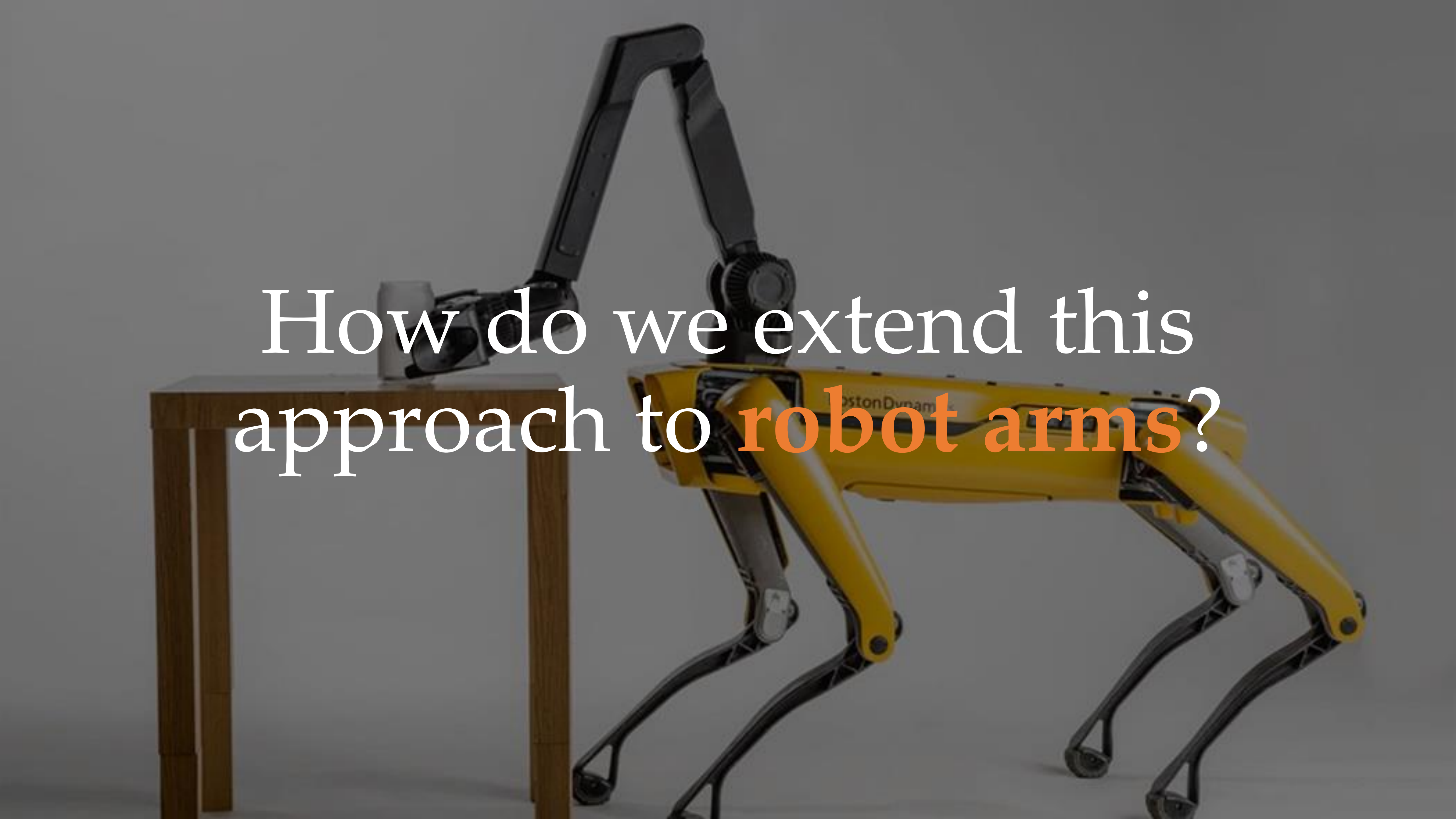

How do we extend this approach to **robot arms**?

Takeaways

$$L(\theta, \dot{\theta}) = K(\theta, \dot{\theta}) - P(\theta)$$

total kinetic (and potential) energy summed across every joint

$$\tau_i = \frac{d}{dt}\frac{\partial L(\theta, \dot{\theta})}{\partial \dot{\theta}_i} - \frac{\partial L(\theta, \dot{\theta})}{\partial \theta_i}$$

torque at i-th joint velocity of i-th joint position of i-th joint

This Lecture

- How do we use kinetic and potential energy to get dynamics?
- What is the Euler-Lagrange equation?

Next Lecture

- How do we find the kinetic and potential energy for a robot arm?