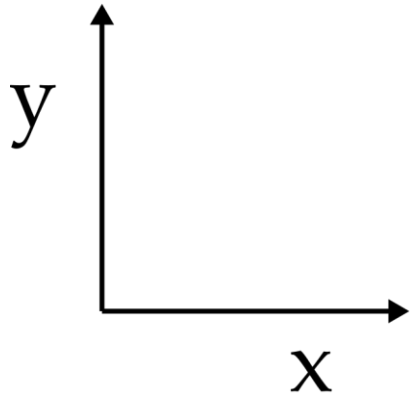
# Degrees of Freedom

Reading: Modern Robotics 2.2

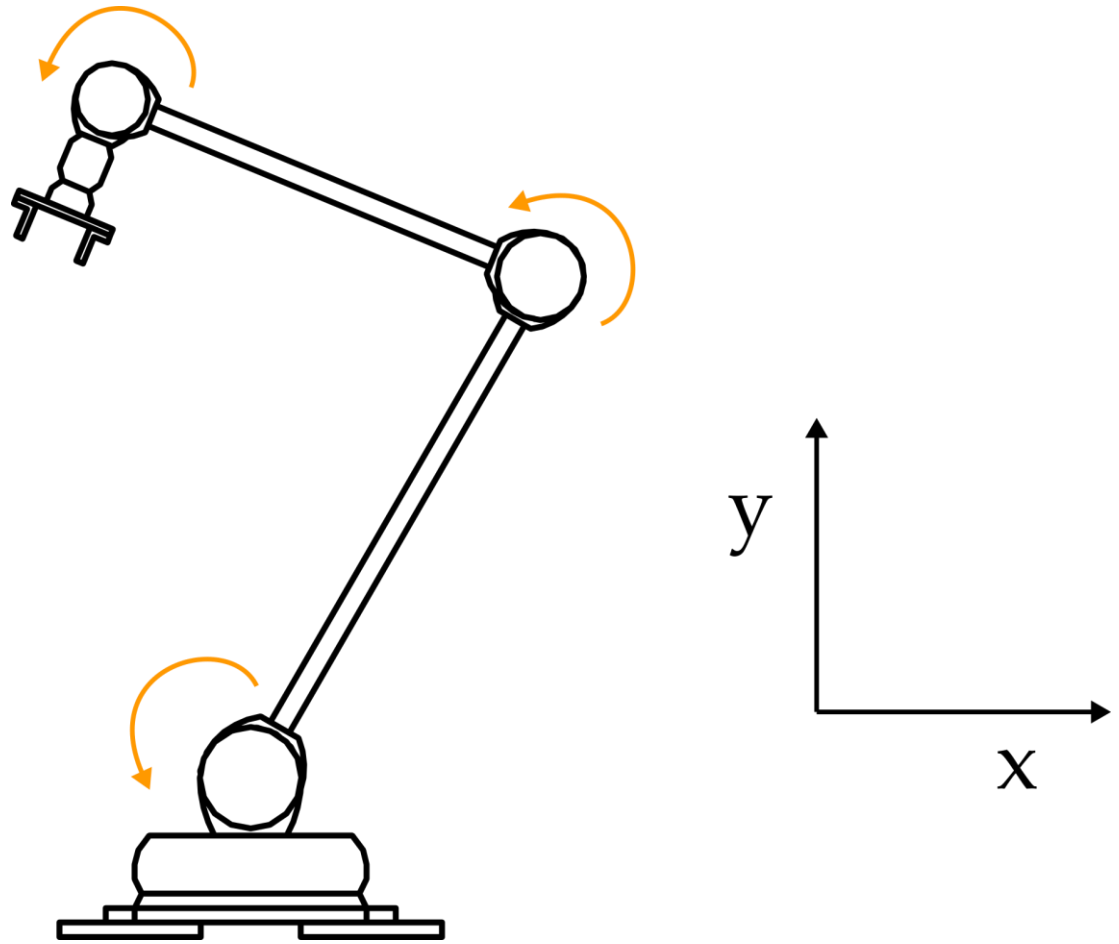
# This Lecture

- What are degrees of freedom?
- How many degrees of freedom does a robot have?

How many **coordinates** do we need to specify car's configuration?




How many **coordinates** do we need to specify robot's configuration?



# Degrees of Freedom

- Minimum number of **coordinates** needed to capture the robot's configuration
- If we need $n$ coordinates, robot has $n$ degrees of freedom

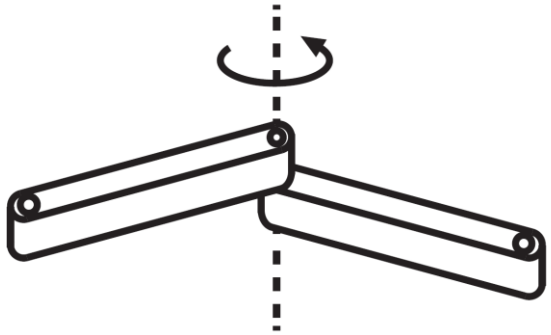

How many degrees of freedom does a robot have?

# Grubler's Formula

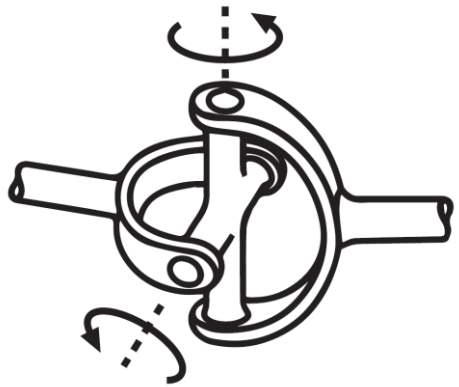
$$\boldsymbol{m(N-1-K)+\sum_{i=1}^{K} f_i}$$

- $N$ is the number of links
- $K$ is the number of joints
- $m = 3$ for robot on 2D plane
- $m = 6$ for robot in 3D space
- $f_i$ is DoFs for the $i$-th joint

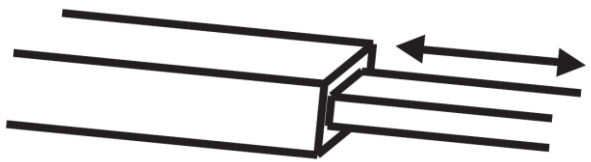
# Grubler's Formula



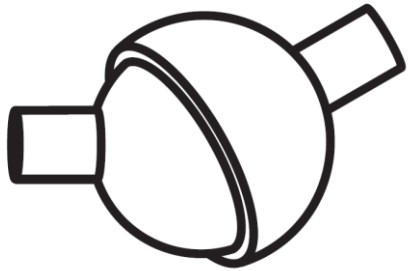
**Revolute.**
1 DoF



**Universal.**
2 DoF



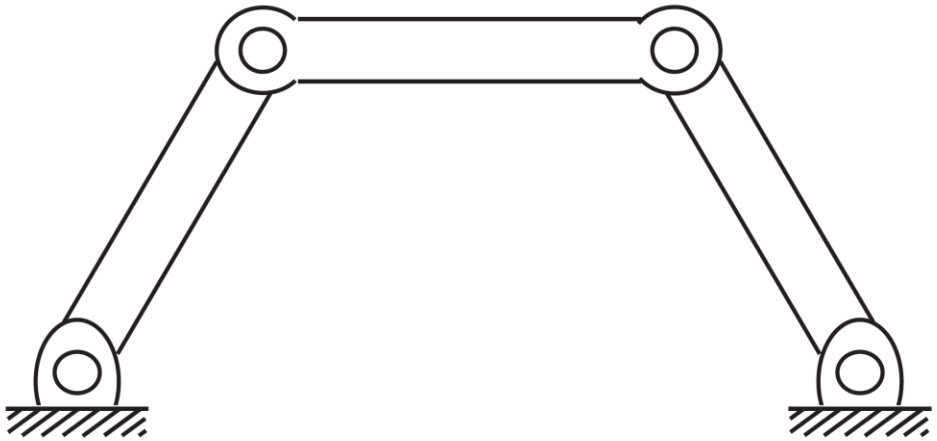
**Prismatic.**
1 DoF



**Spherical.**
3 DoF

# Practice

- $N$ is the number of links
- $K$ is the number of joints
- $m = 3$ for robot on 2D plane
- $m = 6$ for robot in 3D space
- $f_i$ is DoFs for the $i$-th joint



$$\boldsymbol{m(N - 1 - K) + \sum_{i=1}^{K} f_i}$$

# Practice

- $N$ is the number of links
- $K$ is the number of joints
- $m = 3$ for robot on 2D plane
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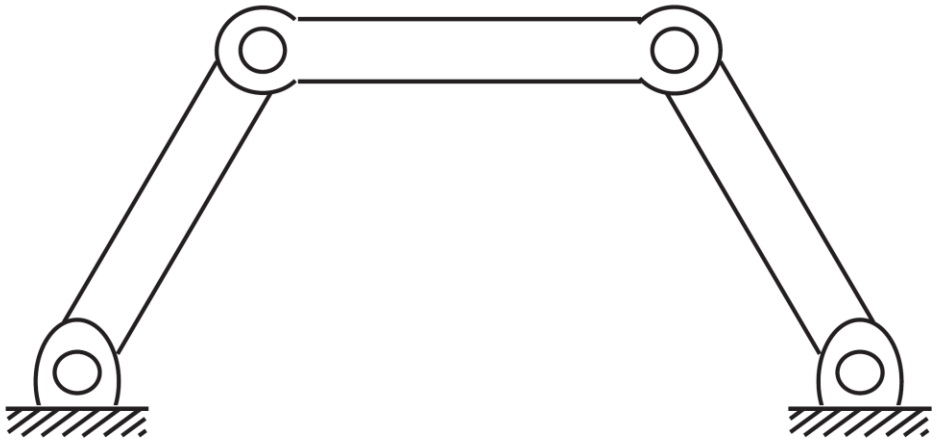
- $f_i$ is DoFs for the $i$-th joint



$$\mathbf{3(4 - 1 - 4) + 4 = 1}$$

Remember to count the ground as a link

$$\boldsymbol{m(N - 1 - K) + \sum_{i=1}^{K} f_i}$$

# Practice

- $N$ is the number of links
- $K$ is the number of joints
- $m = 3$ for robot on 2D plane
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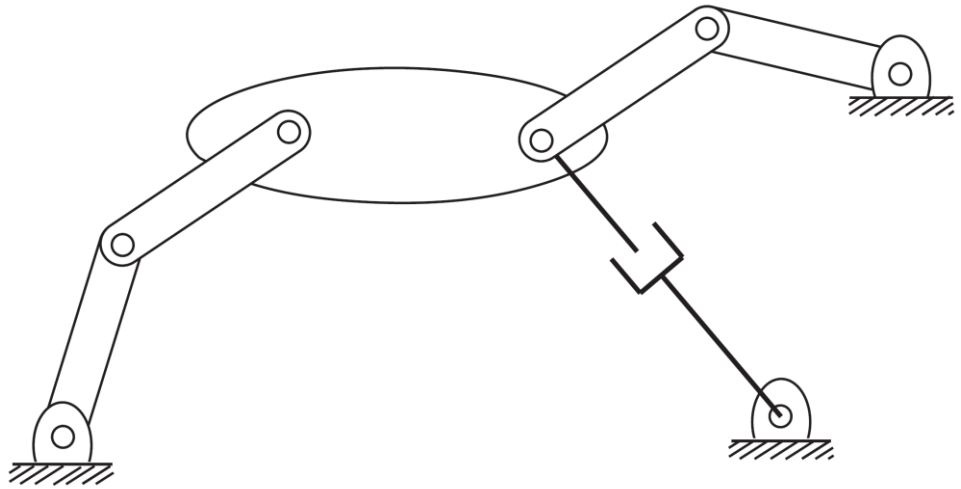
- $f_i$ is DoFs for the $i$-th joint



$$\boldsymbol{m(N - 1 - K) + \sum_{i=1}^{K} f_i}$$

# Practice

- $N$ is the number of links
- $K$ is the number of joints
- $m = 3$ for robot on 2D plane
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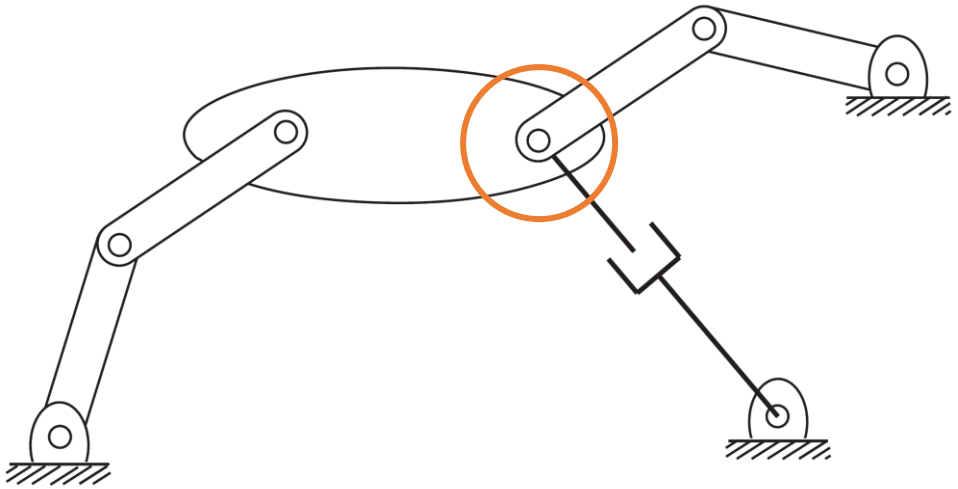
- $f_i$ is DoFs for the $i$-th joint



Hint: joints connect two links, so this is **two** overlapping revolute joints

# Practice

- $N$ is the number of links
- $K$ is the number of joints
- $m = 3$ for robot on 2D plane
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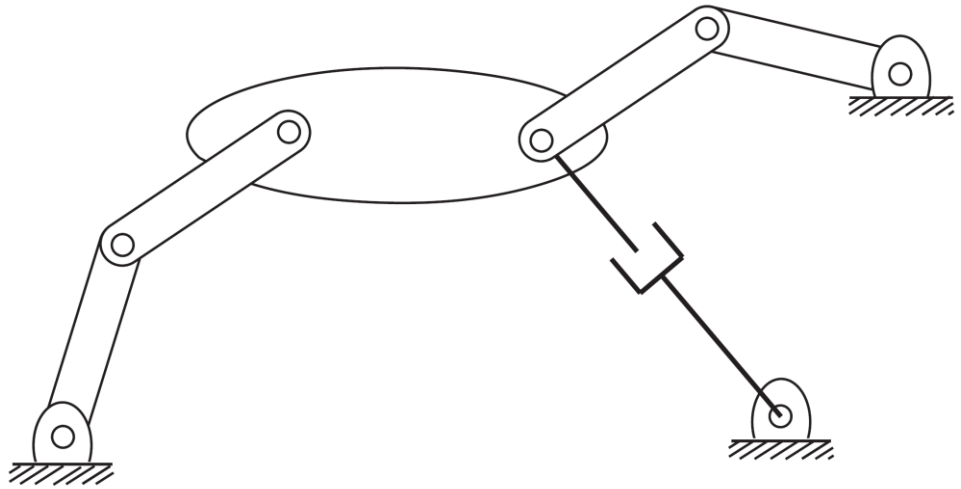
- $f_i$ is DoFs for the $i$-th joint



$$\mathbf{3(8 - 1 - 9) + 9 = 3}$$

$$\boldsymbol{m(N - 1 - K) + \sum_{i=1}^{K} f_i}$$

# This Lecture

- What are degrees of freedom?
- How many degrees of freedom does a robot have?

# Next Lecture

- How do we capture position and rotation?