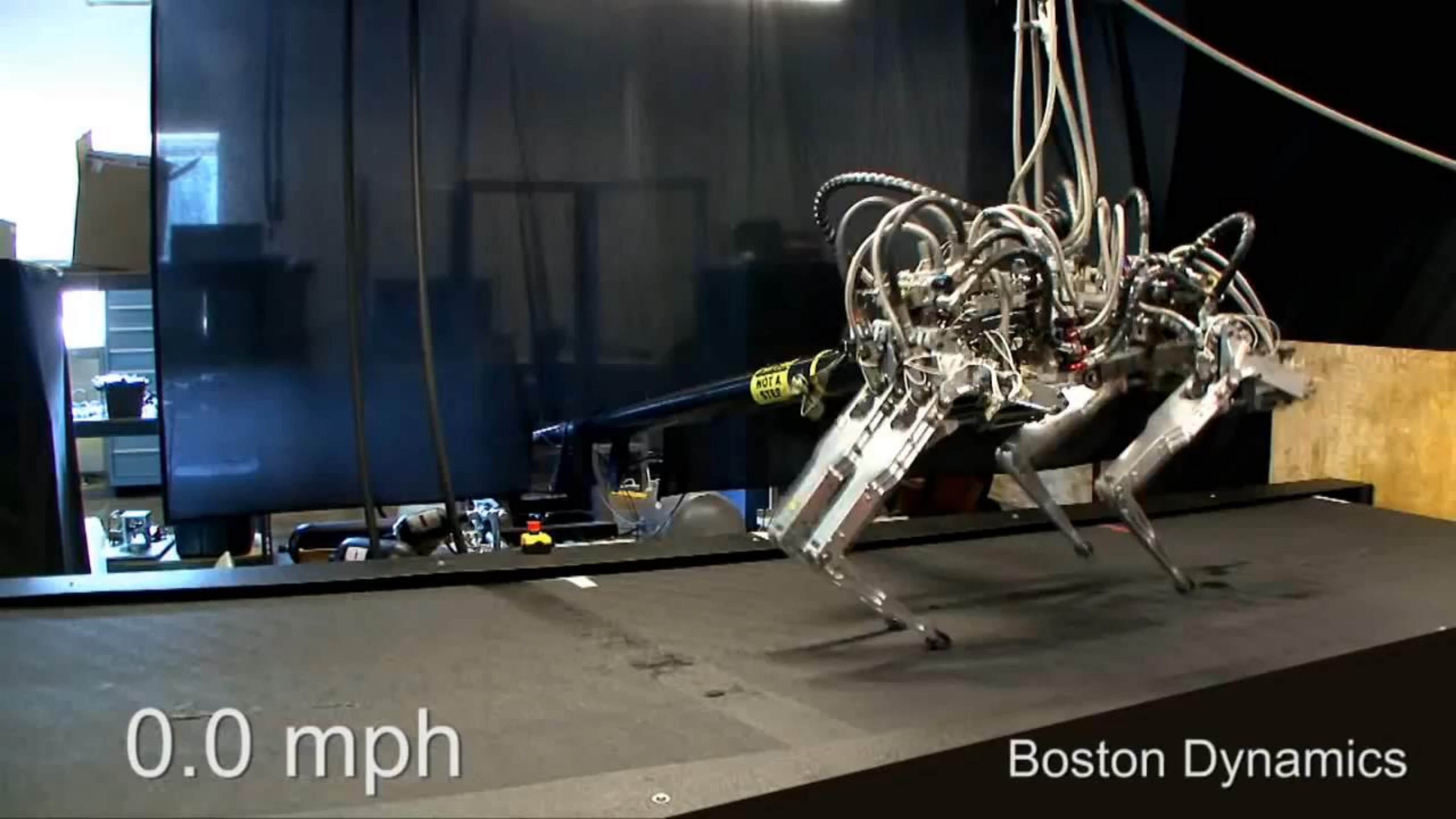
Introducing the Jacobian

Reading: Modern Robotics 5.0 + 3.3.2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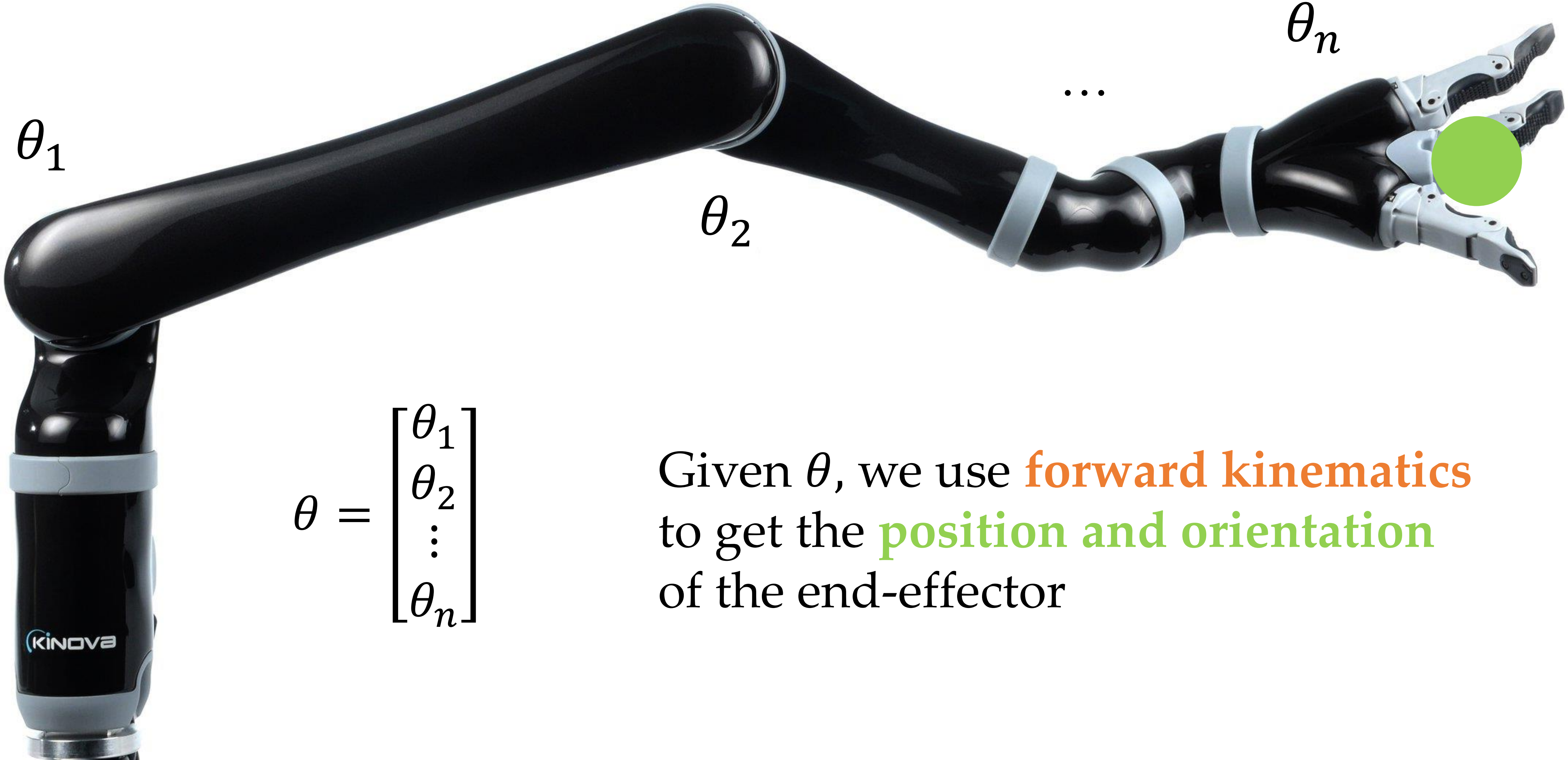
Boston Dynamics

This Lecture

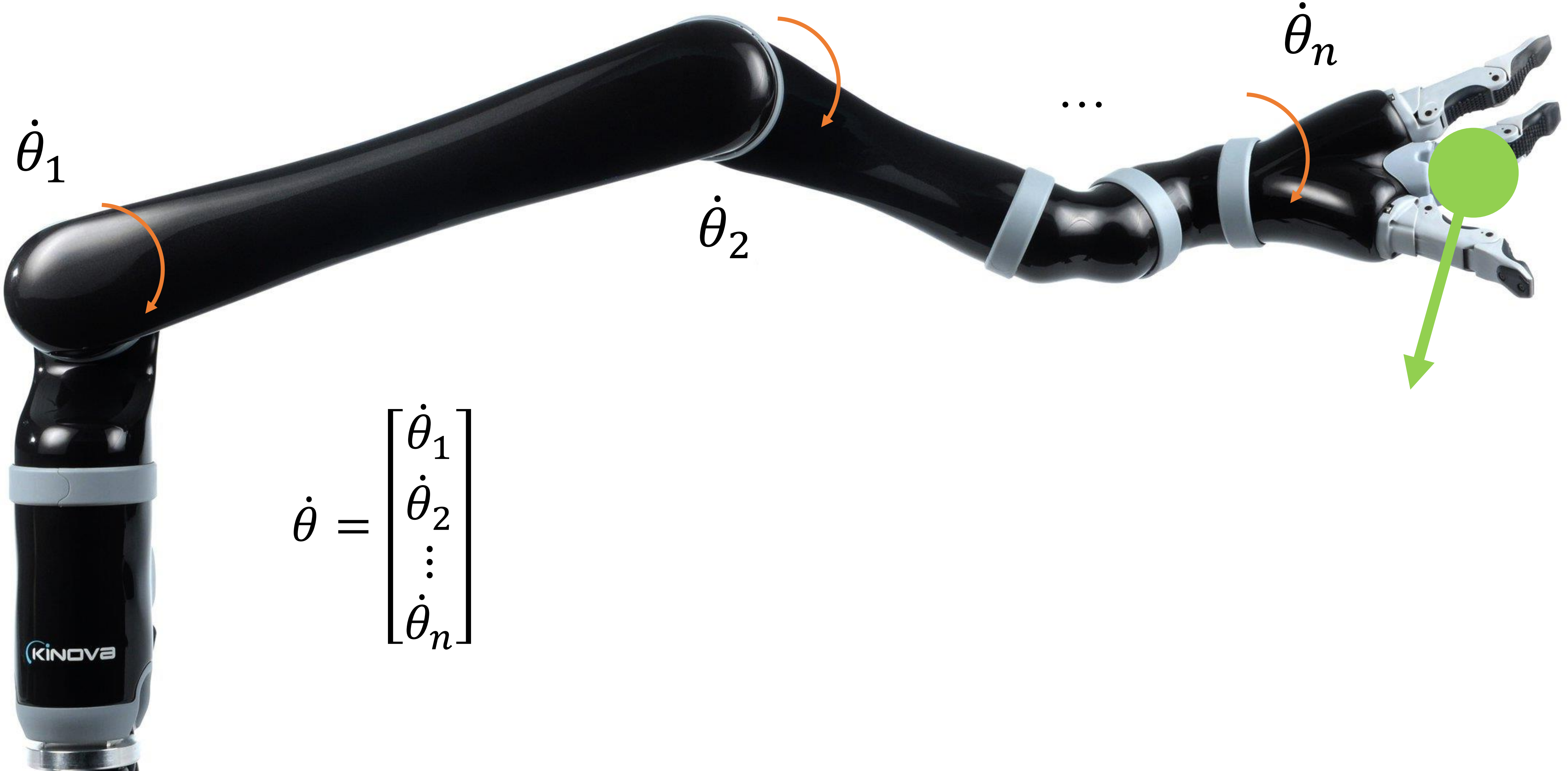
- What is the velocity of our robot arm?
- What is a robot Jacobian?
- How do we relate velocities in different frames?


KINOVA

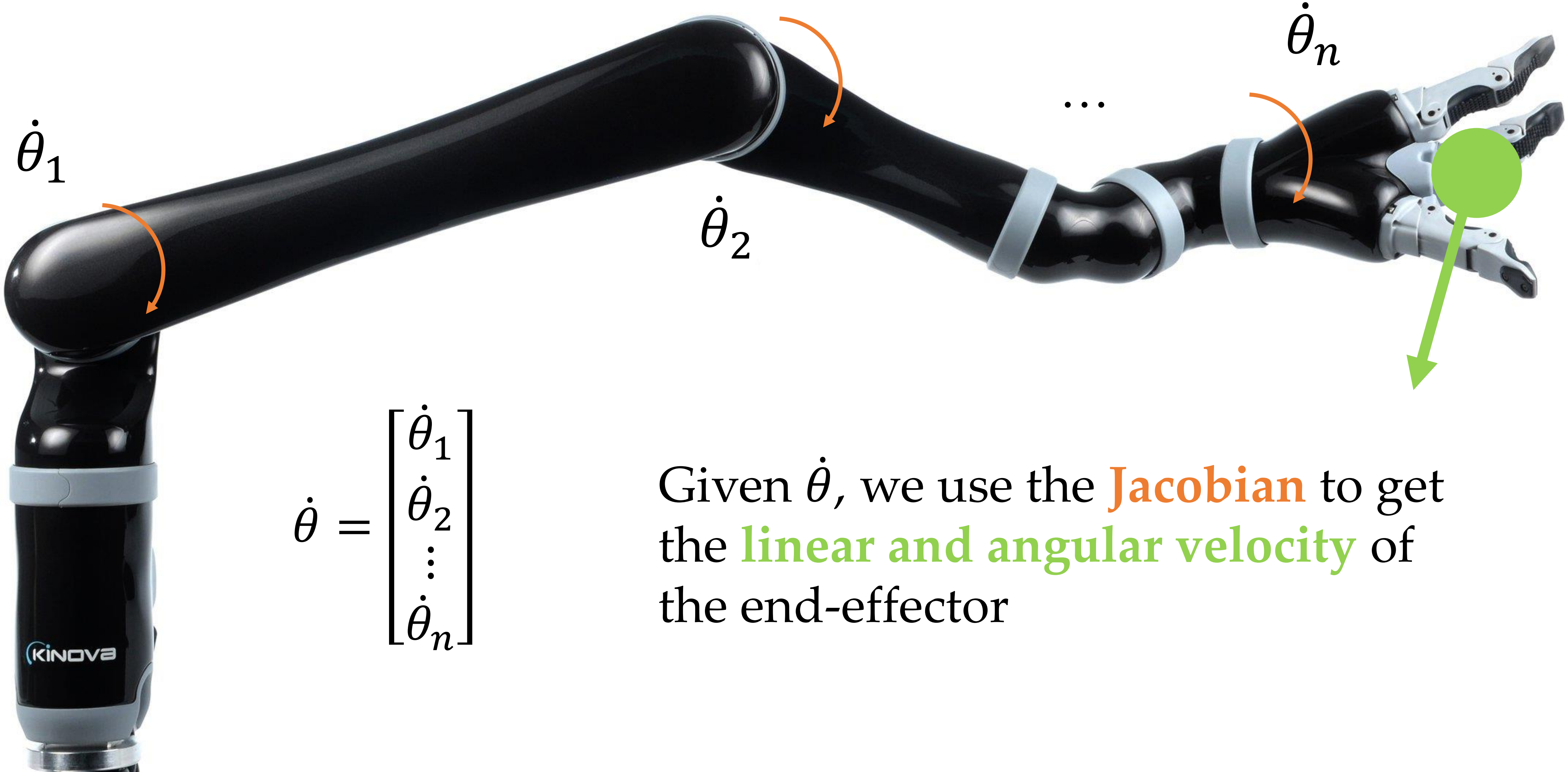


$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

Given θ, we use **forward kinematics** to get the **position and orientation** of the end-effector



$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$



$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

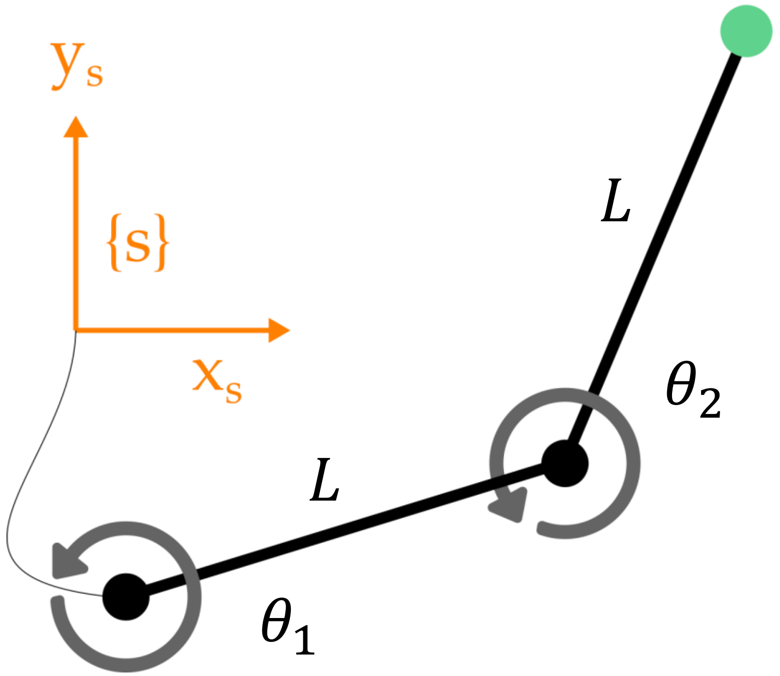
Given $\dot{\theta}$, we use the **Jacobian** to get the **linear and angular velocity** of the end-effector

What is the **Jacobian**?

Example

Here the forward kinematics are:

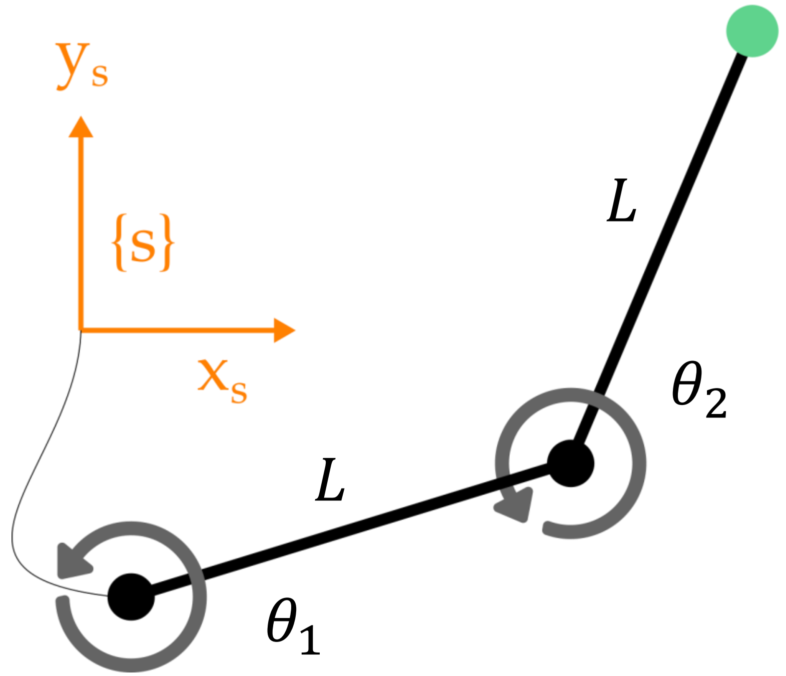
$$p(t) = \underbrace{\begin{bmatrix} L\cos\theta_1(t) + L\cos\big(\theta_1(t) + \theta_2(t)\big) \\ L\sin\theta_1(t) + L\sin\big(\theta_1(t) + \theta_2(t)\big) \end{bmatrix}}_{\text{position of end-effector in } \{s\}} = f\big(\theta(t)\big)$$



Example

The velocity of the end-effector is:

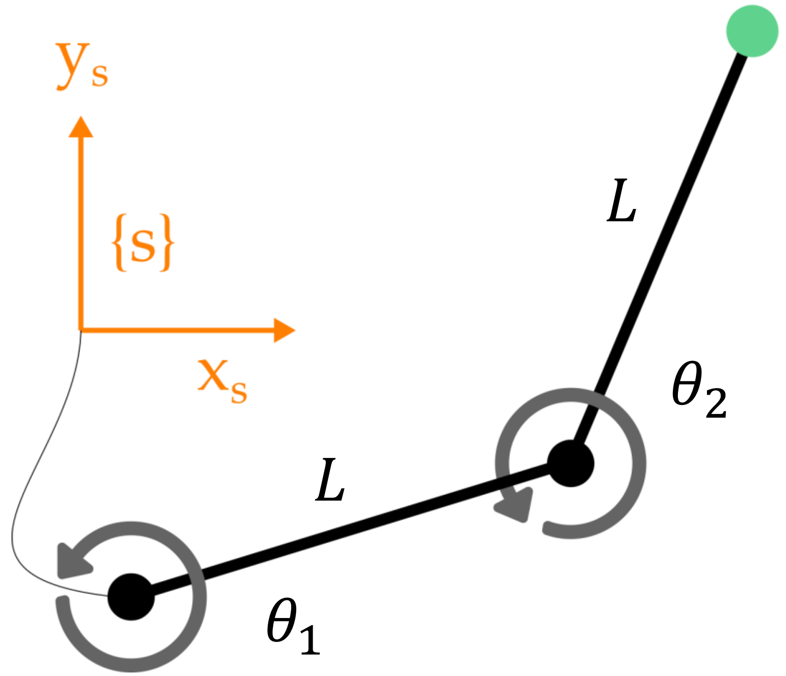
$$\dot{p}(t) = \frac{df}{dt} = \frac{\partial f(\theta)}{\partial \theta}\frac{d\theta(t)}{dt}$$



Example

The velocity of the end-effector is:

$$\dot{p} = \frac{\partial \boldsymbol{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \dot{\theta}$$



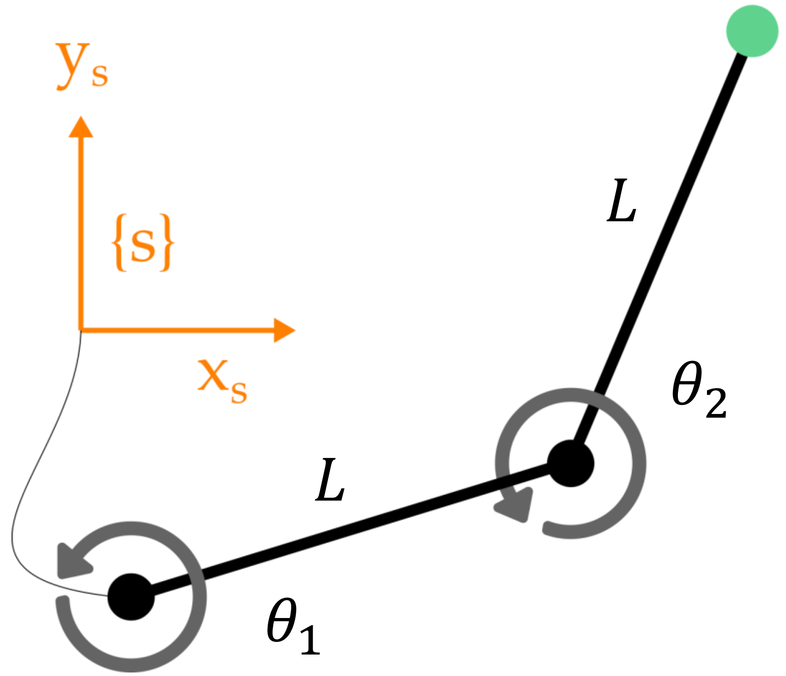
Example

The velocity of the end-effector is:

$$\dot{p} = \frac{\partial \boldsymbol{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \dot{\theta}$$

$$\boxed{\dot{p} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}}$$

We refer to $J(\theta)$ as the **Jacobian**

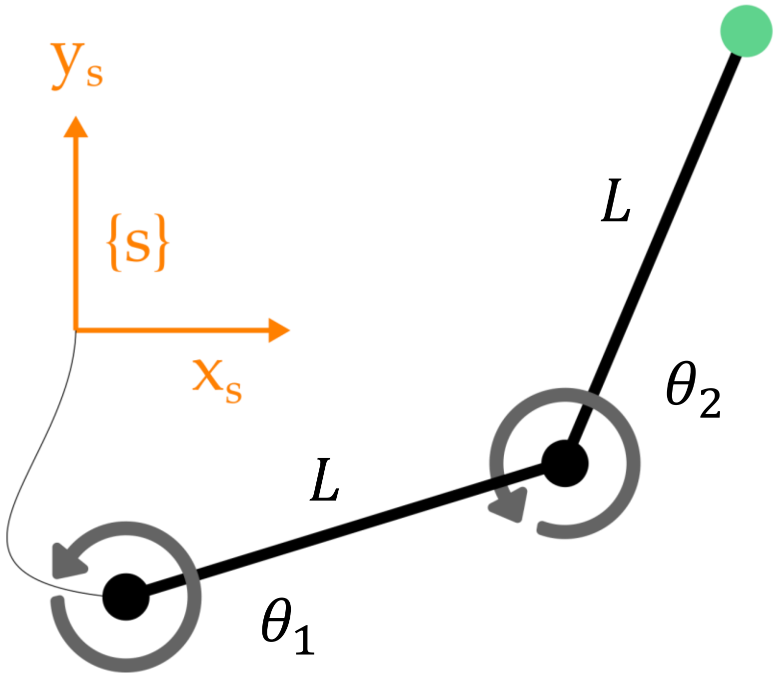


Example

The velocity of the end-effector is:

$$\dot{p} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}$$

$$p(t) = \begin{bmatrix} L\cos\theta_1(t) + L\cos\big(\theta_1(t) + \theta_2(t)\big) \\ L\sin\theta_1(t) + L\sin\big(\theta_1(t) + \theta_2(t)\big) \end{bmatrix}$$

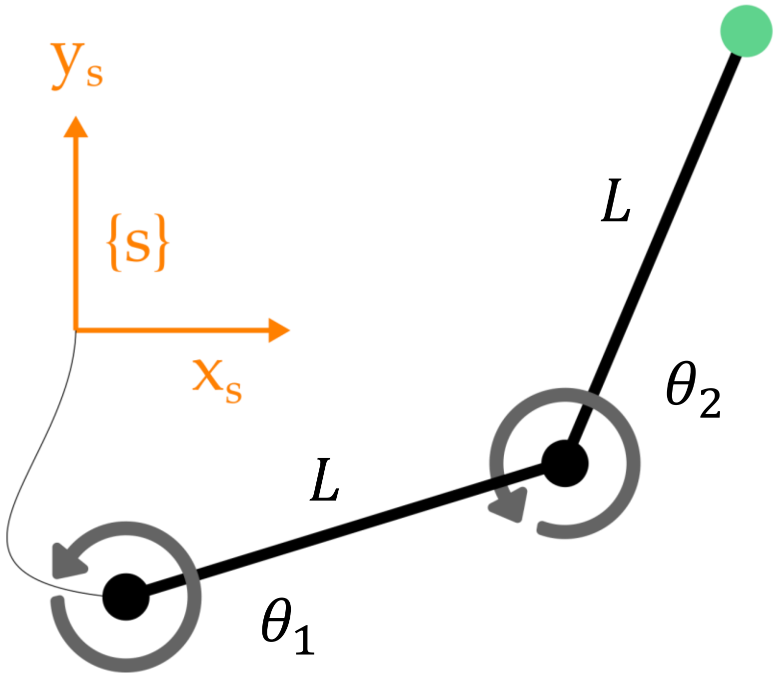


Example

The velocity of the end-effector is:

$$\dot{p} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}$$

$$\dot{p} = \begin{bmatrix} -L\dot{\theta}_1 \sin\theta_1 - L(\dot{\theta}_1 + \dot{\theta}_2)\sin(\theta_1 + \theta_2) \\ L\dot{\theta}_1 \cos\theta_1 + L(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2) \end{bmatrix}$$



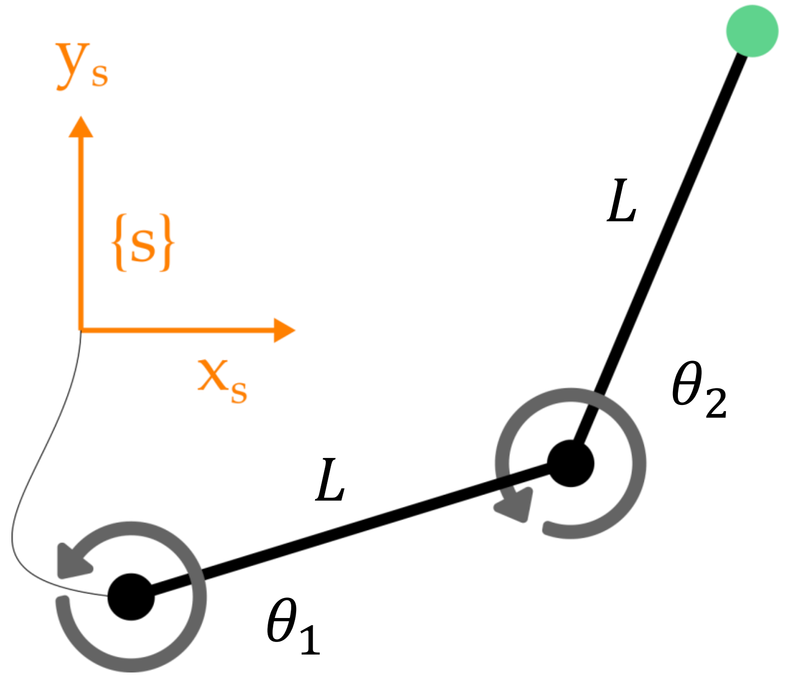
Example

The velocity of the end-effector is:

$$\dot{p} = \boldsymbol{J}(\boldsymbol{\theta})\dot{\theta}$$

$$\dot{p} = \underbrace{\begin{bmatrix} -L\sin\theta_1 - L\sin(\theta_1+\theta_2) & -L\sin(\theta_1+\theta_2) \\ L\cos\theta_1 + L\cos(\theta_1+\theta_2) & L\cos(\theta_1+\theta_2) \end{bmatrix}}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

this 2×2 matrix is this robot's **Jacobian**



The **Jacobian** is a matrix that maps **joint velocity** to **end-effector** velocity

Adjoint

To find the Jacobian, we need to a tool for **relating velocities** in different frames

$$V_s = \begin{bmatrix} \quad \quad \end{bmatrix} V_b$$

Given body twist, how do we get spatial twist (and vice versa)?

Adjoint

To find the Jacobian, we need to a tool for **relating velocities** in different frames

$$\begin{bmatrix} \omega_s \\ v_s \end{bmatrix} = \underline{\begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}} \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}$$

This 6×6 matrix is the **adjoint.**
Obtained by rearranging the
equations for V_s and V_b

Adjoint

Definition. For transformation matrix T the adjoint is:

$$\mathrm{Ad}_T = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}$$

$$(\mathrm{Ad}_T)^{-1} = \mathrm{Ad}_{T^{-1}}$$

Useful property of adjoints

Adjoint

Application. For transformation matrix T_{sb} we have that:

$$\mathcal{V}_s = \mathbf{Ad}_{\boldsymbol{T_{sb}}} \mathcal{V}_b$$

$$\mathcal{V}_b = \mathbf{Ad}_{\boldsymbol{T_{sb}^{-1}}} \mathcal{V}_s$$

This Lecture

- What is the velocity of our robot arm?
- What is a robot Jacobian?
- How do we relate velocities in different frames?

Next Lecture

- How do we get the Jacobian of a robot arm?