Singularities

Reading: Modern Robotics 5.3 – 5.4


19:13:31 05/06/2015 UTC

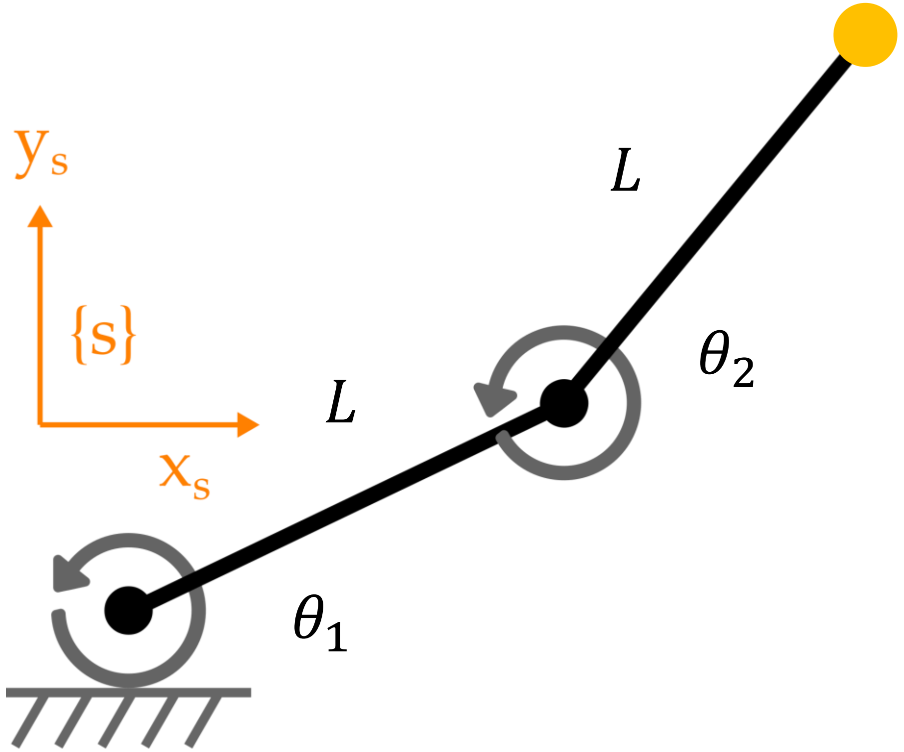
This Lecture

- What is a singularity?
- What directions can robots move in singularities?
- When are we close to or far from singularities?

Here the geometric **Jacobian** is:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

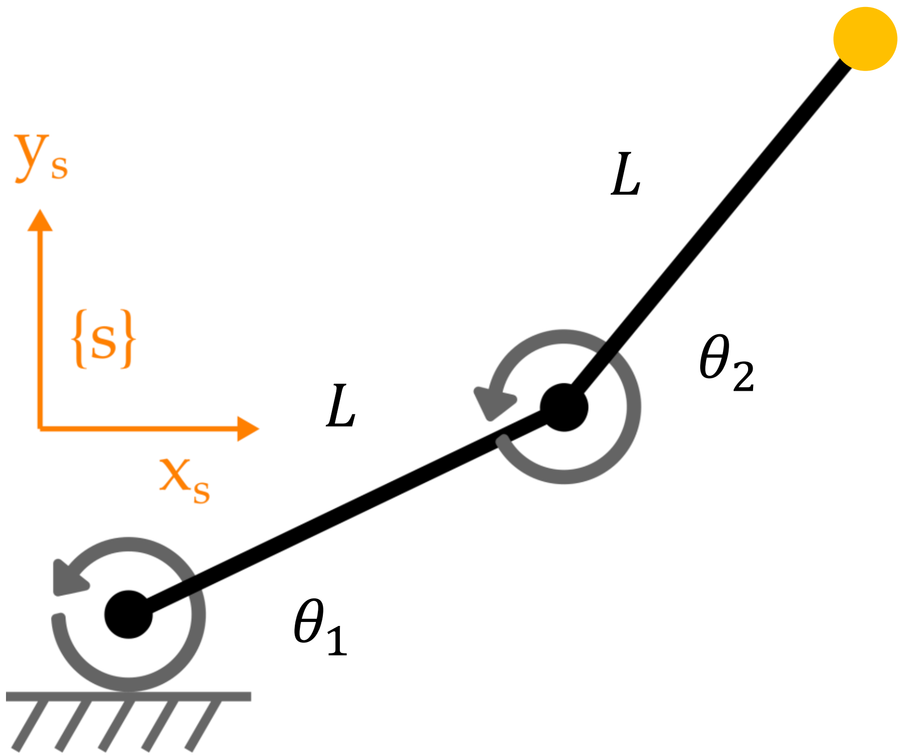
The robot's end-effector is moving in the $x - y$ plane. This is the *task space* for this 2-DoF robot.



Here the geometric **Jacobian** is:

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Let's restrict the Jacobian to rows that contribute motion in x and y (i.e., the robot's *task space*)



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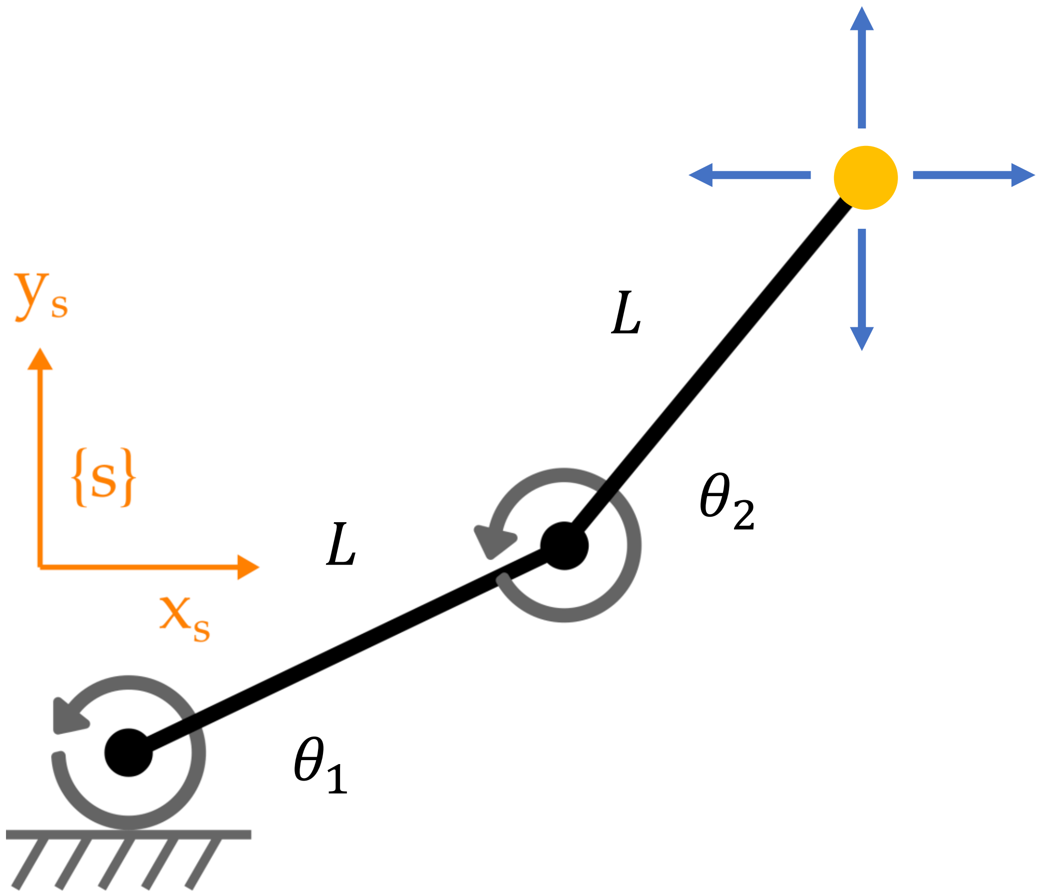
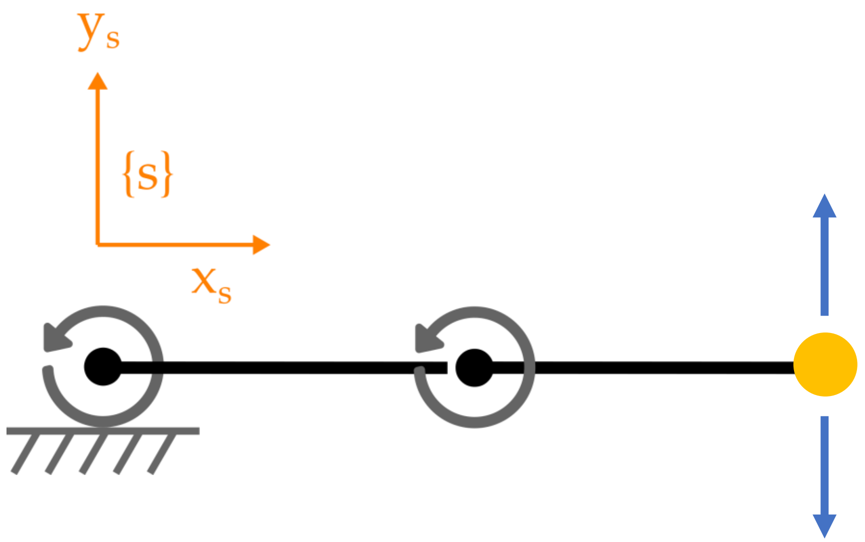
We want to be able to **move** the end-effector **in all directions** of our task space

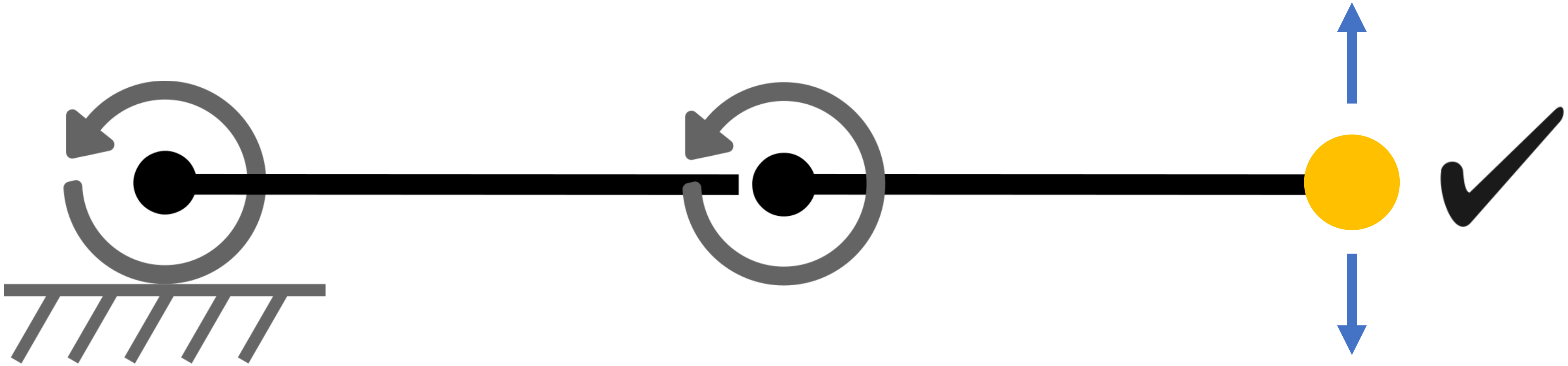


Here the geometric **Jacobian** is:

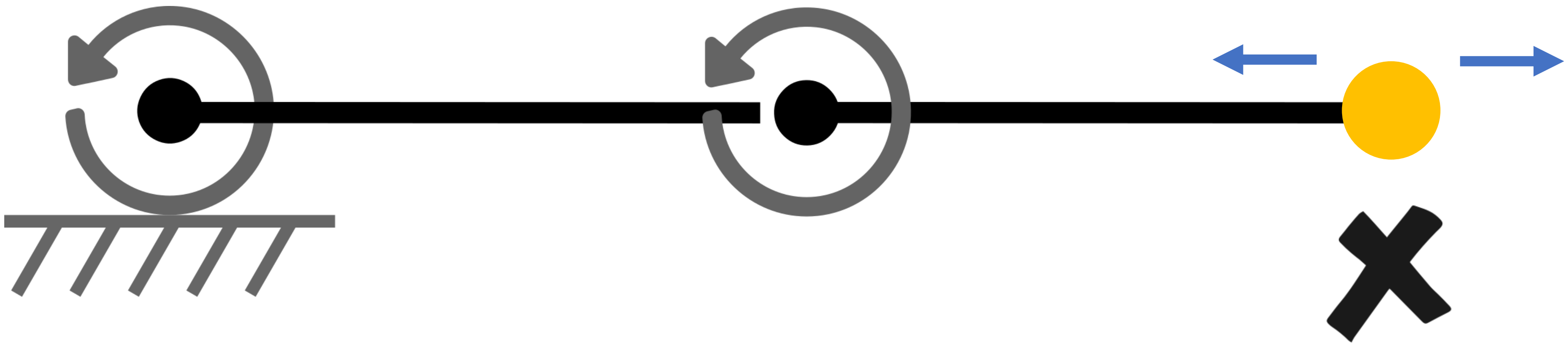
$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2L & L \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ 2\dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}}$$



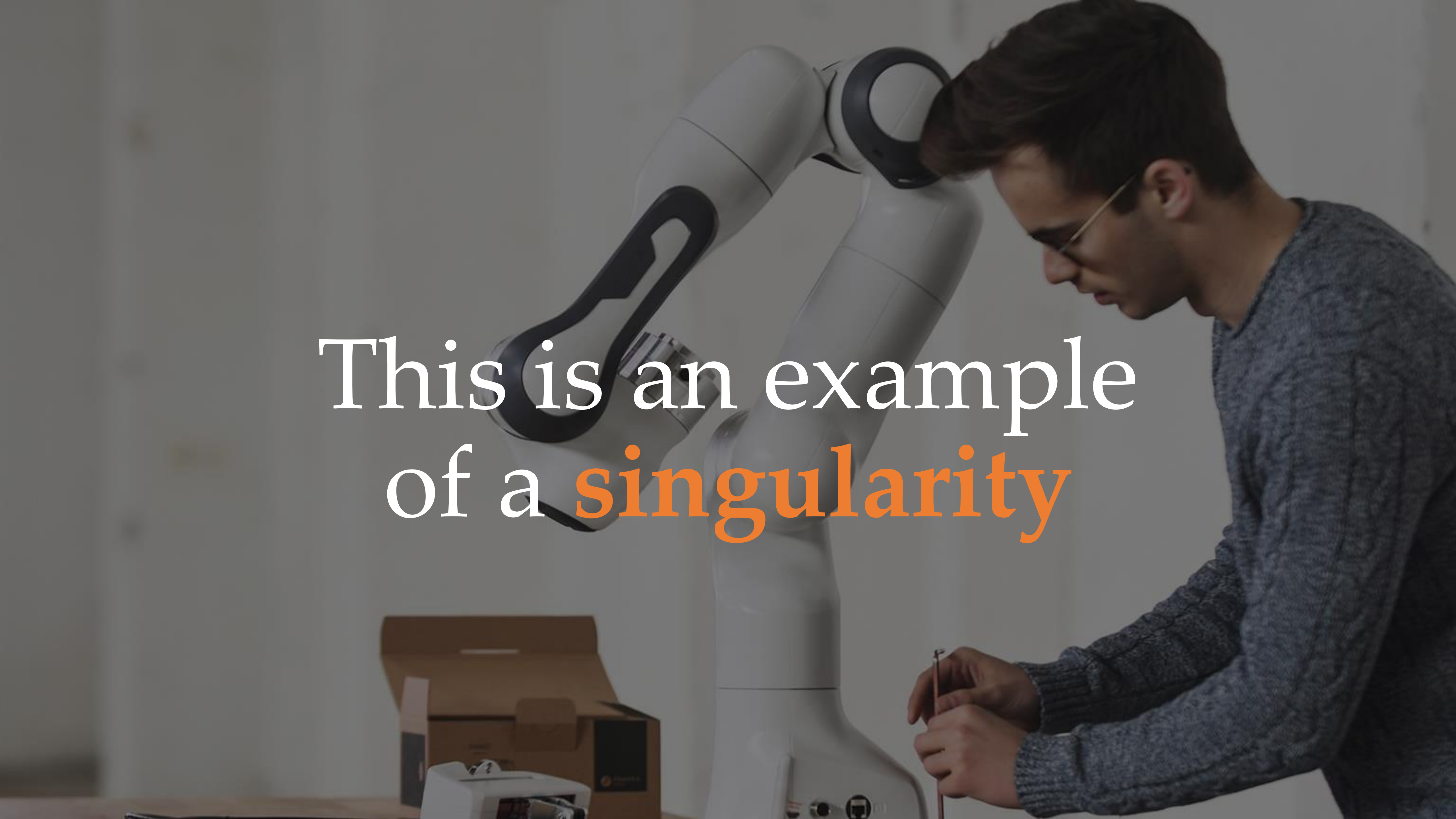
But when $\theta = 0$, we **can't** ***instantaneously*** **move** in x



When the arm is fully extended,
we easily move *up-down*…



…but can’t immediately move *left-right*



This is an example of a **singularity**

Singularity

A **singularity** is a joint position θ where the robot looses the ability to instantaneously move or rotate in one or more directions.

- Let n be the dimensions of the robot's task space.
- Let J be Jacobian, and let $\boldsymbol{J}$ be the rows of the Jacobian associated with the task space
- Robot is in a singularity if the rank of $\boldsymbol{J}(\boldsymbol{\theta})$ drops below n

Singularity

Singularities **do not depend** on the type of Jacobian:

$$\operatorname{rank} \boldsymbol{J_s}(\theta) = \operatorname{rank} \boldsymbol{J_b}(\theta) = \operatorname{rank} \boldsymbol{J}(\theta)$$

Singularities are a result of kinematics,
not how we represent the Jacobian

Singularity

Use determinant to check if the rank of $\boldsymbol{J}(\boldsymbol{\theta})$ drops:

$$\det(\boldsymbol{J}) = 0$$

If $\boldsymbol{J}$ is a square matrix

$$\det(\boldsymbol{J}\boldsymbol{J}^T) = 0$$

If the robot is redundant
and $\boldsymbol{J}$ has more columns than rows

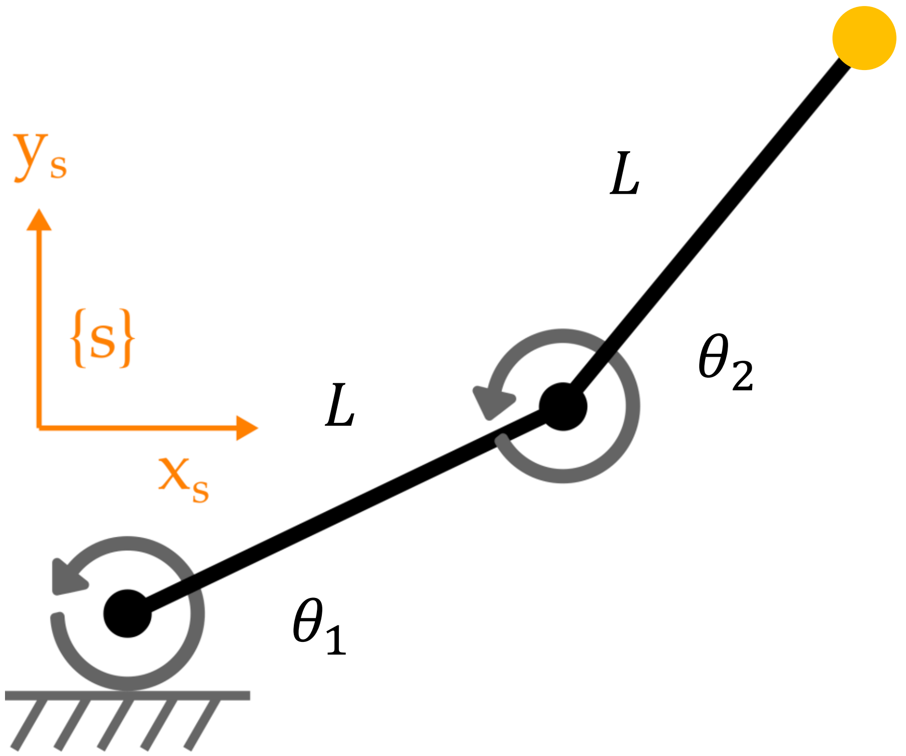
How do we **find singularities**?

Singularity

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Task space

Jacobian ***J*** for task space

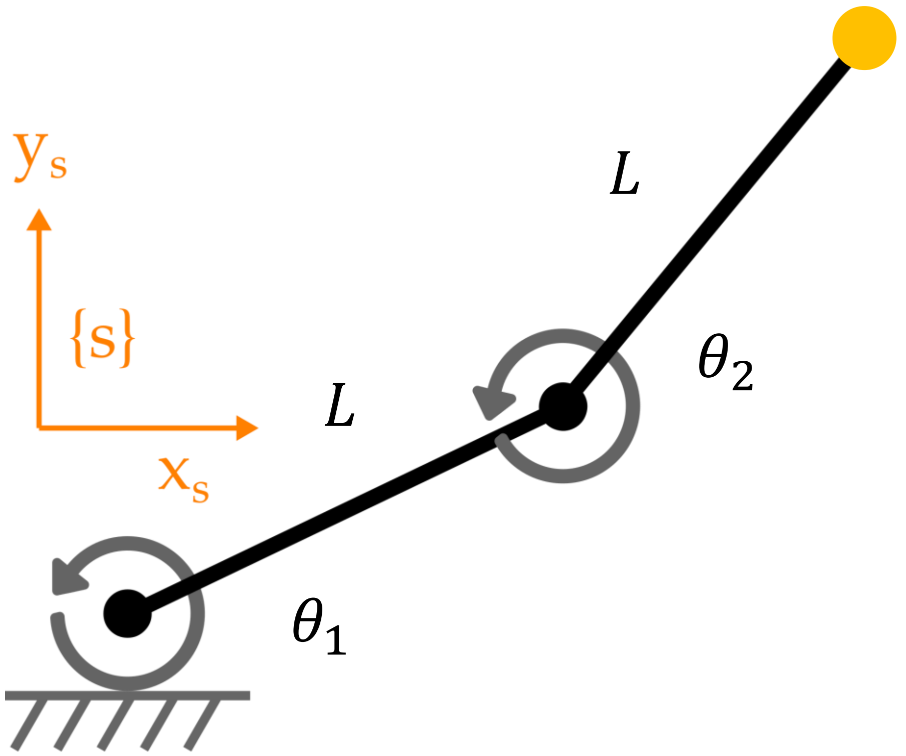


Singularity

$$J = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix}$$

$$\det(\boldsymbol{J}) = L^2 \sin\theta_2$$

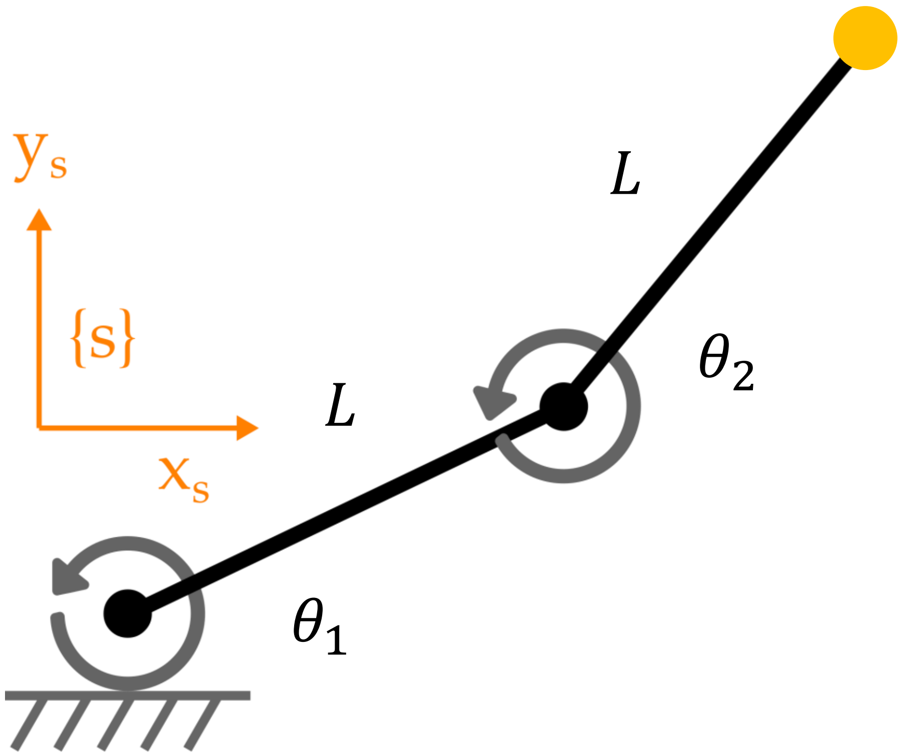
Solve for joint positions where $\det(\boldsymbol{J}) = 0$

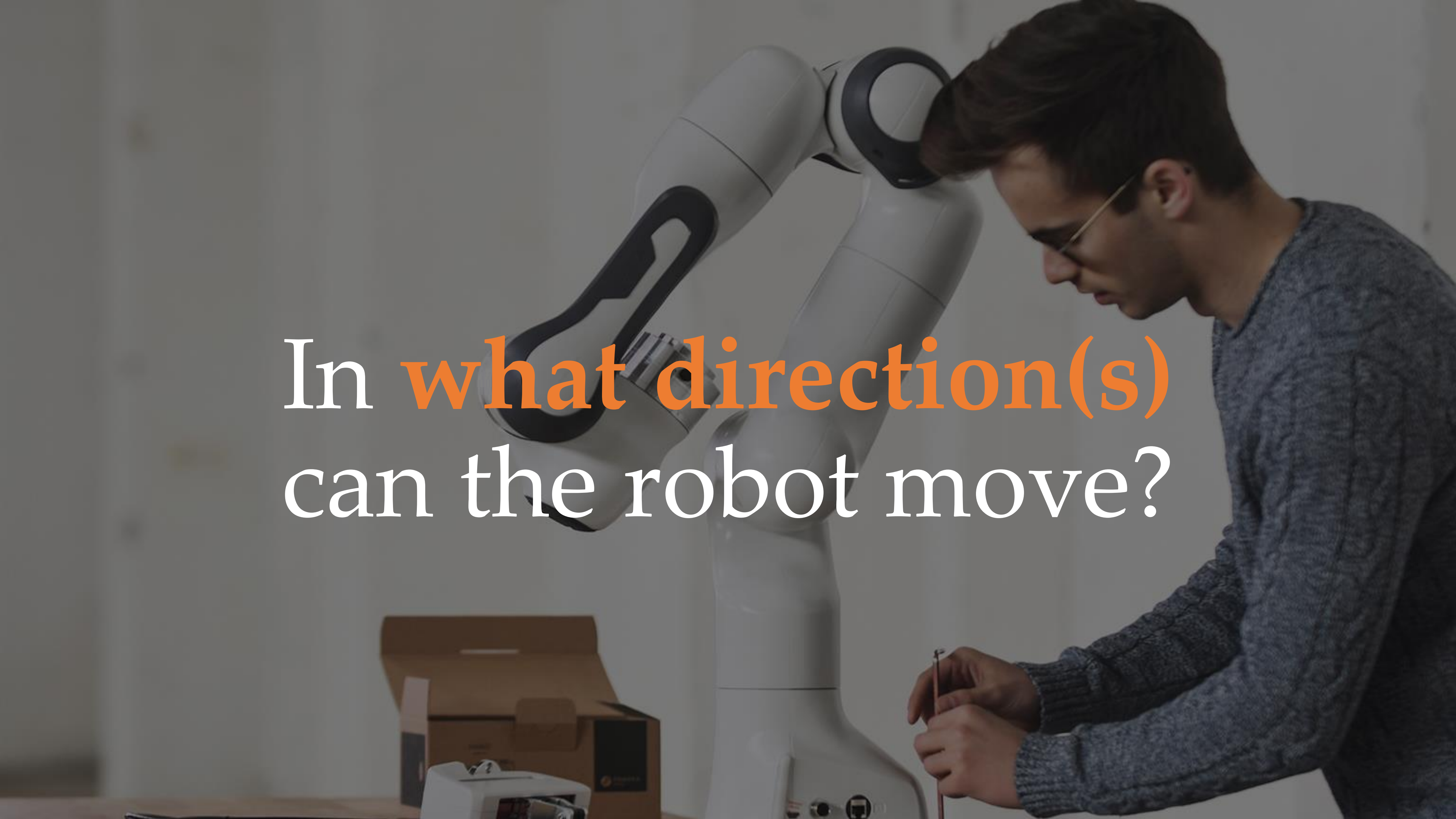


Singularity

$$J = \begin{bmatrix} -Ls_{12} - Ls_1 & -Ls_{12} \\ Lc_{12} + Lc_1 & Lc_{12} \end{bmatrix}$$

Singularity (Jacobian drops rank)
when $\theta_2 = 0$ or $\theta_2 = \pi$





In **what direction(s)** can the robot move?

Singularity

$$\mathrm{range}\,\boldsymbol{J} + \mathrm{null}\,\boldsymbol{J}^T = \mathbb{R}^n$$

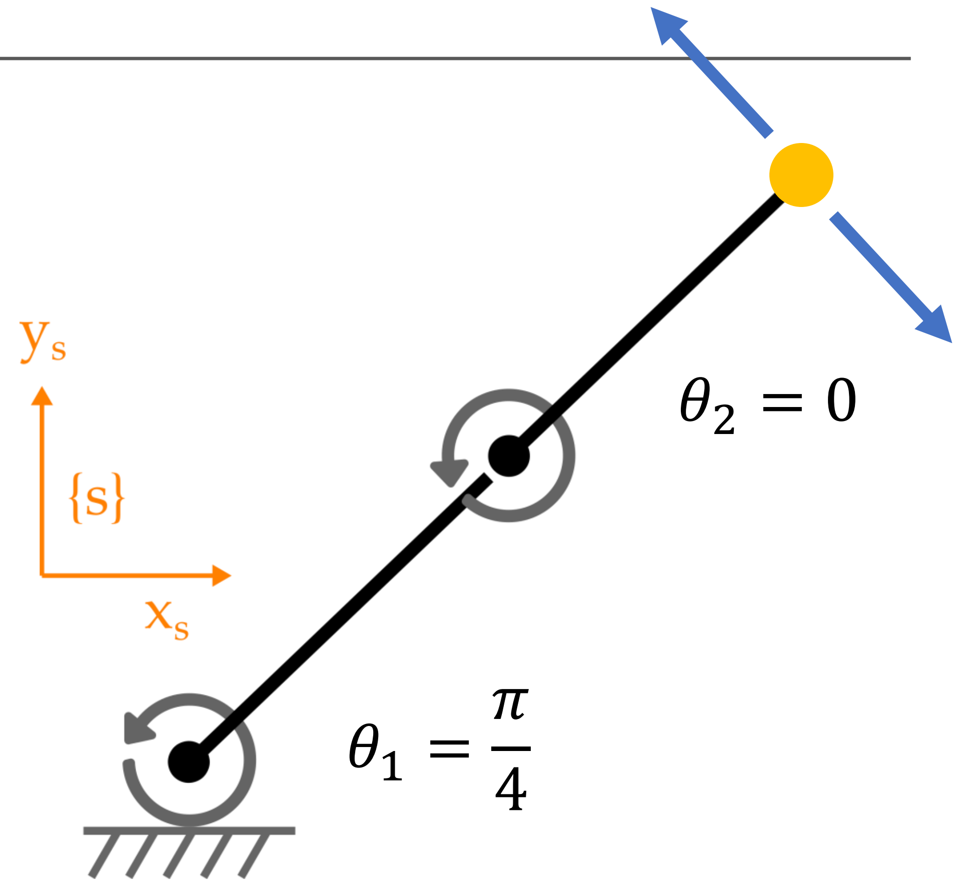
- $\mathbb{R}^n$ are all velocities the task space
- We **can** achieve any velocity in range($\boldsymbol{J}$)
- We **cannot** achieve any velocity in null($\boldsymbol{J}^T$)

Singularity

$$J = \begin{bmatrix} -L\sqrt{2} & -L\sqrt{2}/2 \\ L\sqrt{2} & L\sqrt{2}/2 \end{bmatrix}$$

$$\text{range}\, J = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Use **colspace** or **orth** in Matlab

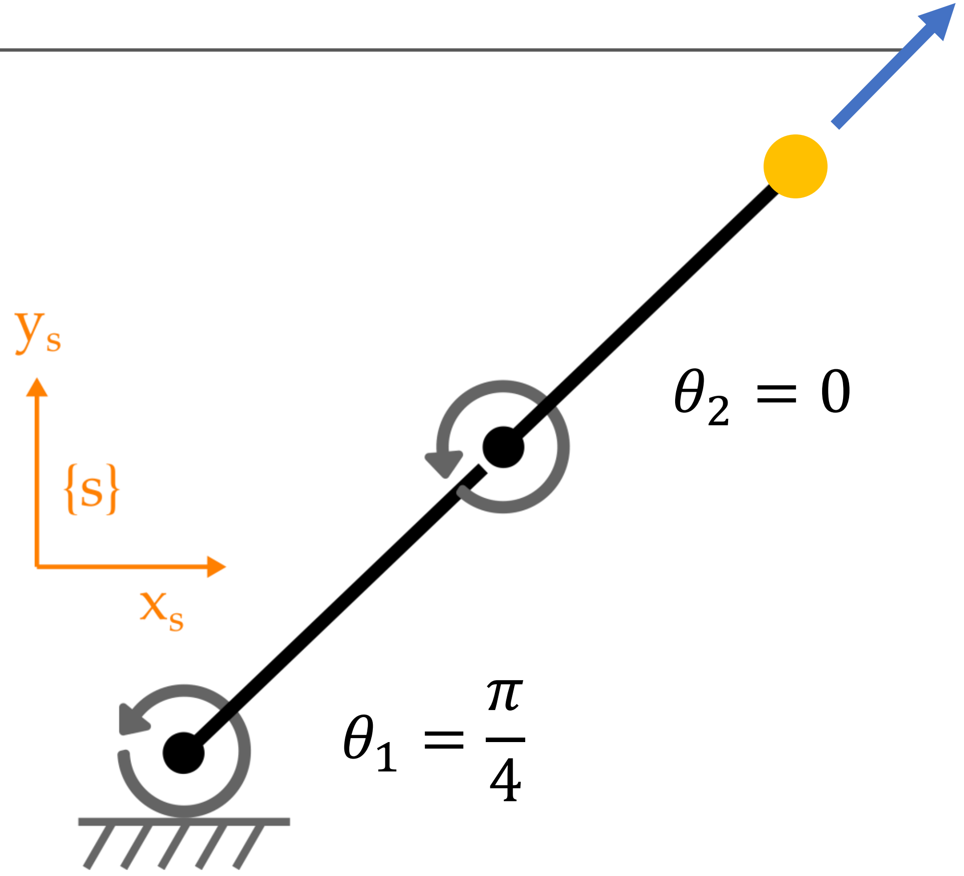


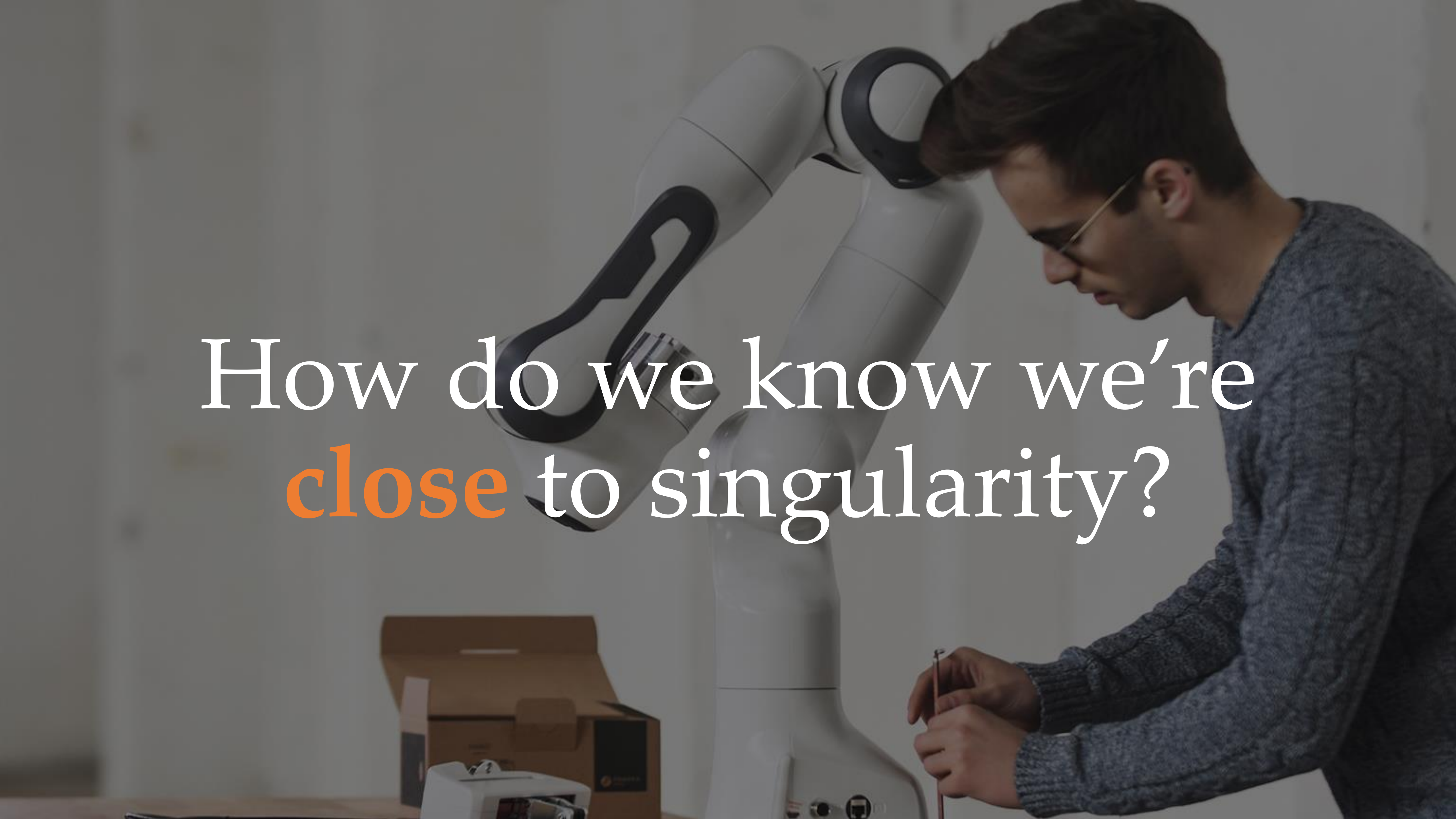
Singularity

$$J = \begin{bmatrix} -L\sqrt{2} & -L\sqrt{2}/2 \\ L\sqrt{2} & L\sqrt{2}/2 \end{bmatrix}$$

$$\text{null}\, J^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Use **null** in Matlab, ‘r’ argument




How do we know we're
close to singularity?

Manipulability

Manipulability measures how easy it is for the end-effector to move in different directions

$$\sqrt{\det(\ \boldsymbol{J}\boldsymbol{J}^T\ )}$$

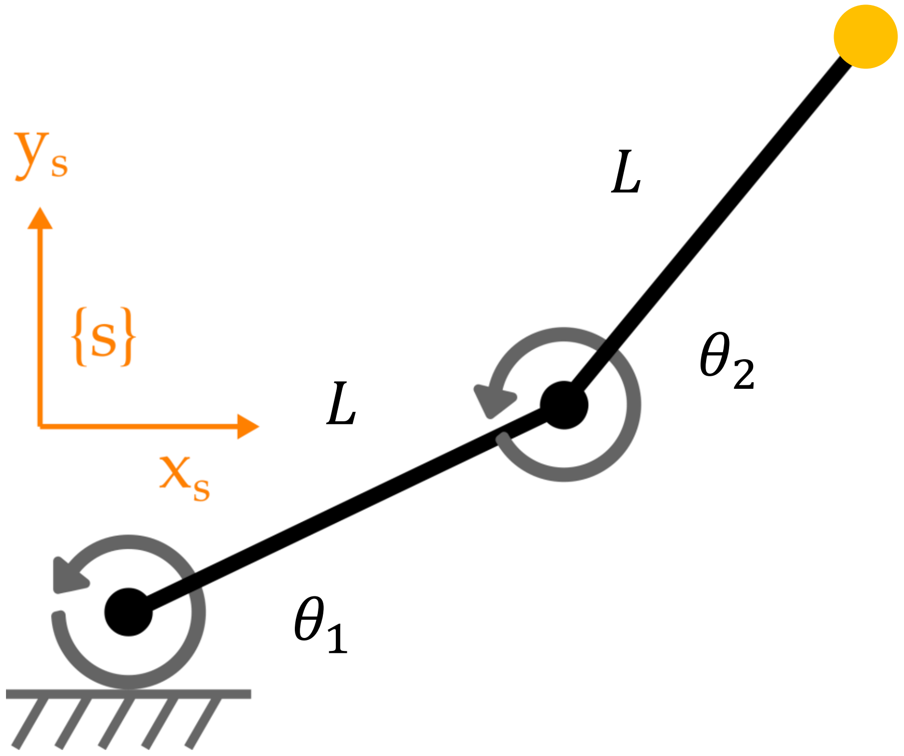
- When $\det JJ^T \to 0$ we are approaching a singularity
- When $\det JJ^T \to \infty$ we are far from singularities

Manipulability

$$J(\boldsymbol{\theta}) = \begin{bmatrix} -Ls_1 - Ls_{12} & -Ls_{12} \\ Lc_1 + Lc_{12} & Lc_{12} \end{bmatrix}$$

$$\sqrt{\det(\, JJ^T \,)} = L^2 |sin(\theta_2)|$$

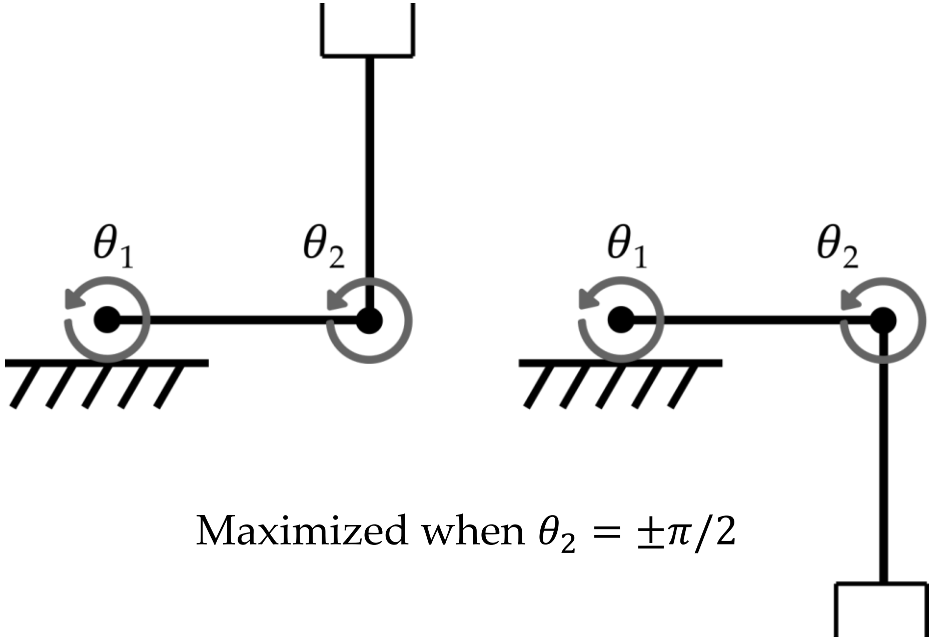
What joint positions
maximize manipulability?



Manipulability

$$J(\boldsymbol{\theta}) = \begin{bmatrix} -Ls_1 - Ls_{12} & -Ls_{12} \\ Lc_1 + Lc_{12} & Lc_{12} \end{bmatrix}$$

$$\sqrt{\det(\ \boldsymbol{J}\boldsymbol{J}^T\ )} = \ L^2|sin(\theta_2)|$$



Maximized when $\theta_2 = \pm\pi/2$

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- What is a singularity?
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- When are we close to or far from singularities?

Next Lecture

- If we know where we want my robot to go, what joint positions get us there?