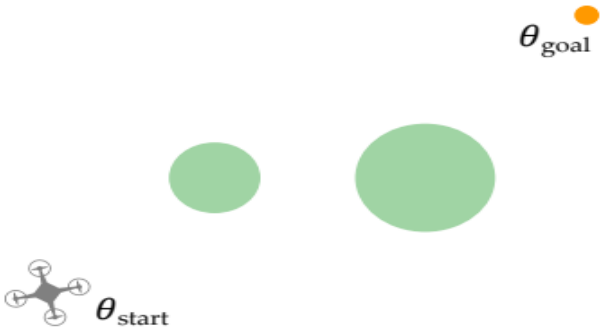
1 Potential Fields



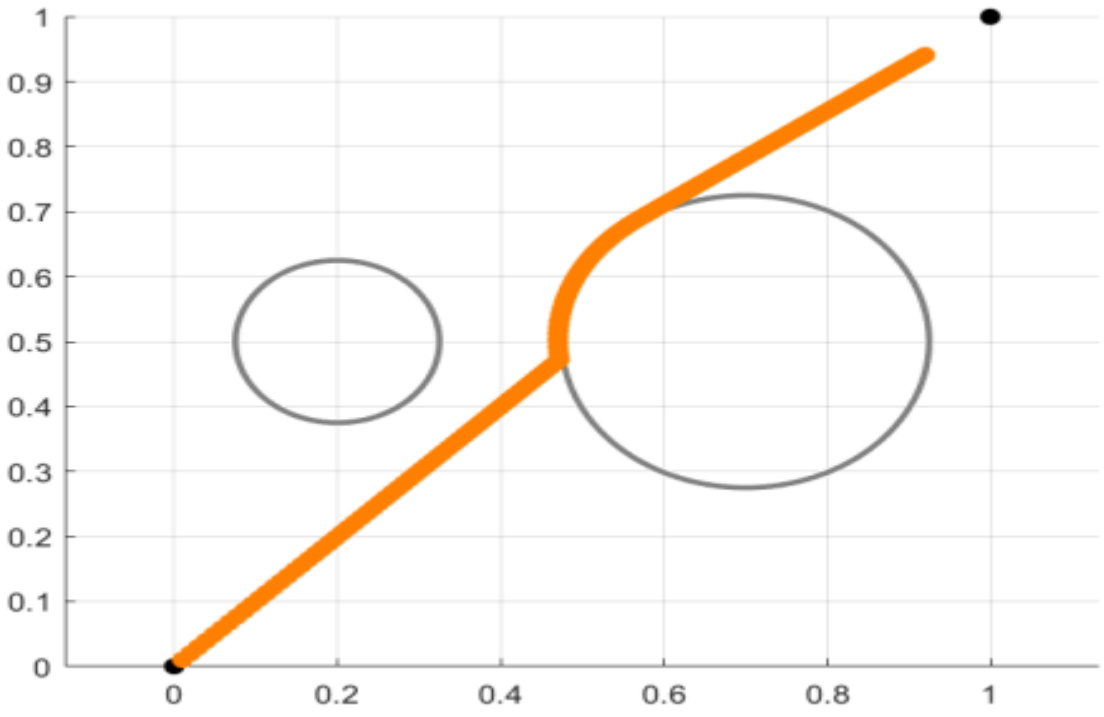
In this problem you will use potential fields to get a motion plan for the 2-DoF environment shown above. Here the drone's position is $\theta = [x, y]^T$.

1.1 (15 points)

Implement the potential fields approach:

- Set $\theta_{start} = [0, 0]^T$ and $\theta_{goal} = [1, 1]^T$
- The first obstacle has center $c_1 = [0.2, 0.5]^T$ and radius $r_1 = 0.125$
- The second obstacle has center $c_2 = [0.7, 0.5]^T$ and radius $r_2 = 0.225$
- **Hint:** Start with a low learning rate α in your gradient descent algorithm. The result shown below was obtained with $\alpha = 0.01$.

The motion plan must reach a final position within 0.1 units of the goal. Turn in your code and a plot of the result using **Publish** in Matlab. Visualize the obstacles in your plot: your solution should look like the example below.




```
clear
```

```
close all

% Start and goal environments
theta_start= [0; 0];
theta_goal = [1; 1];

% Obstacle parameters
obs_c21= [0.2; 0.5];
obs_r21= 0.125;
obs_c22= [0.7; 0.5];
obs_r22= 0.225;

% Visualizing the environment
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(obs_c21', obs_r21, 'Color', [0.5, 0.5, 0.5]);
viscircles(obs_c22', obs_r22, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

%variables
alpha= 0.01;
epsilon = 0.1;
delta= 0.01;

% initial trajectory
theta(:,1) = theta_start;
t=1;
del_Unet=1;

while norm(del_Unet)> epsilon
    del_Ux= U_theta(theta(:,t) + [delta;0]);
    del_Uy= U_theta(theta(:,t)+ [0;delta]);
    del_U= U_theta(theta(:,t));

    del_Unet=[del_Ux-del_U; del_Uy-del_U]/delta;

    theta(:,t+1)= theta(:,t)- alpha*del_Unet;
    t=t+1;
end
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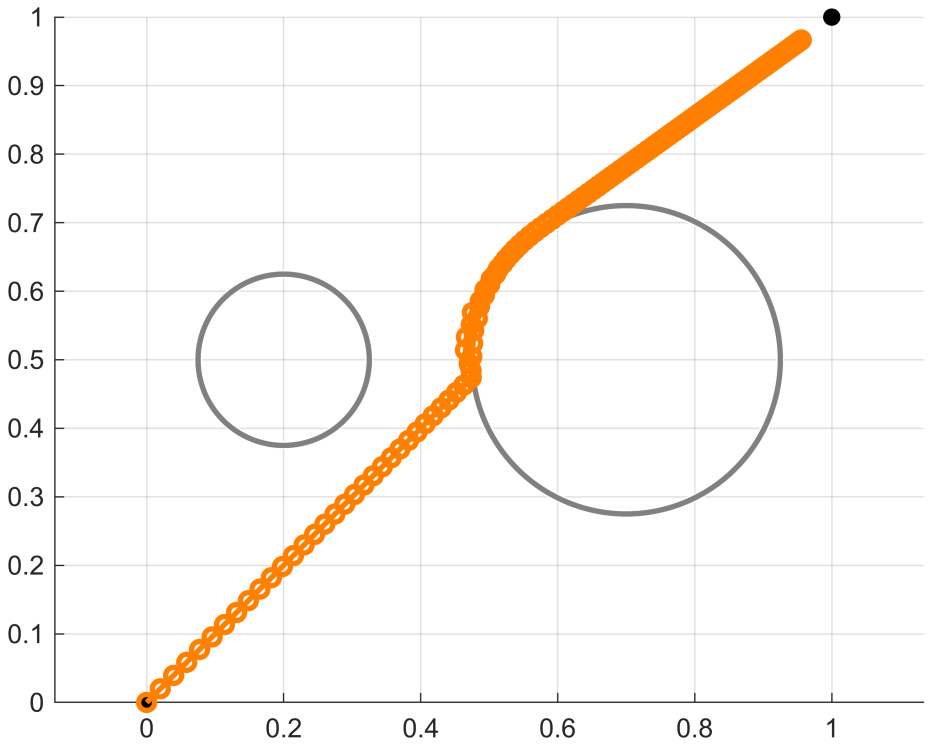
```
grid on
hold on
axis equal
plot(theta(1,:), theta(2,:), 'o-',...
    'Color', [1, 0.5, 0], 'LineWidth', 2);
```




```
function U = U_theta(theta)
    beta=2;
    gamma=1;
    theta_goal = [1; 1];
    obs_c21= [0.2; 0.5];
    obs_r21= 0.125;
    obs_c22= [0.7; 0.5];
```

```
    obs_r22= 0.225;

    Urep1=0;
    Urep2=0;


    Uatt=0.5*beta*norm(theta_goal-theta)^2;

    if norm(obs_c21-theta)<= obs_r21
        Urep1=0.5* gamma*((1/norm(obs_c21-theta))- (1/obs_r21))^2;
    end
    if norm(obs_c22-theta)<= obs_r22
        Urep2= 0.5*gamma*((1/norm(obs_c22-theta))- (1/obs_r22))^2;
    end

    Urep= Urep1+ Urep2;
    disp(Urep)
    U= Uatt + Urep;
end
```

1.2 (10 points)

Modify the position of the obstacles so that a valid plan from θ_{start} to θ_{goal} exists but the potential fields planner fails (i.e., gets stuck). Turn in a **plot** that shows the obstacles and the failed motion plan. **Explain** why potential fields fail in your environment.

```
clear
close all

% Start and goal environments
theta_start= [0; 0];
theta_goal = [1; 1];

% Modified obstacle positions and sizes
obs_c21 = [0.3; 0.5];  % Shifted first obstacle closer to the second
obs_r21 = 0.15;        % Slightly increased radius
obs_c22 = [0.6; 0.5];  % Shifted second obstacle closer to the first
obs_r22 = 0.25;        % Slightly increased radius

% Visualize the environment
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(obs_c21', obs_r21, 'Color', [0.5, 0.5, 0.5]);
viscircles(obs_c22', obs_r22, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k');
plot(1, 1, 'ko', 'MarkerFaceColor', 'k');

% Setting the variables
alpha= 0.01;
epsilon = 0.1;
delta= 0.01;

% Initial trajectory
theta(:,1) = theta_start;
t=1;
del_Unet=1;

while norm(del_Unet)> epsilon
    del_Ux= U_theta(theta(:,t) + [delta;0]);
    del_Uy= U_theta(theta(:,t)+ [0;delta]);
    del_U= U_theta(theta(:,t));

    del_Unet=[del_Ux-del_U; del_Uy-del_U]/delta;

    theta(:,t+1)= theta(:,t)- alpha*del_Unet;
    t=t+1;
```

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end
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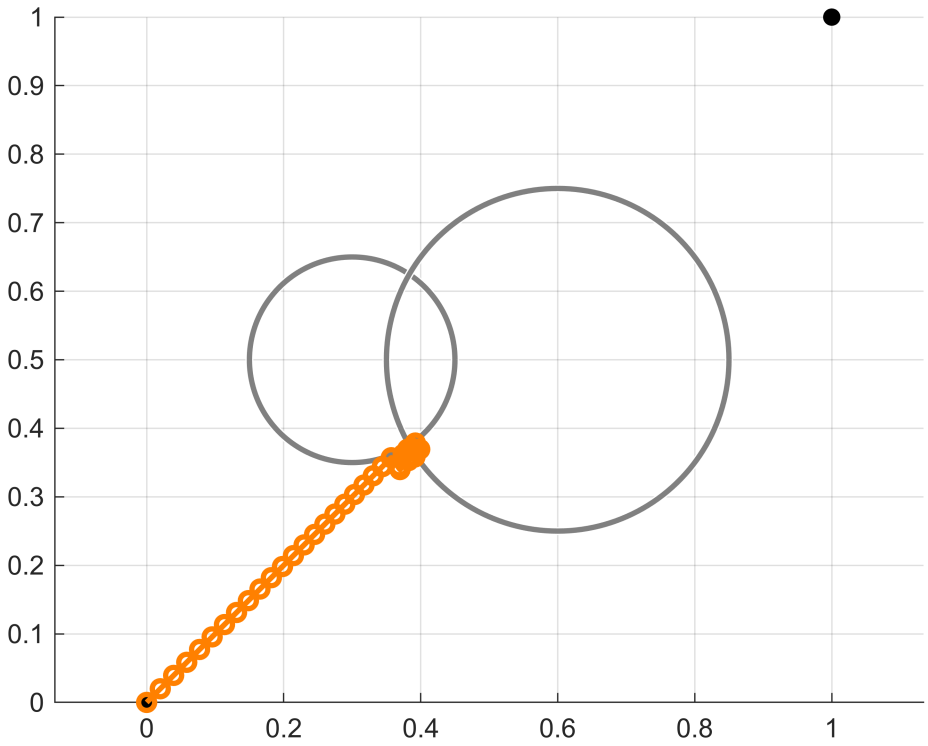
    0.0124
    0.0128
   1.1985e-04
```

```
grid on
hold on
axis equal
plot(theta(1,:), theta(2,:), 'o-',...
    'Color', [1, 0.5, 0], 'LineWidth', 2);
```




```
function U = U_theta(theta)
    beta=2;
    gamma=1;
    theta_goal = [1; 1];
    % Modified obstacle positions and sizes
    obs_c21 = [0.3; 0.5];  % Update obstacle parameters
    obs_r21 = 0.15;
    obs_c22 = [0.6; 0.5];
    obs_r22 = 0.25;

    Urep1=0;
    Urep2=0;

    Uatt=0.5*beta*norm(theta_goal-theta)^2;

    if norm(obs_c21-theta)<= obs_r21
        Urep1=0.5* gamma*((1/norm(obs_c21-theta))- (1/obs_r21))^2;
    end
    if norm(obs_c22-theta)<= obs_r22
        Urep2= 0.5*gamma*((1/norm(obs_c22-theta))- (1/obs_r22))^2;
    end
```

```
    Urep= Urep1+ Urep2;
    disp(Urep);
    U= Uatt + Urep;
end
```

In this modification, the obstacles are placed closer to each other, potentially creating a narrow corridor that the drone might not be able to navigate due to the combined repulsive forces. This setup can result in the planner getting stuck in a **local minimum**, where the drone is unable to progress towards the goal. Thus, we place a local minimim in between the start and goal. Gradient descent can only climb "down" so if **it's surrounded by high gradients**, it can't escape.

2 Trajectory Optimization

In this problem you will use trajectory optimization to perform motion planning in 2-DoF environments. As before, the mobile robot's position is $\theta = [x, y]^T$.

2.1 (15 points)

Implement the trajectory optimization algorithm. Your code should be able to work with an arbitrary number of waypoints and circular obstacles. Set the initial trajectory ξ^0 as:

```
xi_0 = [linspace(theta_start(1), theta_goal(1), k);
        linspace(theta_start(2), theta_goal(2), k)];
```

```
% Problem 2.1

clear
close all

% Start and Goal orientations
theta_start = [0;0];
theta_goal = [1;1];

% Initial trajectory variables
n = 2; % No. of joints / 2-D trajectory
k = 10; % No. of waypoints

% Obstacle Parameters - add obstacles as needed
% Each row: [center_x, center_y, radius]
obstacles = [
    0.55, 0.5, 0.3;  % First obstacle
    % Add more obstacles here, e.g., [x, y, r]
];

xi_0 = [linspace(theta_start(1), theta_goal(1), k);
        linspace(theta_start(2), theta_goal(2), k)];

xi_0_vec = reshape(xi_0, [], 1);

% Equality constraints for start and goal positions
A = [eye(n), zeros(n,n*(k-1)) ;...
     zeros(n,n*(k-1)), eye(n) ];
B = [theta_start;theta_goal];

% Nonlinear optimization
options = optimoptions('fmincon','Display','iter',...
    'Algorithm','sqp','MaxFunctionEvaluations',1e4);
xi_star_vec = fmincon(@(xi) cost(xi, obstacles), xi_0_vec, ...
    [], [], A, B, [], [], @(xi) nonlcon(xi, obstacles), options);
```

```
 Iter  Func-count           Fval   Feasibility   Step Length       Norm of   First-order
                                                                      step    optimality
    0          21    2.406330e+03     2.442e-01     1.000e+00     0.000e+00     9.306e+04
    1          65    1.363684e+03     2.844e-03     2.737e-04     2.567e+01     1.033e+02
    2          88    1.336296e+03     0.000e+00     4.900e-01     3.312e+01     4.955e+01
    3         110    9.947275e+02     0.000e+00     7.000e-01     2.670e+01     3.330e+01
    4         132    9.093469e+02     0.000e+00     7.000e-01     2.865e+01     2.909e+01
    5         154    7.748821e+02     0.000e+00     7.000e-01     2.120e+01     4.364e+01
    6         176    5.391986e+02     0.000e+00     7.000e-01     1.920e+01     3.179e+01
    7         198    3.956808e+02     0.000e+00     7.000e-01     1.285e+01     2.076e+01
    8         220    3.798429e+02     0.000e+00     7.000e-01     9.370e+00     1.782e+01
    9         242    3.623666e+02     0.000e+00     7.000e-01     7.150e+00     2.639e+01
   10         264    3.412506e+02     0.000e+00     7.000e-01     7.378e+00     3.076e+01
   11         285    3.304941e+02     0.000e+00     1.000e+00     5.843e+00     2.229e+01
   12         308    3.226864e+02     0.000e+00     4.900e-01     2.806e+00     2.048e+01
   13         330    3.175471e+02     0.000e+00     7.000e-01     2.991e+00     2.423e+01
   14         353    3.159786e+02     0.000e+00     4.900e-01     1.581e+00     2.583e+01
   15         374    3.159722e+02     0.000e+00     1.000e+00     1.960e+00     2.443e+01
   16         396    3.149716e+02     0.000e+00     7.000e-01     1.337e+00     2.346e+01
   17         417    3.141597e+02     0.000e+00     1.000e+00     1.608e+00     2.410e+01
   18         438    3.135818e+02     0.000e+00     1.000e+00     1.145e+00     2.481e+01
   19         459    3.129914e+02     0.000e+00     1.000e+00     8.481e-01     2.418e+01
   20         480    3.126799e+02     0.000e+00     1.000e+00     3.258e-01     2.413e+01
   21         501    3.120199e+02     0.000e+00     1.000e+00     5.074e-01     2.420e+01
   22         522    3.100772e+02     0.000e+00     1.000e+00     1.502e+00     2.412e+01
   23         543    3.065493e+02     0.000e+00     1.000e+00     2.335e+00     2.396e+01
   24         564    2.990809e+02     0.000e+00     1.000e+00     3.968e+00     2.342e+01
   25         585    2.838973e+02     0.000e+00     1.000e+00     5.688e+00     2.213e+01
   26         606    2.538013e+02     0.000e+00     1.000e+00     7.847e+00     1.958e+01
   27         627    1.982382e+02     0.000e+00     1.000e+00     1.033e+01     1.472e+01
   28         648    1.150687e+02     0.000e+00     1.000e+00     1.392e+01     7.462e+00
   29         669    3.811974e+01     0.000e+00     1.000e+00     1.616e+01     7.956e+00
 Iter  Func-count           Fval   Feasibility   Step Length       Norm of   First-order
                                                                      step    optimality
   30         690    5.580687e+00     0.000e+00     1.000e+00     1.185e+01     4.798e+00
   31         711    6.208616e-01     0.000e+00     1.000e+00     3.783e+00     1.222e+00
   32         732    3.138023e-01     0.000e+00     1.000e+00     6.718e-01     3.266e-01
   33         753    2.879849e-01     0.000e+00     1.000e+00     2.517e-01     1.548e-01
   34         774    2.784429e-01     0.000e+00     1.000e+00     1.505e-01     5.104e-02
   35         795    2.750657e-01     0.000e+00     1.000e+00     6.938e-02     1.499e-02
   36         816    2.742908e-01     0.000e+00     1.000e+00     1.253e-02     3.333e-03
   37         837    2.742592e-01     0.000e+00     1.000e+00     9.383e-04     3.245e-04
   38         858    2.742592e-01     0.000e+00     1.000e+00     2.124e-04     4.465e-05
   39         879    2.742592e-01     0.000e+00     1.000e+00     2.595e-05     1.755e-05
   40         903    2.742592e-01     0.000e+00     3.430e-01     9.282e-06     1.054e-05
   41         924    2.742592e-01     0.000e+00     1.000e+00     9.153e-06     2.044e-06
   42         945    2.742592e-01     0.000e+00     1.000e+00     5.645e-11     3.949e-07

Local minimum found that satisfies the constraints.

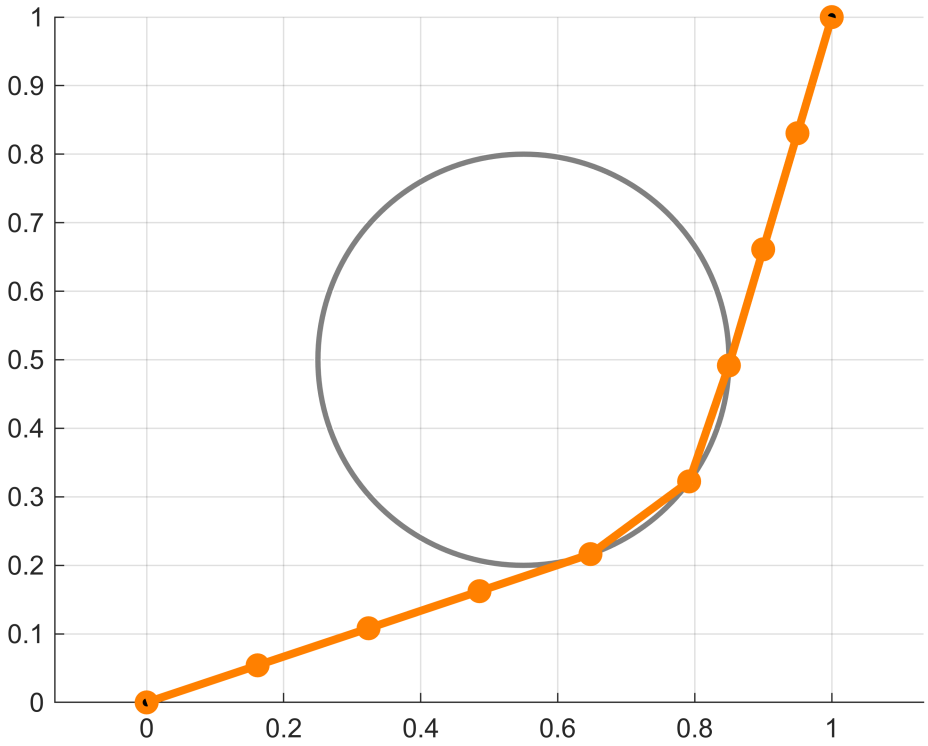
Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
xi_star = reshape(xi_star_vec,2,[]);

% Plotting
figure
grid on
hold on
```

```
axis([0, 1, 0, 1])
axis equal
% Plot Obstacles
for i = 1:size(obstacles, 1)
    viscircles(obstacles(i, 1:2), obstacles(i, 3), 'Color', [0.5, 0.5, 0.5]);
end
plot(theta_start(1), theta_start(2), 'ko', 'MarkerFaceColor', 'k')
plot(theta_goal(1), theta_goal(2), 'ko', 'MarkerFaceColor', 'k')
% Plot Result
plot(xi_star(1,:), xi_star(2,:), 'o-', 'Color', [1, 0.5, 0], 'LineWidth', 3);
```




```
% Cost function to minimize
```

```
function C = cost(xi, obstacles)
    gamma = 20;
    xi = reshape(xi, 2, []);
    C = 0;

    for idx = 2:length(xi)
        Urep = 0;
        for obs_idx = 1:size(obstacles, 1)
            r = obstacles(obs_idx, 3);
            center = obstacles(obs_idx, 1:2)';

            if norm(center - xi(:, idx)) <= r
```

```
                Urep = Urep + 0.5 * gamma * ((1 / norm(center - xi(:, idx))) - (1 /
r))^2;
            end
        end

        C = C + norm(xi(:, idx) - xi(:, idx - 1))^2 + Urep;
    end
end

% Nonlinear constraints (optional)
function [c, ceq] = nonlcon(xi, obstacles)
    xi = reshape(xi, 2, []);
    c = [];
    for idx = 1:length(xi)
        for obs_idx = 1:size(obstacles, 1)
            r = obstacles(obs_idx, 3);
            center = obstacles(obs_idx, 1:2)';
            dist = norm(xi(:, idx) - center);

            % Add a constraint for each obstacle
            c = [c; r - dist];
        end
    end
    ceq = [];
end
```

2.2 (15 points)

Use your code to find a desired trajectory for the following environments. In each environment $\theta_{start} = [0,0]^T$ and $\theta_{goal} = [1,1]^T$.

- **Environment 1.** One obstacle with center $c_1 = [0.55, 0.5]^T$ and radius $r_1 = 0.3$. Your trajectory should have $k = 10$ waypoints.

```
% Problem 2.2 - Environment 1

clear
clc
close all

% Start and Goal orientations
theta_start = [0;0];
theta_goal = [1;1];

% Initial trajectory variables

n = 2; % No. of joints / 2-D trajectory.
k = 10; % No. of waypoints

% Obstacle 1 Paramters
r1 = 0.3;
center1 = [0.55;0.5];

xi_0 = zeros(n, k);
xi_0_vec = reshape(xi_0, [],1);

% Equality constraints for start and goal positions
A = [eye(n), zeros(n,n*(k-1)) ;...
    zeros(n,n*(k-1)), eye(n) ];

B = [theta_start;theta_goal];

% Nonlinear optimization
options = optimoptions('fmincon','Display','iter',...
    'Algorithm','sqp','MaxFunctionEvaluations',1e4);
xi_star_vec = fmincon(@(xi) cost(xi), xi_0_vec, ...
    [], [], A, B, [], [], [], options);
```

```
 Iter  Func-count           Fval   Feasibility   Step Length       Norm of    First-order
                                                                      step     optimality
    0         21    0.000000e+00     1.000e+00     1.000e+00     0.000e+00     2.980e-08
    1         46    1.152960e-01     7.599e-01     2.401e-01     3.396e-01     4.802e-01
    2         74    1.288167e-01     6.973e-01     8.235e-02     1.177e-01     3.858e-01
    3        107    1.301925e-01     6.877e-01     1.384e-02     2.070e-02     3.824e-01
    4        144    1.304400e-01     6.854e-01     3.323e-03     5.499e-03     9.620e-01
    5        167    7.032032e+01     3.495e-01     4.900e-01     8.867e-01     1.115e+03
```

6	189	6.352294e+01	1.049e-01	7.000e-01	9.000e-01	1.427e+03
7	227	4.385346e+01	1.046e-01	2.326e-03	4.696e+00	1.425e+01
8	251	2.747094e+01	6.873e-02	3.430e-01	4.806e+00	8.959e+00
9	273	3.029506e+01	2.062e-02	7.000e-01	3.580e+00	1.101e+01
10	294	2.194269e+01	0.000e+00	1.000e+00	3.772e+00	5.910e+00
11	317	1.691520e+01	0.000e+00	4.900e-01	2.460e+00	6.687e+00
12	338	1.556209e+01	0.000e+00	1.000e+00	2.682e+00	5.474e+00
13	361	1.373612e+01	0.000e+00	4.900e-01	1.497e+00	5.272e+00
14	382	1.356662e+01	0.000e+00	1.000e+00	2.439e+00	5.988e+00
15	404	1.202932e+01	0.000e+00	7.000e-01	1.448e+00	4.338e+00
16	426	1.090450e+01	0.000e+00	7.000e-01	1.709e+00	4.760e+00
17	448	1.017564e+01	0.000e+00	7.000e-01	9.125e-01	4.769e+00
18	470	1.000271e+01	0.000e+00	7.000e-01	1.152e+00	4.435e+00
19	491	9.682184e+00	0.000e+00	1.000e+00	9.631e-01	4.145e+00
20	512	9.261388e+00	0.000e+00	1.000e+00	1.187e+00	4.791e+00
21	533	8.374244e+00	0.000e+00	1.000e+00	8.414e-01	3.774e+00
22	554	7.824443e+00	0.000e+00	1.000e+00	8.498e-01	4.208e+00
23	575	6.749018e+00	0.000e+00	1.000e+00	3.605e-01	3.726e+00
24	596	3.307992e+00	0.000e+00	1.000e+00	1.625e+00	2.179e+00
25	619	2.037926e+00	0.000e+00	4.900e-01	8.710e-01	1.745e+00
26	646	1.927091e+00	0.000e+00	1.176e-01	2.098e-01	2.848e+01
27	671	1.130807e+00	0.000e+00	2.401e-01	1.517e+00	1.308e+00
28	692	9.908110e-01	0.000e+00	1.000e+00	5.148e-01	9.454e-01
29	713	9.585879e-01	0.000e+00	1.000e+00	1.772e-01	8.913e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
30	734	9.504782e-01	0.000e+00	1.000e+00	1.254e-01	8.607e-01
31	755	9.433863e-01	0.000e+00	1.000e+00	1.697e-01	8.485e-01
32	776	9.404222e-01	0.000e+00	1.000e+00	7.222e-02	8.553e-01
33	797	9.358837e-01	0.000e+00	1.000e+00	7.918e-02	8.530e-01
34	818	9.277798e-01	0.000e+00	1.000e+00	8.573e-02	8.466e-01
35	839	9.090249e-01	0.000e+00	1.000e+00	1.152e-01	8.383e-01
36	860	8.698169e-01	0.000e+00	1.000e+00	1.282e-01	8.426e-01
37	881	8.003106e-01	0.000e+00	1.000e+00	1.655e-01	8.889e-01
38	908	7.894592e-01	0.000e+00	1.176e-01	3.813e-02	2.749e+00
39	933	7.491688e-01	0.000e+00	2.401e-01	7.171e-01	1.287e+00
40	954	7.010403e-01	0.000e+00	1.000e+00	2.195e-01	1.167e+00
41	975	6.955670e-01	0.000e+00	1.000e+00	8.705e-02	1.187e+00
42	996	6.935455e-01	0.000e+00	1.000e+00	6.847e-02	1.211e+00
43	1017	6.917963e-01	0.000e+00	1.000e+00	4.121e-02	1.224e+00
44	1038	6.852306e-01	0.000e+00	1.000e+00	1.035e-01	1.249e+00
45	1059	6.715092e-01	0.000e+00	1.000e+00	1.428e-01	1.267e+00
46	1082	6.552153e-01	0.000e+00	4.900e-01	1.079e-01	2.159e+00
47	1110	6.443417e-01	0.000e+00	8.235e-02	8.074e-02	4.873e+00
48	1140	6.384926e-01	0.000e+00	4.035e-02	3.996e-02	3.916e+00
49	1166	6.086512e-01	0.000e+00	1.681e-01	2.875e-01	9.759e-01
50	1190	5.959004e-01	0.000e+00	3.430e-01	8.505e-02	4.651e+00
51	1215	5.695792e-01	0.000e+00	2.401e-01	4.264e-01	9.988e-01
52	1239	5.585371e-01	0.000e+00	3.430e-01	8.067e-02	3.296e+00
53	1261	5.568165e-01	0.000e+00	7.000e-01	1.470e-01	9.637e-01
54	1282	5.536087e-01	0.000e+00	1.000e+00	8.744e-02	9.644e-01
55	1303	5.513904e-01	0.000e+00	1.000e+00	5.739e-03	9.545e-01
56	1325	5.475078e-01	0.000e+00	7.000e-01	2.264e-02	1.724e+00
57	1347	5.473799e-01	0.000e+00	7.000e-01	1.692e-02	9.367e-01
58	1368	5.462014e-01	0.000e+00	1.000e+00	7.459e-03	9.319e-01
59	1391	5.452781e-01	0.000e+00	4.900e-01	1.038e-02	1.244e+00
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
60	1412	5.444957e-01	0.000e+00	1.000e+00	4.100e-03	9.236e-01
61	1433	5.440097e-01	0.000e+00	1.000e+00	2.957e-03	5.819e-01
62	1454	5.427847e-01	0.000e+00	1.000e+00	1.278e-02	9.597e-01
63	1475	5.393796e-01	0.000e+00	1.000e+00	2.612e-02	1.160e+00
64	1497	5.346344e-01	0.000e+00	7.000e-01	2.626e-01	1.009e+00
65	1518	5.237382e-01	0.000e+00	1.000e+00	7.361e-02	9.201e-01

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
66	1539	5.160012e-01	0.000e+00	1.000e+00	7.879e-02	7.991e-01
67	1560	5.147708e-01	0.000e+00	1.000e+00	2.968e-02	7.754e-01
68	1581	5.138298e-01	0.000e+00	1.000e+00	2.674e-02	7.667e-01
69	1602	5.123361e-01	0.000e+00	1.000e+00	2.860e-02	7.607e-01
70	1623	5.086079e-01	0.000e+00	1.000e+00	4.696e-02	7.511e-01
71	1644	5.020822e-01	0.000e+00	1.000e+00	5.351e-02	7.389e-01
72	1665	4.937955e-01	0.000e+00	1.000e+00	5.043e-02	7.277e-01
73	1686	4.880069e-01	0.000e+00	1.000e+00	5.624e-02	7.249e-01
74	1714	4.879379e-01	0.000e+00	8.235e-02	4.223e-03	3.960e-01
75	1735	4.872819e-01	0.000e+00	1.000e+00	1.427e-01	8.078e-01
76	1756	4.833876e-01	0.000e+00	1.000e+00	6.588e-02	7.718e-01
77	1777	4.807187e-01	0.000e+00	1.000e+00	2.329e-02	7.585e-01
78	1798	4.693274e-01	0.000e+00	1.000e+00	6.169e-02	7.276e-01
79	1819	4.540552e-01	0.000e+00	1.000e+00	7.595e-02	7.175e-01
80	1840	4.370382e-01	0.000e+00	1.000e+00	1.362e-01	7.455e-01
81	1861	4.295554e-01	0.000e+00	1.000e+00	1.159e-01	7.910e-01
82	1882	4.277897e-01	0.000e+00	1.000e+00	4.723e-02	8.142e-01
83	1903	4.269821e-01	0.000e+00	1.000e+00	1.677e-02	8.208e-01
84	1924	4.254267e-01	0.000e+00	1.000e+00	2.820e-02	1.690e+00
85	1955	4.246377e-01	0.000e+00	2.825e-02	7.490e-02	8.281e-01
86	1976	4.203536e-01	0.000e+00	1.000e+00	3.101e-02	8.080e-01
87	1997	4.076927e-01	0.000e+00	1.000e+00	5.220e-02	7.196e-01
88	2018	3.892482e-01	0.000e+00	1.000e+00	5.975e-02	5.503e-01
89	2045	3.861608e-01	0.000e+00	1.176e-01	1.255e-02	6.638e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
90	2066	3.805247e-01	0.000e+00	1.000e+00	3.173e-01	5.479e-01
91	2087	3.719592e-01	0.000e+00	1.000e+00	1.127e-01	3.873e-01
92	2108	3.688493e-01	0.000e+00	1.000e+00	2.377e-02	4.061e-01
93	2131	3.640324e-01	0.000e+00	4.900e-01	2.813e-02	1.781e+00
94	2152	3.596278e-01	0.000e+00	1.000e+00	8.230e-02	3.523e-01
95	2173	3.541344e-01	0.000e+00	1.000e+00	2.486e-02	3.749e-01
96	2194	3.387311e-01	0.000e+00	1.000e+00	7.866e-02	4.607e-01
97	2217	3.383436e-01	0.000e+00	4.900e-01	1.616e-02	1.962e+00
98	2247	3.352298e-01	0.000e+00	4.035e-02	4.854e-02	4.237e-01
99	2275	3.351512e-01	0.000e+00	8.235e-02	3.908e-03	4.631e-01
100	2296	3.339114e-01	0.000e+00	1.000e+00	3.917e-02	3.386e-01
101	2317	3.332994e-01	0.000e+00	1.000e+00	1.145e-02	3.456e-01
102	2338	3.324295e-01	0.000e+00	1.000e+00	1.086e-02	3.693e-01
103	2359	3.302767e-01	0.000e+00	1.000e+00	2.633e-02	8.111e-01
104	2380	3.271194e-01	0.000e+00	1.000e+00	3.328e-02	4.405e-01
105	2406	3.261397e-01	0.000e+00	1.681e-01	9.718e-03	4.383e-01
106	2435	3.261058e-01	0.000e+00	5.765e-02	2.029e-03	5.912e-01
107	2457	3.259175e-01	0.000e+00	7.000e-01	6.255e-02	4.027e-01
108	2478	3.251367e-01	0.000e+00	1.000e+00	3.110e-02	4.119e-01
109	2499	3.249058e-01	0.000e+00	1.000e+00	5.306e-03	4.112e-01
110	2520	3.238941e-01	0.000e+00	1.000e+00	5.840e-03	9.391e-01
111	2541	3.207431e-01	0.000e+00	1.000e+00	1.757e-02	2.578e+00
112	2562	3.145294e-01	0.000e+00	1.000e+00	4.801e-02	4.253e+00
113	2588	3.134487e-01	0.000e+00	1.681e-01	1.771e-02	4.142e+00
114	2612	3.131258e-01	0.000e+00	3.430e-01	3.409e-01	2.479e-01
115	2633	2.963018e-01	0.000e+00	1.000e+00	1.948e-01	1.884e-01
116	2654	2.952003e-01	0.000e+00	1.000e+00	1.063e-02	1.580e-01
117	2675	2.935469e-01	0.000e+00	1.000e+00	1.779e-02	1.203e+00
118	2696	2.909305e-01	0.000e+00	1.000e+00	4.284e-02	2.521e-01
119	2717	2.873025e-01	0.000e+00	1.000e+00	4.136e-02	2.433e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
120	2741	2.836733e-01	0.000e+00	3.430e-01	4.304e-02	9.319e-01
121	2766	2.833367e-01	0.000e+00	2.401e-01	1.165e-02	2.861e+00
122	2787	2.802654e-01	0.000e+00	1.000e+00	5.053e-02	2.243e-01
123	2808	2.779226e-01	0.000e+00	1.000e+00	2.259e-02	2.397e-01
124	2829	2.746141e-01	0.000e+00	1.000e+00	5.180e-02	2.895e-01
125	2850	2.736594e-01	0.000e+00	1.000e+00	1.906e-02	3.004e-01

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
126	2874	2.734645e-01	0.000e+00	3.430e-01	9.272e-03	1.259e+00
127	2897	2.733963e-01	0.000e+00	4.900e-01	9.023e-03	1.287e+00
128	2918	2.728705e-01	0.000e+00	1.000e+00	3.618e-02	2.783e-01
129	2939	2.717773e-01	0.000e+00	1.000e+00	1.907e-02	2.780e-01
130	2960	2.709023e-01	0.000e+00	1.000e+00	1.607e-02	2.726e-01
131	2981	2.697157e-01	0.000e+00	1.000e+00	2.034e-02	2.622e-01
132	3002	2.682152e-01	0.000e+00	1.000e+00	2.705e-02	2.309e-01
133	3023	2.673176e-01	0.000e+00	1.000e+00	1.822e-02	2.075e-01
134	3044	2.669710e-01	0.000e+00	1.000e+00	8.992e-03	1.979e-01
135	3065	2.668130e-01	0.000e+00	1.000e+00	8.502e-03	1.947e-01
136	3086	2.667051e-01	0.000e+00	1.000e+00	8.663e-03	1.935e-01
137	3107	2.665770e-01	0.000e+00	1.000e+00	1.032e-02	1.921e-01
138	3128	2.663609e-01	0.000e+00	1.000e+00	1.465e-02	1.897e-01
139	3149	2.659549e-01	0.000e+00	1.000e+00	2.301e-02	1.846e-01
140	3170	2.653537e-01	0.000e+00	1.000e+00	2.929e-02	1.765e-01
141	3191	2.648231e-01	0.000e+00	1.000e+00	2.134e-02	1.687e-01
142	3212	2.645959e-01	0.000e+00	1.000e+00	8.264e-03	1.660e-01
143	3233	2.645226e-01	0.000e+00	1.000e+00	7.392e-03	1.662e-01
144	3254	2.644462e-01	0.000e+00	1.000e+00	7.004e-03	1.665e-01
145	3275	2.642458e-01	0.000e+00	1.000e+00	1.210e-02	1.658e-01
146	3296	2.637635e-01	0.000e+00	1.000e+00	1.826e-02	1.616e-01
147	3317	2.626155e-01	0.000e+00	1.000e+00	2.701e-02	1.471e-01
148	3342	2.625048e-01	0.000e+00	2.401e-01	7.894e-03	1.243e+00
149	3363	2.596185e-01	0.000e+00	1.000e+00	5.284e-02	1.580e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
150	3384	2.578974e-01	0.000e+00	1.000e+00	3.636e-02	1.526e-01
151	3405	2.568010e-01	0.000e+00	1.000e+00	4.611e-02	1.244e-01
152	3426	2.566945e-01	0.000e+00	1.000e+00	1.861e-03	1.227e-01
153	3447	2.561382e-01	0.000e+00	1.000e+00	9.197e-03	1.247e-01
154	3471	2.558216e-01	0.000e+00	3.430e-01	1.086e-02	1.405e-01
155	3501	2.557915e-01	0.000e+00	4.035e-02	1.484e-03	6.151e-02
156	3522	2.557530e-01	0.000e+00	1.000e+00	5.983e-03	1.431e-01
157	3546	2.557466e-01	0.000e+00	3.430e-01	7.618e-04	1.739e-01
158	3567	2.557343e-01	0.000e+00	1.000e+00	5.574e-04	6.748e-02
159	3588	2.557282e-01	0.000e+00	1.000e+00	5.965e-04	6.540e-02
160	3609	2.557145e-01	0.000e+00	1.000e+00	1.657e-03	3.191e-01
161	3630	2.556839e-01	0.000e+00	1.000e+00	4.008e-04	1.009e-01
162	3651	2.556182e-01	0.000e+00	1.000e+00	1.952e-03	9.462e-02
163	3672	2.553050e-01	0.000e+00	1.000e+00	1.519e-02	1.226e-01
164	3693	2.551542e-01	0.000e+00	1.000e+00	1.859e-02	1.406e-01
165	3714	2.551199e-01	0.000e+00	1.000e+00	2.668e-03	1.431e-01
166	3735	2.549032e-01	0.000e+00	1.000e+00	8.075e-03	1.496e-01
167	3761	2.548681e-01	0.000e+00	1.681e-01	5.699e-03	6.169e-01
168	3782	2.546574e-01	0.000e+00	1.000e+00	2.589e-02	1.717e-01
169	3803	2.546060e-01	0.000e+00	1.000e+00	2.955e-03	1.684e-01
170	3827	2.545243e-01	0.000e+00	3.430e-01	1.765e-03	2.165e-01
171	3848	2.544476e-01	0.000e+00	1.000e+00	1.119e-02	1.719e-01
172	3876	2.544463e-01	0.000e+00	8.235e-02	1.210e-03	1.726e-01
173	3897	2.543965e-01	0.000e+00	1.000e+00	5.062e-03	8.554e-02
174	3918	2.543899e-01	0.000e+00	1.000e+00	2.104e-03	8.597e-02
175	3939	2.543781e-01	0.000e+00	1.000e+00	3.085e-03	8.177e-02
176	3963	2.543771e-01	0.000e+00	3.430e-01	2.838e-04	8.784e-02
177	3984	2.543693e-01	0.000e+00	1.000e+00	5.284e-04	7.969e-02
178	4005	2.543265e-01	0.000e+00	1.000e+00	2.922e-03	6.589e-02
179	4026	2.542294e-01	0.000e+00	1.000e+00	1.270e-02	1.482e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
180	4047	2.541919e-01	0.000e+00	1.000e+00	3.827e-03	1.470e-01
181	4068	2.541886e-01	0.000e+00	1.000e+00	5.721e-03	2.946e-01
182	4089	2.541509e-01	0.000e+00	1.000e+00	5.629e-03	8.869e-02
183	4110	2.541473e-01	0.000e+00	1.000e+00	6.916e-04	2.065e-02
184	4131	2.541458e-01	0.000e+00	1.000e+00	1.293e-03	1.744e-02
185	4152	2.541446e-01	0.000e+00	1.000e+00	2.045e-03	1.077e-02

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  186        4173    2.541446e-01     0.000e+00     1.000e+00     3.078e-04     8.010e-03
  187        4194    2.541445e-01     0.000e+00     1.000e+00     1.272e-04     7.914e-03
  188        4215    2.541444e-01     0.000e+00     1.000e+00     2.027e-04     7.708e-03
  189        4236    2.541441e-01     0.000e+00     1.000e+00     2.305e-04     1.536e-02
  190        4257    2.541434e-01     0.000e+00     1.000e+00     2.922e-04     2.660e-02
  191        4278    2.541420e-01     0.000e+00     1.000e+00     5.916e-04     4.001e-02
  192        4299    2.541392e-01     0.000e+00     1.000e+00     1.436e-03     4.939e-02
  193        4320    2.541350e-01     0.000e+00     1.000e+00     2.849e-03     3.459e-02
  194        4341    2.541324e-01     0.000e+00     1.000e+00     2.519e-03     1.703e-02
  195        4362    2.541318e-01     0.000e+00     1.000e+00     6.746e-04     2.440e-03
  196        4383    2.541318e-01     0.000e+00     1.000e+00     1.737e-04     1.544e-03
  197        4404    2.541317e-01     0.000e+00     1.000e+00     6.917e-05     1.443e-03
  198        4425    2.541317e-01     0.000e+00     1.000e+00     6.871e-05     1.472e-03
  199        4446    2.541317e-01     0.000e+00     1.000e+00     5.670e-05     1.485e-03
  200        4467    2.541317e-01     0.000e+00     1.000e+00     9.671e-05     3.055e-03
  201        4488    2.541317e-01     0.000e+00     1.000e+00     1.470e-04     6.135e-03
  202        4509    2.541316e-01     0.000e+00     1.000e+00     2.496e-04     1.114e-02
  203        4530    2.541315e-01     0.000e+00     1.000e+00     4.059e-04     1.849e-02
  204        4551    2.541311e-01     0.000e+00     1.000e+00     6.239e-04     2.735e-02
  205        4572    2.541305e-01     0.000e+00     1.000e+00     7.938e-04     3.151e-02
  206        4593    2.541300e-01     0.000e+00     1.000e+00     6.649e-04     2.061e-02
  207        4614    2.541298e-01     0.000e+00     1.000e+00     2.496e-04     5.262e-03
  208        4635    2.541298e-01     0.000e+00     1.000e+00     1.267e-04     4.221e-04
  209        4656    2.541298e-01     0.000e+00     1.000e+00     4.468e-05     4.099e-05
 Iter   Func-count            Fval   Feasibility   Step Length       Norm of    First-order
                                                                        step     optimality
  210        4677    2.541298e-01     0.000e+00     1.000e+00     5.090e-06     6.840e-06
  211        4698    2.541298e-01     0.000e+00     1.000e+00     5.339e-07     7.413e-07

Local minimum found that satisfies the constraints.

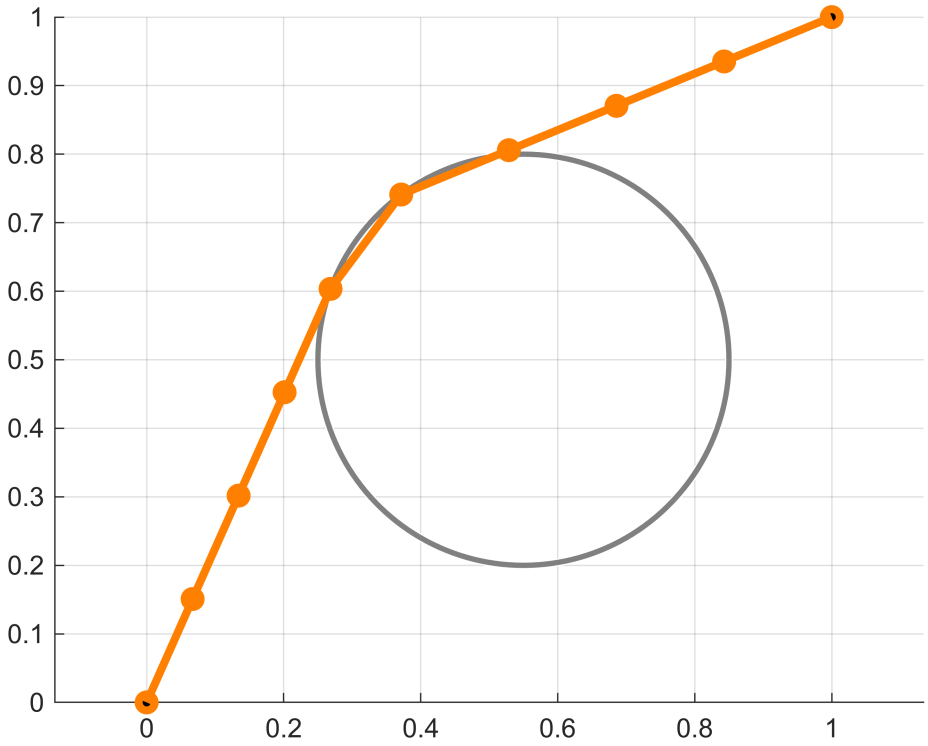
Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
xi_star = reshape(xi_star_vec,2,[]); % to implement

figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center1', r1, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Plot Result
grid on
hold on
axis equal
plot(xi_star(1,:), xi_star(2,:), 'o-',...
    'Color', [1, 0.5, 0], 'LineWidth', 3);
```




```
% Cost function to minimize
```

```
function C = cost(xi)

    gamma = 20;
    xi = reshape(xi,2,[]);
    C = 0;
    r1 = 0.3;
    center1 = [0.55;0.5];

    for idx = 2:length(xi)
        Urep1 = 0;

        if (norm(center1 - xi(:,idx)) <=r1)
            Urep1 = 0.5*gamma*((1/(norm(center1-xi(:,idx)))) - (1/r1))^2;
        end

        C = Urep1  + C + norm(xi(:,idx) - xi(:,idx-1))^2;
    end
end
```

- **Environment 2.** One obstacle with center $c_1 = [0.5, 0.3]^T$ and radius $r_1 = 0.3$. A second obstacle with center $c_2 = [0.5, 0.7]^T$ and radius $r_2 = 0.2$. Set $k = 15$.

```
% Problem 2.2 - Environment 2

clear
clc
close all

% Start and Goal orientations
theta_start = [0;0];
theta_goal = [1;1];

% Initial trajectory variables
n = 2; % No. of joints/ 2-D trajectory
k = 15; % No. of waypoints

% Obstacles 1 and 2 parameters
% First obstacle's radius and center
r1 = 0.3;
center1 = [0.5;0.3];
% Second obstacle's radius and center
r2 = 0.2;
center2 = [0.5;0.7];

xi_0 = zeros(n, k); % Initial trajectory
xi_0_vec = reshape(xi_0, [],1); % Reshape for the need of optimization

% Equality constraints for start and goal positions
A = [eye(n), zeros(n,n*(k-1)) ;...
    zeros(n,n*(k-1)), eye(n) ];

B = [theta_start;theta_goal];

% Nonlinear optimization
options = optimoptions('fmincon','Display','iter',...
    'Algorithm','sqp','MaxFunctionEvaluations',1e5);
xi_star_vec = fmincon(@(xi) cost(xi), xi_0_vec, ...
    [], [], A, B, [], [], [], options);
```

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
0	31	0.000000e+00	1.000e+00	1.000e+00	0.000e+00	2.980e-08
1	63	9.800000e-01	3.000e-01	7.000e-01	9.899e-01	1.400e+00
2	98	8.002409e-01	2.280e-01	2.401e-01	2.482e-01	9.038e-01
3	137	7.199803e-01	2.148e-01	5.765e-02	9.956e-02	8.320e-01
4	182	7.226496e-01	2.134e-01	6.782e-03	1.237e-02	9.556e+00
5	217	7.506090e-01	1.621e-01	2.401e-01	2.927e-01	1.365e+00
6	249	1.018032e+00	4.864e-02	7.000e-01	4.074e-01	1.564e+00
7	280	1.138203e+00	0.000e+00	1.000e+00	3.986e-01	1.730e+00
8	312	1.089659e+00	0.000e+00	7.000e-01	2.766e-01	1.834e+00

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
9	344	1.079187e+00	0.000e+00	7.000e-01	2.754e-01	1.854e+00
10	376	1.052501e+00	0.000e+00	7.000e-01	2.114e-01	1.807e+00
11	408	1.043898e+00	0.000e+00	7.000e-01	2.289e-01	1.780e+00
12	440	1.028618e+00	0.000e+00	7.000e-01	1.743e-01	1.801e+00
13	472	1.020757e+00	0.000e+00	7.000e-01	1.715e-01	1.835e+00
14	504	1.011666e+00	0.000e+00	7.000e-01	1.428e-01	1.841e+00
15	536	1.004509e+00	0.000e+00	7.000e-01	1.374e-01	1.823e+00
16	568	9.966581e-01	0.000e+00	7.000e-01	1.285e-01	1.810e+00
17	600	9.919059e-01	0.000e+00	7.000e-01	1.211e-01	1.815e+00
18	631	9.858665e-01	0.000e+00	1.000e+00	1.539e-01	1.835e+00
19	663	9.830327e-01	0.000e+00	7.000e-01	1.089e-01	1.837e+00
20	694	9.781328e-01	0.000e+00	1.000e+00	1.752e-01	1.821e+00
21	726	9.742403e-01	0.000e+00	7.000e-01	1.055e-01	1.815e+00
22	757	9.724051e-01	0.000e+00	1.000e+00	1.769e-01	1.819e+00
23	789	9.673708e-01	0.000e+00	7.000e-01	9.615e-02	1.823e+00
24	821	9.595208e-01	0.000e+00	7.000e-01	1.198e-01	1.821e+00
25	854	9.563263e-01	0.000e+00	4.900e-01	6.450e-02	1.814e+00
26	885	9.513725e-01	0.000e+00	1.000e+00	9.931e-02	1.795e+00
27	917	9.480460e-01	0.000e+00	7.000e-01	1.100e-01	1.805e+00
28	948	9.342793e-01	0.000e+00	1.000e+00	8.007e-02	1.781e+00
29	979	9.147575e-01	0.000e+00	1.000e+00	1.640e-01	1.760e+00
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
30	1010	8.703939e-01	0.000e+00	1.000e+00	1.240e-01	1.688e+00
31	1041	5.931631e-01	0.000e+00	1.000e+00	9.008e-01	1.050e+00
32	1072	3.789344e-01	0.000e+00	1.000e+00	4.611e-01	4.962e-01
33	1103	2.277291e-01	0.000e+00	1.000e+00	5.351e-01	4.549e-01
34	1142	2.233931e-01	0.000e+00	5.765e-02	3.391e-02	4.374e-01
35	1188	2.229087e-01	0.000e+00	4.748e-03	4.052e-03	4.357e-01
36	1239	2.228200e-01	0.000e+00	7.979e-04	7.422e-04	7.010e-01
37	1277	2.055736e-01	0.000e+00	8.235e-02	1.342e-01	4.193e-01
38	1318	2.038573e-01	0.000e+00	2.825e-02	1.480e-02	4.079e-01
39	1365	2.036490e-01	0.000e+00	3.323e-03	1.862e-03	6.813e-01
40	1401	1.854562e-01	0.000e+00	1.681e-01	2.114e-01	3.726e-01
41	1436	1.832160e-01	0.000e+00	2.401e-01	4.553e-02	2.842e-01
42	1474	1.827084e-01	0.000e+00	8.235e-02	1.335e-02	2.659e+00
43	1507	1.817833e-01	0.000e+00	4.900e-01	1.972e-01	1.547e-01
44	1538	1.804912e-01	0.000e+00	1.000e+00	1.031e-01	9.838e-02
45	1569	1.803867e-01	0.000e+00	1.000e+00	2.001e-02	9.918e-02
46	1600	1.803473e-01	0.000e+00	1.000e+00	9.368e-03	9.832e-02
47	1631	1.803048e-01	0.000e+00	1.000e+00	1.253e-02	9.659e-02
48	1662	1.802746e-01	0.000e+00	1.000e+00	8.701e-03	9.570e-02
49	1693	1.802186e-01	0.000e+00	1.000e+00	1.266e-02	9.483e-02
50	1724	1.801193e-01	0.000e+00	1.000e+00	1.535e-02	9.400e-02
51	1757	1.799917e-01	0.000e+00	4.900e-01	1.123e-02	9.362e-02
52	1798	1.799672e-01	0.000e+00	2.825e-02	1.294e-03	9.356e-02
53	1849	1.799666e-01	0.000e+00	7.979e-04	6.799e-05	3.372e-01
54	1880	1.798228e-01	0.000e+00	1.000e+00	2.163e-02	8.773e-02
55	1911	1.797841e-01	0.000e+00	1.000e+00	7.388e-03	8.821e-02
56	1942	1.797247e-01	0.000e+00	1.000e+00	1.442e-02	8.959e-02
57	1973	1.796623e-01	0.000e+00	1.000e+00	1.241e-02	8.891e-02
58	2006	1.796177e-01	0.000e+00	4.900e-01	8.990e-03	1.876e+00
59	2038	1.794735e-01	0.000e+00	7.000e-01	1.324e-02	7.945e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
60	2080	1.794734e-01	0.000e+00	1.977e-02	1.191e-04	5.677e-01
61	2111	1.791900e-01	0.000e+00	1.000e+00	5.752e-02	9.683e-02
62	2142	1.790400e-01	0.000e+00	1.000e+00	1.711e-02	9.231e-02
63	2173	1.789940e-01	0.000e+00	1.000e+00	8.847e-03	9.332e-02
64	2204	1.789830e-01	0.000e+00	1.000e+00	5.372e-03	9.513e-02
65	2235	1.789815e-01	0.000e+00	1.000e+00	1.631e-03	9.579e-02
66	2266	1.789795e-01	0.000e+00	1.000e+00	2.037e-03	9.637e-02
67	2297	1.789761e-01	0.000e+00	1.000e+00	2.926e-03	9.669e-02
68	2328	1.789710e-01	0.000e+00	1.000e+00	3.262e-03	9.637e-02

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
69	2359	1.789644e-01	0.000e+00	1.000e+00	2.564e-03	9.531e-02
70	2390	1.789550e-01	0.000e+00	1.000e+00	1.785e-03	9.368e-02
71	2421	1.789368e-01	0.000e+00	1.000e+00	1.912e-03	9.134e-02
72	2453	1.789046e-01	0.000e+00	7.000e-01	2.268e-03	8.888e-02
73	2492	1.788959e-01	0.000e+00	5.765e-02	4.112e-04	8.851e-02
74	2523	1.788547e-01	0.000e+00	1.000e+00	5.176e-03	9.274e-02
75	2554	1.787689e-01	0.000e+00	1.000e+00	1.478e-02	1.072e-01
76	2585	1.787344e-01	0.000e+00	1.000e+00	4.417e-03	1.085e-01
77	2618	1.787120e-01	0.000e+00	4.900e-01	2.791e-03	1.092e-01
78	2661	1.787114e-01	0.000e+00	1.384e-02	8.841e-05	1.344e-01
79	2692	1.787000e-01	0.000e+00	1.000e+00	6.772e-03	1.137e-01
80	2723	1.786974e-01	0.000e+00	1.000e+00	1.293e-03	1.133e-01
81	2754	1.786934e-01	0.000e+00	1.000e+00	3.006e-03	1.130e-01
82	2785	1.786918e-01	0.000e+00	1.000e+00	7.728e-04	1.131e-01
83	2816	1.786837e-01	0.000e+00	1.000e+00	2.772e-03	1.137e-01
84	2852	1.786837e-01	0.000e+00	1.681e-01	5.390e-04	5.479e-01
85	2883	1.786762e-01	0.000e+00	1.000e+00	7.154e-03	1.186e-01
86	2914	1.786727e-01	0.000e+00	1.000e+00	2.540e-03	1.176e-01
87	2945	1.786698e-01	0.000e+00	1.000e+00	1.535e-03	1.175e-01
88	2976	1.786664e-01	0.000e+00	1.000e+00	1.358e-03	1.179e-01
89	3007	1.786642e-01	0.000e+00	1.000e+00	2.966e-03	1.197e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
90	3038	1.786601e-01	0.000e+00	1.000e+00	1.511e-03	1.204e-01
91	3069	1.786412e-01	0.000e+00	1.000e+00	5.900e-03	1.224e-01
92	3101	1.786387e-01	0.000e+00	7.000e-01	3.069e-03	1.262e+00
93	3140	1.786268e-01	0.000e+00	5.765e-02	3.013e-03	6.111e-01
94	3176	1.786218e-01	0.000e+00	1.681e-01	7.839e-03	1.259e-01
95	3208	1.786156e-01	0.000e+00	7.000e-01	7.944e-03	1.255e-01
96	3239	1.786097e-01	0.000e+00	1.000e+00	4.440e-03	1.268e-01
97	3272	1.786082e-01	0.000e+00	4.900e-01	2.958e-04	9.989e-02
98	3303	1.785898e-01	0.000e+00	1.000e+00	9.269e-03	1.235e-01
99	3337	1.785837e-01	0.000e+00	3.430e-01	3.634e-04	5.462e-01
100	3368	1.785660e-01	0.000e+00	1.000e+00	7.365e-03	1.202e-01
101	3399	1.785615e-01	0.000e+00	1.000e+00	5.067e-04	1.198e-01
102	3432	1.785604e-01	0.000e+00	4.900e-01	2.372e-04	1.223e+00
103	3463	1.785488e-01	0.000e+00	1.000e+00	3.800e-03	1.176e-01
104	3494	1.785420e-01	0.000e+00	1.000e+00	9.335e-04	1.173e-01
105	3534	1.785418e-01	0.000e+00	4.035e-02	6.529e-05	2.537e-01
106	3565	1.785389e-01	0.000e+00	1.000e+00	1.683e-03	1.161e-01
107	3596	1.785364e-01	0.000e+00	1.000e+00	5.339e-04	1.156e-01
108	3627	1.785351e-01	0.000e+00	1.000e+00	2.267e-03	6.912e-01
109	3660	1.785336e-01	0.000e+00	4.900e-01	2.902e-03	1.118e-01
110	3691	1.785324e-01	0.000e+00	1.000e+00	1.904e-03	2.978e-01
111	3722	1.785312e-01	0.000e+00	1.000e+00	3.062e-04	2.775e-01
112	3753	1.785254e-01	0.000e+00	1.000e+00	6.357e-04	2.315e-01
113	3786	1.785218e-01	0.000e+00	4.900e-01	9.409e-04	2.259e-01
114	3820	1.785211e-01	0.000e+00	3.430e-01	1.777e-03	3.480e-01
115	3855	1.785173e-01	0.000e+00	2.401e-01	7.096e-03	1.069e-01
116	3886	1.785105e-01	0.000e+00	1.000e+00	4.216e-03	5.723e-02
117	3917	1.785093e-01	0.000e+00	1.000e+00	1.031e-03	1.459e-01
118	3948	1.785081e-01	0.000e+00	1.000e+00	2.599e-03	1.020e-01
119	3979	1.785061e-01	0.000e+00	1.000e+00	2.940e-04	2.377e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
120	4010	1.785015e-01	0.000e+00	1.000e+00	1.086e-03	4.306e-01
121	4041	1.784939e-01	0.000e+00	1.000e+00	4.044e-03	4.585e-01
122	4073	1.784921e-01	0.000e+00	7.000e-01	1.240e-03	3.167e-01
123	4104	1.784877e-01	0.000e+00	1.000e+00	2.766e-03	9.626e-02
124	4136	1.784868e-01	0.000e+00	7.000e-01	1.600e-03	1.886e-01
125	4167	1.784836e-01	0.000e+00	1.000e+00	7.805e-04	8.586e-02
126	4198	1.784821e-01	0.000e+00	1.000e+00	1.020e-03	1.273e-01
127	4229	1.784806e-01	0.000e+00	1.000e+00	5.044e-04	1.729e-01
128	4260	1.784784e-01	0.000e+00	1.000e+00	6.838e-04	1.797e-01

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
129	4291	1.784770e-01	0.000e+00	1.000e+00	5.871e-04	1.386e-01
130	4322	1.784742e-01	0.000e+00	1.000e+00	1.606e-03	1.789e-01
131	4355	1.784715e-01	0.000e+00	4.900e-01	1.395e-03	1.854e-01
132	4386	1.784672e-01	0.000e+00	1.000e+00	1.302e-02	1.012e-01
133	4417	1.784660e-01	0.000e+00	1.000e+00	6.964e-03	8.129e-01
134	4448	1.784573e-01	0.000e+00	1.000e+00	1.915e-03	9.941e-02
135	4479	1.784547e-01	0.000e+00	1.000e+00	7.216e-04	9.854e-02
136	4510	1.784464e-01	0.000e+00	1.000e+00	2.769e-03	9.508e-02
137	4541	1.784445e-01	0.000e+00	1.000e+00	1.642e-03	9.365e-02
138	4572	1.784438e-01	0.000e+00	1.000e+00	5.629e-04	9.348e-02
139	4607	1.784436e-01	0.000e+00	2.401e-01	3.941e-04	1.589e-01
140	4638	1.784425e-01	0.000e+00	1.000e+00	2.191e-03	9.449e-02
141	4669	1.784421e-01	0.000e+00	1.000e+00	2.986e-04	7.536e-02
142	4701	1.784417e-01	0.000e+00	7.000e-01	1.226e-03	1.116e-01
143	4732	1.784416e-01	0.000e+00	1.000e+00	4.521e-04	7.697e-02
144	4763	1.784414e-01	0.000e+00	1.000e+00	4.053e-04	7.674e-02
145	4794	1.784413e-01	0.000e+00	1.000e+00	5.853e-04	7.664e-02
146	4825	1.784411e-01	0.000e+00	1.000e+00	6.187e-04	7.668e-02
147	4856	1.784408e-01	0.000e+00	1.000e+00	1.023e-03	7.703e-02
148	4887	1.784400e-01	0.000e+00	1.000e+00	1.417e-03	1.792e-01
149	4923	1.784397e-01	0.000e+00	1.681e-01	1.306e-03	7.804e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
150	4954	1.784389e-01	0.000e+00	1.000e+00	6.984e-04	7.865e-02
151	4987	1.784382e-01	0.000e+00	4.900e-01	9.229e-04	1.698e-01
152	5019	1.784378e-01	0.000e+00	7.000e-01	8.783e-04	2.696e-01
153	5051	1.784375e-01	0.000e+00	7.000e-01	8.143e-04	7.880e-02
154	5082	1.784370e-01	0.000e+00	1.000e+00	6.939e-05	9.530e-02
155	5114	1.784368e-01	0.000e+00	7.000e-01	4.508e-04	2.242e-01
156	5145	1.784359e-01	0.000e+00	1.000e+00	5.426e-04	1.330e-01
157	5176	1.784351e-01	0.000e+00	1.000e+00	1.836e-04	1.355e-01
158	5208	1.784349e-01	0.000e+00	7.000e-01	7.194e-04	4.977e-01
159	5239	1.784336e-01	0.000e+00	1.000e+00	8.940e-04	7.721e-02
160	5270	1.784331e-01	0.000e+00	1.000e+00	3.600e-04	7.683e-02
161	5301	1.784327e-01	0.000e+00	1.000e+00	1.869e-03	3.072e-01
162	5332	1.784324e-01	0.000e+00	1.000e+00	1.359e-03	7.642e-02
163	5363	1.784321e-01	0.000e+00	1.000e+00	1.893e-04	7.628e-02
164	5396	1.784318e-01	0.000e+00	4.900e-01	4.625e-04	1.555e-01
165	5428	1.784317e-01	0.000e+00	7.000e-01	1.970e-04	8.584e-02
166	5459	1.784316e-01	0.000e+00	1.000e+00	6.438e-05	8.045e-02
167	5490	1.784313e-01	0.000e+00	1.000e+00	2.210e-04	6.865e-02
168	5521	1.784310e-01	0.000e+00	1.000e+00	3.357e-04	8.496e-02
169	5552	1.784308e-01	0.000e+00	1.000e+00	3.018e-04	4.520e-02
170	5583	1.784308e-01	0.000e+00	1.000e+00	1.104e-04	1.069e-02
171	5614	1.784308e-01	0.000e+00	1.000e+00	6.445e-05	1.023e-02
172	5645	1.784308e-01	0.000e+00	1.000e+00	3.526e-05	1.968e-02
173	5676	1.784308e-01	0.000e+00	1.000e+00	6.005e-05	3.041e-02
174	5707	1.784307e-01	0.000e+00	1.000e+00	1.120e-04	4.075e-02
175	5738	1.784306e-01	0.000e+00	1.000e+00	2.646e-04	5.397e-02
176	5770	1.784304e-01	0.000e+00	7.000e-01	3.326e-04	6.895e-02
177	5806	1.784303e-01	0.000e+00	1.681e-01	1.301e-04	7.343e-02
178	5844	1.784302e-01	0.000e+00	8.235e-02	9.353e-05	1.664e+02
179	5877	1.784302e-01	0.000e+00	4.900e-01	1.835e-04	5.130e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
180	5908	1.784301e-01	0.000e+00	1.000e+00	8.481e-05	3.530e-02
181	5939	1.784299e-01	0.000e+00	1.000e+00	2.327e-04	2.686e-02
182	5970	1.784298e-01	0.000e+00	1.000e+00	1.107e-04	3.972e-02
183	6001	1.784295e-01	0.000e+00	1.000e+00	3.499e-04	4.317e-02
184	6032	1.784290e-01	0.000e+00	1.000e+00	4.855e-04	4.875e-02
185	6063	1.784286e-01	0.000e+00	1.000e+00	5.053e-04	2.512e-02
186	6094	1.784285e-01	0.000e+00	1.000e+00	3.133e-04	7.506e-03
187	6125	1.784285e-01	0.000e+00	1.000e+00	6.455e-05	4.900e-03
188	6156	1.784285e-01	0.000e+00	1.000e+00	1.822e-05	4.962e-03

```
  189        6187    1.784285e-01     0.000e+00     1.000e+00     3.097e-06     4.811e-03
  190        6218    1.784285e-01     0.000e+00     1.000e+00     2.655e-07     4.752e-03

Local minimum possible. Constraints satisfied.

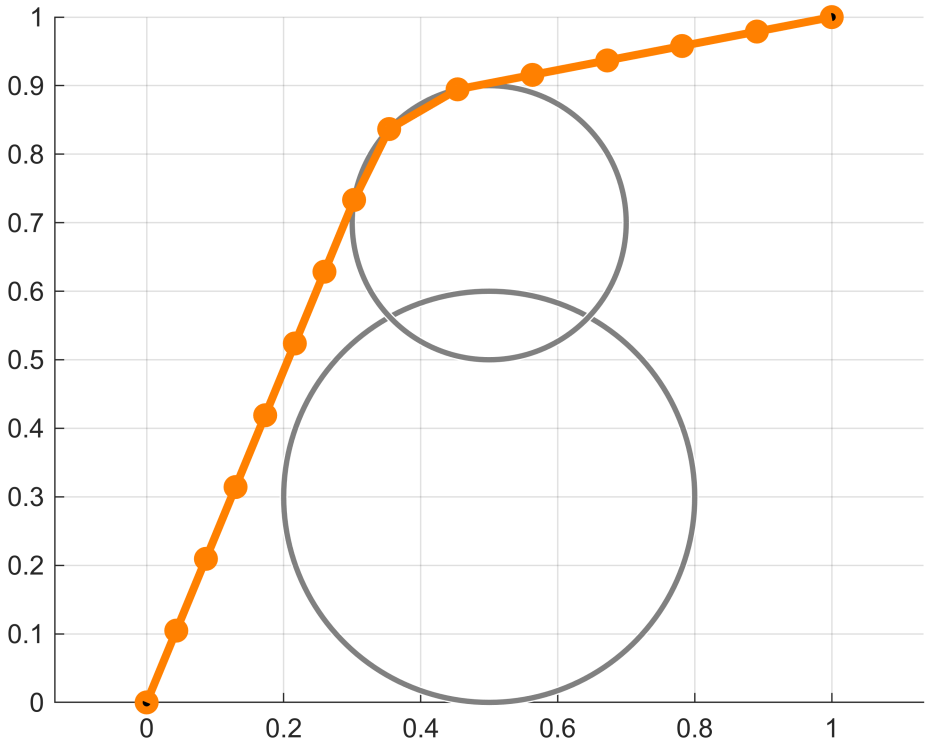
fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
xi_star = reshape(xi_star_vec,2,[]); % final optimized trajectory

% Plot obstacles
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center1', r1, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', r2, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Plot result
grid on
hold on
axis equal
plot(xi_star(1,:), xi_star(2,:), 'o-',...
    'Color', [1, 0.5, 0], 'LineWidth', 3);
```




```
% Cost function to minimize
```

```
function C = cost(xi)
    gamma=20;
    xi = reshape(xi,2,[]);
    C = 0;
    r1 = 0.3;
    center1 = [0.5;0.3];
    r2 = 0.2;
    center2 = [0.5;0.7];
    Urep1 = 0;
    Urep2 = 0;

    for idx = 2:length(xi)

        % First obstacle
        if (norm(center1 - xi(:,idx)) <= r1)
            Urep1 = 0.5*gamma*((1/(norm(center1 - xi(:,idx)))) - (1/r1))^2;
        end

        % Second obstacle
        if (norm(center2 - xi(:,idx)) <= r2)
            Urep2 = 0.5*gamma*((1/(norm(center2 - xi(:,idx)))) - (1/r2))^2;
        end
```

```
        % Total cost
        C = C + norm(xi(:,idx) - xi(:,idx-1))^2 + Urep1 + Urep2;
    end
end
```

- **Environment 3.** One obstacle with center $c_1 = [0.2, 0.35]^T$ and radius $r_1 = 0.2$. A second obstacle with center $c_2 = [0.5, 0.3]^T$ and radius $r_2 = 0.2$. A third obstacle with center $c_3 = [0.7, 0.5]^T$ and radius $r_3 = 0.2$. Set $k = 20$.

```
% Problem 2.2 - Environment 3

clear
clc
close all

% Start and Goal orientations
theta_start = [0;0];
theta_goal = [1;1];

% Initial trajectory variables
n = 2; % No. of joints / 2-D trajectory.
k = 20; % No. of waypoints

% Obstacle paramters
% First obstacle center and radius
r1 = 0.2;
center1 = [0.2;0.35];
% Second obstacle center and radius
r2 = 0.2;
center2 = [0.5;0.3];
% Third obstacle center and radius
r3 = 0.2;
center3 = [0.7;0.5];

xi_0 = zeros(n, k); % Initial trajectory
xi_0_vec = reshape(xi_0, [],1); % Reshape for the requirement of optimization

% Equality constraints for start and goal positions
A = [eye(n), zeros(n,n*(k-1)) ;...
    zeros(n,n*(k-1)), eye(n) ];

B = [theta_start;theta_goal];

% Nonlinear optimization
options = optimoptions('fmincon','Display','iter',...
    'Algorithm','sqp','MaxFunctionEvaluations',1e5);
xi_star_vec = fmincon(@(xi) cost(xi), xi_0_vec, ...
    [], [], A, B, [], [], [], options);
```

```
 Iter  Func-count           Fval   Feasibility   Step Length       Norm of    First-order
                                                                      step     optimality
    0         41   0.000000e+00     1.000e+00     1.000e+00   0.000e+00     2.980e-08
    1         83   9.800000e-01     3.000e-01     7.000e-01   9.899e-01     1.400e+00
    2        128   1.032368e+00     2.280e-01     2.401e-01   2.482e-01     7.826e+01
    3        172   1.166409e+00     1.498e-01     3.430e-01   5.044e-01     1.723e+00
```

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
4	216	1.238832e+00	9.840e-02	3.430e-01	5.114e-01	1.584e+00
5	257	1.427121e+00	0.000e+00	1.000e+00	4.865e-01	2.040e+00
6	299	1.408923e+00	0.000e+00	7.000e-01	4.916e-01	2.230e+00
7	341	1.312863e+00	0.000e+00	7.000e-01	2.971e-01	2.083e+00
8	384	1.247338e+00	0.000e+00	4.900e-01	2.602e-01	1.940e+00
9	427	1.229228e+00	0.000e+00	4.900e-01	1.654e-01	1.974e+00
10	468	1.225647e+00	0.000e+00	1.000e+00	2.516e-01	2.114e+00
11	510	1.207887e+00	0.000e+00	7.000e-01	1.934e-01	2.127e+00
12	552	1.192803e+00	0.000e+00	7.000e-01	1.652e-01	2.025e+00
13	594	1.182451e+00	0.000e+00	7.000e-01	1.626e-01	1.956e+00
14	636	1.170593e+00	0.000e+00	7.000e-01	1.538e-01	2.003e+00
15	678	1.157015e+00	0.000e+00	7.000e-01	1.828e-01	2.083e+00
16	720	1.149073e+00	0.000e+00	7.000e-01	1.842e-01	2.085e+00
17	761	1.141037e+00	0.000e+00	1.000e+00	2.279e-01	1.987e+00
18	802	1.137807e+00	0.000e+00	1.000e+00	2.874e-01	1.898e+00
19	843	1.133690e+00	0.000e+00	1.000e+00	2.867e-01	1.975e+00
20	885	1.100406e+00	0.000e+00	7.000e-01	2.269e-01	2.049e+00
21	927	1.078630e+00	0.000e+00	7.000e-01	2.332e-01	2.003e+00
22	968	1.051276e+00	0.000e+00	1.000e+00	2.396e-01	1.902e+00
23	1009	1.037429e+00	0.000e+00	1.000e+00	2.775e-01	1.899e+00
24	1051	1.023951e+00	0.000e+00	7.000e-01	3.545e-01	1.915e+00
25	1092	9.978490e-01	0.000e+00	1.000e+00	5.106e-01	1.891e+00
26	1134	9.857902e-01	0.000e+00	7.000e-01	3.486e-01	1.807e+00
27	1175	9.762593e-01	0.000e+00	1.000e+00	2.605e-01	1.703e+00
28	1217	9.526427e-01	0.000e+00	7.000e-01	1.988e-01	1.695e+00
29	1259	9.297930e-01	0.000e+00	7.000e-01	1.926e-01	1.737e+00
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
30	1301	9.143387e-01	0.000e+00	7.000e-01	1.930e-01	1.740e+00
31	1342	9.044437e-01	0.000e+00	1.000e+00	2.130e-01	1.651e+00
32	1383	8.837833e-01	0.000e+00	1.000e+00	2.895e-01	1.536e+00
33	1424	8.810792e-01	0.000e+00	1.000e+00	3.333e-01	1.531e+00
34	1465	8.288351e-01	0.000e+00	1.000e+00	3.872e-01	1.570e+00
35	1507	7.715599e-01	0.000e+00	7.000e-01	3.329e-01	1.484e+00
36	1549	6.825512e-01	0.000e+00	7.000e-01	3.061e-01	1.285e+00
37	1591	6.280434e-01	0.000e+00	7.000e-01	3.896e-01	1.144e+00
38	1635	5.430207e-01	0.000e+00	3.430e-01	1.642e-01	1.100e+00
39	1684	5.169726e-01	0.000e+00	5.765e-02	4.499e-02	1.106e+00
40	1739	5.158960e-01	0.000e+00	6.782e-03	7.576e-03	6.695e+00
41	1798	4.922219e-01	0.000e+00	1.628e-03	4.094e-02	5.018e+00
42	1850	4.250983e-01	0.000e+00	1.977e-02	1.309e-01	1.010e+01
43	1904	3.332649e-01	0.000e+00	9.689e-03	2.581e-01	7.195e-01
44	1956	3.294927e-01	0.000e+00	1.977e-02	1.462e-02	3.819e+00
45	2014	3.262540e-01	0.000e+00	2.326e-03	2.342e-02	8.597e+00
46	2072	3.102756e-01	0.000e+00	2.326e-03	6.295e-02	1.444e+01
47	2124	3.056068e-01	0.000e+00	1.977e-02	1.758e-01	2.652e+01
48	2183	3.047742e-01	0.000e+00	1.628e-03	3.668e-02	1.991e+01
49	2246	2.982285e-01	0.000e+00	3.910e-04	3.296e-02	1.708e+01
50	2299	2.964518e-01	0.000e+00	1.384e-02	1.952e-01	3.449e+01
51	2362	2.961624e-01	0.000e+00	3.910e-04	8.322e-03	3.089e+01
52	2415	2.716860e-01	0.000e+00	1.384e-02	2.817e-01	7.151e-01
53	2458	2.311177e-01	0.000e+00	4.900e-01	3.884e-01	3.494e-01
54	2499	2.180983e-01	0.000e+00	1.000e+00	1.136e-01	3.815e-01
55	2540	2.151691e-01	0.000e+00	1.000e+00	4.525e-02	4.276e-01
56	2586	2.151170e-01	0.000e+00	1.681e-01	7.905e-03	1.413e+00
57	2633	2.139996e-01	0.000e+00	1.176e-01	9.096e-02	4.462e-01
58	2678	2.138465e-01	0.000e+00	2.401e-01	6.002e-03	4.430e-01
59	2728	2.138210e-01	0.000e+00	4.035e-02	1.805e-03	5.542e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
60	2774	2.134354e-01	0.000e+00	1.681e-01	4.897e-02	4.114e-01
61	2815	2.131783e-01	0.000e+00	1.000e+00	1.441e-02	4.199e-01
62	2856	2.128902e-01	0.000e+00	1.000e+00	1.050e-02	4.247e-01
63	2897	2.116399e-01	0.000e+00	1.000e+00	2.664e-02	4.291e-01

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
64	2938	2.092965e-01	0.000e+00	1.000e+00	3.863e-02	4.237e-01
65	2979	2.048284e-01	0.000e+00	1.000e+00	5.170e-02	7.691e-01
66	3027	2.047079e-01	0.000e+00	8.235e-02	1.721e-02	4.702e+00
67	3078	2.036215e-01	0.000e+00	2.825e-02	5.226e-02	2.976e+00
68	3131	2.035357e-01	0.000e+00	1.384e-02	6.002e-03	2.316e+00
69	3175	2.035218e-01	0.000e+00	3.430e-01	2.567e-02	2.889e-01
70	3216	2.001322e-01	0.000e+00	1.000e+00	6.030e-02	2.985e-01
71	3257	1.992216e-01	0.000e+00	1.000e+00	1.564e-02	2.715e-01
72	3299	1.989130e-01	0.000e+00	7.000e-01	2.308e-02	2.623e+00
73	3340	1.986879e-01	0.000e+00	1.000e+00	1.508e-02	2.533e-01
74	3381	1.984363e-01	0.000e+00	1.000e+00	3.180e-03	2.520e-01
75	3422	1.981057e-01	0.000e+00	1.000e+00	1.525e-02	3.003e+00
76	3463	1.980505e-01	0.000e+00	1.000e+00	1.918e-02	3.669e+00
77	3506	1.975034e-01	0.000e+00	4.900e-01	2.519e-02	1.976e+00
78	3547	1.966637e-01	0.000e+00	1.000e+00	3.367e-02	2.338e-01
79	3588	1.961426e-01	0.000e+00	1.000e+00	7.325e-03	2.265e-01
80	3629	1.952796e-01	0.000e+00	1.000e+00	1.801e-02	2.167e-01
81	3670	1.947995e-01	0.000e+00	1.000e+00	1.100e-02	2.131e-01
82	3712	1.947855e-01	0.000e+00	7.000e-01	1.618e-02	4.566e+00
83	3753	1.930827e-01	0.000e+00	1.000e+00	2.379e-02	1.941e-01
84	3794	1.921475e-01	0.000e+00	1.000e+00	9.998e-03	1.922e-01
85	3835	1.907505e-01	0.000e+00	1.000e+00	2.459e-02	2.018e-01
86	3876	1.904790e-01	0.000e+00	1.000e+00	5.582e-03	2.084e-01
87	3917	1.895965e-01	0.000e+00	1.000e+00	2.252e-02	2.353e-01
88	3958	1.892330e-01	0.000e+00	1.000e+00	9.181e-03	2.433e-01
89	3999	1.886308e-01	0.000e+00	1.000e+00	1.316e-02	2.487e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
90	4040	1.877134e-01	0.000e+00	1.000e+00	1.749e-02	2.501e-01
91	4081	1.863029e-01	0.000e+00	1.000e+00	2.920e-02	2.483e-01
92	4123	1.851641e-01	0.000e+00	7.000e-01	2.494e-02	2.624e+00
93	4164	1.846451e-01	0.000e+00	1.000e+00	6.708e-02	2.185e-01
94	4205	1.831891e-01	0.000e+00	1.000e+00	4.166e-02	2.302e-01
95	4246	1.829963e-01	0.000e+00	1.000e+00	9.062e-03	2.317e-01
96	4287	1.828049e-01	0.000e+00	1.000e+00	9.791e-03	2.326e-01
97	4328	1.827039e-01	0.000e+00	1.000e+00	5.615e-03	2.322e-01
98	4369	1.826070e-01	0.000e+00	1.000e+00	6.538e-03	2.310e-01
99	4410	1.824815e-01	0.000e+00	1.000e+00	8.579e-03	2.294e-01
100	4451	1.822053e-01	0.000e+00	1.000e+00	1.769e-02	2.261e-01
101	4492	1.816840e-01	0.000e+00	1.000e+00	3.145e-02	2.209e-01
102	4533	1.808944e-01	0.000e+00	1.000e+00	4.479e-02	2.142e-01
103	4574	1.801095e-01	0.000e+00	1.000e+00	3.807e-02	2.090e-01
104	4615	1.795151e-01	0.000e+00	1.000e+00	1.646e-02	2.072e-01
105	4656	1.789950e-01	0.000e+00	1.000e+00	1.034e-02	1.478e+00
106	4697	1.784290e-01	0.000e+00	1.000e+00	4.905e-02	2.090e-01
107	4738	1.774766e-01	0.000e+00	1.000e+00	8.391e-03	2.065e-01
108	4780	1.762277e-01	0.000e+00	7.000e-01	3.181e-02	6.370e+00
109	4821	1.746588e-01	0.000e+00	1.000e+00	1.212e-01	1.723e-01
110	4862	1.742038e-01	0.000e+00	1.000e+00	5.717e-02	4.272e+00
111	4905	1.737577e-01	0.000e+00	4.900e-01	3.255e-02	1.891e-01
112	4946	1.722159e-01	0.000e+00	1.000e+00	3.144e-02	1.740e-01
113	4987	1.706296e-01	0.000e+00	1.000e+00	2.060e-02	1.577e-01
114	5035	1.705530e-01	0.000e+00	8.235e-02	3.939e-03	2.779e+00
115	5076	1.694870e-01	0.000e+00	1.000e+00	1.558e-02	1.510e-01
116	5117	1.686177e-01	0.000e+00	1.000e+00	2.775e-02	1.482e-01
117	5158	1.680972e-01	0.000e+00	1.000e+00	1.543e-02	1.420e-01
118	5199	1.669887e-01	0.000e+00	1.000e+00	2.852e-02	4.721e-01
119	5240	1.663867e-01	0.000e+00	1.000e+00	1.669e-02	3.765e+00
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
120	5281	1.649704e-01	0.000e+00	1.000e+00	3.368e-02	2.051e-01
121	5322	1.638646e-01	0.000e+00	1.000e+00	2.373e-02	4.000e+00
122	5364	1.626223e-01	0.000e+00	7.000e-01	7.860e-02	9.445e+00
123	5405	1.616939e-01	0.000e+00	1.000e+00	5.883e-02	6.076e+00

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
124	5446	1.592708e-01	0.000e+00	1.000e+00	4.131e-02	4.542e-01
125	5487	1.565081e-01	0.000e+00	1.000e+00	1.948e-02	3.188e-01
126	5528	1.508531e-01	0.000e+00	1.000e+00	9.086e-02	4.580e+00
127	5569	1.498331e-01	0.000e+00	1.000e+00	1.943e-02	1.005e-01
128	5610	1.496818e-01	0.000e+00	1.000e+00	7.420e-03	9.608e-02
129	5651	1.495678e-01	0.000e+00	1.000e+00	6.632e-03	9.156e-02
130	5692	1.494120e-01	0.000e+00	1.000e+00	7.546e-03	8.746e-02
131	5737	1.493882e-01	0.000e+00	2.401e-01	4.539e-03	1.147e+00
132	5778	1.492917e-01	0.000e+00	1.000e+00	4.431e-03	8.898e-02
133	5819	1.492052e-01	0.000e+00	1.000e+00	4.085e-03	9.106e-02
134	5860	1.489393e-01	0.000e+00	1.000e+00	1.675e-02	9.775e-02
135	5901	1.487239e-01	0.000e+00	1.000e+00	1.427e-02	1.007e-01
136	5942	1.482010e-01	0.000e+00	1.000e+00	3.342e-02	1.023e-01
137	5983	1.476912e-01	0.000e+00	1.000e+00	3.714e-02	9.777e-02
138	6024	1.473994e-01	0.000e+00	1.000e+00	2.076e-02	8.976e-02
139	6065	1.472971e-01	0.000e+00	1.000e+00	7.228e-03	8.687e-02
140	6106	1.472564e-01	0.000e+00	1.000e+00	5.343e-03	8.721e-02
141	6147	1.472140e-01	0.000e+00	1.000e+00	5.220e-03	8.741e-02
142	6188	1.471456e-01	0.000e+00	1.000e+00	5.655e-03	8.757e-02
143	6229	1.470070e-01	0.000e+00	1.000e+00	7.323e-03	8.775e-02
144	6270	1.466989e-01	0.000e+00	1.000e+00	1.124e-02	8.808e-02
145	6311	1.460374e-01	0.000e+00	1.000e+00	1.959e-02	8.880e-02
146	6352	1.448720e-01	0.000e+00	1.000e+00	3.362e-02	9.021e-02
147	6393	1.435530e-01	0.000e+00	1.000e+00	4.362e-02	9.203e-02
148	6434	1.428449e-01	0.000e+00	1.000e+00	3.117e-02	9.303e-02
149	6475	1.426962e-01	0.000e+00	1.000e+00	1.174e-02	9.295e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
150	6516	1.426730e-01	0.000e+00	1.000e+00	4.298e-03	9.268e-02
151	6557	1.426483e-01	0.000e+00	1.000e+00	3.498e-03	9.227e-02
152	6598	1.425765e-01	0.000e+00	1.000e+00	6.540e-03	9.103e-02
153	6639	1.424109e-01	0.000e+00	1.000e+00	9.650e-03	8.807e-02
154	6680	1.420268e-01	0.000e+00	1.000e+00	1.538e-02	8.101e-02
155	6724	1.417335e-01	0.000e+00	3.430e-01	7.874e-03	7.635e-02
156	6767	1.410427e-01	0.000e+00	4.900e-01	2.409e-02	9.778e-01
157	6808	1.406825e-01	0.000e+00	1.000e+00	3.057e-02	7.193e-02
158	6849	1.403376e-01	0.000e+00	1.000e+00	1.522e-02	7.987e-02
159	6893	1.402265e-01	0.000e+00	3.430e-01	5.763e-03	1.356e-01
160	6942	1.402189e-01	0.000e+00	5.765e-02	1.427e-03	4.558e-01
161	6993	1.401938e-01	0.000e+00	2.825e-02	2.195e-02	8.818e-02
162	7034	1.400913e-01	0.000e+00	1.000e+00	1.071e-02	8.519e-02
163	7075	1.400109e-01	0.000e+00	1.000e+00	7.792e-03	8.299e-02
164	7123	1.400066e-01	0.000e+00	8.235e-02	9.677e-04	3.476e-01
165	7167	1.399779e-01	0.000e+00	3.430e-01	2.348e-03	2.133e-01
166	7217	1.399772e-01	0.000e+00	4.035e-02	4.003e-04	2.448e-01
167	7258	1.399685e-01	0.000e+00	1.000e+00	1.983e-03	8.297e-02
168	7299	1.399633e-01	0.000e+00	1.000e+00	8.201e-04	8.323e-02
169	7340	1.399351e-01	0.000e+00	1.000e+00	2.152e-03	8.359e-02
170	7381	1.398554e-01	0.000e+00	1.000e+00	7.338e-03	1.184e+00
171	7422	1.396612e-01	0.000e+00	1.000e+00	1.130e-02	7.566e-01
172	7471	1.396216e-01	0.000e+00	5.765e-02	1.494e-03	6.693e-01
173	7526	1.396213e-01	0.000e+00	6.782e-03	3.152e-04	6.598e-01
174	7567	1.392797e-01	0.000e+00	1.000e+00	3.264e-02	5.332e-01
175	7608	1.390628e-01	0.000e+00	1.000e+00	1.676e-02	2.599e-01
176	7649	1.389639e-01	0.000e+00	1.000e+00	1.643e-02	1.010e-01
177	7690	1.389404e-01	0.000e+00	1.000e+00	2.332e-03	5.620e-02
178	7731	1.389011e-01	0.000e+00	1.000e+00	3.988e-03	5.145e-02
179	7772	1.388261e-01	0.000e+00	1.000e+00	5.963e-03	5.203e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
180	7814	1.387806e-01	0.000e+00	7.000e-01	5.034e-03	9.548e-01
181	7857	1.386439e-01	0.000e+00	4.900e-01	1.363e-02	5.416e-02
182	7898	1.385190e-01	0.000e+00	1.000e+00	7.591e-03	5.383e-02
183	7939	1.382995e-01	0.000e+00	1.000e+00	2.427e-02	5.831e-02

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
184	7980	1.382478e-01	0.000e+00	1.000e+00	3.589e-03	5.824e-02
185	8022	1.381301e-01	0.000e+00	7.000e-01	9.081e-03	1.009e+00
186	8067	1.380655e-01	0.000e+00	2.401e-01	3.116e-03	3.240e-01
187	8115	1.380627e-01	0.000e+00	8.235e-02	4.627e-04	3.064e-01
188	8168	1.380626e-01	0.000e+00	1.384e-02	5.221e-05	3.047e-01
189	8211	1.380613e-01	0.000e+00	4.900e-01	7.238e-03	5.798e-02
190	8252	1.380375e-01	0.000e+00	1.000e+00	3.764e-03	1.643e-01
191	8293	1.380265e-01	0.000e+00	1.000e+00	9.388e-04	1.990e-01
192	8334	1.379606e-01	0.000e+00	1.000e+00	4.237e-03	8.963e-02
193	8376	1.379598e-01	0.000e+00	7.000e-01	1.067e-02	1.373e+00
194	8417	1.378728e-01	0.000e+00	1.000e+00	9.766e-03	6.792e-02
195	8458	1.378460e-01	0.000e+00	1.000e+00	2.144e-03	6.846e-02
196	8499	1.378085e-01	0.000e+00	1.000e+00	4.478e-03	7.194e-02
197	8542	1.378079e-01	0.000e+00	4.900e-01	9.438e-04	2.425e-01
198	8583	1.378009e-01	0.000e+00	1.000e+00	1.707e-03	7.299e-02
199	8624	1.377969e-01	0.000e+00	1.000e+00	5.675e-04	7.286e-02
200	8665	1.377725e-01	0.000e+00	1.000e+00	2.741e-03	7.103e-02
201	8706	1.377344e-01	0.000e+00	1.000e+00	3.228e-03	6.829e-02
202	8747	1.376524e-01	0.000e+00	1.000e+00	5.938e-03	6.264e-02
203	8793	1.376280e-01	0.000e+00	1.681e-01	1.382e-03	6.132e-02
204	8844	1.376269e-01	0.000e+00	2.825e-02	4.000e-04	3.240e-01
205	8885	1.375258e-01	0.000e+00	1.000e+00	1.122e-02	6.340e-02
206	8926	1.375110e-01	0.000e+00	1.000e+00	2.597e-03	6.250e-02
207	8967	1.375051e-01	0.000e+00	1.000e+00	1.254e-03	6.165e-02
208	9008	1.374978e-01	0.000e+00	1.000e+00	1.756e-03	9.091e-02
209	9049	1.374935e-01	0.000e+00	1.000e+00	1.293e-03	5.832e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
210	9090	1.374881e-01	0.000e+00	1.000e+00	4.176e-03	7.922e-02
211	9131	1.374858e-01	0.000e+00	1.000e+00	5.925e-04	6.056e-02
212	9172	1.374812e-01	0.000e+00	1.000e+00	9.752e-04	6.046e-02
213	9213	1.374617e-01	0.000e+00	1.000e+00	2.157e-03	6.144e-02
214	9255	1.374398e-01	0.000e+00	7.000e-01	3.398e-03	4.606e-01
215	9296	1.374280e-01	0.000e+00	1.000e+00	6.399e-03	3.371e-01
216	9342	1.374234e-01	0.000e+00	1.681e-01	1.607e-03	6.536e-02
217	9383	1.374144e-01	0.000e+00	1.000e+00	1.738e-03	2.507e-01
218	9436	1.374021e-01	0.000e+00	1.384e-02	6.336e-03	3.479e-01
219	9477	1.373960e-01	0.000e+00	1.000e+00	3.254e-03	6.694e-02
220	9518	1.373896e-01	0.000e+00	1.000e+00	7.653e-04	6.741e-02
221	9562	1.373832e-01	0.000e+00	3.430e-01	1.312e-03	2.432e-01
222	9603	1.373823e-01	0.000e+00	1.000e+00	6.296e-04	7.218e-02
223	9644	1.373789e-01	0.000e+00	1.000e+00	3.153e-04	6.824e-02
224	9687	1.373765e-01	0.000e+00	4.900e-01	7.271e-04	2.828e-01
225	9728	1.373708e-01	0.000e+00	1.000e+00	3.121e-04	1.778e-01
226	9769	1.373482e-01	0.000e+00	1.000e+00	2.665e-03	3.824e-01
227	9811	1.373480e-01	0.000e+00	7.000e-01	8.173e-04	6.793e-02
228	9852	1.373422e-01	0.000e+00	1.000e+00	5.313e-04	6.784e-02
229	9896	1.373422e-01	0.000e+00	3.430e-01	1.046e-03	3.995e-01
230	9937	1.373374e-01	0.000e+00	1.000e+00	2.890e-04	1.135e-01
231	9978	1.373360e-01	0.000e+00	1.000e+00	2.711e-04	8.866e-02
232	10019	1.373290e-01	0.000e+00	1.000e+00	1.463e-03	1.145e-01
233	10060	1.373169e-01	0.000e+00	1.000e+00	4.262e-03	1.264e-01
234	10101	1.373147e-01	0.000e+00	1.000e+00	8.402e-04	7.087e-02
235	10142	1.373138e-01	0.000e+00	1.000e+00	4.749e-04	2.399e-02
236	10183	1.373134e-01	0.000e+00	1.000e+00	1.891e-04	3.260e-02
237	10224	1.373112e-01	0.000e+00	1.000e+00	9.802e-04	6.298e-02
238	10265	1.373075e-01	0.000e+00	1.000e+00	1.826e-03	8.428e-02
239	10306	1.373045e-01	0.000e+00	1.000e+00	1.831e-03	6.909e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
240	10347	1.373030e-01	0.000e+00	1.000e+00	8.062e-04	6.136e-02
241	10388	1.373022e-01	0.000e+00	1.000e+00	8.073e-04	1.403e-01
242	10433	1.373019e-01	0.000e+00	2.401e-01	2.116e-03	6.212e-02
243	10474	1.372980e-01	0.000e+00	1.000e+00	1.544e-03	6.187e-02

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
244	10515	1.372946e-01	0.000e+00	1.000e+00	2.907e-04	6.200e-02
245	10556	1.372763e-01	0.000e+00	1.000e+00	1.101e-03	6.270e-02
246	10603	1.372758e-01	0.000e+00	1.176e-01	5.405e-04	3.336e-01
247	10644	1.372567e-01	0.000e+00	1.000e+00	3.705e-03	6.919e-02
248	10685	1.372521e-01	0.000e+00	1.000e+00	1.240e-03	6.937e-02
249	10726	1.372450e-01	0.000e+00	1.000e+00	1.583e-03	1.839e-01
250	10767	1.372420e-01	0.000e+00	1.000e+00	1.240e-03	1.046e-01
251	10810	1.372403e-01	0.000e+00	4.900e-01	3.499e-04	1.128e-01
252	10851	1.372343e-01	0.000e+00	1.000e+00	2.978e-03	2.433e-01
253	10892	1.372275e-01	0.000e+00	1.000e+00	1.261e-03	1.312e-01
254	10933	1.372217e-01	0.000e+00	1.000e+00	1.049e-03	6.426e-02
255	10974	1.372161e-01	0.000e+00	1.000e+00	2.697e-03	6.522e-02
256	11015	1.372151e-01	0.000e+00	1.000e+00	9.433e-04	7.352e-02
257	11056	1.372142e-01	0.000e+00	1.000e+00	8.357e-04	6.693e-02
258	11097	1.372136e-01	0.000e+00	1.000e+00	1.278e-04	6.693e-02
259	11138	1.372103e-01	0.000e+00	1.000e+00	2.965e-04	6.683e-02
260	11180	1.372072e-01	0.000e+00	7.000e-01	2.612e-04	1.507e-01
261	11222	1.372072e-01	0.000e+00	7.000e-01	4.762e-04	1.558e-01
262	11265	1.372070e-01	0.000e+00	4.900e-01	4.406e-04	6.673e-02
263	11306	1.372060e-01	0.000e+00	1.000e+00	3.000e-04	6.675e-02
264	11351	1.372057e-01	0.000e+00	2.401e-01	2.339e-04	7.170e-02
265	11392	1.372054e-01	0.000e+00	1.000e+00	6.727e-05	1.426e-02
266	11433	1.372054e-01	0.000e+00	1.000e+00	7.899e-05	1.290e-02
267	11474	1.372051e-01	0.000e+00	1.000e+00	3.889e-04	1.452e-02
268	11519	1.372051e-01	0.000e+00	2.401e-01	8.391e-05	3.606e-02
269	11560	1.372050e-01	0.000e+00	1.000e+00	8.345e-05	1.448e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
270	11601	1.372047e-01	0.000e+00	1.000e+00	1.181e-04	1.487e-02
271	11642	1.372040e-01	0.000e+00	1.000e+00	3.282e-04	3.032e-02
272	11683	1.372017e-01	0.000e+00	1.000e+00	1.383e-03	6.656e-02
273	11724	1.372006e-01	0.000e+00	1.000e+00	5.410e-04	5.643e-02
274	11765	1.371981e-01	0.000e+00	1.000e+00	1.871e-03	6.304e-02
275	11806	1.371972e-01	0.000e+00	1.000e+00	5.394e-04	4.325e-02
276	11847	1.371936e-01	0.000e+00	1.000e+00	2.392e-03	5.806e-02
277	11888	1.371897e-01	0.000e+00	1.000e+00	1.679e-03	5.530e-02
278	11929	1.371836e-01	0.000e+00	1.000e+00	3.834e-03	6.576e-02
279	11970	1.371829e-01	0.000e+00	1.000e+00	1.587e-03	1.241e-01
280	12011	1.371818e-01	0.000e+00	1.000e+00	5.398e-04	1.784e-02
281	12052	1.371817e-01	0.000e+00	1.000e+00	1.654e-04	1.795e-02
282	12093	1.371817e-01	0.000e+00	1.000e+00	1.021e-04	1.808e-02
283	12134	1.371816e-01	0.000e+00	1.000e+00	1.733e-04	1.933e-02
284	12175	1.371816e-01	0.000e+00	1.000e+00	2.108e-04	1.166e-02
285	12216	1.371815e-01	0.000e+00	1.000e+00	5.102e-05	6.724e-03
286	12257	1.371814e-01	0.000e+00	1.000e+00	1.697e-04	1.361e-02
287	12298	1.371812e-01	0.000e+00	1.000e+00	1.433e-04	2.346e-02
288	12339	1.371807e-01	0.000e+00	1.000e+00	3.976e-04	4.197e-02
289	12380	1.371797e-01	0.000e+00	1.000e+00	7.196e-04	5.665e-02
290	12421	1.371779e-01	0.000e+00	1.000e+00	1.238e-03	6.247e-02
291	12462	1.371752e-01	0.000e+00	1.000e+00	1.799e-03	6.112e-02
292	12503	1.371725e-01	0.000e+00	1.000e+00	1.686e-03	1.137e-01
293	12544	1.371703e-01	0.000e+00	1.000e+00	1.543e-03	1.123e-01
294	12585	1.371679e-01	0.000e+00	1.000e+00	1.612e-03	2.187e-01
295	12626	1.371641e-01	0.000e+00	1.000e+00	1.722e-03	1.142e-01
296	12667	1.371594e-01	0.000e+00	1.000e+00	2.910e-03	1.221e-01
297	12708	1.371574e-01	0.000e+00	1.000e+00	1.034e-03	6.103e-02
298	12749	1.371562e-01	0.000e+00	1.000e+00	1.112e-03	4.706e-02
299	12790	1.371556e-01	0.000e+00	1.000e+00	1.440e-03	2.701e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
300	12831	1.371554e-01	0.000e+00	1.000e+00	2.787e-04	1.948e-02
301	12872	1.371553e-01	0.000e+00	1.000e+00	2.193e-04	1.952e-02
302	12913	1.371551e-01	0.000e+00	1.000e+00	2.549e-04	1.946e-02
303	12954	1.371548e-01	0.000e+00	1.000e+00	6.646e-04	4.821e-02

```
 304       12995    1.371548e-01     0.000e+00     1.000e+00     3.591e-04     1.952e-02
 305       13036    1.371546e-01     0.000e+00     1.000e+00     6.658e-05     1.952e-02
 306       13079    1.371545e-01     0.000e+00     4.900e-01     1.512e-04     5.105e-02
 307       13120    1.371543e-01     0.000e+00     1.000e+00     1.544e-04     1.956e-02
 308       13161    1.371542e-01     0.000e+00     1.000e+00     6.743e-05     9.449e-03
 309       13202    1.371542e-01     0.000e+00     1.000e+00     1.172e-04     1.794e-02
 310       13243    1.371541e-01     0.000e+00     1.000e+00     9.859e-05     2.019e-02
 311       13284    1.371538e-01     0.000e+00     1.000e+00     4.425e-04     2.636e-02
 312       13325    1.371534e-01     0.000e+00     1.000e+00     4.367e-04     4.496e-02
 313       13366    1.371524e-01     0.000e+00     1.000e+00     9.273e-04     6.722e-02
 314       13407    1.371508e-01     0.000e+00     1.000e+00     1.487e-03     6.742e-02
 315       13448    1.371492e-01     0.000e+00     1.000e+00     1.818e-03     6.717e-02
 316       13489    1.371484e-01     0.000e+00     1.000e+00     7.643e-04     6.676e-02
 317       13530    1.371478e-01     0.000e+00     1.000e+00     4.549e-04     6.637e-02
 318       13571    1.371475e-01     0.000e+00     1.000e+00     3.495e-04     4.370e-02
 319       13612    1.371473e-01     0.000e+00     1.000e+00     4.685e-04     2.192e-02
 320       13653    1.371472e-01     0.000e+00     1.000e+00     1.660e-04     1.056e-02
 321       13694    1.371471e-01     0.000e+00     1.000e+00     2.890e-04     8.361e-03
 322       13735    1.371471e-01     0.000e+00     1.000e+00     8.681e-05     8.408e-03
 323       13776    1.371471e-01     0.000e+00     1.000e+00     2.255e-04     9.460e-03
 324       13817    1.371470e-01     0.000e+00     1.000e+00     2.121e-04     1.713e-02
 325       13858    1.371467e-01     0.000e+00     1.000e+00     4.367e-04     2.857e-02
 326       13899    1.371460e-01     0.000e+00     1.000e+00     7.076e-04     4.628e-02
 327       13940    1.371445e-01     0.000e+00     1.000e+00     1.129e-03     6.763e-02
 328       13982    1.371430e-01     0.000e+00     7.000e-01     1.037e-03     1.250e-01
 329       14023    1.371420e-01     0.000e+00     1.000e+00     2.553e-03     9.850e-02
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
 330       14064    1.371406e-01     0.000e+00     1.000e+00     8.141e-04     3.074e-02
 331       14105    1.371397e-01     0.000e+00     1.000e+00     8.626e-04     1.339e-02
 332       14146    1.371393e-01     0.000e+00     1.000e+00     5.349e-04     4.886e-03
 333       14187    1.371390e-01     0.000e+00     1.000e+00     2.025e-04     2.482e-02
 334       14233    1.371387e-01     0.000e+00     1.681e-01     4.932e-04     3.485e-02
 335       14278    1.371386e-01     0.000e+00     2.401e-01     1.480e-04     3.582e-02
 336       14319    1.371384e-01     0.000e+00     1.000e+00     9.982e-04     1.138e-02
 337       14360    1.371383e-01     0.000e+00     1.000e+00     1.956e-04     4.679e-03
 338       14404    1.371383e-01     0.000e+00     3.430e-01     9.149e-06     1.304e-02
 339       14445    1.371383e-01     0.000e+00     1.000e+00     4.209e-05     8.014e-03
 340       14486    1.371383e-01     0.000e+00     1.000e+00     3.884e-05     4.684e-03
 341       14527    1.371383e-01     0.000e+00     1.000e+00     6.511e-05     4.704e-03
 342       14568    1.371382e-01     0.000e+00     1.000e+00     1.091e-04     7.302e-03
 343       14612    1.371382e-01     0.000e+00     3.430e-01     1.162e-04     1.891e-02
 344       14653    1.371382e-01     0.000e+00     1.000e+00     1.295e-04     7.012e-03
 345       14694    1.371381e-01     0.000e+00     1.000e+00     1.908e-04     6.606e-03
 346       14735    1.371379e-01     0.000e+00     1.000e+00     7.985e-04     5.368e-03
 347       14776    1.371379e-01     0.000e+00     1.000e+00     2.334e-04     4.323e-03
 348       14817    1.371378e-01     0.000e+00     1.000e+00     1.063e-04     4.292e-03
 349       14858    1.371378e-01     0.000e+00     1.000e+00     5.056e-05     4.183e-03
 350       14901    1.371378e-01     0.000e+00     4.900e-01     1.960e-05     4.235e-03
 351       14942    1.371378e-01     0.000e+00     1.000e+00     2.802e-05     3.160e-03
 352       14983    1.371378e-01     0.000e+00     1.000e+00     1.180e-05     3.159e-03
 353       15024    1.371378e-01     0.000e+00     1.000e+00     3.514e-05     4.331e-03
 354       15065    1.371378e-01     0.000e+00     1.000e+00     4.671e-05     4.335e-03
 355       15111    1.371378e-01     0.000e+00     1.681e-01     1.720e-05     4.334e-03
 356       15152    1.371378e-01     0.000e+00     1.000e+00     6.193e-05     4.320e-03
 357       15193    1.371378e-01     0.000e+00     1.000e+00     1.160e-04     4.291e-03
 358       15234    1.371378e-01     0.000e+00     1.000e+00     3.970e-05     4.299e-03
 359       15275    1.371378e-01     0.000e+00     1.000e+00     3.775e-05     1.455e-03
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
 360       15281    1.371378e-01     0.000e+00     1.176e-01     2.290e-06     1.455e-03

Local minimum possible. Constraints satisfied.
```

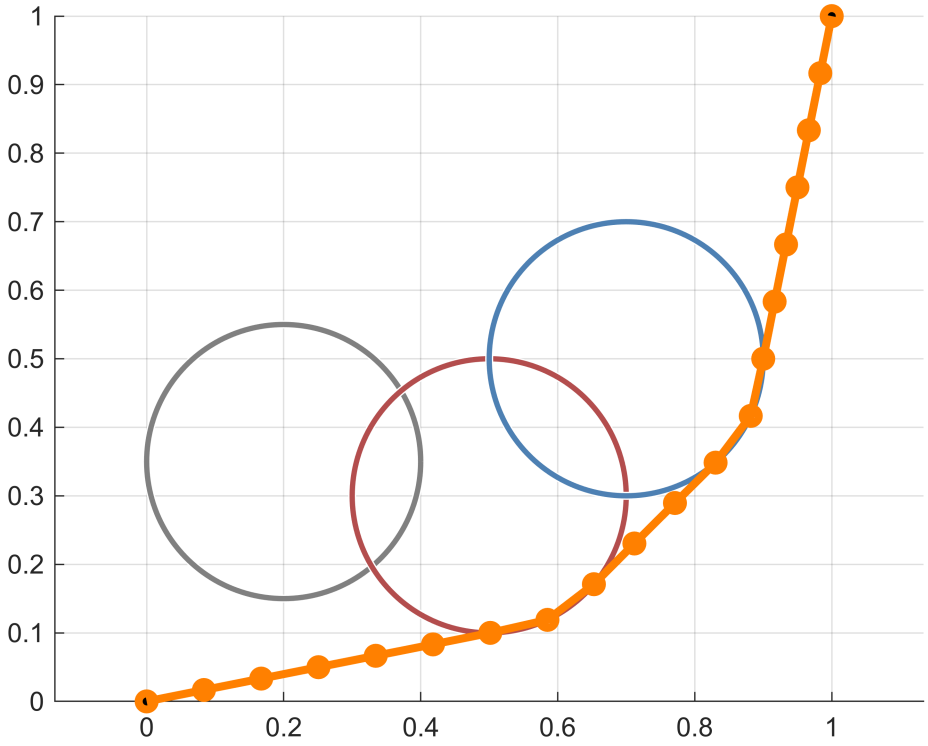
```
fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
xi_star = reshape(xi_star_vec,2,[]); % final optimized trajectory

% Plot obstacles
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center1', r1, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', r2, 'Color', [0.7, 0.3, 0.3]);
viscircles(center3', r3, 'Color', [0.3, 0.5, 0.7]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Plot result
grid on
hold on
axis equal
plot(xi_star(1,:), xi_star(2,:), 'o-',...
    'Color', [1, 0.5, 0], 'LineWidth', 3);
```



```
% Cost function to minimize
```

```
function C = cost(xi)
    gamma = 20;
    xi = reshape(xi,2,[]);
    C = 0;
    r1 = 0.2;
    center1 = [0.2;0.35];
    r2 = 0.2;
    center2 = [0.5;0.3];
    r3 = 0.2;
    center3 = [0.7;0.5];

    for idx = 2:length(xi)
        Urep1 = 0;
        Urep2 = 0;
        Urep3 = 0;

        % First obstacle
        if (norm(center1 - xi(:,idx)) <= r1)
            Urep1 = 0.5*gamma*((1/(norm(center1 - xi(:,idx)))) - (1/r1))^2;
        end

        % Second obstacle
        if (norm(center2 - xi(:,idx)) <= r2)
            Urep2 = 0.5*gamma*((1/(norm(center2 - xi(:,idx)))) - (1/r2))^2;
        end

        % Third obstacle
        if (norm(center3 - xi(:,idx)) <= r3)
            Urep3 = 0.5*gamma*((1/(norm(center3 - xi(:,idx)))) - (1/r3))^2;
        end

        % Total cost
        C = C + norm(xi(:,idx) - xi(:,idx-1))^2 + Urep1 + Urep2 + Urep3;
    end
end
```

2.3 (10 points)

Consider an environment with two obstacles:

- $\theta_{start} = [0,0]^T$ and $\theta_{goal} = [1,1]^T$
- First obstacle with center $c_1 = [0.4, 0.6]^T$ and radius $r_1 = 0.2$
- Second obstacle with center $c_2 = [0.6, 0.4]^T$ and radius $r_2 = 0.2$
- The trajectory ξ should have $k = 20$ waypoints

Modify the initial trajectory ξ^0 so that the optimal trajectory goes around both obstacles. Submit a **plot** of your result and **list** the initial trajectory that you used.

```
% Problem 2.3

clear
clc
close all

% Start and Goal orientations
theta_start = [0;0];
theta_goal = [1;1];

% Initial trajectory variables
n = 2; % No. of joints/ 2-D trajectory
k = 20; % No. of waypoints

% Obstacles 1 and 2 parameters
% First obstacle's radius and center
r1 = 0.2;
center1 = [0.4;0.6];
% Second obstacle's radius and center
r2 = 0.2;
center2 = [0.6;0.4];


% MY INITIAL TRAJECTORY
% Create initial trajectory that arches above the first and below the second
obstacle
mid_point1 = [0.5; 0.7]; % A point above the first obstacle
mid_point2 = [0.5; 0.3]; % A point below the second obstacle

% Construct initial trajectory xi_0
xi_0 = [linspace(theta_start(1), mid_point1(1), floor(k/4)), ...
        linspace(mid_point1(1), mid_point2(1), floor(k/4)), ...
        linspace(mid_point2(1), theta_goal(1), floor(k/2));
        linspace(theta_start(2), mid_point1(2), floor(k/4)), ...
```

```
        linspace(mid_point1(2), mid_point2(2), floor(k/4)), ...
        linspace(mid_point2(2), theta_goal(2), floor(k/2))];
xi_0_vec = reshape(xi_0, [],1); % Reshape for the need of optimization

% Equality constraints for start and goal positions
A = [eye(n), zeros(n,n*(k-1)) ;...
    zeros(n,n*(k-1)), eye(n) ];

B = [theta_start;theta_goal];

% Nonlinear optimization
options = optimoptions('fmincon','Display','iter',...
    'Algorithm','sqp','MaxFunctionEvaluations',1e5);
xi_star_vec = fmincon(@(xi) cost(xi), xi_0_vec, ...
    [], [], A, B, [], [], [], options);
```

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
0	41	5.259554e+03	0.000e+00	1.000e+00	0.000e+00	1.096e+05
1	105	4.223578e+03	0.000e+00	2.737e-04	4.063e+01	1.276e+02
2	148	3.412082e+03	0.000e+00	4.900e-01	5.225e+01	6.299e+01
3	192	3.360310e+03	0.000e+00	3.430e-01	3.162e+01	5.050e+01
4	235	2.233704e+03	0.000e+00	4.900e-01	4.734e+01	4.379e+01
5	277	2.038745e+03	0.000e+00	7.000e-01	4.050e+01	4.369e+01
6	319	1.634861e+03	0.000e+00	7.000e-01	3.146e+01	4.701e+01
7	361	1.474206e+03	0.000e+00	7.000e-01	2.980e+01	4.114e+01
8	403	1.231450e+03	0.000e+00	7.000e-01	2.418e+01	3.427e+01
9	445	1.088206e+03	0.000e+00	7.000e-01	2.383e+01	2.818e+01
10	487	8.968974e+02	0.000e+00	7.000e-01	1.633e+01	3.109e+01
11	529	8.627004e+02	0.000e+00	7.000e-01	1.607e+01	3.496e+01
12	570	8.115329e+02	0.000e+00	1.000e+00	1.311e+01	3.023e+01
13	612	7.938560e+02	0.000e+00	7.000e-01	1.131e+01	2.958e+01
14	654	7.564874e+02	0.000e+00	7.000e-01	8.699e+00	2.532e+01
15	696	7.526249e+02	0.000e+00	7.000e-01	1.009e+01	3.439e+01
16	738	7.157127e+02	0.000e+00	7.000e-01	6.660e+00	2.719e+01
17	781	6.971369e+02	0.000e+00	4.900e-01	5.646e+00	2.547e+01
18	823	6.915785e+02	0.000e+00	7.000e-01	5.370e+00	2.606e+01
19	864	6.850450e+02	0.000e+00	1.000e+00	5.187e+00	2.610e+01
20	907	6.800388e+02	0.000e+00	4.900e-01	3.493e+00	2.574e+01
21	948	6.734686e+02	0.000e+00	1.000e+00	4.238e+00	2.490e+01
22	990	6.702804e+02	0.000e+00	7.000e-01	2.494e+00	2.499e+01
23	1032	6.700025e+02	0.000e+00	7.000e-01	2.931e+00	2.569e+01
24	1073	6.678741e+02	0.000e+00	1.000e+00	2.594e+00	2.246e+01
25	1116	6.659580e+02	0.000e+00	4.900e-01	1.618e+00	2.449e+01
26	1157	6.653640e+02	0.000e+00	1.000e+00	2.031e+00	2.418e+01
27	1199	6.646424e+02	0.000e+00	7.000e-01	1.344e+00	2.494e+01
28	1241	6.641004e+02	0.000e+00	7.000e-01	1.438e+00	2.383e+01
29	1283	6.634615e+02	0.000e+00	7.000e-01	1.177e+00	2.363e+01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
30	1325	6.629003e+02	0.000e+00	7.000e-01	1.062e+00	2.427e+01
31	1366	6.628434e+02	0.000e+00	1.000e+00	1.375e+00	2.398e+01
32	1407	6.619820e+02	0.000e+00	1.000e+00	1.157e+00	2.400e+01
33	1450	6.614425e+02	0.000e+00	4.900e-01	9.162e-01	2.403e+01
34	1491	6.606951e+02	0.000e+00	1.000e+00	1.131e+00	2.384e+01
35	1532	6.604600e+02	0.000e+00	1.000e+00	1.532e+00	2.336e+01
36	1573	6.588206e+02	0.000e+00	1.000e+00	1.592e+00	2.433e+01
37	1614	6.573849e+02	0.000e+00	1.000e+00	2.664e+00	2.437e+01
38	1655	6.518431e+02	0.000e+00	1.000e+00	1.921e+00	2.343e+01
39	1696	6.489891e+02	0.000e+00	1.000e+00	4.700e+00	2.312e+01

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
40	1737	6.336788e+02	0.000e+00	1.000e+00	3.550e+00	2.315e+01
41	1778	6.033210e+02	0.000e+00	1.000e+00	1.219e+01	2.257e+01
42	1819	5.694327e+02	0.000e+00	1.000e+00	2.000e+01	2.132e+01
43	1860	5.419120e+02	0.000e+00	1.000e+00	2.405e+01	2.087e+01
44	1901	5.130550e+02	0.000e+00	1.000e+00	1.533e+01	1.806e+01
45	1942	4.815863e+02	0.000e+00	1.000e+00	6.172e+00	1.772e+01
46	1983	4.172338e+02	0.000e+00	1.000e+00	1.844e+01	1.641e+01
47	2024	3.660126e+02	0.000e+00	1.000e+00	1.398e+01	1.472e+01
48	2065	2.799702e+02	0.000e+00	1.000e+00	2.394e+01	1.187e+01
49	2106	1.993409e+02	0.000e+00	1.000e+00	2.148e+01	8.503e+00
50	2147	1.067375e+02	0.000e+00	1.000e+00	2.682e+01	6.370e+00
51	2188	4.091122e+01	0.000e+00	1.000e+00	2.207e+01	4.335e+00
52	2229	1.048690e+01	0.000e+00	1.000e+00	1.308e+01	1.630e+00
53	2274	8.018335e+00	0.000e+00	2.401e-01	1.430e+00	8.118e+01
54	2328	7.347262e+00	0.000e+00	9.689e-03	1.375e+01	1.669e+00
55	2370	1.097989e+00	0.000e+00	7.000e-01	5.140e+00	7.712e-01
56	2415	7.610732e-01	0.000e+00	2.401e-01	5.129e-01	6.305e-01
57	2465	7.142845e-01	0.000e+00	4.035e-02	7.061e-02	5.781e-01
58	2508	3.783916e-01	0.000e+00	4.900e-01	2.229e+00	9.066e-01
59	2550	2.837937e-01	0.000e+00	7.000e-01	4.384e-01	6.868e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
60	2598	2.761461e-01	0.000e+00	8.235e-02	3.710e-02	6.671e-01
61	2660	2.761138e-01	0.000e+00	5.585e-04	3.755e-04	6.498e-01
62	2701	2.478416e-01	0.000e+00	1.000e+00	6.738e-01	4.716e-01
63	2742	2.409657e-01	0.000e+00	1.000e+00	1.254e-01	5.045e-01
64	2783	2.397934e-01	0.000e+00	1.000e+00	5.216e-02	5.065e-01
65	2824	2.388920e-01	0.000e+00	1.000e+00	8.271e-02	5.010e-01
66	2865	2.384171e-01	0.000e+00	1.000e+00	3.175e-02	4.974e-01
67	2906	2.366505e-01	0.000e+00	1.000e+00	7.353e-02	4.862e-01
68	2947	2.330986e-01	0.000e+00	1.000e+00	8.224e-02	4.681e-01
69	2988	2.239409e-01	0.000e+00	1.000e+00	1.234e-01	4.277e-01
70	3029	2.044408e-01	0.000e+00	1.000e+00	1.961e-01	3.485e-01
71	3070	1.695681e-01	0.000e+00	1.000e+00	3.956e-01	2.111e-01
72	3117	1.632345e-01	0.000e+00	1.176e-01	7.236e-02	1.931e-01
73	3169	1.613184e-01	0.000e+00	1.977e-02	2.029e-02	1.891e-01
74	3225	1.608086e-01	0.000e+00	4.748e-03	5.210e-03	1.882e-01
75	3287	1.607826e-01	0.000e+00	5.585e-04	6.126e-04	9.220e-01
76	3340	1.576940e-01	0.000e+00	1.384e-02	2.306e-02	3.361e-01
77	3398	1.575542e-01	0.000e+00	2.326e-03	4.382e-03	2.892e+00
78	3450	1.530685e-01	0.000e+00	1.977e-02	3.235e-02	1.156e+00
79	3513	1.527830e-01	0.000e+00	3.910e-04	5.222e-03	2.748e+00
80	3566	1.488220e-01	0.000e+00	1.384e-02	3.137e-02	1.653e+00
81	3620	1.485773e-01	0.000e+00	9.689e-03	4.376e-03	2.114e+00
82	3667	1.374998e-01	0.000e+00	1.176e-01	1.254e-01	8.428e-02
83	3716	1.370526e-01	0.000e+00	5.765e-02	6.377e-03	8.142e-02
84	3771	1.370384e-01	0.000e+00	6.782e-03	5.299e-04	6.803e-01
85	3816	1.363245e-01	0.000e+00	2.401e-01	2.885e-02	3.660e+00
86	3858	1.358079e-01	0.000e+00	7.000e-01	5.693e-02	6.425e-02
87	3899	1.354108e-01	0.000e+00	1.000e+00	2.733e-02	6.110e-02
88	3940	1.353238e-01	0.000e+00	1.000e+00	4.185e-03	6.199e-02
89	3981	1.352314e-01	0.000e+00	1.000e+00	6.652e-03	2.948e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
90	4022	1.352247e-01	0.000e+00	1.000e+00	3.110e-03	5.841e-01
91	4063	1.351975e-01	0.000e+00	1.000e+00	3.594e-03	1.137e-01
92	4104	1.351787e-01	0.000e+00	1.000e+00	4.676e-03	5.817e-02
93	4145	1.351556e-01	0.000e+00	1.000e+00	9.711e-03	6.129e-02
94	4186	1.351515e-01	0.000e+00	1.000e+00	1.984e-03	2.771e-01
95	4227	1.351451e-01	0.000e+00	1.000e+00	1.355e-03	1.289e-01
96	4268	1.351332e-01	0.000e+00	1.000e+00	2.024e-03	5.817e-02
97	4309	1.351067e-01	0.000e+00	1.000e+00	2.590e-03	6.245e-02
98	4350	1.349986e-01	0.000e+00	1.000e+00	6.796e-03	6.328e-02
99	4398	1.349758e-01	0.000e+00	8.235e-02	7.998e-04	6.343e-02

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
100	4455	1.349745e-01	0.000e+00	3.323e-03	7.367e-05	1.255e-01
101	4496	1.349109e-01	0.000e+00	1.000e+00	6.940e-03	6.938e-02
102	4537	1.348704e-01	0.000e+00	1.000e+00	7.177e-03	7.343e-02
103	4578	1.348570e-01	0.000e+00	1.000e+00	2.583e-03	7.289e-02
104	4619	1.348484e-01	0.000e+00	1.000e+00	2.501e-03	2.266e-01
105	4660	1.348408e-01	0.000e+00	1.000e+00	1.604e-03	7.255e-02
106	4701	1.348253e-01	0.000e+00	1.000e+00	1.468e-03	7.316e-02
107	4742	1.347446e-01	0.000e+00	1.000e+00	3.980e-03	7.400e-02
108	4787	1.346992e-01	0.000e+00	2.401e-01	3.591e-03	1.056e+00
109	4828	1.346650e-01	0.000e+00	1.000e+00	2.307e-03	3.534e-01
110	4869	1.346184e-01	0.000e+00	1.000e+00	1.993e-03	2.572e-01
111	4910	1.343492e-01	0.000e+00	1.000e+00	7.262e-03	1.047e-01
112	4954	1.338188e-01	0.000e+00	3.430e-01	1.902e-02	6.290e-02
113	5003	1.337811e-01	0.000e+00	5.765e-02	3.580e-03	1.285e+00
114	5047	1.332554e-01	0.000e+00	3.430e-01	4.998e-02	2.335e+00
115	5088	1.332550e-01	0.000e+00	1.000e+00	2.496e-02	6.907e-02
116	5129	1.328924e-01	0.000e+00	1.000e+00	3.013e-02	5.675e-02
117	5170	1.328221e-01	0.000e+00	1.000e+00	5.945e-03	5.771e-02
118	5211	1.327140e-01	0.000e+00	1.000e+00	1.843e-02	8.661e-01
119	5255	1.326857e-01	0.000e+00	3.430e-01	7.882e-03	3.943e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
120	5299	1.326816e-01	0.000e+00	3.430e-01	8.766e-03	6.336e-02
121	5340	1.326646e-01	0.000e+00	1.000e+00	1.853e-03	6.425e-02
122	5382	1.326294e-01	0.000e+00	7.000e-01	3.863e-03	2.170e-01
123	5424	1.326281e-01	0.000e+00	7.000e-01	7.558e-04	6.625e-02
124	5465	1.326165e-01	0.000e+00	1.000e+00	8.699e-04	1.337e-01
125	5506	1.326124e-01	0.000e+00	1.000e+00	3.070e-03	6.615e-02
126	5549	1.326104e-01	0.000e+00	4.900e-01	2.433e-03	2.039e-01
127	5590	1.326089e-01	0.000e+00	1.000e+00	4.985e-04	6.610e-02
128	5631	1.326070e-01	0.000e+00	1.000e+00	5.233e-04	5.521e-02
129	5674	1.326062e-01	0.000e+00	4.900e-01	6.620e-04	2.265e-01
130	5715	1.325987e-01	0.000e+00	1.000e+00	1.487e-03	8.263e-02
131	5756	1.325803e-01	0.000e+00	1.000e+00	2.052e-03	9.593e-02
132	5797	1.325565e-01	0.000e+00	1.000e+00	1.991e-03	6.397e-02
133	5838	1.324966e-01	0.000e+00	1.000e+00	1.193e-02	5.768e-02
134	5879	1.324875e-01	0.000e+00	1.000e+00	1.414e-03	5.865e-02
135	5922	1.324837e-01	0.000e+00	4.900e-01	1.285e-03	3.863e-01
136	5963	1.324589e-01	0.000e+00	1.000e+00	2.071e-03	4.183e-01
137	6004	1.324050e-01	0.000e+00	1.000e+00	5.423e-03	4.190e-01
138	6045	1.323216e-01	0.000e+00	1.000e+00	1.141e-02	1.536e-01
139	6086	1.322846e-01	0.000e+00	1.000e+00	1.298e-02	5.423e-01
140	6130	1.322814e-01	0.000e+00	3.430e-01	7.826e-03	4.648e-01
141	6171	1.322640e-01	0.000e+00	1.000e+00	5.919e-03	1.559e-01
142	6212	1.322471e-01	0.000e+00	1.000e+00	6.140e-03	2.388e-01
143	6253	1.322138e-01	0.000e+00	1.000e+00	3.740e-03	7.074e-02
144	6294	1.321767e-01	0.000e+00	1.000e+00	1.019e-02	8.938e-01
145	6335	1.320802e-01	0.000e+00	1.000e+00	1.669e-02	6.824e-02
146	6376	1.319997e-01	0.000e+00	1.000e+00	7.566e-03	6.682e-02
147	6417	1.318375e-01	0.000e+00	1.000e+00	3.686e-02	1.504e+00
148	6463	1.317930e-01	0.000e+00	1.681e-01	4.448e-02	5.508e-02
149	6504	1.316783e-01	0.000e+00	1.000e+00	4.334e-02	5.772e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
150	6545	1.316363e-01	0.000e+00	1.000e+00	1.208e-02	5.534e-02
151	6586	1.316105e-01	0.000e+00	1.000e+00	9.372e-03	5.355e-02
152	6627	1.316028e-01	0.000e+00	1.000e+00	3.012e-03	5.345e-02
153	6668	1.315921e-01	0.000e+00	1.000e+00	3.226e-03	5.380e-02
154	6709	1.315715e-01	0.000e+00	1.000e+00	3.839e-03	5.481e-02
155	6752	1.315641e-01	0.000e+00	4.900e-01	1.735e-03	4.688e-01
156	6793	1.315536e-01	0.000e+00	1.000e+00	5.503e-03	5.486e-02
157	6834	1.315446e-01	0.000e+00	1.000e+00	2.373e-03	5.523e-02
158	6875	1.315318e-01	0.000e+00	1.000e+00	7.598e-03	4.115e-01
159	6916	1.315277e-01	0.000e+00	1.000e+00	2.477e-03	5.707e-02

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
160	6957	1.315238e-01	0.000e+00	1.000e+00	6.287e-04	5.780e-02
161	6999	1.315213e-01	0.000e+00	7.000e-01	1.758e-03	3.106e-01
162	7040	1.315174e-01	0.000e+00	1.000e+00	5.865e-04	5.959e-02
163	7081	1.315163e-01	0.000e+00	1.000e+00	5.606e-04	6.020e-02
164	7122	1.315152e-01	0.000e+00	1.000e+00	9.284e-04	6.132e-02
165	7163	1.315149e-01	0.000e+00	1.000e+00	1.527e-04	6.152e-02
166	7204	1.315132e-01	0.000e+00	1.000e+00	5.911e-04	6.221e-02
167	7253	1.315131e-01	0.000e+00	5.765e-02	6.512e-05	4.250e-02
168	7294	1.315072e-01	0.000e+00	1.000e+00	1.649e-03	6.148e-02
169	7335	1.314774e-01	0.000e+00	1.000e+00	7.751e-03	5.613e-02
170	7376	1.314580e-01	0.000e+00	1.000e+00	3.943e-03	4.992e-02
171	7417	1.314545e-01	0.000e+00	1.000e+00	2.191e-03	4.834e-02
172	7458	1.314528e-01	0.000e+00	1.000e+00	1.624e-03	4.835e-02
173	7499	1.314502e-01	0.000e+00	1.000e+00	1.798e-03	4.849e-02
174	7540	1.314434e-01	0.000e+00	1.000e+00	2.851e-03	4.880e-02
175	7587	1.314419e-01	0.000e+00	1.176e-01	4.900e-04	6.165e-02
176	7628	1.314370e-01	0.000e+00	1.000e+00	3.648e-03	4.791e-02
177	7669	1.314324e-01	0.000e+00	1.000e+00	4.363e-03	4.696e-02
178	7710	1.314294e-01	0.000e+00	1.000e+00	2.514e-03	4.673e-02
179	7754	1.314290e-01	0.000e+00	3.430e-01	7.735e-04	8.150e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
180	7795	1.314283e-01	0.000e+00	1.000e+00	6.412e-04	4.645e-02
181	7836	1.314279e-01	0.000e+00	1.000e+00	4.671e-04	4.626e-02
182	7877	1.314267e-01	0.000e+00	1.000e+00	1.067e-03	4.576e-02
183	7918	1.314244e-01	0.000e+00	1.000e+00	1.080e-03	4.519e-02
184	7959	1.314185e-01	0.000e+00	1.000e+00	1.823e-03	4.396e-02
185	8001	1.314102e-01	0.000e+00	7.000e-01	1.988e-03	6.251e-02
186	8042	1.314097e-01	0.000e+00	1.000e+00	1.705e-03	7.835e-02
187	8083	1.314053e-01	0.000e+00	1.000e+00	2.404e-03	9.585e-02
188	8128	1.314049e-01	0.000e+00	2.401e-01	7.786e-04	1.758e-01
189	8169	1.314026e-01	0.000e+00	1.000e+00	3.760e-04	4.089e-02
190	8210	1.314007e-01	0.000e+00	1.000e+00	1.660e-03	3.991e-02
191	8251	1.313978e-01	0.000e+00	1.000e+00	2.895e-03	4.771e-02
192	8292	1.313944e-01	0.000e+00	1.000e+00	7.428e-04	4.849e-02
193	8333	1.313756e-01	0.000e+00	1.000e+00	2.408e-03	2.095e-01
194	8374	1.312825e-01	0.000e+00	1.000e+00	8.847e-03	3.612e-01
195	8421	1.311865e-01	0.000e+00	1.176e-01	1.120e-02	6.012e-02
196	8476	1.311854e-01	0.000e+00	6.782e-03	1.527e-04	6.018e-02
197	8517	1.311080e-01	0.000e+00	1.000e+00	1.897e-02	5.915e-02
198	8558	1.310675e-01	0.000e+00	1.000e+00	2.273e-02	5.804e-02
199	8599	1.310645e-01	0.000e+00	1.000e+00	2.145e-03	5.793e-02
200	8640	1.310610e-01	0.000e+00	1.000e+00	2.391e-03	5.781e-02
201	8681	1.310566e-01	0.000e+00	1.000e+00	1.833e-03	5.773e-02
202	8722	1.310456e-01	0.000e+00	1.000e+00	3.826e-03	2.308e-01
203	8763	1.310431e-01	0.000e+00	1.000e+00	7.277e-03	5.732e-02
204	8804	1.310345e-01	0.000e+00	1.000e+00	1.368e-03	5.736e-02
205	8845	1.309973e-01	0.000e+00	1.000e+00	6.340e-03	5.730e-02
206	8889	1.309921e-01	0.000e+00	3.430e-01	6.756e-03	7.952e-01
207	8931	1.309615e-01	0.000e+00	7.000e-01	2.564e-02	5.648e-02
208	8972	1.309499e-01	0.000e+00	1.000e+00	1.236e-02	5.638e-02
209	9013	1.309438e-01	0.000e+00	1.000e+00	4.252e-04	5.626e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
210	9054	1.309181e-01	0.000e+00	1.000e+00	1.336e-03	2.420e-01
211	9095	1.308842e-01	0.000e+00	1.000e+00	1.999e-02	5.499e-02
212	9145	1.308832e-01	0.000e+00	4.035e-02	2.168e-04	5.497e-02
213	9202	1.308830e-01	0.000e+00	3.323e-03	4.944e-05	9.254e-02
214	9246	1.308735e-01	0.000e+00	3.430e-01	4.298e-03	1.029e+00
215	9287	1.308557e-01	0.000e+00	1.000e+00	4.807e-03	5.684e-02
216	9328	1.308427e-01	0.000e+00	1.000e+00	8.681e-04	5.665e-02
217	9370	1.308296e-01	0.000e+00	7.000e-01	4.052e-03	6.904e-01
218	9411	1.308206e-01	0.000e+00	1.000e+00	1.888e-03	5.590e-02
219	9452	1.308095e-01	0.000e+00	1.000e+00	1.338e-03	5.648e-02

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
220	9493	1.308008e-01	0.000e+00	1.000e+00	4.130e-03	6.366e-01
221	9534	1.307838e-01	0.000e+00	1.000e+00	1.749e-03	4.755e-01
222	9580	1.307648e-01	0.000e+00	1.681e-01	2.980e-03	3.541e-01
223	9621	1.307593e-01	0.000e+00	1.000e+00	1.282e-03	4.911e-02
224	9665	1.307573e-01	0.000e+00	3.430e-01	3.762e-04	1.065e-01
225	9706	1.307560e-01	0.000e+00	1.000e+00	1.465e-03	4.990e-02
226	9747	1.307545e-01	0.000e+00	1.000e+00	9.728e-04	5.067e-02
227	9788	1.307526e-01	0.000e+00	1.000e+00	3.132e-04	1.039e-01
228	9830	1.307435e-01	0.000e+00	7.000e-01	1.317e-03	2.494e-01
229	9871	1.307382e-01	0.000e+00	1.000e+00	1.322e-03	1.091e-01
230	9914	1.307329e-01	0.000e+00	4.900e-01	1.261e-03	1.525e-01
231	9956	1.307327e-01	0.000e+00	7.000e-01	1.470e-03	5.721e-02
232	9997	1.307315e-01	0.000e+00	1.000e+00	4.729e-04	5.686e-02
233	10044	1.307310e-01	0.000e+00	1.176e-01	3.663e-04	5.682e-02
234	10085	1.307305e-01	0.000e+00	1.000e+00	3.219e-04	5.678e-02
235	10126	1.307300e-01	0.000e+00	1.000e+00	2.117e-04	5.677e-02
236	10168	1.307292e-01	0.000e+00	7.000e-01	2.208e-04	9.346e-02
237	10209	1.307288e-01	0.000e+00	1.000e+00	5.770e-04	5.713e-02
238	10250	1.307282e-01	0.000e+00	1.000e+00	1.098e-04	4.524e-02
239	10294	1.307282e-01	0.000e+00	3.430e-01	1.924e-04	7.055e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
240	10335	1.307279e-01	0.000e+00	1.000e+00	1.019e-04	2.345e-02
241	10376	1.307277e-01	0.000e+00	1.000e+00	9.715e-05	2.344e-02
242	10417	1.307270e-01	0.000e+00	1.000e+00	3.635e-04	5.135e-02
243	10458	1.307255e-01	0.000e+00	1.000e+00	6.331e-04	5.808e-02
244	10499	1.307191e-01	0.000e+00	1.000e+00	3.418e-03	5.944e-02
245	10540	1.307140e-01	0.000e+00	1.000e+00	3.756e-03	6.041e-02
246	10581	1.307103e-01	0.000e+00	1.000e+00	3.718e-03	6.074e-02
247	10622	1.307088e-01	0.000e+00	1.000e+00	1.887e-03	6.052e-02
248	10663	1.307080e-01	0.000e+00	1.000e+00	8.023e-04	6.028e-02
249	10704	1.307064e-01	0.000e+00	1.000e+00	9.788e-04	5.996e-02
250	10748	1.307063e-01	0.000e+00	3.430e-01	5.011e-04	9.820e-02
251	10789	1.307044e-01	0.000e+00	1.000e+00	1.489e-03	6.048e-02
252	10830	1.307029e-01	0.000e+00	1.000e+00	1.230e-03	6.107e-02
253	10871	1.307001e-01	0.000e+00	1.000e+00	2.250e-03	6.219e-02
254	10912	1.306990e-01	0.000e+00	1.000e+00	1.134e-03	6.263e-02
255	10953	1.306963e-01	0.000e+00	1.000e+00	2.549e-03	6.342e-02
256	11001	1.306961e-01	0.000e+00	8.235e-02	2.800e-04	6.687e-02
257	11042	1.306950e-01	0.000e+00	1.000e+00	1.185e-03	4.862e-02
258	11083	1.306940e-01	0.000e+00	1.000e+00	1.173e-03	7.113e-02
259	11124	1.306934e-01	0.000e+00	1.000e+00	7.057e-04	2.324e-02
260	11165	1.306931e-01	0.000e+00	1.000e+00	1.458e-03	2.329e-02
261	11206	1.306929e-01	0.000e+00	1.000e+00	6.141e-04	2.328e-02
262	11247	1.306928e-01	0.000e+00	1.000e+00	4.213e-04	2.327e-02
263	11288	1.306922e-01	0.000e+00	1.000e+00	8.927e-04	2.319e-02
264	11329	1.306909e-01	0.000e+00	1.000e+00	1.114e-03	3.249e-02
265	11370	1.306880e-01	0.000e+00	1.000e+00	1.456e-03	9.447e-02
266	11411	1.306844e-01	0.000e+00	1.000e+00	2.295e-03	6.549e-02
267	11452	1.306724e-01	0.000e+00	1.000e+00	1.464e-02	6.196e-02
268	11493	1.306631e-01	0.000e+00	1.000e+00	3.640e-03	6.096e-02
269	11534	1.306630e-01	0.000e+00	1.000e+00	6.257e-03	6.083e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
270	11580	1.306531e-01	0.000e+00	1.681e-01	2.553e-03	3.486e-01
271	11629	1.306524e-01	0.000e+00	5.765e-02	9.251e-04	2.641e-01
272	11677	1.306512e-01	0.000e+00	8.235e-02	5.867e-03	5.820e-02
273	11718	1.306469e-01	0.000e+00	1.000e+00	2.252e-03	5.807e-02
274	11759	1.306468e-01	0.000e+00	1.000e+00	3.914e-03	2.747e-01
275	11800	1.306433e-01	0.000e+00	1.000e+00	2.674e-03	5.842e-02
276	11841	1.306416e-01	0.000e+00	1.000e+00	2.907e-04	5.852e-02
277	11882	1.306388e-01	0.000e+00	1.000e+00	1.164e-03	1.928e-01
278	11923	1.306361e-01	0.000e+00	1.000e+00	2.583e-03	5.793e-02
279	11964	1.306347e-01	0.000e+00	1.000e+00	6.615e-04	5.811e-02

```
  280       12008    1.306334e-01     0.000e+00     3.430e-01     8.278e-04     7.544e-02
  281       12049    1.306332e-01     0.000e+00     1.000e+00     5.557e-04     6.702e-02
  282       12090    1.306329e-01     0.000e+00     1.000e+00     2.378e-04     3.331e-02
  283       12131    1.306329e-01     0.000e+00     1.000e+00     1.261e-04     1.738e-02
  284       12172    1.306327e-01     0.000e+00     1.000e+00     3.175e-04     7.882e-03
  285       12213    1.306325e-01     0.000e+00     1.000e+00     1.024e-03     2.337e-02
  286       12254    1.306324e-01     0.000e+00     1.000e+00     3.314e-04     1.400e-02
  287       12295    1.306323e-01     0.000e+00     1.000e+00     4.633e-04     7.764e-03
  288       12336    1.306323e-01     0.000e+00     1.000e+00     2.313e-04     7.824e-03
  289       12377    1.306323e-01     0.000e+00     1.000e+00     3.206e-04     7.955e-03
  290       12418    1.306322e-01     0.000e+00     1.000e+00     1.985e-04     7.974e-03
  291       12459    1.306322e-01     0.000e+00     1.000e+00     1.010e-04     7.989e-03
  292       12500    1.306322e-01     0.000e+00     1.000e+00     1.313e-04     7.956e-03
  293       12541    1.306322e-01     0.000e+00     1.000e+00     1.937e-04     7.896e-03
  294       12582    1.306322e-01     0.000e+00     1.000e+00     2.567e-04     1.286e-02
  295       12625    1.306321e-01     0.000e+00     4.900e-01     1.962e-04     1.677e-02
  296       12680    1.306321e-01     0.000e+00     6.782e-03     1.791e-05     1.712e-02
  297       12721    1.306321e-01     0.000e+00     1.000e+00     1.054e-04     1.262e-02
  298       12762    1.306319e-01     0.000e+00     1.000e+00     3.476e-04     7.859e-03
  299       12803    1.306319e-01     0.000e+00     1.000e+00     1.258e-04     7.899e-03
 Iter   Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                        step    optimality
  300       12844    1.306318e-01     0.000e+00     1.000e+00     5.684e-04     2.269e-02
  301       12885    1.306317e-01     0.000e+00     1.000e+00     1.176e-03     1.123e-02
  302       12926    1.306317e-01     0.000e+00     1.000e+00     6.812e-04     1.314e-02
  303       12967    1.306317e-01     0.000e+00     1.000e+00     2.771e-05     1.540e-02
  304       13015    1.306317e-01     0.000e+00     8.235e-02     4.358e-05     2.296e-02
  305       13056    1.306316e-01     0.000e+00     1.000e+00     2.064e-04     8.264e-03
  306       13097    1.306316e-01     0.000e+00     1.000e+00     1.826e-04     6.522e-03
  307       13138    1.306316e-01     0.000e+00     1.000e+00     1.140e-04     5.881e-03
  308       13179    1.306316e-01     0.000e+00     1.000e+00     5.220e-05     7.956e-03
  309       13220    1.306316e-01     0.000e+00     1.000e+00     4.432e-05     1.129e-02
  310       13261    1.306315e-01     0.000e+00     1.000e+00     6.967e-05     1.956e-02
  311       13302    1.306315e-01     0.000e+00     1.000e+00     1.183e-04     3.241e-02
  312       13343    1.306313e-01     0.000e+00     1.000e+00     2.447e-04     5.180e-02
  313       13384    1.306307e-01     0.000e+00     1.000e+00     8.555e-04     8.655e-02
  314       13425    1.306301e-01     0.000e+00     1.000e+00     6.550e-04     9.446e-02
  315       13466    1.306288e-01     0.000e+00     1.000e+00     1.497e-03     7.034e-02
  316       13507    1.306283e-01     0.000e+00     1.000e+00     1.162e-03     1.804e-02
  317       13548    1.306282e-01     0.000e+00     1.000e+00     1.844e-04     8.669e-03
  318       13589    1.306282e-01     0.000e+00     1.000e+00     1.043e-04     8.549e-03
  319       13630    1.306282e-01     0.000e+00     1.000e+00     1.170e-04     8.433e-03
  320       13671    1.306281e-01     0.000e+00     1.000e+00     2.371e-04     1.140e-02
  321       13712    1.306280e-01     0.000e+00     1.000e+00     3.620e-04     3.253e-02
  322       13754    1.306280e-01     0.000e+00     7.000e-01     2.036e-04     1.654e-02
  323       13795    1.306279e-01     0.000e+00     1.000e+00     1.069e-04     1.343e-02
  324       13837    1.306279e-01     0.000e+00     7.000e-01     2.846e-04     2.406e-02
  325       13879    1.306279e-01     0.000e+00     7.000e-01     5.796e-05     1.301e-02
  326       13920    1.306278e-01     0.000e+00     1.000e+00     7.094e-05     3.839e-03
  327       13927    1.306278e-01     0.000e+00     8.235e-02     2.523e-06     3.839e-03

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

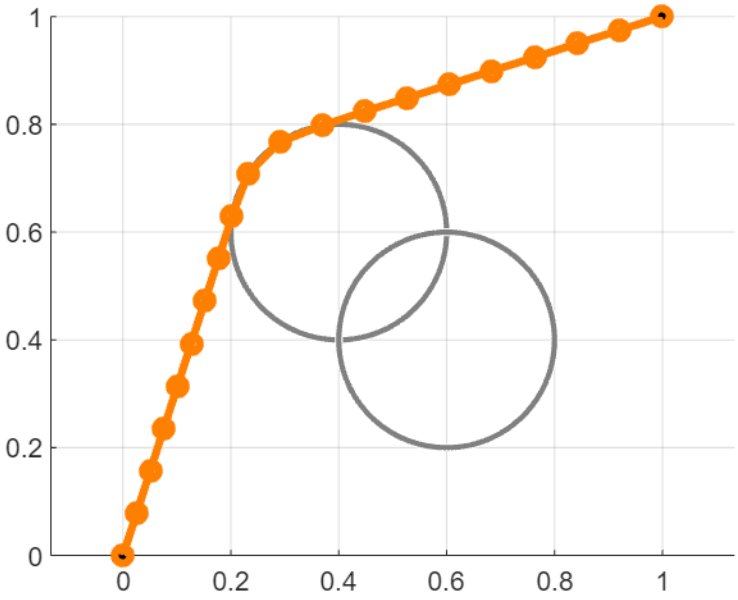
<stopping criteria details>
```

```
xi_star = reshape(xi_star_vec,2,[]); % final optimized trajectory

% Plot obstacles
```

```
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center1', r1, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', r2, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Plot result
grid on
hold on
axis equal
plot(xi_star(1,:), xi_star(2,:), 'o-',...
    'Color', [1, 0.5, 0], 'LineWidth', 3);
```




```
% Cost function to minimize
```

```
function C = cost(xi)
    gamma = 20; % Repulsion coefficient
    xi = reshape(xi, 2, []);
    C = 0;
    epsilon = 1e-6; % Small constant to prevent division by zero

    % Define obstacles with correct centers and radii
    obstacles = [
        0.4, 0.6, 0.2; % First obstacle
        0.6, 0.4, 0.2; % Second obstacle
        % Add more obstacles if needed
```

```
    ];

    for idx = 2:length(xi)
        Urep = 0;
        for obs = 1:size(obstacles, 1)
            center = obstacles(obs, 1:2)';
            r = obstacles(obs, 3);
            dist_to_center = norm(xi(:, idx) - center);

            % Calculate repulsive potential if within the obstacle's influence
            if dist_to_center < r
                % Add a small value epsilon to prevent division by zero
                Urep = Urep + 0.5 * gamma * ((1 / (dist_to_center + epsilon)) -
(1 / r))^2;
            end
        end

        % Sum the repulsive potential and the path length cost
        C = C + norm(xi(:, idx) - xi(:, idx - 1))^2 + Urep;
    end
end


% MY INITIAL TRAJECTORY
% % Create initial trajectory that arches above the first and below the second
obstacle
% mid_point1 = [0.5; 0.7]; % A point above the first obstacle
% mid_point2 = [0.5; 0.3]; % A point below the second obstacle
%
% % Construct initial trajectory xi_0
% xi_0 = [linspace(theta_start(1), mid_point1(1), floor(k/4)), ...
%         linspace(mid_point1(1), mid_point2(1), floor(k/4)), ...
%         linspace(mid_point2(1), theta_goal(1), floor(k/2));
%         linspace(theta_start(2), mid_point1(2), floor(k/4)), ...
%         linspace(mid_point1(2), mid_point2(2), floor(k/4)), ...
%         linspace(mid_point2(2), theta_goal(2), floor(k/2))];
```

3 RRT Algorithm

In this problem you will use the RRT algorithm to perform
environments. As before, the mobile robot's position is $\theta = [x,$

3.1 (10 points)

Implement the RRT algorithm. Your code should be able to work
of circular obstacles.

- The bounds of the workspace are $x \in [0, 1]$, $y \in [0, 1]$
- The motion plan must end within $\epsilon = 0.1$ units of the g

```
% Problem 3.1

clear
close all

% Define start and goal positions
theta_start.coord = [0; 0];
theta_goal = [1; 1];

% Workspace bounds
x_bounds = [0, 1];
y_bounds = [0, 1];

% Define obstacles
% Each row is an obstacle with format: [center_x, center_y, radius]
obstacles = [
    0.5, 0.3, 0.2;
    0.7, 0.7, 0.2;
    0.6, 0.4, 0.1;
    0.4, 0.3, 0.2;
    0.1, 0.75, 0.3;
    % Add as many obstacles required, satisfying the condition of arbitrary
    % number of obstacles' inclusion
];

% RRT parameters
epsilon = 0.1; % Goal threshold
delta = 0.05;  % Step size
N = 1000;      % Number of iterations
```

```
% Visualize environment
figure
hold on
grid on
axis([x_bounds, y_bounds])
axis equal

% Plot obstacles
for i = 1:size(obstacles, 1)
    viscircles(obstacles(i, 1:2), obstacles(i, 3), 'Color', [0.5, 0.5, 0.5]);
end

% Initialize tree
theta_start.parent = 0;
G(1) = theta_start;

% Main RRT loop
for idx = 1:N
    if norm(G(end).coord - theta_goal) < epsilon
        break
    end

    % Random sample
    theta_rand = rand(2,1);

    % Nearest node
    [min_dist, theta_near_index] = min(vecnorm([G.coord] - theta_rand));
    theta_near = G(theta_near_index);

    % Step towards random sample
    vec_to_rand = theta_rand - theta_near.coord;
    if norm(vec_to_rand) > delta
        vec_to_rand = delta * vec_to_rand / norm(vec_to_rand);
    end
    theta_new.coord = theta_near.coord + vec_to_rand;

    % Collision check with all obstacles
    if isCollision(theta_new.coord, obstacles)
        continue;
    end

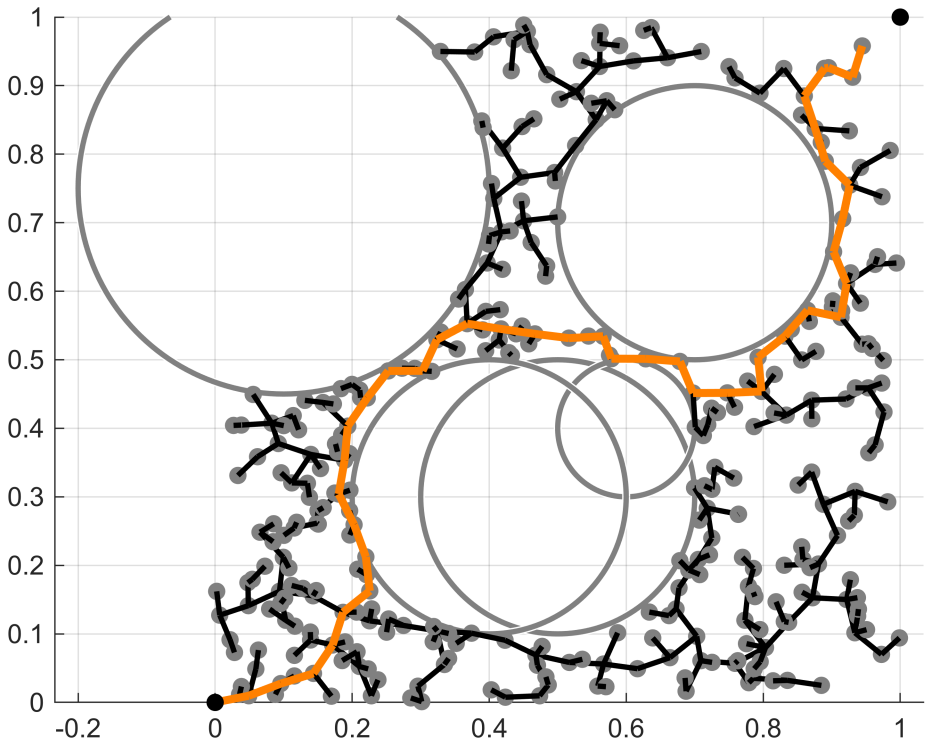
    % Add new node to tree
    theta_new.parent = theta_near_index;
    G = [G, theta_new];

    % Plotting
    plot(theta_new.coord(1), theta_new.coord(2), 'o', 'Color', [0.5, 0.5, 0.5], ...
    'MarkerFaceColor', [0.5, 0.5, 0.5]);
    line([theta_near.coord(1), theta_new.coord(1)], [theta_near.coord(2),
theta_new.coord(2)], 'Color', 'k', 'LineWidth', 2);
```

```
end

% Trace back path
child_theta = G(end);
while child_theta.parent ~= 0
    parent_theta = G(child_theta.parent);
    line([child_theta.coord(1), parent_theta.coord(1)], [child_theta.coord(2),
parent_theta.coord(2)], 'Color', [1, 0.5, 0], 'LineWidth', 3);
    child_theta = parent_theta;
end

% Plot start and goal
plot(theta_start.coord(1), theta_start.coord(2), 'ko', 'MarkerFaceColor', 'k');
plot(theta_goal(1), theta_goal(2), 'ko', 'MarkerFaceColor', 'k');
```




```
% Collision checking function
```

```
function collision = isCollision(coord, obstacles)
    collision = any(arrayfun(@(idx) norm(coord - obstacles(idx, 1:2)') <
obstacles(idx, 3), 1:size(obstacles, 1)));
end
```

3.2 (15 points)

Use your code to find a desired trajectory for the following environments. In each environment $\theta_{start} = [0,0]^T$ and $\theta_{goal} = [1,1]^T$.

- **Environment 1.** One obstacle with center $c_1 = [0.55, 0.5]^T$ and radius $r_1 = 0.3$.

```
% Problem 3.2 - Environment 1

clear
close all

% Define start and goal positions
theta_start.coord = [0; 0];
theta_goal = [1; 1];

% Define obstacles
center = [0.55; 0.5];
radius = 0.3;

% Parameters
epsilon = 0.1;
delta = 0.1;
N = 1000;

% Visualize environment
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center', radius, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Initialize tree
theta_start.parent = 0;
G(1) = theta_start;

for idx = 1:N

    % Stop if the last node in G is close to theta_goal

    if norm(G(end).coord  - theta_goal) < epsilon
        break
    end

    % Sample random joint position
    theta_rand = rand(2,1);
```

```
% % theta_rand = theta_goal;
% % if rand() < 0.8
% %     theta_rand = rand(2,1);
% % end

% Find node in G nearest to theta_rand
min_dist = inf;
theta_near_index = 0;

for jdx = 1:length(G)
    coord = G(jdx).coord;
    dist = norm(theta_rand - coord);
    if dist < min_dist
        min_dist = dist;
        theta_near_index = jdx;
    end
end

% dist = zeros(length(G), 1);
% for jdx = 1:1:length(G)
%     dist(jdx) = norm(G(jdx).coord - theta_rand);
% end
% [~, theta_near_index] = min(dist);

theta_near = G(theta_near_index);
% Take a step from theta_near towards theta_rand
vec_to_rand = theta_rand - theta_near.coord;
dist_to_rand = norm(vec_to_rand);
if dist_to_rand < delta
    theta_new.coord = theta_rand;
else
    theta_new.coord = theta_near.coord + delta * ...
    vec_to_rand/dist_to_rand;
end
% Check if theta_new is collision free

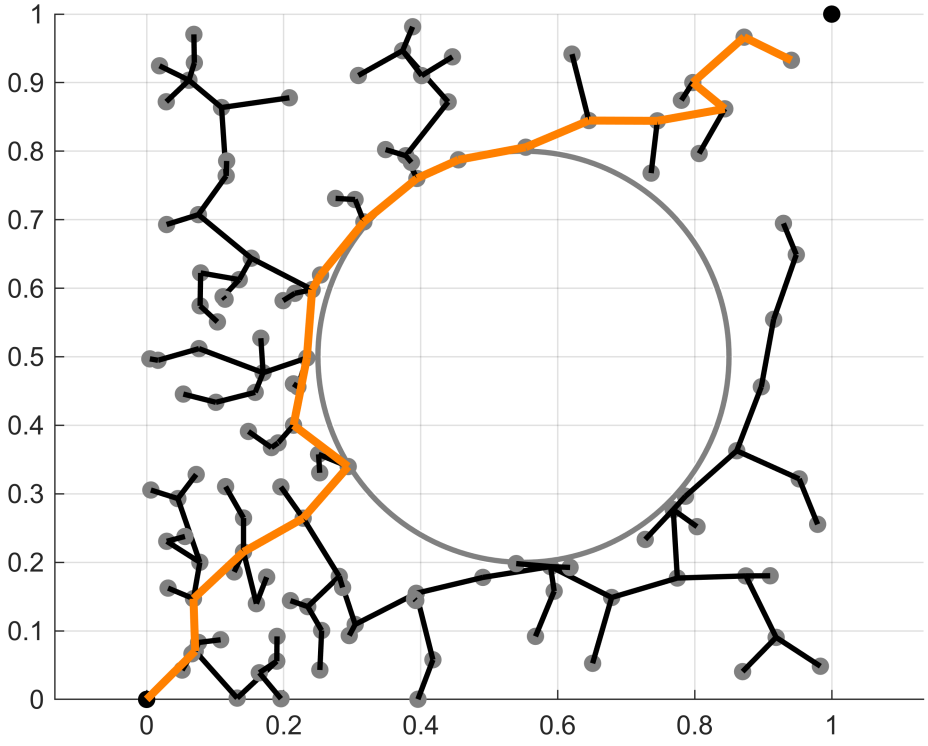
dist_to_obs = norm(theta_new.coord - center);
if dist_to_obs < radius
    continue
end

% If collision free, add theta_new to tree with parent theta_near
theta_new.parent = theta_near_index;
G = [G, theta_new];
% Plot node and edge
plot(theta_new.coord(1), theta_new.coord(2), 'o', 'Color', [0.5, 0.5, 0.5], ...
'MarkerFaceColor', [0.5, 0.5, 0.5])
line([theta_near.coord(1), theta_new.coord(1)], [theta_near.coord(2), ...
theta_new.coord(2)], 'Color', 'k', 'LineWidth', 2);
```

```
    drawnow

end
    % work backwards from the final node to the root of the tree
    child_theta = G(end);
    while child_theta.parent ~= 0

        parent_theta_index = child_theta.parent; % implemented
        parent_theta = G(parent_theta_index); % implemented
        line([child_theta.coord(1), parent_theta.coord(1)], ...
            [child_theta.coord(2), parent_theta.coord(2)], ...
            'Color', [1, 0.5, 0], 'LineWidth', 3);
        child_theta = parent_theta;
    end
```



- **Environment 2.** One obstacle with center $c_1 = [0.5, 0.3]^T$ and radius $r_1 = 0.3$. A second obstacle with center $c_2 = [0.5, 0.7]^T$ and radius $r_2 = 0.2$.

```
% Problem 3.2 - Environment 2

clear
close all

% Define start and goal positions
theta_start.coord = [0; 0];
theta_goal = [1; 1];

% Define obstacles
center = [0.5; 0.3];
radius = 0.3;
center2 = [0.5; 0.7]; % Second obstacle center
radius2 = 0.2;        % Second obstacle radius

% Parameters
epsilon = 0.1;
delta = 0.1;
N = 1000;

% Visualize environment
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center', radius, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', radius2, 'Color', [0.7, 0.3, 0.3]); % Visualizing the second
obstacle
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Initialize tree
theta_start.parent = 0;
G(1) = theta_start;

for idx = 1:N

    % Stop if the last node in G is close to theta_goal
    if norm(G(end).coord  - theta_goal) < epsilon
        break
    end

    % sample random joint position
    theta_rand = rand(2,1);
```

```
    % % theta_rand = theta_goal;
    % % if rand() < 0.8
    % %     theta_rand = rand(2,1);
    % % end

    % Find node in G nearest to theta_rand
    min_dist = inf;
    theta_near_index = 0;

    for jdx = 1:length(G)
        coord = G(jdx).coord;
        dist = norm(theta_rand - coord);
        if dist < min_dist
            min_dist = dist;
            theta_near_index = jdx;
        end
    end

    theta_near = G(theta_near_index);
    % Take a step from theta_near towards theta_rand
    vec_to_rand = theta_rand - theta_near.coord;
    dist_to_rand = norm(vec_to_rand);
    if dist_to_rand < delta
        theta_new.coord = theta_rand;
    else
        theta_new.coord = theta_near.coord + delta * ...
        vec_to_rand/dist_to_rand;
    end

    % Check if theta_new is collision free with both obstacles
    dist_to_obs1 = norm(theta_new.coord - center);
    dist_to_obs2 = norm(theta_new.coord - center2);
    if dist_to_obs1 < radius || dist_to_obs2 < radius2
        continue
    end

    % If collision free, add theta_new to tree with parent theta_near
    theta_new.parent = theta_near_index;
    G = [G, theta_new];

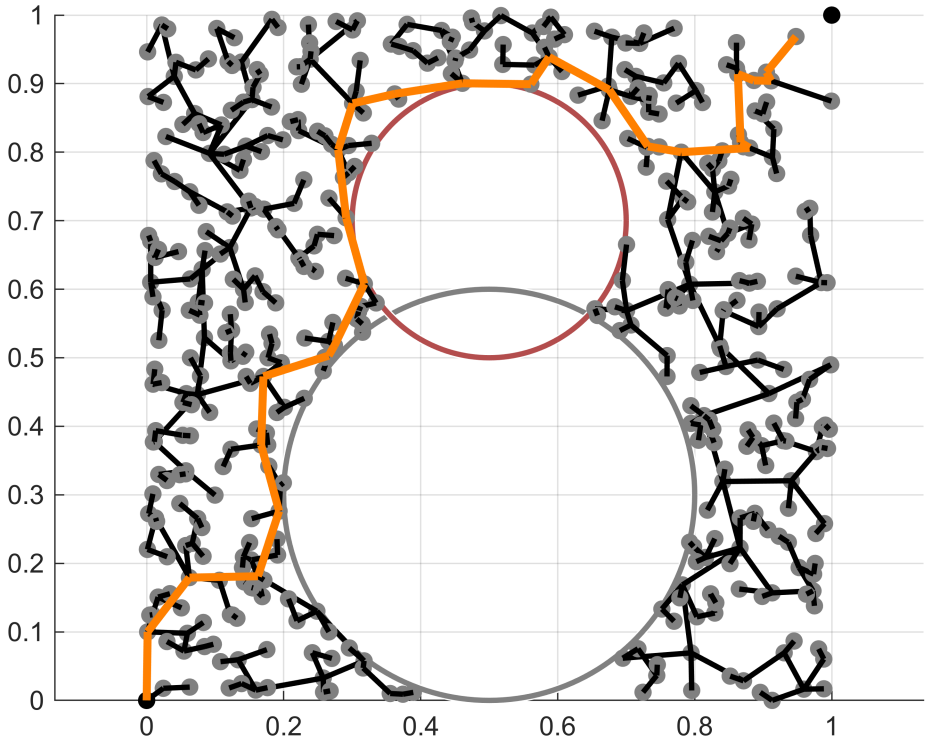
    % Plot node and edge
    plot(theta_new.coord(1), theta_new.coord(2), 'o', 'Color', [0.5, 0.5, 0.5], ...
    'MarkerFaceColor', [0.5, 0.5, 0.5])
    line([theta_near.coord(1), theta_new.coord(1)], [theta_near.coord(2), ...
    theta_new.coord(2)], 'Color', 'k', 'LineWidth', 2);
    drawnow

end

% Work backwards from the final node to the root of the tree
```

```
child_theta = G(end);
while child_theta.parent ~= 0

    parent_theta_index = child_theta.parent;
    parent_theta = G(parent_theta_index);
    line([child_theta.coord(1), parent_theta.coord(1)], ...
        [child_theta.coord(2), parent_theta.coord(2)], ...
        'Color', [1, 0.5, 0], 'LineWidth', 3);
    child_theta = parent_theta;
end
```



- **Environment 3.** One obstacle with center $c_1 = [0.2, 0.35]^T$ and radius $r_1 = 0.2$. A second obstacle with center $c_2 = [0.5, 0.3]^T$ and radius $r_2 = 0.2$. A third obstacle with center $c_3 = [0.7, 0.5]^T$ and radius $r_3 = 0.2$.

```
% Problem 3.2 - Environment 3
clear
close all
% Environment initiation
theta_start.coord = [0; 0];
theta_goal = [1; 1];

% Obstacle Parameters
% First obstacle center and radius
center = [0.2; 0.35];
radius = 0.2;
% Second obstacle center and radius
center2 = [0.5; 0.3];
radius2 = 0.2;
% Third obstacle center and radius
center3 = [0.7; 0.5];
radius3 = 0.2;

% Specifying parameters
epsilon = 0.1;
delta = 0.1;
N = 1000;

% Visualizing the environment
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center', radius, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', radius2, 'Color', [0.7, 0.3, 0.3]); % Visualizing the second
obstacle
viscircles(center3', radius3, 'Color', [0.3, 0.5, 0.7]); % Visualizing the third
obstacle
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Initializing the tree
theta_start.parent = 0;
G(1) = theta_start;

for idx = 1:N

    % Stop if the last node in G is close to theta_goal
```

```
    if norm(G(end).coord  - theta_goal) < epsilon
        break
    end

    % Sample random joint position
    theta_rand = rand(2,1);
    % % theta_rand = theta_goal;
    % % if rand() < 0.8
    % %     theta_rand = rand(2,1);
    % % end

    % Find node in G nearest to theta_rand
    min_dist = inf;
    theta_near_index = 0;

    for jdx = 1:length(G)
        coord = G(jdx).coord;
        dist = norm(theta_rand - coord);
        if dist < min_dist
            min_dist = dist;
            theta_near_index = jdx;
        end
    end

    theta_near = G(theta_near_index);
    % take a step from theta_near towards theta_rand
    vec_to_rand = theta_rand - theta_near.coord;
    dist_to_rand = norm(vec_to_rand);
    if dist_to_rand < delta
        theta_new.coord = theta_rand;
    else
        theta_new.coord = theta_near.coord + delta * ...
        vec_to_rand/dist_to_rand;
    end

    % Check if theta_new is collision free with all obstacles
    dist_to_obs1 = norm(theta_new.coord - center);
    dist_to_obs2 = norm(theta_new.coord - center2);
    dist_to_obs3 = norm(theta_new.coord - center3);
    if dist_to_obs1 < radius || dist_to_obs2 < radius2 || dist_to_obs3 < radius3
        continue
    end

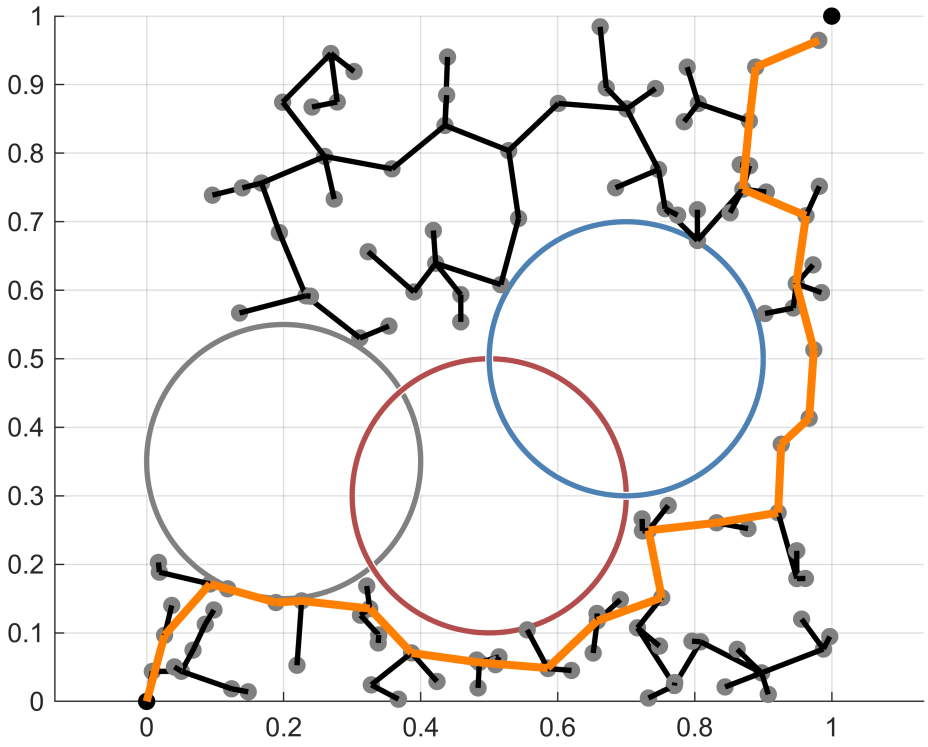
    % If collision free, add theta_new to tree with parent theta_near
    theta_new.parent = theta_near_index;
    G = [G, theta_new];
    % Plot node and edge
    plot(theta_new.coord(1), theta_new.coord(2), 'o', 'Color', [0.5, 0.5, 0.5], ...
    'MarkerFaceColor', [0.5, 0.5, 0.5])
    line([theta_near.coord(1), theta_new.coord(1)], [theta_near.coord(2), ...
```

```
    theta_new.coord(2)], 'Color', 'k', 'LineWidth', 2);
    drawnow

end

% Work backwards from the final node to the root of the tree
child_theta = G(end);
while child_theta.parent ~= 0

    parent_theta_index = child_theta.parent;
    parent_theta = G(parent_theta_index);
    line([child_theta.coord(1), parent_theta.coord(1)], ...
        [child_theta.coord(2), parent_theta.coord(2)], ...
        'Color', [1, 0.5, 0], 'LineWidth', 3);
    child_theta = parent_theta;
end
```



3.3 (10 points)

Using **Environment 3** from the previous part, compare two versions of RRT. The first version is the standard RRT algorithm you have implemented (let's refer to this as the **baseline**). The second version will sample the goal more frequently (let's refer to this as **goal bias**). For **goal bias**, with probability 0.2 set θ_{rand} as θ_{goal}. Otherwise sample randomly as normal.

Run your code 10 times for **baseline** and 10 times for **goal bias**. Write down how many samples it takes on average to find a motion plan. Which approach is more sample-efficient: **baseline** or **goal bias**? Write a few sentences to explain and support your answer.

```
% Problem 3.3 - baseline

clear
close all
% Environment initiation
theta_start.coord = [0; 0];
theta_goal = [1; 1];

% Obstacle Parameters
% First obstacle center and radius
center = [0.2; 0.35];
radius = 0.2;
% Second obstacle center and radius
center2 = [0.5; 0.3];
radius2 = 0.2;
% Third obstacle center and radius
center3 = [0.7; 0.5];
radius3 = 0.2;

% Specifying parameters
epsilon = 0.1;
delta = 0.1;
N = 1000;

% Visualizing the environment
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center', radius, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', radius2, 'Color', [0.7, 0.3, 0.3]); % Visualizing the second
obstacle
viscircles(center3', radius3, 'Color', [0.3, 0.5, 0.7]); % Visualizing the third
obstacle
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')
```

```
% Initializing the tree
theta_start.parent = 0;
G(1) = theta_start;

for idx = 1:N

    % Stop if the last node in G is close to theta_goal
    if norm(G(end).coord  - theta_goal) < epsilon
        break
    end

    % Sample random joint position
    theta_rand = rand(2,1);
    % % theta_rand = theta_goal;
    % % if rand() < 0.8
    % %     theta_rand = rand(2,1);
    % % end

    % Find node in G nearest to theta_rand
    min_dist = inf;
    theta_near_index = 0;

    for jdx = 1:length(G)
        coord = G(jdx).coord;
        dist = norm(theta_rand - coord);
        if dist < min_dist
            min_dist = dist;
            theta_near_index = jdx;
        end
    end

    theta_near = G(theta_near_index);
    % take a step from theta_near towards theta_rand
    vec_to_rand = theta_rand - theta_near.coord;
    dist_to_rand = norm(vec_to_rand);
    if dist_to_rand < delta
        theta_new.coord = theta_rand;
    else
        theta_new.coord = theta_near.coord + delta * ...
        vec_to_rand/dist_to_rand;
    end

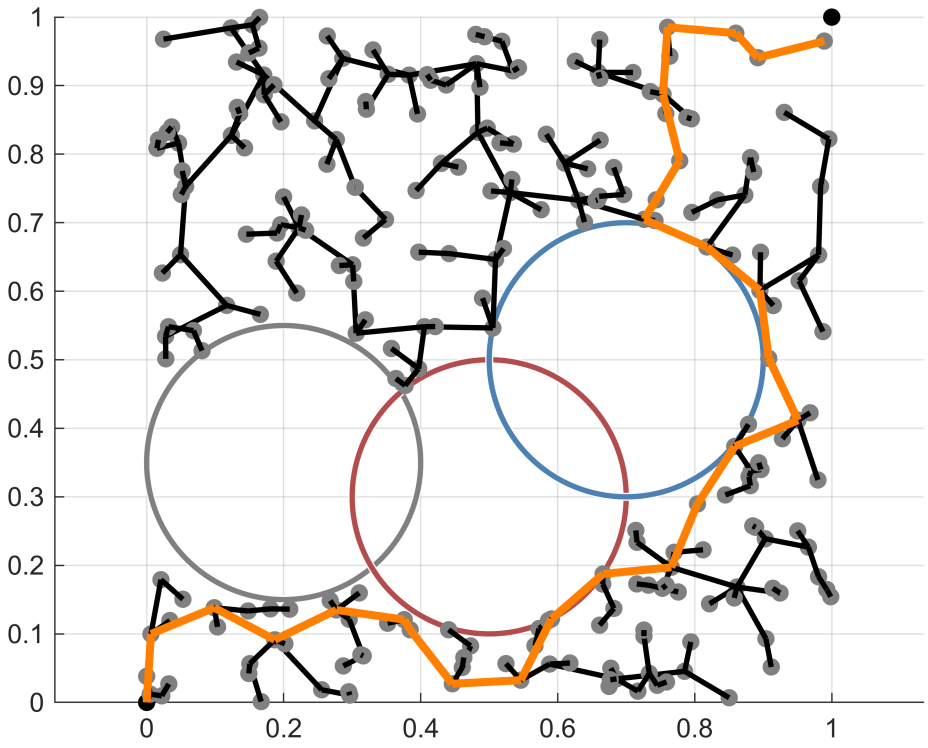
    % Check if theta_new is collision free with all obstacles
    dist_to_obs1 = norm(theta_new.coord - center);
    dist_to_obs2 = norm(theta_new.coord - center2);
    dist_to_obs3 = norm(theta_new.coord - center3);
    if dist_to_obs1 < radius || dist_to_obs2 < radius2 || dist_to_obs3 < radius3
        continue
    end
```

```
    % If collision free, add theta_new to tree with parent theta_near
    theta_new.parent = theta_near_index;
    G = [G, theta_new];
    % Plot node and edge
    plot(theta_new.coord(1), theta_new.coord(2), 'o', 'Color', [0.5, 0.5, 0.5], ...
    'MarkerFaceColor', [0.5, 0.5, 0.5])
    line([theta_near.coord(1), theta_new.coord(1)], [theta_near.coord(2), ...
    theta_new.coord(2)], 'Color', 'k', 'LineWidth', 2);
    drawnow

end

% Work backwards from the final node to the root of the tree
child_theta = G(end);
while child_theta.parent ~= 0

    parent_theta_index = child_theta.parent;
    parent_theta = G(parent_theta_index);
    line([child_theta.coord(1), parent_theta.coord(1)], ...
        [child_theta.coord(2), parent_theta.coord(2)], ...
        'Color', [1, 0.5, 0], 'LineWidth', 3);
    child_theta = parent_theta;
end
```



3.3 (10 points)

Using **Environment 3** from the previous part, compare two versions of RRT. The first version is the standard RRT algorithm you have implemented (let's refer to this as the **baseline**). The second version will sample the goal more frequently (let's refer to this as **goal bias**). For **goal bias**, with probability 0.2 set θ_{rand} as θ_{goal}. Otherwise sample randomly as normal.

Run your code 10 times for **baseline** and 10 times for **goal bias**. Write down how many samples it takes on average to find a motion plan. Which approach is more sample-efficient: **baseline** or **goal bias**? Write a few sentences to explain and support your answer.

```
% Problem 3.3 - Goal Bias

clear
close all
% Environment initiation
theta_start.coord = [0; 0];
theta_goal = [1; 1];

% Obstacle Parameters
% First obstacle center and radius
center = [0.2; 0.35];
radius = 0.2;
% Second obstacle center and radius
center2 = [0.5; 0.3];
radius2 = 0.2;
% Third obstacle center and radius
center3 = [0.7; 0.5];
radius3 = 0.2;

% Specifying parameters
epsilon = 0.1;
delta = 0.1;
N = 1000;

% Visualizing the environment
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center', radius, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', radius2, 'Color', [0.7, 0.3, 0.3]); % Visualizing the second
obstacle
viscircles(center3', radius3, 'Color', [0.3, 0.5, 0.7]); % Visualizing the third
obstacle
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')
```

```
% Initializing the tree
theta_start.parent = 0;
G(1) = theta_start;

for idx = 1:N

    % Stop if the last node in G is close to theta_goal
    if norm(G(end).coord  - theta_goal) < epsilon
        break
    end

    % sample random joint position: probability 0.2

    if rand() < 0.2
        theta_rand = theta_goal;
    else
        theta_rand = rand(2,1);
    end

    % find node in G nearest to theta_rand
    min_dist = inf;
    theta_near_index = 0;

    for jdx = 1:length(G)
        coord = G(jdx).coord;
        dist = norm(theta_rand - coord);
        if dist < min_dist
            min_dist = dist;
            theta_near_index = jdx;
        end
    end

    theta_near = G(theta_near_index);
    % take a step from theta_near towards theta_rand
    vec_to_rand = theta_rand - theta_near.coord;
    dist_to_rand = norm(vec_to_rand);
    if dist_to_rand < delta
        theta_new.coord = theta_rand;
    else
        theta_new.coord = theta_near.coord + delta * ...
        vec_to_rand/dist_to_rand;
    end

    % check if theta_new is collision free with all obstacles
    dist_to_obs1 = norm(theta_new.coord - center);
    dist_to_obs2 = norm(theta_new.coord - center2);
    dist_to_obs3 = norm(theta_new.coord - center3);
    if dist_to_obs1 < radius || dist_to_obs2 < radius2 || dist_to_obs3 < radius3
        continue
```

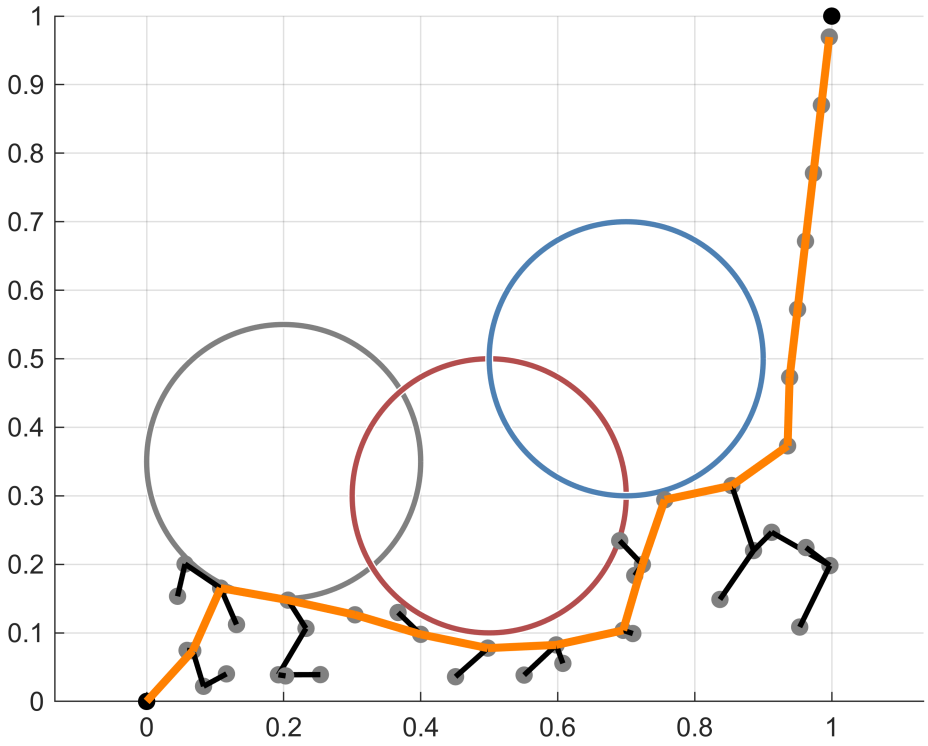
```
    end

    % if collision free, add theta_new to tree with parent theta_near
    theta_new.parent = theta_near_index;
    G = [G, theta_new];
    % plot node and edge
    plot(theta_new.coord(1), theta_new.coord(2), 'o', 'Color', [0.5, 0.5, 0.5], ...
    'MarkerFaceColor', [0.5, 0.5, 0.5])
    line([theta_near.coord(1), theta_new.coord(1)], [theta_near.coord(2), ...
    theta_new.coord(2)], 'Color', 'k', 'LineWidth', 2);
    drawnow

end

% work backwards from the final node to the root of the tree
child_theta = G(end);
while child_theta.parent ~= 0

    parent_theta_index = child_theta.parent;
    parent_theta = G(parent_theta_index);
    line([child_theta.coord(1), parent_theta.coord(1)], ...
        [child_theta.coord(2), parent_theta.coord(2)], ...
        'Color', [1, 0.5, 0], 'LineWidth', 3);
    child_theta = parent_theta;
end
```



I determine Goal-Biased RRT to be more sample-efficient than the baseline but there are compensations which are mentioned below in the comparison.

It took around **1-3 samples** on average to find a motion plan for **goal-biased** to find a motion plan, whereas it took **4-6 samples** on average for the **baseline** to get an ~accurate motion plan.

To quantitatively determine which is more sample-efficient, I ran multiple simulations (10 times) of each method and averaged the number of samples taken to find a successful path. The method with the lower average would be considered more sample-efficient which is shown above.

Sample Efficiency Comparison:

- **Goal-biased RRT** seemed to be **more sample-efficient** in open or less complex environments because it directs its exploration towards the goal, thereby potentially finding a path with fewer samples.

- In contrast, the **baseline RRT** might be more effective in **highly cluttered environments** where a direct path to the goal is less likely, and a more uniform exploration of the space is beneficial.

We need to understand the key differences between the two methods and their impact on sample efficiency:

1. Baseline RRT: This approach uniformly samples the entire configuration space **without any bias** towards the goal. It explores the space in a **more scattered** manner, which can be beneficial in complex environments with many obstacles. However, it might take **more samples to find a path to the goal**, especially in large or open spaces, because the sampling is entirely random and not directed towards the goal.

2. Goal-Biased RRT: In this approach, the algorithm is biased to sample near the goal state more frequently. This bias can significantly **reduce the number of samples** required to find a path to the goal in many cases, especially in less complex environments or when the goal is not surrounded by obstacles. The downside is that in highly cluttered environments, **this bias might lead to more samples being wasted** near the goal where paths are infeasible.