Practice Set 13

Robotics & Automation
Dylan Losey, Virginia Tech

Using your textbook and what we covered in lecture, try solving the following problems. For some problems you may find it convenient to use Matlab (or another programming language of your choice). The solutions are on the next page.

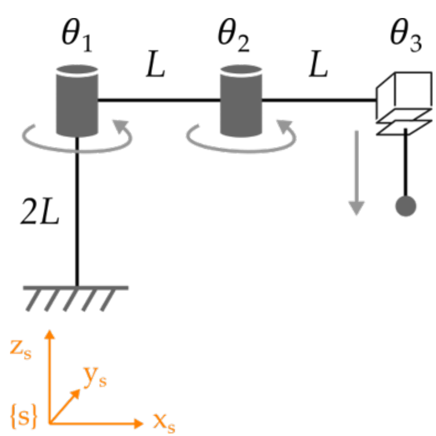
Problem 1

You have a robot with 8 joints. If the space Jacobian gives $V_s = J_s(\theta)\dot{\theta}$, what is the size (i.e., the dimensions) of the Jacobian matrix?

Problem 2

Write a function that computes the space Jacobian. Your function should take in a matrix $S = [S_1, S_2, \ldots, S_n]$, where S_i is the screw for the i-th joint, and a vector $\theta = [\theta_1, \theta_2, \ldots \theta_n]^T$, where θ_i is the position of the i-th joint. The output should be J_s.

Problem 3



Find the space Jacobian for this robot in the following cases:

- Leave L and θ_1, θ_2, θ_3 as symbolic variables. You should get the same result as what we found by hand in lecture.
- Let $L = 1$ and $\theta_1 = \pi/2$, $\theta_2 = 0$, $\theta_3 = 2L$

Problem 1

You have a robot with 8 joints. If the space Jacobian gives $V_s = J_s(\theta)\dot{\theta}$, what is the size (i.e., the dimensions) of the Jacobian matrix?

> The Jacobian is $m \times n$, where n is the number of joints and m is the dimension of the task space. Here the Jacobian is 6×8.

Problem 2

Write a function that computes the space Jacobian. Your function should take in a matrix $S = [S_1, S_2, \ldots, S_n]$, where S_i is the screw for the i-th joint, and a vector $\theta = [\theta_1, \theta_2, \ldots \theta_n]^T$, where θ_i is the position of the i-th joint. The output should be J_s.

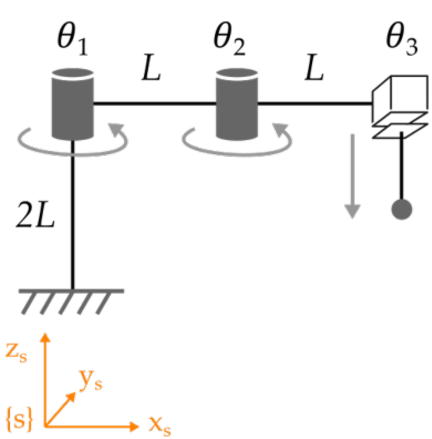
```
function Js = JacobianSpace(S, theta)

    T = eye(4);
    for i = 1:length(theta)
        Si = S(:, i);
        Js(:, i) = Adjoint(T) * Si;
        T = T * expm(bracket(Si) * theta(i));
    end

end
```

> See the code above. This code implements the equations for the space Jacobian we discussed in lecture. Here `bracket(Si)` is our code for $[S_i]$. It's up to you to implement `Adjoint()` and `bracket()`.

Problem 3



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First get the screws for every joint. You should be able to find these yourself:

$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -L \\ 0 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tag{1}$$

Now plug $S = [S_1, S_2, S_3]$ and θ into your `JacobianSpace` code. To declare L and θ as symbolic variables in matlab, use: `syms L theta1 theta2 theta3 real`.
Answers shown in the following figures.

```
Js =

[0,              0,   0]
[0,              0,   0]
[1,              1,   0]
[0,  L*sin(theta1),   0]
[0, -L*cos(theta1),   0]
[0,              0,  -1]
```

Figure 1: Here L and θ_1, θ_2, θ_3 are left as symbolic variables.

```
Js =

         0         0         0
         0         0         0
    1.0000    1.0000         0
         0    1.0000         0
         0   -0.0000         0
         0         0   -1.0000
```

Figure 2: Here $L = 1$ and $\theta_1 = \pi/2$, $\theta_2 = 0$, $\theta_3 = 2L$. You might notice that the value of θ_2 and θ_3 has no effect on the space Jacobian for this robot.