- **Environment 2.** One obstacle with center $c_1 = [0.5, 0.3]^T$ and radius $r_1 = 0.3$. A second obstacle with center $c_2 = [0.5, 0.7]^T$ and radius $r_2 = 0.2$. Set $k = 15$.

```
clear
clc
close all

% Start and Goal orientations
theta_start = [0;0];
theta_goal = [1;1];

% Initial trajectory variables
n = 2; % No. of joints/ 2-D trajectory
k = 15; % No. of waypoints

% Obstacles 1 and 2 parameters
% First obstacle's radius and center
r1 = 0.3;
center1 = [0.5;0.3];

% Second obstacle's radius and center
r2 = 0.2;
center2 = [0.5;0.7];

xi_0 = zeros(n, k); % Initial trajectory

xi_0_vec = reshape(xi_0, [],1); % Reshape for the need of optimization

% Equality constraints for start and goal positions
A = [eye(n), zeros(n,n*(k-1)) ;...
 zeros(n,n*(k-1)), eye(n) ];
B = [theta_start;theta_goal];

% Nonlinear optimization
options = optimoptions('fmincon','Display','iter',...
 'Algorithm','sqp','MaxFunctionEvaluations',1e5);

xi_star_vec = fmincon(@(xi) cost(xi), xi_0_vec, ...
 [], [], A, B, [], [], [], options);
```

```
 Iter  Func-count           Fval   Feasibility   Step Length       Norm of    First-order
                                                                      step     optimality
    0         31    0.000000e+00     1.000e+00     1.000e+00     0.000e+00     2.980e-08
    1         63    9.800000e-01     3.000e-01     7.000e-01     9.899e-01     1.400e+00
    2         98    8.002409e-01     2.280e-01     2.401e-01     2.482e-01     9.038e-01
    3        137    7.199803e-01     2.148e-01     5.765e-02     9.956e-02     8.320e-01
    4        182    7.226496e-01     2.134e-01     6.782e-03     1.237e-02     9.556e+00
    5        217    7.506090e-01     1.621e-01     2.401e-01     2.927e-01     1.365e+00
    6        249    1.018032e+00     4.864e-02     7.000e-01     4.074e-01     1.564e+00
    7        280    1.138203e+00     0.000e+00     1.000e+00     3.986e-01     1.730e+00
```

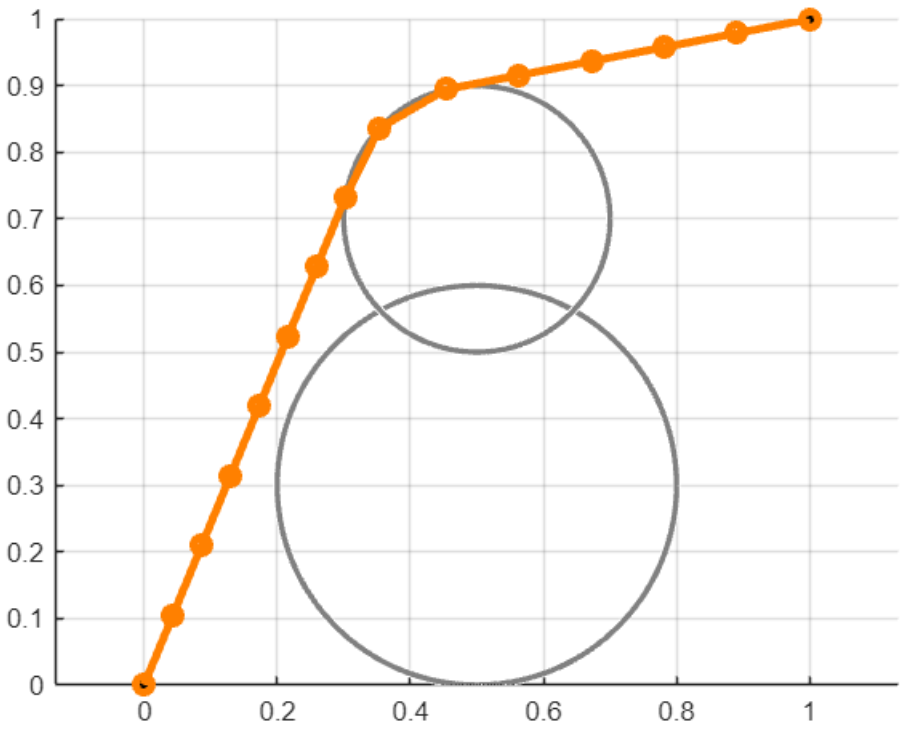
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
8	312	1.089659e+00	0.000e+00	7.000e-01	2.766e-01	1.834e+00
9	344	1.079187e+00	0.000e+00	7.000e-01	2.754e-01	1.854e+00
10	376	1.052501e+00	0.000e+00	7.000e-01	2.114e-01	1.807e+00
11	408	1.043898e+00	0.000e+00	7.000e-01	2.289e-01	1.780e+00
12	440	1.028618e+00	0.000e+00	7.000e-01	1.743e-01	1.801e+00
13	472	1.020757e+00	0.000e+00	7.000e-01	1.715e-01	1.835e+00
14	504	1.011666e+00	0.000e+00	7.000e-01	1.428e-01	1.841e+00
15	536	1.004509e+00	0.000e+00	7.000e-01	1.374e-01	1.823e+00
16	568	9.966581e-01	0.000e+00	7.000e-01	1.285e-01	1.810e+00
17	600	9.919059e-01	0.000e+00	7.000e-01	1.211e-01	1.815e+00
18	631	9.858665e-01	0.000e+00	1.000e+00	1.539e-01	1.835e+00
19	663	9.830327e-01	0.000e+00	7.000e-01	1.089e-01	1.837e+00
20	694	9.781328e-01	0.000e+00	1.000e+00	1.752e-01	1.821e+00
21	726	9.742403e-01	0.000e+00	7.000e-01	1.055e-01	1.815e+00
22	757	9.724051e-01	0.000e+00	1.000e+00	1.769e-01	1.819e+00
23	789	9.673708e-01	0.000e+00	7.000e-01	9.615e-02	1.823e+00
24	821	9.595208e-01	0.000e+00	7.000e-01	1.198e-01	1.821e+00
25	854	9.563263e-01	0.000e+00	4.900e-01	6.450e-02	1.814e+00
26	885	9.513725e-01	0.000e+00	1.000e+00	9.931e-02	1.795e+00
27	917	9.480460e-01	0.000e+00	7.000e-01	1.100e-01	1.805e+00
28	948	9.342793e-01	0.000e+00	1.000e+00	8.007e-02	1.781e+00
29	979	9.147575e-01	0.000e+00	1.000e+00	1.640e-01	1.760e+00
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
30	1010	8.703939e-01	0.000e+00	1.000e+00	1.240e-01	1.688e+00
31	1041	5.931631e-01	0.000e+00	1.000e+00	9.008e-01	1.050e+00
32	1072	3.789344e-01	0.000e+00	1.000e+00	4.611e-01	4.962e-01
33	1103	2.277291e-01	0.000e+00	1.000e+00	5.351e-01	4.549e-01
34	1142	2.233931e-01	0.000e+00	5.765e-02	3.391e-02	4.374e-01
35	1188	2.229087e-01	0.000e+00	4.748e-03	4.052e-03	4.357e-01
36	1239	2.228200e-01	0.000e+00	7.979e-04	7.422e-04	7.010e-01
37	1277	2.055736e-01	0.000e+00	8.235e-02	1.342e-01	4.193e-01
38	1318	2.038573e-01	0.000e+00	2.825e-02	1.480e-02	4.079e-01
39	1365	2.036490e-01	0.000e+00	3.323e-03	1.862e-03	6.813e-01
40	1401	1.854562e-01	0.000e+00	1.681e-01	2.114e-01	3.726e-01
41	1436	1.832160e-01	0.000e+00	2.401e-01	4.553e-02	2.842e-01
42	1474	1.827084e-01	0.000e+00	8.235e-02	1.335e-02	2.659e+00
43	1507	1.817833e-01	0.000e+00	4.900e-01	1.972e-01	1.547e-01
44	1538	1.804912e-01	0.000e+00	1.000e+00	1.031e-01	9.838e-02
45	1569	1.803867e-01	0.000e+00	1.000e+00	2.001e-02	9.918e-02
46	1600	1.803473e-01	0.000e+00	1.000e+00	9.368e-03	9.832e-02
47	1631	1.803048e-01	0.000e+00	1.000e+00	1.253e-02	9.659e-02
48	1662	1.802746e-01	0.000e+00	1.000e+00	8.701e-03	9.570e-02
49	1693	1.802186e-01	0.000e+00	1.000e+00	1.266e-02	9.483e-02
50	1724	1.801193e-01	0.000e+00	1.000e+00	1.535e-02	9.400e-02
51	1757	1.799917e-01	0.000e+00	4.900e-01	1.123e-02	9.362e-02
52	1798	1.799672e-01	0.000e+00	2.825e-02	1.294e-03	9.356e-02
53	1849	1.799666e-01	0.000e+00	7.979e-04	6.799e-05	3.372e-01
54	1880	1.798228e-01	0.000e+00	1.000e+00	2.163e-02	8.773e-02
55	1911	1.797841e-01	0.000e+00	1.000e+00	7.388e-03	8.821e-02
56	1942	1.797247e-01	0.000e+00	1.000e+00	1.442e-02	8.959e-02
57	1973	1.796623e-01	0.000e+00	1.000e+00	1.241e-02	8.891e-02
58	2006	1.796177e-01	0.000e+00	4.900e-01	8.990e-03	1.876e+00
59	2038	1.794735e-01	0.000e+00	7.000e-01	1.324e-02	7.945e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
60	2080	1.794734e-01	0.000e+00	1.977e-02	1.191e-04	5.677e-01
61	2111	1.791900e-01	0.000e+00	1.000e+00	5.752e-02	9.683e-02
62	2142	1.790400e-01	0.000e+00	1.000e+00	1.711e-02	9.231e-02
63	2173	1.789940e-01	0.000e+00	1.000e+00	8.847e-03	9.332e-02
64	2204	1.789830e-01	0.000e+00	1.000e+00	5.372e-03	9.513e-02
65	2235	1.789815e-01	0.000e+00	1.000e+00	1.631e-03	9.579e-02
66	2266	1.789795e-01	0.000e+00	1.000e+00	2.037e-03	9.637e-02
67	2297	1.789761e-01	0.000e+00	1.000e+00	2.926e-03	9.669e-02

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
68	2328	1.789710e-01	0.000e+00	1.000e+00	3.262e-03	9.637e-02
69	2359	1.789644e-01	0.000e+00	1.000e+00	2.564e-03	9.531e-02
70	2390	1.789550e-01	0.000e+00	1.000e+00	1.785e-03	9.368e-02
71	2421	1.789368e-01	0.000e+00	1.000e+00	1.912e-03	9.134e-02
72	2453	1.789046e-01	0.000e+00	7.000e-01	2.268e-03	8.888e-02
73	2492	1.788959e-01	0.000e+00	5.765e-02	4.112e-04	8.851e-02
74	2523	1.788547e-01	0.000e+00	1.000e+00	5.176e-03	9.274e-02
75	2554	1.787689e-01	0.000e+00	1.000e+00	1.478e-02	1.072e-01
76	2585	1.787344e-01	0.000e+00	1.000e+00	4.417e-03	1.085e-01
77	2618	1.787120e-01	0.000e+00	4.900e-01	2.791e-03	1.092e-01
78	2661	1.787114e-01	0.000e+00	1.384e-02	8.841e-05	1.344e-01
79	2692	1.787000e-01	0.000e+00	1.000e+00	6.772e-03	1.137e-01
80	2723	1.786974e-01	0.000e+00	1.000e+00	1.293e-03	1.133e-01
81	2754	1.786934e-01	0.000e+00	1.000e+00	3.006e-03	1.130e-01
82	2785	1.786918e-01	0.000e+00	1.000e+00	7.728e-04	1.131e-01
83	2816	1.786837e-01	0.000e+00	1.000e+00	2.772e-03	1.137e-01
84	2852	1.786837e-01	0.000e+00	1.681e-01	5.390e-04	5.479e-01
85	2883	1.786762e-01	0.000e+00	1.000e+00	7.154e-03	1.186e-01
86	2914	1.786727e-01	0.000e+00	1.000e+00	2.540e-03	1.176e-01
87	2945	1.786698e-01	0.000e+00	1.000e+00	1.535e-03	1.175e-01
88	2976	1.786664e-01	0.000e+00	1.000e+00	1.358e-03	1.179e-01
89	3007	1.786642e-01	0.000e+00	1.000e+00	2.966e-03	1.197e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
90	3038	1.786601e-01	0.000e+00	1.000e+00	1.511e-03	1.204e-01
91	3069	1.786412e-01	0.000e+00	1.000e+00	5.900e-03	1.224e-01
92	3101	1.786387e-01	0.000e+00	7.000e-01	3.069e-03	1.262e+00
93	3140	1.786268e-01	0.000e+00	5.765e-02	3.013e-03	6.111e-01
94	3176	1.786218e-01	0.000e+00	1.681e-01	7.839e-03	1.259e-01
95	3208	1.786156e-01	0.000e+00	7.000e-01	7.944e-03	1.255e-01
96	3239	1.786097e-01	0.000e+00	1.000e+00	4.440e-03	1.268e-01
97	3272	1.786082e-01	0.000e+00	4.900e-01	2.958e-04	9.989e-02
98	3303	1.785898e-01	0.000e+00	1.000e+00	9.269e-03	1.235e-01
99	3337	1.785837e-01	0.000e+00	3.430e-01	3.634e-04	5.462e-01
100	3368	1.785660e-01	0.000e+00	1.000e+00	7.365e-03	1.202e-01
101	3399	1.785615e-01	0.000e+00	1.000e+00	5.067e-04	1.198e-01
102	3432	1.785604e-01	0.000e+00	4.900e-01	2.372e-04	1.223e+00
103	3463	1.785488e-01	0.000e+00	1.000e+00	3.800e-03	1.176e-01
104	3494	1.785420e-01	0.000e+00	1.000e+00	9.335e-04	1.173e-01
105	3534	1.785418e-01	0.000e+00	4.035e-02	6.529e-05	2.537e-01
106	3565	1.785389e-01	0.000e+00	1.000e+00	1.683e-03	1.161e-01
107	3596	1.785364e-01	0.000e+00	1.000e+00	5.339e-04	1.156e-01
108	3627	1.785351e-01	0.000e+00	1.000e+00	2.267e-03	6.912e-01
109	3660	1.785336e-01	0.000e+00	4.900e-01	2.902e-03	1.118e-01
110	3691	1.785324e-01	0.000e+00	1.000e+00	1.904e-03	2.978e-01
111	3722	1.785312e-01	0.000e+00	1.000e+00	3.062e-04	2.775e-01
112	3753	1.785254e-01	0.000e+00	1.000e+00	6.357e-04	2.315e-01
113	3786	1.785218e-01	0.000e+00	4.900e-01	9.409e-04	2.259e-01
114	3820	1.785211e-01	0.000e+00	3.430e-01	1.777e-03	3.480e-01
115	3855	1.785173e-01	0.000e+00	2.401e-01	7.096e-03	1.069e-01
116	3886	1.785105e-01	0.000e+00	1.000e+00	4.216e-03	5.723e-02
117	3917	1.785093e-01	0.000e+00	1.000e+00	1.031e-03	1.459e-01
118	3948	1.785081e-01	0.000e+00	1.000e+00	2.599e-03	1.020e-01
119	3979	1.785061e-01	0.000e+00	1.000e+00	2.940e-04	2.377e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
120	4010	1.785015e-01	0.000e+00	1.000e+00	1.086e-03	4.306e-01
121	4041	1.784939e-01	0.000e+00	1.000e+00	4.044e-03	4.585e-01
122	4073	1.784921e-01	0.000e+00	7.000e-01	1.240e-03	3.167e-01
123	4104	1.784877e-01	0.000e+00	1.000e+00	2.766e-03	9.626e-02
124	4136	1.784868e-01	0.000e+00	7.000e-01	1.600e-03	1.886e-01
125	4167	1.784836e-01	0.000e+00	1.000e+00	7.805e-04	8.586e-02
126	4198	1.784821e-01	0.000e+00	1.000e+00	1.020e-03	1.273e-01
127	4229	1.784806e-01	0.000e+00	1.000e+00	5.044e-04	1.729e-01

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
128	4260	1.784784e-01	0.000e+00	1.000e+00	6.838e-04	1.797e-01
129	4291	1.784770e-01	0.000e+00	1.000e+00	5.871e-04	1.386e-01
130	4322	1.784742e-01	0.000e+00	1.000e+00	1.606e-03	1.789e-01
131	4355	1.784715e-01	0.000e+00	4.900e-01	1.395e-03	1.854e-01
132	4386	1.784672e-01	0.000e+00	1.000e+00	1.302e-02	1.012e-01
133	4417	1.784660e-01	0.000e+00	1.000e+00	6.964e-03	8.129e-01
134	4448	1.784573e-01	0.000e+00	1.000e+00	1.915e-03	9.941e-02
135	4479	1.784547e-01	0.000e+00	1.000e+00	7.216e-04	9.854e-02
136	4510	1.784464e-01	0.000e+00	1.000e+00	2.769e-03	9.508e-02
137	4541	1.784445e-01	0.000e+00	1.000e+00	1.642e-03	9.365e-02
138	4572	1.784438e-01	0.000e+00	1.000e+00	5.629e-04	9.348e-02
139	4607	1.784436e-01	0.000e+00	2.401e-01	3.941e-04	1.589e-01
140	4638	1.784425e-01	0.000e+00	1.000e+00	2.191e-03	9.449e-02
141	4669	1.784421e-01	0.000e+00	1.000e+00	2.986e-04	7.536e-02
142	4701	1.784417e-01	0.000e+00	7.000e-01	1.226e-03	1.116e-01
143	4732	1.784416e-01	0.000e+00	1.000e+00	4.521e-04	7.697e-02
144	4763	1.784414e-01	0.000e+00	1.000e+00	4.053e-04	7.674e-02
145	4794	1.784413e-01	0.000e+00	1.000e+00	5.853e-04	7.664e-02
146	4825	1.784411e-01	0.000e+00	1.000e+00	6.187e-04	7.668e-02
147	4856	1.784408e-01	0.000e+00	1.000e+00	1.023e-03	7.703e-02
148	4887	1.784400e-01	0.000e+00	1.000e+00	1.417e-03	1.792e-01
149	4923	1.784397e-01	0.000e+00	1.681e-01	1.306e-03	7.804e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
150	4954	1.784389e-01	0.000e+00	1.000e+00	6.984e-04	7.865e-02
151	4987	1.784382e-01	0.000e+00	4.900e-01	9.229e-04	1.698e-01
152	5019	1.784378e-01	0.000e+00	7.000e-01	8.783e-04	2.696e-01
153	5051	1.784375e-01	0.000e+00	7.000e-01	8.143e-04	7.880e-02
154	5082	1.784370e-01	0.000e+00	1.000e+00	6.939e-05	9.530e-02
155	5114	1.784368e-01	0.000e+00	7.000e-01	4.508e-04	2.242e-01
156	5145	1.784359e-01	0.000e+00	1.000e+00	5.426e-04	1.330e-01
157	5176	1.784351e-01	0.000e+00	1.000e+00	1.836e-04	1.355e-01
158	5208	1.784349e-01	0.000e+00	7.000e-01	7.194e-04	4.977e-01
159	5239	1.784336e-01	0.000e+00	1.000e+00	8.940e-04	7.721e-02
160	5270	1.784331e-01	0.000e+00	1.000e+00	3.600e-04	7.683e-02
161	5301	1.784327e-01	0.000e+00	1.000e+00	1.869e-03	3.072e-01
162	5332	1.784324e-01	0.000e+00	1.000e+00	1.359e-03	7.642e-02
163	5363	1.784321e-01	0.000e+00	1.000e+00	1.893e-04	7.628e-02
164	5396	1.784318e-01	0.000e+00	4.900e-01	4.625e-04	1.555e-01
165	5428	1.784317e-01	0.000e+00	7.000e-01	1.970e-04	8.584e-02
166	5459	1.784316e-01	0.000e+00	1.000e+00	6.438e-05	8.045e-02
167	5490	1.784313e-01	0.000e+00	1.000e+00	2.210e-04	6.865e-02
168	5521	1.784310e-01	0.000e+00	1.000e+00	3.357e-04	8.496e-02
169	5552	1.784308e-01	0.000e+00	1.000e+00	3.018e-04	4.520e-02
170	5583	1.784308e-01	0.000e+00	1.000e+00	1.104e-04	1.069e-02
171	5614	1.784308e-01	0.000e+00	1.000e+00	6.445e-05	1.023e-02
172	5645	1.784308e-01	0.000e+00	1.000e+00	3.526e-05	1.968e-02
173	5676	1.784308e-01	0.000e+00	1.000e+00	6.005e-05	3.041e-02
174	5707	1.784307e-01	0.000e+00	1.000e+00	1.120e-04	4.075e-02
175	5738	1.784306e-01	0.000e+00	1.000e+00	2.646e-04	5.397e-02
176	5770	1.784304e-01	0.000e+00	7.000e-01	3.326e-04	6.895e-02
177	5806	1.784303e-01	0.000e+00	1.681e-01	1.301e-04	7.343e-02
178	5844	1.784302e-01	0.000e+00	8.235e-02	9.353e-05	1.664e+02
179	5877	1.784302e-01	0.000e+00	4.900e-01	1.835e-04	5.130e-02
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
180	5908	1.784301e-01	0.000e+00	1.000e+00	8.481e-05	3.530e-02
181	5939	1.784299e-01	0.000e+00	1.000e+00	2.327e-04	2.686e-02
182	5970	1.784298e-01	0.000e+00	1.000e+00	1.107e-04	3.972e-02
183	6001	1.784295e-01	0.000e+00	1.000e+00	3.499e-04	4.317e-02
184	6032	1.784290e-01	0.000e+00	1.000e+00	4.855e-04	4.875e-02
185	6063	1.784286e-01	0.000e+00	1.000e+00	5.053e-04	2.512e-02
186	6094	1.784285e-01	0.000e+00	1.000e+00	3.133e-04	7.506e-03
187	6125	1.784285e-01	0.000e+00	1.000e+00	6.455e-05	4.900e-03

```
  188        6156    1.784285e-01     0.000e+00     1.000e+00     1.822e-05     4.962e-03
  189        6187    1.784285e-01     0.000e+00     1.000e+00     3.097e-06     4.811e-03
  190        6218    1.784285e-01     0.000e+00     1.000e+00     2.655e-07     4.752e-03

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
xi_star = reshape(xi_star_vec,2,[]); % final optimized trajectory

% Plot obstacles
figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center1', r1, 'Color', [0.5, 0.5, 0.5]);
viscircles(center2', r2, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')
% Plot result
grid on
hold on
axis equal
plot(xi_star(1,:), xi_star(2,:), 'o-',...
 'Color', [1, 0.5, 0], 'LineWidth', 3);
```




```
% Cost function to minimize
```

```
function C = cost(xi)
 gamma=20;
 xi = reshape(xi,2,[]);
 C = 0;
 r1 = 0.3;
 center1 = [0.5;0.3];
 r2 = 0.2;
 center2 = [0.5;0.7];
 Urep1 = 0;
 Urep2 = 0;
 for idx = 2:length(xi)

 % First obstacle
 if (norm(center1 - xi(:,idx)) <= r1)
 Urep1 = 0.5*gamma*((1/(norm(center1 - xi(:,idx)))) - (1/r1))^2;
 end
 % Second obstacle
 if (norm(center2 - xi(:,idx)) <= r2)
 Urep2 = 0.5*gamma*((1/(norm(center2 - xi(:,idx)))) - (1/r2))^2;
 end

 % Total cost
 C = C + norm(xi(:,idx) - xi(:,idx-1))^2 + Urep1 + Urep2;
```

```
  end
end
```