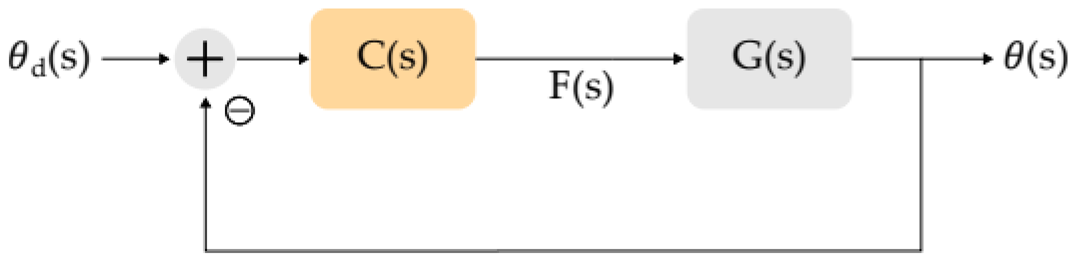
2 Stability

Imagine that you purchased three separate 1-DoF robots. Each system comes with its own links, joint, motor, and motor controller, and some of the systems are acting in strange ways. Your job is to design controllers that will result in closed-loop stability.

Throughout this problem assume that you can directly measure θ. Use the block diagram above as a reference when determining closed-loop stability.




```
syms C G theta_d theta real

eqn = G*(C*(theta_d - theta)) == theta;
eqn = solve(eqn, theta)/theta_d
```

eqn =

$$\frac{C\,G}{C\,G+1}$$

```
[~,denom] = numden(eqn)
```

denom = $C\,G+1$

2.1 (5 points)

The first 1-DoF robot is a mass-damper:

$$f(t) = m\ddot{\theta} - b\dot{\theta} \tag{2}$$

Design a controller that results in closed-loop stability.

```
syms theta m b s F real

laplacian = m*s^2 - b*s
```

laplacian = $m\,s^2 - b\,s$

```
g = 1/laplacian
```

g =

$$-\frac{1}{b\,s - m\,s^2}$$

```
c = 7*b*s + 4 % Choosing a controller
```

c = $7\,b\,s + 4$

```
eqn = subs(denom, [C,G], [c,g]) == 0
```

eqn =

$$1 - \frac{7\,b\,s + 4}{b\,s - m\,s^2} = 0$$

```
eqn = subs(eqn, [m,b], [1,1])
```

eqn =

$$1 - \frac{7\,s + 4}{s - s^2} = 0$$

```
verification = round(vpasolve(eqn, s), 4) % Both are real negative values
```

verification =

$$\begin{pmatrix} -5.2361 \\ -0.7639 \end{pmatrix}$$

2.2 (10 points)

The second 1-DoF robot has plant dynamics:

$$G(s) = \frac{1}{s(s-3)(s-5)} \tag{3}$$

Design a controller that results in closed-loop stability.

```
laplacian = (s*(s-3)*(s-5))
```

laplacian = $s\,(s-3)\,(s-5)$

```
g = 1/laplacian
```

g =

$$\frac{1}{s\,(s-3)\,(s-5)}$$

```
c = 20*s^2 + 19 % Choosing a controller
```

c = $20\,s^2 + 19$

```
eqn = subs(denom, [C,G], [c,g]) == 0
```

eqn =

$$\frac{20\,s^2+19}{s\,(s-3)\,(s-5)}+1=0$$

```
verification = round(vpasolve(eqn, s), 4) % Both are negative values that lie on
the left side of the plane which is correct in controls
```

verification =

$$\begin{pmatrix} -10.7712 \\ -0.6144+1.1775\,\mathrm{i} \\ -0.6144-1.1775\,\mathrm{i} \end{pmatrix}$$

2.3 (5 points)

The third 1-DoF robot is a mass-spring-damper:

$$f(t)=5\ddot{\theta}+2\dot{\theta}-15\theta \tag{4}$$

Design a controller that places both poles at $s=-5$. **Aside –** When both poles of a mass-spring-damper are equal negative real numbers, the system is *critically damped*.

```
laplacian = 5*s^2+ 2*s - 15
```

laplacian = $5\,s^2+2\,s-15$

```
g = 1/laplacian
```

g =

$$\frac{1}{5\,s^2+2\,s-15}$$

```
c = 48*s + 140 % Choosing a controller
```

c = $48\,s+140$

```
eqn = subs(denom, [C,G], [c,g]) == 0
```

eqn =

$$\frac{48\,s+140}{5\,s^2+2\,s-15}+1=0$$

```
verification = solve(eqn, s) % Verification that the poles are -5
```

verification = -5