

## 2.2 (15 points)

Use your code to find a desired trajectory for the following environments. In each environment  $\theta_{start} = [0, 0]^T$  and  $\theta_{goal} = [1, 1]^T$ .

- **Environment 1.** One obstacle with center  $c_1 = [0.55, 0.5]^T$  and radius  $r_1 = 0.3$ . Your trajectory should have  $k = 10$  waypoints.

```
clear
clc
close all

% Start and Goal orientations
theta_start = [0;0];
theta_goal = [1;1];

% Initial trajectory variables

n = 2; % No. of joints / 2-D trajectory.
k = 10; % No. of waypoints

% Obstacle 1 Paramters
r1 = 0.3;
center1 = [0.55;0.5];

xi_0 = zeros(n, k);
xi_0_vec = reshape(xi_0, [],1);

% Equality constraints for start and goal positions
A = [eye(n), zeros(n,n*(k-1)) ;...
     zeros(n,n*(k-1)), eye(n) ];

B = [theta_start;theta_goal];

% Nonlinear optimization
options = optimoptions('fmincon','Display','iter',...
    'Algorithm','sqp','MaxFunctionEvaluations',1e4);
xi_star_vec = fmincon(@(xi) cost(xi), xi_0_vec, ...
    [], [], A, B, [], [], options);
```

Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
0	21	0.000000e+00	1.000e+00	1.000e+00	0.000e+00	2.980e-08
1	46	1.152960e-01	7.599e-01	2.401e-01	3.396e-01	4.802e-01
2	74	1.288167e-01	6.973e-01	8.235e-02	1.177e-01	3.858e-01
3	107	1.301925e-01	6.877e-01	1.384e-02	2.070e-02	3.824e-01
4	144	1.304400e-01	6.854e-01	3.323e-03	5.499e-03	9.620e-01
5	167	7.032032e+01	3.495e-01	4.900e-01	8.867e-01	1.115e+03
6	189	6.352294e+01	1.049e-01	7.000e-01	9.000e-01	1.427e+03
7	227	4.385346e+01	1.046e-01	2.326e-03	4.696e+00	1.425e+01
8	251	2.747094e+01	6.873e-02	3.430e-01	4.806e+00	8.959e+00

9	273	3.029506e+01	2.062e-02	7.000e-01	3.580e+00	1.101e+01
10	294	2.194269e+01	0.000e+00	1.000e+00	3.772e+00	5.910e+00
11	317	1.691520e+01	0.000e+00	4.900e-01	2.460e+00	6.687e+00
12	338	1.556209e+01	0.000e+00	1.000e+00	2.682e+00	5.474e+00
13	361	1.373612e+01	0.000e+00	4.900e-01	1.497e+00	5.272e+00
14	382	1.356662e+01	0.000e+00	1.000e+00	2.439e+00	5.988e+00
15	404	1.202932e+01	0.000e+00	7.000e-01	1.448e+00	4.338e+00
16	426	1.090450e+01	0.000e+00	7.000e-01	1.709e+00	4.760e+00
17	448	1.017564e+01	0.000e+00	7.000e-01	9.125e-01	4.769e+00
18	470	1.000271e+01	0.000e+00	7.000e-01	1.152e+00	4.435e+00
19	491	9.682184e+00	0.000e+00	1.000e+00	9.631e-01	4.145e+00
20	512	9.261388e+00	0.000e+00	1.000e+00	1.187e+00	4.791e+00
21	533	8.374244e+00	0.000e+00	1.000e+00	8.414e-01	3.774e+00
22	554	7.824443e+00	0.000e+00	1.000e+00	8.498e-01	4.208e+00
23	575	6.749018e+00	0.000e+00	1.000e+00	3.605e-01	3.726e+00
24	596	3.307992e+00	0.000e+00	1.000e+00	1.625e+00	2.179e+00
25	619	2.037926e+00	0.000e+00	4.900e-01	8.710e-01	1.745e+00
26	646	1.927091e+00	0.000e+00	1.176e-01	2.098e-01	2.848e+01
27	671	1.130807e+00	0.000e+00	2.401e-01	1.517e+00	1.308e+00
28	692	9.908110e-01	0.000e+00	1.000e+00	5.148e-01	9.454e-01
29	713	9.585879e-01	0.000e+00	1.000e+00	1.772e-01	8.913e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
30	734	9.504782e-01	0.000e+00	1.000e+00	1.254e-01	8.607e-01
31	755	9.433863e-01	0.000e+00	1.000e+00	1.697e-01	8.485e-01
32	776	9.404222e-01	0.000e+00	1.000e+00	7.222e-02	8.553e-01
33	797	9.358837e-01	0.000e+00	1.000e+00	7.918e-02	8.530e-01
34	818	9.277798e-01	0.000e+00	1.000e+00	8.573e-02	8.466e-01
35	839	9.090249e-01	0.000e+00	1.000e+00	1.152e-01	8.383e-01
36	860	8.698169e-01	0.000e+00	1.000e+00	1.282e-01	8.426e-01
37	881	8.003106e-01	0.000e+00	1.000e+00	1.655e-01	8.889e-01
38	908	7.894592e-01	0.000e+00	1.176e-01	3.813e-02	2.749e+00
39	933	7.491688e-01	0.000e+00	2.401e-01	7.171e-01	1.287e+00
40	954	7.010403e-01	0.000e+00	1.000e+00	2.195e-01	1.167e+00
41	975	6.955670e-01	0.000e+00	1.000e+00	8.705e-02	1.187e+00
42	996	6.935455e-01	0.000e+00	1.000e+00	6.847e-02	1.211e+00
43	1017	6.917963e-01	0.000e+00	1.000e+00	4.121e-02	1.224e+00
44	1038	6.852306e-01	0.000e+00	1.000e+00	1.035e-01	1.249e+00
45	1059	6.715092e-01	0.000e+00	1.000e+00	1.428e-01	1.267e+00
46	1082	6.552153e-01	0.000e+00	4.900e-01	1.079e-01	2.159e+00
47	1110	6.443417e-01	0.000e+00	8.235e-02	8.074e-02	4.873e+00
48	1140	6.384926e-01	0.000e+00	4.035e-02	3.996e-02	3.916e+00
49	1166	6.086512e-01	0.000e+00	1.681e-01	2.875e-01	9.759e-01
50	1190	5.959004e-01	0.000e+00	3.430e-01	8.505e-02	4.651e+00
51	1215	5.695792e-01	0.000e+00	2.401e-01	4.264e-01	9.988e-01
52	1239	5.585371e-01	0.000e+00	3.430e-01	8.067e-02	3.296e+00
53	1261	5.568165e-01	0.000e+00	7.000e-01	1.470e-01	9.637e-01
54	1282	5.536087e-01	0.000e+00	1.000e+00	8.744e-02	9.644e-01
55	1303	5.513904e-01	0.000e+00	1.000e+00	5.739e-03	9.545e-01
56	1325	5.475078e-01	0.000e+00	7.000e-01	2.264e-02	1.724e+00
57	1347	5.473799e-01	0.000e+00	7.000e-01	1.692e-02	9.367e-01
58	1368	5.462014e-01	0.000e+00	1.000e+00	7.459e-03	9.319e-01
59	1391	5.452781e-01	0.000e+00	4.900e-01	1.038e-02	1.244e+00
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
60	1412	5.444957e-01	0.000e+00	1.000e+00	4.100e-03	9.236e-01
61	1433	5.440097e-01	0.000e+00	1.000e+00	2.957e-03	5.819e-01
62	1454	5.427847e-01	0.000e+00	1.000e+00	1.278e-02	9.597e-01
63	1475	5.393796e-01	0.000e+00	1.000e+00	2.612e-02	1.160e+00
64	1497	5.346344e-01	0.000e+00	7.000e-01	2.626e-01	1.009e+00
65	1518	5.237382e-01	0.000e+00	1.000e+00	7.361e-02	9.201e-01
66	1539	5.160012e-01	0.000e+00	1.000e+00	7.879e-02	7.991e-01
67	1560	5.147708e-01	0.000e+00	1.000e+00	2.968e-02	7.754e-01
68	1581	5.138298e-01	0.000e+00	1.000e+00	2.674e-02	7.667e-01

69	1602	5.123361e-01	0.000e+00	1.000e+00	2.860e-02	7.607e-01
70	1623	5.086079e-01	0.000e+00	1.000e+00	4.696e-02	7.511e-01
71	1644	5.020822e-01	0.000e+00	1.000e+00	5.351e-02	7.389e-01
72	1665	4.937955e-01	0.000e+00	1.000e+00	5.043e-02	7.277e-01
73	1686	4.880069e-01	0.000e+00	1.000e+00	5.624e-02	7.249e-01
74	1714	4.879379e-01	0.000e+00	8.235e-02	4.223e-03	3.960e-01
75	1735	4.872819e-01	0.000e+00	1.000e+00	1.427e-01	8.078e-01
76	1756	4.833876e-01	0.000e+00	1.000e+00	6.588e-02	7.718e-01
77	1777	4.807187e-01	0.000e+00	1.000e+00	2.329e-02	7.585e-01
78	1798	4.693274e-01	0.000e+00	1.000e+00	6.169e-02	7.276e-01
79	1819	4.540552e-01	0.000e+00	1.000e+00	7.595e-02	7.175e-01
80	1840	4.370382e-01	0.000e+00	1.000e+00	1.362e-01	7.455e-01
81	1861	4.295554e-01	0.000e+00	1.000e+00	1.159e-01	7.910e-01
82	1882	4.277897e-01	0.000e+00	1.000e+00	4.723e-02	8.142e-01
83	1903	4.269821e-01	0.000e+00	1.000e+00	1.677e-02	8.208e-01
84	1924	4.254267e-01	0.000e+00	1.000e+00	2.820e-02	1.690e+00
85	1955	4.246377e-01	0.000e+00	2.825e-02	7.490e-02	8.281e-01
86	1976	4.203536e-01	0.000e+00	1.000e+00	3.101e-02	8.080e-01
87	1997	4.076927e-01	0.000e+00	1.000e+00	5.220e-02	7.196e-01
88	2018	3.892482e-01	0.000e+00	1.000e+00	5.975e-02	5.503e-01
89	2045	3.861608e-01	0.000e+00	1.176e-01	1.255e-02	6.638e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
90	2066	3.805247e-01	0.000e+00	1.000e+00	3.173e-01	5.479e-01
91	2087	3.719592e-01	0.000e+00	1.000e+00	1.127e-01	3.873e-01
92	2108	3.688493e-01	0.000e+00	1.000e+00	2.377e-02	4.061e-01
93	2131	3.640324e-01	0.000e+00	4.900e-01	2.813e-02	1.781e+00
94	2152	3.596278e-01	0.000e+00	1.000e+00	8.230e-02	3.523e-01
95	2173	3.541344e-01	0.000e+00	1.000e+00	2.486e-02	3.749e-01
96	2194	3.387311e-01	0.000e+00	1.000e+00	7.866e-02	4.607e-01
97	2217	3.383436e-01	0.000e+00	4.900e-01	1.616e-02	1.962e+00
98	2247	3.352298e-01	0.000e+00	4.035e-02	4.854e-02	4.237e-01
99	2275	3.351512e-01	0.000e+00	8.235e-02	3.908e-03	4.631e-01
100	2296	3.339114e-01	0.000e+00	1.000e+00	3.917e-02	3.386e-01
101	2317	3.332994e-01	0.000e+00	1.000e+00	1.145e-02	3.456e-01
102	2338	3.324295e-01	0.000e+00	1.000e+00	1.086e-02	3.693e-01
103	2359	3.302767e-01	0.000e+00	1.000e+00	2.633e-02	8.111e-01
104	2380	3.271194e-01	0.000e+00	1.000e+00	3.328e-02	4.405e-01
105	2406	3.261397e-01	0.000e+00	1.681e-01	9.718e-03	4.383e-01
106	2435	3.261058e-01	0.000e+00	5.765e-02	2.029e-03	5.912e-01
107	2457	3.259175e-01	0.000e+00	7.000e-01	6.255e-02	4.027e-01
108	2478	3.251367e-01	0.000e+00	1.000e+00	3.110e-02	4.119e-01
109	2499	3.249058e-01	0.000e+00	1.000e+00	5.306e-03	4.112e-01
110	2520	3.238941e-01	0.000e+00	1.000e+00	5.840e-03	9.391e-01
111	2541	3.207431e-01	0.000e+00	1.000e+00	1.757e-02	2.578e+00
112	2562	3.145294e-01	0.000e+00	1.000e+00	4.801e-02	4.253e+00
113	2588	3.134487e-01	0.000e+00	1.681e-01	1.771e-02	4.142e+00
114	2612	3.131258e-01	0.000e+00	3.430e-01	3.409e-01	2.479e-01
115	2633	2.963018e-01	0.000e+00	1.000e+00	1.948e-01	1.884e-01
116	2654	2.952003e-01	0.000e+00	1.000e+00	1.063e-02	1.580e-01
117	2675	2.935469e-01	0.000e+00	1.000e+00	1.779e-02	1.203e+00
118	2696	2.909305e-01	0.000e+00	1.000e+00	4.284e-02	2.521e-01
119	2717	2.873025e-01	0.000e+00	1.000e+00	4.136e-02	2.433e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
120	2741	2.836733e-01	0.000e+00	3.430e-01	4.304e-02	9.319e-01
121	2766	2.833367e-01	0.000e+00	2.401e-01	1.165e-02	2.861e+00
122	2787	2.802654e-01	0.000e+00	1.000e+00	5.053e-02	2.243e-01
123	2808	2.779226e-01	0.000e+00	1.000e+00	2.259e-02	2.397e-01
124	2829	2.746141e-01	0.000e+00	1.000e+00	5.180e-02	2.895e-01
125	2850	2.736594e-01	0.000e+00	1.000e+00	1.906e-02	3.004e-01
126	2874	2.734645e-01	0.000e+00	3.430e-01	9.272e-03	1.259e+00
127	2897	2.733963e-01	0.000e+00	4.900e-01	9.023e-03	1.287e+00
128	2918	2.728705e-01	0.000e+00	1.000e+00	3.618e-02	2.783e-01

129	2939	2.717773e-01	0.000e+00	1.000e+00	1.907e-02	2.780e-01
130	2960	2.709023e-01	0.000e+00	1.000e+00	1.607e-02	2.726e-01
131	2981	2.697157e-01	0.000e+00	1.000e+00	2.034e-02	2.622e-01
132	3002	2.682152e-01	0.000e+00	1.000e+00	2.705e-02	2.309e-01
133	3023	2.673176e-01	0.000e+00	1.000e+00	1.822e-02	2.075e-01
134	3044	2.669710e-01	0.000e+00	1.000e+00	8.992e-03	1.979e-01
135	3065	2.668130e-01	0.000e+00	1.000e+00	8.502e-03	1.947e-01
136	3086	2.667051e-01	0.000e+00	1.000e+00	8.663e-03	1.935e-01
137	3107	2.665770e-01	0.000e+00	1.000e+00	1.032e-02	1.921e-01
138	3128	2.663609e-01	0.000e+00	1.000e+00	1.465e-02	1.897e-01
139	3149	2.659549e-01	0.000e+00	1.000e+00	2.301e-02	1.846e-01
140	3170	2.653537e-01	0.000e+00	1.000e+00	2.929e-02	1.765e-01
141	3191	2.648231e-01	0.000e+00	1.000e+00	2.134e-02	1.687e-01
142	3212	2.645959e-01	0.000e+00	1.000e+00	8.264e-03	1.660e-01
143	3233	2.645226e-01	0.000e+00	1.000e+00	7.392e-03	1.662e-01
144	3254	2.644462e-01	0.000e+00	1.000e+00	7.004e-03	1.665e-01
145	3275	2.642458e-01	0.000e+00	1.000e+00	1.210e-02	1.658e-01
146	3296	2.637635e-01	0.000e+00	1.000e+00	1.826e-02	1.616e-01
147	3317	2.626155e-01	0.000e+00	1.000e+00	2.701e-02	1.471e-01
148	3342	2.625048e-01	0.000e+00	2.401e-01	7.894e-03	1.243e+00
149	3363	2.596185e-01	0.000e+00	1.000e+00	5.284e-02	1.580e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
150	3384	2.578974e-01	0.000e+00	1.000e+00	3.636e-02	1.526e-01
151	3405	2.568010e-01	0.000e+00	1.000e+00	4.611e-02	1.244e-01
152	3426	2.566945e-01	0.000e+00	1.000e+00	1.861e-03	1.227e-01
153	3447	2.561382e-01	0.000e+00	1.000e+00	9.197e-03	1.247e-01
154	3471	2.558216e-01	0.000e+00	3.430e-01	1.086e-02	1.405e-01
155	3501	2.557915e-01	0.000e+00	4.035e-02	1.484e-03	6.151e-02
156	3522	2.557530e-01	0.000e+00	1.000e+00	5.983e-03	1.431e-01
157	3546	2.557466e-01	0.000e+00	3.430e-01	7.618e-04	1.739e-01
158	3567	2.557343e-01	0.000e+00	1.000e+00	5.574e-04	6.748e-02
159	3588	2.557282e-01	0.000e+00	1.000e+00	5.965e-04	6.540e-02
160	3609	2.557145e-01	0.000e+00	1.000e+00	1.657e-03	3.191e-01
161	3630	2.556839e-01	0.000e+00	1.000e+00	4.008e-04	1.009e-01
162	3651	2.556182e-01	0.000e+00	1.000e+00	1.952e-03	9.462e-02
163	3672	2.553050e-01	0.000e+00	1.000e+00	1.519e-02	1.226e-01
164	3693	2.551542e-01	0.000e+00	1.000e+00	1.859e-02	1.406e-01
165	3714	2.551199e-01	0.000e+00	1.000e+00	2.668e-03	1.431e-01
166	3735	2.549032e-01	0.000e+00	1.000e+00	8.075e-03	1.496e-01
167	3761	2.548681e-01	0.000e+00	1.681e-01	5.699e-03	6.169e-01
168	3782	2.546574e-01	0.000e+00	1.000e+00	2.589e-02	1.717e-01
169	3803	2.546060e-01	0.000e+00	1.000e+00	2.955e-03	1.684e-01
170	3827	2.545243e-01	0.000e+00	3.430e-01	1.765e-03	2.165e-01
171	3848	2.544476e-01	0.000e+00	1.000e+00	1.119e-02	1.719e-01
172	3876	2.544463e-01	0.000e+00	8.235e-02	1.210e-03	1.726e-01
173	3897	2.543965e-01	0.000e+00	1.000e+00	5.062e-03	8.554e-02
174	3918	2.543899e-01	0.000e+00	1.000e+00	2.104e-03	8.597e-02
175	3939	2.543781e-01	0.000e+00	1.000e+00	3.085e-03	8.177e-02
176	3963	2.543771e-01	0.000e+00	3.430e-01	2.838e-04	8.784e-02
177	3984	2.543693e-01	0.000e+00	1.000e+00	5.284e-04	7.969e-02
178	4005	2.543265e-01	0.000e+00	1.000e+00	2.922e-03	6.589e-02
179	4026	2.542294e-01	0.000e+00	1.000e+00	1.270e-02	1.482e-01
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
180	4047	2.541919e-01	0.000e+00	1.000e+00	3.827e-03	1.470e-01
181	4068	2.541886e-01	0.000e+00	1.000e+00	5.721e-03	2.946e-01
182	4089	2.541509e-01	0.000e+00	1.000e+00	5.629e-03	8.869e-02
183	4110	2.541473e-01	0.000e+00	1.000e+00	6.916e-04	2.065e-02
184	4131	2.541458e-01	0.000e+00	1.000e+00	1.293e-03	1.744e-02
185	4152	2.541446e-01	0.000e+00	1.000e+00	2.045e-03	1.077e-02
186	4173	2.541446e-01	0.000e+00	1.000e+00	3.078e-04	8.010e-03
187	4194	2.541445e-01	0.000e+00	1.000e+00	1.272e-04	7.914e-03
188	4215	2.541444e-01	0.000e+00	1.000e+00	2.027e-04	7.708e-03

189	4236	2.541441e-01	0.000e+00	1.000e+00	2.305e-04	1.536e-02
190	4257	2.541434e-01	0.000e+00	1.000e+00	2.922e-04	2.660e-02
191	4278	2.541420e-01	0.000e+00	1.000e+00	5.916e-04	4.001e-02
192	4299	2.541392e-01	0.000e+00	1.000e+00	1.436e-03	4.939e-02
193	4320	2.541350e-01	0.000e+00	1.000e+00	2.849e-03	3.459e-02
194	4341	2.541324e-01	0.000e+00	1.000e+00	2.519e-03	1.703e-02
195	4362	2.541318e-01	0.000e+00	1.000e+00	6.746e-04	2.440e-03
196	4383	2.541318e-01	0.000e+00	1.000e+00	1.737e-04	1.544e-03
197	4404	2.541317e-01	0.000e+00	1.000e+00	6.917e-05	1.443e-03
198	4425	2.541317e-01	0.000e+00	1.000e+00	6.871e-05	1.472e-03
199	4446	2.541317e-01	0.000e+00	1.000e+00	5.670e-05	1.485e-03
200	4467	2.541317e-01	0.000e+00	1.000e+00	9.671e-05	3.055e-03
201	4488	2.541317e-01	0.000e+00	1.000e+00	1.470e-04	6.135e-03
202	4509	2.541316e-01	0.000e+00	1.000e+00	2.496e-04	1.114e-02
203	4530	2.541315e-01	0.000e+00	1.000e+00	4.059e-04	1.849e-02
204	4551	2.541311e-01	0.000e+00	1.000e+00	6.239e-04	2.735e-02
205	4572	2.541305e-01	0.000e+00	1.000e+00	7.938e-04	3.151e-02
206	4593	2.541300e-01	0.000e+00	1.000e+00	6.649e-04	2.061e-02
207	4614	2.541298e-01	0.000e+00	1.000e+00	2.496e-04	5.262e-03
208	4635	2.541298e-01	0.000e+00	1.000e+00	1.267e-04	4.221e-04
209	4656	2.541298e-01	0.000e+00	1.000e+00	4.468e-05	4.099e-05
Iter	Func-count	Fval	Feasibility	Step Length	Norm of step	First-order optimality
210	4677	2.541298e-01	0.000e+00	1.000e+00	5.090e-06	6.840e-06
211	4698	2.541298e-01	0.000e+00	1.000e+00	5.339e-07	7.413e-07

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the value of the optimality tolerance, and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>

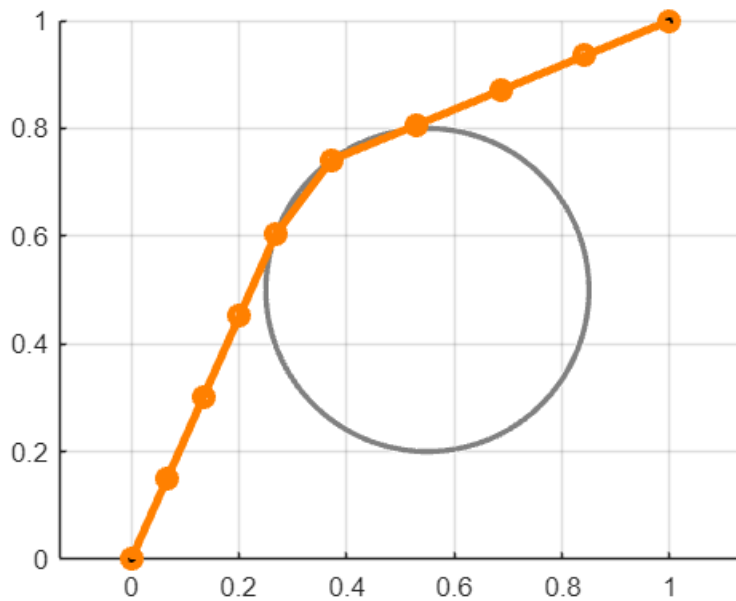
```

xi_star = reshape(xi_star_vec,2,[]); % to implement

figure
grid on
hold on
axis([0, 1, 0, 1])
axis equal
viscircles(center1', r1, 'Color', [0.5, 0.5, 0.5]);
plot(0, 0, 'ko', 'MarkerFaceColor', 'k')
plot(1, 1, 'ko', 'MarkerFaceColor', 'k')

% Plot Result
grid on
hold on
axis equal
plot(theta_start(1), theta_start(2), 'ko', 'MarkerFaceColor', 'k')
plot(theta_goal(1), theta_goal(2), 'ko', 'MarkerFaceColor', 'k')
% Plot Result
plot(xi_0(1,:), xi_0(2,:), 'o-', 'Color', [0.3, 0.3, ...
0.3], 'LineWidth', 3);
plot(xi_star(1,:), xi_star(2,:), 'o-',...
'Color', [1, 0.5, 0], 'LineWidth', 3);

```



```
% Cost function to minimize
```

```
function C = cost(xi)

    gamma = 20;
    xi = reshape(xi,2,[]);
    C = 0;
    r1 = 0.3;
    center1 = [0.55;0.5];

    for idx = 2:length(xi)
        Urep1 = 0;

        if (norm(center1 - xi(:,idx)) <= r1)
            Urep1 = 0.5*gamma*((1/(norm(center1-xi(:,idx)))) - (1/r1))^2;
        end

        C = Urep1 + C + norm(xi(:,idx) - xi(:,idx-1))^2;
    end
end
```