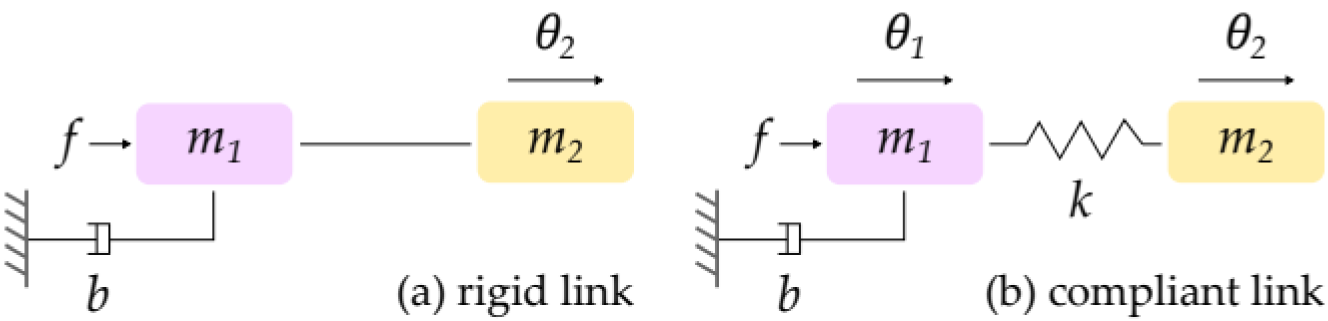
3 Compliant Joints



We often introduce mechanical compliance to make the robot soft and safe during interaction. The drawing above compares a rigid link and (Left) and a compliant link (Right). Here m_1 is the motor mass and m_2 is the link mass. For rigid systems we have a rigid connection between motor output and the link, and the total mass is $m_1 + m_2$. For compliant systems we introduce a spring k between the motor output and the link. In both systems we control the actuator force f to regulate θ_2, the position of the link.

```
syms F s m1 m2 theta_1 theta_2 k b real
```

3.1 (5 points)

Find the plant dynamics $G_1(s)$ and $G_2(s)$. Here $G_1(s)$ is the plant for the rigid 1-DoF robot and $G_2(s)$ is the plant for the compliant 1-DoF robot. Both plants should be of the form:

$$G(s) = \frac{\theta_2(s)}{F(s)} \tag{5}$$

```
% G1 evaluation
eqn = F == (m1 + m2)*s^2*theta_2 + b*s*theta_2
```

eqn = $F = \theta_2\,(m_1 + m_2)\,s^2 + b\,\theta_2\,s$

```
G1 = solve(eqn, theta_2)/F
```

G1 =

$$\frac{1}{(m_1 + m_2)\,s^2 + b\,s}$$

```
% G2 evaluation

eqn = F - b*s*theta_1 == m1*s^2*theta_1 + k*(theta_1 - theta_2)
```

eqn = $F - b\,s\,\theta_1 = m_1\,\theta_1\,s^2 + k\,(\theta_1 - \theta_2)$

```
theta_1 = simplify(solve(eqn, theta_1))
```

theta_1 =

$$\frac{F + k\,\theta_2}{m_1\,s^2 + b\,s + k}$$

```
eqn = -k*(theta_2 - theta_1) == m2*s^2*theta_2
```

eqn =

$$-k\,\left(\theta_2 - \frac{F + k\,\theta_2}{m_1\,s^2 + b\,s + k}\right) = m_2\,s^2\,\theta_2$$

```
G2 = simplify(solve(eqn, theta_2))/F
```

G2 =

$$\frac{k}{s\,\left(b\,k + b\,m_2\,s^2 + m_1\,m_2\,s^3 + k\,m_1\,s + k\,m_2\,s\right)}$$

3.2 (10 points)

Assume we measure θ_2 in real-time and the closed-loop transfer function is:

$$\frac{\theta(s)}{\theta_d(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} \tag{6}$$

We will use a proportional controller $C(s) = k_p$. Let $m_1 = 1$ kg, $m_2 = 2$ kg, $b = 0.1$ Ns/m, and $k = 100$ N/m.

- For what range of k_p is the **rigid** robot stable?
- For what range of k_p is the **compliant** robot stable?

```
% 3.2 (a)
```

- For what range of k_p is the **rigid** robot stable?

```
syms C G theta_d theta real

eqn = G*(C*(theta_d - theta)) == theta;
eqn = solve(eqn, theta)/theta_d
```

eqn =

$$\frac{C\,G}{C\,G + 1}$$

```
[~,den] = numden(eqn)
```

den = $C\,G + 1$

```
syms kp real
```

```
c = kp;

g = G1;

eqn = subs(den, [C,G], [c,g]) == 0
```

eqn =

$$\frac{\mathrm{kp}}{(m_1+m_2)\,s^2+b\,s}+1=0$$

We will use a proportional controller $C(s) = k_p$. Let $m_1 = 1$ kg, $m_2 = 2$ kg, $b = 0.1$ Ns/m, and $k = 100$ N/m.

```
eqn = simplify(subs(eqn, [m1, m2, b, k], [1, 2, 0.1, 100])) % Further range is
provided in the written section, this is just to get the simplified characteristic
equation with s and Kp
```

eqn =

$$30\,s^2+s+10\,\mathrm{kp}=0 \land s \neq 0 \land s \neq -\frac{1}{30}$$

```
% 3.2 (b)
```

- For what range of k_p is the **compliant** robot stable?

```
c = kp;

g = G2;

eqn = subs(den, [C,G], [c,g]) == 0
```

eqn =

$$\frac{k\,\mathrm{kp}}{s\,\left(b\,k+b\,m_2\,s^2+m_1\,m_2\,s^3+k\,m_1\,s+k\,m_2\,s\right)}+1=0$$

We will use a proportional controller $C(s) = k_p$. Let $m_1 = 1$ kg, $m_2 = 2$ kg, $b = 0.1$ Ns/m, and $k = 100$ N/m.

```
eqn = subs(eqn, [m1, m2, b, k], [1, 2, 0.1, 100]) % Further range is provided in
the written section, this is just to get the simplified characteristic equation
with s and Kp
```

eqn =

$$\frac{100\,\mathrm{kp}}{s\,\left(2\,s^3+\frac{s^2}{5}+300\,s+10\right)}+1=0$$