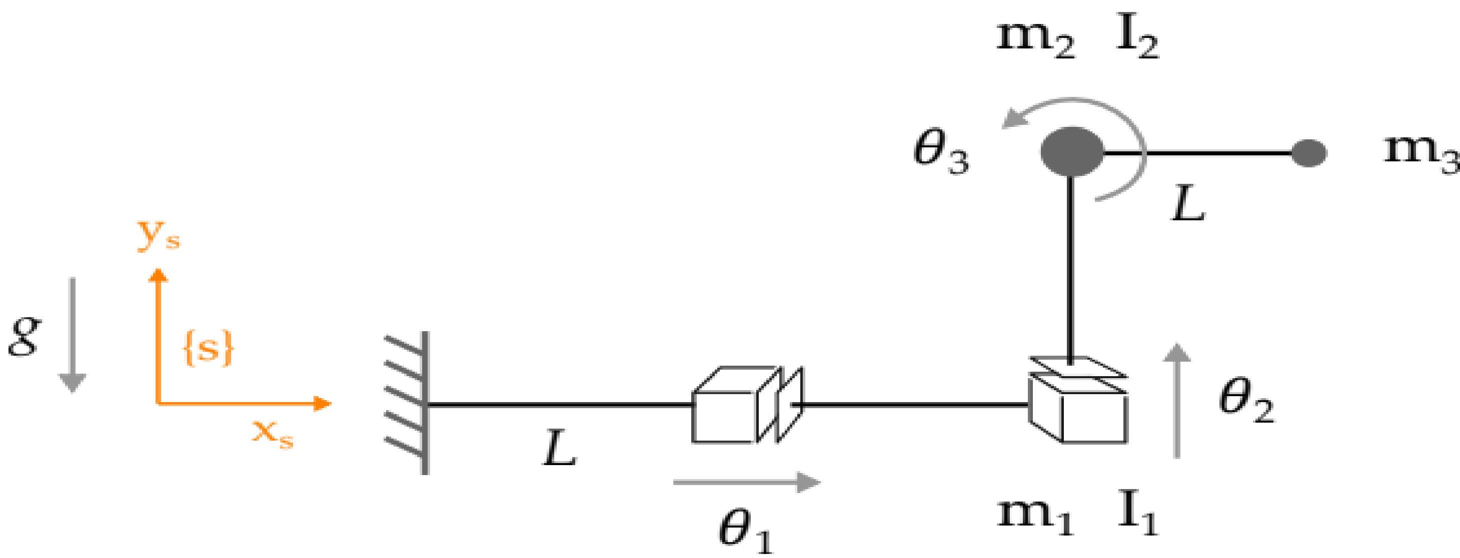

Find the dynamics for the robot shown above. Your answer should be of the form:

$$\tau = \mathrm{M}(\Theta) * \ddot{\theta} + C\left(\theta, \dot{\theta}\right) * \dot{\theta} + g(\theta)$$

List the Mass matrix, the Coriolis matrix, and the Gravity vector. Each center of mass is at the end of the link.

```
clc;
clear;

syms L g m1 m2 m3 theta1 theta2 theta3 theta1_dot theta2_dot theta3_dot
theta1_dot_dot theta2_dot_dot theta3_dot_dot Ix1 Ix2 Ix3 Iy1 Iy2 Iy3 Iz1 Iz2 Iz3
real

theta = [theta1; theta2; theta3];
thetadot = [theta1_dot; theta2_dot; theta3_dot];
thetadotdot = [theta1_dot_dot; theta2_dot_dot; theta3_dot_dot];

% Home matrix till m1 center of mass
M1 = [eye(3),[L;0;0]; 0 0 0 1];
% Home matrix till m2 center of mass
M2 = [eye(3), [L;0;0]; 0 0 0 1];
% Home matrix till m3 center of mass
M3 = [eye(3), [2*L;0;0]; 0 0 0 1];

% Screw for joint 1
S1 = [0;0;0;1;0;0];
% Screw for joint 2
S2 = [0;0;0;0;1;0];
% Screw for joint 3
S3 = [0;0;1;0;0;0];
```

```
% Considering m1 center of mass as end-effector
S_eq1 = [S1, zeros(6, 1), zeros(6, 1)];
% Considering m2 center of mass as end-effector
S_eq2 = [S1, S2, zeros(6, 1)];
% Considering m3 center of mass as end-effector
S_eq3 = [S1, S2, S3];

% For center of mass m1
% Forward kinematics
T_1 = fk(M1, S_eq1, theta)
```

T_1 =

$$\begin{pmatrix} 1 & 0 & 0 & L+\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
R_1 = T_1(1:3, 1:3);
% Space Jacobian
Js_1 = simplify(expand(JacS(S_eq1, theta)));
% Body Jacobian
Jb_1 = adjointM(inv(T_1))*Js_1;
% Geometric Jacobian
J_geometric_1 = simplify(expand([R_1, zeros(3); zeros(3), R_1] * Jb_1));
% NOTE: For Jw(x1:y1, x2:y2) and Jv(x1:y1, x2:y2), number of columns y1 and
% y2 vary according to the number of joints, thus we change accordingly
Jw1 = J_geometric_1(1:3,1:3)
```

Jw1 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Jv1 = J_geometric_1(4:6, 1:3)
```

Jv1 =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Inertia_1 = [[Ix1 0 0]
    [0 Iy1 0]
    [0 0 Iz1]];

% For m2 center of mass
T_2 = fk(M2, S_eq2, theta)
```

T_2 =

$$\begin{pmatrix} 1 & 0 & 0 & L+\theta_1 \\ 0 & 1 & 0 & \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
R_2 = T_2(1:3, 1:3);
Js_2 = simplify(expand(JacS(S_eq2, theta))); % Space Jacobian
Jb_2 = adjointM(inv(T_2))*Js_2; % Body Jacobian
J_geometric_2 = simplify(expand([R_2, zeros(3); zeros(3), R_2] * Jb_2)); %
Geometric Jacobian
Jw2 = J_geometric_2(1:3,1:3)
```

Jw2 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Jv2 = J_geometric_2(4:6, 1:3)
```

Jv2 =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Inertia_2 = [[Ix2 0 0]
    [0 Iy2 0]
    [0 0 Iz2]];

% For m3 center of mass
T_3 = fk(M3, S_eq3, theta)
```

T_3 =

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & \theta_1 + 2\,L\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & \theta_2 + 2\,L\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
R_3 = T_3(1:3, 1:3);
Js_3 = simplify(expand(JacS(S_eq3, theta))); % Space Jacobian
Jb_3 = adjointM(inv(T_3))*Js_3; % Body Jacobian
J_geometric_3 = simplify(expand([R_3, zeros(3); zeros(3), R_3] * Jb_3)); %
Geometric Jacobian
Jw3 = J_geometric_3(1:3,1:3)
```

Jw3 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Jv3 = J_geometric_3(4:6, 1:3)
```

Jv3 =

$$\begin{pmatrix} 1 & 0 & -2\,L\sin(\theta_3) \\ 0 & 1 & 2\,L\cos(\theta_3) \\ 0 & 0 & 0 \end{pmatrix}$$

```
Inertia_3 = [[Ix3 0 0]
    [0 Iy3 0]
    [0 0 Iz3]];


% Mass Matrix
Mass_Matrix = simplify(expand(m1*(Jv1'*Jv1) + Jw1'*R_1*Inertia_1*R_1'*Jw1
+ m2*(Jv2'*Jv2) + Jw2'*R_2*Inertia_2*R_2'*Jw2 + m3*(Jv3'*Jv3) +
Jw3'*R_3*Inertia_3*R_3'*Jw3))
```

Mass_Matrix =

$$\begin{pmatrix} m_1+m_2+m_3 & 0 & -2\,L\,m_3\sin(\theta_3) \\ 0 & m_2+m_3 & 2\,L\,m_3\cos(\theta_3) \\ -2\,L\,m_3\sin(\theta_3) & 2\,L\,m_3\cos(\theta_3) & 4\,m_3\,L^2+\mathrm{Iz}_3 \end{pmatrix}$$

```
% Coriolis Matrix
Coriolis_Matrix = coriolis(Mass_Matrix, theta, thetadot)
```

Coriolis_Matrix =

$$\begin{pmatrix} 0 & 0 & -2\,L\,m_3\,\dot{\theta}_3\cos(\theta_3) \\ 0 & 0 & -2\,L\,m_3\,\dot{\theta}_3\sin(\theta_3) \\ 0 & 0 & 0 \end{pmatrix}$$

```
% Height of each center of mass
h1 = T_1(2, 4)
```

h1 = 0

```
h2 = T_2(2, 4)
```

h2 = θ_2

```
h3 = T_3(2, 4)
```

`h3 =` $\theta_2 + 2\,L\,\sin(\theta_3)$

```
% Potential Energy
P = g*m1*h1 + g*m2*h2 + g*m3*h3
```

`P =` $g\,m_3\,(\theta_2 + 2\,L\,\sin(\theta_3)) + g\,m_2\,\theta_2$

```
% Gravity vector
gravity_vector = simplify(expand([diff(P, theta(1)); diff(P, theta(2)); diff(P,
theta(3))]))
```

`gravity_vector =`

$$\begin{pmatrix} 0 \\ g\,(m_2 + m_3) \\ 2\,L\,g\,m_3\cos(\theta_3) \end{pmatrix}$$

```
% Tau calculation
tau = simplify(expand(Mass_Matrix*thetadotdot + Coriolis_Matrix*thetadot +
gravity_vector))
```

`tau =`

$$\begin{pmatrix} -2\,L\,m_3\cos(\theta_3)\,\theta_{\substack{:\\3}}^{2} + m_1\,\theta_{\substack{:\\1}} + m_2\,\theta_{\substack{:\\1}} + m_3\,\theta_{\substack{:\\1}} - 2\,L\,m_3\,\theta_{\substack{:\\3}}\sin(\theta_3) \\ -2\,L\,m_3\sin(\theta_3)\,\theta_{\substack{:\\3}}^{2} + g\,m_2 + g\,m_3 + m_2\,\theta_{\substack{:\\2}} + m_3\,\theta_{\substack{:\\2}} + 2\,L\,m_3\,\theta_{\substack{:\\3}}\cos(\theta_3) \\ \mathrm{Iz}_3\,\theta_{\substack{:\\3}} + 4\,L^2\,m_3\,\theta_{\substack{:\\3}} + 2\,L\,g\,m_3\cos(\theta_3) + 2\,L\,m_3\,\theta_{\substack{:\\2}}\cos(\theta_3) - 2\,L\,m_3\,\theta_{\substack{:\\1}}\sin(\theta_3) \end{pmatrix}$$