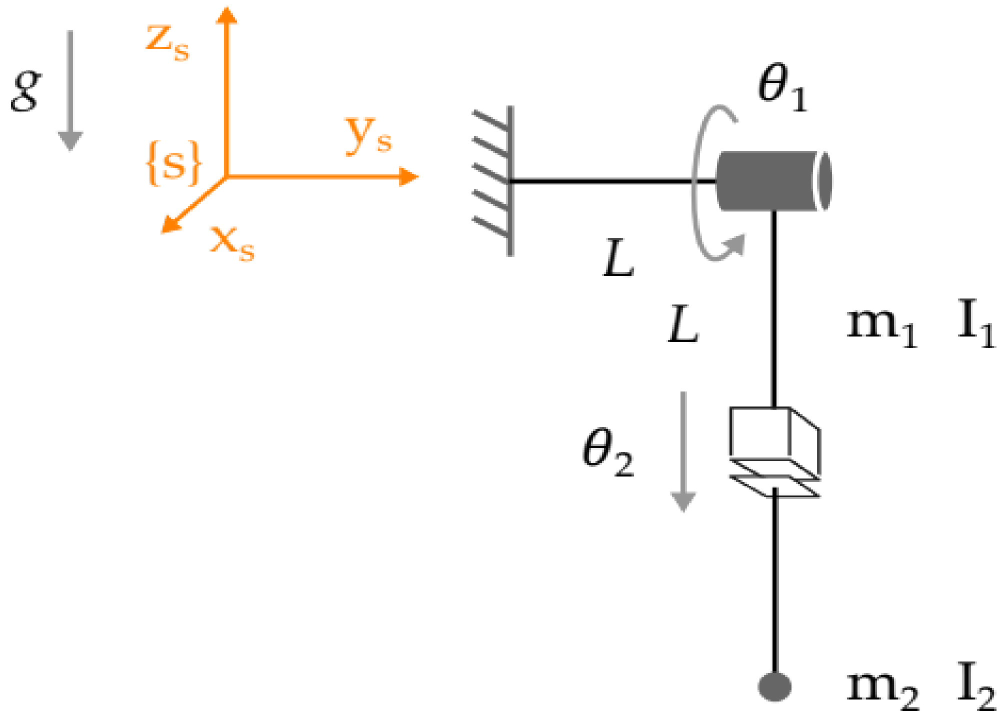

Find the dynamics for the robot shown above. Your answer should be of the form:

$$\tau = \mathrm{M}(\Theta) * \dot{\dot{\theta}} + C\left(\theta, \dot{\theta}\right) * \dot{\theta} + g(\theta)$$

List the Mass matrix, the Coriolis matrix, and the Gravity vector. The center of mass for m_1 is located halfway between the revolute joint and the prismatic joint. The center of mass for m_2 is at the robot's end-effector. Inertia matrices I_1 and I_2 are:

$$I_1 = \begin{bmatrix} I_{x1} & 0 & 0 \\ 0 & I_{y1} & 0 \\ 0 & 0 & I_{z1} \end{bmatrix}$$

$$I_2 = \begin{bmatrix} I_{x2} & 0 & 0 \\ 0 & I_{y2} & 0 \\ 0 & 0 & I_{z2} \end{bmatrix}$$

```
clc;
clear;
```

```
syms L g m1 m2 theta1 theta2 theta1_dot theta2_dot theta1_dot_dot theta2_dot_dot
Ix1 Ix2 Ix3 Iy1 Iy2 Iy3 Iz1 Iz2 real

theta = [theta1; theta2];
thetadot = [theta1_dot; theta2_dot];
thetadotdot = [theta1_dot_dot; theta2_dot_dot];

% home matrix for center of mass m1
M1 = [roty(0),[0;L;-L/2]; 0 0 0 1];

% home matrix for center of mass m2
M2 = [eye(3), [0;L;-L]; 0 0 0 1];

S1 = [0;1;0;0;0;0];
S2 = [0;0;0;0;0;-1];

S_eq1 = [S1, [0;0;0;0;0;0]];
S_eq2 = [S1, S2];

% For center of mass m1
T_1 = fk(M1, S_eq1, theta)
```

T_1 =

$$\begin{pmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & -\frac{L\sin(\theta_1)}{2} \\ 0 & 1 & 0 & L \\ -\sin(\theta_1) & 0 & \cos(\theta_1) & -\frac{L\cos(\theta_1)}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
R_1 = T_1(1:3, 1:3);
JS_1 = simplify(expand(JacS(S_eq1, theta))); % Space Jacobian
Jb_1 = adjointM(inv(T_1))*JS_1; % Body Jacobian
J_geometric_1 = simplify(expand([R_1, zeros(3); zeros(3), R_1] * Jb_1)); %
Geometric Jacobian
Jw1 = J_geometric_1(1:3,1:2)
```

Jw1 =

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

```
Jv1 = J_geometric_1(4:6, 1:2)
```

Jv1 =

$$\begin{pmatrix} -\frac{L\cos(\theta_1)}{2} & 0 \\ 0 & 0 \\ \frac{L\sin(\theta_1)}{2} & 0 \end{pmatrix}$$

```
Inertia_1 = [[Ix1 0 0]
    [0 Iy1 0]
    [0 0 Iz1]];


T_2 = fk(M2, S_eq2, theta)
```

T_2 =

$$\begin{pmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & -L\sin(\theta_1) - \theta_2\sin(\theta_1) \\ 0 & 1 & 0 & L \\ -\sin(\theta_1) & 0 & \cos(\theta_1) & -L\cos(\theta_1) - \theta_2\cos(\theta_1) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
R_2 = T_2(1:3, 1:3);
JS_2 = simplify(expand(JacS(S_eq2, theta))); % Space Jacobian
Jb_2 = adjointM(inv(T_2))*JS_2; % Body Jacobian
J_geometric_2 = simplify(expand([R_2, zeros(3); zeros(3), R_2] * Jb_2)); %
Geometric Jacobian
Jw2 = J_geometric_2(1:3,1:2)
```

Jw2 =

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

```
Jv2 = J_geometric_2(4:6, 1:2)
```

Jv2 =

$$\begin{pmatrix} -\cos(\theta_1)\,(L+\theta_2) & -\sin(\theta_1) \\ 0 & 0 \\ \sin(\theta_1)\,(L+\theta_2) & -\cos(\theta_1) \end{pmatrix}$$

```
Inertia_2 = [[Ix2 0 0]
    [0 Iy2 0]
    [0 0 Iz2]];

% Mass matrix evaluation
Mass_Matrix = simplify(expand(m1*(Jv1'*Jv1) + Jw1'*R_1*Inertia_1*R_1'*Jw1 +
m2*(Jv2'*Jv2) + Jw2'*R_2*Inertia_2*R_2'*Jw2))
```

Mass_Matrix =

$$\begin{pmatrix} \mathrm{Iy}_1 + \mathrm{Iy}_2 + \frac{L^2\, m_1}{4} + L^2\, m_2 + m_2\, \theta_2{}^2 + 2\, L\, m_2\, \theta_2 & 0 \\ 0 & m_2 \end{pmatrix}$$

```
% Coriolis matrix evaluation
Coriolis_Matrix = coriolis(Mass_Matrix, theta, thetadot)
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```
Coriolis_Matrix =
```

$$\begin{pmatrix} \theta_{\dot{2}}\, (L\, m_2 + m_2\, \theta_2) & \theta_{\dot{1}}\, (L\, m_2 + m_2\, \theta_2) \\ -\theta_{\dot{1}}\, (L\, m_2 + m_2\, \theta_2) & 0 \end{pmatrix}$$

```
% Height evaluations as it is acting along the z-axis
h1 = T_1(3, 4);
h2 = T_2(3, 4);

% Potential energy evaluation
P = g*m1*h1 + g*m2*h2;

% Gravity vector evaluation
gravity_vector = simplify(expand([diff(P, theta1); diff(P, theta2)]))
```

```
gravity_vector =
```

$$\begin{pmatrix} \frac{g\, \sin(\theta_1)\, (L\, m_1 + 2\, L\, m_2 + 2\, m_2\, \theta_2)}{2} \\ -g\, m_2 \cos(\theta_1) \end{pmatrix}$$

```
% Final tau (joint torques) evaluation
tau = simplify(expand(Mass_Matrix*thetadotdot + Coriolis_Matrix*thetadot +
gravity_vector))
```

```
tau =
```

$$\left(\begin{array}{c} \mathrm{Iy}_1\, \theta_{\ddot{1}} + \mathrm{Iy}_2\, \theta_{\ddot{1}} + m_2\, \theta_2{}^2\, \theta_{\ddot{1}} + \frac{L^2\, m_1\, \theta_{\ddot{1}}}{4} + L^2\, m_2\, \theta_{\ddot{1}} + 2\, L\, m_2\, \theta_2\, \theta_{\ddot{1}} + 2\, L\, m_2\, \theta_{\dot{1}}\, \theta_{\dot{2}} + 2\, m_2\, \theta_2\, \theta_{\dot{1}}\, \theta_{\dot{2}} + \frac{L\, g\, m_1 \sin(}{2} \\ -m_2\, \left(L\, \theta_{\dot{1}}{}^2 - \theta_{\ddot{2}} + g\, \cos(\theta_1) + \theta_2\, \theta_{\dot{1}}{}^2 \right) \end{array} \right.$$