

Defn: Set A is decidable if there exists a TTM that accepts all strings in A and rejects all strings in $\Sigma^* \setminus A$.

Otherwise, set A is undecidable.

Halting set H is undecidable.

Given p , is there an input on which TM p does not halt?

Define $\hat{H} = \{ p \mid \text{TM } p \text{ halts on all inputs} \}$

Is \hat{H} decidable?

No

Suppose \hat{H} is decidable. Let q be the description of TM that accepts \hat{H} .

↓
 M_q

On input (p, x) , define a TM that works as follows:

↓
 M_r

On input y , ignores y , writes x on tape,
and simulates M_p on input x .

Runs M_q on input r and accepts (p, x) iff
 M_q accepts r .

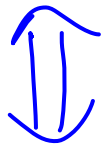
M_s

M_p

M_s accepts (p, x)



M_q accepts r



M_r halts on all inputs $\Leftrightarrow M_p$ halts on x