Consider any property of computable sets.

Define it as :

$$P : \underset{\text{all computable sets}}{\text{collection of}} \longrightarrow \{\text{true}, \text{false}\}.$$

Property P is non-trivial if it does not assign same value to all computable sets.

Rice's Theorem: For any non-trivial property P, checking if it holds given a TM M_p as input, is undecidable.

proof: Suppose $P(\emptyset) = \text{false}$.
Let $X \subseteq \{0,1\}^*$ such that $P(X) = \text{true}$.

Suppose M_Q decides property P.

Define TM M_S as:

On input (P, x), construct description of a TM r that works as follows:

On input y, run M_P on x. If it halts, run TM for X on y and accept iff $y \in X$.

Run M_q on r and accepts (p, x) iff

M_q accepts r.

M_S accepts $(p, x) \iff M_q$ accepts r

$\iff P(\text{set accepted by } M_r) = \text{true}$

$\iff \text{set accepted by } M_r = X$

$\iff M_p$ halts on x

solves Halting problem

Not possible.

Reduction

Given sets $A, B \subseteq \{0,1\}^*$, we say that A reduces to B if there exists a function

reduction

$f : \{0,1\}^* \to \{0,1\}^*$ such that:

1) f is computable
2) $x \in A$ iff $f(x) \in B$.

Lemma: Suppose A reduces to B and A is undecidable. Then B is also undecidable.

Halting problem $\longrightarrow$ $P_{set} = \{ r \mid P(\text{set accepted by } M_r) = \text{true} \}$.

$$(p, x) \xrightarrow{f} r$$