

Time-bounded TMs

What can be computed in reasonable time?

What is this?

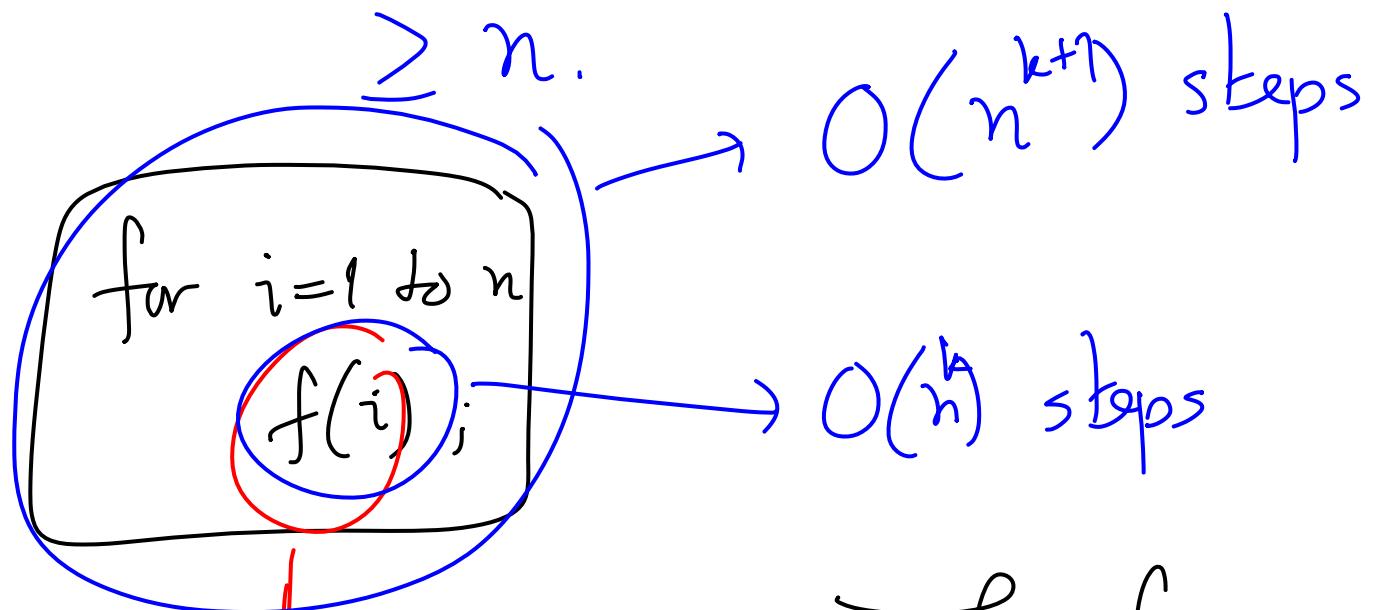
Polynomial-time computations

A TM that on input of size n , runs
for at most $c \cdot n^k$ steps, for constants c & k .

$c \cdot n^k$
" "
 $O(n^k)$

Why is this reasonable time?

Any non-trivial computation will take time



Can be done in
reasonable time

Therefore, any computation taking
 $O(n^k)$ steps for any $k > 0$ is ⁱⁿ reasonable time.

Let P be set of all sets that can be solved in polynomial time.

What sets are in P ?

Let FP be the set of all functions that can be computed in polynomial time.

Examples

Sorting \in FP

Matrix Multiplication \in FP

Shortest Path \in FP

Max Flow \in FP

Primality Testing \in P

Integer Factoring ??

Problems not in P

$$D = \{(i, x) \mid \text{TM } M_i \text{ rejects } (i, x) \text{ within } |x|^i \text{ steps}\}$$

D is a computable set.

Is $D \in P$?

length of x
↑
size

Suppose $D \in P$. \Rightarrow Therefore, $D \notin P$.

$\Rightarrow \exists \text{ TM } M_j$ that accepts D and runs for at most $c \cdot n^k$ steps.

Is $(j, x) \in D$?

$$(j, x) \in D \Leftrightarrow$$

$$\Leftrightarrow$$

$$\Leftrightarrow$$

M_j rejects (j, x) within $b(j)$ steps
 M_j rejects (j, x) within $c|x|$ steps
 $(j, x) \notin D$ for $|x| \geq c$

provided
 $|x| \geq c$