Context Free Grammers

- Specified by a finite set of terminals, a finite set of non-terminals, and a finite set of productions connecting terminals & non-terminals

Terminals $= \Sigma \cup \{\epsilon\}$ → alphabet

represented by small letters

Non-terminals → intermediate symbols representing a set of strings over terminals

represented as capital letters

S is a special non-terminal called "Start" symbol

Productions → tells us how each non-terminal is translated to a set of terminals

$$A \rightarrow \gamma \;,\; \gamma \in (\text{Terminals} + \text{Non-terminals})^*$$

<u>Example</u>: Consider $\{0^n 1^n \mid n \geq 1\}$

$$S \longrightarrow 0S1$$
$$S \longrightarrow 01$$

$0^5 1^5$

S
$\downarrow$
$0S1$
$\downarrow$
$00S11$
$\downarrow$
$000S111$
$\downarrow$
$0000S1111$
$\downarrow$
0000011111

Example: Arithmetic expressions using $+, -, *, /$ over integers

$5 + - 8$

$-(2 + 3 * 5)$

$(2+3) * 5$

$S \rightarrow S + S$

$S \rightarrow S * S$

$S \rightarrow S - S$

$S \rightarrow S / S$

$S \rightarrow I$

$S \rightarrow (S)$

$S \rightarrow - S$

$I \rightarrow 0I$

$I \rightarrow 1I$

$\vdots$

$I \rightarrow 9I$

$I \rightarrow 0$

$I \rightarrow 1$

$\vdots$

$I \rightarrow 9$

Defn : The set of strings of terminals generated by a context-free grammer (CFG in short) is called a context-free language (CFL in short)

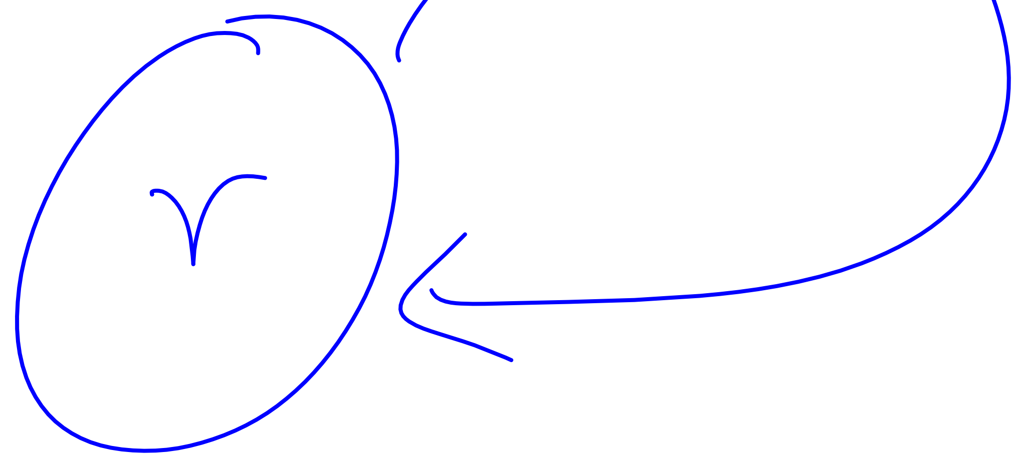
Theorem: A set of Σ^* is a CFL if and only if it is accepted by a PDA.

proof: Let $G = (\underset{\text{terminals}}{\Sigma}, \underset{\text{non-terminals}}{\Gamma}, \underset{\text{productions}}{P})$ be a CFG.

For each $a \in \Sigma$, introduce a new non-terminal N_a in Γ and new production $N_a \to a$ in P

Convert every production $A \to \gamma$ of P by replacing any terminal a in γ except the one that is first letter of γ, by N_a.

$A \to a B_1 B_2 \cdots B_k$



$a, A, B_1 \cdots B_k$