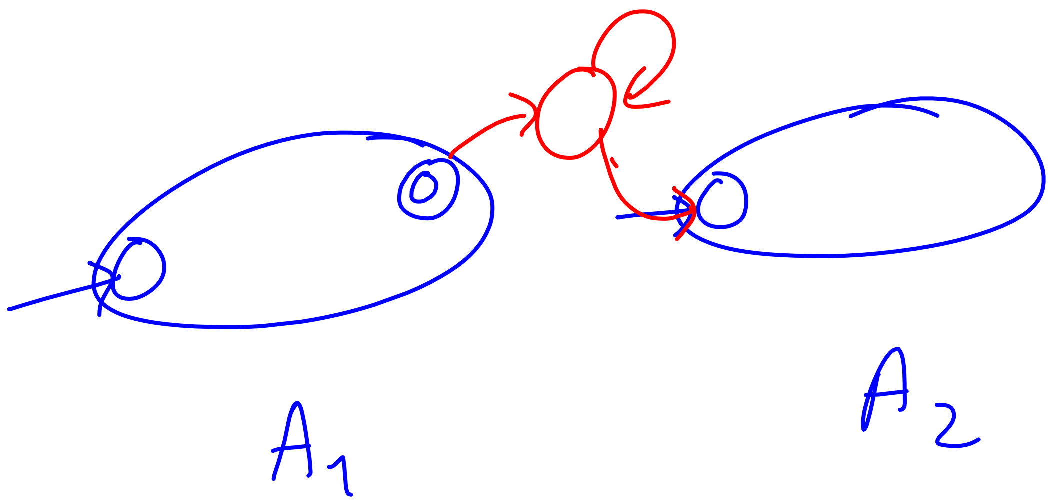
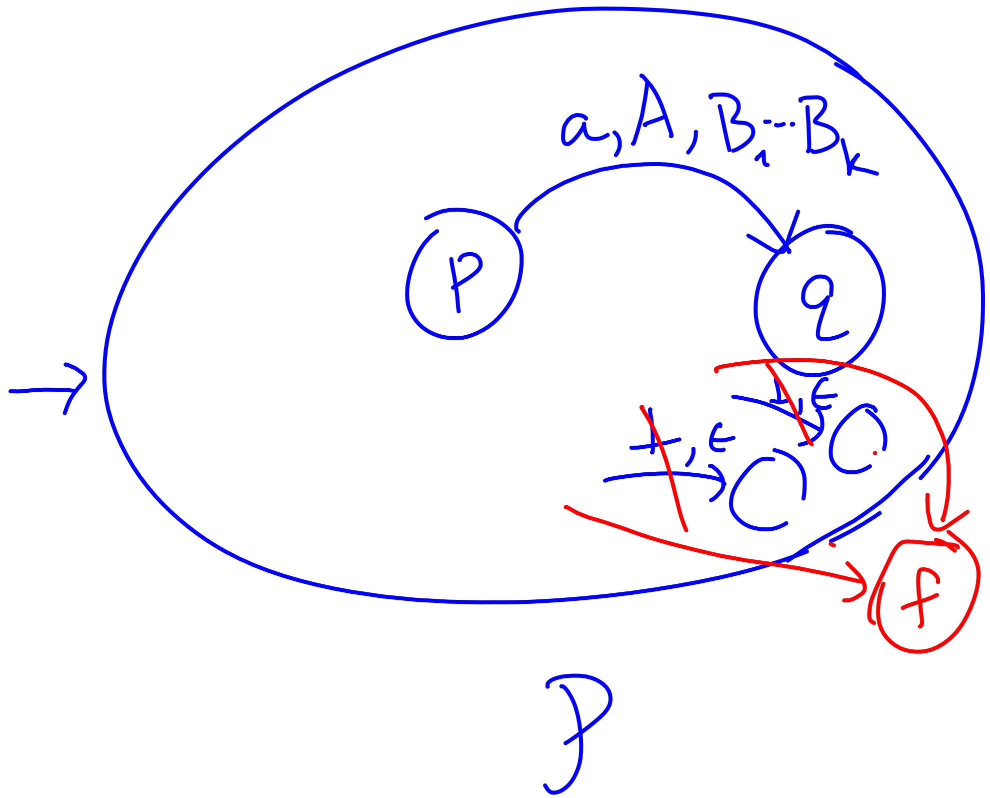
Closure under concatenation & *



<u>Theorem</u>: Any PDA can be simulated by a PDA with one state.

proof: Given $(Q, \Sigma, \Gamma, s_0, \perp, \delta, F)$.
Assume that PDA accepts by empty stack,
so $F = \emptyset$.



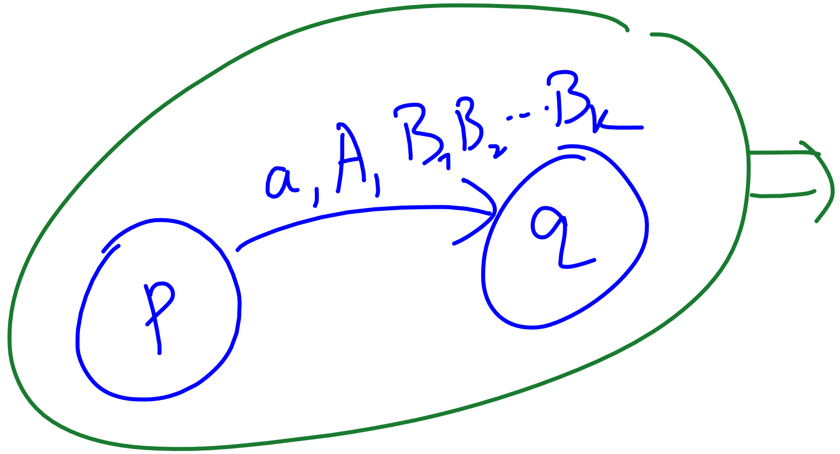
Define $\Gamma' = Q \times \Gamma \times Q$

(q_1, A, q_2) means P moves from state q_1 on suitable input to state q_2 (after a series of transitions) and stack no longer has A at the top (rest of it remains the same)

Define a new PDA as:

$$\left(\{r\}, \Sigma, \Gamma', r, \perp', \delta', \phi \right)$$

$$\perp' = (s_0, \perp, f)$$

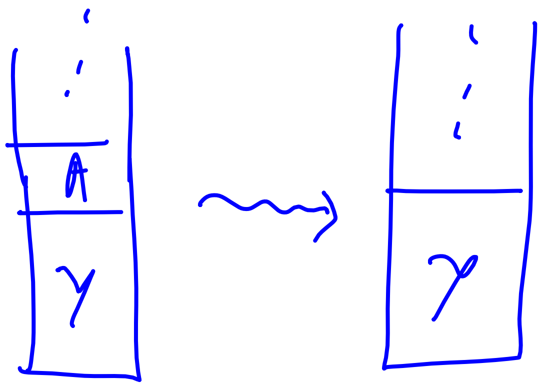


B_1 is popped $\quad$ B_2 is popped

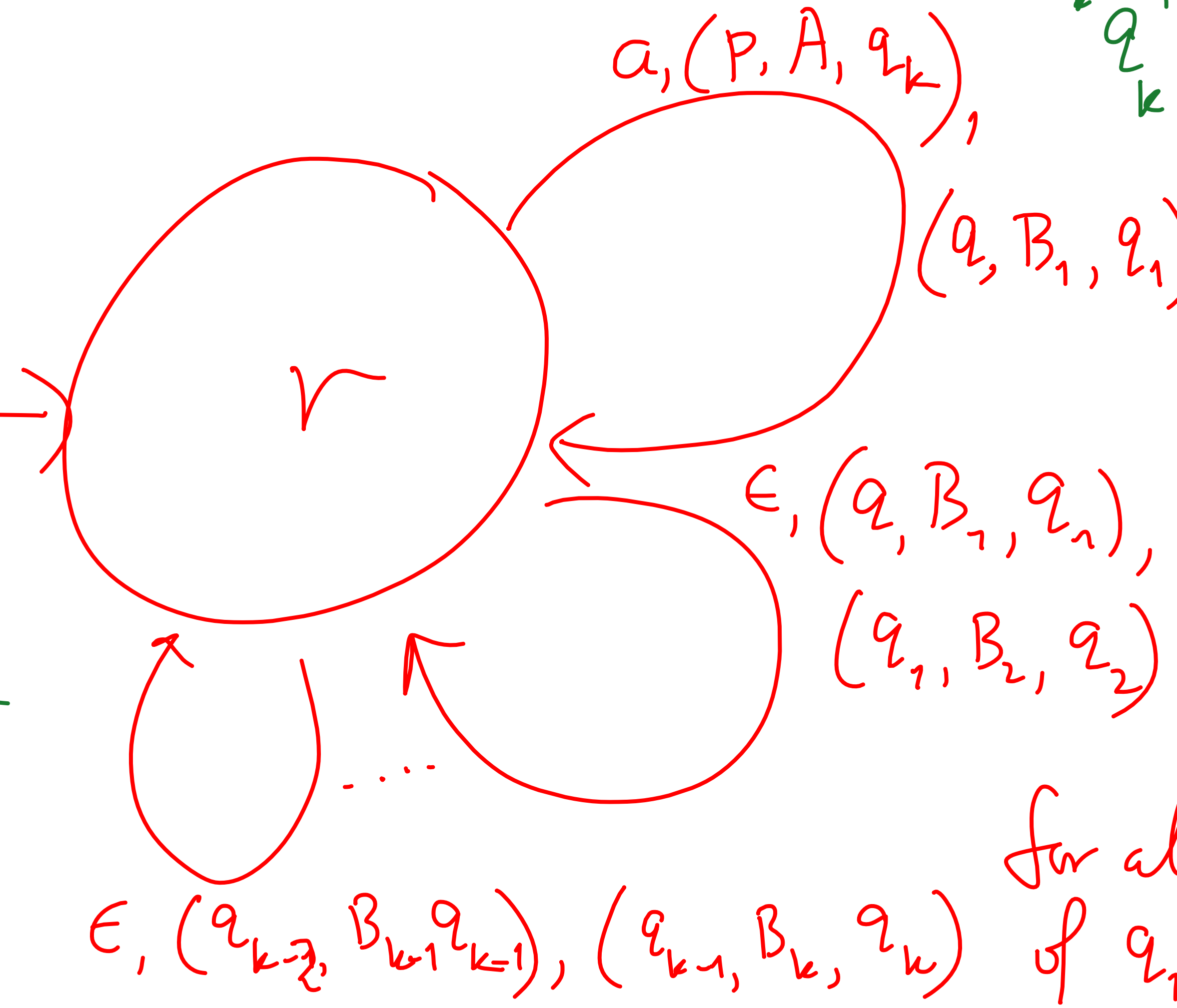
$p \rightsquigarrow q_1 \rightsquigarrow q_2 \rightsquigarrow \cdots$

B_k is popped

q_k



at state p $\qquad$ at state $q' = q_k$



for all values of $q_1, \ldots, q_k \in Q$