Examples of problems not known to be in P

1) Satisfiability

- Boolean variables $x_1, x_2, \dots, x_n$ take values true / false
- A clause is a disjunction of boolean variables and their negations

$$x_1 \vee \neg x_7 \vee x_3 \vee x_{11} \vee \neg x_5$$

▷ A conjunctive-normal formula is a conjunction of clauses
form (CNF)

$$F = C_1 \wedge C_2 \wedge C_3 \cdots \wedge C_m$$

$$\mathrm{SAT} = \{ F \mid \exists x_1 \exists x_2 \cdots \exists x_n \, F \}$$

2) Hamiltonian cycle

Given a graph $G = (V, E)$, is there a cycle in G passing through all vertices?

Hamiltonian cycle

$HAM = \{ G \mid G$ has a Hamiltonian cycle $\}$

3) Traveling Salesman problem

Given a map of cities and connecting roads, a salesman wants to start at a city, visit all other cities, and come back to starting city in shortest time. Find such a path.

4) Sudoku ($n^2 \times n^2$)

5) Vertex cover

vertex cover

has a vertex cover

Given graph $G = (V, E)$ and number k, find $U \subseteq V$, $|U| \le k$, such that every edge in E has an endpoint in U.

$$SUD = \{\text{puzzles that have a solution}\}$$

$$VC = \{(G, k) \mid G \text{ has a vertex cover of size} \leq k\}$$

The class NP

Set $A \in NP$ if there exists a non-deterministic polynomial-time TM accepting it.

Theorem: SAT, HAM, SUD, VC are in NP.

proof: Easy to construct TMs that guess a solution and verify its correctness in polynomial time.

We know that $P \subseteq NP$.

It is conjectured that $P \neq NP$.