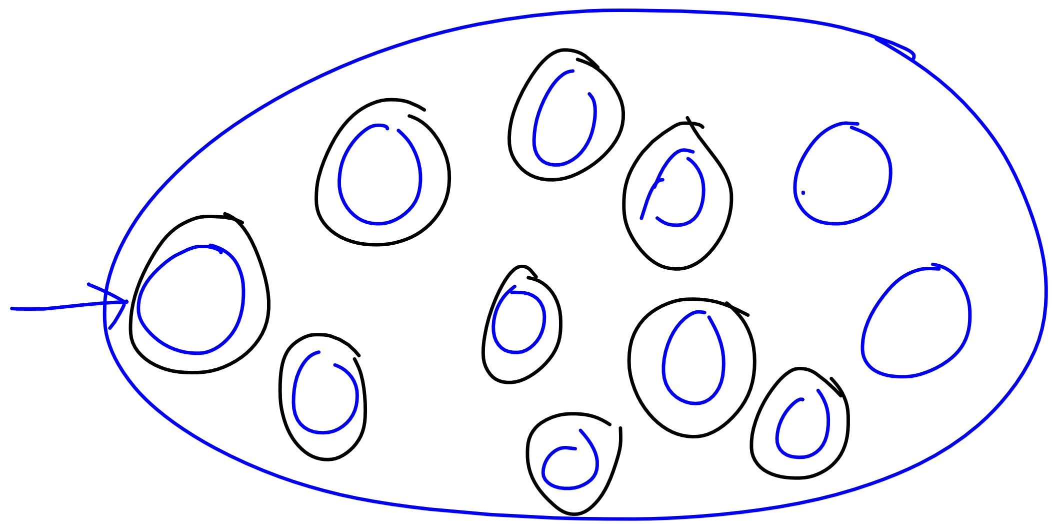
Let F be a finite automata. The set of strings accepted by F is called a <u>regular</u> set.

Properties of regular sets

1) Closed under union & intersection
2) Closed under complement

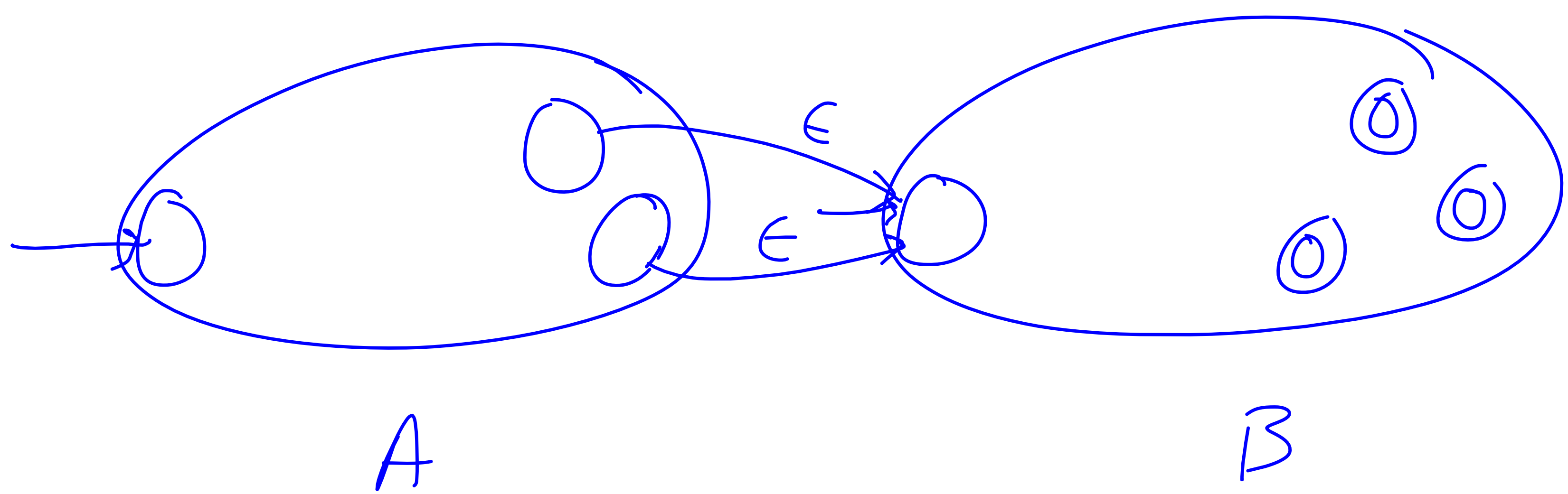
$\Rightarrow$ If $A \subseteq \Sigma^*$ is a regular set then so is $\Sigma^* - A$.



DFA for $\overline{A}$

3) Regular sets are closed under concatenation

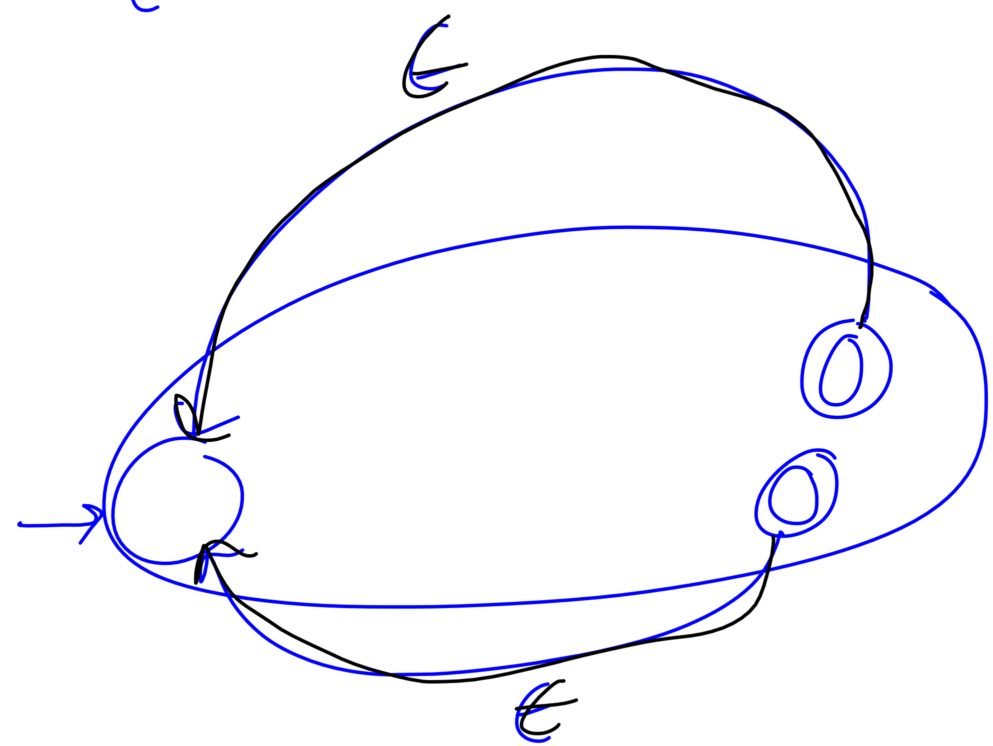
$$A \cdot B = \{ xy \mid x \in A, y \in B \}$$



4) Closed under * operation

$$A^* = \{ y \mid y = x_1 x_2 \cdots x_k,\ x_i \in A \}$$

Σ^*



Regular Expressions

Regular expression over alphabet Σ is defined as:

(i) $a \in \Sigma$ is a regular expression

(ii) ϵ is r.e.

(iii) If r_1, r_2 are r.e., so is $r_1 + r_2$

(iv) If r_1, r_2 are r.e., so is $r_1 \circ r_2$

(v) If r is r.e., so is r^*.

A regular expression r defines a set $L(r)$ in Σ^* as follows:

(i) $r = a \in \Sigma \longmapsto L(r) = \{a\}$

(ii) $r = \epsilon \longmapsto L(r) = \{\epsilon\}$

(iii) $r = r_1 + r_2 \longmapsto L(r) = L(r_1) \cup L(r_2)$

(iv) $r = r_1 \circ r_2 \longmapsto L(r) = L(r_1) \circ L(r_2)$

(v) $r = r_1^* \longmapsto L(r) = L(r_1)^*$

Examples

1) $(0+1)^*$: $\{0,1\}^*$

2) $1(0+1)^*0$: all strings starting with 1 & ending with 0

$$a^* \equiv a\Sigma^*$$

Theorem: Set A is regular if and only if $A = L(r)$ for some regular expression r.

proof: Suppose $A = L(r)$.

1) $r = a \in \Sigma$, $L(r) = \{a\}$

2) $r = \epsilon$, $L(r) = \{\epsilon\}$

3) $r = r_1 + r_2$, $L(r) = L(r_1) \cup L(r_2)$

4) $r = r_1 \circ r_2$, $L(r) = L(r_1) \circ L(r_2)$

5) $r = r_1^*$, $L(r) = L(r_1)^*$