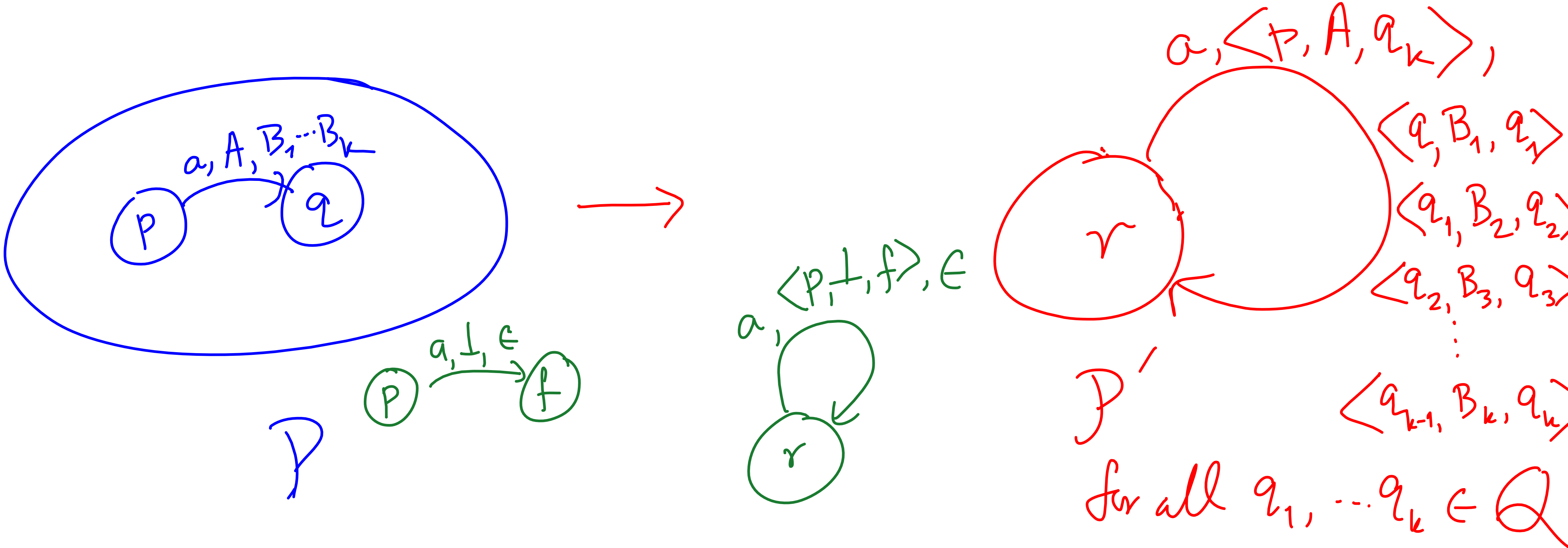

$$[p, x, C_1 C_2 \cdots C_m] \xrightarrow[\mathcal{P}]{n} [q, \epsilon, \epsilon]$$

PDA $\mathcal{P}$, in stat p, on reading input x with stack being $C_1 \cdots C_m$, in n steps moves to stat q with no input left and empty stack

<u>Theorem</u>: PDA P accepts a string x iff P' accepts x.

proof: By induction on number of steps n to show following:

for any $p \in Q$, $x \in \Sigma^*$, $C_1 \cdots C_m \in \Gamma^*$:

$$[p, x, C_1 \cdots C_m] \xrightarrow[P]{n} [f, \epsilon, \epsilon] \text{ iff}$$

$$\exists q_1, q_2, \ldots, q_{m-1} \in Q : [r, x, \langle p, C_1, q_1\rangle \langle q_1, C_2, q_2\rangle \cdots \langle q_{m-1}, C_m, f\rangle] \xrightarrow[P']{n} [r, \epsilon, \epsilon]$$

If above is true, then:

$$[q_0, x, \perp] \xrightarrow[P]{n} [f, \epsilon, \epsilon] \quad \text{iff}$$

$$[r, x, \langle q_0, \perp, f\rangle] \xrightarrow[P']{n} [r, \epsilon, \epsilon] \text{ for some } n > 0.$$

Base Case : $n = 1$

Condition reduces to :

$[p, a, \perp] \xrightarrow[P]{1} [f, \epsilon, \epsilon]$ iff $\quad (a \in \Sigma \cup \{\epsilon\})$

$[r, a, \langle p, \perp, f \rangle] \xrightarrow[P']{1} [r, \epsilon, \epsilon]$

Induction Step: Assume for n.

Need to show:

$$[p, x, C_1 \cdots C_m] \xrightarrow[P]{n+1} [f, \epsilon, \epsilon] \text{ iff}$$

$$\exists q_1 \ldots q_{m-1} \, [r, x, \langle p, C_1, q_1 \rangle \cdots \langle q_{m-1}, C_m, q_m \rangle] \xrightarrow[P']{n+1} [r, \epsilon, \epsilon]$$

Suppose $[p, x, C_1 \cdots C_m] \xrightarrow[P]{n+1} [f, \epsilon, \epsilon]$.

This implies there exist $a \in \Sigma \cup \{\epsilon\}$, state $q \in Q$, and $B_1, \ldots, B_k \in \Gamma$ such that:

(i) $x = ay$

(ii) $[p, ay, C_1 C_2 \cdots C_m] \xrightarrow[P]{1} [q, y, B_1 \cdots B_k C_2 \cdots C_m]$

(iii) $[q, y, B_1 \cdots B_k C_2 \cdots C_m] \xrightarrow[P]{n} [f, \epsilon, \epsilon]$