$A \quad \to E = \{ [x] \}$ $\quad y, z \in [x]$ iff $\forall w \; yw \in A$ iff $zw \in A$

DFA based on equivalence classes:

$$\mathcal{F} = (E, \Sigma, [\epsilon], \delta, F)$$
$$F = \{ [x] \mid [x] \cap A \neq \emptyset$$

$$\delta([x], a) = [xa]$$

Lemma: $\mathcal{F}$ on input x ends up in state $[x]$.

proof: By induction on $|x|$ = length of x

Base step: $|x| = 0 \Leftrightarrow x = \epsilon$, automata is in state $[\epsilon]$

Induction step: assume for $|x| < n$.

Consider x, $|x| = n$. Let $x = ya$, $|y| = n-1$.

On reading y, $\mathcal{F}$ ends up in $[y]$.

$\Rightarrow \delta([y], a) = [ya] = [x]$. $\boxtimes$

Therefore, on reading $x \in A$, $\mathcal{F}$ ends up in $[x]$ & $[x] \in F$. On reading $x \notin A$, $\mathcal{F}$ ends up in $[x]$ & $[x] \notin F$.

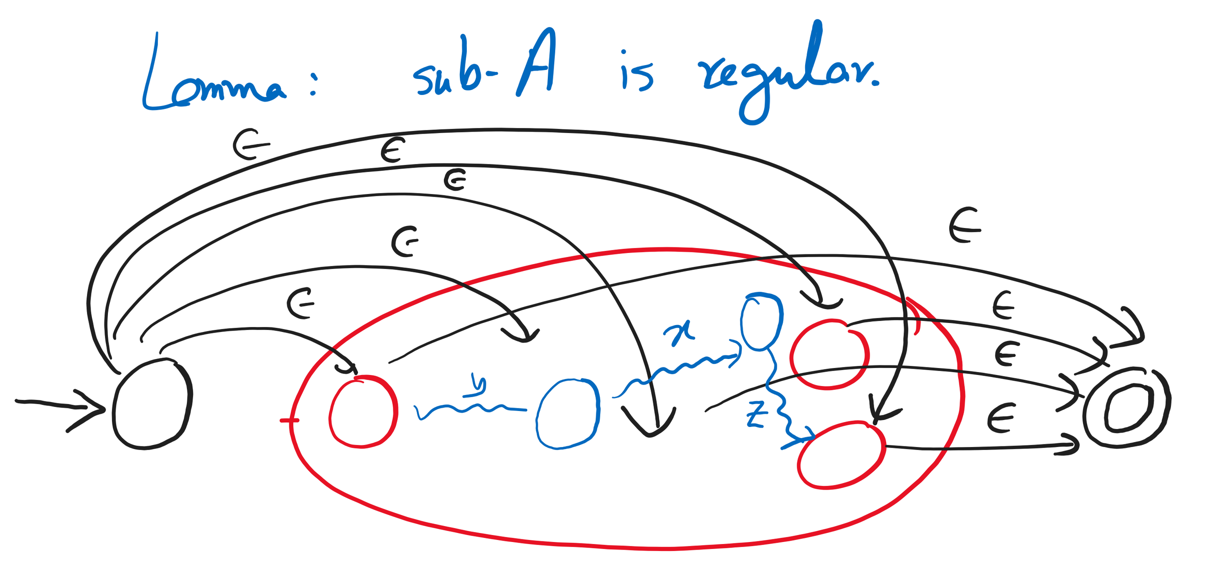
We know that any deterministic automata accepting A will have number of states $\geq |E|$.

Therefore, $\mathcal{F}$ is a DFA with minimum number of states accepting A. $\boxtimes$

Let A be a regular set.

Define sub-A = $\{ x \mid \exists y, z \in \Sigma^* \; \& \; yxz \in A \}$

Lemma: sub-A is regular.

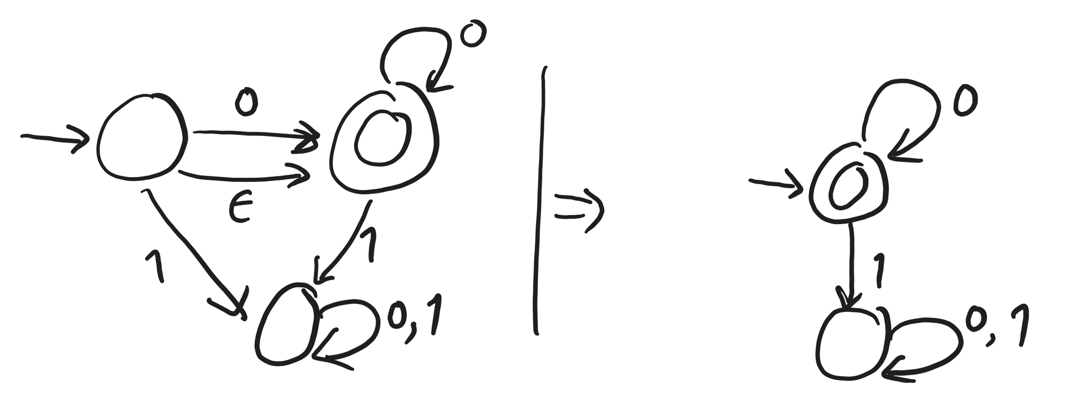


Minimum DFA for A

Transition from NFA to DFA.

If set of states of NFA is Q, then set of states of corresponding DFA is 2^Q

If $|Q| = n$, then $\Downarrow$ # states of DFA $= 2^n$

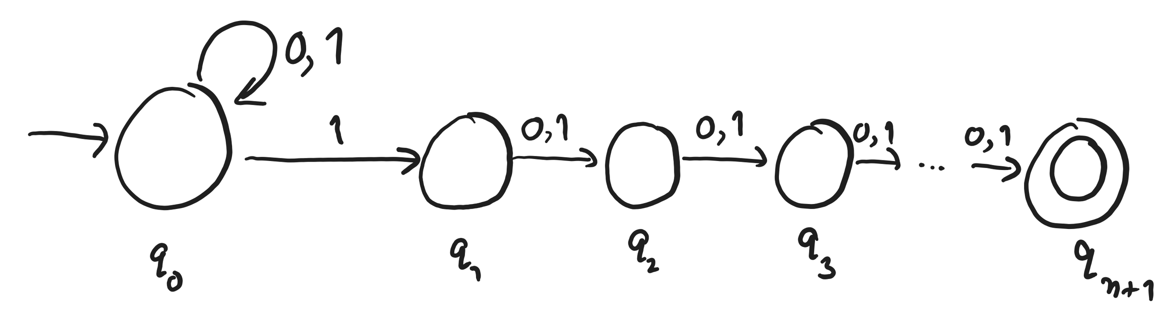


Lemma: There is a regular set with an accepting NFA having n states and any DFA that accepts the set has at least 2^{n-1} states.

proof: Let

$$A = \{ x1y \mid |y| = n \}, \quad \Sigma = \{0, 1\}$$

NFA of $n+2$ states



Consider A and equivalence classes induced by A.

xRy iff for all z, $xz \in A$ iff $yz \in A$

Let $w \in \{0,1\}^*$, $|w| = n+1$

$$E_w = \{ xw \mid x \in \{0,1\}^* \}$$

▷ Each E_w is contained in an equivalence class induced by A.

Let $xw, yw \in E_w$.

For any z, $xwz \in A$ iff $wz \in A$
iff $ywz \in A$.

▷ Different E_w's are contained in different equivalence classes

Consider E_w & $E_{w'}$ for $w \neq w'$.

Let $w \in E_w$ & $w' \in E_{w'}$

Suppose w, w' differ on k^{th} bit from right. $\downarrow \leq n+1$

k^{th} bit from right for $w = 1$
for $w' = 0$

Consider $\underset{\in A}{w0^{n+1-k}}$ & $\underset{\notin A}{w'0^{n+1-k}}$

$\Rightarrow$ No of equivalence classes $\geq$ No of E_w's
$= 2^{n+1}$ $\boxtimes$