TM $M_{s(y)}$: Erase input, write y and halt.

TMs computing functions

TM M_a : On input y, output $s(y)$.

TM M_b : On input p, output $\cancel{p \| s(p)}$.

$s(p) \| p$

$p \quad f \xrightarrow{\epsilon} s \quad s(p)$

$$(Q, s_0, \delta, \Sigma, \cancel{B}, F)$$

$\downarrow$

use 11 as seperator

represent 0 as 00

represent 1 as 01

What does $M_{s(b) \| s(s(b))}$ do?

Erases input, outputs $s(b)$.

What does $M_{s(b) \| b}$ do?

Self-reproducing TM

Outputs $s(b) \| b$

Recursion Theorem: Let f be any computable function. Then there exists a p such that $M_p = M_{f(p)}$.

Computable function: there is a TM computing f which halts on all inputs.

First-order logic

Predicates : $Q(\underbrace{x_1, x_2, \dots, x_k}_{\text{variables taking value from } D})$

Logical operations : $\wedge, \vee, \neg$

Quantifiers : $\forall, \exists$

Given a formula F_n and a valuation of predicates, decide if it is true.

If D is infinite, how does one represent input? Represent each predicate as a TM.

Define predicate H as:

$H(P, x, t)$: TM M_P halts on input x within t steps.

M_p halts on x iff $\exists t\ H(p, x, t)$