State(q, t) : state at time t is q

Head(j, t) : head at time t is at j^{th} cell

Tape(j, s, t) : j^{th} cell at time t has symbol s

<u>For $t = 1$:</u>

size $= p(n)+2$

$$\left[\begin{array}{l} \text{State}(q_0, 0) \wedge \text{Head}(1, 0) \wedge \text{Tape}(1, x[1], 0) \wedge \\ \quad \ldots\ldots \wedge \text{Tape}(n, x[n], 0) \wedge \text{Tape}(n+1, \not{B}, 0) \wedge \\ \quad \ldots \wedge \text{Tape}(p(n), \not{B}, 0) \end{array}\right.$$

For t

$$\equiv \neg State(q,t) \vee \neg State(q',t)$$

$$\forall q: \quad State(q,t) \Rightarrow \neg State(q',t) \quad \text{for } q' \neq q$$

$$\forall j: \quad Head(j,t) \Rightarrow \neg Head(j',t) \quad \text{for } j' \neq j$$

$$\forall j \, \forall s: \quad Tape(j,s,t) \Rightarrow \neg Tape(j,s',t) \quad \text{for } s' \neq s$$

$$\text{Size} = p(n) \cdot \left(|Q|^2 + p^2(n) + p(n) \cdot |\Sigma|^2 \right)$$

$$= O(p^3(n))$$

For $t \geq 1$

$$\forall j \forall q \forall s \left[State(q, t) \wedge Head(j, t) \wedge Tape(j, s, t) \right]$$

$$\Rightarrow \left[State(q_1, t+1) \wedge Head(j_1, t+1) \wedge Tape(j, s_1, t+1) \right]$$

$$\vee \left[State(q_2, t+1) \wedge Head(j_2, t+1) \wedge Tape(j, s_2, t+1) \right]$$

$$\vee \; \ldots$$

$$\text{size} \leq p^2(n) * |Q| * |\Sigma| * |Q|^2 = O(p^2(n))$$

$$\left[\forall j\ \forall j' \neq j\ \forall s:\ \mathrm{Head}(j,t) \Rightarrow \left[\mathrm{Tape}(j',s,t+1) \Leftrightarrow \mathrm{Tape}(j',s,t)\right]\right.$$

size $= O(p^3(n))$

$$\left.\bigvee_{q \in F} \mathrm{State}(q, p(n))\right]$$ size $= 1$

Theorem: TM M accepts x, $|x| = n$, in $p(n)$ steps iff formula F is satisfiable.

$|F| = O(p^3(n))$

Therefore, if SAT $\in$ P then NP = P.

$$\Downarrow$$

SAT is NP-hard.

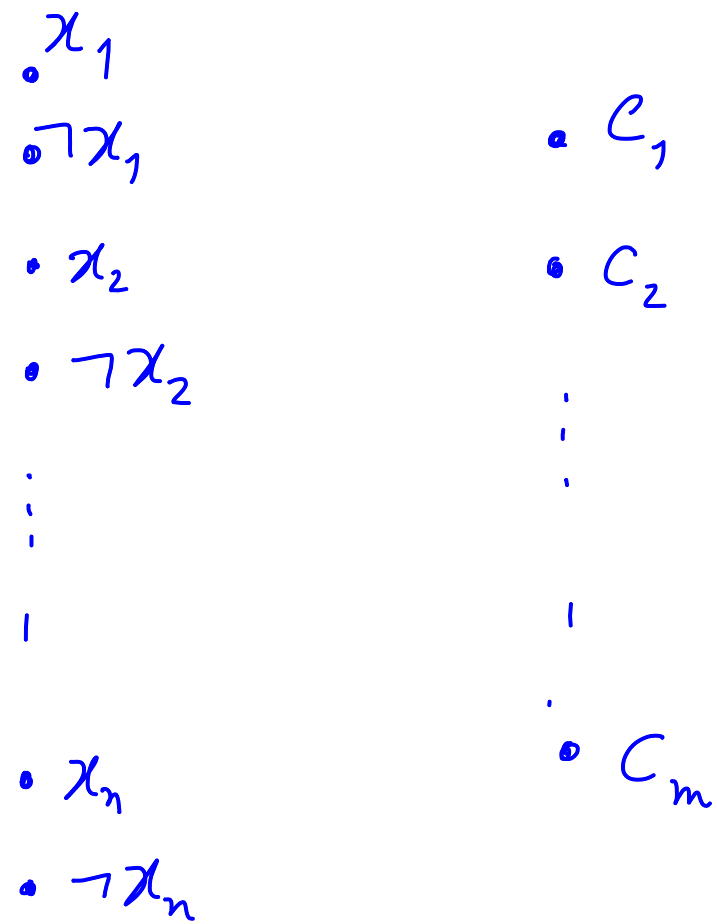
<u>Defn</u>: Set A is <u>NP-complete</u> if A is NP-hard and A $\in$ NP.

<u>Theorem</u> : VC in NP-complete.

proof : Reduce SAT to VC.

Given formula $F = C_1 \wedge C_2 \wedge \cdots \wedge C_m$

over n variables $x_1, x_2, \ldots, x_n$.



x_1
$\neg x_1$
x_2
$\neg x_2$
x_n
$\neg x_n$
C_1
C_2
C_m