$\mathrm{IND} = \{(G,k) \mid G$ has an independent set of size $\geq k\}$.

Δ $\mathrm{IND} \in \mathrm{NP}$.

Theorem: 3SAT is NP-complete where

$$3SAT = \{F = c_1 \wedge c_2 \wedge \cdots \wedge c_m \mid \text{each } c_i \text{ has exactly 3 variables}\}$$

proof: It is easily seen to be in NP.
We give a reduction from SAT.
Let $F = c_1 \wedge c_2 \wedge \cdots \wedge c_m$ be a formula with clause c_i containing r variables:

$$c_i = x_{i_1} \vee \neg x_{i_2} \vee \neg x_{i_3} \vee \cdots \vee x_{i_r}$$

$$\Downarrow$$

$$c_{i_1} = x_{i_1} \vee \neg x_{i_2} \vee y_1$$
$$c_{i_2} = \neg y_1 \vee \neg x_{i_3} \vee y_2$$
$$c_{i_3} = \neg y_2 \vee \cdots \vee y_3$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$c_{i_{r-2}} = \neg y_{r-3} \vee \neg x_{r-1} \vee x_r$$

Suppose $c_i = x_1 \vee \neg x_2$

$$\Downarrow$$

$$c_{i_1} = x_1 \vee \neg x_2 \vee x_1$$

The new formula with new set of clauses is satisfiable iff F is. $\boxtimes$

Theorem: VC is NP-complete.

proof: $VC = \{(G,k) \mid G$ has a vertex cover of size $\leq k.\}$

It is easily seen to be in NP.

Reduce IND to VC.

(G, k), $G = (V,E)$, $|V| = n$

$\downarrow$

$(G, n-k)$

G has an independent set of size $\geq k$
$\Updownarrow$
G has a vertex cover of size $\leq n-k$. $\boxtimes$

SAT $\rightarrow$ 3SAT $\rightarrow$ IND $\rightarrow$ VC

What about other variants of 3SAT?

Define

$$kSAT = \{F = c_1 \wedge \cdots \wedge c_m \mid \text{each } c_i \text{ has exactly } k \text{ variables } \& \ F \in SAT\}$$

Since 3SAT is NP-complete, so is kSAT for $k \geq 4$.

What about 2SAT?

Theorem: $2SAT \in P$.