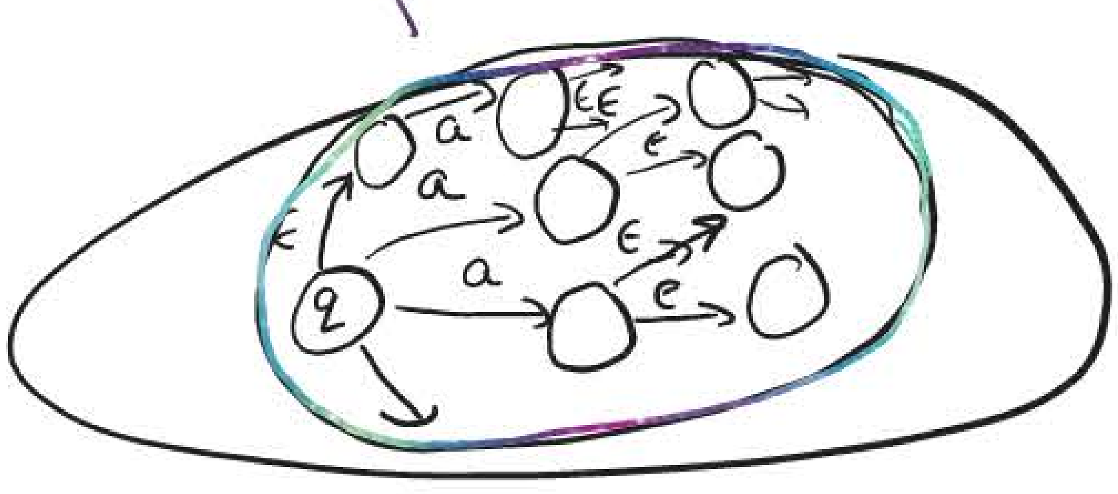
NFA $\mathcal{F} = (Q, q_0, \Sigma, \delta, F)$

DFA $\mathcal{F}_D = (2^Q, \{q_0\}, \Sigma, \delta_D, F_D)$

$F_D = \{ H \subseteq Q \mid H \cap F \neq \phi \}$

reach (q, a) = set of states reachable from q in $\mathcal{F}$ on reading symbol a



$$\delta_D : 2^Q \times \Sigma \longrightarrow 2^Q$$

$$\delta_D(H, a) = \bigcup_{q \in H} \text{reach}(q, a)$$

Consider $x \in \Sigma^*$ accepted by $\mathcal{F}$.

State transitions in $\mathcal{F}$ on reading x in an accepting path:

$$q_0 \xrightarrow{b_1} q_1 \xrightarrow{b_2} q_2 \xrightarrow{b_3} \cdots \xrightarrow{b_m} q_m \in F$$

$$b_1 b_2 b_3 \cdots b_m = x$$

State transitions in $\mathcal{F}_D$ on reading x:

$$\{q_0\} \xrightarrow{a_1} H_1 \xrightarrow{a_2} H_2 \xrightarrow{a_3} \cdots \xrightarrow{a_n} H_n, \quad n = |x|, \; x = a_1 a_2 \cdots a_n$$

$H_1 = \delta_D(\{q_0\}, a_1)$

Suppose $b_1 = \epsilon = b_2$ and $b_3 = a_1$

$\Rightarrow q_3 \in H_1$

Suppose $b_4 = \epsilon$ & $b_5 = a_2$

$\Rightarrow q_5 \in H_2$

$\vdots$

Therefore, $q_m \in H_n \Rightarrow H_n \cap F \neq \phi$.

$\Rightarrow H_n \in F_D$

Suppose x is accepted by DFA $\mathcal{F}_D$.

Suppose the states that $\mathcal{F}_D$ goes through on reading x are:

$$\{q_0\} \xrightarrow{a_1} H_1 \xrightarrow{a_2} H_2 \xrightarrow{a_3} \cdots H_{n-1} \xrightarrow{a_n} H_n \in F_D$$

$\Downarrow$

$H_n \cap F \neq \phi$

$$q_0 \overset{a_1}{\rightsquigarrow} q_1 \overset{a_2}{\rightsquigarrow} q_2 \rightsquigarrow \cdots q_{n-2} \overset{a_{n-1}}{\rightsquigarrow} q_{n-1} \overset{a_n}{\rightsquigarrow} q_n \in F$$

Therefore, $\mathcal{F}$ accepts same set as $\mathcal{F}_D$.