Theorem: For every set $X \subseteq \Sigma^*$, X is accepted by a PDA iff it is generated by a CFG.

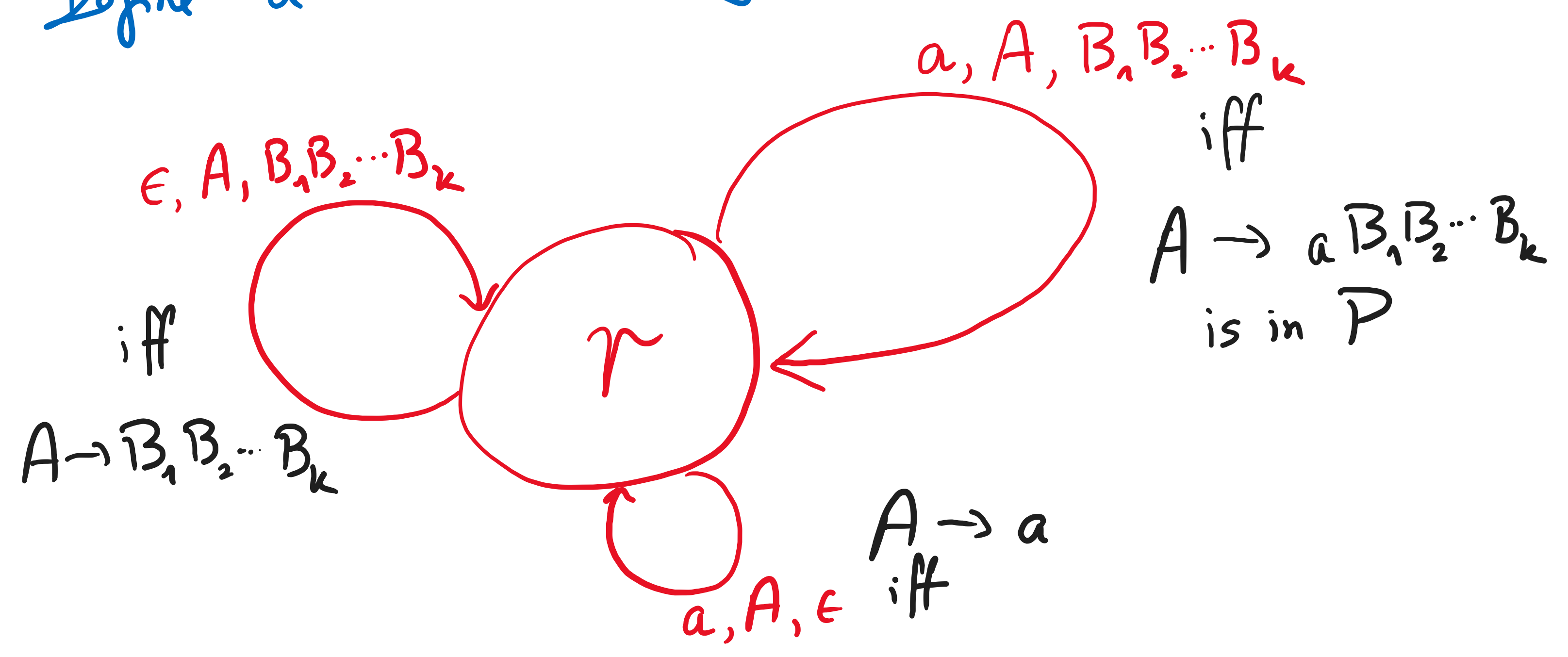
proof: Suppose X is generated by CFG (Σ, Γ, P).

Convert productions in P to a special form:

Given $A \to \gamma$, $\gamma \in (\Sigma \cup \Gamma)^*$, replace all occurances of $a \in \Sigma$ in γ by non-terminal N_a except when γ begins with a.

Include in P, productions $N_a \to a$ for all $a \in \Sigma$.

Define a PDA as follows:



Consider $x \in X$.

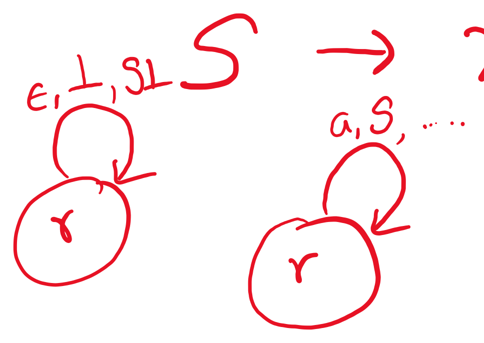
Suppose x is generated by CFG as follows:

$$
\begin{array}{lll}
 & \vdots & S \\
 & \gamma_i & \downarrow \\
 & \downarrow \text{ not leftmost derivation} & \gamma_1 \\
 & \gamma_{i+1} & \downarrow \\
 & \downarrow & \gamma_2 \\
 & \vdots & \vdots \\
 & yA\alpha B\beta = \gamma_{j-1} \leftarrow \text{not leftmost derivation} & \downarrow \\
B \to \delta & yA\alpha\delta\beta = \gamma_j \leftarrow \text{leftmost non-terminal of } \gamma_i \text{ is replaced} & \gamma_m = x \\
A \to \delta' & y\delta'\alpha\delta\beta = \gamma_{j+1} &
\end{array}
$$

Leftmost derivation is one where leftmost non-terminal in γ_i is replaced to obtain γ_{i+1}

This way, we can convert the derivation sequence to all leftmost derivations. Suppose the new sequence is:

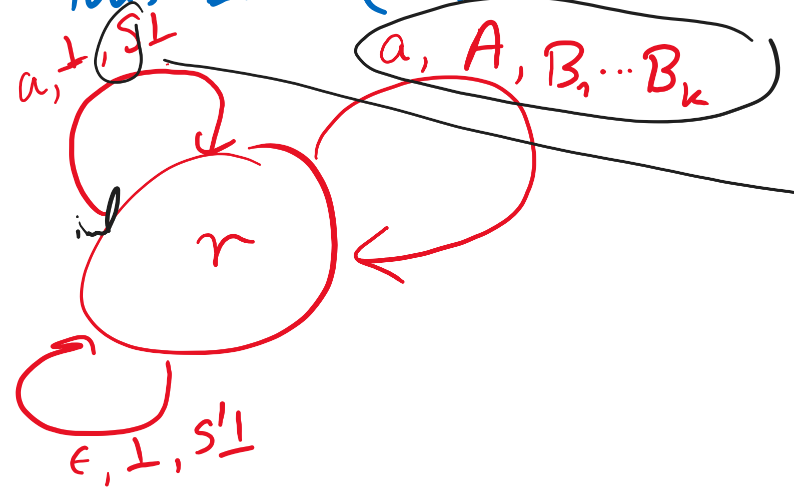
$$S \to \gamma_1' \to \gamma_2' \to \cdots \to \gamma_m' = x$$



$$\gamma_i = \underbrace{a_1a_2 \cdots a_u}_{\text{input read}} \underbrace{B_1B_2 \cdots B_v}_{\text{stack}}$$

Suppose X is accepted by PDA. We can convert the PDA to a single state PDA.

Let this be $(r, \Sigma, \Gamma, \perp, \delta)$.



$A \to aB_1 \cdots B_k$

$\hat{S} \to aS$

$\hat{S} \to S'$

$\boxtimes$