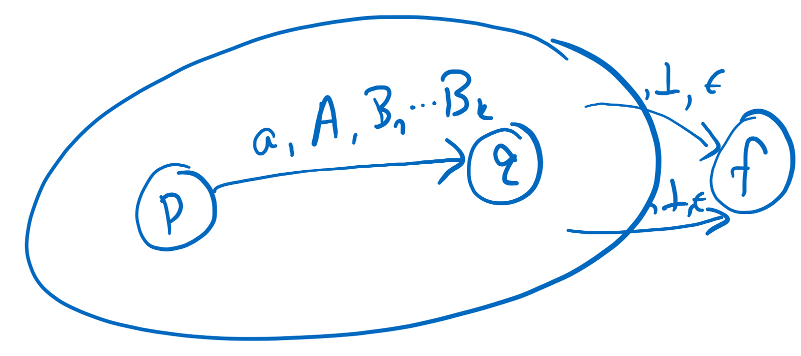
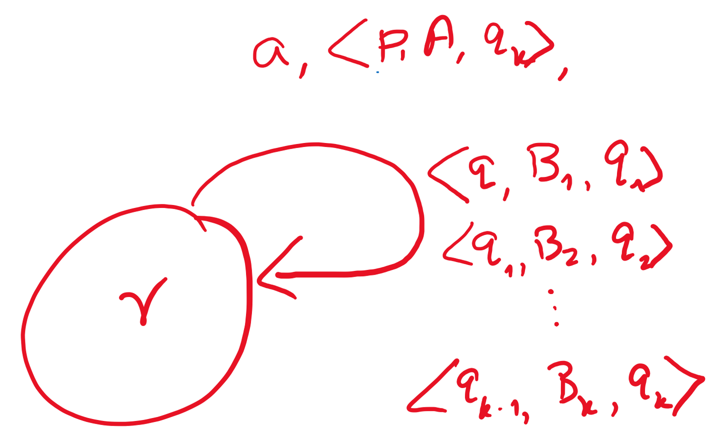




$\mathcal{P}$ $\qquad\qquad\qquad$ $\mathcal{P}'$

Notation: A configuration of $\mathcal{P}$ is a 3-tuple

$$[p, x, \gamma]$$

state, input to be read, stack contents

Definition: $[p, x, \gamma] \xrightarrow[\mathcal{P}]{n} [q, y, \gamma']$ if

(i) $x = x'y$

(ii) $\mathcal{P}$ moves from configuration $[p, x, \gamma]$ to configuration $[q, y, \gamma']$ in n steps

Theorem: $[p, x, C_1 C_2 \cdots C_m] \xrightarrow[\mathcal{P}]{n} [f, \epsilon, \epsilon]$ iff $\exists q_1 \cdots q_{m-1} \in Q$

$$[r, x, \langle p, C_1, q_1\rangle \langle q_1, C_2, q_2\rangle \cdots \langle q_{m-1}, C_m, f\rangle] \xrightarrow[\mathcal{P}']{n} [r, \epsilon, \epsilon]$$

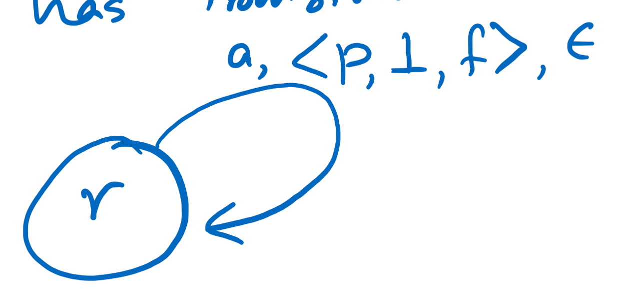
proof: By induction on n.

Base Step: $n = 1$

$[p, a, \perp] \xrightarrow[\mathcal{P}]{1} [f, \epsilon, \epsilon]$ iff $[r, a, \langle p, \perp, f\rangle] \xrightarrow[\mathcal{P}']{1} [r, \epsilon, \epsilon]$

$a \in \Sigma \cup \{\epsilon\}$

$[p, a, \perp] \xrightarrow[\mathcal{P}]{1} [f, \epsilon, \epsilon]$ iff $\mathcal{P}'$ has transition



iff $[r, a, \langle p, \perp, f\rangle] \xrightarrow[\mathcal{P}']{1} [r, \epsilon, \epsilon]$.

Induction Step: Assume for n.

Need to prove:

$[p, x, C_1 \cdots C_m] \xrightarrow[\mathcal{P}]{n+1} [f, \epsilon, \epsilon]$ iff $\exists q_1 \cdots q_{m-1} \in Q$

$[r, x, \langle p, C_1, q_1\rangle \cdots \langle q_{m-1}, C_m, f\rangle] \xrightarrow[\mathcal{P}']{n+1} [r, \epsilon, \epsilon]$

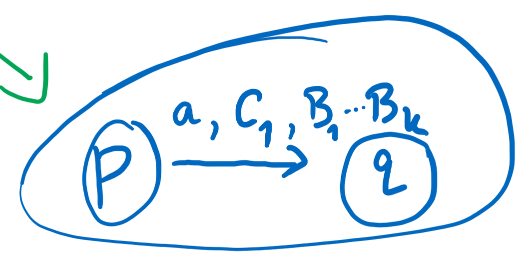
iff

$[p, x, C_1 \cdots C_m] \xrightarrow[\mathcal{P}]{1} [q, y, B_1 \cdots B_k C_2 \cdots C_m] \xrightarrow[\mathcal{P}]{n} [f, \epsilon, \epsilon]$

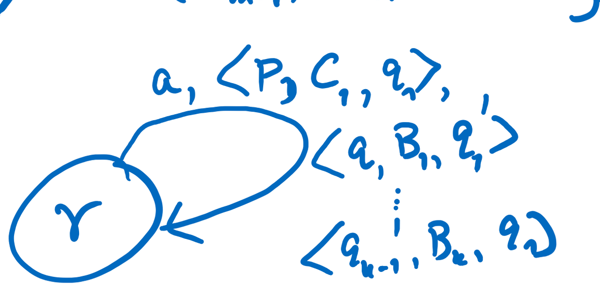
$x = ay$

iff

$\exists \underline{q_1' \cdots q_{k-1}'} q_1 \cdots q_{m-1}$ $[r, y, \langle q, B_1, q_1'\rangle \langle q_1', B_2, q_2'\rangle \cdots \langle q_{k-1}', B_k, q_1\rangle \langle q_1, C_2, q_2\rangle \cdots \langle q_{m-1}, C_m, f\rangle] \xrightarrow[\mathcal{P}']{n} [r, \epsilon, \epsilon]$



$\rightsquigarrow$



$$\Rightarrow [r, x, \langle p, C_1, q_1\rangle \langle q_1, C_2, q_2\rangle \cdots \langle q_{m-1}, C_m, f\rangle]$$
$$\mathcal{P}' \downarrow 1$$
$$[r, y, \langle q, B_1, q_1'\rangle \cdots \langle q_{k-1}', B_k, q_1\rangle \langle q_1, C_2, q_2\rangle \cdots \langle q_{m-1}, C_m, f\rangle]$$
$$\mathcal{P}' \downarrow n$$
$$[r, \epsilon, \epsilon] \quad \text{(by induction hypothesis)}$$

Same proof works when we start with $n+1$ step transition in $\mathcal{P}'$. $\boxtimes$