

What sets are not regular?

How to prove if a set is not regular?

↓  
Use finiteness of equivalence  
classes number

Example:  $A = \{0^n 1^n \mid n > 0\}$ .

Eq classes induced by A on  $\{0, 1\}^*$ :

For  $x, y$  in an eq class,  $xz \in A$  iff  $yz \in A$ .

Consider  $\{0^m \mid m > 0\}$ .

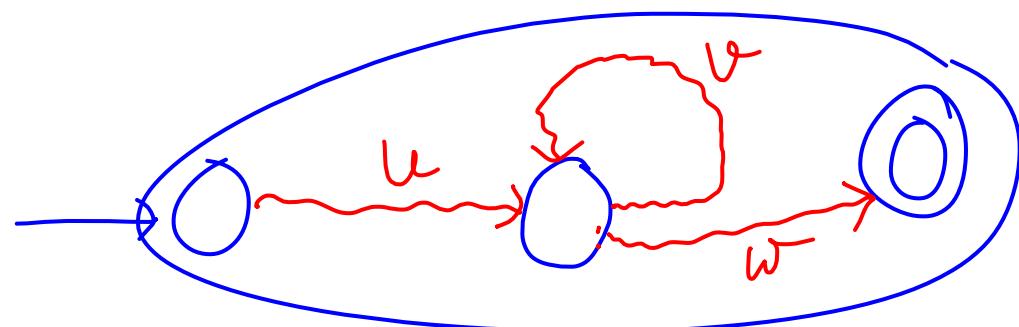
$0^{m_1} 1^{m_1} \in A$

$0^{m_2} 1^{m_1} \notin A$

Pumping Lemma: Suppose  $A$  is regular. Then & infinite.

there exists  $n > 0$  such that for all  $x \in \Sigma^*$ ,  
 $|x| > n$ ,  $|uv| \leq n$  &  $|v| \geq 1$  if  $uvw \in A$  then  $uv^iw \in A$  for  
 $x = uvw$ , all  $i \geq 0$ .

Proof:



DFA for  $A$

Consider  $x \in A$  &  $|x| > n$ .  
where  $n = \text{no of states in DFA}$

$A$  regular  $\Rightarrow \left\{ \begin{array}{l} (\exists n > 0) (\forall x, |x| > n) (\exists u, v, w \text{ s.t. } x = uvw, \\ |uv| \leq n, |v| \geq 1) \\ uvw \in A \Rightarrow \forall i \geq 0 \quad uv^i w \in A \end{array} \right\}$

$\Downarrow$

$\left\{ \begin{array}{l} (\forall n > 0) (\exists x, |x| > n) (\forall u, v, w, \text{ s.t. } x = uvw, |uv| \leq n, |v| \geq 1) \\ uvw \in A \quad \& \quad \exists i \geq 0 \quad uv^i w \notin A \end{array} \right\} \Rightarrow$

$A$  is not regular

$$A = \{0^m 1^n \mid m \geq 1\}.$$

Adversary chooses  $n > 0$ .

Choose  $0^n 1^n \in A$ .

Adversary splits  $0^n 1^n = uvw$ ,  $|uv| \leq n$ ,  $|v| \geq 1$

$$uv = 0^j, v = 0^k$$

Choose  $i = 0$ .

$$uw = 0^j 0^{n-j-k} 1^n = 0^{n-k} 1^n \notin A.$$

$$A = \{ 3^m \mid m \geq 0 \} \text{ over } \{0,1\}.$$

Adversary chooses  $n > 0$ .

Choose  $x = 3^n \in A$

$x = u v w, \quad |uv| \leq n, |v| \geq 1.$

||  $|w|=k, |v|=l$

$$N_u 2^{k+l} + N_v 2^k + N_w$$

Pumping  $v_n$  gives number

$$N_u 2^{k+i l} + N_v 2^{k+(i-1)l} + \\ N_g 2^{k+(i-2)l} + \dots + N_v 2^k + N_w$$