

## Myhill - Nerode Theorem :

For any regular set  $A$ , there is a unique deterministic automata with minimum number of states accepting it.

proof: Let  $A$  be a regular set.

Define a relation  $R$  on it as follows:  
 $xRy$  if for every  $z$ ,  $xz \in A$  iff  $yz \in A$ .

$R$  is an equivalence relation:

(reflexive)  $xRx$

(symmetric)  $xRy \Rightarrow yRx$

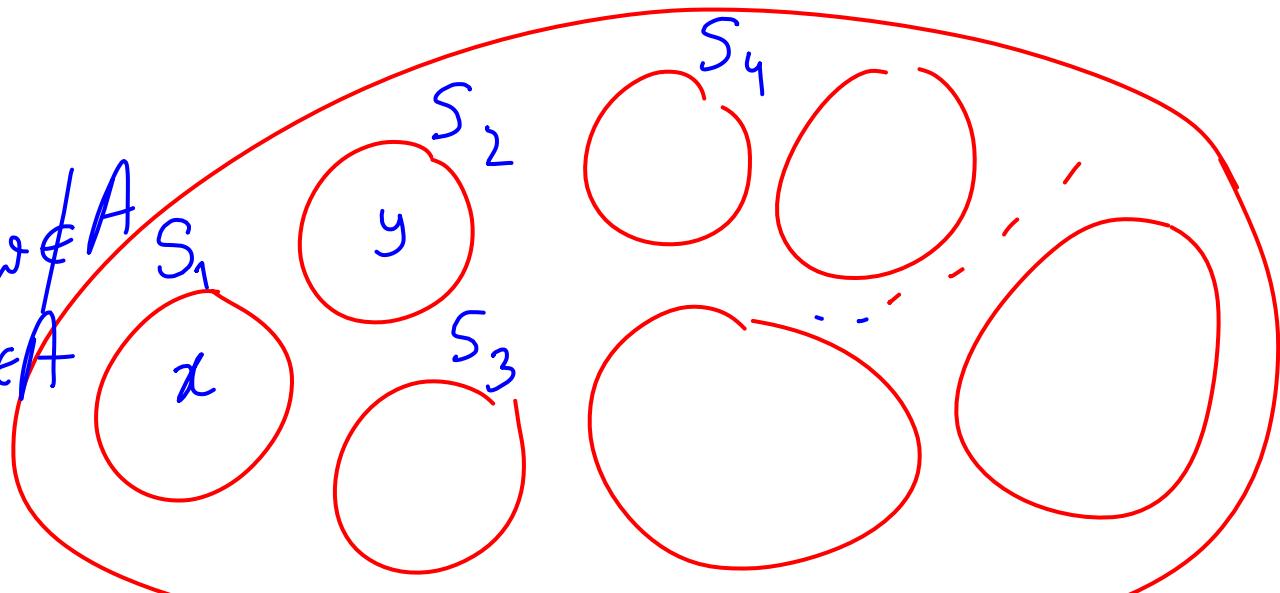
(transitive)  $xRy \& yRz \Rightarrow xRz$

Consider any  $w$  &  $xw, zw$

$xw \in A$  iff  $yw \in A$  iff  $zw \in A$

Consider equivalence classes formed by R  
on  $\Sigma^*$

$\exists w$  s.t.  
either  $xw \in A$  &  $yw \notin A$   
or  $xw \notin A$  &  $yw \in A$



$\Rightarrow$  Different eq classes will take a DFA accepting  
A to different states.

$S_i$  : set of states DFA ends up in on reading  
strings in  $i^{\text{th}}$  eq class.

$$S_i \cap S_j = \emptyset \text{ for } i \neq j$$

$$\& \bigcup S_i = Q \text{ (the set of states of DFA)}$$

A new DFA based on equivalence classes:

Let  $E$  = set of all eq classes,  
each represented as  $[x]$  where  $x$   
belongs to the class

Define  $F = \{E, [e], \delta, F, \Sigma\}$

