Game Theory

1 What is Game theory about?

A game is played whenever people interact with each other. If you drive a car on a busy city street, you are playing a game with the other drivers. When you bid at an auction, you are playing a game with the other bidders. When a supermarket manager decides the price at which she will try to sell cans of beans, she is playing a game with her customers and with the managers of rival supermarkets. When a firm and a union negotiate the next year's wage contract, they are playing a game. The prosecuting and defense attorneys are playing a game when each decides which arguments to present to the jury.

If all these situations are games, then game theory is clearly something important. Indeed, one could argue that all social sciences are nothing more than subdisciplines of game theory. The branch of social science that studies strategic decision-making is called game theory. The games in this theory range from chess to child-rearing, from tennis to takeovers, and from advertising to arms control. Playing these games requires many different kinds of skills. We develop the ideas and principles of strategic thinking; to apply them to a specific situation you face and to find the right choice there, you will have to do some more work.

2 Zero-sum game

A zero sum game is one in which any winnings for one player count as a loss for the other player. In such situations, every advantage you secure, such as winning chips in a poker match, comes directly at the expense of someone else's resources.

Example: Football game between two teams. Loss for team A is a win for team B.

When multiple candidates apply for a single job opening, if one candidate is hired, all the others lose out on that exact opportunity. There's no "win-win" scenario.

However, not all games are zero-sum games in real life.

3 Most famous strategic game

Imagine two people, Alex and Taylor, who are accused of committing a crime together. The police don't have enough evidence to convict either of them unless one of them confesses. They separate Alex and Taylor into different interrogation rooms and offer them the following deal:

- If both stay silent, the police can only charge them with a minor offense, and they'll each get 1 year in jail.
- If one confesses and the other stays silent, the confessor gets a deal and walks free, while the silent one gets 5 years in jail.

- If both confess, the police have enough evidence to convict them both, and they will each get 2 years in jail.

Now, let's consider Alex's thought process. Alex knows that Taylor will either confess or stay silent. If Taylor confesses, Alex gets 5 years in jail by staying silent and 2 years in jail if they confess too. If Taylor stays silent, Alex gets 3 months in jail by staying silent, but no jail time if they confess. So, no matter what Taylor does, Alex sees that confessing is the better choice.

Meanwhile, Taylor is sitting in their interrogation room, thinking the same thing. No matter what Alex does, it's better for Taylor to confess too.

The result? Both confess, and they each end up with 2 years in jail. But if they had both stayed silent, they could have gotten away with just 1 year each.

Here's a table that summarizes the possible outcomes:

Alex's Choice	**Taylor Stays Silent**	**Taylor Confesses**
Stay Silent	1 year, 1 year	5 years, Free
Confess	Free, 5 years	2 years, 2 years

4 Iterated Prisoners Dilemma

So far, we've focused on the Prisoner's Dilemma as a one-time encounter - just one isolated decision moment between two players who may never interact again. But what happens when the very same players face this dilemma repeatedly, round after round, aware that their choices today could influence the responses they receive tomorrow? By shifting our perspective from a single-shot scenario to what's known as the Iterated Prisoner's Dilemma, we'll see how patterns of trust, retaliation, and cooperation emerge when decisions become part of an ongoing relationship rather than a one-off gamble.

In 1980, Robert Axelrod, political scientist at the University of Michigan, held a computer-based competition to see which strategies worked best in an iteraed version of the Prisoner's Dilemma. He invited experts, many who had studied this problem deeply, to each write a simple computer program that decided, at every step, to play the game. These programs could remember what happened in previous rounds and use that information to guide future decisions. These computer programs were called strategies. All the strategies played against each other multiple times — 200 rounds for each pairing - and the entire contest was repeated five times to make sure no one just got lucky once. By doing this, Axelrod could figure out which ways of playing led to the most success in the long run. The payoff matrix is as follows:

Fourteen such strategies were submitted and a computer program named RANDOM, that

	Cooperate	**Defect**
Cooperate	3,3	0,5
Defect	5,0	1,1

randomly cooperates and defects with equal probability, was also included. Let us some of these strategies:

1. Tit for Tat: Tit for Tat starts by cooperating and then copies exactly what its opponent did in the last move.

2. FRIEDMAN: FRIEDMAN cooperates until the opponent defects, then defects forever.

3. SHUBIK: SHUBIK cooperates until the opponent defects, then retaliates with an escalating number of defections for each new betrayal.

4. JOSS: JOSS starts by cooperating, and then just copies what opponent did in the last move. But JOSS gets sneaky and defects 10% of the time.

5. GRASSKAMP: GRASSKAMP starts like Tit-for-Tat, defects once on move 51, and then plays based on whether it detects randomness or recognizes specific strategies.

At the end of the game, it was a surprise that the most simplest program ended up winning, i.e. Tit for Tat. Axelrod found that all the best-performing strategies, including Tit for Tat, shared four qualities.

- **Be nice:** Do not be the first to defect. Out of the 15 strategies in the tournament, eight were nice and seven nasty. The top eight strategies were all nice and even the worst performing nice strategy still far outscored the best performing nasty one.

- **Be forgiving:** A forgiving strategy retaliates when an opponent defects but doesn't hold grudges, allowing cooperation to resume. Tit-for-Tat exemplifies this approach, punishing defections immediately but resetting with each new round. In contrast, Friedman is completely unforgiving, defecting permanently after a single betrayal. While being unforgiving may feel satisfying, it ultimately proves less effective in the long run, as Axelrod's findings show that niceness and forgiveness lead to better outcomes.

5 Axelrod's second tournament

After publishing his analysis of the first tournament, Axelrod conducted a second round. This time, he made one crucial change: the number of rounds per game was no longer fixed at 200 but determined randomly, with an average of 200 rounds. This uncertainty was key. In a fixed-length game, players know when the final round is, and rationally, there's no incentive to cooperate in the last round, which leads to a backward unraveling of cooperation throughout the entire game. By keeping the endpoint uncertain, players are compelled to keep cooperating because the game might continue, and they may still need their opponent's cooperation in future rounds.

In the second tournament, the game theorists were divided into two camps. One group embraced the winning traits of niceness and forgiveness, submitting strategies built on these principles, including a version called "Tit-for-Two-Tats," which defected only after two consecutive betrayals. The second group sought to exploit this generosity, creating nasty strategies designed to manipulate overly forgiving opponents. One such strategy, "Tester," defected on the first move to gauge the opponent's response. If the opponent retaliated, Tester switched to Tit-for-Tat for the remainder of the game. If not, Tester exploited the opponent by defecting repeatedly.

Despite the attempts at trickery, nasty strategies once again failed to dominate. Tit-for-Tat remained the most effective, with nice strategies overwhelmingly outperforming nasty ones. In the top 15 strategies, only one was not nice, and in the bottom 15, only one was not nasty. Axelrod's second tournament reaffirmed the importance of niceness, and forgiveness further proving that in the long game, cooperation wins. After the second tournament, Axelrod identified the other two qualities that distinguished the better performing strategies.

- **Be retaliatory:** If your opponent defects, strike back immediately, don't be a pushover. Always cooperate is a total pushover, and it's very easy to take advantage of. Tit-for-Tat, on the other hand, is very hard to take advantage of.

- **Be clear:** The last quality that Axelrod identified is being clear. Programs that were too opaque didn't perform well as it was very hard to establish any pattern of trust. Hence, it's important to be clear in life.

6 Some important life lessons from iterated Prisoner's dilemma

These four principles being nice, forgiving, retaliatory, and clear is a lot like the morality that has evolved around the world. What's interesting is that while Tit-for-Two-Tats would've won the first tournament, it only came 24th in the second tournament. This highlights an important fact that in the repeated prisoner's dilemma, there is no single best strategy. The strategy that performs best always depends on the other strategies it's interacting with. For example, if you put Tit-for-Tat in an environment with only the ultimate bullies of always defect, then Tit-for-Tat comes in last.

To explore this further, Axelrod ran simulations where successful strategies reproduced, and unsuccessful ones faded out. Over time, only nice strategies survived, with Tit-for-Tat dominating at 14.5% of the population. This process mirrors evolution, although without mutations, making it more of an ecological simulation. **Axelrod also showed that even in a nasty world dominated by defectors, a small cluster of cooperative players, like Tit-for-Tat, could thrive, spread, and eventually take over.**

6.1 How can cooperation emerge in a population of players who are self-interested?

Individuals don't have to be good because they're good-hearted i.e. you don't have to be altruistic. You could be looking out for yourself and your own interests, and yet cooperation can still emerge. Some argue that this could explain how we went from a world full of completely selfish organisms where every organism only cared about themselves to one where cooperation emerged and flourished. For instance, we have seen fish cleaning sharks in the nature. Axelrod's insights can be applied to areas like evolutionary biology and international conflicts, but there was one aspect that his original tournaments didn't cover. What happens if there is a little bit of random error/noise in the system? For example, one player tries to cooperate, but it comes across as a defection. Little errors like this happen in the real world all the time.

7 Real world scenario

In noisy environments, Tit-for-Tat struggled. A single mistake could set off a chain reaction of retaliation, leading to mutual defection. This flaw was not apparent in the artificial setting of a computer tournament, because misperceptions did not arise. But when tit-for-tat is applied to real-world problems, misperceptions cannot be avoided and the result can be disastrous.

For instance, in 1987 the United States responded to the Soviet spying and wiretapping of the U.S. embassy in Moscow by reducing the number of Soviet diplomats permitted to work in the United States. The Soviets responded by withdrawing the native support staff employed at the U.S. Moscow embassy and placed tighter limits on the size of the American delegation. As a result, both sides found it more difficult to carry out their diplomatic functions.

For example, Another series of tit-for-tat retaliations occurred in 1988, when the Canadians discovered spying on the part of the visiting Soviet diplomats. They reduced the size of the Soviet delegation and the Soviets reduced the Canadian representation in the Soviet Union. In the end, both countries were bitter, and future diplomatic cooperation was more difficult.

The problem with tit-for-tat is that any mistake "echoes" back and forth. One side punishes the other for a defection, and this sets off a chain reaction. The rival responds to the punishment by hitting back. This response calls for a second punishment. At no point does the strategy accept a punishment without hitting back. Example: historical conflicts between two nations such as Israel vs Palestine, Armenia and Azerbaijan, etc.

What tit-for-tat lacks is a way of saying "Enough is enough," and is too easily provoked. You should be more forgiving when a defection seems to be a mistake rather than the rule. Even if the defection was intentional, after a long-enough cycle of punishments it may still be time to call it quits and try reestablishing cooperation. At the same time, you don't want to be too forgiving and risk exploitation. How do you make this trade-off? A useful way to evaluate a strategy is to measure how well it performs against itself. If one thinks in terms of evolution, the "fittest strategies" will become dominant in the population. As a result, they will encounter each other often. Unless a strategy performs well against itself, any initial success will eventually become self-defeating. At first glance, tit-for-tat does very well against itself. Two tit-for-tatters will start off cooperating, and since each is responding in kind, this cooperation seems destined to go on forever. The pair of strategies appears to completely avoid the problem of the prisoners' dilemma.

The basic properties of clarity, niceness, provocability, and forgivingness seem likely to be true of any good rule of behavior for extricating oneself from a prisoners' dilemma. But tit-for-tat is too quick to punish someone who has a history of cooperating. We need to find a strategy that is more discriminating: it should be more forgiving when a defection appears to be an exception, and it should punish when defection appears to be the rule. You can consider the following guidelines as a step in that direction.

- Begin cooperating.
- Continue cooperating.
- Keep count of how many times the other side appears to have defected while you have cooperated.
- If this percentage becomes unacceptable, revert to tit-for-tat.

In real-world situations, mistakes and misunderstandings happen, so we need a strategy that is flexible and fair. This approach builds on *Tit-for-Tat* by adding clear rules for when defections become unacceptable, based on four timeframes:

1. **First Impression:** If the opponent defects on the first move, stop cooperating and switch to *Tit-for-Tat*.

2. **Short Term:** Two defections within three turns are unacceptable - switch to *Tit-for-Tat*.

3. **Medium Term:** Three defections out of the last 20 turns are unacceptable - switch to *Tit-for-Tat*.

4. **Long Term:** Five defections out of the last 100 turns are unacceptable - switch to *Tit-for-Tat*.

If any rule is broken:

- Use *Tit-for-Tat* for 20 turns as punishment.
- After punishment, put the opponent on probation with stricter rules.
- If the opponent behaves for 50 turns, clear their record and return to normal.
- If the opponent violates probation, switch to permanent *Tit-for-Tat*.

7.1 Why can this be a better strategy?

This strategy is better than simple retaliation because:

- It allows for occasional mistakes without immediate punishment.
- It distinguishes between accidental and repeated bad behavior.
- It balances forgiveness and firmness, ensuring cooperation while punishing opportunism.

This approach is ideal for real-world situations where cooperation and misunderstandings both occur, making it more effective than basic *Tit-for-Tat*. When the eventual misperceptions arise you will no longer be inclined to let the incident pass. Opportunism on the part of your opponent will be self-defeating.