Chapter 12: Comparing Two Population Means

1 Comparing Two Population Means

In previous chapter, we studied the difference between two proportions, we mean that we are going to subtract one proportion from the other. The same is true when examining differences of means. For instance, we might subtract the mean amount of time Americans spend watching TV every day from the mean amount of time they spend exercising. From subtraction, we learn that: **if the result is positive, the first mean is greater than the second. if the result is negative, the first mean is less than the second. If the result is 0, the means are equal.**

2 Dependent and Independent samples

When comparing two populations, it is important to pay attention to whether the data sampled from the populations are two independent samples or are, in fact, one sample of related pairs (dependent samples). With paired (dependent) samples, if you know the value that a subject has in one group, then you know something about the other group, too.

Examples: Here are four descriptions of research studies. Identify whether they are dependent sample or independent samples.

1. People chosen in a random sample were asked how many minutes they had spent the day before watching television and how many minutes they had spent exercising. Researchers want to know how different the mean amounts of times are for these two activities. **Dependent samples**

2. Men and women each had their sense of smell measured. Researchers want to know whether, typically, men and women differ in their ability to sense smells. **Independent samples**

3. Researchers randomly assigned overweight people to one of two diets: Weight Watchers and Atkins. Researchers want to know whether the mean weight loss on Weight Watchers was different from that on Atkins. **Independent samples**

4. The numbers of years of education for husbands and wives are compared to see whether the means are different. **Dependent samples**

3 Estimating the Difference of Means with Confidence Intervals for Independent Samples

We have two population samples. Let's say we have the first sample mean $\bar{x}_1$, standard deviation s_1, and sample size of n_1. Similarly, we have the second sample mean $\bar{x}_2$, standard deviation s_2, and sample size of n_2. The difference between the two sample mean is written as $\bar{x}_1 - \bar{x}_2$. The step to calculate the confidence intervals are as follows:

- Check if all the conditions of CLT are met which are:
  1. *Random Samples and Independence.* Both samples are randomly taken from their populations, and each observation is independent of any other.
  2. *Independent Samples.* The two samples are independent of each other.
  3. *Large Samples.* The populations are approximately Normal, or the sample size in each sample is 25 or more.
- Find the estimated standard error calculated as $SE_{est} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
- Find the margin of error. Margin of error $(\mathrm{m}) = t^* SE_{est}$
- We will always use technology to calculate the t^* value which is based on an approximate $t-$distribution.
- Find the confidence interval using: $(\bar{x}_1 - \bar{x}_2) \pm m$

4 Interpreting Confidence Intervals of Differences

The most important thing to look for is whether or not the interval includes 0.

If the confidence interval contains 0: no significant difference between the means.

If a confidence interval does not contain 0: significant difference between the means.

5 Some Examples:

I will demonstrate one example with steps. After that, we will use technology to calculate it. Our focus will be on interpreting the results, not on calculating the confidence intervals.

Example: Do people in the United States sleep more on holidays and weekends than on weekdays? The Bureau of Labor Statistics carries out a "time use" survey, in which randomly chosen people are asked to record every activity they do on a randomly chosen day of the year. For instance, you might be chosen to take part in the survey on Tuesday, April 18, while someone else will be chosen to take part on Sunday, December 5. Because we have two separate groups of people reporting their amount of sleep — one group that reported only on weekends and holidays, and another group that reported only on weekdays —these data are two independent samples. The summary statistics follow.

Weekday: $\bar{x} = 499.7$ minutes, $s = 126.9$ minutes, $n = 6007$
Weekend/Holiday: $\bar{x} = 555.9$ minutes, $s = 140.9$ minutes, $n = 6436$

Find a 95% confidence interval for the mean difference in time spent sleeping on a weekend/holiday compared to time spent sleeping on a weekday. Interpret this interval.

Step 1: We are told that people are selected randomly, and it also seems reasonable that the amount of sleep they get is independent. The question explained that the samples were themselves independent of each other. We have large sample sizes, we do not have to worry and can proceed.

step 2: We'll use the Weekend/Holiday group as our sample 1 and the Weekday group as sample 2. $\bar{x}_1 - \bar{x}_2 = 555.9 - 499.7 = 56.2$ minutes

Step 3:

$$m = t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = t^* \sqrt{\frac{140.9^2}{6436} + \frac{126.9^2}{6007}} = t^* \times 2.401137$$

We have a large sample. So, using $t^* = 1.96$, we have $m = 1.96 \times 2.401137 = 4.706229$ minutes.

Step 4: Therefore, a 95% confidence interval is

$$(\bar{x}_1 - \bar{x}_2) \pm m = 56.2 \pm 4.706229, \text{ or about } (51.5, 60.9) \text{ minutes.}$$

Step 5: We are 95% confident that the true mean difference in amount of time spent sleeping on weekends/holidays and amount of time spent sleeping on weekdays is between 51.5 minutes and 60.9 minutes.

Step 6: The most important part: Our confidence interval doesn't contain 0, hence there is a significant difference between the means. The interval contains all positive values, so we are confident that people typically sleep longer on weekends and holidays than they do on weekdays.

Example 2: Using data from NHANES, we looked at the pulse rate for nearly 800 people to see whether it is plausible that men and women have the same population mean. NHANES data are random and independent. The results are as follows:

Two-Sample T: CI

Sample	N	Mean	StDev	SE Mean
Women	384	76.3	12.8	0.65
Men	372	72.1	13.0	0.67

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Difference = μ(1) - μ(2)
Estimate for difference: 4.200
95% CI for difference:  (2.357, 6.043)
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- Are the conditions for using a confidence interval for the difference between two means met?
  Yes, the samples are random, independent, and large ($n_1 = 384 > 25$ and $n_2 = 372 > 25$), so the conditions are met

- State the interval in a clear and correct sentence.
  I am 95% confident that the mean difference of pulse rate between women and men is between 2.4 and 6.0 beats per minute.

- Does the interval capture 0? Explain what that shows.
  Because the interval does not capture 0, there is a significant difference between the mean pulse rates between men and women. Because the interval is entirely positive and it is from the women's mean minus the men's mean, it shows that women tend to have a higher pulse rate than men.

Example 3: In a market research study, you want to compare the mean satisfaction scores of two different brands of shampoo (Brand A and Brand B) among a sample of customers. In a sample of 307, Brand A received an average satisfaction rating of 5.8 with a standard deviation of 1.68 and in a similar sample of 252 customers Brand B received an average satisfaction rating of 5.6 with standard deviation 2.39. Satisfaction was recording on a scale of 1 to 5.

Is the difference in mean satisfaction scores between Brand A and Brand B statistically significant at the 95% confidence level?

First of all, we will calculate the 95% confidence interval using technology. The results are attached below.

Descriptive Statistics:

Group	Sample Size	Mean	Std. Dev.
Group 1	307	5.80	1.28
Group 2	252	5.60	2.39

Estimate of Difference of Means:

Point Estimate	Standard Error	Margin of Error
0.200	0.167	± 0.329

Confidence Interval:

Population Parameter	Lower Bound	Upper Bound	Confidence Level
Difference $\mu_1 - \mu_2$	-0.129	0.529	95%

Thus, we are 95% confident that the true mean difference in customer satisfaction index for shampoo between Brand A and Brand B is between -0.129 and 0.529. Since the confidence interval captures 0, the difference in mean satisfaction scores between Brand A and Brand B is not statistically significant.

6 Hypothesis testing involving two means

It is similar to two sample proportions hypothesis test, but you are given two means.

Step 1: Firstly, figure out μ_1, μ_2, s_1,s_2,n_1,and n_2 from the problem.

Step 2: Then, write down the pair of hypotheses as:

Two-sided	One-sided (Left)	One-sided (Right)
$H_0 : \mu_1 = \mu_2$ $H_a : \mu_1 \neq \mu_2$	$H_0 : \mu_1 = \mu_2$ $H_a : \mu_1 < \mu_2$	$H_0 : \mu_1 = \mu_2$ $H_a : \mu_1 > \mu_2$

Step 3: Check if the following conditions are satisfied:

i. *Random Samples and Independence.* Both samples are randomly taken from their populations, and each observation is independent of any other.

ii. *Independent Samples.* The two samples are independent of each other.

iii. *Large Samples.* The populations are approximately Normal, or the sample size in each sample is 25 or more.

Step 4: Compute the value of $t-$stat:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE_{EST}}$$

where,

$$SE_{EST} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Step 5: Find the corresponding p-value of your t-test statistics.

Step 6: Compare your p-value with significance level α
If p-value $< \alpha$, you will reject H_o
If p-value $> \alpha$, you will fail to reject H_o