**Optimization Project Proposal**

**Title:** *Multi-Objective Network Flow Optimization for Internet Bandwidth Allocation*
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## Problem Statement

Modern networks have limited bandwidth that must be shared among many users with different requirements. The objective is to allocate this bandwidth efficiently while achieving several goals: maximizing throughput, maintaining fairness, reducing delay, and staying within the physical limits of the network. To represent the system, we model the network as a graph consisting of nodes (routers) and links (edges) with fixed capacities. Each data flow must follow these capacity limits and satisfy flow conservation rules at intermediate nodes.

**Mathematical Formulation:**

$$
\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{n} w_i \log(x_i) - \lambda \cdot \text{Latency}(x) \\
\text{subject to} \quad & \sum_{i} f_{i,e} \leq c_e \ \ \forall e \in E \\
& \text{Flow conservation at each node} \\
& x_{i,\min} \leq x_i \leq x_{i,\max}, \ \ f_{i,e} \geq 0
\end{aligned}
\tag{1}
$$

Here, $x_i$ is the bandwidth for user $i$, $f_{i,e}$ is the flow on link $e$, $c_e$ is link capacity, and $E$ is the set of links.

## Proposed Approach

### 1. Convexity Analysis

The objective combines concave utility (log fairness) with a convex latency penalty, and all constraints are linear. Thus the feasible region is convex and the optimization problem admits a global optimum. Convexity ensures that KKT conditions fully characterize the optimal allocation.

### 2. Solution Methodology

**(a) Lagrangian and KKT Conditions**
We construct the Lagrangian:

$$\mathcal{L}(x, \mu, \nu) = -\sum_{i} w_i \log(x_i) + \mu^T(\text{capacity}) + \nu^T(\text{bounds}),$$

and apply KKT: stationarity, primal/dual feasibility, and complementary slackness. This links the mathematical model directly to the numerical solution procedures.

**(b) Interior Point Methods**
Inequality constraints are incorporated using logarithmic barrier functions. Newton's method solves each relaxed subproblem as the barrier parameter decreases, giving a fast, globally convergent solution.

**(c) Multi-Objective Optimization**
Trade offs between throughput, fairness, and latency are explored using: Weighted Sum and $\epsilon$-Constraint methods. Pareto frontiers show how performance shifts with different priorities. Fairness is quantified using:

$$J(x) = \frac{(\sum_i x_i)^2}{n \sum_i x_i^2}.$$

### 3. Tools

Python, CVXPY, NumPy, Matplotlib, Pandas, Seaborn.