

Perceptron Learning II

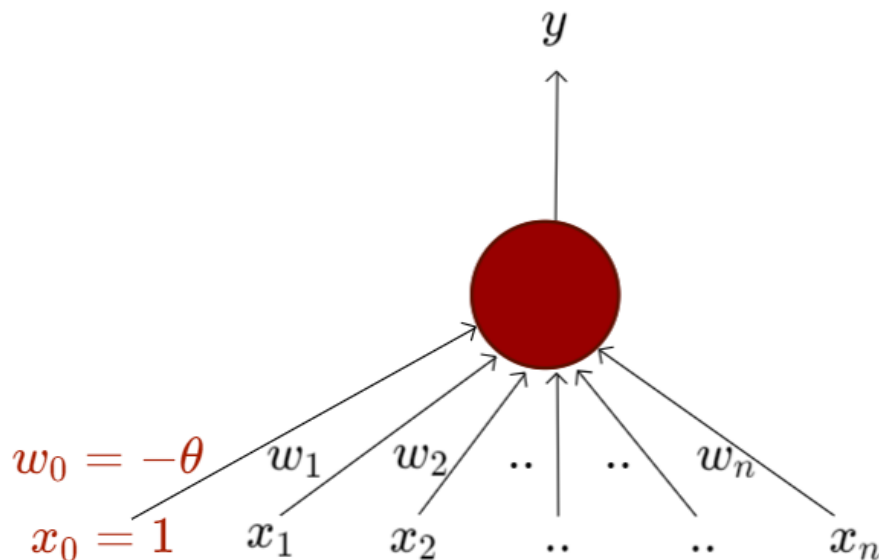
Last Lecture

- Perceptron Learning Algorithm

Today's Topics

- Numerical Example
- Convergence theorem

Perceptron



A more accepted convention,

$$\begin{aligned} y &= 1 \quad \text{if} \sum_{i=0}^n w_i * x_i \geq 0 \\ &= 0 \quad \text{if} \sum_{i=0}^n w_i * x_i < 0 \end{aligned}$$

where, $x_0 = 1$ and $w_0 = -\theta$

$$\begin{aligned} y &= 1 \quad \text{if} \sum_{i=1}^n w_i * x_i \geq \theta \\ &= 0 \quad \text{if} \sum_{i=1}^n w_i * x_i < \theta \end{aligned}$$

Rewriting the above,

$$\begin{aligned} y &= 1 \quad \text{if} \sum_{i=1}^n w_i * x_i - \theta \geq 0 \\ &= 0 \quad \text{if} \sum_{i=1}^n w_i * x_i - \theta < 0 \end{aligned}$$

Perceptron Learning Algorithm

Algorithm: Perceptron Learning Algorithm

$P \leftarrow$ inputs with label 1;

$N \leftarrow$ inputs with label 0;

Initialize \mathbf{w} randomly;

while !convergence **do**

 Pick random $\mathbf{x} \in P \cup N$;

if $\mathbf{x} \in P$ and $\sum_{i=0}^n w_i * x_i < 0$ **then**

$\mathbf{w} = \mathbf{w} + \mathbf{x}$;

end

if $\mathbf{x} \in N$ and $\sum_{i=0}^n w_i * x_i \geq 0$ **then**

$\mathbf{w} = \mathbf{w} - \mathbf{x}$;

end

end

//the algorithm converges when all the inputs
are classified correctly

- If the perceptron makes a **mistake (incorrect prediction)** and the **target = 1**, we **update weights as:**
 $\mathbf{w} = \mathbf{w} + \mathbf{x}$
- If the perceptron makes a **mistake (incorrect prediction)** and the **target = 0**, we **update weights as:**
 $\mathbf{w} = \mathbf{w} - \mathbf{x}$

Numerical Example: OR

Input (x1, x2)	Output (OR)
(0, 0)	0
(0, 1)	1
(1, 0)	1
(1, 1)	1

Parameters:

- Initial Weights:

$w_1=0, w_2=0, b=0$

- Threshold: 0

• Activation Function:

$y=1$ if $w_1 \cdot x_1 + w_2 \cdot x_2 + b \geq 0$

$Y=0$ if $w_1 \cdot x_1 + w_2 \cdot x_2 + b < 0$

- If the perceptron makes a **mistake (incorrect prediction)** and the **target = 1**, we **update weights as:**

$$W=W+X$$

- If the perceptron makes a **mistake (incorrect prediction)** and the **target = 0**, we **update weights as:**

$$W=W-X$$

Perceptron Example

Epoch 1:

1. Input (0, 0) -> Target = 0

- Net input = $0 \cdot 0 + 0 \cdot 0 + 0 = 0$
- Output = 1 (because net input ≥ 0)
- Mistake (output 1, target 0), so update rule $w = w - x$:
 - $w1 = 0 - 0 = 0$
 - $w2 = 0 - 0 = 0$
 - $b = 0 - 1 = -1$

2. Input (0, 1) -> Target = 1

- Net input = $0 \cdot 0 + 0 \cdot 1 + (-1) = -1$
- Output = 0 (because net input < 0)
- Mistake (output 0, target 1), so update rule $w = w + x$:
 - $w1 = 0 + 0 = 0$
 - $w2 = 0 + 1 = 1$
 - $b = -1 + 1 = 0$

Perceptron Example

Epoch 1:

Step	Input (x1, x2)	Target (t)	Net Input (w1x1 + w2x2 + b)	Output (y)	Update Rule	Updated w1	Updated w2	Updated b
1	(0, 0)	0	$0 \cdot 0 + 0 \cdot 0 + 0 = 0$	1	$w = w - x$	0	0	-1
2	(0, 1)	1	$0 \cdot 0 + 0 \cdot 1 + (-1) = -1$	0	$w = w + x$	0	1	0
3	(1, 0)	1	$0 \cdot 1 + 1 \cdot 0 + 0 = 0$	1	No update	0	1	0
4	(1, 1)	1	$0 \cdot 1 + 1 \cdot 1 + 0 = 1$	1	No update	0	1	0

Perceptron Example

Epoch 2:

Step	Input (x1, x2)	Target (t)	Net Input ($w_1x_1 + w_2x_2 + b$)	Output (y)	Update Rule	Updated w1	Updated w2	Updated b
1	(0, 0)	0	$0 \cdot 0 + 1 \cdot 0 + 0 = 0$	1	$w = w - x$	0	1	-1
2	(0, 1)	1	$0 \cdot 0 + 1 \cdot 1 + (-1) = 0$	1	No update	0	1	-1
3	(1, 0)	1	$0 \cdot 1 + 1 \cdot 0 + (-1) = -1$	0	$w = w + x$	1	1	0
4	(1, 1)	1	$1 \cdot 1 + 1 \cdot 1 + 0 = 2$	1	No update	1	1	0

Perceptron Example

Input (x1, x2)	Target (OR)	Net Input ($w_1 \cdot x_1 + w_2 \cdot x_2 + b$)	Output (y)	Correct?
(0, 0)	0	$1 \cdot 0 + 1 \cdot 0 + 0 = 0$	0	Yes
(0, 1)	1	$1 \cdot 0 + 1 \cdot 1 + 0 = 1$	1	Yes
(1, 0)	1	$1 \cdot 1 + 1 \cdot 0 + 0 = 1$	1	Yes
(1, 1)	1	$1 \cdot 1 + 1 \cdot 1 + 0 = 2$	1	Yes

Perceptron Learning on Iris Dataset (Binary Class)

- `self.weights = np.zeros(n_features)`
- `self.bias = 0`
- `for _ in range(self.n_iters):`
- `for idx, x_i in enumerate(X):`
 - `linear_output = np.dot(x_i, self.weights) + self.bias`
 - `y_predicted = self.activation_function(linear_output)`
 - `update = self.learning_rate * (y[idx] - y_predicted)`
 - `self.weights += update * x_i`
 - `self.bias += update`

Proof of Convergence

Proof of Convergence

Theorem

Definition: Two sets P and N of points in an n -dimensional space are called absolutely linearly separable if $n + 1$ real numbers w_0, w_1, \dots, w_n exist such that every point $(x_1, x_2, \dots, x_n) \in P$ satisfies $\sum_{i=1}^n w_i * x_i > w_0$ and every point $(x_1, x_2, \dots, x_n) \in N$ satisfies $\sum_{i=1}^n w_i * x_i < w_0$.

Proposition: If the sets P and N are finite and linearly separable, the perceptron learning algorithm updates the weight vector \mathbf{w}_t a finite number of times. In other words: if the vectors in P and N are tested cyclically one after the other, a weight vector \mathbf{w}_t is found after a finite number of steps t which can separate the two sets.

Proof of Convergence

- If $x \in N$ then $-x \in P$ ($\because w^T x < 0 \implies w^T(-x) \geq 0$)
- We can thus consider a single set $P' = P \cup N^-$ and for every element $p \in P'$ ensure that $w^T p \geq 0$
- Further we will normalize all the p 's so that $\|p\| = 1$ (notice that this does not affect the solution \because if $w^T \frac{p}{\|p\|} \geq 0$ then $w^T p \geq 0$)

Algorithm: Perceptron Learning Algorithm

```
 $P \leftarrow$  inputs with label 1;  
 $N \leftarrow$  inputs with label 0;  
 $N^-$  contains negations of all points in  $N$ ;  
 $P' \leftarrow P \cup N^-$ ;  
Initialize  $\mathbf{w}$  randomly;  
while !convergence do  
    Pick random  $\mathbf{p} \in P'$  ;  
     $\mathbf{p} \leftarrow \frac{\mathbf{p}}{\|\mathbf{p}\|}$  (so now,  $\|\mathbf{p}\| = 1$ ) ;  
    if  $\mathbf{w} \cdot \mathbf{p} < 0$  then  
        |  $\mathbf{w} = \mathbf{w} + \mathbf{p}$  ;  
    end  
end
```

Proof of Convergence

- w^* is some optimal solution which exists but we don't know what it is
- We make a correction only if $w^T \cdot p_i \leq 0$ at that time step

Proof of Convergence

Proof:

- Now suppose at time step t we inspected the point p_i and found that $w^T \cdot p_i \leq 0$
- We make a correction $w_{t+1} = w_t + p_i$
- Let β be the angle between w^* and w_{t+1}

$$\cos\beta = \frac{w^* \cdot w_{t+1}}{\|w_{t+1}\|}$$

$$\begin{aligned} \text{Numerator} &= w^* \cdot w_{t+1} = w^* \cdot (w_t + p_i) \\ &= w^* \cdot w_t + w^* \cdot p_i \\ &\geq w^* \cdot w_t + \delta \quad (\delta = \min\{w^* \cdot p_i | \forall i\}) \\ &\geq w^* \cdot (w_{t-1} + p_j) + \delta \\ &\geq w^* \cdot w_{t-1} + w^* \cdot p_j + \delta \\ &\geq w^* \cdot w_{t-1} + 2\delta \\ &\geq w^* \cdot w_0 + (k)\delta \quad (\text{By induction}) \end{aligned}$$

- We make a correction only if $w^T \cdot p_i \leq 0$ at that time step
 - So at time-step t we would have made only k ($\leq t$) corrections
 - Every time we make a correction a quantity δ gets added to the numerator
 - So by time-step t , a quantity $k\delta$ gets added to the numerator
-

Proof of Convergence

Proof (continued:)

So far we have, $w^T \cdot p_i \leq 0$ (and hence we made the correction)

$$\cos\beta = \frac{w^* \cdot w_{t+1}}{\|w_{t+1}\|} \quad (\text{by definition})$$

$$\text{Numerator} \geq w^* \cdot w_0 + k\delta \quad (\text{proved by induction})$$

$$\begin{aligned} \text{Denominator}^2 &= \|w_{t+1}\|^2 \\ &= (w_t + p_i) \cdot (w_t + p_i) \\ &= \|w_t\|^2 + 2w_t \cdot p_i + \|p_i\|^2 \\ &\leq \|w_t\|^2 + \|p_i\|^2 \quad (\because w_t \cdot p_i \leq 0) \\ &\leq \|w_t\|^2 + 1 \quad (\because \|p_i\|^2 = 1) \\ &\leq (\|w_{t-1}\|^2 + 1) + 1 \\ &\leq \|w_{t-1}\|^2 + 2 \\ &\leq \|w_0\|^2 + (k) \quad (\text{By same observation that we made about } \delta) \end{aligned}$$

Proof of Convergence

Proof (continued:)

So far we have, $w^T \cdot p_i \leq 0$ (and hence we made the correction)

$$\cos\beta = \frac{w^* \cdot w_{t+1}}{\|w_{t+1}\|} \quad (\text{by definition})$$

$$\text{Numerator} \geq w^* \cdot w_0 + k\delta \quad (\text{proved by induction})$$

$$\text{Denominator}^2 \leq \|w_0\|^2 + k \quad (\text{By same observation that we made about } \delta)$$

$$\cos\beta \geq \frac{w^* \cdot w_0 + k\delta}{\sqrt{\|w_0\|^2 + k}}$$

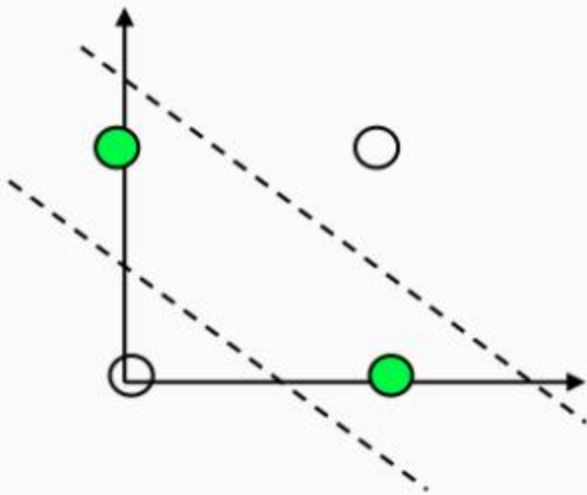
- $\cos\beta$ thus grows proportional to \sqrt{k}
- As k (number of corrections) increases $\cos\beta$ can become arbitrarily large
- But since $\cos\beta \leq 1$, k must be bounded by a maximum number
- Thus, there can only be a finite number of corrections (k) to w and the algorithm will converge!

Multi-layer Perceptrons

Perceptron Limitations

For a linearly not-separable problem:

- Would it help if we use **more layers of neurons**?
- What could be the learning rule for each neuron?



Boolean XOR

Solution: Multilayer networks
and the backpropagation
learning algorithm

Boolean functions from 2 inputs

x_1	x_2	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

- Of these, how many are linearly separable ? (turns out all except XOR and !XOR - feel free to verify)

--

what do we do about functions which are not linearly separable ?

x_1	x_2	XOR	
0	0	0	$w_0 + \sum_{i=1}^2 w_i x_i < 0$
1	0	1	$w_0 + \sum_{i=1}^2 w_i x_i \geq 0$
0	1	1	$w_0 + \sum_{i=1}^2 w_i x_i \geq 0$
1	1	0	$w_0 + \sum_{i=1}^2 w_i x_i < 0$

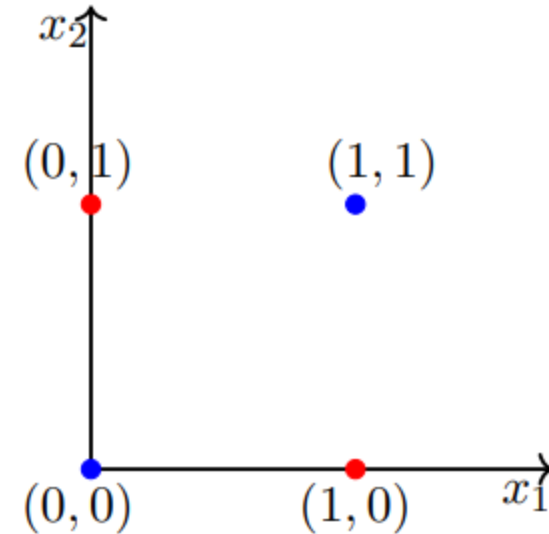
$$w_0 + w_1 \cdot 0 + w_2 \cdot 0 < 0 \implies w_0 < 0$$

$$w_0 + w_1 \cdot 0 + w_2 \cdot 1 \geq 0 \implies w_2 \geq -w_0$$

$$w_0 + w_1 \cdot 1 + w_2 \cdot 0 \geq 0 \implies w_1 \geq -w_0$$

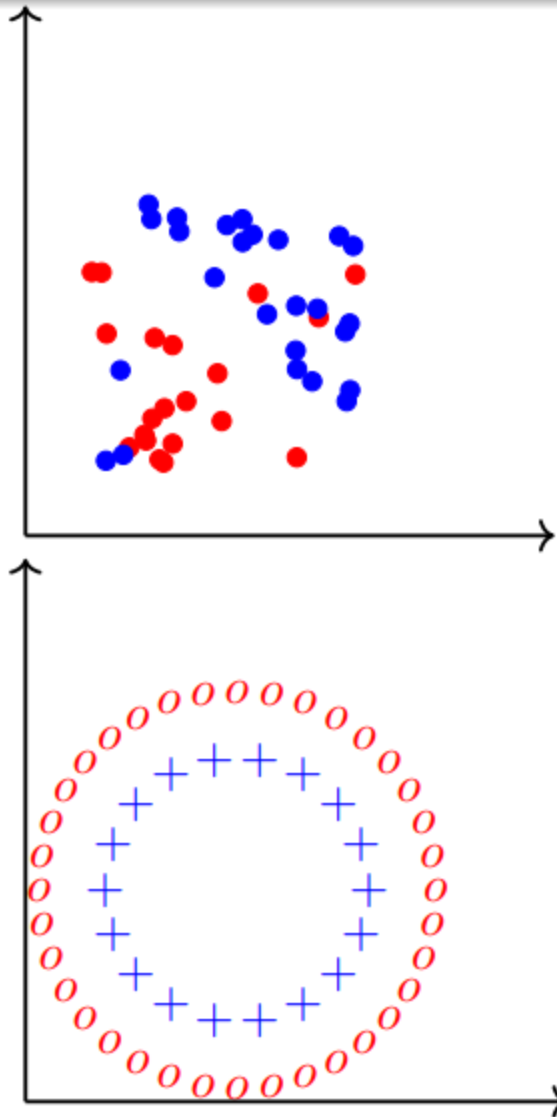
$$w_0 + w_1 \cdot 1 + w_2 \cdot 1 < 0 \implies w_1 + w_2 < -w_0$$

- The fourth condition contradicts conditions 2 and 3
- Hence we cannot have a solution to this set of inequalities



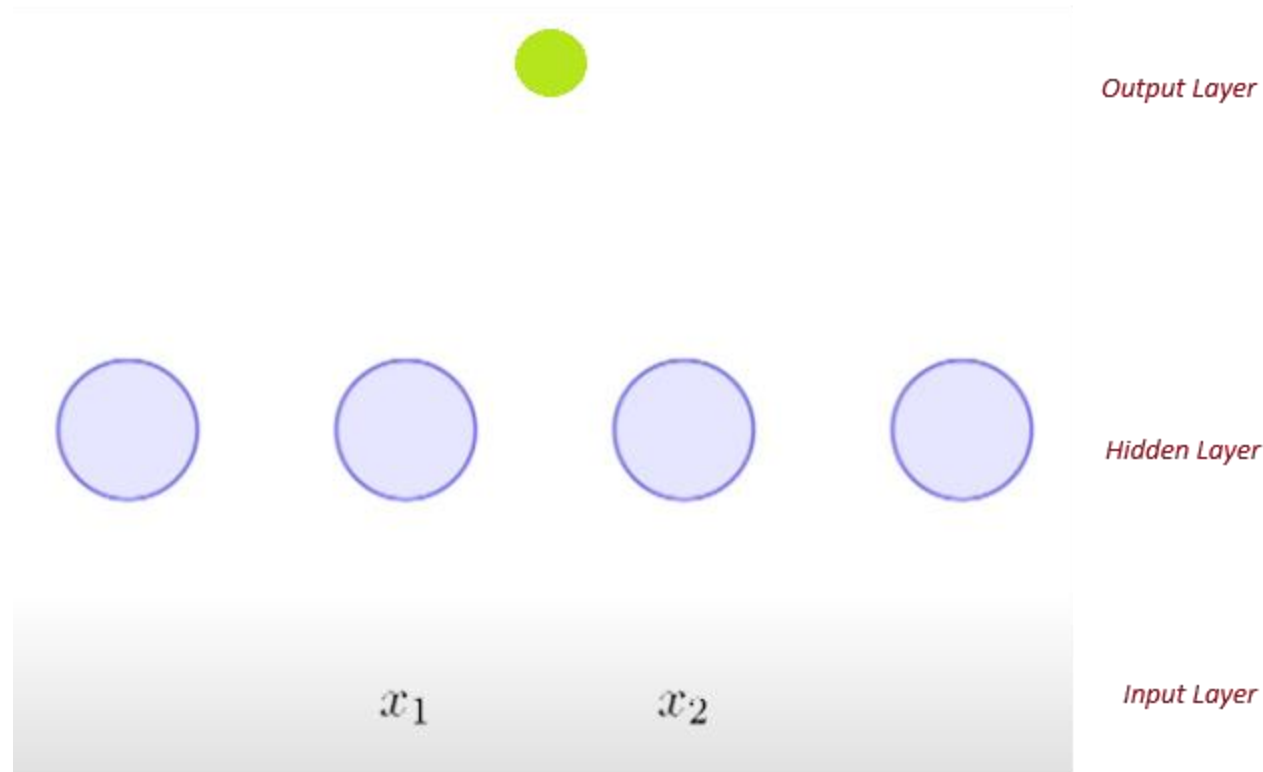
- And indeed you can see that it is impossible to draw a line which separates the red points from the blue points

Non-linearly separable data



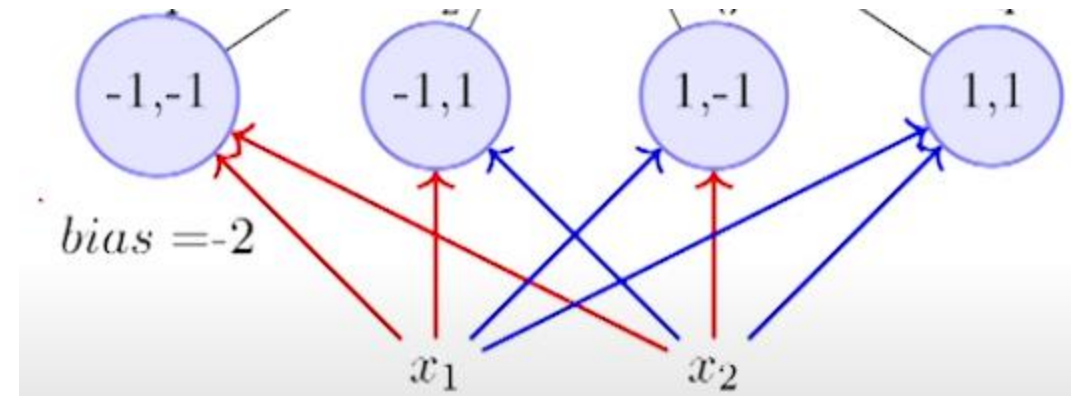
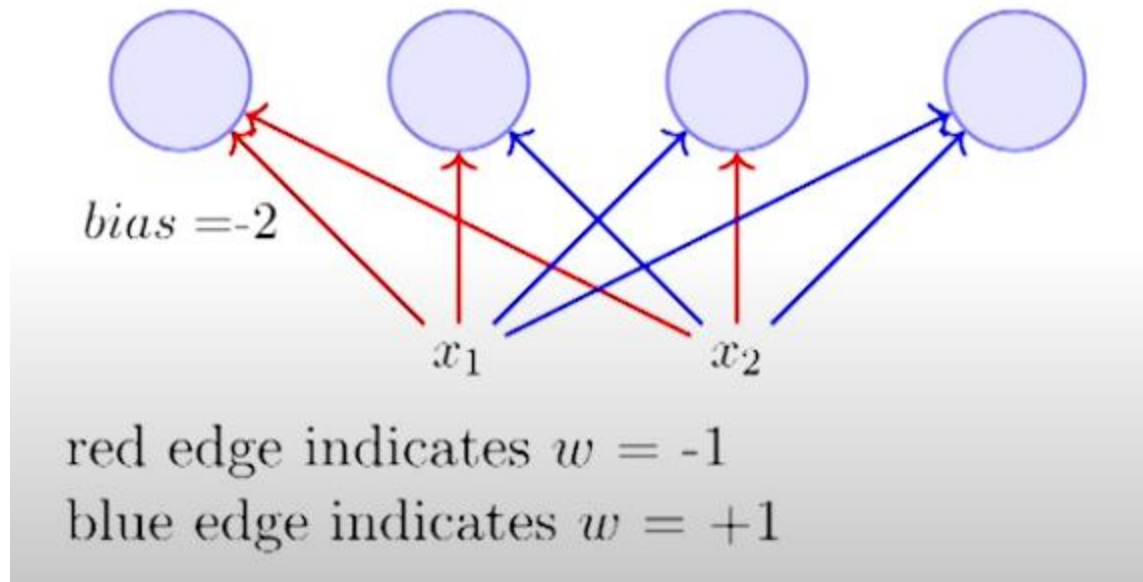
- Most real world data is not linearly separable and will always contain some outliers
- In fact, sometimes there may not be any outliers but still the data may not be linearly separable
- We need computational units (models) which can deal with such data
- While a single perceptron cannot deal with such data, we will show that a network of perceptrons can indeed deal with such data

Network of Perceptrons

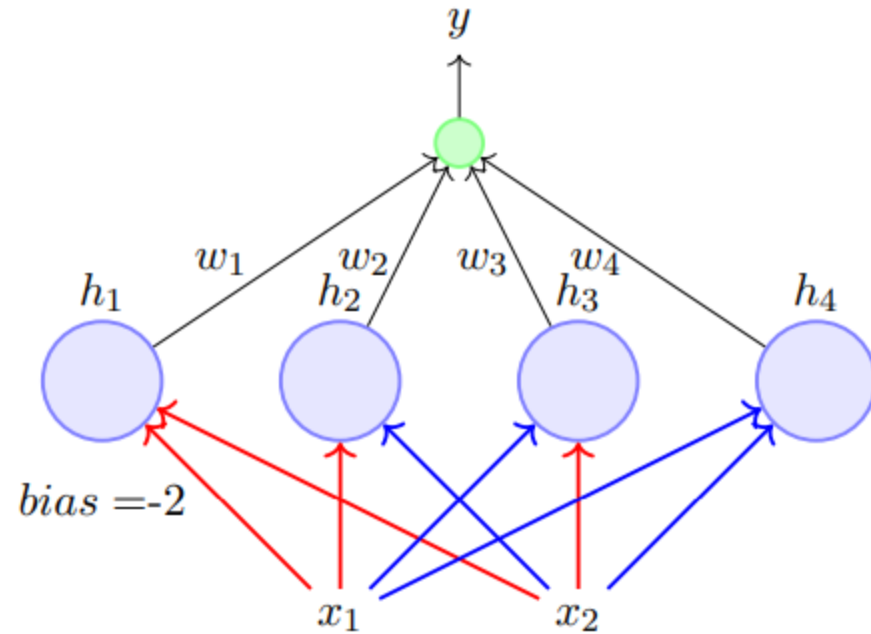


- This network contains 3 layers
- The layer containing the inputs (x_1, x_2) is called the **input layer**
- The middle layer containing the 4 perceptrons is called the **hidden layer**
- The final layer containing one output neuron is called the **output layer**

Network of Perceptrons



Network of Perceptrons

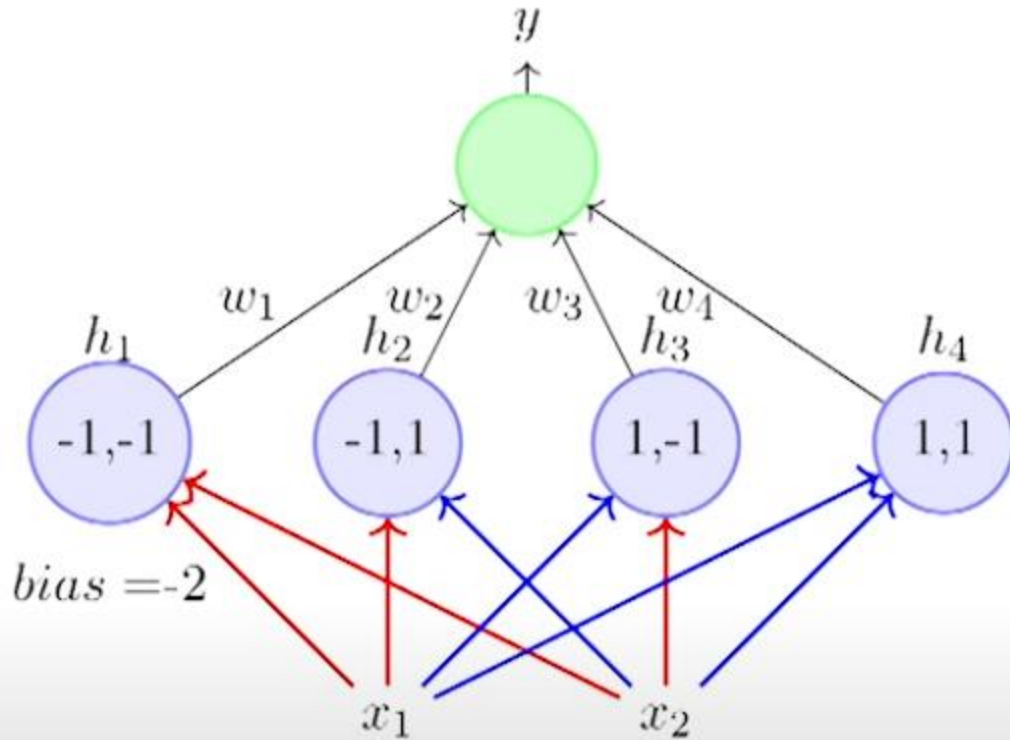


red edge indicates $w = -1$
blue edge indicates $w = +1$

- This network contains 3 layers
- The layer containing the inputs (x_1, x_2) is called the **input layer**
- The middle layer containing the 4 perceptrons is called the **hidden layer**
- The final layer containing one output neuron is called the **output layer**
- The outputs of the 4 perceptrons in the hidden layer are denoted by h_1, h_2, h_3, h_4
- The red and blue edges are called layer 1 weights
- w_1, w_2, w_3, w_4 are called layer 2 weights

Network of Perceptrons

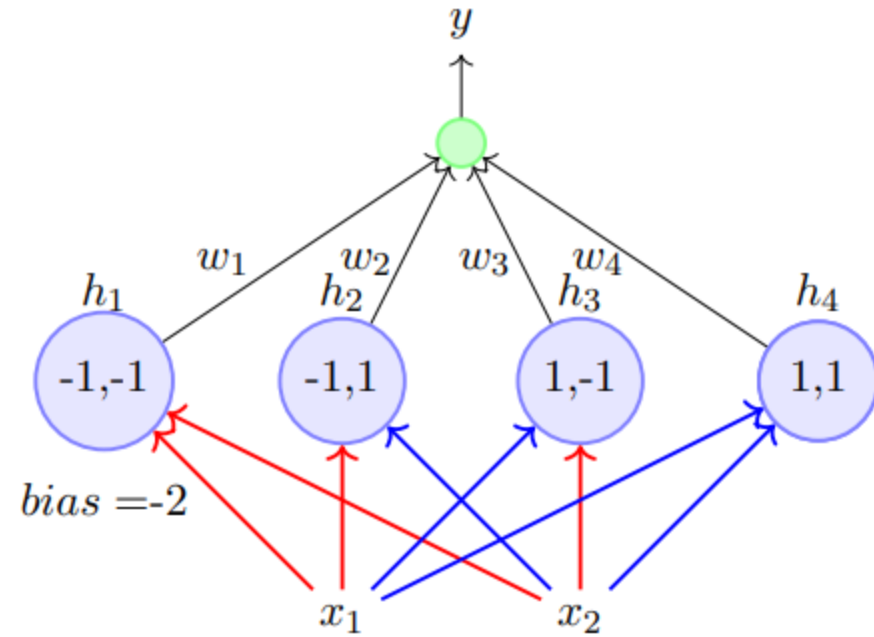
- Let w_0 be the bias output of the neuron (i.e., it will fire if $\sum_{i=1}^4 w_i h_i \geq w_0$)



red edge indicates $w = -1$
blue edge indicates $w = +1$

x_1	x_2	XOR	h_1	h_2	h_3	h_4	$\sum_{i=1}^4 w_i h_i$
0	0	0	1	0	0	0	w_1

Network of Perceptrons



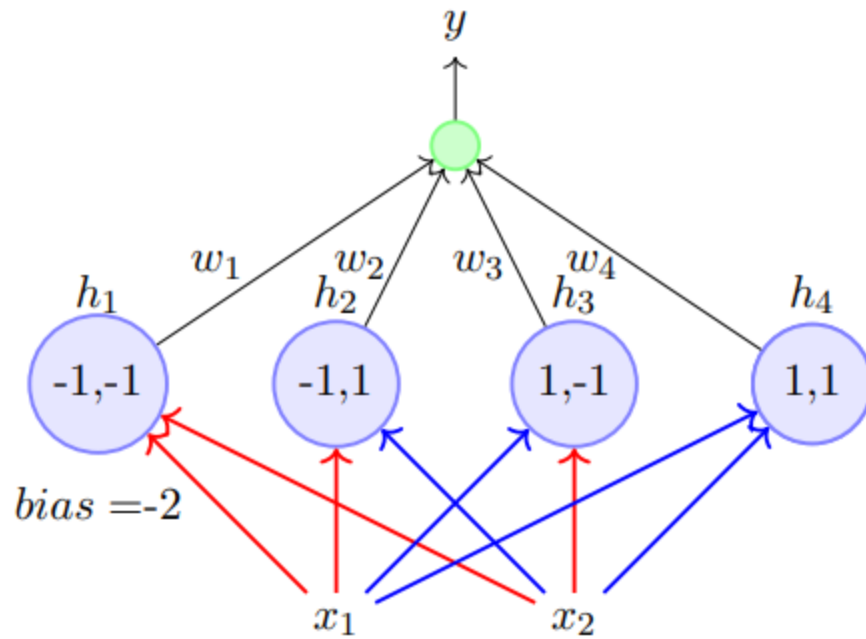
red edge indicates $w = -1$
blue edge indicates $w = +1$

- Let w_0 be the bias output of the neuron (*i.e.*, it will fire if $\sum_{i=1}^4 w_i h_i \geq w_0$)

x_1	x_2	XOR	h_1	h_2	h_3	h_4	$\sum_{i=1}^4 w_i h_i$
0	0	0	1	0	0	0	w_1
0	1	1	0	1	0	0	w_2
1	0	1	0	0	1	0	w_3
1	1	0	0	0	0	1	w_4

- This results in the following four conditions to implement XOR: $w_1 < w_0, w_2 \geq w_0, w_3 \geq w_0, w_4 < w_0$
- Unlike before, there are no contradictions now and the system of inequalities can be satisfied
- Essentially each w_i is now responsible for one of the 4 possible inputs and can be adjusted to get the desired output for that input

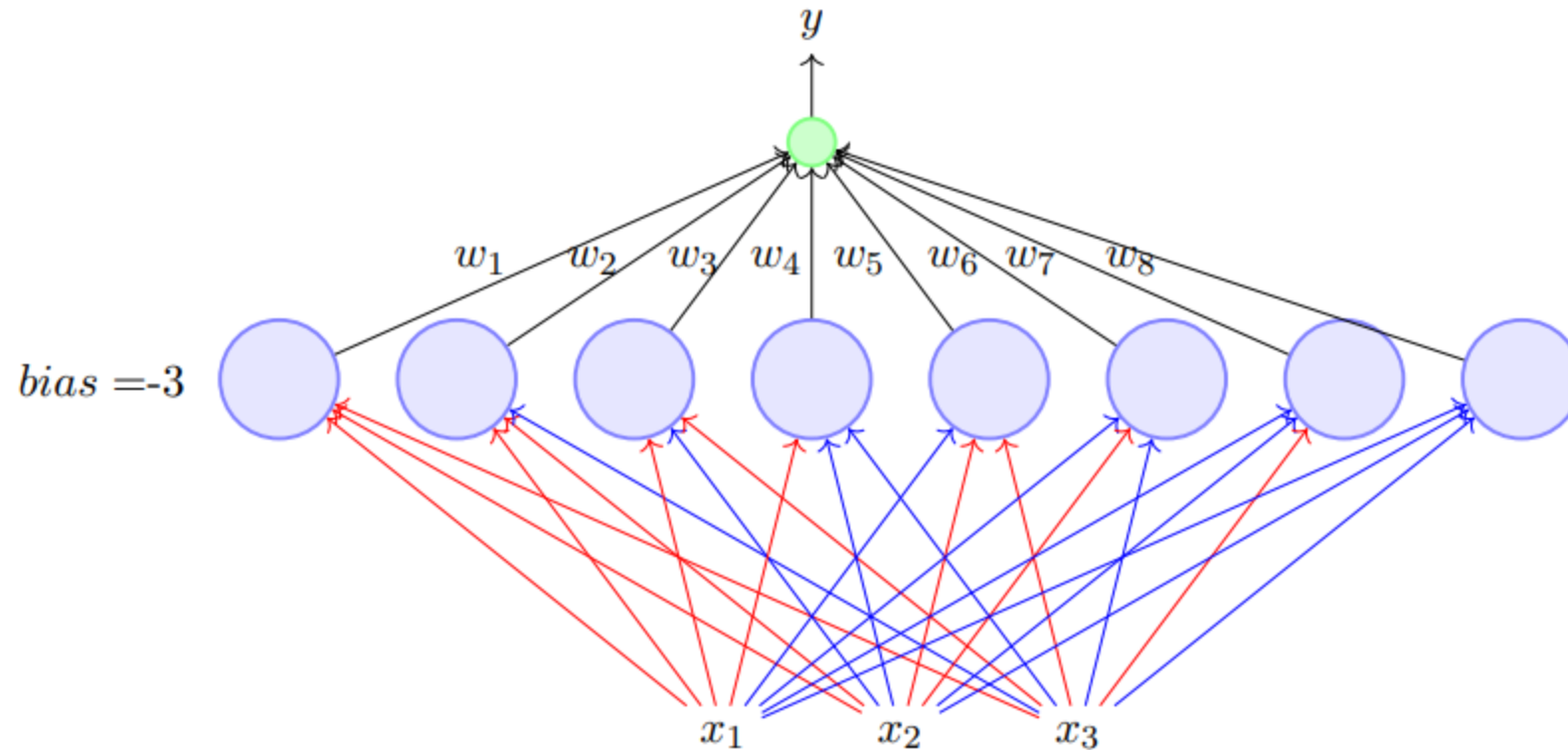
Network of Perceptrons



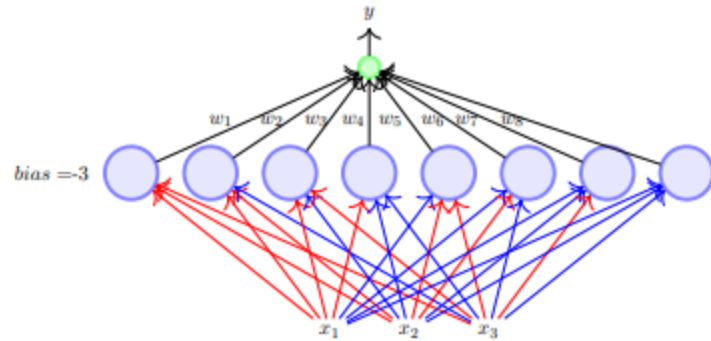
- It should be clear that the same network can be used to represent the remaining 15 boolean functions also
- Each boolean function will result in a different set of non-contradicting inequalities which can be satisfied by appropriately setting w_1, w_2, w_3, w_4

more than 3 inputs

- Again each of the 8 perceptrons will fire only for one of the 8 inputs
- Each of the 8 weights in the second layer is responsible for one of the 8 inputs and can be adjusted to produce the desired output for that input



How to solve real world problem?



$$\begin{array}{l}
 p_1 \\
 p_2 \\
 \vdots \\
 n_1 \\
 n_2 \\
 \vdots
 \end{array}
 \begin{bmatrix}
 x_{11} & x_{12} & \dots & x_{1n} & y_1 = 1 \\
 x_{21} & x_{22} & \dots & x_{2n} & y_2 = 1 \\
 \vdots & \vdots & \vdots & \vdots & \vdots \\
 x_{k1} & x_{k2} & \dots & x_{kn} & y_i = 0 \\
 x_{j1} & x_{j2} & \dots & x_{jn} & y_j = 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots
 \end{bmatrix}$$

n inputs

Theorem

Any boolean function of n inputs can be represented exactly by a network of perceptrons containing 1 hidden layer with 2^n perceptrons and one output layer containing 1 perceptron

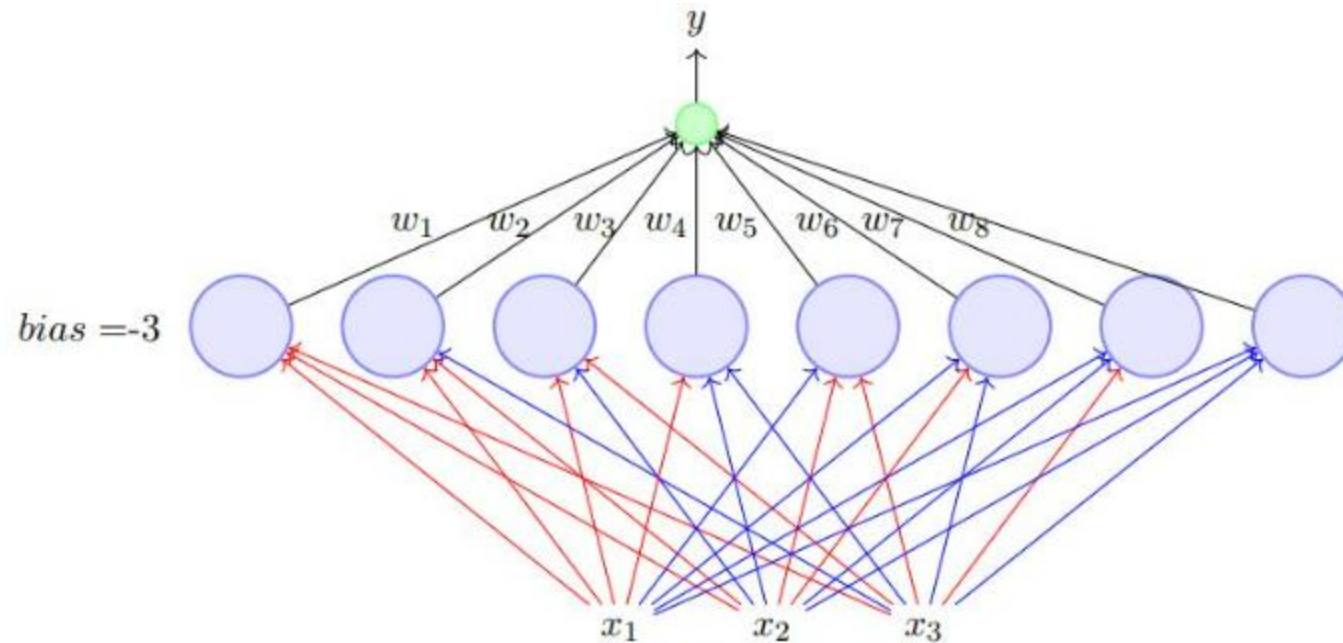
Proof (informal:) We just saw how to construct such a network

Note: A network of $2^n + 1$ perceptrons is not necessary but sufficient. For example, we already saw how to represent AND function with just 1 perceptron

Catch: As n increases the number of perceptrons in the hidden layers obviously increases exponentially

Last Lecture

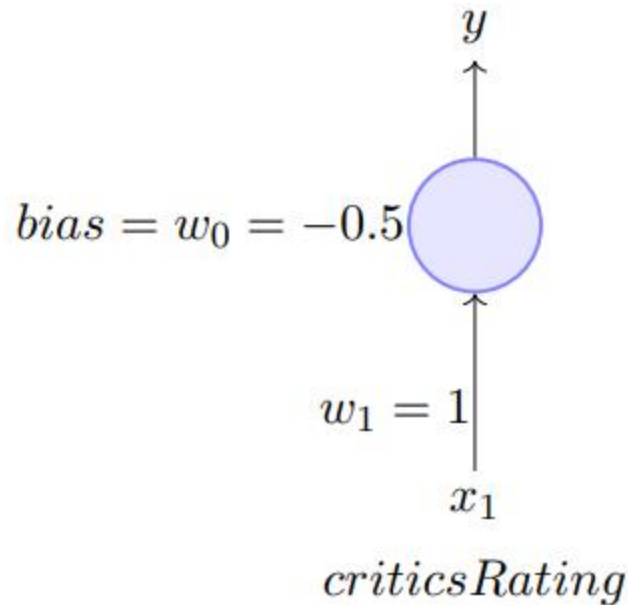
- MLP
- Any boolean function can be represented using an MLP



Sigmoid Neuron

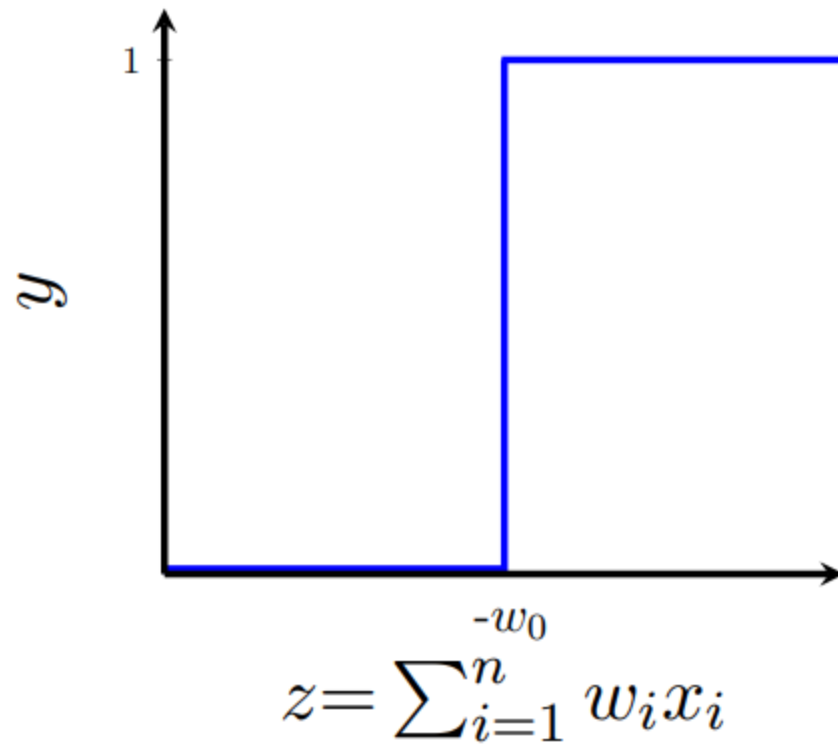
- Negatives of thresholding logic in perceptron
- How sigmoid neuron overcomes this limitation?
- Learning algorithm of sigmoid neuron
- How sigmoid neurons represents arbitrary functions?

Thresholding Logic of Perceptron



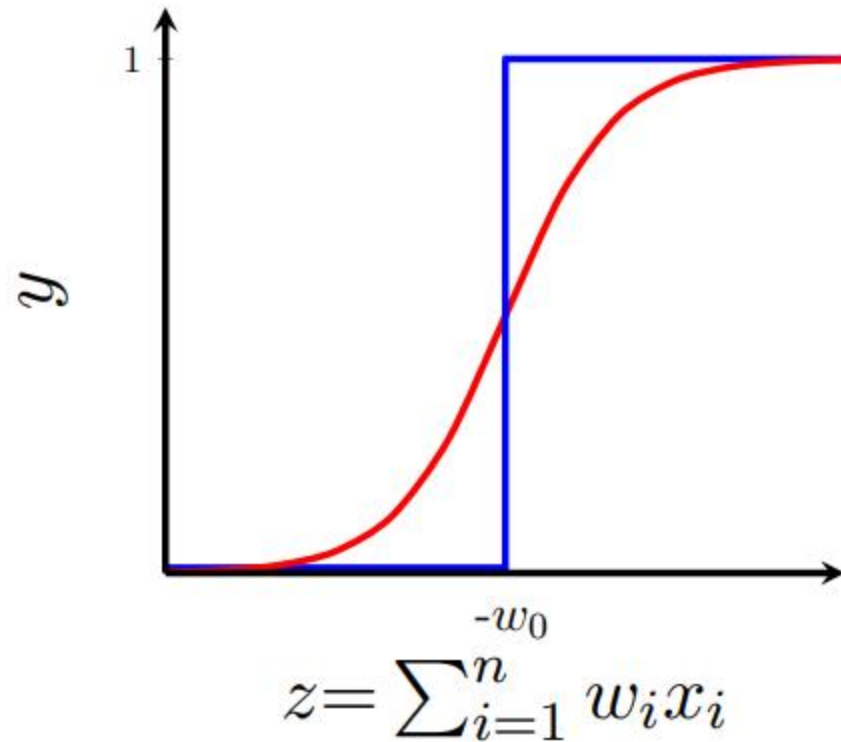
- The thresholding logic used by a perceptron is very harsh !
- For example, let us return to our problem of deciding whether we will like or dislike a movie
- Consider that we base our decision only on one input ($x_1 = criticsRating$ which lies between 0 and 1)
- If the threshold is 0.5 ($w_0 = -0.5$) and $w_1 = 1$ then what would be the decision for a movie with $criticsRating = 0.51$? (like)
- What about a movie with $criticsRating = 0.49$? (dislike)
- It seems harsh that we would like a movie with rating 0.51 but not one with a rating of 0.49

Thresholding Logic of Perceptron



- This behavior is not a characteristic of the specific problem we chose or the specific weight and threshold that we chose
- It is a characteristic of the perceptron function itself which behaves like a step function
- There will always be this sudden change in the decision (from 0 to 1) when $\sum_{i=1}^n w_i x_i$ crosses the threshold ($-w_0$)
- For most real world applications we would expect a smoother decision function which gradually changes from 0 to 1

Sigmoid Neuron



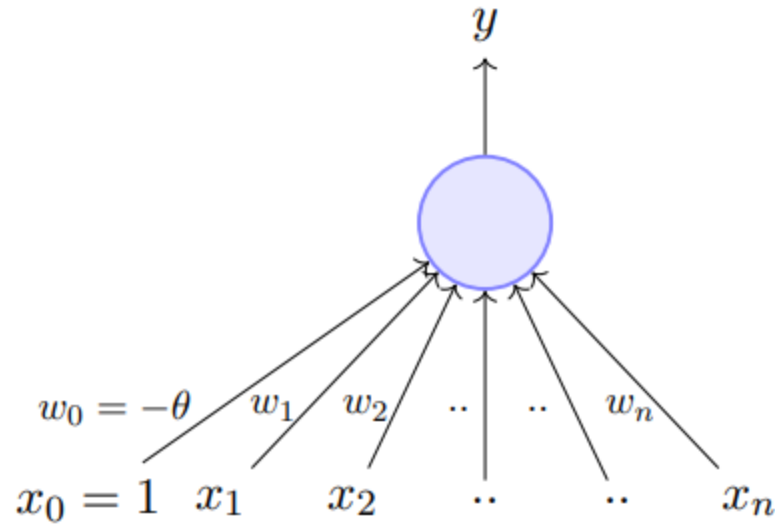
- Introducing sigmoid neurons where the output function is much smoother than the step function
- Here is one form of the sigmoid function called the logistic function

$$y = \frac{1}{1 + e^{-(w_0 + \sum_{i=1}^n w_i x_i)}}$$

- We no longer see a sharp transition around the threshold $-w_0$
- Also the output y is no longer binary but a real value between 0 and 1 which can be interpreted as a probability
- Instead of a like/dislike decision we get the probability of liking the movie

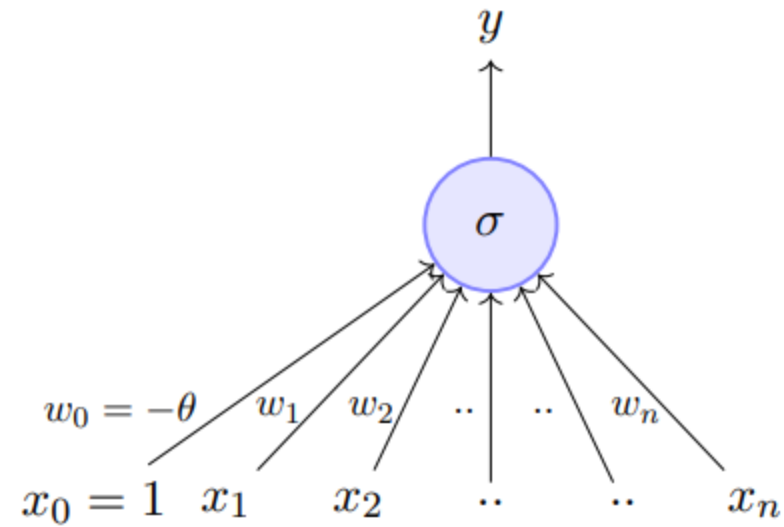
Perceptron Vs Sigmoid

Perceptron



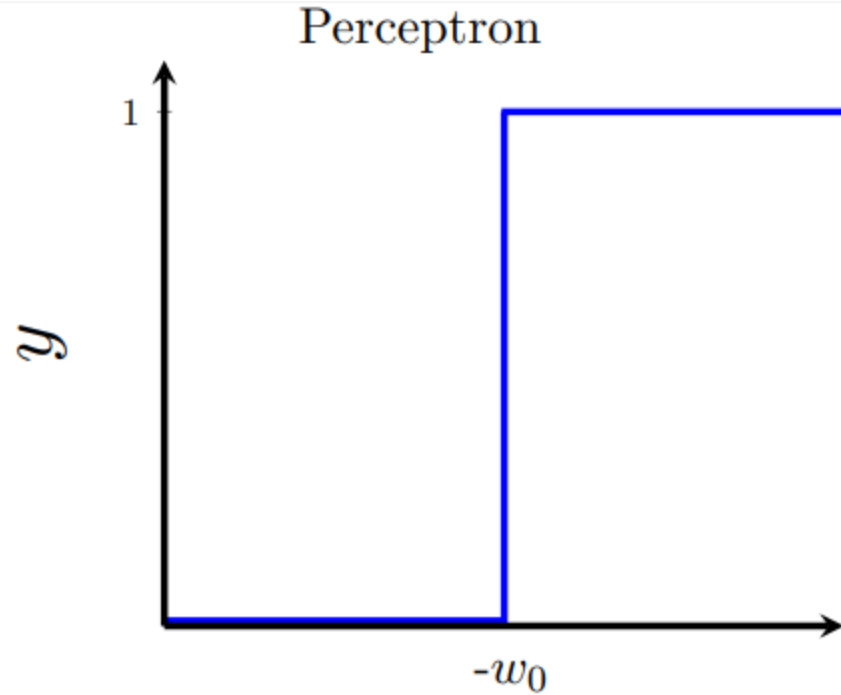
$$y = 1 \quad \text{if } \sum_{i=0}^n w_i * x_i \geq 0$$
$$= 0 \quad \text{if } \sum_{i=0}^n w_i * x_i < 0$$

Sigmoid (logistic) Neuron



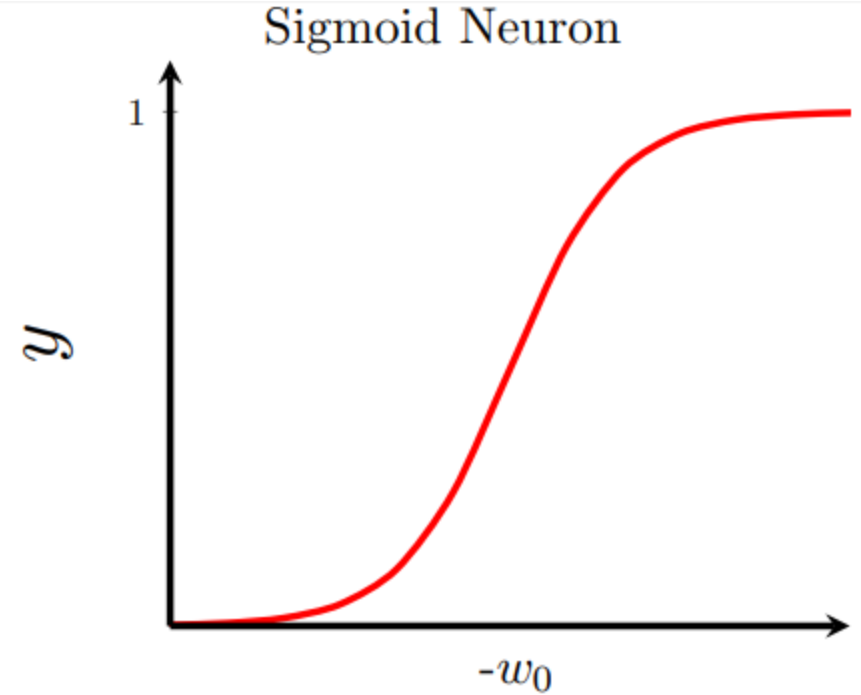
$$y = \frac{1}{1 + e^{-(\sum_{i=0}^n w_i x_i)}}$$

Perceptron Vs Sigmoid



$$z = \sum_{i=1}^n w_i x_i$$

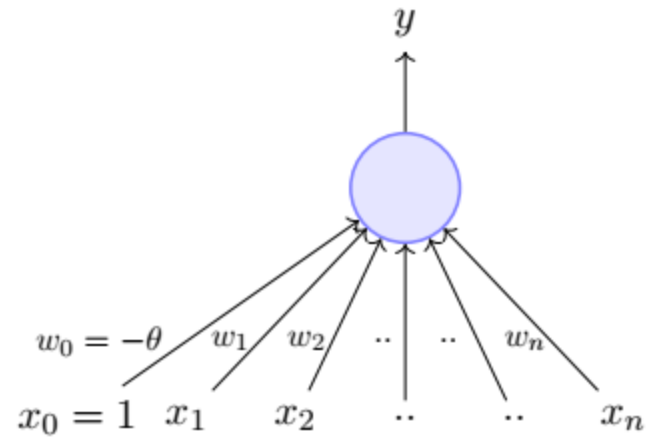
Not smooth, not continuous (at w_0), **not**
differentiable



$$z = \sum_{i=1}^n w_i x_i$$

Smooth, continuous, **differentiable**

How do we learn weights of Sigmoid Neuron?



Learning Setup

- **Data:** $\{x_i, y_i\}_{i=1}^n$
- **Model:** Our approximation of the relation between \mathbf{x} and y . For example,

$$\hat{y} = \frac{1}{1 + e^{-(\mathbf{w}^T \mathbf{x})}}$$

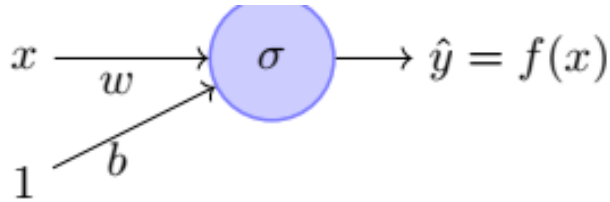
or $\hat{y} = \mathbf{w}^T \mathbf{x}$

or $\hat{y} = \mathbf{x}^T \mathbf{W} \mathbf{x}$

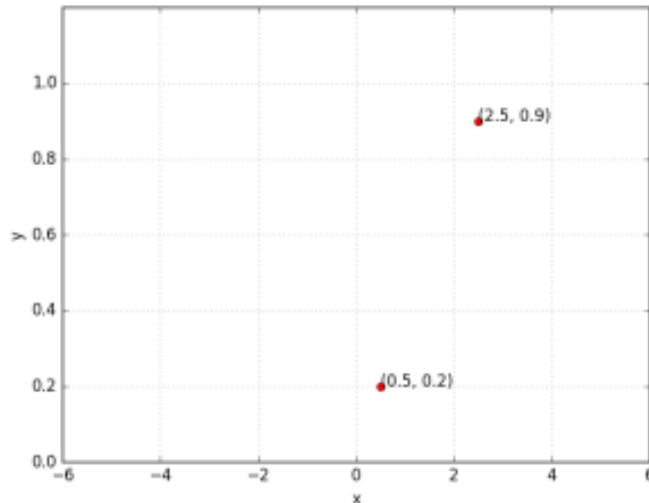
or just about any function

- **Parameters:** In all the above cases, w is a parameter which needs to be learned from the data
- **Learning algorithm:** An algorithm for learning the parameters (w) of the model (for example, perceptron learning algorithm, gradient descent, etc.)
- **Objective/Loss/Error function:** To guide the learning algorithm - the learning algorithm should aim to minimize the loss function

Example



$$f(x) = \frac{1}{1 + e^{-(w \cdot x + b)}}$$



Input for training

$\{x_i, y_i\}_{i=1}^N \rightarrow N$ pairs of (x, y)

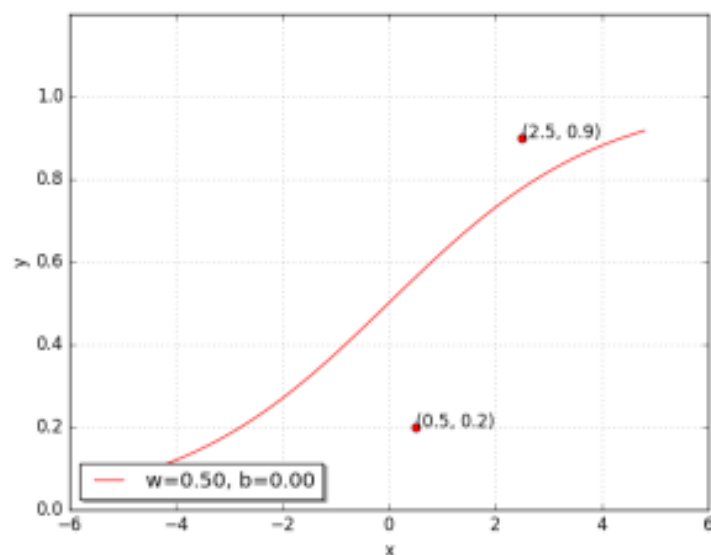
Training objective

Find w and b such that:

$$\underset{w, b}{\text{minimize}} \mathcal{L}(w, b) = \sum_{i=1}^N (y_i - f(x_i))^2$$

What does it mean to train the network?

- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$
- At the end of training we expect to find w^* , b^* such that:
- $f(0.5) \rightarrow 0.2$ and $f(2.5) \rightarrow 0.9$



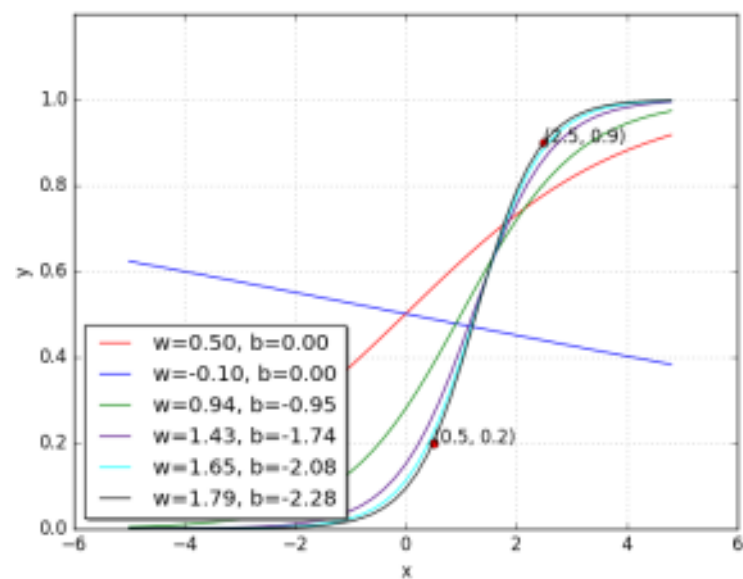
$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

- Can we try to find such a w^*, b^* manually
- Let us try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?
- Let us revisit $\mathcal{L}(w, b)$ to see how bad it is ...

$$\begin{aligned} \mathcal{L}(w, b) &= \frac{1}{2} * \sum_{i=1}^N (y_i - f(x_i))^2 \\ &= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\ &= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2 \\ &= 0.073 \end{aligned}$$

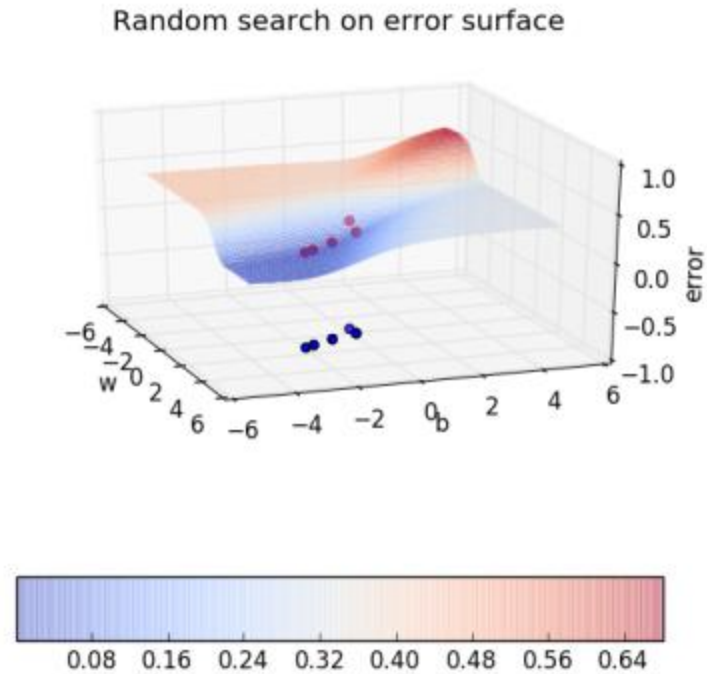
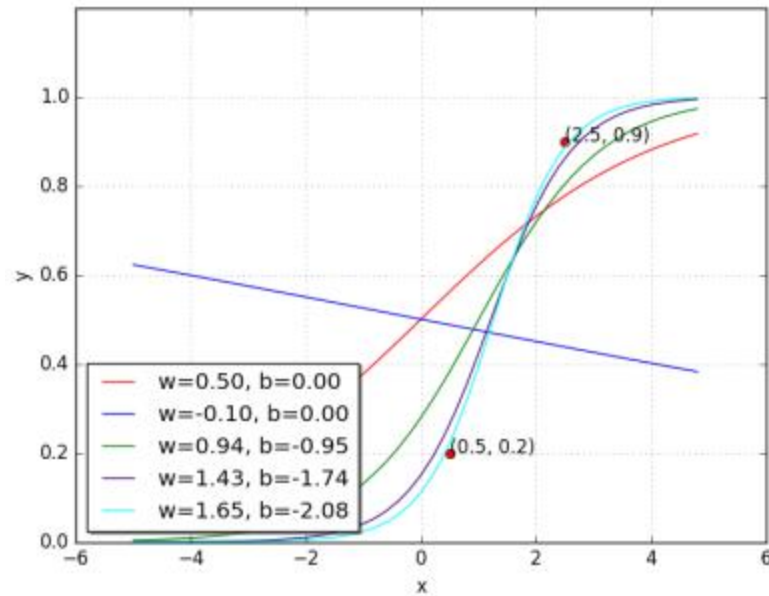
We want $\mathcal{L}(w, b)$ to be as close to 0 as possible

Let us try some other values of w , b



w	b	$\mathcal{L}(w, b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028
1.65	-2.08	0.0003
1.78	-2.27	0.0000

Error Surface



Gradient Descent

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

Algorithm: `gradient_descent()`

$t \leftarrow 0;$

$max_iterations \leftarrow 1000;$

while $t < max_iterations$ **do**

$w_{t+1} \leftarrow w_t - \eta \nabla w_t;$

$b_{t+1} \leftarrow b_t - \eta \nabla b_t;$

$t \leftarrow t + 1;$

end

Multi Layer Network of Sigmoid Neurons

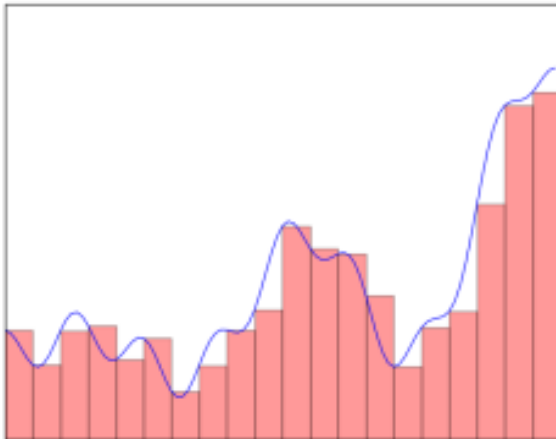
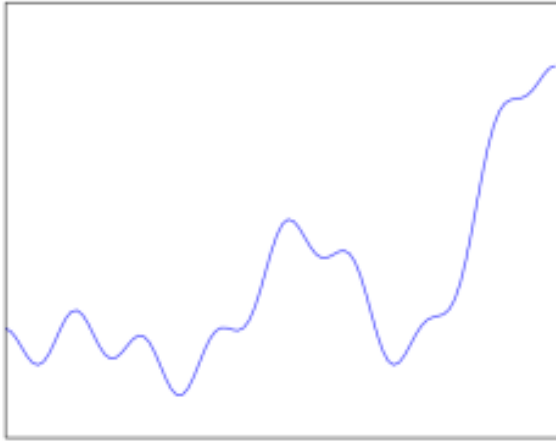
Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

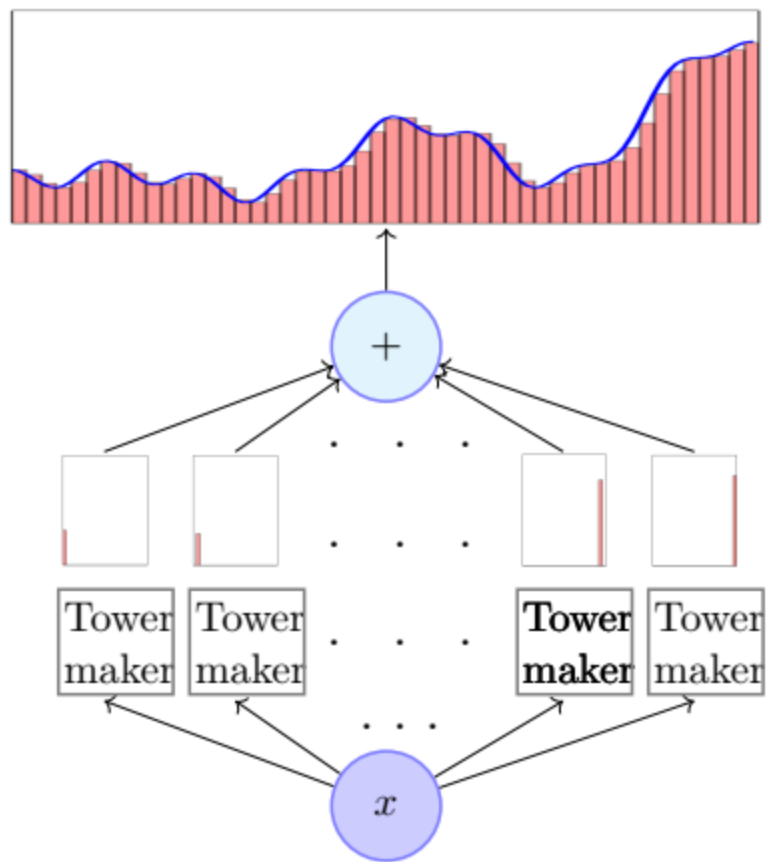
Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision

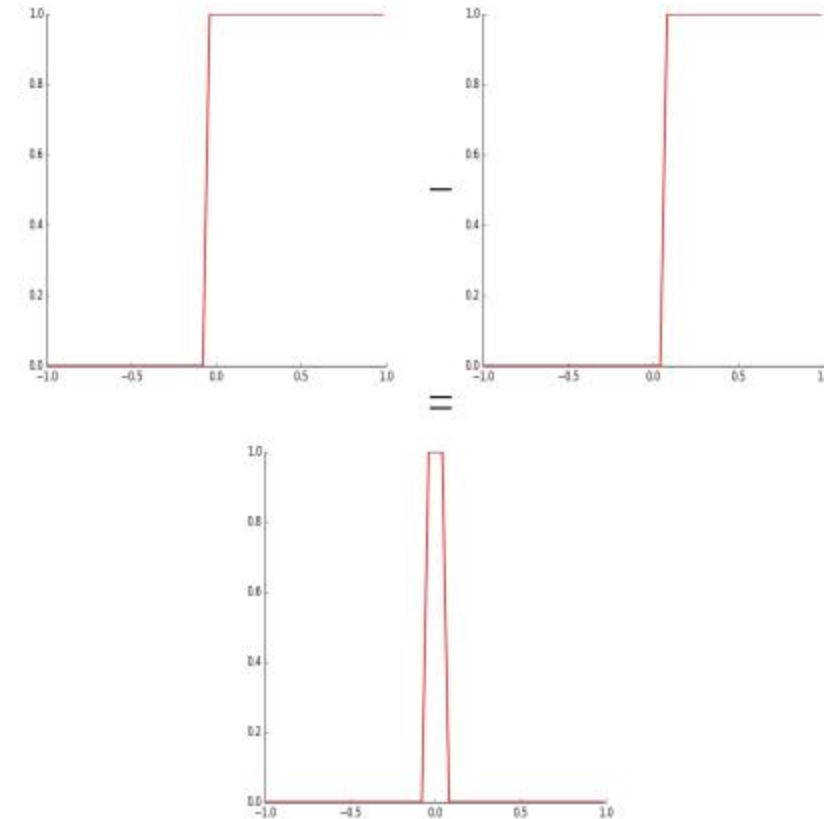
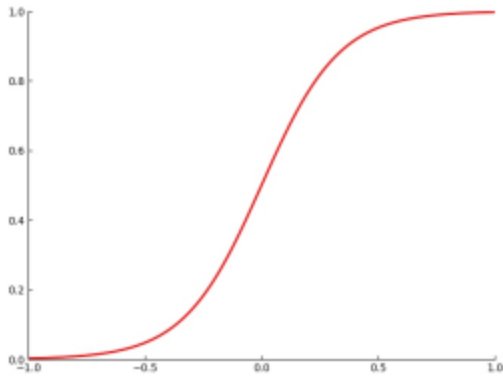
How to represent an arbitrary function?

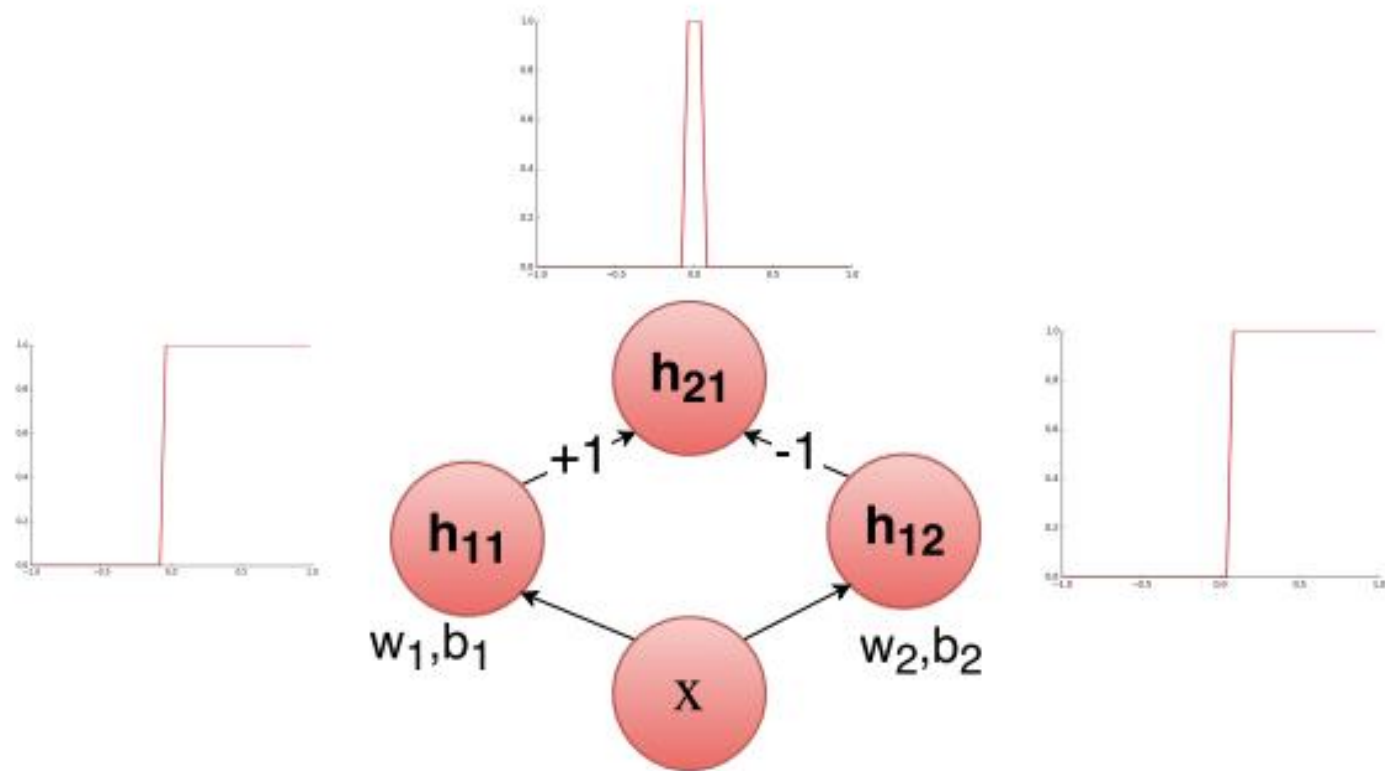


- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)
- We observe that such an arbitrary function can be approximated by several “tower” functions
- More the number of such “tower” functions, better the approximation
- To be more precise, we can approximate any arbitrary function by a sum of such “tower” functions



How do we generate tower functions using sigmoid neuron?





2 inputs

