Tutorial 1

MATLAB Tutorial on Common Probability Distributions and Theorems

In this section, we'll discuss how to work with different probability distributions and concepts using MATLAB, including binomial, Gaussian, Poisson, Bernoulli distributions, and fundamental functions like PMF, PDF, and CDF. We'll also explore Bayes' Theorem and its practical implementation.

---

1. Binomial Distribution

The binomial distribution models the number of successes in a fixed number of independent Bernoulli trials.

Probability Mass Function (PMF)

The PMF of a binomial distribution gives the probability of having exactly k successes in n trials, each with success probability p.

Example Problem: Calculate PMF and Plot

Matlab code

```
n = 10; % Number of trials
p = 0.5; % Probability of success
k = 0:n; % Possible number of successes

% Calculate PMF using binopdf
pmf = binopdf(k, n, p);

% Plot the PMF
stem(k, pmf, 'filled');
title('Binomial Distribution PMF');
xlabel('Number of Successes (k)');
ylabel('Probability');
```

---

2. Gaussian (Normal) Distribution

The Gaussian distribution is characterized by its mean μ and standard deviation σ.

Probability Density Function (PDF) and Cumulative Distribution Function (CDF)

- The PDF describes the likelihood of a continuous random variable taking a specific value.
- The CDF gives the probability that the random variable is less than or equal to a certain value.

Example Problem: Plot PDF and CDF

matlab

```
mu = 0; % Mean
sigma = 1; % Standard deviation
x = -5:0.1:5; % Range of values

% Calculate PDF and CDF
pdf = normpdf(x, mu, sigma);
cdf = normcdf(x, mu, sigma);

% Plot PDF
figure;
plot(x, pdf);
title('Gaussian Distribution PDF');
xlabel('x');
ylabel('Probability Density');

% Plot CDF
figure;
plot(x, cdf);
title('Gaussian Distribution CDF');
xlabel('x');
ylabel('Cumulative Probability');
```

---

3. Poisson Distribution

The Poisson distribution models the number of events occurring in a fixed interval of time or space, given the average rate λ.

Example Problem: Calculate PMF and Plot

matlab

```
lambda = 5; % Average rates of events
k = 0:15; % Number of events

% Calculate PMF using poisspdf
pmf = poisspdf(k, lambda);

% Plot the PMF
stem(k, pmf, 'filled');
title('Poisson Distribution PMF');
xlabel('Number of Events (k)');
ylabel('Probability');
```

---

4. Bernoulli Distribution

The Bernoulli distribution is the simplest discrete distribution, modeling a random experiment with only two outcomes: success (1) and failure (0).

Example Problem: Simulate a Bernoulli Trial

matlab

```
p = 0.7; % Probability of success
N = 1000; % Number of trials

% Generate Bernoulli random variables
bernoulli_trials = rand(N, 1) < p;

% Calculate and display the proportion of successes
successes = sum(bernoulli_trials);
disp(['Proportion of Successes: ', num2str(successes / N)]);
```

---

5. PMF, PDF, and CDF Overview

- **PMF (Probability Mass Function)**: Used for discrete random variables. Example: binopdf() for binomial, poisspdf() for Poisson.
- **PDF (Probability Density Function)**: Used for continuous random variables. Example: normpdf() for Gaussian.
- **CDF (Cumulative Distribution Function)**: Used for both discrete and continuous random variables to describe the cumulative probability. Example: binocdf(), poisscdf(), normcdf().

Example: Compare PMF, PDF, and CDF

matlab

```
% Binomial Distribution Example
n = 10; p = 0.5; k = 0:n;
pmf = binopdf(k, n, p);
cdf = binocdf(k, n, p);

figure;
subplot(1, 2, 1);
stem(k, pmf, 'filled');
title('Binomial PMF');
xlabel('k');
ylabel('Probability');

subplot(1, 2, 2);
stairs(k, cdf);
title('Binomial CDF');
xlabel('k');
ylabel('Cumulative Probability');
```

---

6. Bayes' Theorem

Bayes' Theorem is used to update the probability of a hypothesis given new evidence. The formula is:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Example Problem: Applying Bayes' Theorem

Consider a medical test:

- $P(\text{Disease}) = 0.01$
- $P(\text{Positive Test}|\text{Disease}) = 0.95$
- $P(\text{Positive Test}|\text{No Disease}) = 0.1$

We want to find $P(\text{Disease}|\text{Positive Test})$.

Implementation

matlab

```
P_Disease = 0.01;
P_Positive_given_Disease = 0.95;
P_Positive_given_NoDisease = 0.1;
P_NoDisease = 1 - P_Disease;

% Calculate P(Positive)
P_Positive = P_Positive_given_Disease * P_Disease +
P_Positive_given_NoDisease * P_NoDisease;

% Apply Bayes' Theorem
P_Disease_given_Positive = (P_Positive_given_Disease * P_Disease) / P_Positive;

disp(['Probability of Disease given a Positive Test: ',
num2str(P_Disease_given_Positive)]);
```