22AIE114 : Introduction to Electrical and Electronic Engineering (2-0-3-3)

Unit 1, Lecture 4

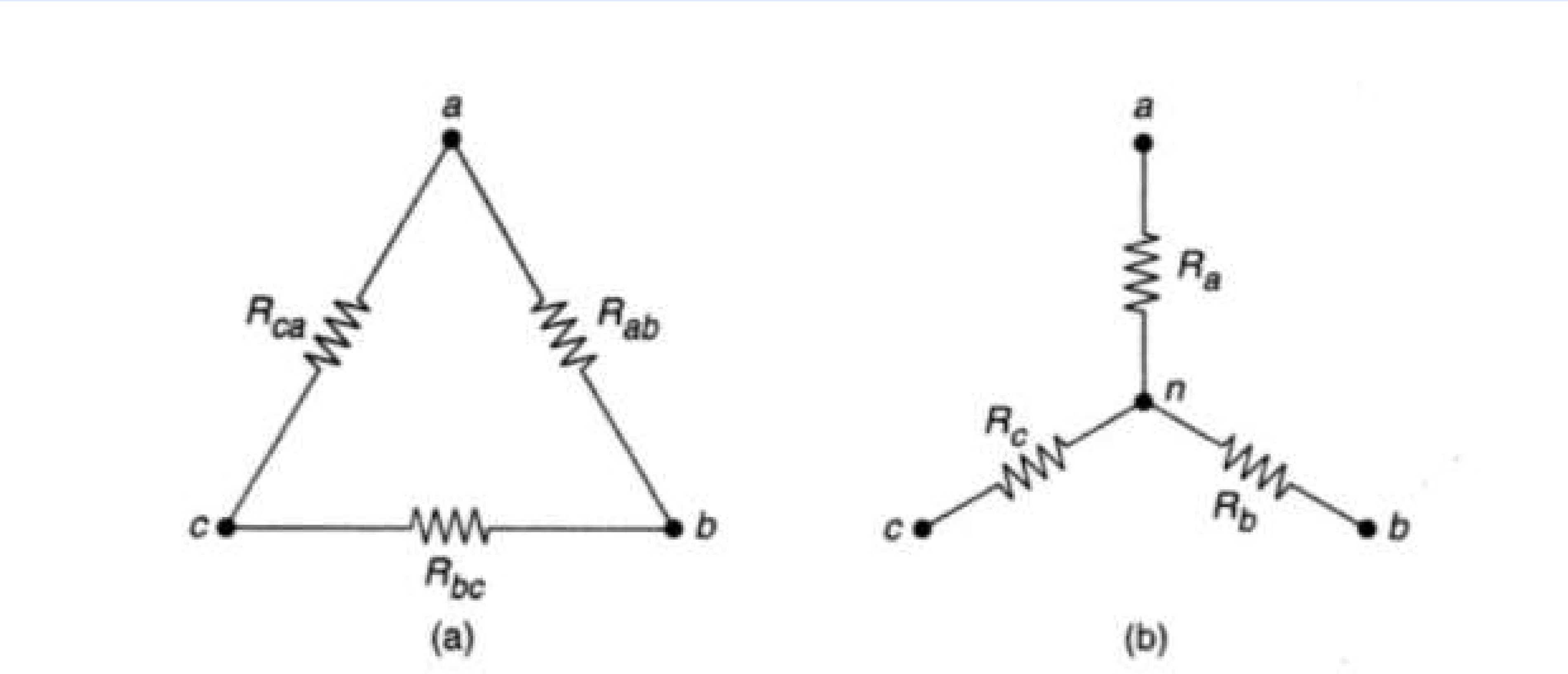
Topic: Star Delta Transformation and Series – parallel capacitors, inductors

Outline

- To understand star-delta transformations.
- To understand series and parallel combinations of capacitors
- To understand series and parallel combinations of Inductors
- To perform network reductions and solving circuits.

Star (Y) - Delta Transformation

STAR (Y)–DELTA (Δ) CONVERSION



(a) Delta-connected and (b) Star-connected networks

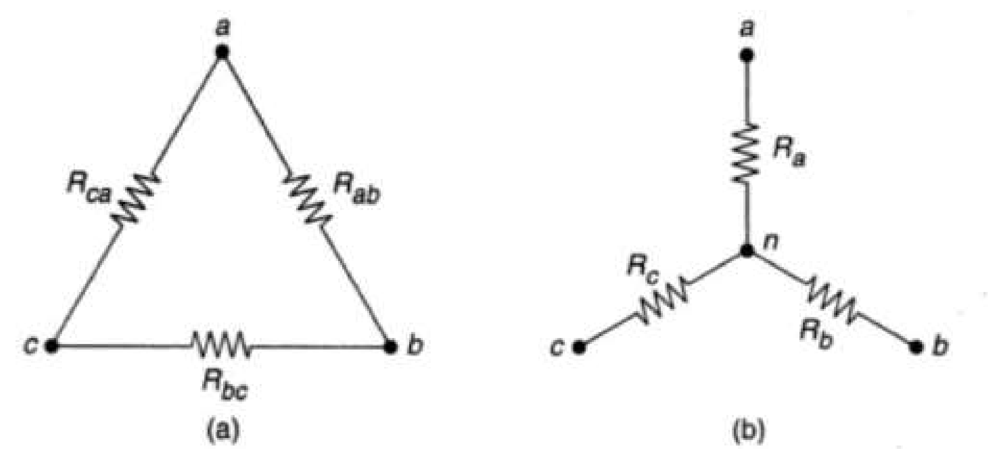
Star - Delta Transformation

Certain network configurations cannot be resolved by series–parallel combinations alone. Such configurations are handled by Y–Δ *transformations*.

Figure (a) shows three Δ–connected resistances connected between three nodes a, b and c.

Fig. (b), there are three Y-connected resistances.
Y-connection has an extra node n that gets eliminated upon converting it to Δ.
Y–Δ conversion is therefore a *node reduction* technique.

Star - Delta Transformation



(a) Delta-connected and (b) Star-connected networks

Equating resistance between node pairs:

Node pair *ab*

$$R_a + R_b = R_{ab} \parallel (R_{bc} + R_{ca})$$

Node pair *bc*

$$R_b + R_c = R_{bc} \parallel (R_{ca} + R_{ab})$$

Node pair *ca*

$$R_c + R_a = R_{ca} \parallel (R_{ab} + R_{bc})$$

Star - Delta Transformation

Solving above 3 equations, we get:

Y–Δ Conversion

$$R_{ab} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_{bc} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_{ca} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

Δ–Y Conversion

$$R_a = \frac{R_{ab} R_{ac}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_b = \frac{R_{bc} R_{ba}}{R_{ab} + R_{bc} + R_{ca}}$$

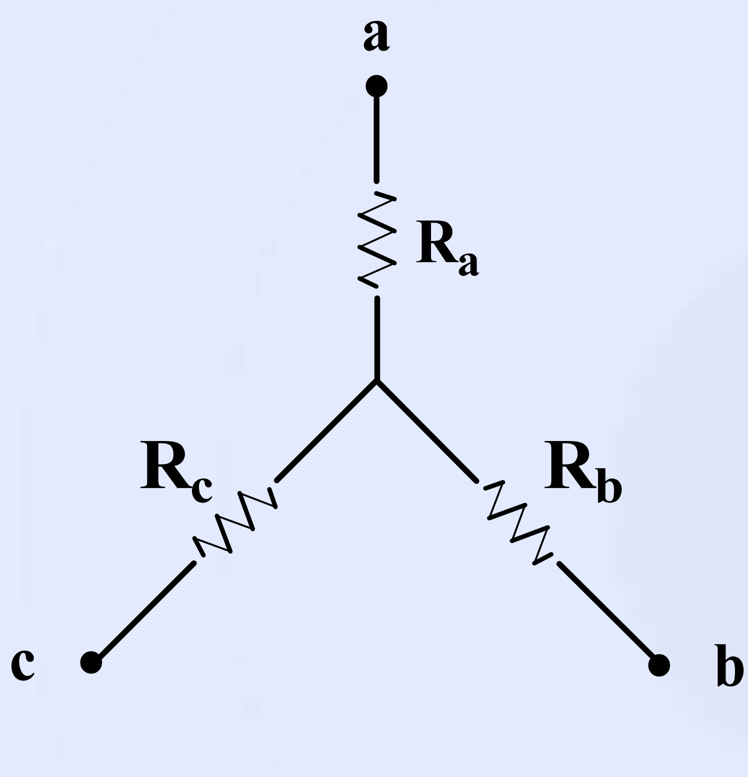
$$R_c = \frac{R_{ca} R_{cb}}{R_{ab} + R_{bc} + R_{ca}}$$

Balanced Y–Δ

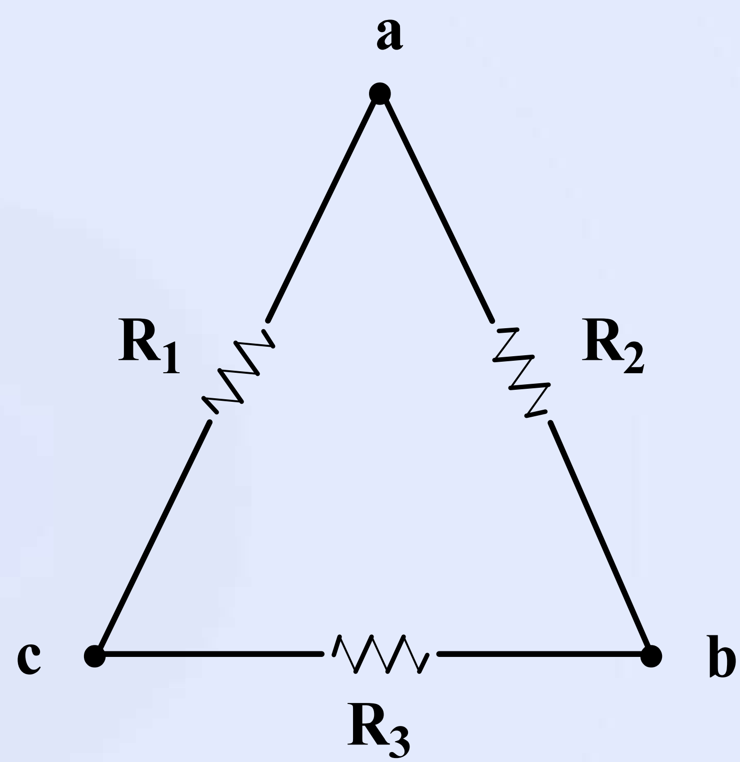
A balanced Y ($R_a = R_b = R_c = R_Y$) leads to balanced Δ ($R_{ab} = R_{bc} = R_{ca} = R_\Delta$) wherein

$$R_\Delta = 3R_Y$$

Star - Delta Transformation



(a) wye configuration



(b) delta configuration

Star - Delta Transformation

Go to wye

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

Go to delta

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

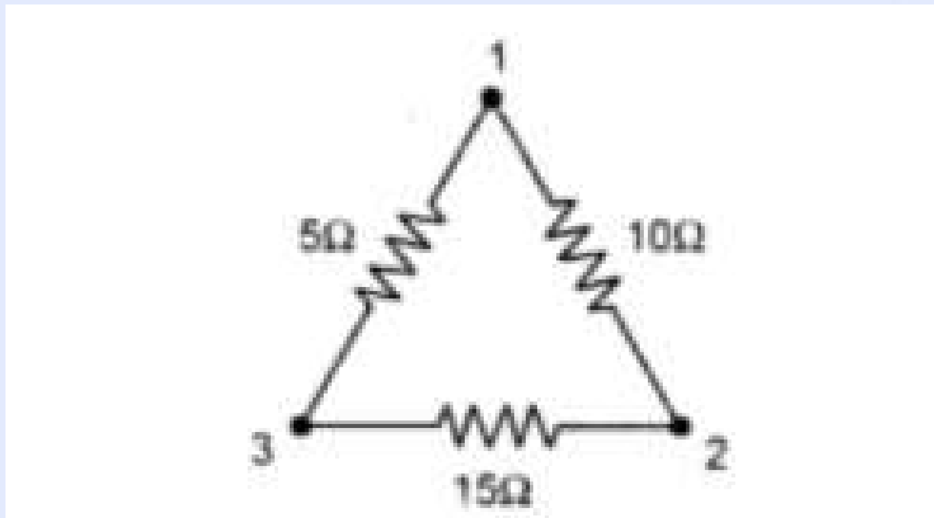
$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

We note that the denominator for R_a, R_b, R_c is the same.
We note that the numerator for R_1, R_2, R_3 is the same.

Example 1

Convert given delta circuit to equivalent star

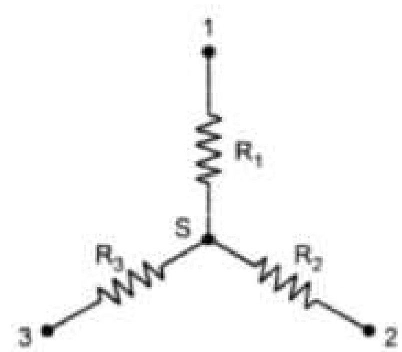


Solution :

$$R_1 = \frac{10 \times 5}{5+10+15} = 1.67\ \Omega$$

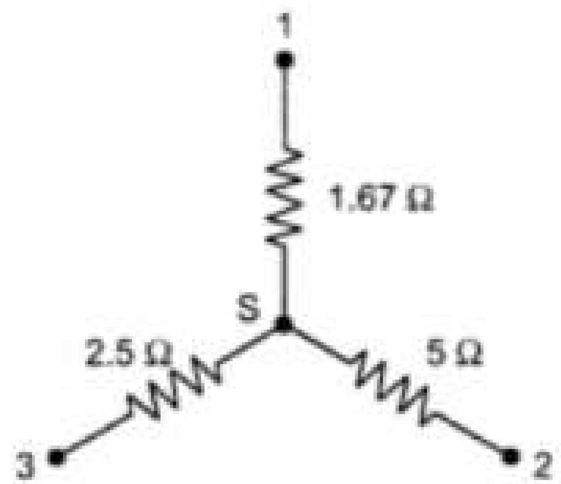
$$R_2 = \frac{15 \times 10}{5+10+15} = 5\ \Omega$$

$$R_3 = \frac{5 \times 15}{5+10+15} = 2.5\ \Omega$$

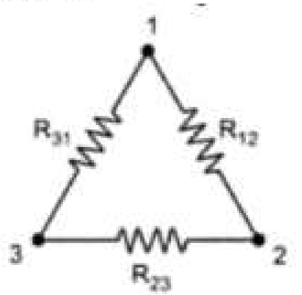


Example 2

Convert given Star circuit to equivalent Delta



Solution :



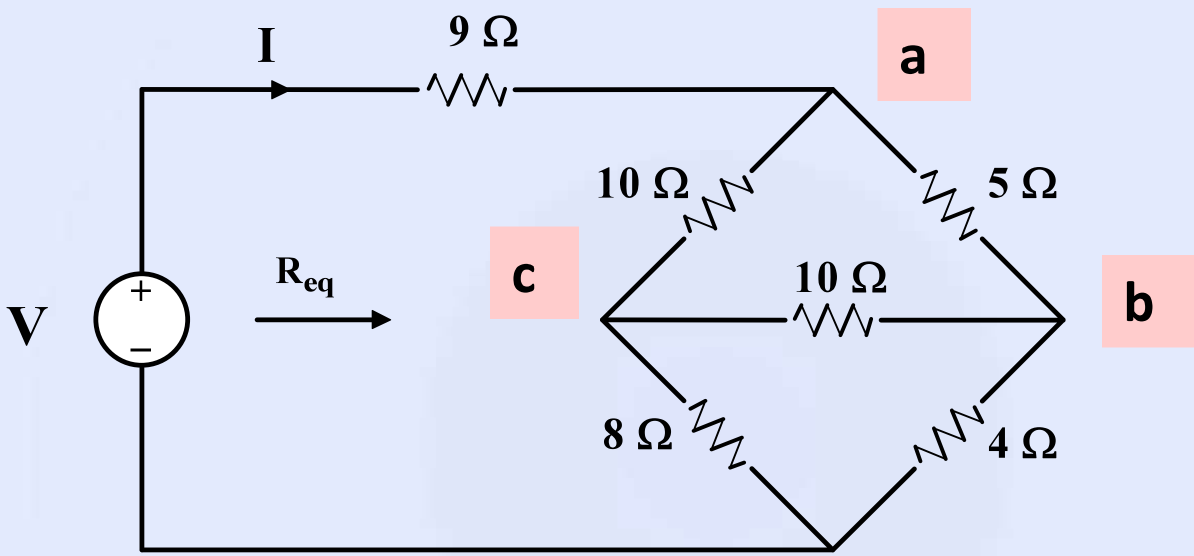
$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = 10\ \Omega$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = 15\ \Omega$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833 = 5\ \Omega$$

Example 3

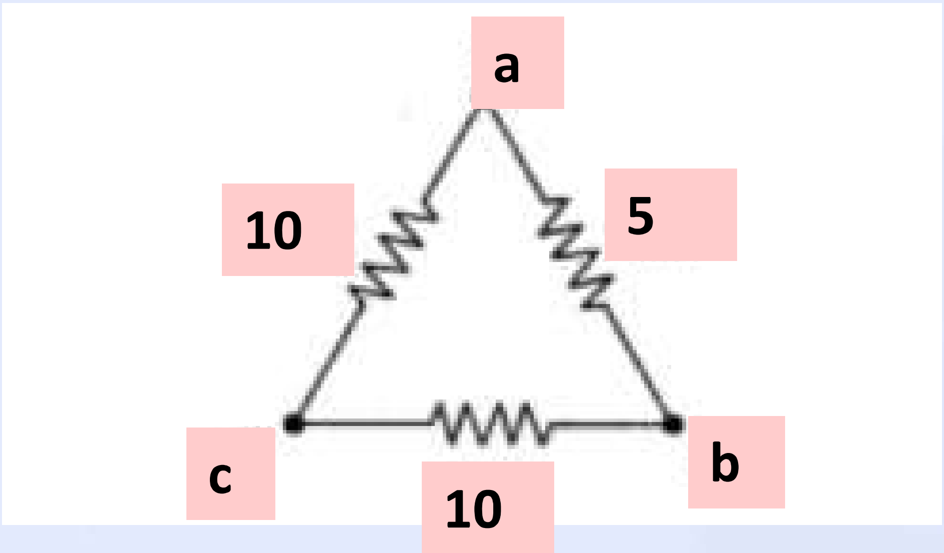
Find R_{eq} of the circuit in the given Figure



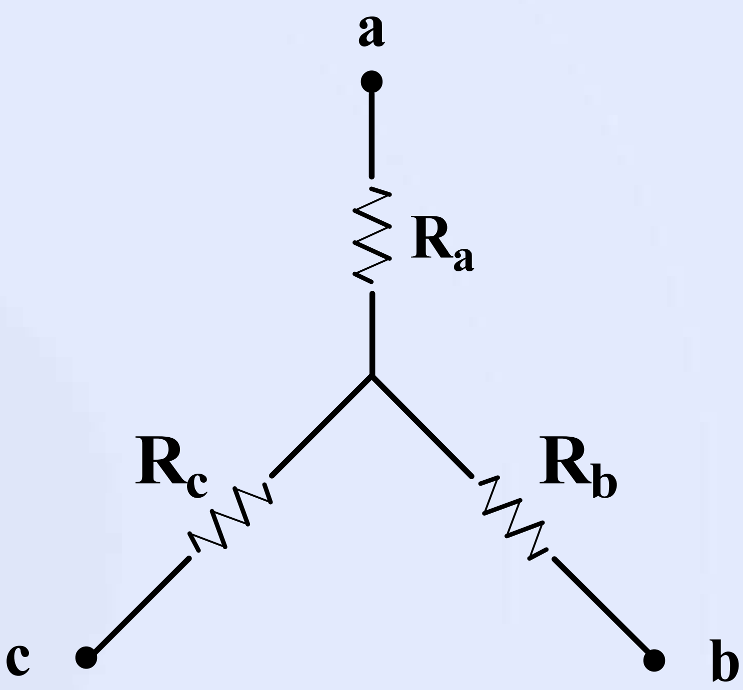
Convert the delta around a – b – c to a wye.

Example 3

Find R_{eq} of the circuit in the given Figure



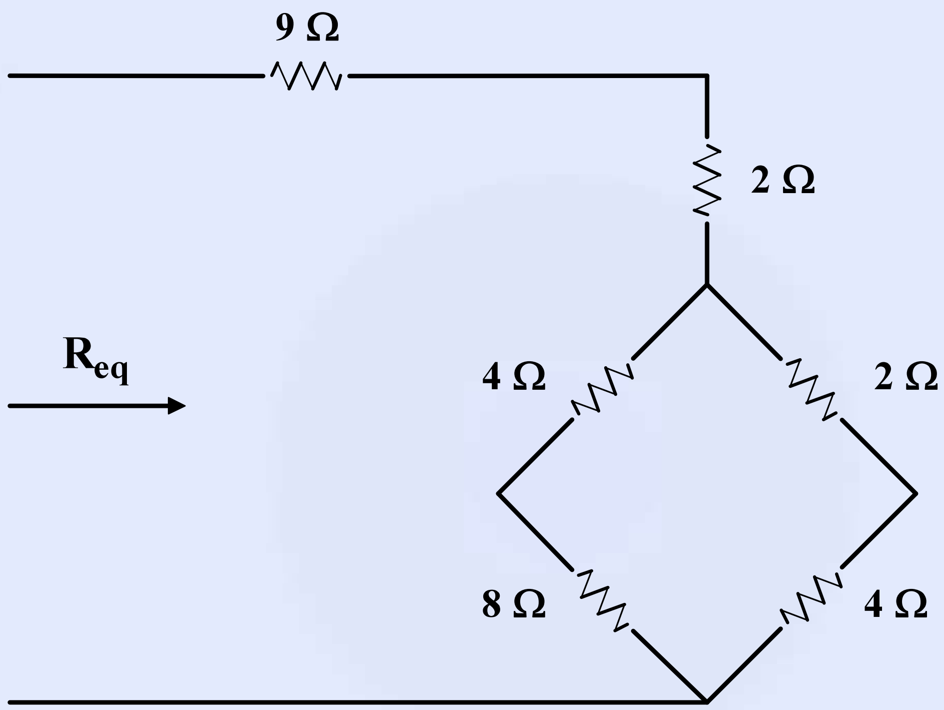
Convert the delta around a – b – c to a wye.



Ra = 2 Ohms
Rb =2 Ohms
Rc =4 Ohms

Example 3

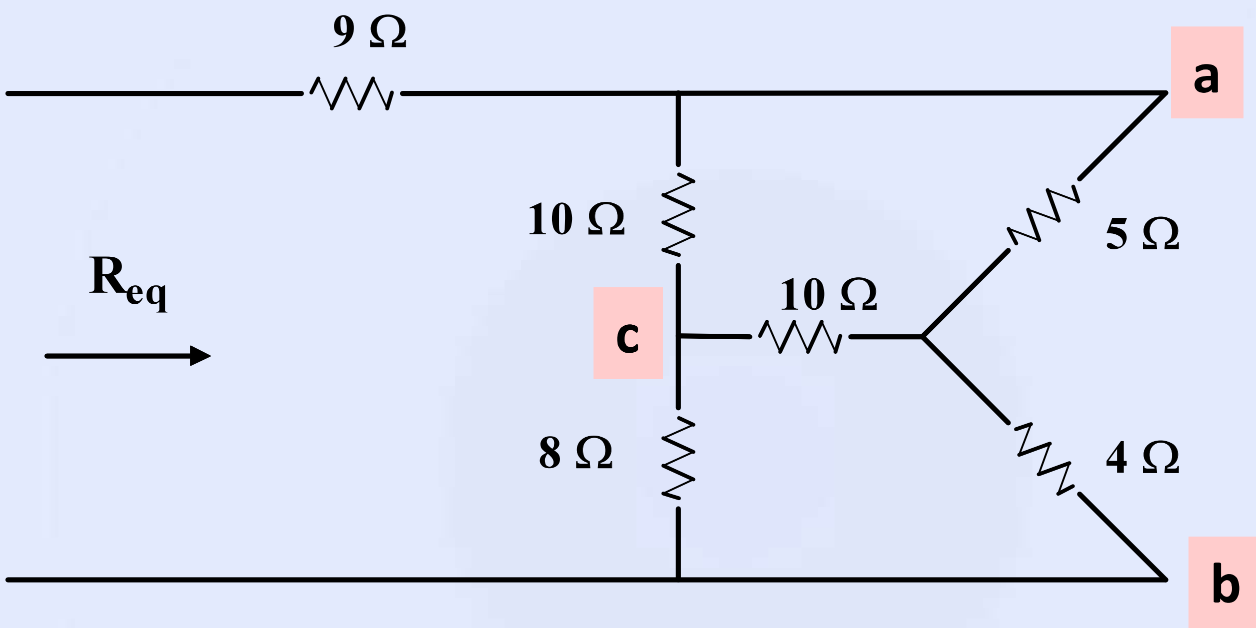
Find R_{eq} of the circuit in the given Figure



It is easy to see that R_{eq} = 15 Ω

Example 4

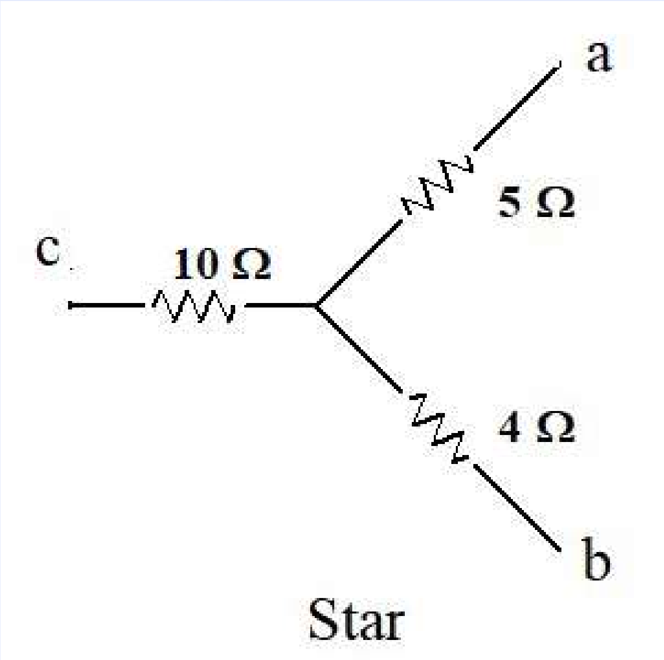
Using wye to delta, find Req

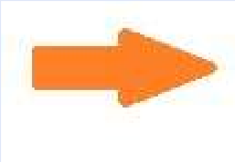


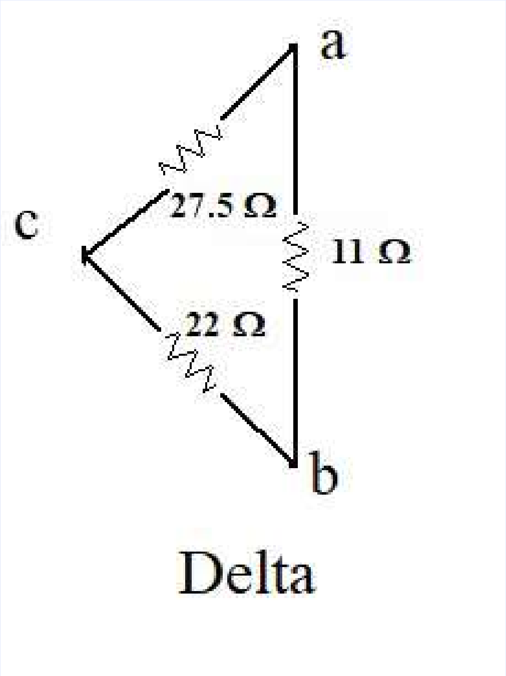
Convert star to delta at a-b-c

Example 4

Using wye to delta, find Req

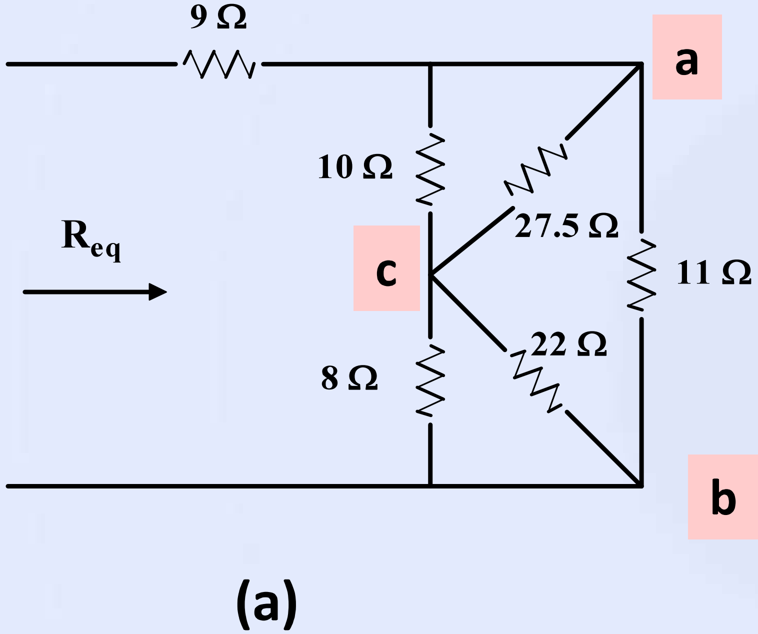


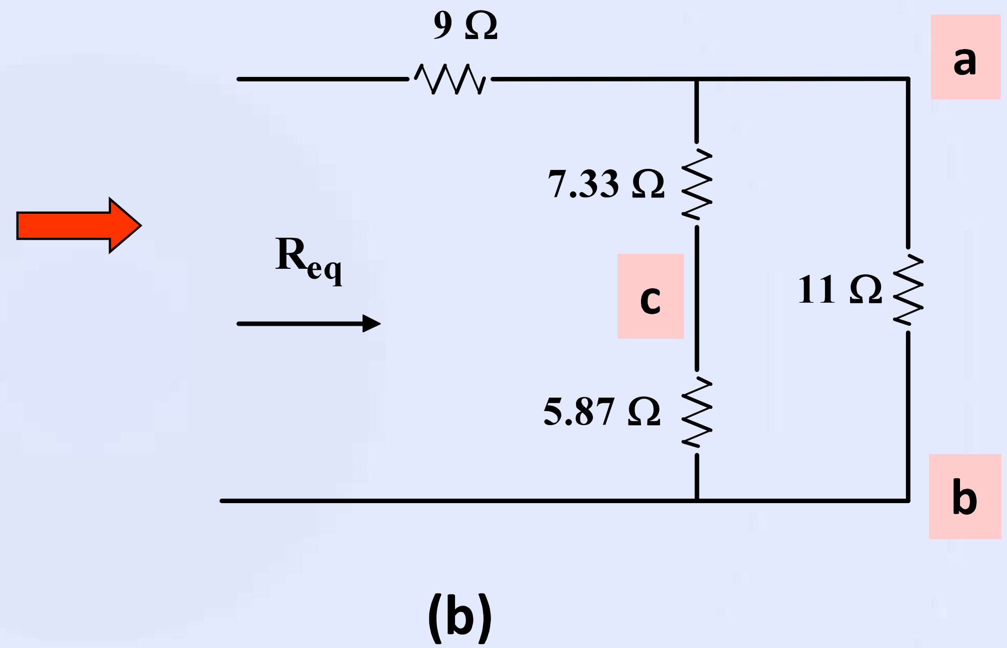




Example 4

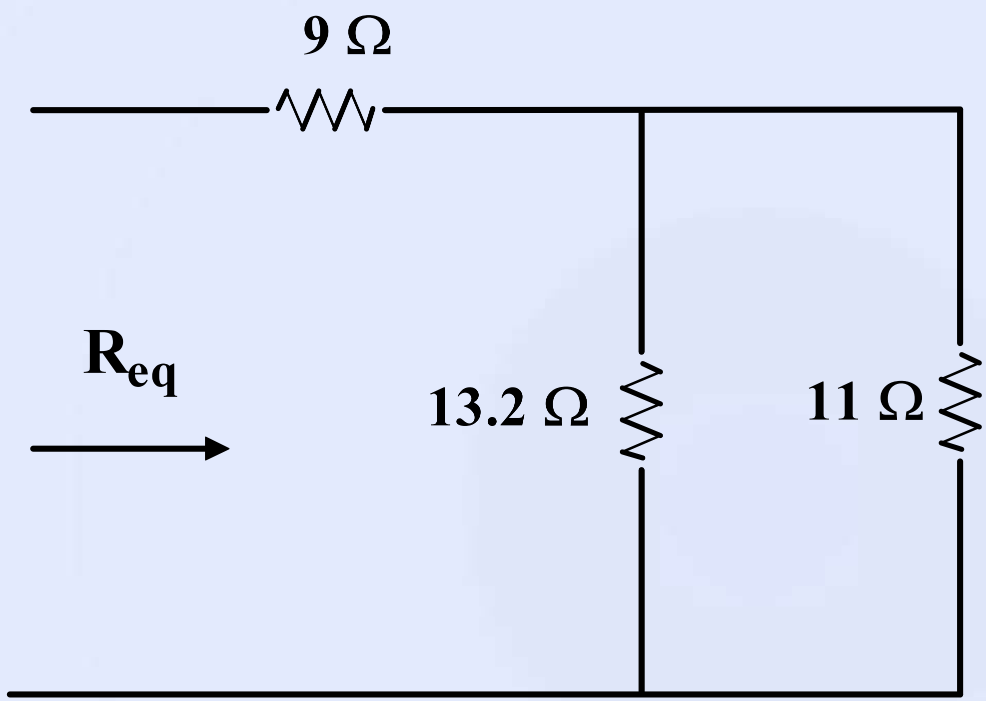
Using wye to delta, find Req





Example 4

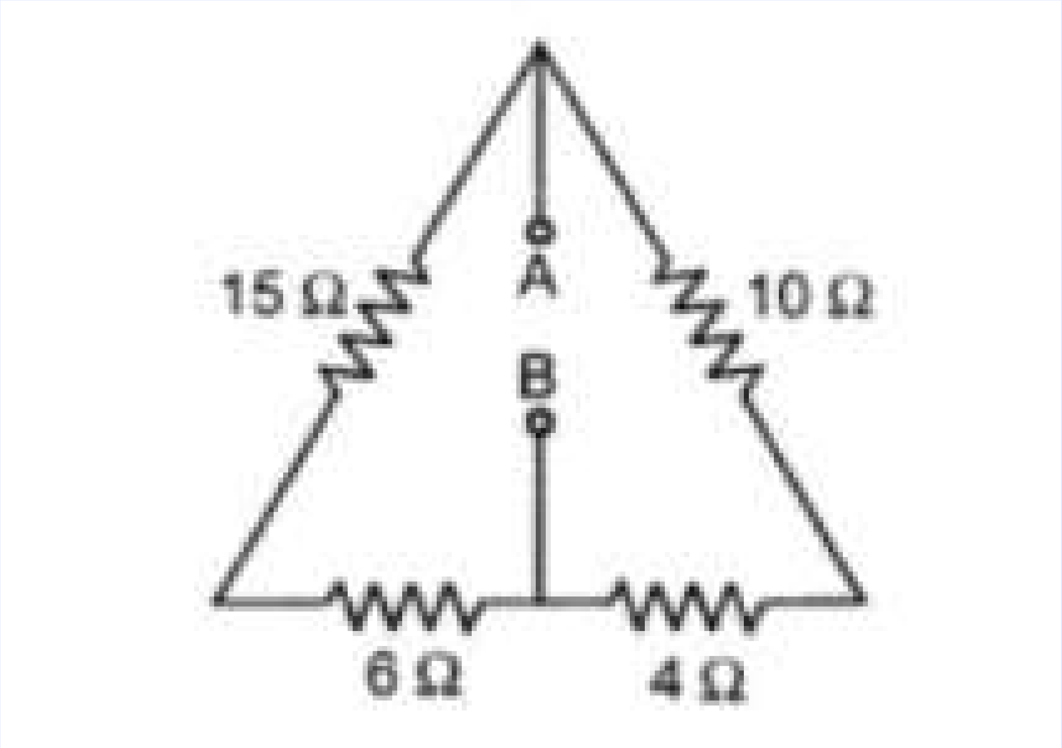
Using wye to delta, find Req



$R_{eq} = 15\ \Omega$

Example 5

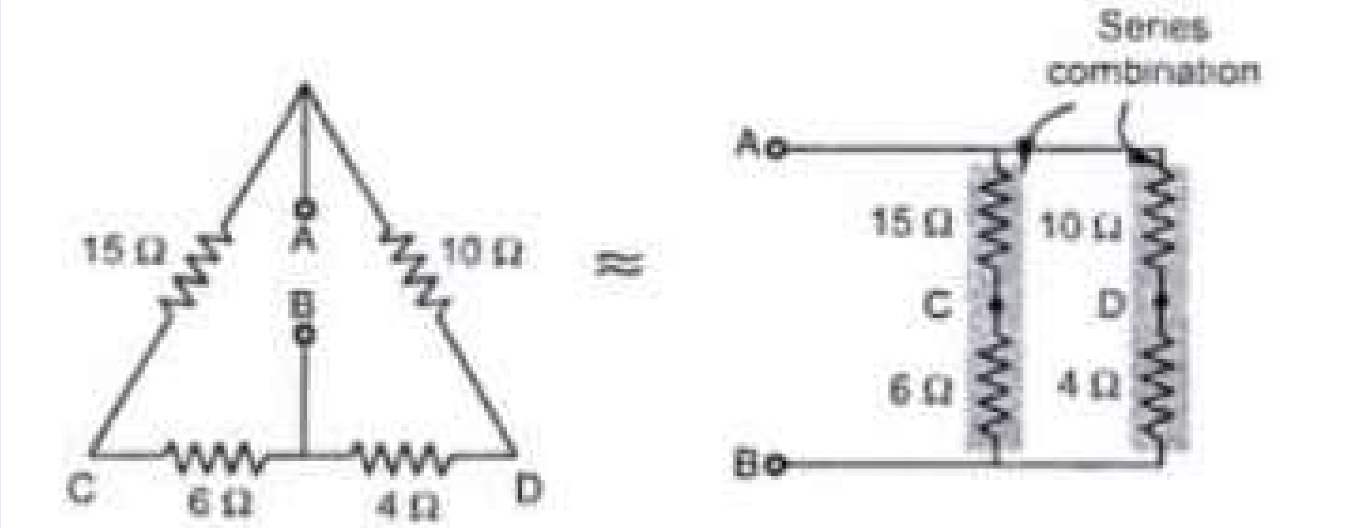
Find equivalent resistance between points A-B



Example 5

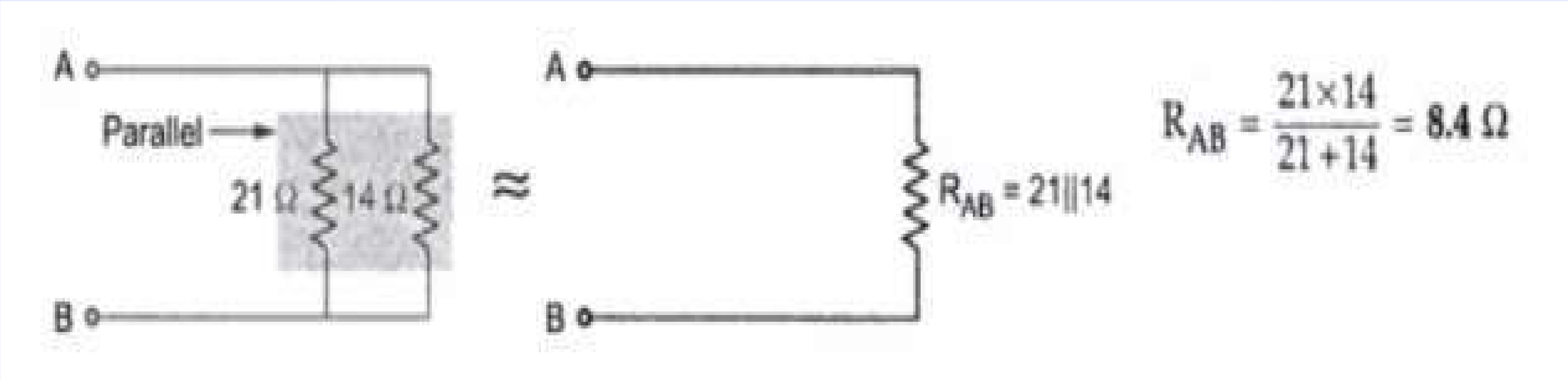
Find equivalent resistance between points A-B

Redrawing the circuit



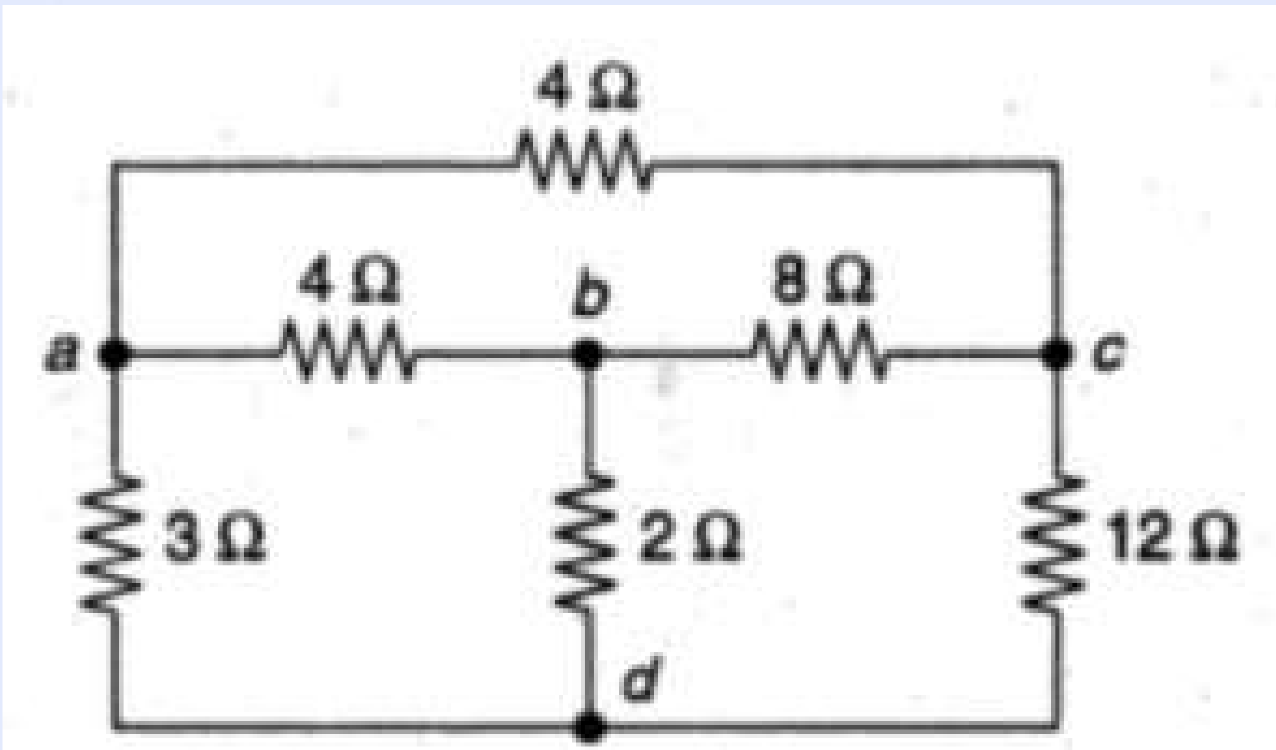
Example 5

Find equivalent resistance between points A-B



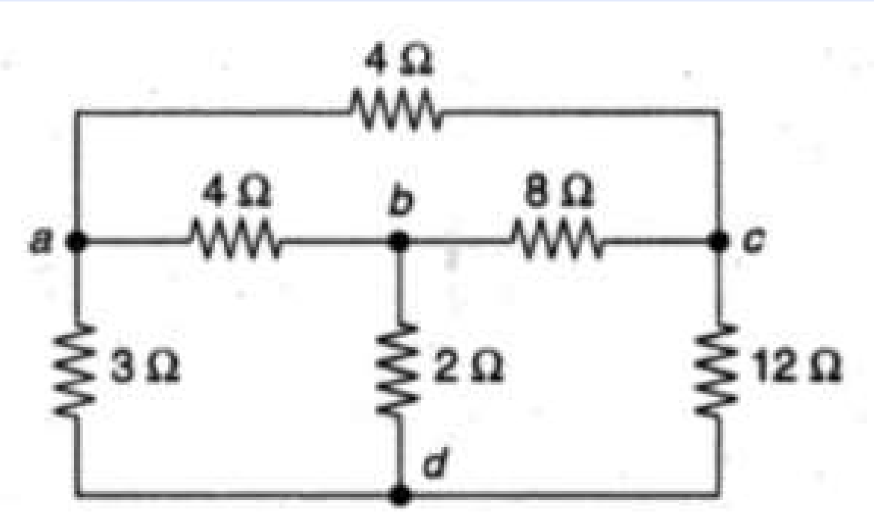
Example 6

Calculate the equivalent resistance between nodes ad



Example 6

Calculate the equivalent resistance between nodes ad



Solution

Converting the Y at node b to Δ,

$$R_x = \frac{4\times 8 + 8\times 2 + 2\times 4}{2} = 28\ \Omega$$

$$R_y = \frac{4\times 8 + 8\times 2 + 2\times 4}{8} = 7\ \Omega$$

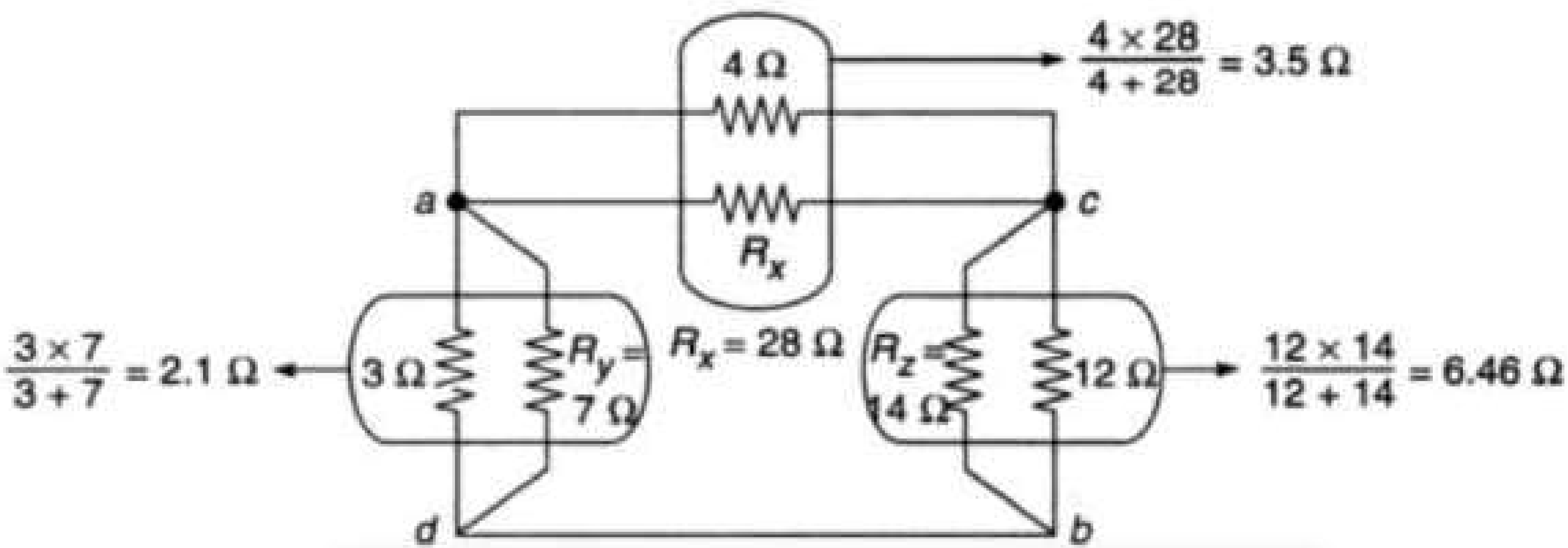
$$R_z = \frac{4\times 8 + 8\times 2 + 2\times 4}{4} = 14\ \Omega$$

Example 6

Calculate the equivalent resistance between nodes ad

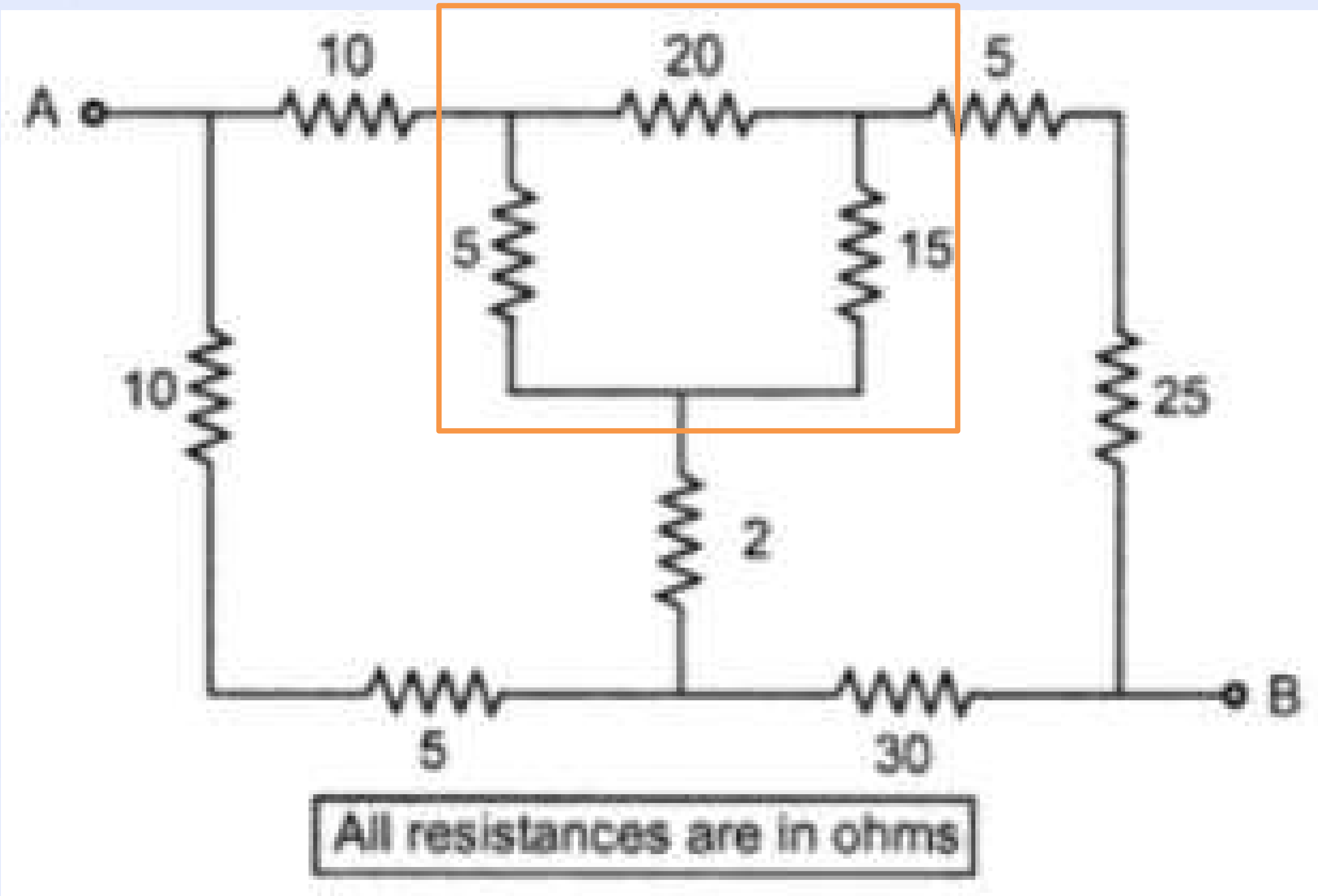
By series–parallel combination, we get

$$R_{ad}\,(\text{eq}) = \frac{2.1 \times (3.5 + 6.46)}{2.1 + 3.5 + 6.46} = 1.734\ \Omega$$



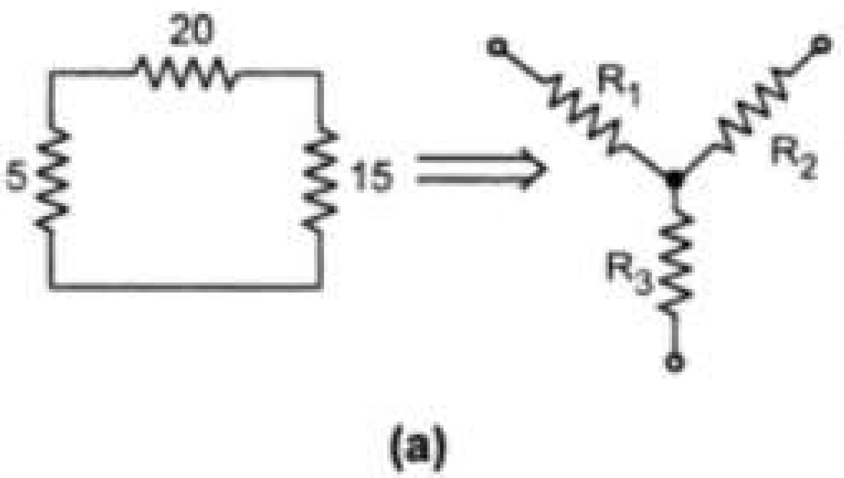
Example 7

Find resistance between the terminals A-B of the circuit



Example 7

Find resistance between the terminals A-B of the circuit



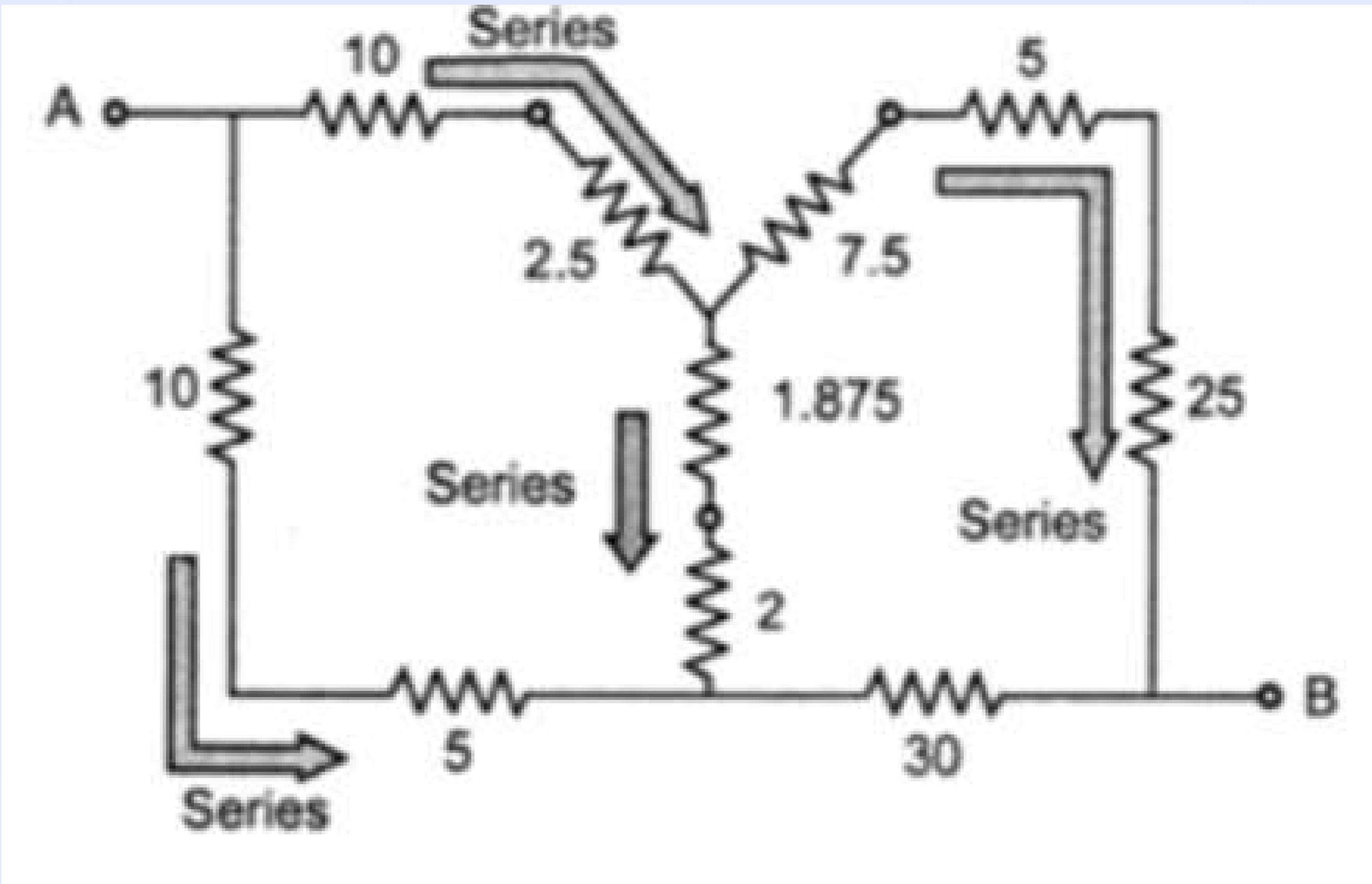
$$R_1 = \frac{20\times 5}{20+5+15} = 2.5\ \Omega$$

$$R_2 = \frac{20\times 15}{20+5+15} = 7.5\ \Omega$$

$$R_3 = \frac{5\times 15}{20+5+15} = 1.875\ \Omega$$

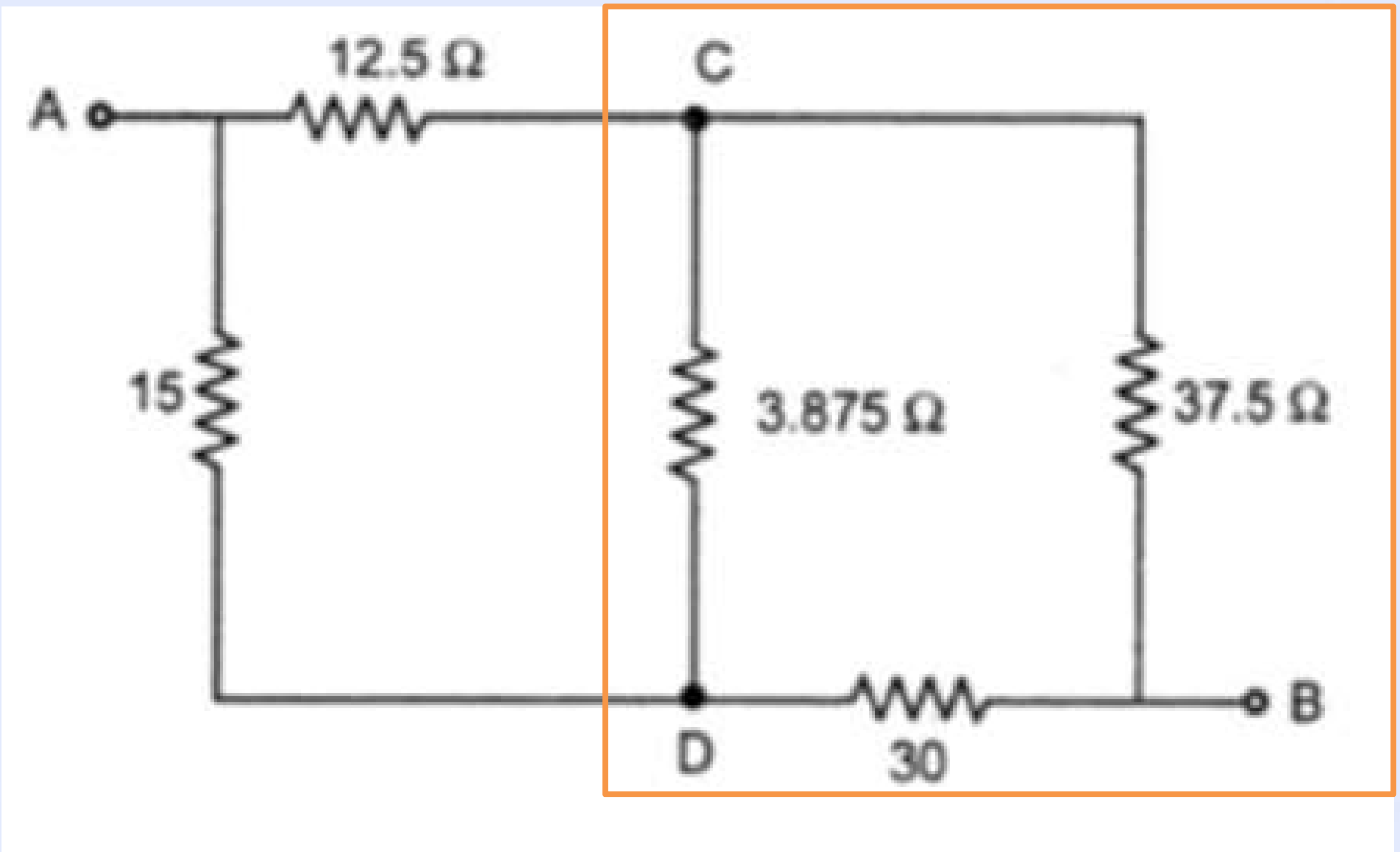
Example 7

Find resistance between the terminals A-B of the circuit



Example 7

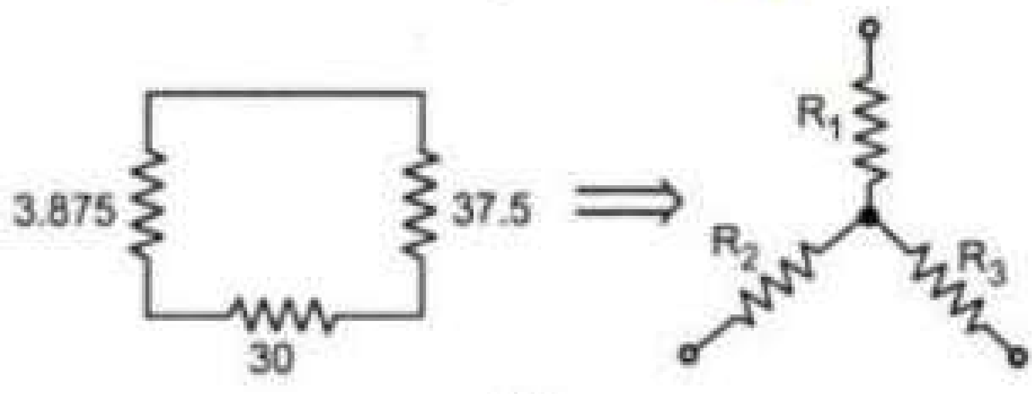
Find resistance between the terminals A-B of the circuit



Example 7

Find resistance between the terminals A-B of the circuit

Convert Δ CDB to equivalent star,



(d)

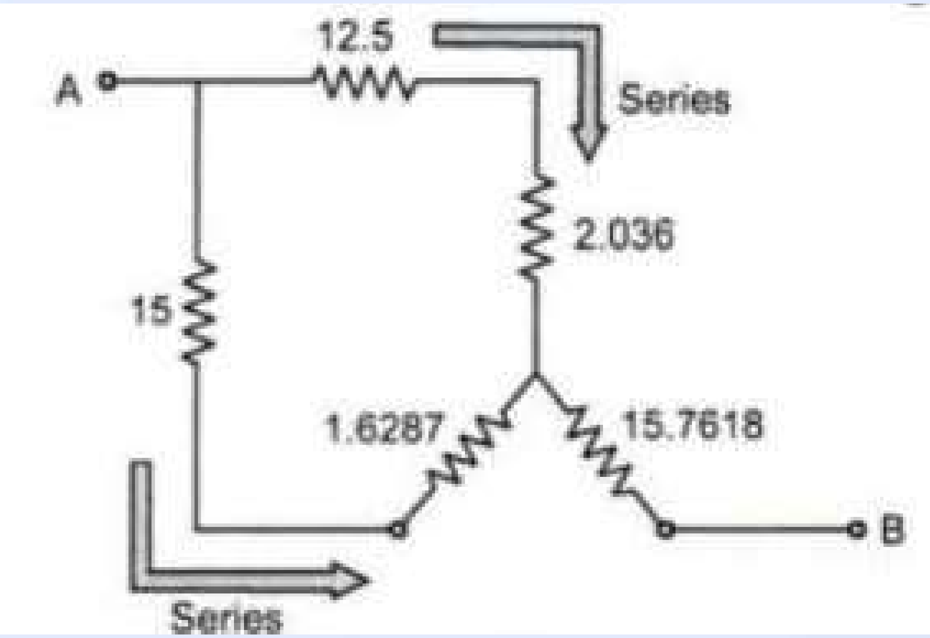
$$R_1 = \frac{3.875 \times 37.5}{3.875 + 37.5 + 30} = 2.036\,\Omega$$

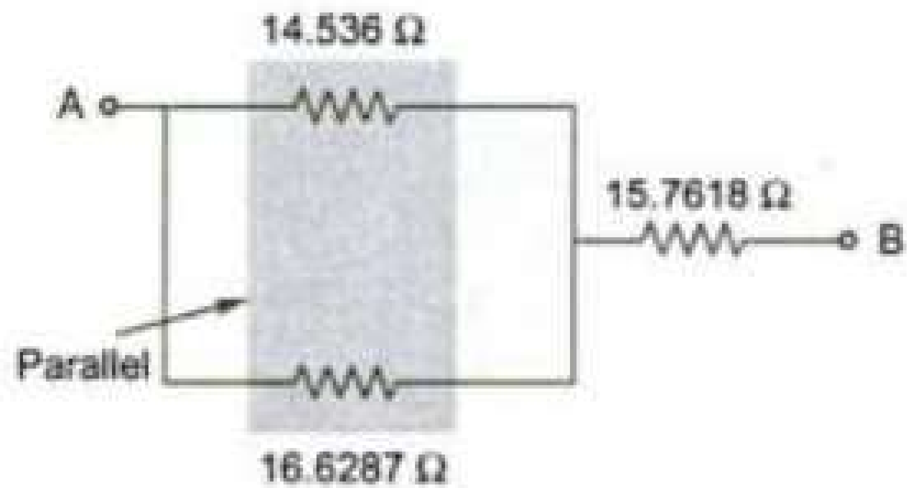
$$R_2 = \frac{3.875 \times 30}{3.875 + 37.5 + 30} = 1.6287\,\Omega$$

$$R_3 = \frac{30 \times 37.5}{3.875 + 37.5 + 30} = 15.7618\,\Omega$$

Example 7

Find resistance between the terminals A-B of the circuit



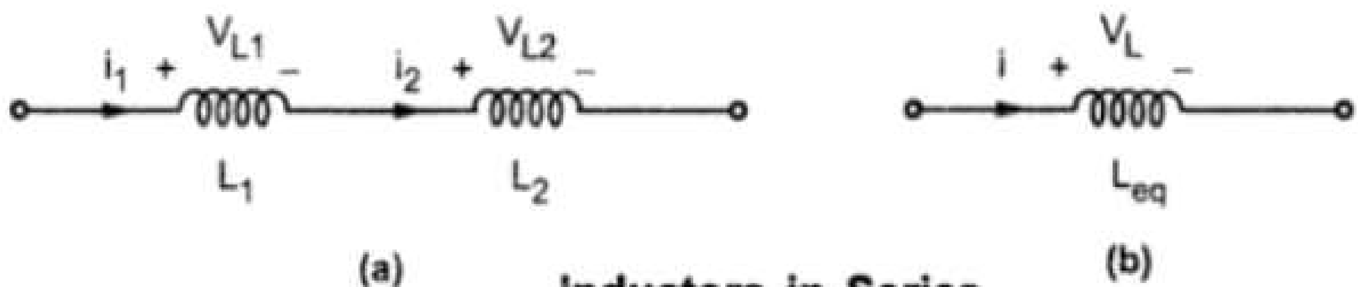


$$\frac{14.536 \times 16.6287}{14.536 + 16.6287} = 7.7557\ \Omega$$

$$R_{AB} = 7.7557 + 15.7618$$
$$= \mathbf{23.5175\ \Omega}$$

Inductors in series

The currents flowing through L_1 and L_2 are i_1 and i_2 while voltages developed across L_1 and L_2 are V_{L1} and V_{L2} respectively.



Inductors in Series

We have, $V_{L1} = L_1 \frac{di_1}{dt}$ and $V_{L2} = L_2 \frac{di_2}{dt}$ while $V_L = L_{eq} \frac{di}{dt}$

Inductors in series

For series combination,

$$i = i_1 = i_2$$

and $$V_L = V_{L1} + V_{L2}$$

$$\therefore \quad L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

$$\therefore \quad L_{eq}\frac{di}{dt} = (L_1 + L_2)\frac{di}{dt}$$

$$\therefore \quad \boxed{L_{eq} = L_1 + L_2}$$

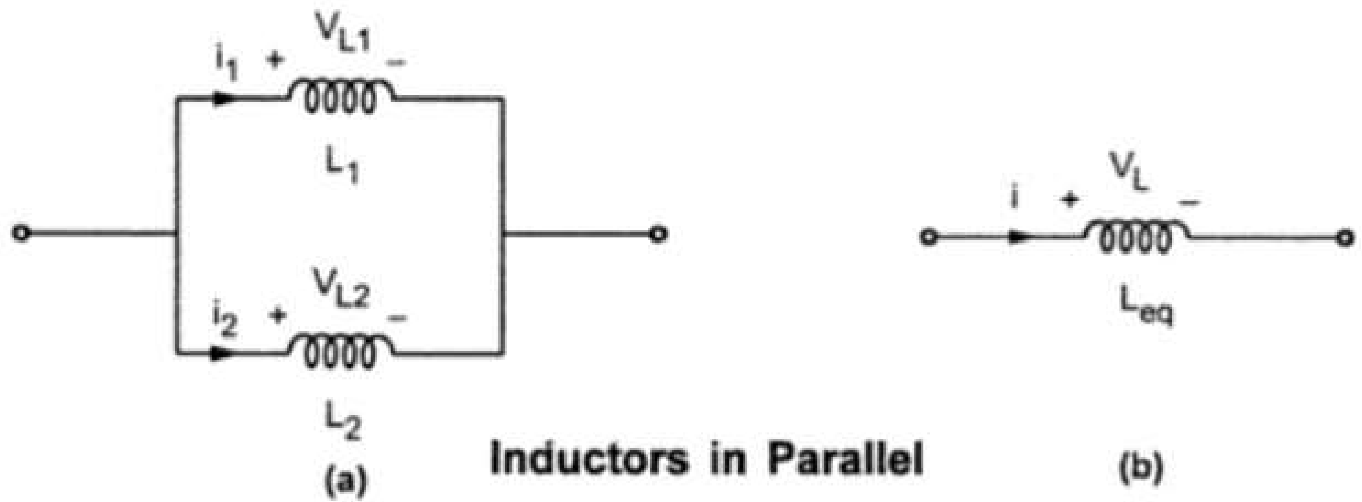
That means, equivalent inductance of the series combination of two inductances is the sum of inductances connected in series.

For n inductances in series,

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \dots + L_n}$$

Inductors in parallel

The currents flowing through L_1 and L_2 are i_1 and i_2 respectively. The voltage developed across L_1 and L_2 are V_{L1} and V_{L2} respectively.



Inductors in Parallel

For inductor we have,

$$i_1 = \frac{1}{L_1}\int_{-\infty}^{t} V_{L1}\,dt, \qquad i_2 = \frac{1}{L_2}\int_{-\infty}^{t} V_{L2}\,dt, \quad \text{while} \quad i = \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L\,dt$$

Inductors in parallel

For parallel combination,

$$V_L = V_{L1} = V_{L2} \quad \text{and}$$

$$i = i_1 + i_2$$

$$\therefore \quad \frac{1}{L_{eq}} \int_{-\infty}^{t} V_L dt = \frac{1}{L_1} \int_{-\infty}^{t} V_L\, dt + \frac{1}{L_2} \int_{-\infty}^{t} V_L\, dt = \left(\frac{1}{L_1} + \frac{1}{L_2}\right) \int_{-\infty}^{t} V_L\, dt$$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

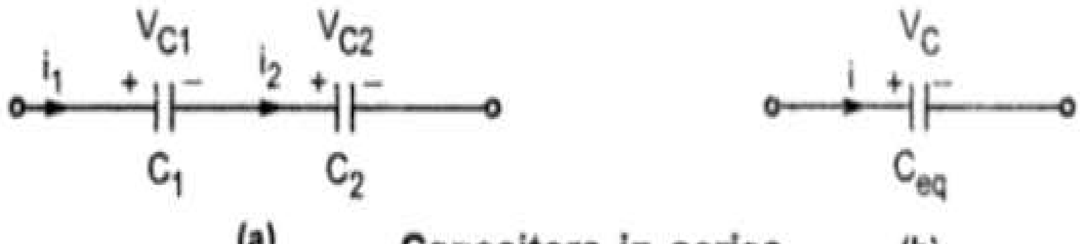
That means, reciprocal of equivalent inductance of the parallel combination is the sum of reciprocals of the individual inductances.

For n inductances in parallel,

$$\therefore \quad \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_n}}$$

Capacitors in series

The currents flowing through and voltages developed across C_1 and C_2 are i_1, i_2 and V_{C1} and V_{C2} respectively.



Capacitors in series

We have, $V_{C1} = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt$, $V_{C2} = \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$ while $V = \frac{1}{C_{eq}}\int_{-\infty}^{t} i\,dt$

Capacitors in series

For series combination,

$$i = i_1 = i_2 \quad \text{and}$$

$$V_C = V_{C1} + V_{C2}$$

$$\frac{1}{C_{eq}} \int_{-\infty}^{t} i\,dt = \frac{1}{C_1} \int_{-\infty}^{t} i_1 dt + \frac{1}{C_2} \int_{-\infty}^{t} i_2 dt$$

But $\quad i = i_1 = i_2$

$$\therefore \quad \frac{1}{C_{eq}} \int_{-\infty}^{t} i\,dt = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) \int_{-\infty}^{t} i\,dt$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \quad \boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

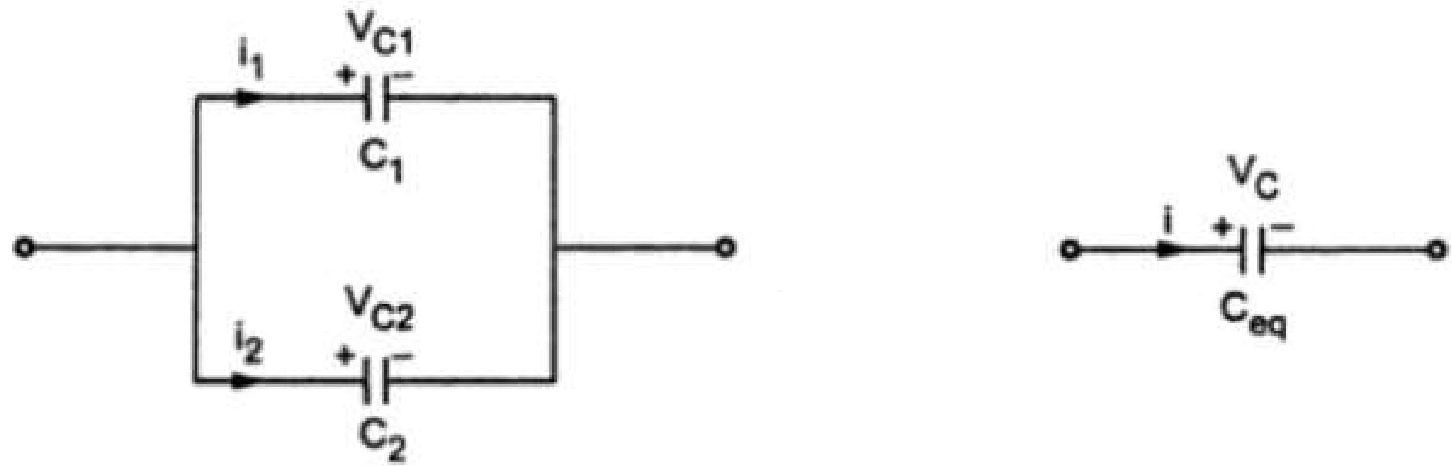
That means, reciprocal of equivalent capacitor of the series combination is the sum of the reciprocal of individual capacitances.

For n capacitors in series,

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \frac{1}{C_n}}$$

Capacitors in parallel

The currents flowing through C_1 and C_2 are i_1 and i_2 respectively and voltages developed across C_1, C_2 are V_{C1} and V_{C2} respectively.



Capacitors in Parallel

For capacitor we have, $i_1 = C_1 \frac{dV_{C1}}{dt}$, $i_2 = C_2 \frac{dV_{C2}}{dt}$, while $i = C_{eq} \frac{dV_C}{dt}$

Capacitors in parallel

For parallel combination,

$$V_{C1} = V_{C2} = V_C \quad \text{and}$$

$$i = i_1 + i_2$$

$$C_{eq} \frac{dV_C}{dt} = C_1 \frac{dV_{C1}}{dt} + C_2 \frac{dV_{C2}}{dt}$$

$$\therefore \quad C_{eq} \frac{dV_C}{dt} = (C_1 + C_2) \frac{dV_C}{dt}$$

$$\therefore \quad C_{eq} = C_1 + C_2$$

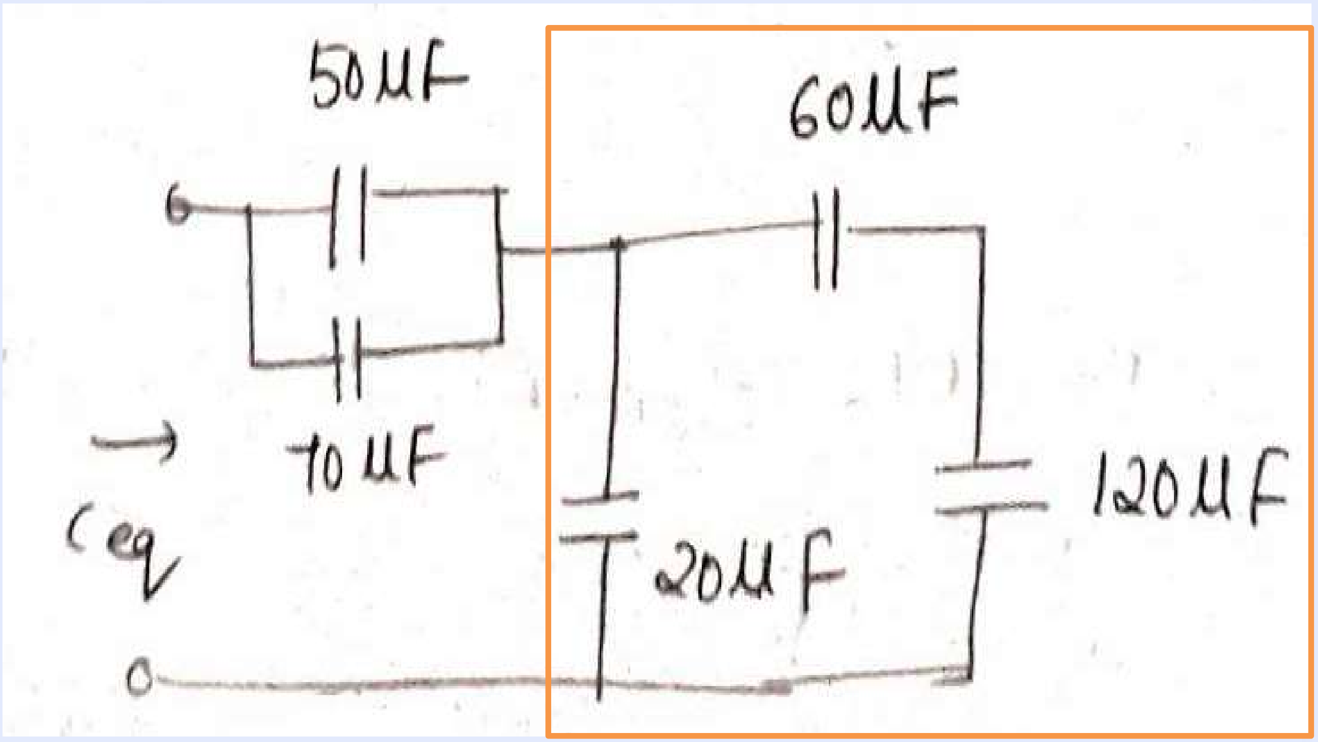
That means, equivalent capacitance of the parallel combination of the capacitances is the sum of the individual capacitances connected in series.

For n capacitors in parallel,

$$\therefore \quad \boxed{C_{eq} = C_1 + C_2 + \ldots + C_n}$$

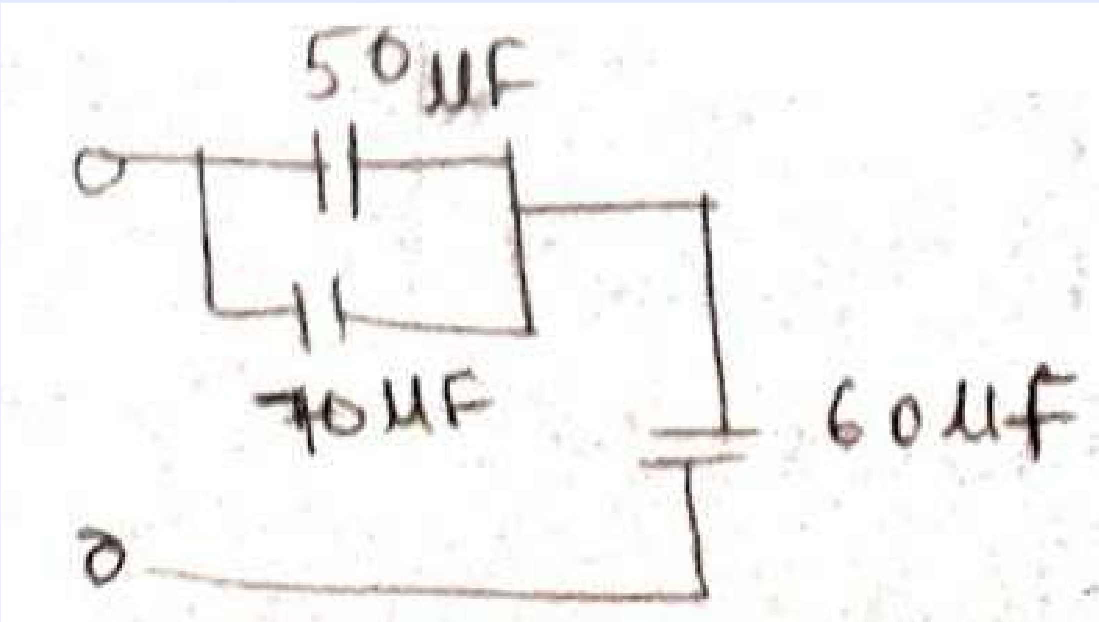
Example 8

Find the equivalent capacitance



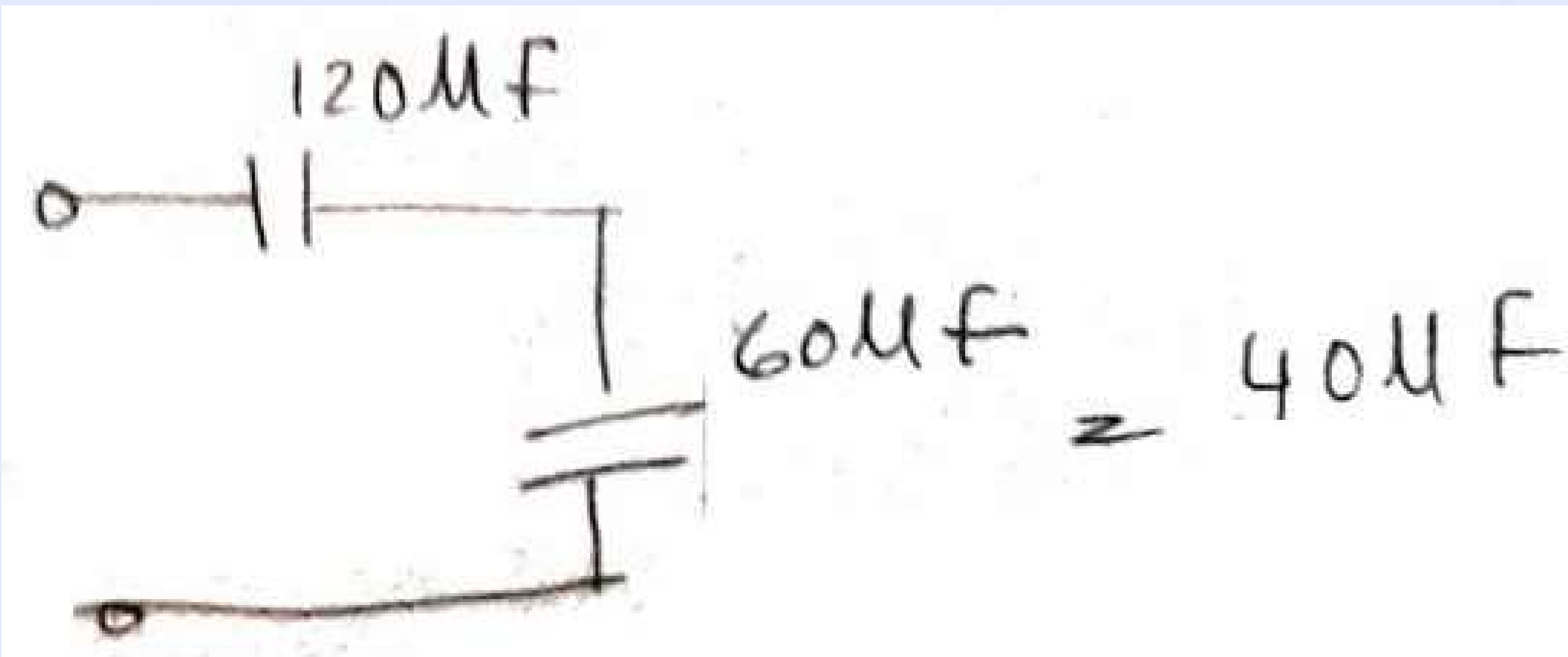
Example 8

Find the equivalent capacitance



Example 8

Find the equivalent capacitance



Example 9

Calculate the combined capacitance in micro-Farads (μF) of the following capacitors when they are connected together in a parallel combination:

a) two capacitors each with a capacitance of 47nF

b) one capacitor of 470nF connected in parallel to a capacitor of 1μF

Solution:

a) Total Capacitance,

$C_T = C_1 + C_2 = 47nF + 47nF = 94nF$ or $0.094\mu F$

b) Total Capacitance,

$C_T = C_1 + C_2 = 470nF + 1\mu F$

therefore, $C_T = 470nF + 1000nF = 1470nF$ or $1.47\mu F$

Example 10

Find the overall capacitance and the individual rms voltage drops across the following sets of two capacitors in **series** when connected to a 12V AC supply.

a) two capacitors each with a capacitance of 47nF

b) one capacitor of 470nF connected in series to a capacitor of 1μF

Example 10

a) Total Equal Capacitance,

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{47nF \times 47nF}{47nF + 47nF} = 23.5nF$$

Voltage drop across the two identical 47nF capacitors,

$$V_{C1} = \frac{C_T}{C_1} \times V_T = \frac{23.5nF}{47nF} \times 12V = 6volts$$

$$V_{C2} = \frac{C_T}{C_2} \times V_T = \frac{23.5nF}{47nF} \times 12V = 6volts$$

Example 10

b) Total Unequal Capacitance,

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{470nF \times 1uF}{470nF + 1uF} = 320nF$$

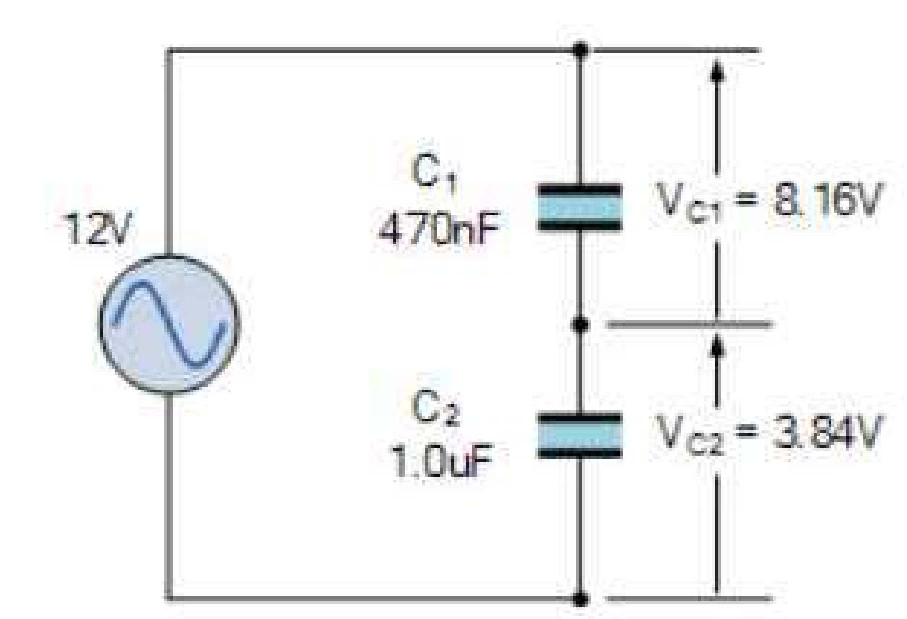
Voltage drop across the two non-identical Capacitors: $C_1 = 470nF$ and $C_2 = 1\mu F$.

$$V_{C1} = \frac{C_T}{C_1} \times V_T = \frac{320nF}{470nF} \times 12 = 8.16 volts$$

$$V_{C2} = \frac{C_T}{C_2} \times V_T = \frac{320nF}{1uF} \times 12 = 3.84 volts$$

Example 10

The difference in voltage allows the capacitors to maintain the same amount of charge, Q on the plates of each capacitors as shown.

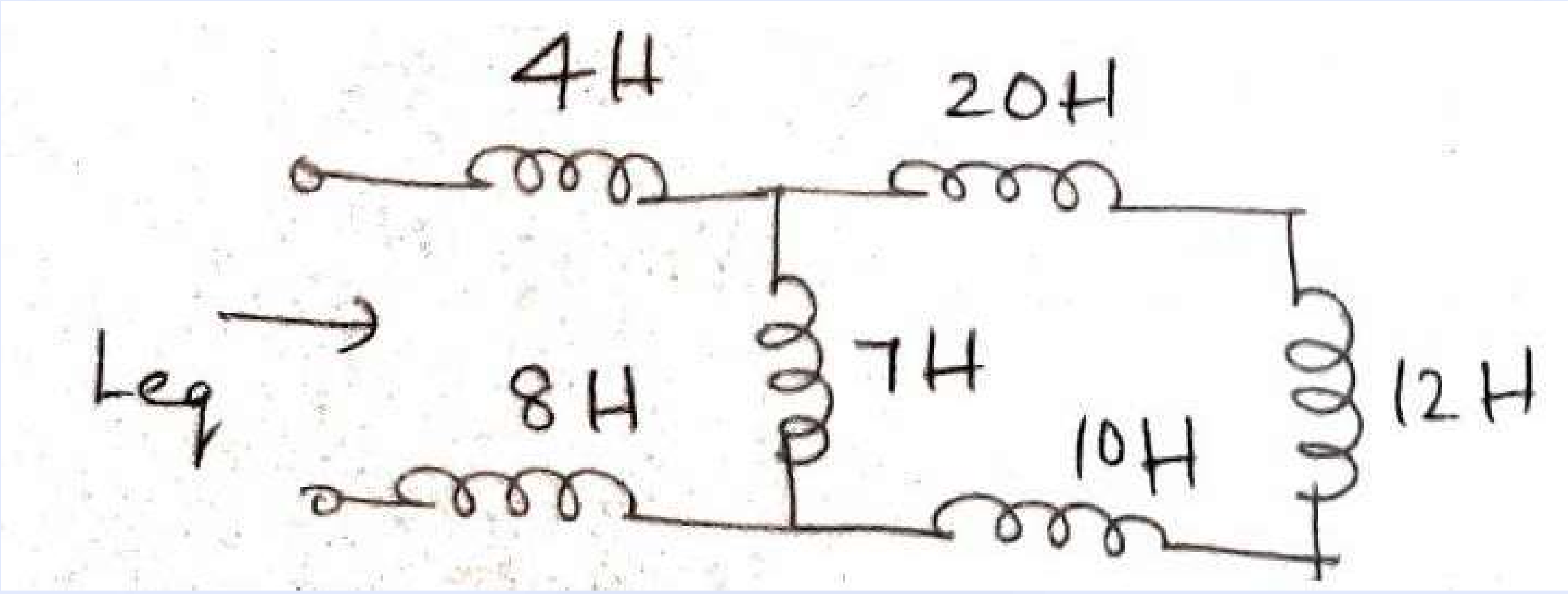


$$Q_{C1} = V_{C1} \times C_1 = 8.16V \times 470nF = 3.84\mu C$$

$$Q_{C2} = V_{C2} \times C_2 = 3.84V \times 1uF = 3.84\mu C$$

Example 11

Find the equivalent Inductance of the given circuit



$L_{eq} = 18\ H$

Example 12

Three inductors of 60mH, 120mH and 75mH respectively, are connected together in a parallel combination with no mutual inductance between them. Calculate the total inductance of the parallel combination in millihenries.

Solution:

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$\therefore L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} = \frac{1}{\frac{1}{60\mathrm{mH}} + \frac{1}{120\mathrm{mH}} + \frac{1}{75\mathrm{mH}}}$$

$$L_T = \frac{1}{38.333} = 26\mathrm{mH}$$

"Amrita Vishwa Vidyapeetham has a major role to play in transforming our society into a knowledge society through its unique value-added education system."

Dr. A.P.J. Abdul Kalam
Former President of India

THANK YOU