22AIE114 : Introduction to Electrical and Electronic Engineering (2-0-3-3)

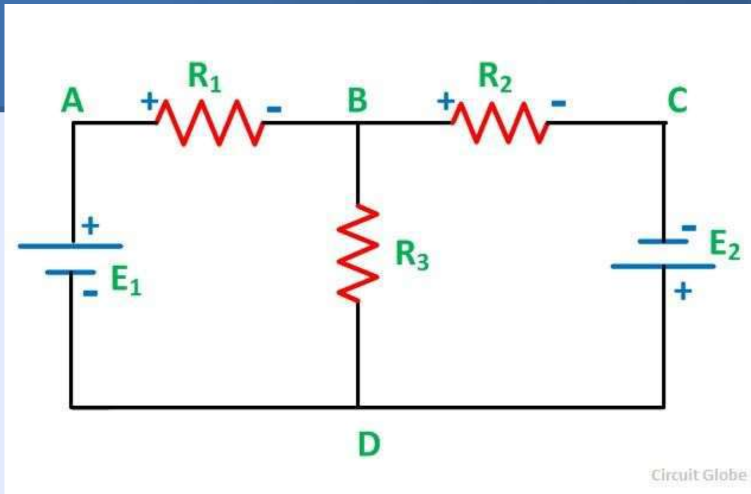
Unit 1, Lecture 5

Topic: Basic Circuit Laws KCL and KVL

Outline

- Circuit Terms
- KCL (Kirchhoff's Current Law)
- KVL (Kirchhoff's Voltage Law)
- Current division method
- Voltage division method

Circuit Terms



- **Node**
  - A node is a point in the network or circuit where two or more circuit elements are joined. For example, in the above circuit diagram, A and C is the node points.
- **Junction**
  - A junction is a point in the network where three or more circuit elements are joined. It is a point where the current is divided. In the above circuit, B and D are the junctions.
- **Branch**

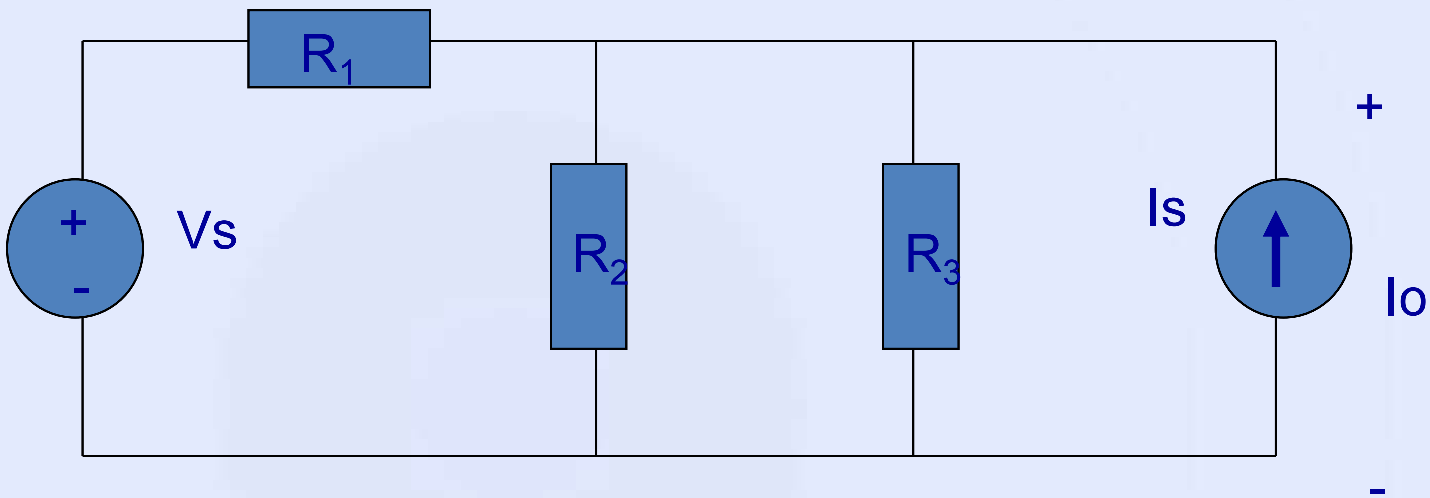
  The part of a network, which lies between the two junction points is called a Branch. In the above circuit AB, CD and BD are the branches of the circuit.
- **Loop**

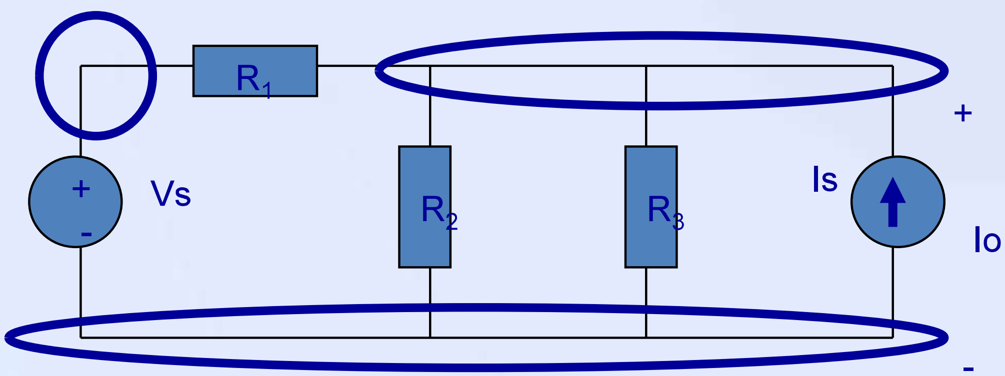
  A closed path of a network is called a loop. ABDA, BCDB are loops in the above circuit diagram shown.
- **Mesh**

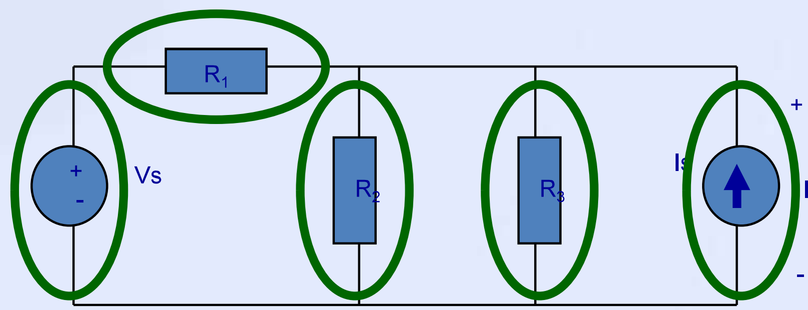
  The most elementary form of a loop which cannot be further divided is called a mesh.

Example

- How many nodes, branches?







Kirchhoff´s Laws

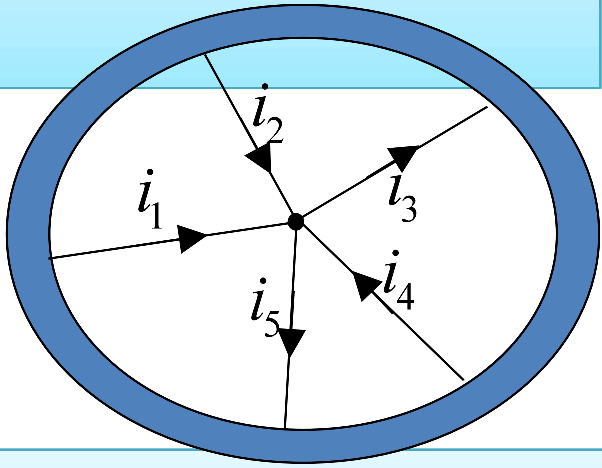
- **Kirchoffs First Law – The Current Law, (KCL)**
- **Kirchoffs Second Law – The Voltage Law, (KVL)**
- Basic method for analysing any complex electrical circuit, there are different ways of improving upon this method by using **Mesh Current Analysis** or **Nodal Voltage Analysis** that results in a lessening of the math's involved and when large networks are involved this reduction in maths can be a big advantage.

Kirchhoff's Current Law (KCL)

- Based on law of Conservation of charge.

The algebraic sum of the currents entering (or leaving) a node is zero.

Σ currents in - Σ currents out = 0



Entering: $$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

Leaving: $$-i_1 - i_2 + i_3 - i_4 + i_5 = 0$$

The sum of the currents entering a node is equal to the sum of the currents leaving a node.

Σ currents in = Σ currents out

$$i_1 + i_2 + i_4 = i_3 + i_5$$

Example 2.1

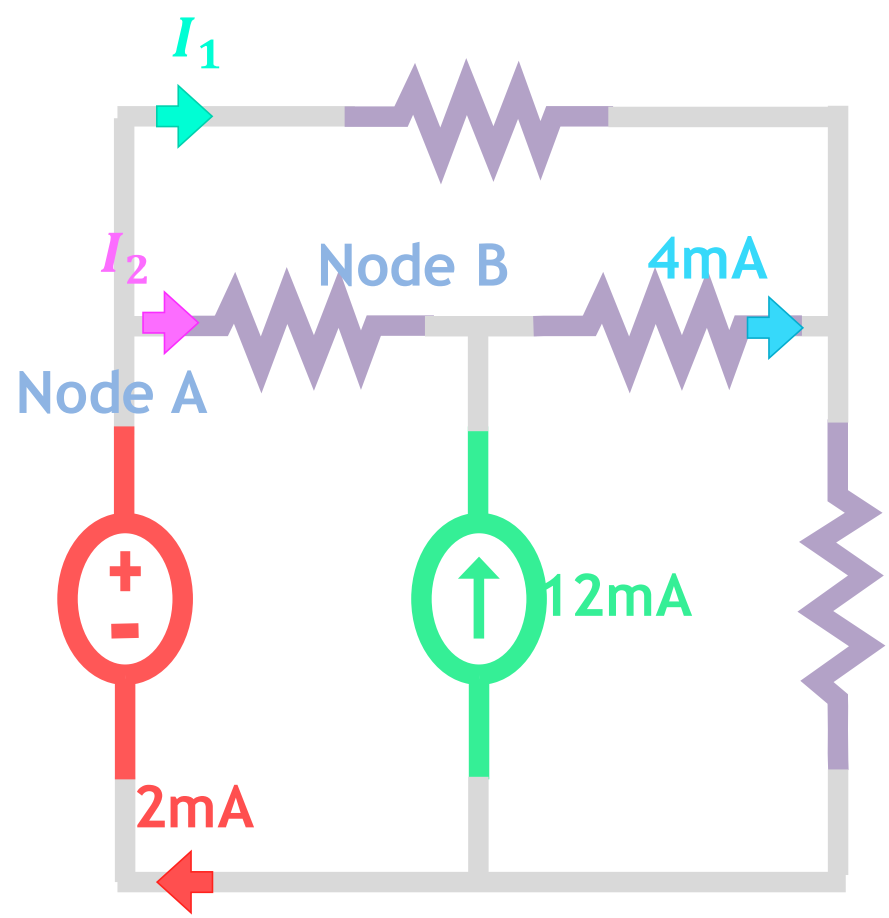
- Find I_1 and I_2 using KCL.

- ***Sign convention**: Currents entering the node are positive, currents leaving the node are negative.*
- *Identify nodes, current entering and leaving at each nodes.*

- KCL at Node B

$$I_2 + 12mA - 4mA = 0$$

$$I_2 = -8mA$$



Example 2.1

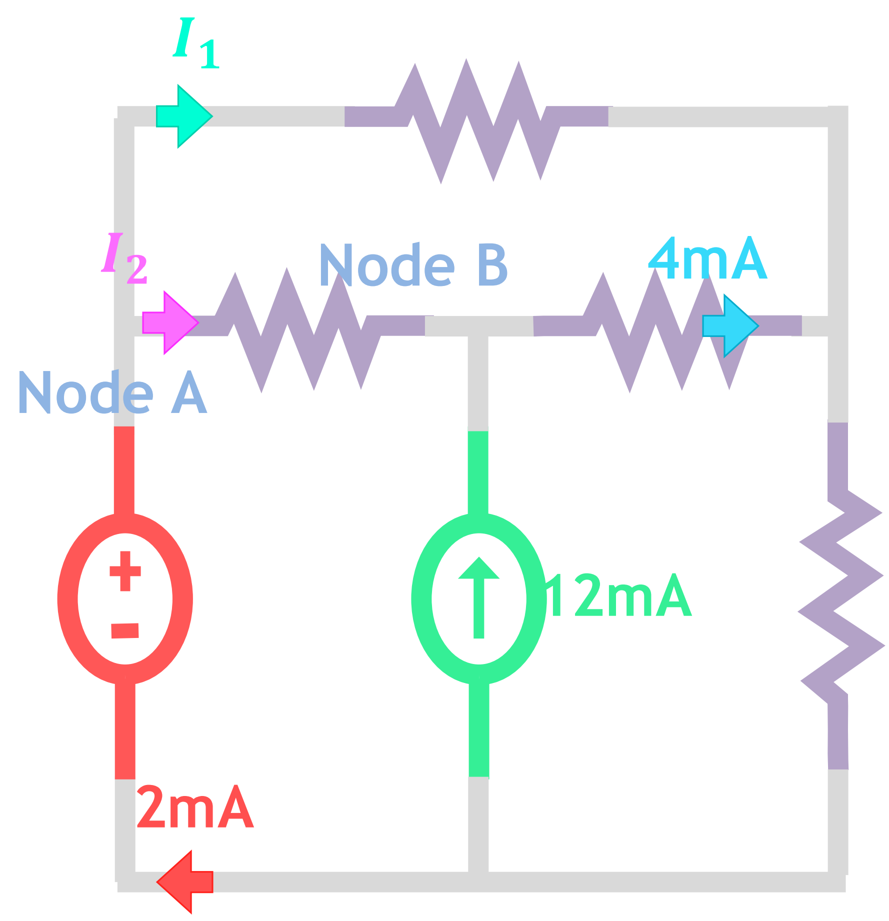
- KCL at Node A

$$-I_1 - I_2 + 2mA = 0$$

$$-I_1 + 8mA + 2mA = 0$$

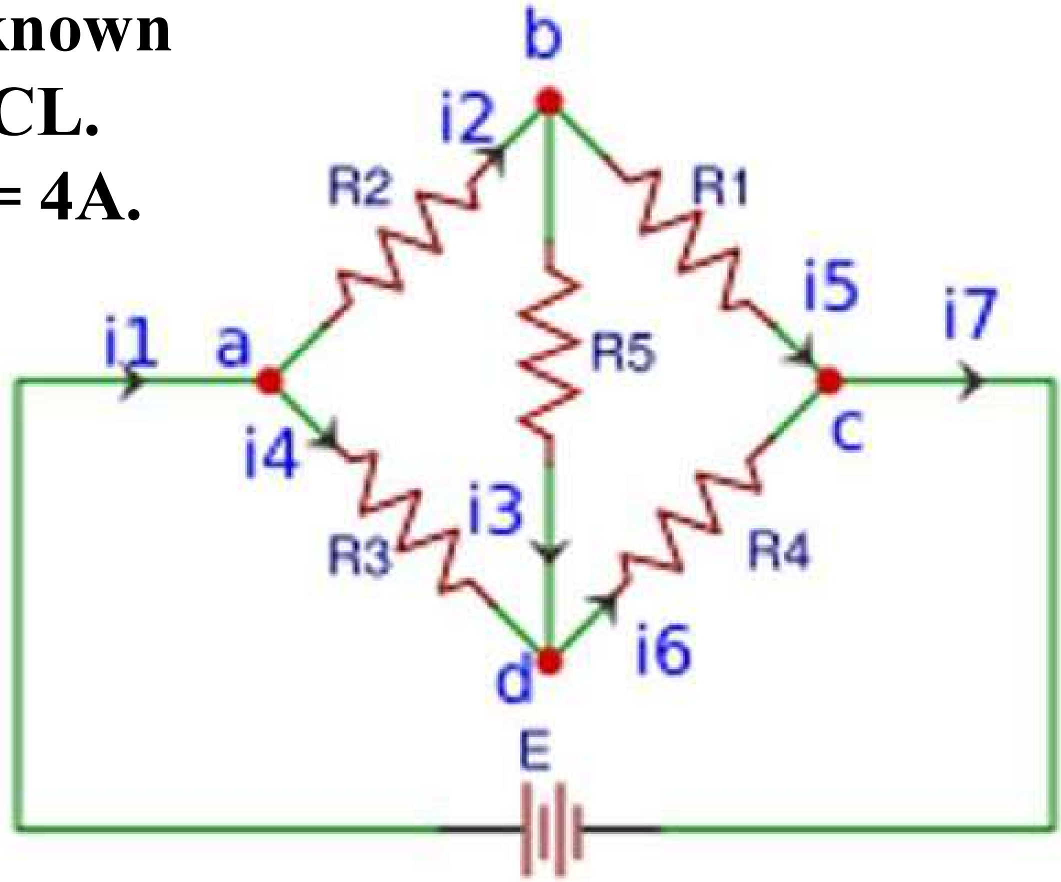
$$I_1 = 10mA$$

The negative sign for I_2 means that the actual direction of current flow is opposite to initially chosen one.



Example 2.2

- **Find the magnitude and direction of the unknown currents in figure using KCL.**
- **Given $i_1 = 10A$, $i_2 = 6A$, $i_5 = 4A$.**



Example 2.2

- By observing, it is evident that

  $i_1 = i_7$

- Therefore, $i_7 = 10A$
- At node "a", from KCL,

  $i_1 = i_2 + i_4$

  $10 = 6 + i_4$

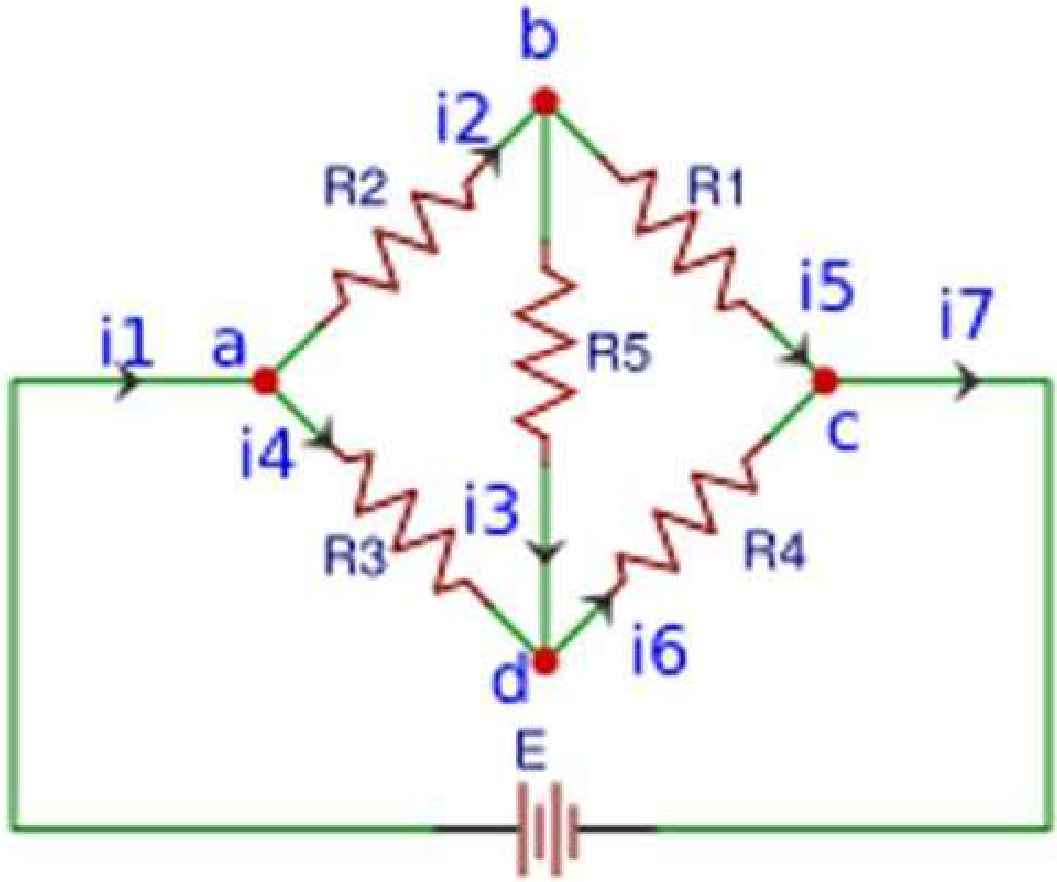
  Or, $i_4 = 4A$

- At node "b", utilizing KCL,

  $i_2 = i_3 + i_5$

  Or, $i_3 = i_2 - i_5 = 6 - 4 = 2A$

  i.e., $i_3 = 2A$



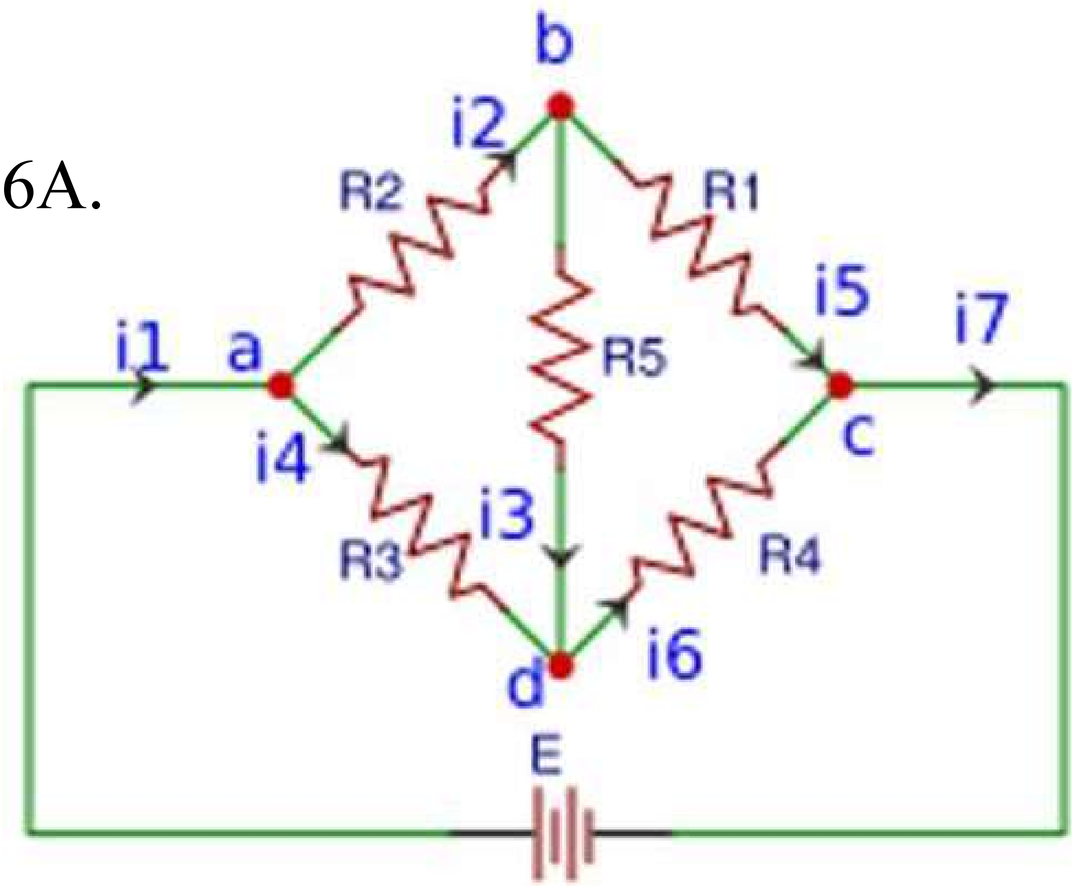
Example 2.2

- Similarly, at node "C",

$$i_7 = i_5 + i_6$$

- giving $i_6 = i_7 - i_5 = 10 - 4 = 6A$.

Therefore, $i_6 = 6A$.



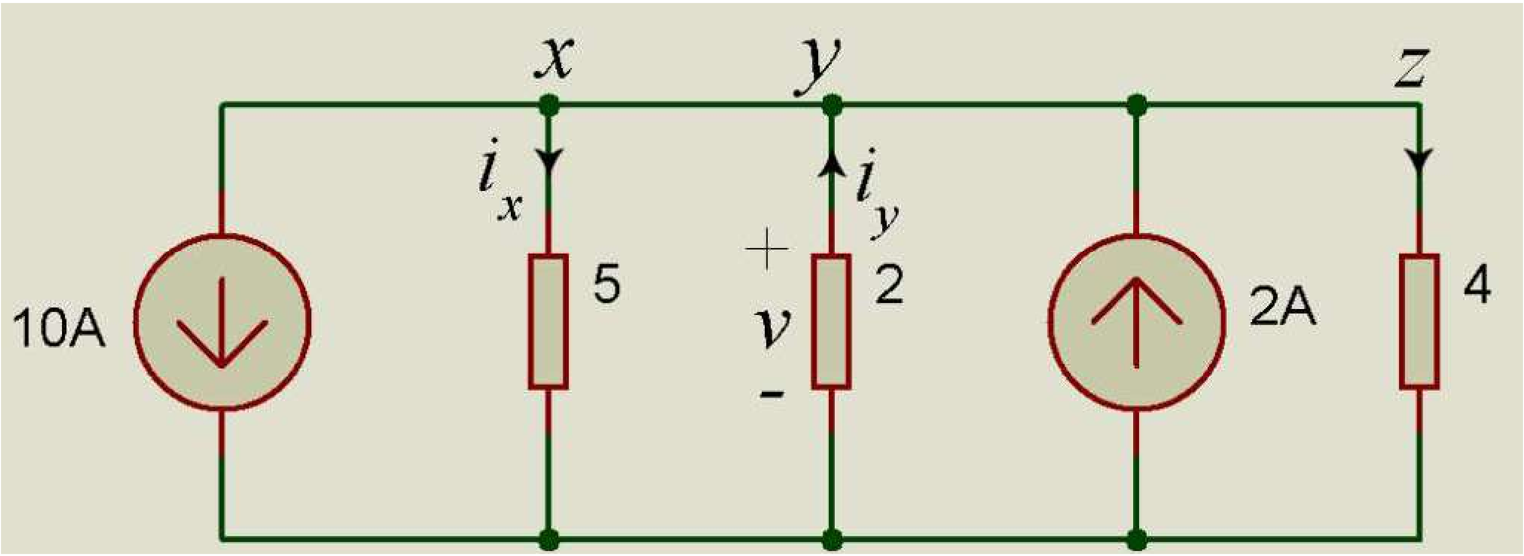
i1=i7=10A

i2=6A, i3=2A, i4=4A

i5=4A, i6=6A

Example 2.3

- Find "v" across 2 Ohms resistor and the magnitude and direction of the unknown currents in the branches xn, yn and zn using KCL.

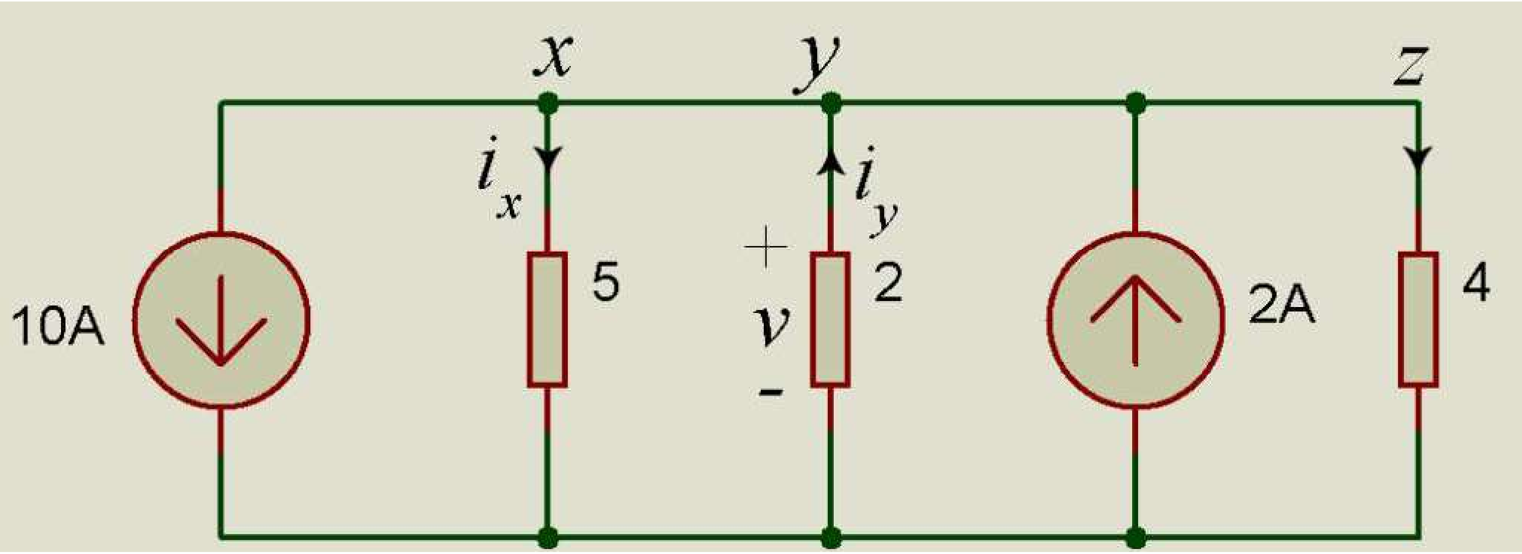


Example 2.3

- Let the unknown currents in branches xn, yn and zn be ix, iy and iz respectively and their directions have been shown in figure.
- At node "y" from KCL,

$$10 + i_x + i_z = i_y + 2$$

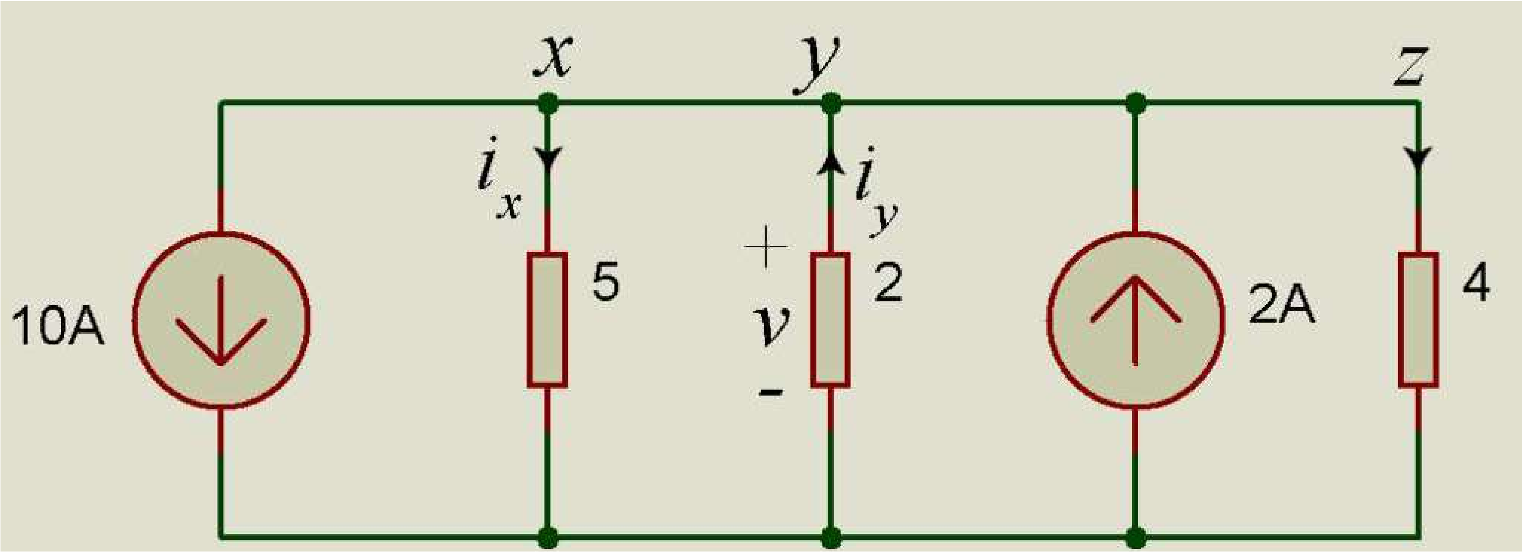
$$\text{Or, } i_x - i_y + i_z = -8 \qquad \text{.........(i)}$$



Example 2.3

- Again, from Ohm's law,
- $i_x = \frac{v}{5}A, i_y = \frac{-v}{2}A$ and $i_z = \frac{v}{4}A$

$$\frac{v}{5} + \frac{v}{2} + \frac{v}{4} = -8 \qquad \text{OR} \qquad v = -8 \times \frac{20}{19} = -8.42V$$

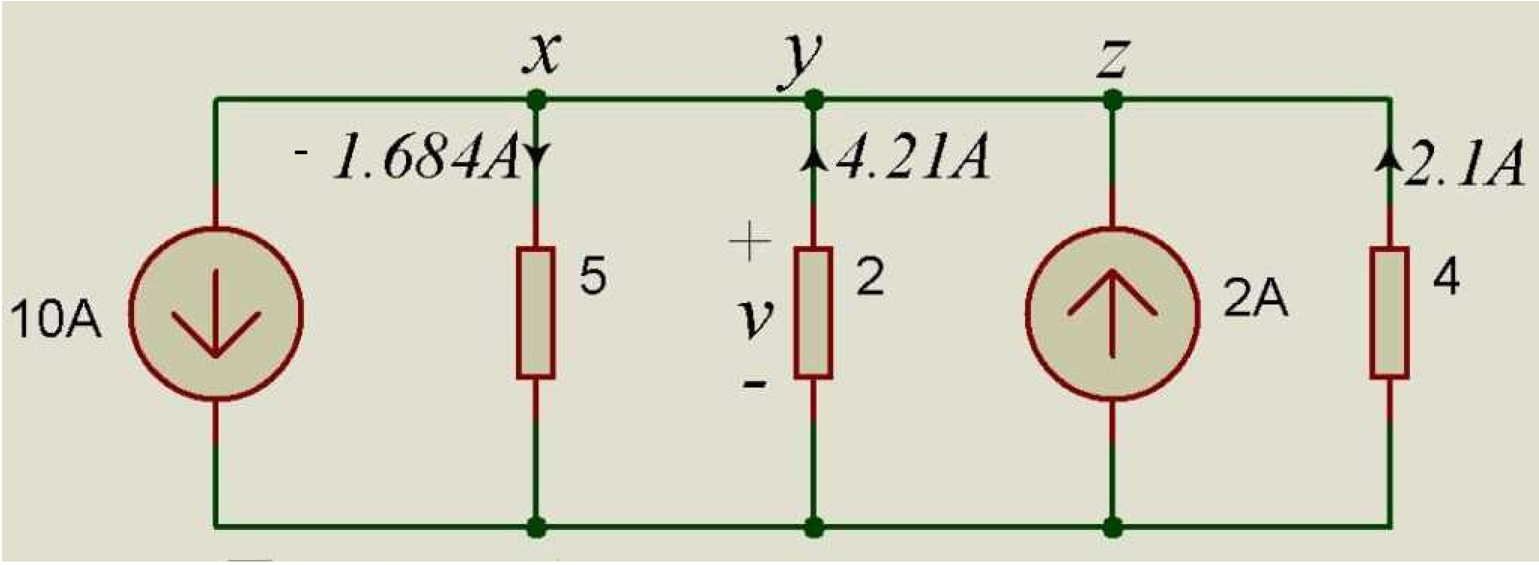


Example 2.3

current flow from “n” to “x” will be 8.42/5 = 1.684A

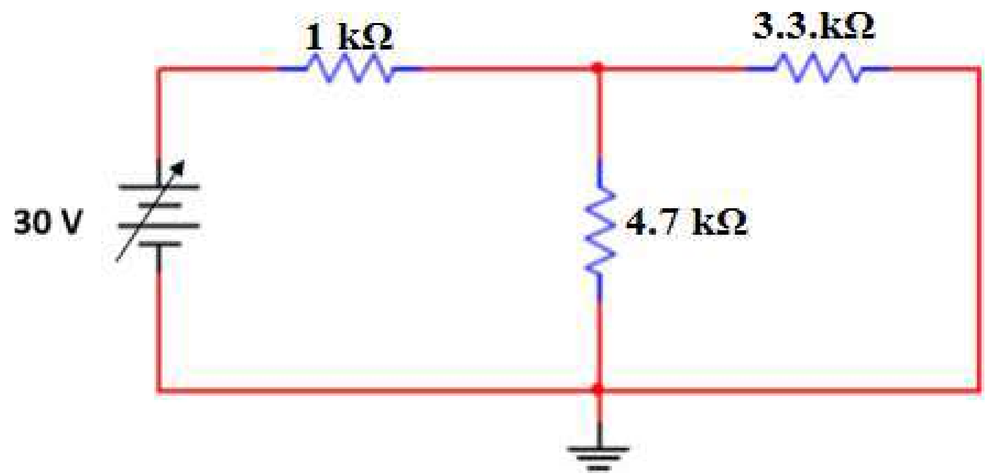
current flow from “n” to “y” will be 4.21A

current flow from “n” to “z” will be 2.1A



Example 2.4

- Find the current flowing through 1k Ohm resistor using KCL.



- How to solve?
- Any Idea?

- Think of it.
- We will discuss the solution method later section.

Kirchhoff's Voltage Law (KVL)

- **Law of Conservation of Energy**

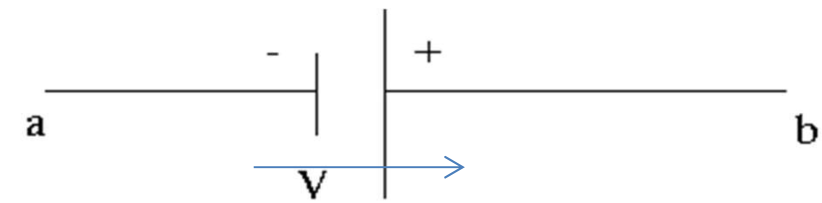
  The algebraic sum of voltages around each loop is zero.

- In other words, in a closed circuit, the algebraic sum of all the EMFs and the algebraic sum of all the voltage drops (product of current (I) and resistance (R)) is zero.

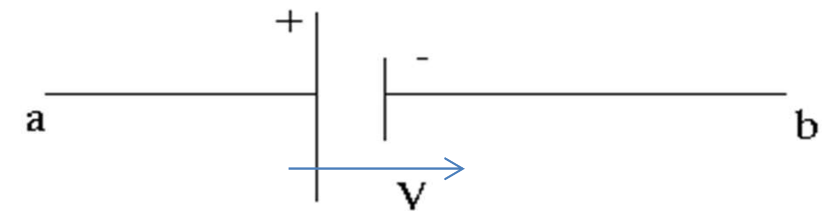
- Σ voltage drops - Σ voltage rises = 0

Or Σ voltage drops = Σ voltage rises

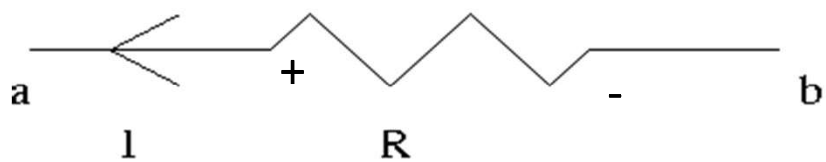
Sign conventions for Kirchoff's loop rule



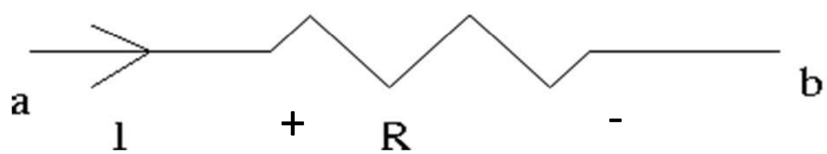
$V_b - V_a = +V$



$V_b - V_a = -V$

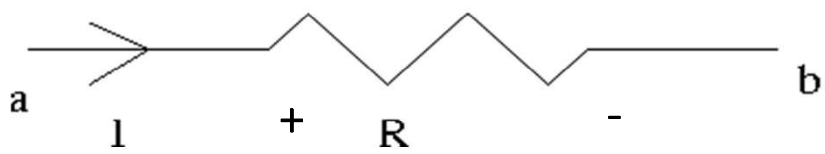


$V_b - V_a = +IR$



$V_b - V_a = -IR$

Sign conventions for Kirchoff's loop rule



$V_b - V_a = -IR$

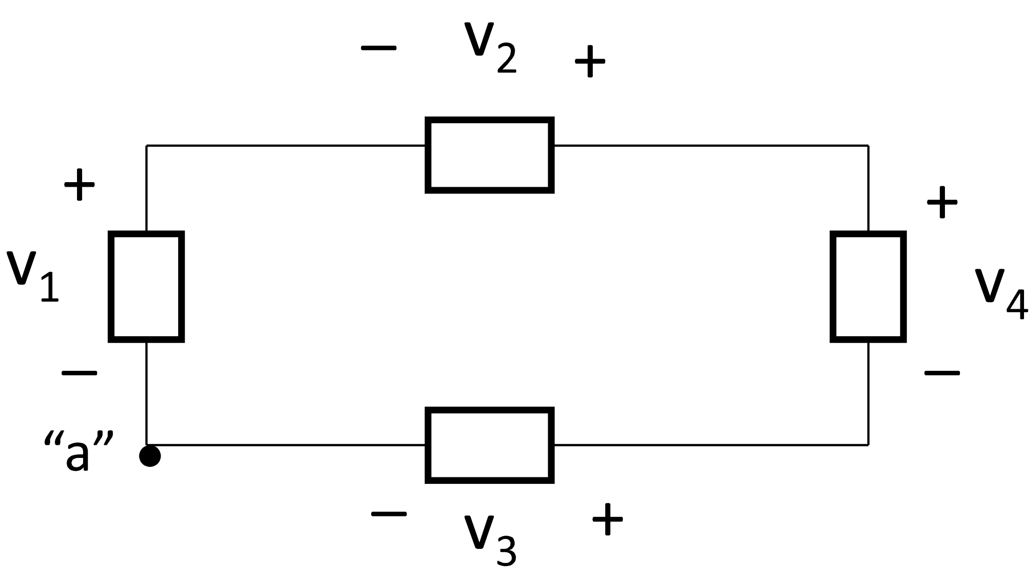
Common Practice:

When a current flows through a resistor, the terminal through which the current enters into the resistor will be considered as positive terminal. The other terminal through which the current leaves out of the resistor will be then negative terminal.

Example 3- KVL

Write KVL equation for the circuit given below.

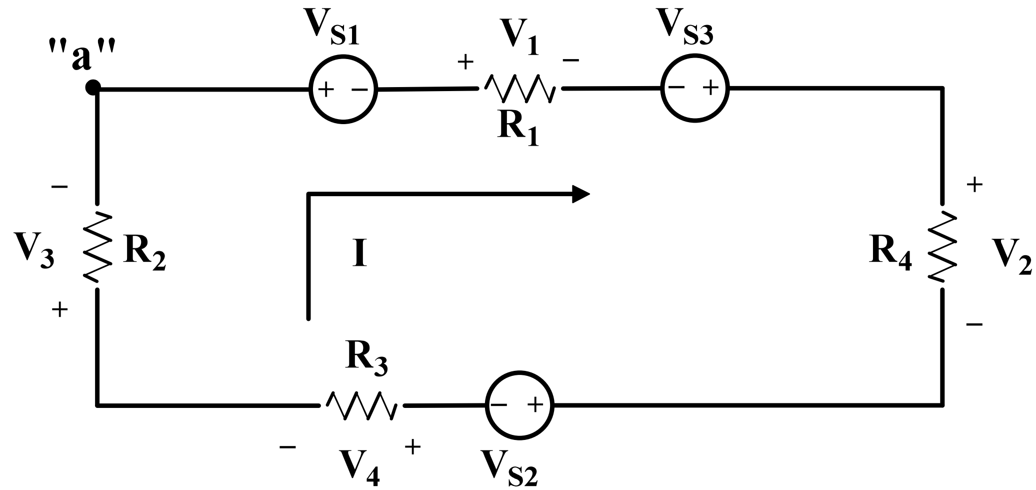
$- v_3 - v_4 + v_2 + v_1 = 0$ drops in Clock Wise direction starting at “a”



$- v_1 - v_2 + v_4 + v_3 = 0$ drops in Counter CW direction starting at “a”

Example 4- KVL

Write KVL equation for the circuit given below.



Starting at point “a”, apply KVL going clockwise, we have

$$-V_{S1} - V_1 + V_{S3} - V_2 - V_{S2} - V_4 - V_3 = 0$$

or

$$-V_{S1} - V_{S2} + V_{S3} = I(R_1 + R_2 + R_3 + R_4)$$

To Analyse circuits using Kirchhoff's laws

1. Draw the circuit.
2. Assign labels to the known and unknown quantities(V1, V2,… R1, R2, etc.) .
3. Label each branch with a branch current. (I1, I2, I3 etc.)Assigns a current to each branch or mesh (clockwise or anticlockwise). (Don't worry if you guess incorrectly the direction of a particular unknown current, as the answer resulting from the analysis in this case will simply come out negative, but with the right magnitude.).
4. Apply the junction rule (KCL) to as many junctions in the circuit as possible to obtain the maximum number of independent relations. Follow the sign conventions.

To Analyse circuits using Kirchhoff's laws

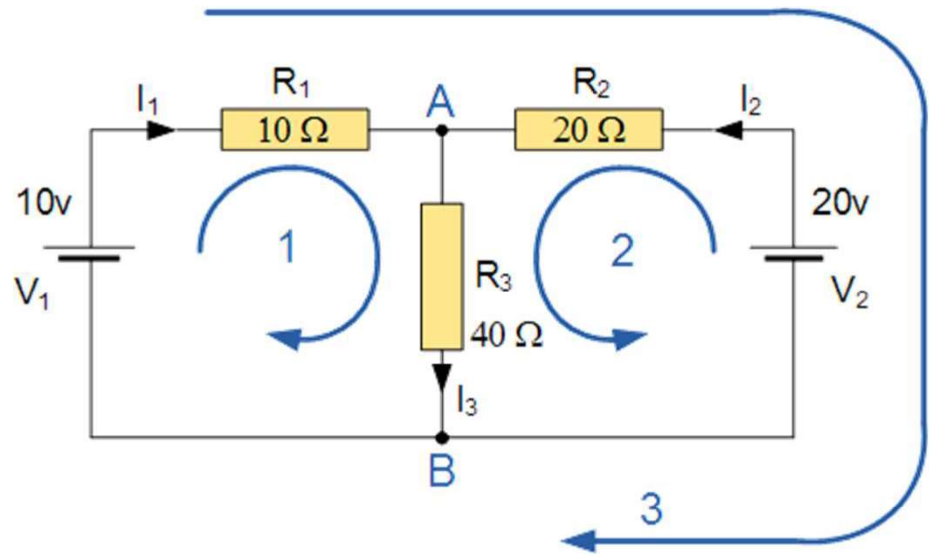
5. Apply the loop rule (KVL) to as many loops in the circuit as necessary in order to solve for the unknowns. Follow the sign convention properly.

   Note that if one has n unknowns in a circuit one will need n independent equations. In general there will be more loops present in a circuit than one needs to solve for all the unknowns; the relations resulting from these ``extra'' loops can be used as a consistency check on your final answers.

6. Solve the resulting set of simultaneous equations for the unknown quantities.

Example 6- KVL +KCL

Using Kirchoffs Circuit Law, find the current flowing in the 40Ω Resistor, R3



The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using **Kirchoffs Current Law**, **KCL** the equations are given as:

At node A : $I_1 + I_2 = I_3$

At node B : $I_3 = I_1 + I_2$

Using **Kirchoffs Voltage Law**, **KVL** the equations are given as:

Loop 1 is given as : $10 = R_1 I_1 + R_3 I_3 = 10I_1 + 40I_3$

Loop 2 is given as : $20 = R_2 I_2 + R_3 I_3 = 20I_2 + 40I_3$

Loop 3 is given as : $10 - 20 = 10I_1 - 20I_2$

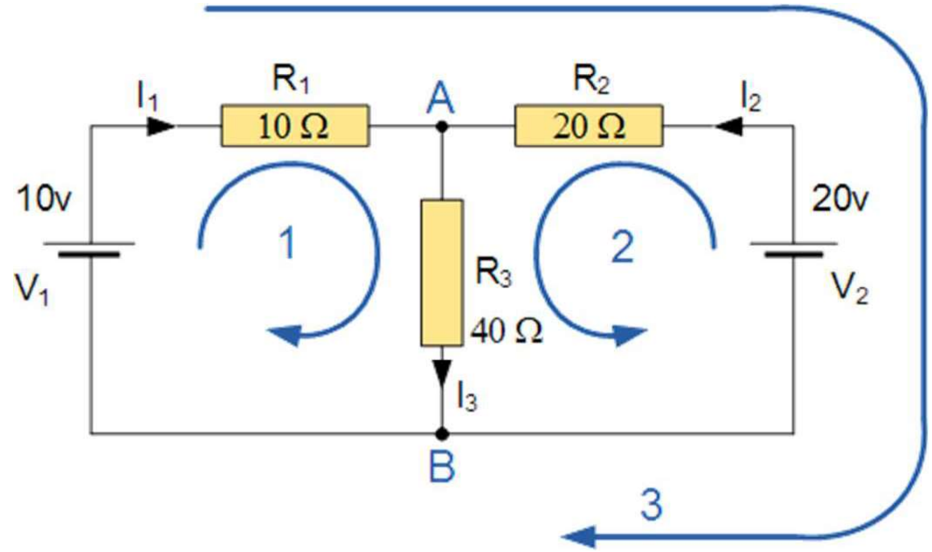
Example 6- KVL +KCL

As I_3 is the sum of $I_1 + I_2$ we can rewrite the equations as;
Eq. No 1 : $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$
Eq. No 2 : $20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2$

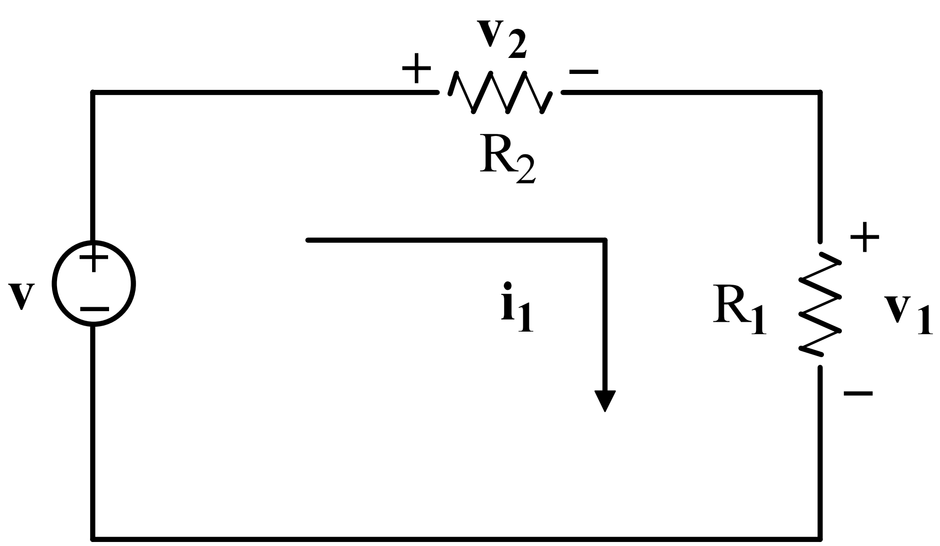
We now have two "**Simultaneous Equations**" that can be reduced to give us the values of I_1 and I_2
Substitution of I_1 in terms of I_2 gives us the value of I_1 as -0.143 Amps
Substitution of I_2 in terms of I_1 gives us the value of I_2 as +0.429 Amps
As : $I_3 = I_1 + I_2$

The current flowing in resistor R_3 is given as :
-0.143 + 0.429 = 0.286 Amps
and the voltage across the resistor R_3 is given as : 0.286 x 40 = 11.44 volts

The negative sign for I_1 means that the actual direction of current flow is opposite to the initially chosen.



Voltage Divider Rule



Using KVL : $V = v_1 + v_2$

$v_1 = i_1R_1, \quad v_2 = i_1R_2$

$V = i_1(R_1 + R_2)$

$$i_1 = \frac{V}{(R_1 + R_2)}$$

$$\boxed{v_1 = \frac{V * R_1}{(R_1 + R_2)}}$$

Voltage across resistor (R_x) in single loop series circuit = input voltage * R_x/R_{total}

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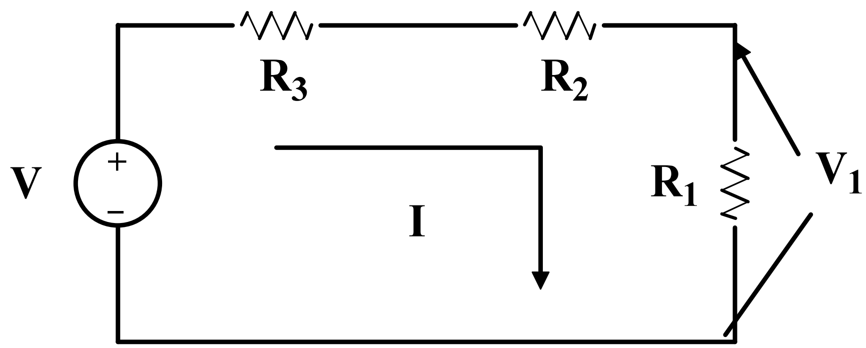
Example 7 -Voltage Divider Rule

Find V_1 in the circuit shown using voltage divider rule.

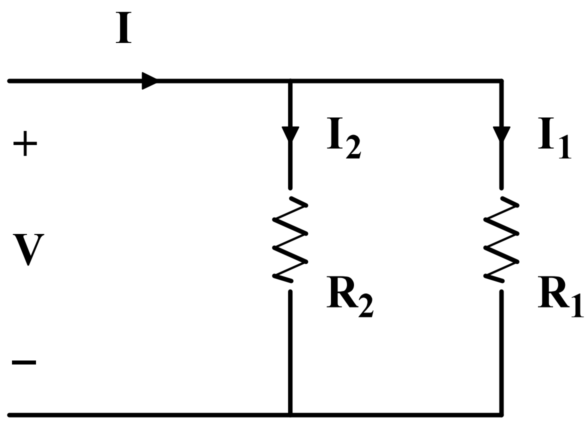


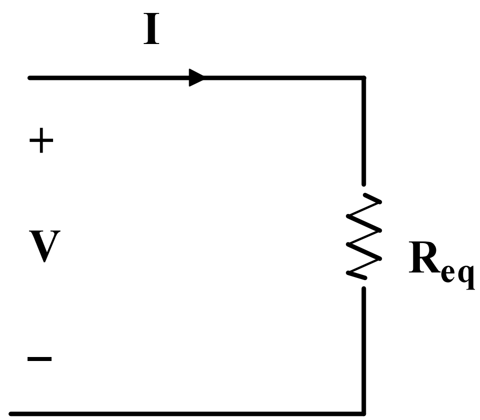
Voltage across resistor (R_x) in single loop series circuit = input voltage * R_x/R_{total}

$$V_1 = \frac{VR_1}{(R_1 + R_2 + R_3)}$$

Current division Rule

Single Node Parallel Circuits



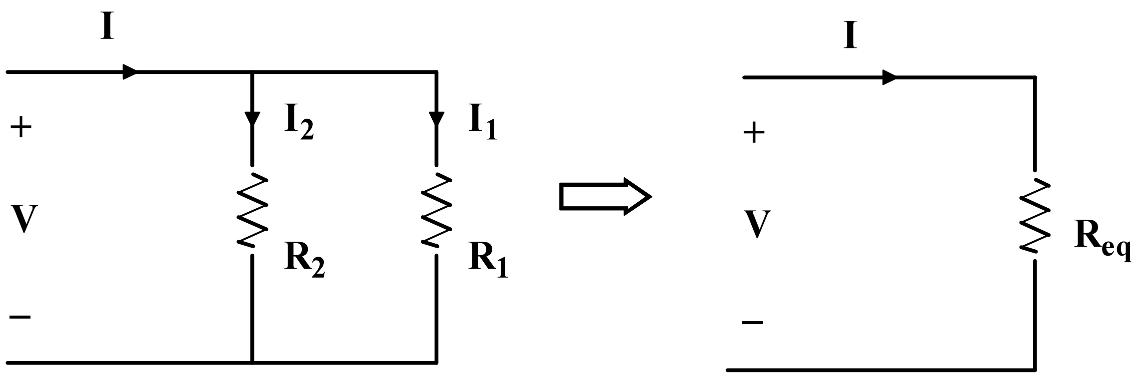


Apply KCL $\quad I = I_1 + I_2 \quad = \dfrac{V}{R_1} + \dfrac{V}{R_2}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \qquad I = \frac{V}{R_{eq}} \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Current division Rule

$$I_1 = \frac{V}{R_1} = \frac{IR_{eq}}{R_1} = \frac{IR_2}{R_1 + R_2}$$



$$I_1 = \frac{IR_2}{R_1 + R_2}$$

$$I_2 = \frac{IR_1}{R_1 + R_2}$$

General case for current division

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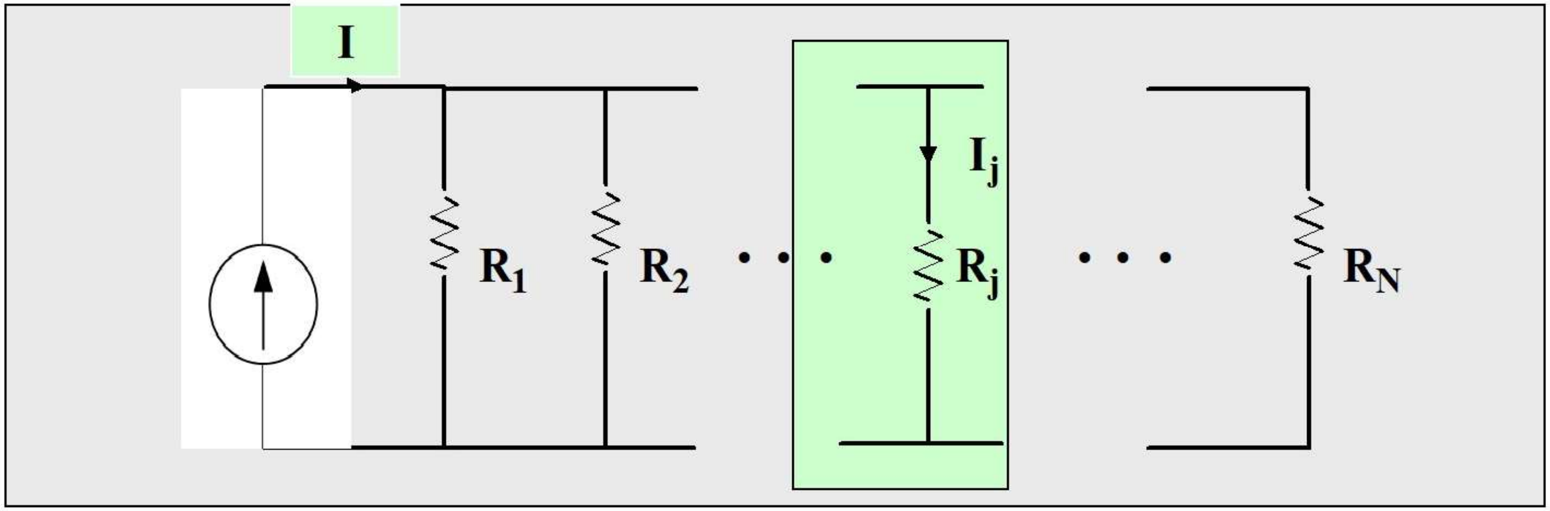


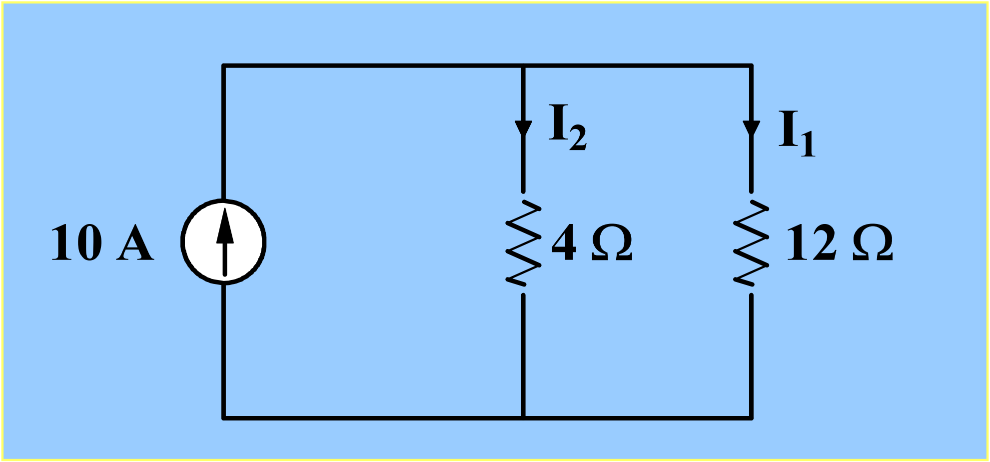
In general, if we have N resistors in parallel and we want to find the current in, say, the jth resistor

$$I_j = \frac{IR_{eq}}{R_j}$$

Req : Equivalent parallel resistance.

Example 8 -Current Division Rule

Find the currents I_1 and I2 using the current division.



By direct application of current division:

$$I_j = \frac{IR_{eq}}{R_j}$$

$$I_1 = \frac{10(4)}{12+4} = 2.5\,A$$

$$I_2 = \frac{10(12)}{12+4} = 7.5\,A$$

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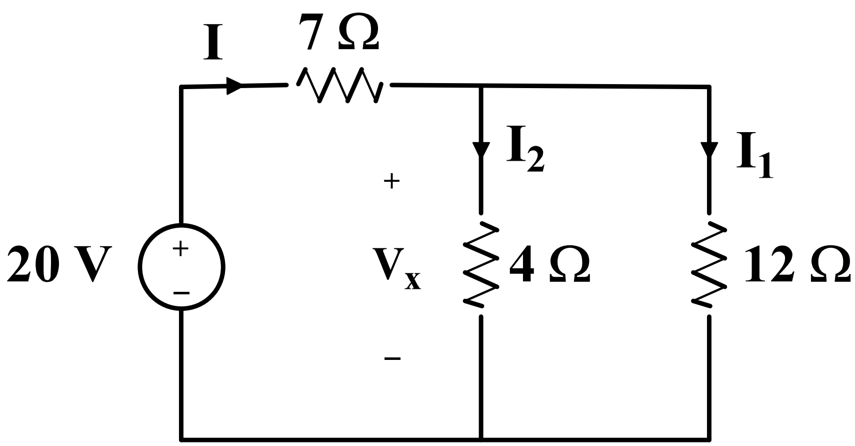
Example 9 -Current Division Rule

Find the currents I_1 and I_2 in the circuit using current division. Also, find the voltage V_x



$$R_{eq} = 7 + \frac{4(12)}{12+4} = 7+3 = 10\ \Omega$$

$$I = \frac{20}{R_{eq}} = \frac{20}{10} = 2\ A$$

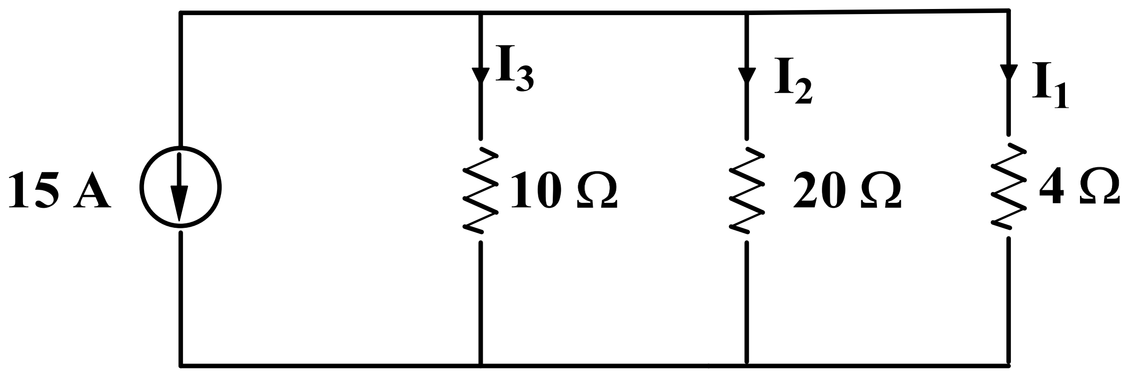
$$I_1 = \frac{2(4)}{12+4} = 0.5\ A$$

$$I_2 = \frac{2(12)}{12+4} = 1.5\ A$$

$V_x = I_1 x12$ or $I_2 x4 = 6$ V.

Example 10 -Current Division Rule

Find the currents I_1, I_2, and I_3 using the current division rule.



$$I_j = \frac{IR_{eq}}{R_j}$$

$$I_1 = \frac{(-15)(R_{eq})}{4}, \qquad I_2 = \frac{(-15)(R_{eq})}{20}, \qquad I_3 = \frac{(-15)(R_{eq})}{10},$$

$$I_1 = \frac{(-15)(2.5)}{4} = -9.375\,A$$

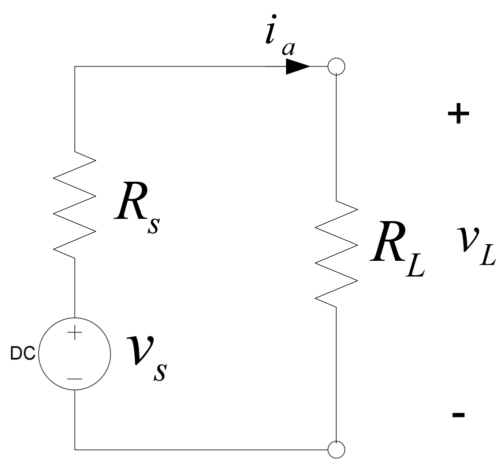
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4} + \frac{1}{20} + \frac{1}{10} = 0.25 + 0.05 + 0.1 = 0.4\,S$$

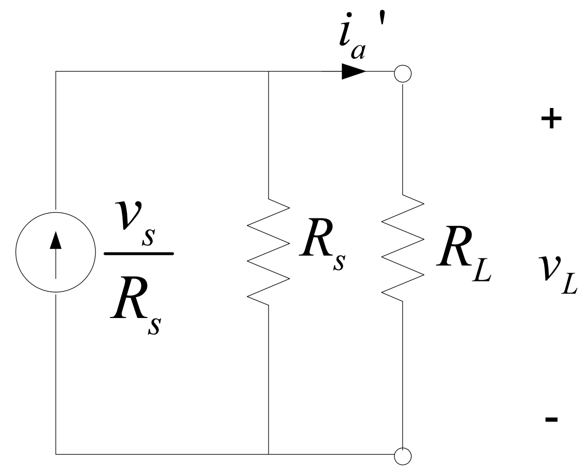
$$I_2 = \frac{(-15)(2.5)}{20} = -1.875\,A$$

$$I_3 = \frac{(-15)(2.5)}{10} = -3.75\,A$$

Notice that $I_1 + I_2 + I_3 = -15$ A

Source Transformation Proof





$$i_a = \frac{v_s}{(R_s + R_L)}$$

$$i_a' = \frac{R_s}{(R_s + R_L)} \frac{v_s}{R_s} = i_a$$

$$v_L = \frac{R_L}{(R_s + R_L)} v_s$$

$$v_L = i_a' R_L = \frac{R_L}{(R_s + R_L)} v_s$$

Voltage across and current through any load are the same

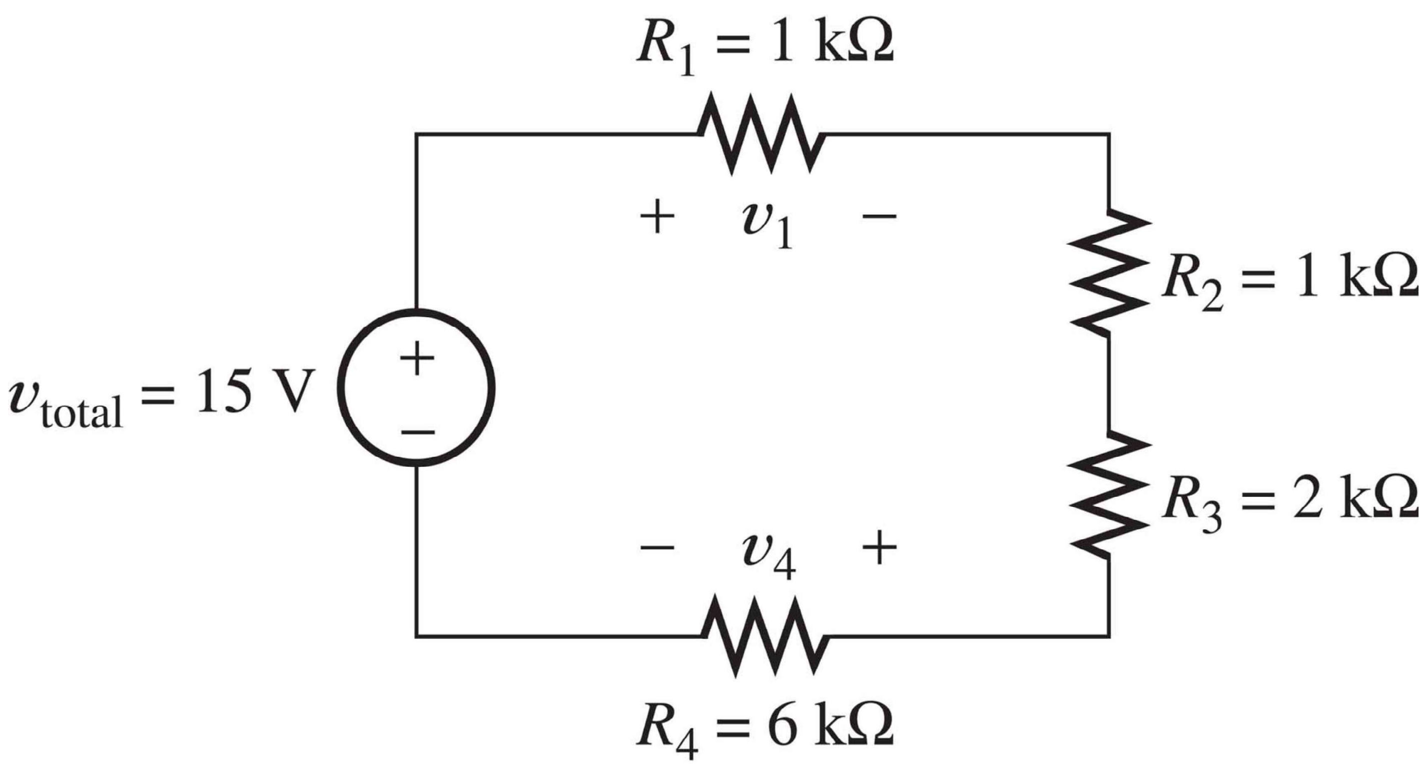
Recap

Junction/Node Law	KCL: $\Sigma I = 0$
Loop Law	KVL: $\Sigma V = 0$

- Voltage across resistor (R_x) in single loop series circuit = input voltage * R_x/R_{total}.
- If we have N resistors in parallel and we want to find the current in, say, the jth resistor

$$I_j = \frac{IR_{eq}}{R_j}$$

KVL + Voltage Division



- Apply KVL and find v1 + v4
- Find v1 using voltage division rule.
- Find v4 using voltage division rule.

Voltage Division

Determine the voltage between the following points in the voltage divider of below circuit.

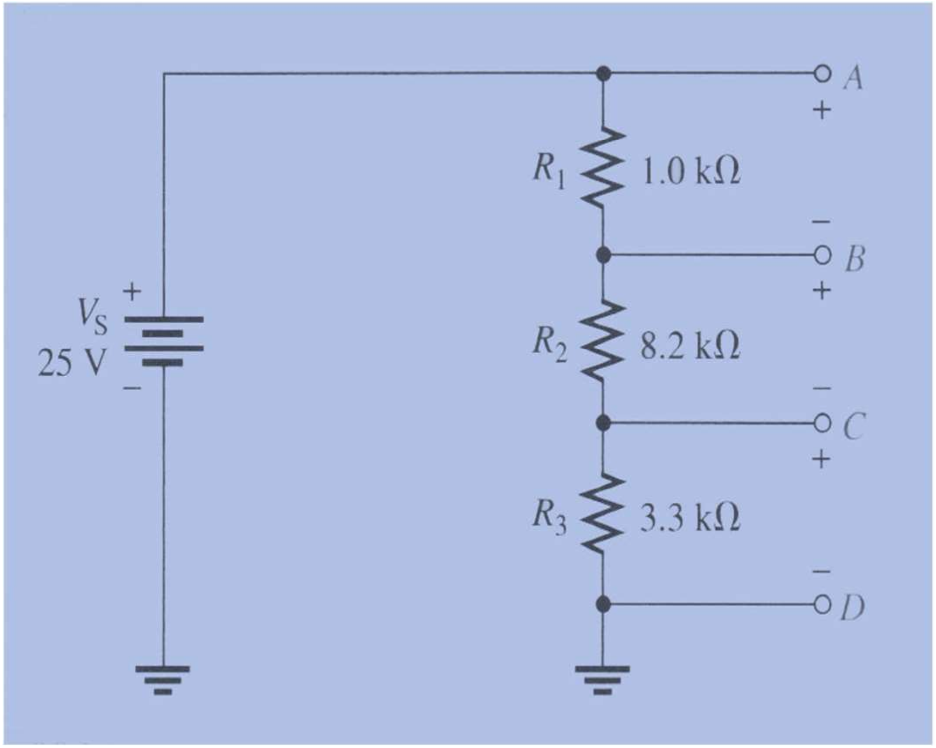
a) A to B

b) A to C

c) B to C

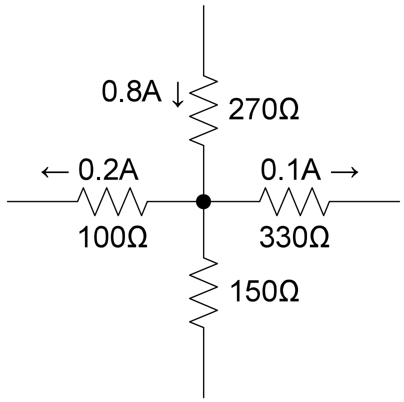
d) B to D

e) C to D



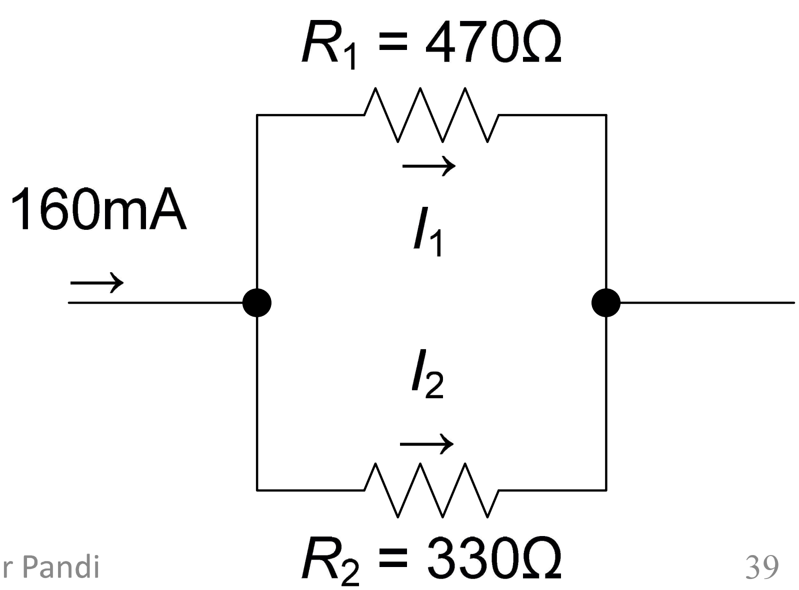
KCL

- Find the current in the 150Ω resistor using KCL.



CURRENT DIVIDER RULE

- Find the current in the 330Ω resistor using the current divider rule, and verify the result using Kirchhoff's current law.
- Find the current in the 470Ω resistor using the current divider rule, and verify the result using Kirchhoff's current law.
- Find current in 470 Ω resistor , if 330Ω resistor is shorted.
- Find current in 330Ω resistor , if 470 Ω resistor is shorted.



> "Amrita Vishwa Vidyapeetham has a major role to play in transforming our society into a knowledge society through its unique value-added education system."

Dr. A.P.J. Abdul Kalam
Former President of India

THANK YOU