Problem 4.5

i) If $\lambda < 0$,

Aug error $E_{aug}(w) = E_{in}(w) + \lambda w^T w$

The soft order constraint is given by

$$\sum_{q=0}^{Q} w_q^2 \leq 0$$

The insample optimization problem is given by

$$\min_{w} E_{in}(w) \text{ subject to } w^T w \leq C \longrightarrow (1)$$

The augumented error of the soft order constraint is given by

$$E_{aug} = E_{in}(w) + \lambda w^T w \longrightarrow (2)$$

for the unconstrained optimization problem from (3) to be minimizing augmented error the soft order constraint 'C' constant term should be $\lambda_C = -\lambda C$. Hence $w^T w \leq C$.

Problem-4.6

→ From example 4.2 from text book the aug error is

$$E_{aug} = (w - w_{in})^T Z^T Z (w - w_{in}) + \lambda w^T w + y^T (I - H) y$$

$$N$$

Given, $w_{reg} = (Z^T Z + \lambda I)^{-1} Z^T y$

For suppose $\|w_{reg}\| > \|w_{in}\|$.

$$\|w_{reg}\| = (w_{reg})^2 \quad \rightarrow (1)$$

$$E_{aug}(w_{reg}) = E_{in}(w_{reg}) + \lambda \|w_{reg}\|^2$$

$$E_{aug}(w_{in}) = E_{in}(w_{in}) + \lambda \|w_{in}\|^2 \quad \rightarrow (2)$$

We know that $E_{aug}(w_{reg}) = \arg\min_{w} E_{aug}(w)$

Hence, from equations (1) and (2) based on the hypothesis

$||w_{reg}||$ cannot be greater than $||w_{ein}||$. So $||w_{reg}|| < ||w_{ein}||$.

(b) $w_{reg}^T w_{reg} = u^T (Z^T Z + \lambda I)^{-2} u$

Also given that $u = Z^T y$.

It can also be written as.

$$||w_{reg}||^2 = u^T (Z^T Z + \lambda I)^{-2} u$$

Substituting $u = Z^T y$.

$$u^T = (Z^T y)^T = y^T Z$$

$$||w_{ein}||^2 = y^T Z (Z^T Z + \lambda I)^{-2} Z^T y.$$