AI1110
Assignment 2

Nishanth Bhoomi
CS21BTECH11040

Outline

1. Question
2. Solution
3. Answer

Question 12

A cone is inscribed in a sphere of radius 12 cm.If the volume of the cone is maximum,find its height.

Solution

(1)

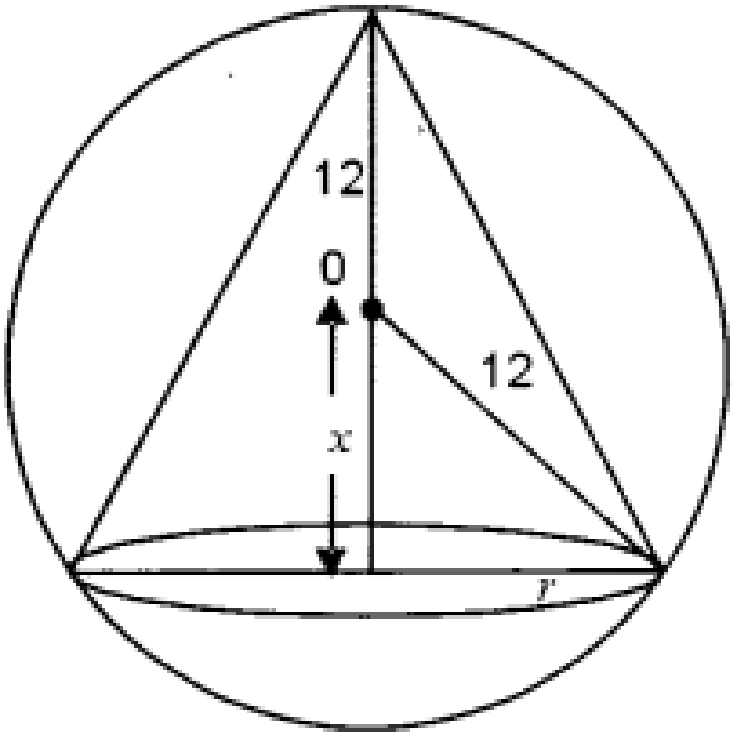

Figure 1: Cone inscribed in a sphere

Let radius of cone be r and its height be h.

Here, $r^2 + x^2 = 12^2$

$r^2 = 144 - x^2$

$Now, the volume of the cone = \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3} \times \pi \times (144 - x^2) \times (12 + x)$

$= \frac{1}{3} \times \pi \times (12 - x) \times (12 + x)^2$

Now, $\frac{dV}{dx} = \frac{d}{dx}(\frac{1}{3} \times \pi \times (12 - x) \times (12 + x)^2)$

$= \frac{1}{3} \times \pi \times (-(12 + x)^2 + 2 \times (144 - x^2))$

$\frac{d^2V}{dx^2} = \frac{1}{3} \times \pi \times (-2 \times (12 + x) + 2 \times (0 - 2x))$

$= \frac{1}{3} \times \pi \times (-24 - 2x - 4x)$

$= \frac{1}{3} \times \pi \times -(24 + 6x)$

Here,second derivative is negative,implies volume of cone is maximum.

Now, put $\frac{dV}{dx} = 0$, we obtain,

$\frac{\pi}{3} \times -(12 + x)^2 + 2 \times (144 - x^2)$

$\implies (12 + x) \times (-(12 + x) + 2 \times (12 - x) = 0$

$\implies (12 + x) \times -12 - x + 24 - 2 \times x = 0$

$\implies (12 + x) \times (12 - 3x) = 0$

$\implies$ (12+x =0) or (12-3x =0)

$\implies$ x = -12 or x = 4

Answer

As distance cannot be negative, x = 4.
Hence, for maximum volume of cone, the height of the cone is h=12+x where x=4cm
So, height of the cone is 16cm.