Assignment 2

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A cone is inscribed in a sphere of radius 12 cm.If the volume of the cone is maximum,find its height.

Solution:

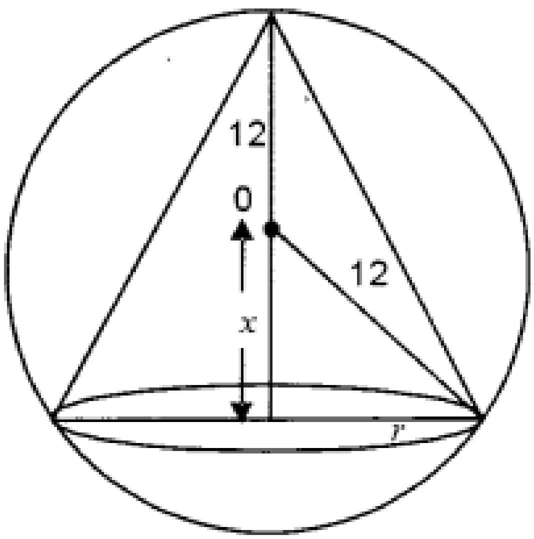

Fig. 0. Cone inscribed in a sphere

Let radius of cone be r and its height be h.
Here, r$^2 + x^2 = 12^2$
$r^2 = 144 - x^2$
Now, the volume of the cone = $\frac{1}{3} \times \pi \times r^2 \times h$
$= \frac{1}{3} \times \pi \times (144 - x^2) \times (12 + x)$
= $\frac{1}{3} \times \pi \times (12 - x) \times (12 + x)^2$

Now,$\frac{dV}{dx} = \frac{d}{dx}(\frac{1}{3} \times \pi \times (12 - x) \times (12 + x)^2)$
$= \frac{1}{3} \times \pi \times (-(12 + x)^2 + 2 \times (144 - x^2))$

$\frac{d^2V}{dx^2} = \frac{1}{3} \times \pi \times (-2 \times (12 + x) + 2 \times (0 - 2x))$

=$\frac{1}{3} \times \pi \times (-24 - 2x - 4x)$
=$\frac{1}{3} \times \pi \times -(24 + 6x)$

Here,second derivative is negative,implies volume of cone is maximum. Now, put $\frac{dV}{dx} = 0$, we obtain,

$\frac{\pi}{3} \times -(12 + x)^2 + 2 \times (144 - x^2)$
$\implies (12 + x) \times (-(12 + x) + 2 \times (12 - x) = 0$
$\implies (12 + x) \times -12 - x + 24 - 2 \times x = 0$
$\implies (12 + x) \times (12 - 3x) = 0$
$\implies$ (12+x =0) or (12-3x =0)
$\implies$ x = -12 or x = 4

As distance cannot be negative, x = 4.
Hence, for maximum volume of cone, the height of the cone is h=12+x where x=4cm
So, height of the cone is 16cm.