



DEPARTMENT OF MATHEMATICS  
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

End-Sem Assignment Odd Semester of 2015-2016

**MA322: SCIENTIFIC COMPUTING**

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**Instructions:**

1. This question paper contains 2 questions in parts.
  2. Be advised that the assignment part accounts for 25% of your overall grade.
  3. The Statements you make use of to solve the problems **MUST** be clearly written.
  4. You must return the assignments (**printed or written hard-copies as well as the programmes through e-mail to your TA**) before the commencement of your end-sem exam on 22nd November 2015.
  5. The front-page of the assignment report must bear your group name along with the names, roll numbers and signatures of the group members.
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1. A thin rectangular homogeneous thermally conducting plate lies in the  $xy$ -plane defined by  $0 \leq x \leq 4$ ,  $0 \leq y \leq 1$ . The edge  $y = 0$  is maintained at temperature  $200x(x - 4)$ , while the remaining edges are held at  $0^\circ$ . The other faces are insulated and no sources and sink are present.

Set the partial differential equation governing the steady-state temperature  $T(x, y)$  along with the boundary conditions. Then solve it numerically using a known finite difference scheme. Use the Gauss-Seidel method as the iterative solver. You must experiment

(a) with  $\Delta x = 0.1, 0.05, 0.025$

(b) with three different grid aspect ratios given by  $\beta = \frac{\Delta x}{\Delta y}$  (preferably  $1, < 1, > 1$ ).

The analytical solution of this problem is given by

$$T(x, y) = \sum_{n=1}^{\infty} \operatorname{cosech} \left( -\frac{n\pi}{4} \right) \frac{12800}{n^3 \pi^3} [(-1)^n - 1] \sin \left( \frac{n\pi}{4} x \right) \sinh \left[ \frac{n\pi}{4} (y - 1) \right]$$

Compare graphically the analytical solution along the vertical centerline with the numerical ones. (There should be three graphs, one for each  $\beta$ . Each figure will have the analytical solution along the vertical centerline with the numerical ones obtained for  $\Delta x = 0.1, 0.05, 0.025$ .)

What can you comment upon the efficiency of the iterative solvers? Perform further experiment on the Gauss-Seidel iterative solver by using **SOR** and comment upon the optimum  $\omega$  for each of the grids.

(25)+(5)+(20).

2. Solve numerically the following convection-diffusion equation for  $Re = 10, 50$  and  $100$ . Use central differences for discretizing the derivatives and then solve the system of the linear algebraic equations by Thomas Algorithm.

$$-\frac{\partial^2 u}{\partial x^2} + Re \frac{\partial u}{\partial x} = 0, \quad 0 \leq x \leq 1$$

with boundary conditions  $u(0) = 0$  and  $u(1) = 1$ . The analytical solution to this problem is

$$u(x) = \frac{e^{Re x} - 1}{e^{Re} - 1}$$

Use  $\Delta x = 0.1, 0.05, 0.025$ . Compare graphically (There should be three graphs, one for each  $Re$ . Each figure will have the analytical solution with the numerical ones obtained for  $\Delta x = 0.1, 0.05, 0.025$ .) the analytical and the numerical solutions obtained by you. What observation can be made based upon your numerical results with increasing  $Re$  and decreasing  $\Delta x$ ? (30)+(20).

**NOTE:** Your report must include write-ups on the finite difference methods being constructed and used for solving the two partial differential equations numerically.

HAPPY COMPUTING.