Image Processing Lab

Frequency Filtering

GROUP-10

Submitted by:

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Objective

Write C++/Image-J modular functions to perform the following operations on the given test image, ricegrains_mono.bmp. All functions must support binary images.

1. Make separate functions for erosion, dilation, opening, and closing of binary images
   a. **ErodeBinary, DilateBinary**
      Input: Binary image, structuring element
      Output: Eroded/dilated image
   b . **OpenBinary, CloseBinary**
      Input: Binary image, structuring element
      Output: Opened/closed image

Use structuring elements:

1	1

0	1	0
1	1	1
0	1	0

and 3x3, 9 × 9, 15 × 15 kernels of grayvalue = 1 (reference point – centre pixel).

Introduction

Mathematical Morphology

Mathematical morphology (MM) is a theory and technique for the analysis and processing of geometrical structures, based on set theory, lattice theory, topology, and random functions. MM is most commonly applied to digital images, but it can be employed as well on graphs, surface meshes, solids, and many other spatial structures.

The basic morphological operators are erosion, dilation, opening and closing.

MM was originally developed for binary images, and was later extended to grayscale functions and images. The subsequent generalization to complete lattices is widely accepted today as MM's theoretical foundation.

Binary Morphology

In binary morphology, an image is viewed as a subset of an Euclidean space $\mathbf{R}^{\mathbf{d}}$ or the integer grid $\mathbf{Z}^{\mathbf{d}}$, for some dimension d.
The image is assumes to be composed f only two intensity values corresponding to black and white (0 and 255).
A most important factor which defines the type of morphological operation is the structuring element.

Structuring Element

The basic idea in binary morphology is to probe an image with a simple, pre-defined shape, drawing conclusions on how this shape fits or misses the shapes in the image. This simple "probe" is called the structuring element, and is itself a binary image (i.e., a subset of the space or grid).

Here are some examples of widely used structuring elements (denoted by B):

Let $\mathbf{E} = \mathbf{R}^2$: B is an open disk of radius r, centered at the origin.
Let $\mathbf{E} = \mathbf{Z}^2$ **:** B is a 3x3 square, that is, B={(-1,-1), (-1,0), (-1,1), (0,-1), (0,0), (0,1), (1,-1), (1,0), (1,1)}.

Let **E = Z²** **:** B is the "cross" given by: B={(-1,0), (0,-1), (0,0), (0,1), (1,0)}.

Now let’s have a look at the basic operators in Mathematical Morphology.

Let E be a Euclidean space or an integer grid, and **A** a binary image in **E**.

Erosion

The erosion of the binary image A by the structuring element B is defined by:

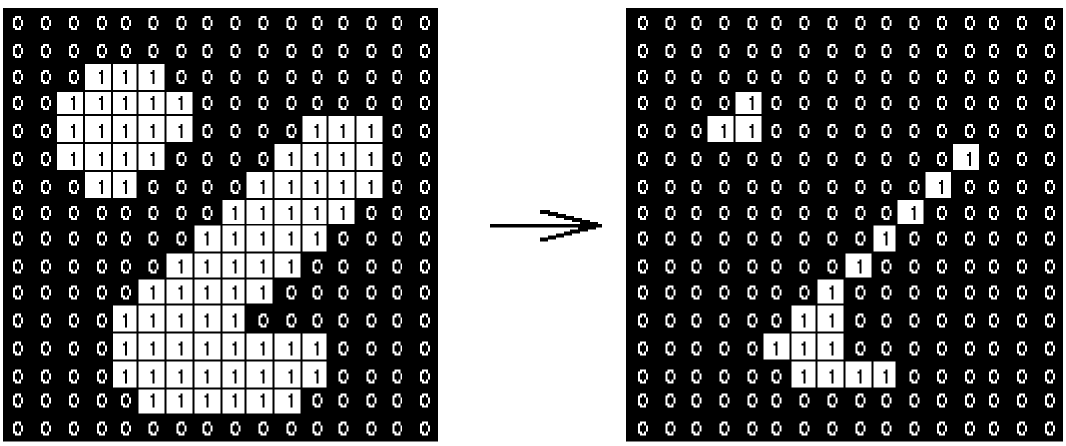
$$A \ominus B = \{z \in E | B_z \subseteq A\}$$

where $\mathbf{B_z}$ is the translation of **B** by the vector **z**, i.e., $B_z = \{b + z | b \in B\} \quad \forall z \in E$

When the structuring element B has a center (e.g., B is a disk or a square), and this center is located on the origin of E, then the erosion of A by B can be understood as the locus of points reached by the center of B when B moves inside A.

Erosion is one of the two basic operators in the area of mathematical morphology, the other being dilation. It is typically applied to binary images, but there are versions that work on grayscale images. The basic effect of the operator on a binary image is to erode away the boundaries of regions of foreground pixels (i.e. white pixels, typically).

To compute the erosion of a binary input image by this structuring, for each foreground pixel (which we will call the input pixel) we superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel coordinates. If for every pixel in the structuring element, the corresponding pixel in the image underneath is a foreground pixel, then the input pixel is left as it is. If any of the corresponding pixels in the image are background, however, the input pixel is also set to background value.



Dilation

The dilation of **A** by the structuring element **B** is defined by:

$$A \oplus B = \bigcup_{b \in B} A_b$$

If B has a center on the origin, as before, then the dilation of A by B can be understood as the locus of the points covered by B when the center of B moves inside A. In the above example, the dilation of the square of side 10 by the disk of radius 2 is a square of side 14, with rounded corners, centered at the origin. The radius of the rounded corners is 2.

The dilation can also be obtained by:

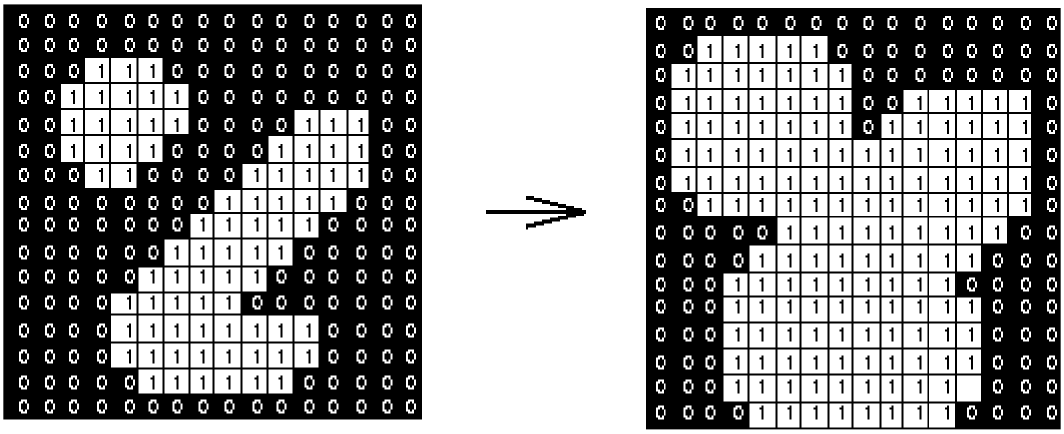
$$A \oplus B = \{z \in E | (B^s)_z \cap A \neq \varnothing\}$$

where Bs denotes the symmetric of B, that is,

$$B^s = \{x \in E | -x \in B\}$$

Dilation is one of the two basic operators in the area of mathematical morphology, the other being erosion. It is typically applied to binary images, but there are versions that work on grayscale images. The basic effect of the operator on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (i.e. white pixels, typically).

To compute the dilation of a binary input image by this structuring element, for each background pixel (which we will call the input pixel) we superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position. If at least one pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value. If all the corresponding pixels in the image are background, however, the input pixel is left at the background value.



Opening

The opening of A by B is obtained by the erosion of A by B, followed by dilation of the resulting image by B:

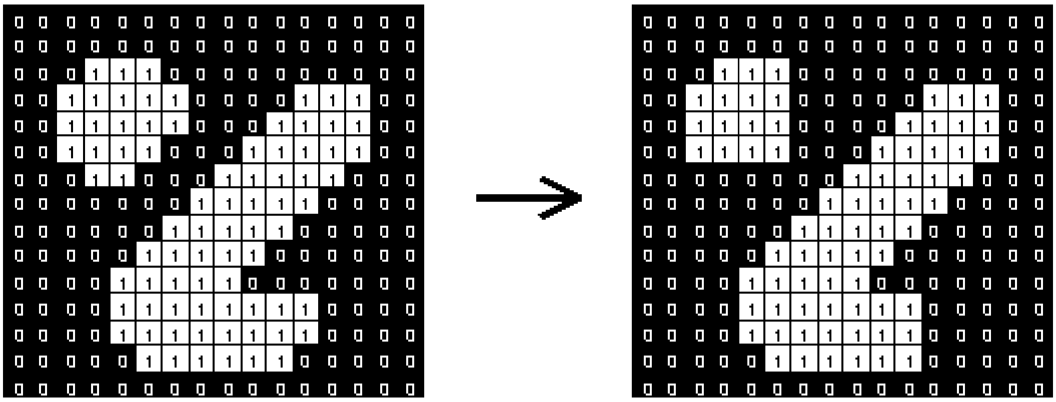
$$A \circ B = (A \ominus B) \oplus B$$

Opening and closing are two important operators from mathematical morphology. They are both derived from the fundamental operations of erosion and dilation. Like those operators they are normally applied to binary images, although there are also graylevel versions. The basic effect of an opening is somewhat like erosion in that it tends to remove some of the foreground (bright) pixels from the edges of regions of foreground pixels. However it is less destructive than erosion in general.

While erosion can be used to eliminate small clumps of undesirable foreground pixels, e.g. `salt noise', quite effectively, it has the big disadvantage that it will affect all regions of foreground pixels indiscriminately. Opening gets around this by performing both an erosion and a dilation on the image. The effect of opening can be quite easily visualized. Imagine taking the structuring element and sliding it around inside each foreground region, without changing its orientation.

All pixels which can be covered by the structuring element with the structuring element being entirely within the foreground region will be preserved. However, all foreground pixels which cannot be reached by the structuring element without parts of it moving out of the foreground region will be eroded away. After the opening has been carried out, the new boundaries of foreground regions will all be such that the structuring element fits inside them, and so further openings with the same element have no effect.

The property is known as idempotence. The effect of an opening on a binary image using a 3×3 square structuring element is illustrated in Figure.



Closing

The closing of A by B is obtained by the dilation of A by B, followed by erosion of the resulting structure by B:

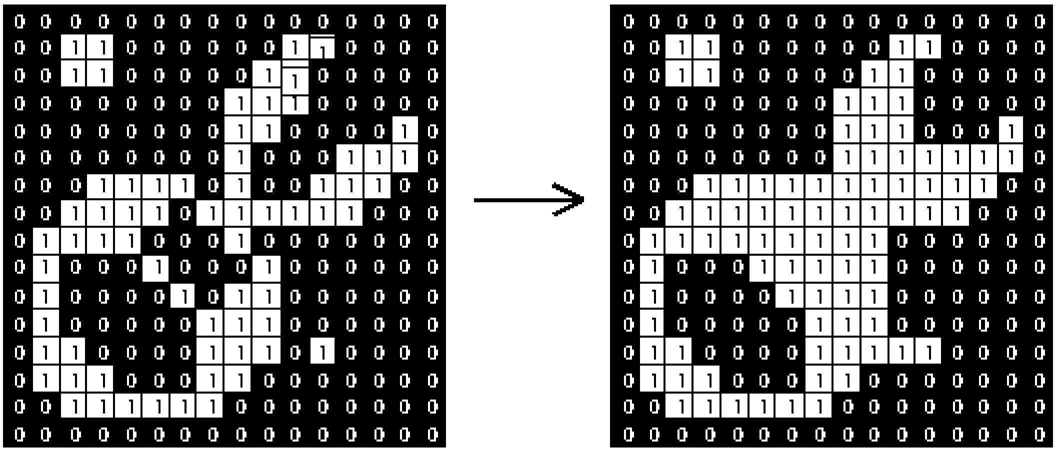
$$A \bullet B = (A \oplus B) \ominus B$$

Closing is an important operator from the field of mathematical morphology. Like its dual operator opening, it can be derived from the fundamental operations of erosion and dilation. Closing is similar in some ways to dilation in that it tends to enlarge the boundaries of foreground (bright) regions in an image (and shrink background color holes in such regions), but it is less destructive of the original boundary shape.

One of the uses of dilation is to fill in small background color holes in images, e.g. `pepper noise'. One of the problems with doing this, however, is that the dilation will also distort all regions of pixels indiscriminately. By performing an erosion on the image after the dilation, i.e. a closing, we reduce some of this effect.

The effect of closing can be quite easily visualized. Imagine taking the structuring element and sliding it around outside each foreground region, without changing its orientation. For any background boundary point, if the structuring element can be made to touch that point, without any part of the element being inside a foreground region, then that point remains background. If this is not possible, then the pixel is set to foreground.

Closing also exhibits the property of idempotence.
The effect of a closing on a binary image using a 3×3 square structuring element is illustrated in Figure.



Algorithm

Depending on the choice in the trackbar system the code executes on the selected image, with the selected operation employing the selected structuring element.
The Graphical User Interface is user-friendly and allows real-time changes in any of the parameters, displaying the outputs automatically.
The code has structures and typedefs used to represent the various data types used.
The functions which carry out the morphological operations are described as below:

erode: Performs erosion of the input image based on the supplied structuring element

Input: Input Image, Structuring element kernel
Output: Output eroded image

$$\mathtt{dst}(x, y) = \min_{(x', y'):\, \mathtt{element}(x', y') \neq 0} \mathtt{src}(x + x', y + y')$$

dilate: Performs dilation of the input image based on the supplied structuring element

Input: Input Image, Structuring element kernel
Output: Output dilated image

$$\mathtt{dst}(x, y) = \max_{(x', y'):\, \mathtt{element}(x', y') \neq 0} \mathtt{src}(x + x', y + y')$$

open: Performs opening of the input image based on the supplied structuring element

Input: Input Image, Structuring element kernel
Output: Output opened image

- `Erode input image`
- `Dilate eroded input image`

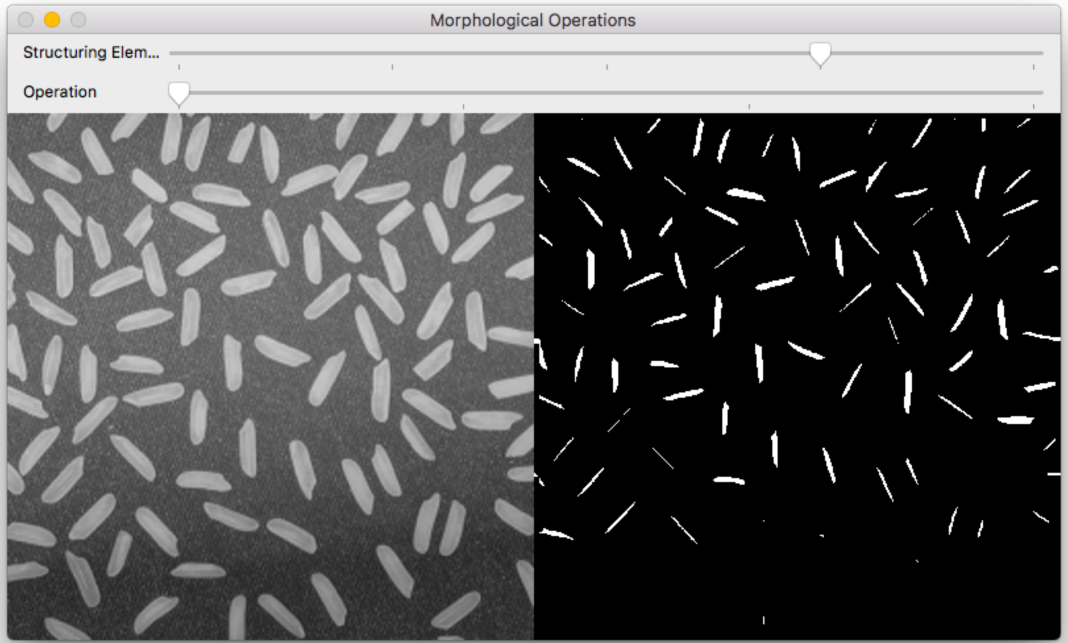
close: Performs closing of the input image based on the supplied structuring element

Input: Input Image, Structuring element kernel
Output: Output closed image

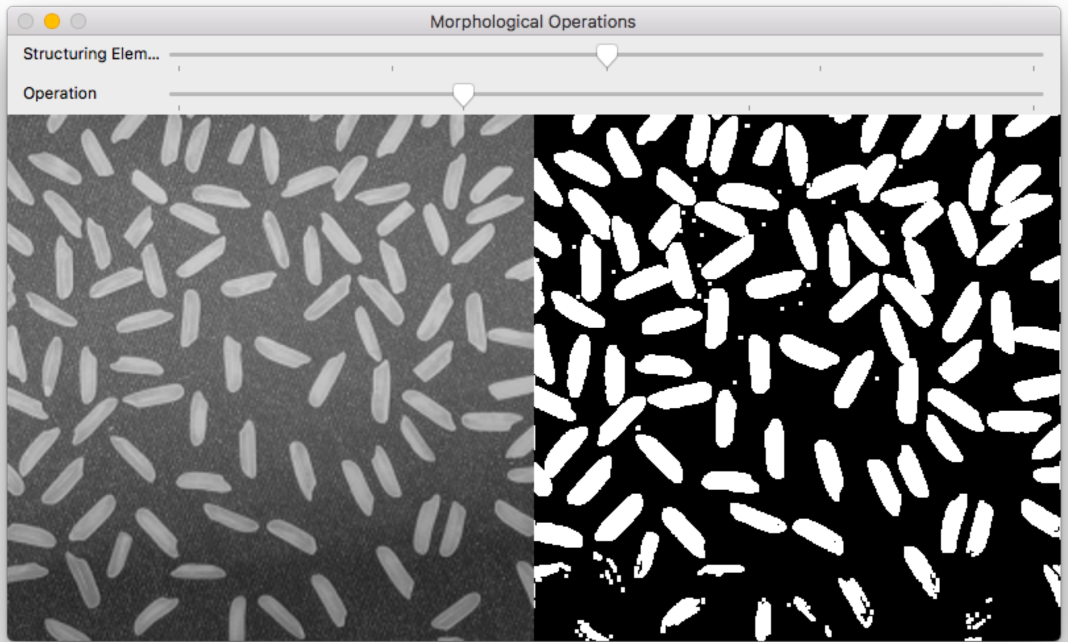
- `Dilate input image`
- `Erode dilated input image`

Results

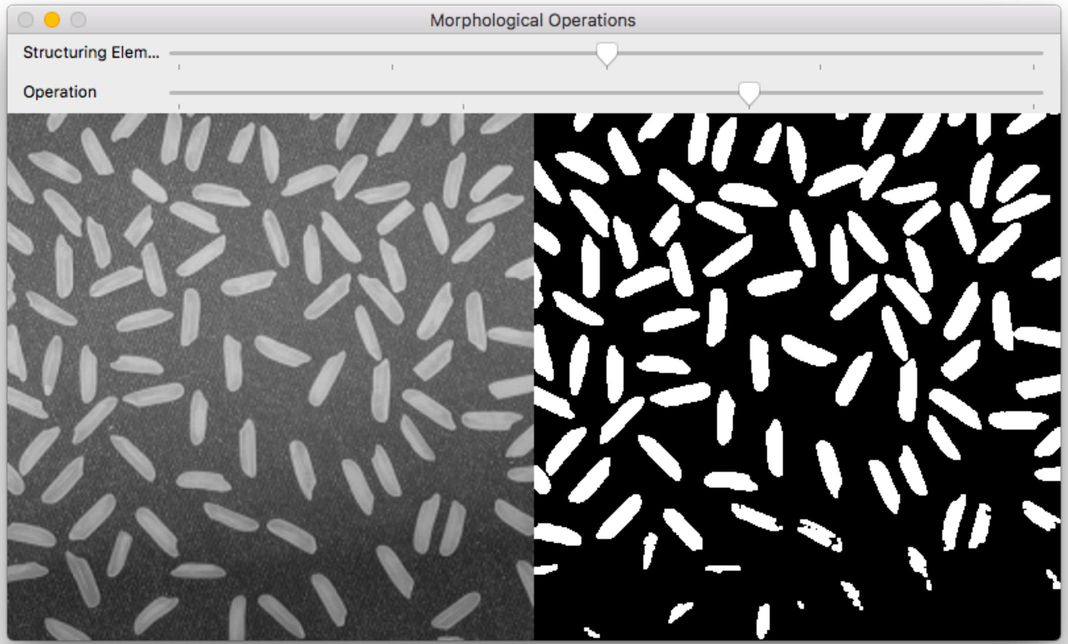
Erosion



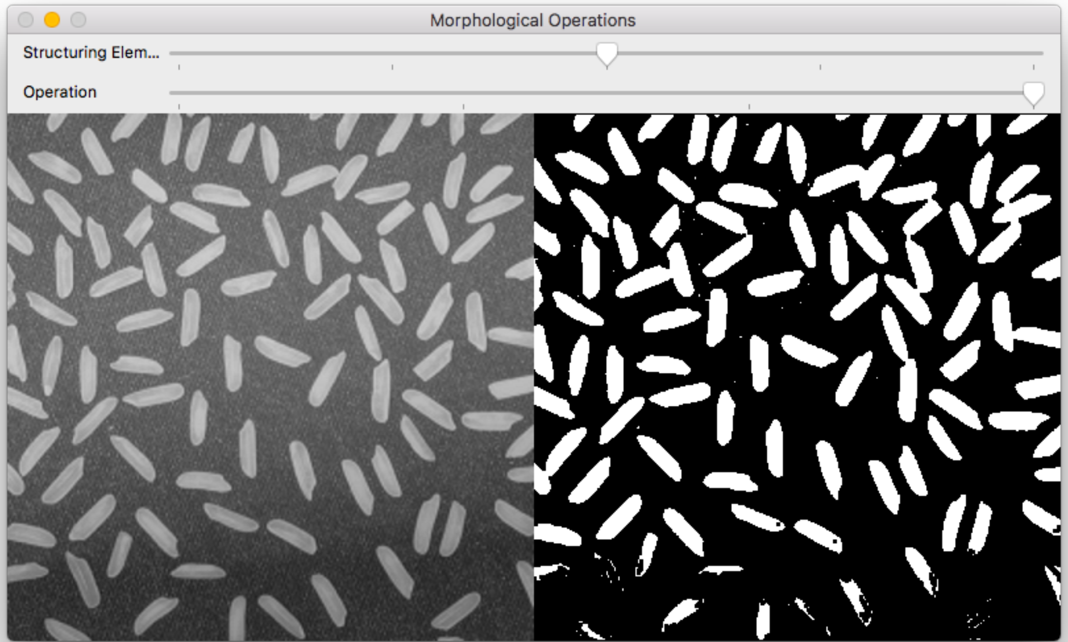
Dilation



Opening



Closing



Analysis Aditya Narayan [13EC35010]

1. Morphological operators are the set of mathematical operators that operate on point sets, including binary images. It makes use of set operations like union, intersection, translation and negation.

2. Erosion is used to reduce the size of an object in the image. The image pixels at the edge of the object surface are trimmed out. Protrusions from the object are thus eliminated.

3. Dilation is used to increase the size of an object in the image. The cavities inside the object boundaries are filled up.

4. The opening operation aims to achieve erosion without incurring the cost of object size reduction. Thus the dilation operation is performed after an erosion operation to achieve opening.

5. The closing operation aims to fill in the cavities inside the object without incurring the cost of image size enlargement. The erosion operation is performed after a dilation operation to achieve closing.

6. Thus we see that erosion and dilation aren't commutative. However, a closing operation negates the effects of an opening operation. Thus they are commutative.

7. We can achieve an improvement in the asymptotic algorithmic performance with large-size structuring elements. Instead of opting for a bigger sized structuring element (and thus incurring polynomial time complexity), we can apply a smaller structuring element repeatedly (and thus incur a linear time complexity).

Analysis Adarsh Kumar Kosta [13EC35008]

1. Morphological operators often take a binary image and a structuring element as input and combine them using a set operator (intersection, union, inclusion, complement). They process objects in the input image based on characteristics of its shape, which are encoded in the structuring element.
2. The effect of Erosion is to remove any foreground pixel that is not completely surrounded by other white pixels (assuming 8-connectedness). Such pixels must lie at the edges of white regions, and so the practical upshot is that foreground regions shrink (and holes inside a region grow).
3. The effect of DIlation is to set to the foreground color any background pixels that have a neighboring foreground pixel (assuming 8-connectedness). Such pixels must lie at the edges of white regions, and so the practical upshot is that foreground regions grow (and holes inside a region shrink).
4. The thing with erosion and dilation is that if these operations are applied multiple times on the image the effects keep on adding up, i.e in case of erosion the foreground pixels will keep on shrinking until all vanish and in case of dilation the foreground region will keep on growing until complete image is occupied by it.
5. The operation of opening involves erosion followed by dilation for the same structuring element. The effect of this operation is that all pixels which can be covered by the structuring element with the structuring element being entirely within the foreground region will be preserved. However, all foreground pixels which cannot be reached by the structuring element without parts of it moving out of the foreground region will be eroded away.
6. The operation of closing involves dilation followed by erosion for the same structuring element. The effect of this operation is that for any background boundary point, if the structuring element can be made to touch that point, without any part of the element being inside a foreground region, then that point remains background. If this is not possible, then the pixel is set to foreground.
7. The unique property of opening and closing is idempotence, which states that After the opening/closing has been carried out the foreground/background region will be such that the structuring element can be made to cover any point in the foreground/background without any part of it also covering a

background/foreground point, and so further openings/closings will have no effect.

8. To achieve the effect of a closing with a larger structuring element, it is possible to perform multiple dilations followed by the same number of erosions.