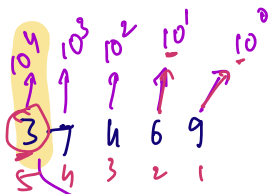


All right \Rightarrow 400 coins

$$\begin{array}{r} 8764 = 8000 \\ + 700 \\ + 60 \\ + 4 \\ \hline = 8 \times 10^3 + 7 \times 10^2 + 6 \times 10^1 + 4 \times 10^0 \end{array}$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $10^3 \quad 10^2 \quad 10^1 \quad 10^0$
Base value of the digits



Base \Rightarrow 10

Decimal Number System

Base value of 5^{th} digit = 10^4

Base value of x^{th} digit = 10^{x-1}

Unique Digits : $[0, 9]$

Binary Number System (Base = 2)

Unique digits = {0, 1}

$$\begin{array}{c}
 1 \ 0 \ 1 \ 1 \ 0 \\
 \downarrow \downarrow \downarrow \downarrow \downarrow \\
 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0
 \end{array}
 \Rightarrow 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2^1 = 16 + 4 + 2 = \boxed{22}$$

$$\begin{array}{c}
 1 \ 0 \ 1 \ 0 \\
 \downarrow \downarrow \downarrow \downarrow \\
 2^3 \ 2^2 \ 2^1 \ 2^0
 \end{array}
 = 2^3 + 2^1 = 8 + 2 = \boxed{10}$$

Decimal to Binary

$$\begin{array}{r}
 2 \overline{) 28} \ 0 \\
 2 \overline{) 14} \ 0 \\
 2 \overline{) 7} \ 1 \\
 2 \overline{) 3} \ 1 \\
 2 \overline{) 1} \ 1 \\
 \hline
 0
 \end{array}$$

$$\boxed{11100}$$

$$28 = 2^4 + 2^3 + 2^2 = 16 + 12 = 28$$

$$\begin{array}{l}
 28 - 16 = 12 \\
 12 - 8 = 4 \\
 4 - 4 = 0
 \end{array}$$

$$\begin{array}{cccccc}
 0 & 1 & 1 & 1 & 0 & 0 \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0
 \end{array}$$

Quiz:

$$\begin{array}{r}
 2 \overline{) 37} \ 1 \\
 2 \overline{) 18} \ 0 \\
 2 \overline{) 9} \ 1 \\
 2 \overline{) 4} \ 0 \\
 2 \overline{) 2} \ 0 \\
 2 \overline{) 1} \ 1 \\
 \hline
 0
 \end{array}$$

$$100101$$

$$37 = 2^5 + 2^2 + 2^0 = 32 + 5 = 37$$

$$\begin{array}{l}
 37 - 32 = 5 \\
 5 - 4 = 1
 \end{array}$$

$$\begin{array}{cccccc}
 1 & 0 & 0 & 1 & 0 & 1 \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0
 \end{array}$$

$$2^5 = 2^4 + 2^3 + 2^0$$

$$2^5 - 16 = 9$$

$$9 - 8 = 1$$

$$\frac{1}{4} \quad \frac{1}{3} \quad \frac{0}{2} \quad \frac{0}{1} \quad \frac{1}{0}$$

Addition:

$10/10$	$13/10$	$10/10$	$10/10$	$10/10$
6	3	4	5	9
3		8	4	3
$1\%10$	$10\%10$	$13\%10$	$10\%10$	$12\%10$
1	0	3	0	2

$$\begin{array}{c} 12 \\ \swarrow \searrow \\ 12/10 \quad 2^2 (2\%10) \end{array}$$

Result \Rightarrow $\boxed{\text{sum} \% 10}$
 Carry \Rightarrow $\boxed{\text{sum} / 10}$

Result \Rightarrow sum % base =

Carry \Rightarrow sum / base

Adding

2 binary

$2/2$	$3/2$	$2/2$	$2/2$	$1/2$	$1/2$	0
1	1	1	1	0	0	0
$1\%2$	$2\%2$	$3\%2$	$2\%2$	$2\%2$	$1\%2$	$1\%2$
1	0	1	0	0	1	1

$2^6 + 2^4 + 2^1 + 2^0 \Rightarrow 64 + 16 + 2 + 1 = 83$

sum % 2
 carry = sum / 2

ex2:

$$\begin{array}{r}
 \begin{array}{c} \frac{1}{2} \\ a = \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{c} \frac{1}{2} \\ \begin{array}{c} 0 \\ 1 \end{array} \end{array} \begin{array}{c} \frac{2}{2} \\ \begin{array}{c} 0 \\ 0 \end{array} \end{array} \begin{array}{c} \frac{2}{2} \\ \begin{array}{c} 1 \\ 0 \end{array} \end{array} \begin{array}{c} \begin{array}{c} 1 \\ 1 \end{array} \end{array} \\
 \begin{array}{c} \frac{1}{2} \\ b = \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \\
 \hline
 \begin{array}{c} \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{2}{2} \quad \frac{2}{2} \end{array} \\
 \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 0 & 0 \\ \hline \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} \frac{2}{2} \\ a_1 = \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{c} \frac{4}{2} \\ \begin{array}{c} 0 \\ 0 \end{array} \end{array} \begin{array}{c} \frac{3}{2} \\ \begin{array}{c} 1 \\ 1 \end{array} \end{array} \begin{array}{c} \frac{3}{2} \\ \begin{array}{c} 0 \\ 1 \end{array} \end{array} \begin{array}{c} \begin{array}{c} 1 \\ 0 \end{array} \end{array} \\
 \begin{array}{c} \frac{2}{2} \\ a_2 = \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \\
 \begin{array}{c} \frac{2}{2} \\ a_3 = \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \\
 \begin{array}{c} \frac{2}{2} \\ a_4 = \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline \end{array} \\
 \hline
 \begin{array}{c} \frac{1}{2} \quad \frac{2}{2} \quad \frac{4}{2} \quad \frac{3}{2} \quad \frac{3}{2} \end{array} \\
 \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 0 & 1 & 1 \\ \hline \end{array}
 \end{array}$$

$$\begin{array}{l}
 \text{result} = \text{sum} \% \text{base} \\
 \text{carry} = \text{sum} / \text{base}
 \end{array}$$

$$\left(\frac{4}{2} \right)$$

Bitwise Operators

AND (&)

OR (|)

XOR (^)

NOT (~)

Left Shift (<<)

Right Shift (>>)

A	B	A & B	A B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

if all bits are 1 (set), then result is 1 (set), else result is 0 (unset)

if we have odd no. of set bits, result is set. Else, result is unset

1 → set bit
0 → unset bit

if atleast one bit is 1 (set), then result is 1 (set), else result is 0 (unset)

$x \& y \mid a \mid b \mid \dots$

$1^1 0^1 1^1 1^1 0^1 0^1 0^1 1^1 \Rightarrow 1^1 1^1 1^1 1^1 0^1 0^1 0^1 0^1$
 $0^1 0^1 0^1 0^1 = 0$

$$\Rightarrow \begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 0 \\ \hline 0 & 1 & 0 & 1 & 1 & 0 \end{array} \Rightarrow 0^1 1 \Rightarrow 1$$

2)

$$a^1 b = b^1 a \rightarrow (\text{Commutative})$$

$$a^1 b^1 c \Rightarrow b^1 c^1 a \Rightarrow c^1 a^1 b \Rightarrow c^1 b^1 a$$

$$\text{int } x = (5) \& (8)$$

$$5 \& 8 \Rightarrow 0$$

$$\begin{array}{cccc} 0 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \end{array}$$

$$\text{int } x = 5 | 8$$

$$\text{OR} \begin{array}{cccc} 0 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \end{array}$$

$$\text{OR} = \begin{array}{cccc} 1 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 0 & 0 \end{array}$$

Quiz:

$$a = 13$$

1 1 0 1

$$b = 10$$

1 0 1 0

$$a \& b$$

$$\underline{1} \ \underline{0} \ \underline{0} \ \underline{0} \Rightarrow 8$$

$$a | b$$

$$\underline{1} \ \underline{1} \ \underline{1} \ \underline{1} \Rightarrow 15$$

$$a \wedge b$$

$$\underline{0} \ \underline{1} \ \underline{1} \ \underline{1} \Rightarrow 7$$

NOT operator

a	$\sim a$
0	1
1	0

$$\sim(0100) \Rightarrow 1011$$

Quiz:

Right most bit of even number

$$2: 10$$

$$4: 100$$

$$6: 110$$

$$8: 1000$$

$$10: 1010$$

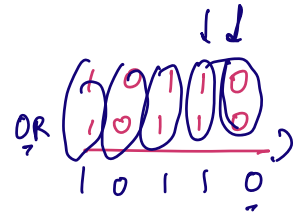
$$12: 1100$$

$$14: 1110$$

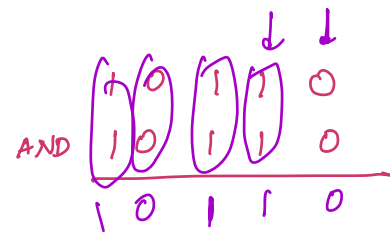
Ans = 0

Properties

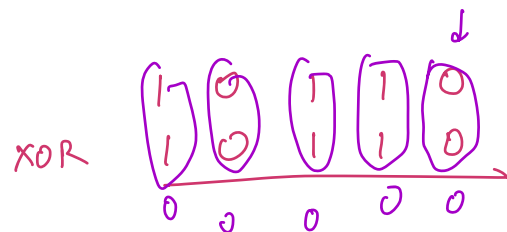
$$a \vee a = a$$



$$a \wedge a = a$$

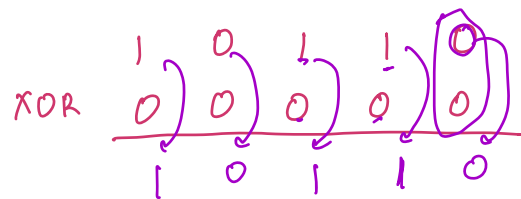


$$a^1 a = 0$$



$$\begin{aligned} 0^1 0 &= 0 \\ 1^1 1 &= 0 \end{aligned}$$

$$a^1 0 = a$$



$$\begin{aligned} 0^1 0 &\Rightarrow 0 \\ 1^1 0 &\Rightarrow 1 \end{aligned}$$

$$\begin{aligned} 4^1 0 &= 4 \\ 120^1 120 &= 0 \\ 6^1 6^1 6^1 6 &= 0 \end{aligned}$$

$$9^1 9^1 9^1 9^1 (9) = 9$$

Commutative :

$$\begin{aligned} a \& b &= b \& a \\ a | b &= b | a \\ a^{-1} b &= b^{-1} a \end{aligned}$$

$$a - b \neq b - a$$

Associative :

$$a^{-1} b^{-1} c = \begin{aligned} &(a^{-1} b)^{-1} c \\ &(b^{-1} c)^{-1} a \\ &(c^{-1} a)^{-1} b \end{aligned}$$



Quiz:

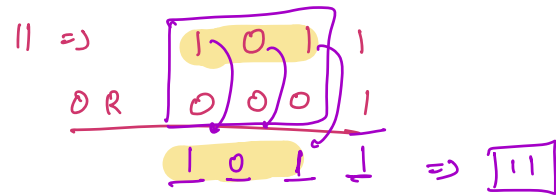
$$\begin{aligned} &\rightarrow 120^1 5^1 6^1 6^1 120^1 5^1 \\ \Rightarrow & \begin{array}{cccccc} 120^1 & 120^1 & 5^1 & 5^1 & 6^1 & 6^1 \\ \hline & 0 & 1 & 0 & 1 & 0 & 1 \end{array} \Rightarrow \boxed{0} \end{aligned}$$

8mins

Q417:

$$a = 11$$

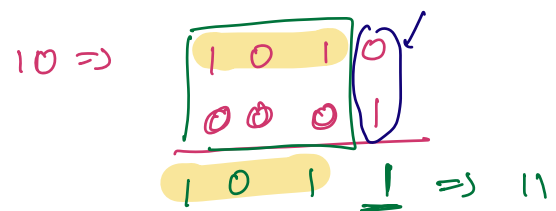
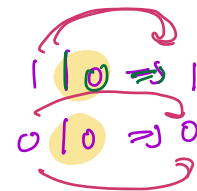
$$a \mid 1 \Rightarrow$$



Q417

$$a = 10$$

$$a \mid 1 \downarrow$$

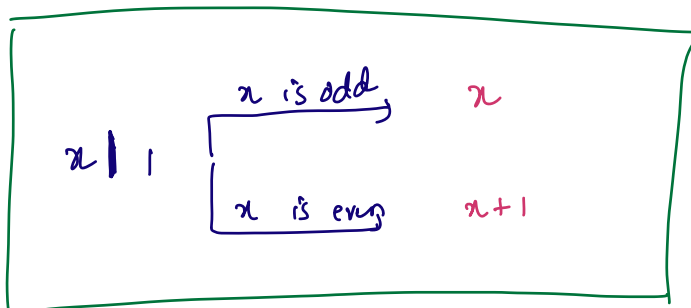
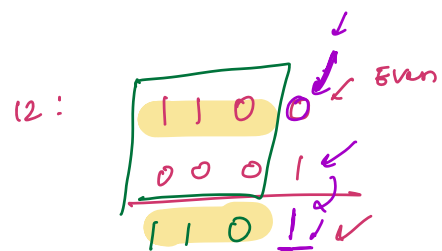


$$10 \mid 1 = 11$$

$$11 \mid 1 = 11 \checkmark$$

$$12 \mid 1 = 13$$

$$13 \mid 1 = 13 \checkmark$$



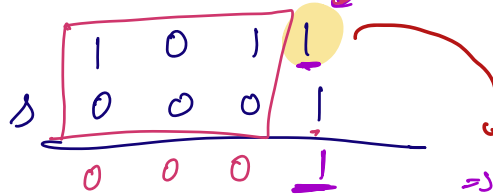
Quiz :

int a = x

what is a % 1

(odd)

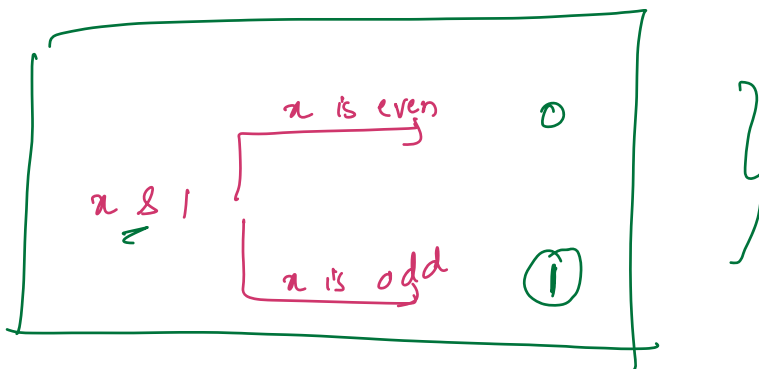
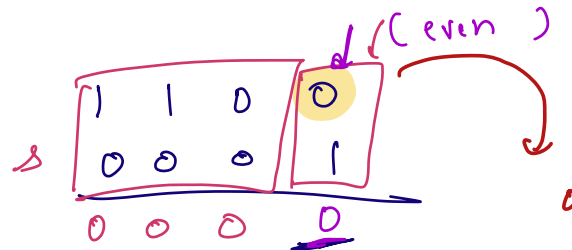
a = 11



180 \Rightarrow 0

080 \Rightarrow 0

a = 12



Approach 1

if (a % 2 == 1)

"odd"

else

"EVEN"

✓

✓

Approach 2 :

if (a % 2 == 1)

"odd"

else

"EVEN"

Quiz:

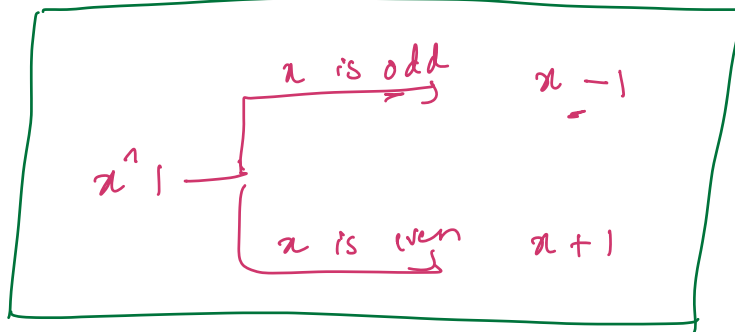
int a = 1
print(a ^ 1)

11:
$$\begin{array}{r} 1011 \\ \text{XOR } 0001 \\ \hline 1010 \end{array} \Rightarrow 10$$

$$\begin{array}{l} 1 \oplus 0 = 1 \\ 0 \oplus 1 = 1 \end{array}$$

12:
$$\begin{array}{r} 1100 \\ \text{XOR } 0001 \\ \hline 1101 \end{array} \Rightarrow 13$$

$$\begin{array}{l} 1 \oplus 1 = 0 \\ 1 \oplus 0 = 1 \end{array}$$



10:
$$\begin{array}{r} 1011 \\ \text{XOR } 0001 \\ \hline 1010 \end{array} \Rightarrow x + 1$$

$2^0 = 1$
Even

odd
$$\begin{array}{r} 1011 \\ \text{XOR } 0001 \\ \hline 1010 \end{array} \Rightarrow x - 1$$

Question: Single Number ✓

Given an array where all the numbers appear even no. of times except one number which appears odd no. of times.

Find the odd number!

A = 2 8 3 1 2 1 2 3 2 8 1
 0 1 2 3 4 5 6 7 8 9 10

$$2^1 \Rightarrow 4$$

$$8^1 \Rightarrow 2$$

$$3^1 \Rightarrow 2$$

$$1^1 \Rightarrow 3$$

$$2^1 2^1 2^1 2^1 \Rightarrow 0$$

$$8^1 8^1 \Rightarrow 0$$

$$3^1 3^1 \Rightarrow 0$$

$$1^1 1^1 1^1 \Rightarrow 1$$

$$A = 2^1 8^1 3^1 1^1 2^1 1^1 2^1 3^1 2^1 8^1 1$$
$$\Rightarrow (2^1 2^1 2^1 2^1)^1 (8^1 8^1)^1 (3^1 3^1)^1 (1^1 1^1 1^1)^1 = 1$$

ans = 0;

for (i=0; i<N; i++){

 ans = ans ^ A[i];

}

return ans;

Quiz: Min $\log_2 N$ bits required to represent N in binary

$N = 10$ $1010 \Rightarrow \underbrace{0001010}_{1 \text{ bit}} \Rightarrow 4 \text{ bits}$

$N = 25$ $11001 \Rightarrow 000011001 \Rightarrow 5 \text{ bits}$

$O(\log N)$ steps

2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

Left Shift \leftarrow

$a = 10$

	7	6	5	4	3	2	1	0	
$a = 10$	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	$\Rightarrow 10 = 10 \times 2^0$
$a \ll 1$	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	$\Rightarrow 20 = 10 \times 2^1$
$a \ll 2$	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	$\Rightarrow 40 = 10 \times 2^2$
$a \ll 3$	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	$\Rightarrow 80 = 10 \times 2^3$

$$a \ll \frac{1}{\epsilon}$$

✓ Overflow

$$a \ll 1 = a \times 2^0$$

15 $\angle \angle 2$

$$15 \times 2^2 =$$

$$15 \times 4 = \boxed{60}$$

$$\Rightarrow 2^1$$

$2^1 \quad 2^2$

→ 23

$$\Rightarrow 2^4$$

$\Rightarrow 25$

$$\Rightarrow 2^6$$

2^1

T.C: $O(1)$

2ⁱ using power
function
↓
T.C: $O(\log i)$

10
5
Binary

Right Shift

	7	6	5	4	3	2	1	0	
$a = 20$	0	0	0	1	0	1	0	0	underflow = 20
$a >> 1$	0	0	0	0	1	0	1	0	$\Rightarrow 10 = \frac{20}{2^1}$
$a >> 2$	0	0	0	0	0	1	0	1	$\Rightarrow 5 = \frac{20}{2^2}$
$a >> 3$	0	0	0	0	0	0	1	0	$\Rightarrow 2 = \frac{20}{2^3}$
$a >> 4$	0	0	0	0	0	0	0	1	$\Rightarrow 1 = \frac{20}{2^4}$
$a >> 5$	0	0	0	0	0	0	0	0	$\Rightarrow 0 = \frac{20}{2^5}$

0000

$$a >> i = \frac{a}{2^i}$$

$$29 >> 2 = \frac{29}{2^2} = \frac{29}{4} = 7$$

$$29 << 2 = 29 \times 2^2 = 29 \times 4 = 116$$

$N = 47$

$32 + 8 + 4 + 2 + 1$

$M.S.B$

$a =$

0	0	1	0	1	1	1	1
7	6	5	4	3	2	1	0

Find index of Left Most One (MSB)

Task: Find Index of Left most One.

$$n/2 = 43$$

$$0 < 10 \Rightarrow 0$$

$$1 < 1000 \Rightarrow 0$$

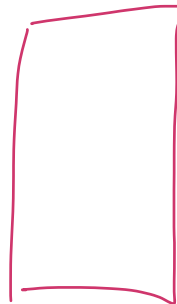
$$n \ll 32 \Rightarrow \underbrace{00000000}_{\text{}} \\ \swarrow \quad \downarrow \quad \searrow \\ \text{---} \quad \text{---} \quad \text{---} \\ 00000 \dots \dots \dots 00$$

$$0 \ll 10 = 0$$

int : 4 bytes \Rightarrow 32 bits
 long : 8 bytes \Rightarrow 64 bits

10000000

$N = 13 \Rightarrow 1101$
 int 13
 \downarrow

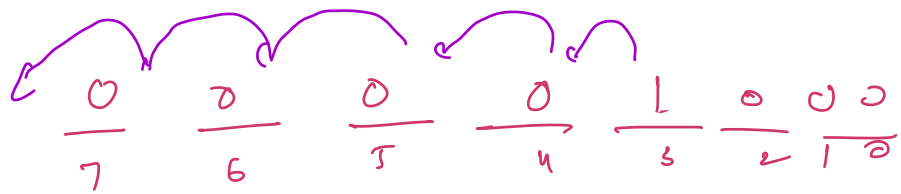


\ll
 --- --- --- 1 --- --- $\pm [2^{10}] (2^{32} - 1) >$

$$N = 10 \ll 1 = 10 \times 2^1$$

$$40 \times 2^1$$

int N = 1000



$$\# \text{ bits} = 8$$

$$\text{index of leftmost } 1 = 3$$

$$\boxed{\begin{matrix} 8-5 \\ 8 - \text{floor}(\log N) \end{matrix}}$$