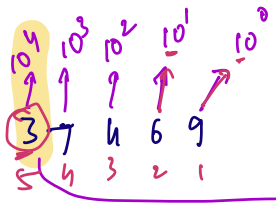
All right => 400 coins

8764 = 8000
+ 700
+ 60
+ 4

= $8 \times 10^3 + 7 \times 10^2 + 6 \times 10^1 + 4 \times 10^0$

(Thousands, Hundreds, Tens, Units)

Base value of the digits



Base => 10

Decimal Number System

Base value of 5th digit = 10^4

Base value of nth digit = 10^{n-1}

Unique Digits : [0, 9]

Binary Number System (Base = 2)

Unique digits = {0, 1}

1 0 1 1 0 ↗ $\Rightarrow$ $1\cdot2^4 + 1\cdot2^2 + 1\cdot2^1$ ✓

$16 + 4 + 2 = \boxed{22}$

1 ~~0~~ 1 0 $= 2^3 + 2^1 = 8 + 2 = \boxed{10}$

Decimal to Binary

2	28	0
2	14	0
2	7	1
2	3	1
2	1	1
	0	

$\boxed{1\ 1\ 1\ 0\ 0}$

$28 = \boxed{2^4 + 2^3 + 2^2} \Rightarrow$

$28 - 16 = 12$

$12 - 8 = 4$

$4 - 4 = 0$

0 1 1 1 0 0

$2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

Quiz:

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

1 0 0 1 0 1

$37 = 2^5 + 2^2 + 2^0$

$37 - 32 = 5$

$5 - 4 = 1$

1 0 0 1 0 1

$2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

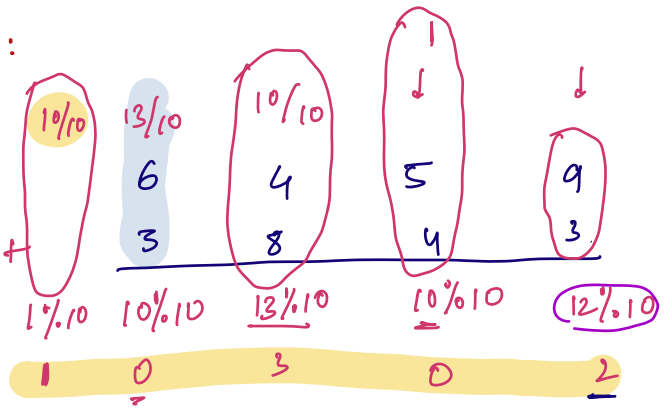
$25 = 2^4 + 2^3 + 2^0$

$25 - 16 = 9$

$9 - 8 = 1$

1	1	0	0	1
4	3	2	1	0

Addition :



Result => sum % 10

carry => sum / 10

Result => sum % base

carry => sum / base

Adding 2 binary

$2^5 + 2^4 + 2^2 + 2^0 = 53$

$2^4 + 2^3 + 2^2 + 2^1 = 30$

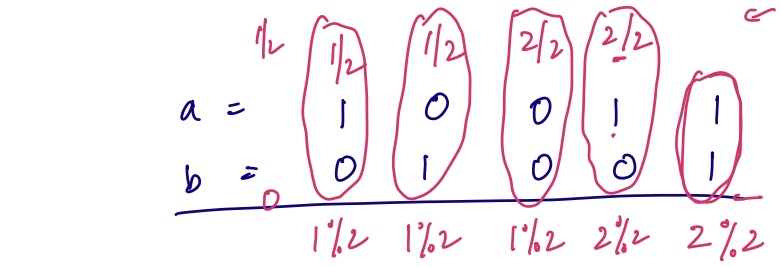
83

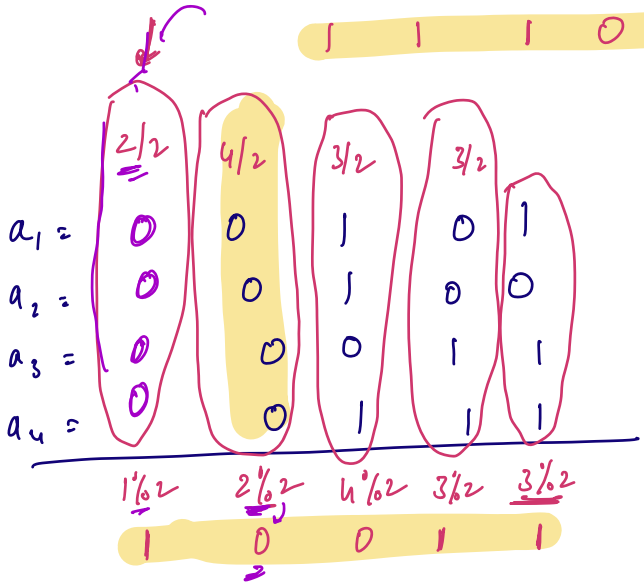
sum % 2

carry : sum / 2

$2^6 + 2^4 + 2^1 + 2^0 \Rightarrow 64 + 16 + 2 + 1 = 83$

Ex2:





2/2

result = sum % base

carry = sum / base

Bitwise Operators

AND (&)

OR (|)

XOR (^)

NOT (~)

Left Shift (<<)

Right Shift (>>)

A	B	A & B	A | B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

If all bits are 1(set), then result is 1(set), else result is 0(unset)

If we have odd no. of set bits, result is set. Else, result is unset

If atleast one bit is 1(set), then result is 1(set), else result is 0(unset)

1 → set bit

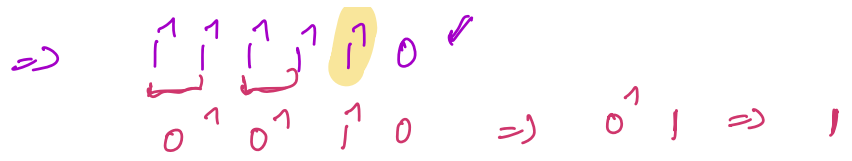
0 → unset bit

x | y | a | b | q ....

1 ^ 0 ^ 1 ^ 1 ^ 1 ^ 0 ^ 0 ^ 0 ^ 1 ⇒ 1 ^ 1 ^ 1 ^ 1 ^ 1 ^ 0 ^ 0 ^ 0 ^ 0

0 ^ 0 ^ 0

0 ^ 0 = 0



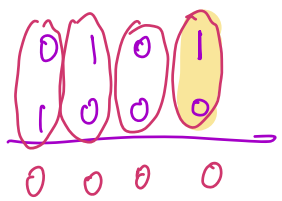
=>

$a \wedge b = b \wedge a$ → (Commutative)

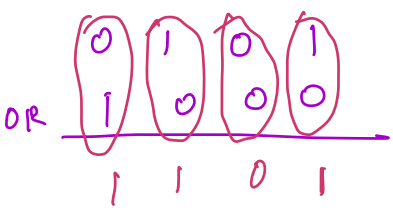
$a \wedge b \wedge c \Rightarrow b \wedge c \wedge a \Rightarrow c \wedge a \wedge b \Rightarrow c \wedge b \wedge a$

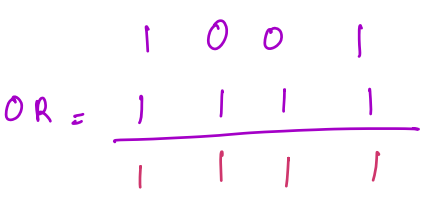
int x = 5 & 8

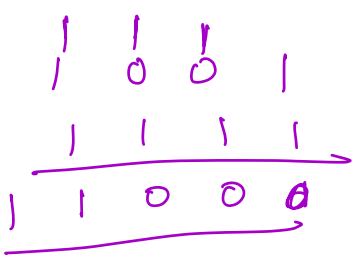
5 & 8 => 0



int x = 5 | 8







Quiz:

a = 13 → 1 1 0 1

b = 10 → 1 0 1 0

a & b → 1 0 0 0 => 8

a | b → 1 1 1 1 => 15

a ^ b → 0 1 1 1 => 7

NOT operator

a	~a
0	1
1	0

~(0100) => 1011

Quiz: Right most bit of even number

2 : 1 0

4 : 1 0 0

6 : 1 1 0

8 : 1 0 0 0

10 : 1 0 1 0

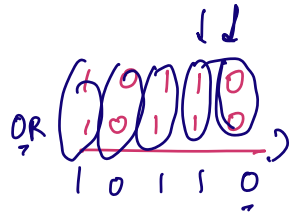
12 : 1 1 0 0

14 : 1 1 1 0

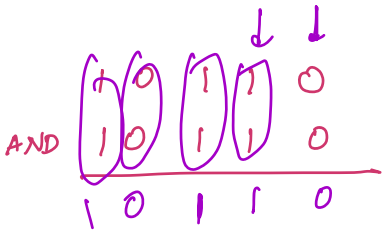
Ans = 0

Properties

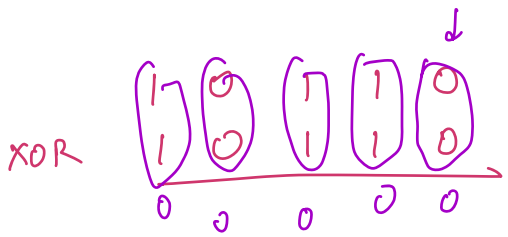
$a \mid a = a$



$a \,\&\, a = a$



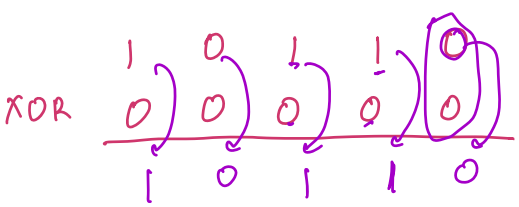
$a \wedge a = 0$

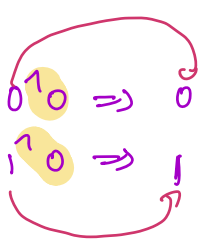


$0 \wedge 0 = 0$

$1 \wedge 1 = 0$

$a \wedge 0 = a$





$4 \wedge 0 = 4$

$120 \wedge 120 = 0$

$6 \wedge 6 \wedge 6 \wedge 6 = 0$

$9 \wedge 9 \wedge 9 \wedge 9 \wedge 9 = 9$

Commutative:

$a - b \neq b - a$

$a \,\&\, b = b \,\&\, a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

Associative:

$a \wedge b \wedge c = (a \wedge b) \wedge c$

$(b \wedge c) \wedge a$

$(c \wedge a) \wedge b$

Quiz:

$120 \wedge 5 \wedge 6 \wedge 6 \wedge 120 \wedge 5$

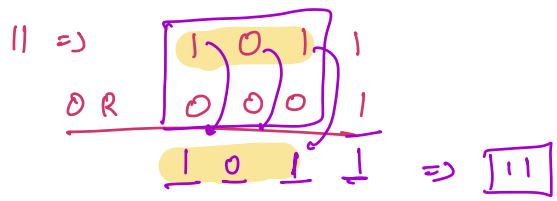
$\Rightarrow 120 \wedge 120 \wedge 5 \wedge 5 \wedge 6 \wedge 6$

$0 \wedge 0 \wedge 0 \Rightarrow \boxed{0}$

8mins

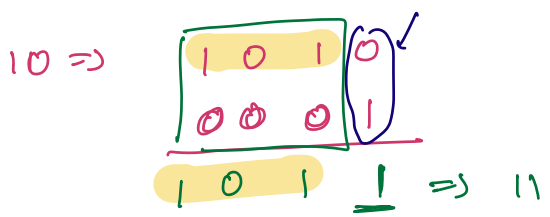
Quiz: $a = 11$

$a \mid 1 \Rightarrow$



Quiz $a = 10$

$a \mid 1$

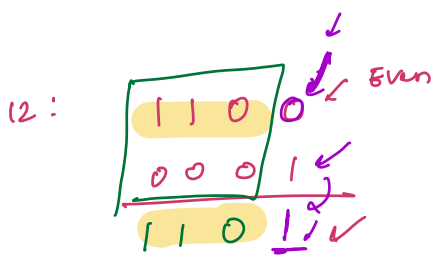


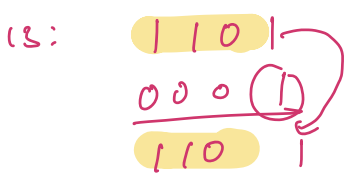
$10 \mid 1 = 11$

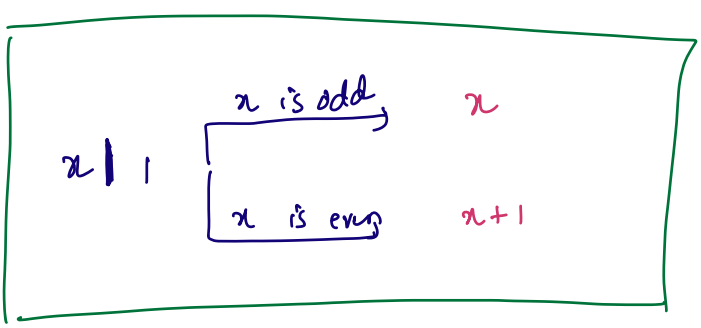
$11 \mid 1 = 11$ ✓

$12 \mid 1 = 13$

$13 \mid 1 \quad 13$ ✓

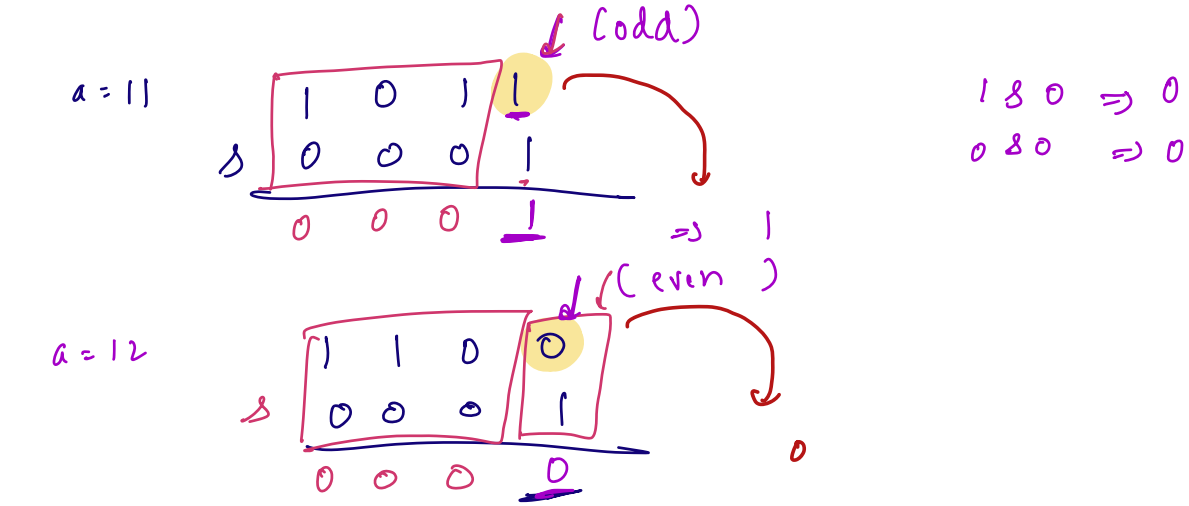


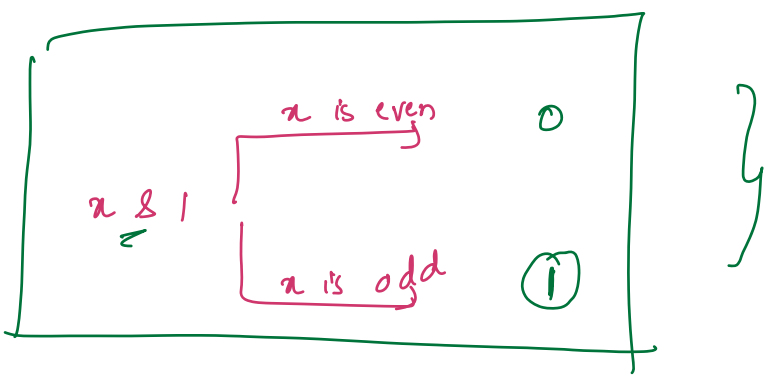




Quiz : int a = x

what is a & 1





Approach1

if (x & 1 == 1)

"ODD"

else

"EVEN"

✓

✓

Approach2 :

if (x % 2 == 1)

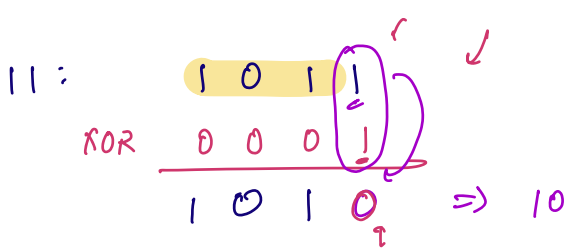
"ODD"

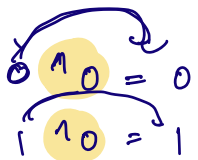
else

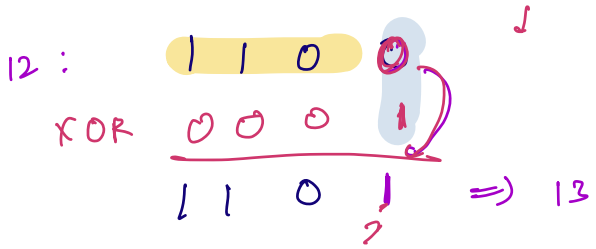
"EVEN"

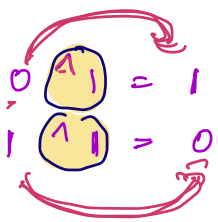
Quiz: int a = x

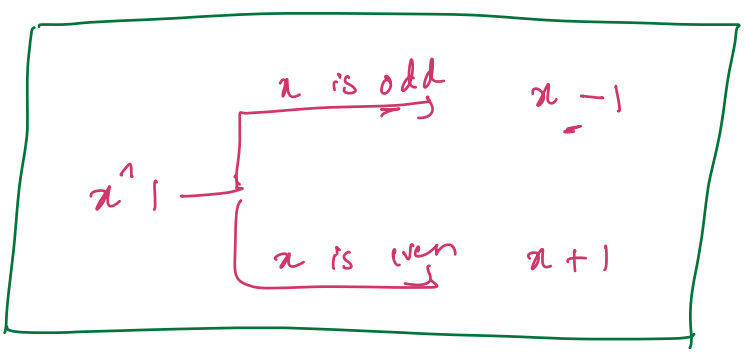
print (a ^ 1)

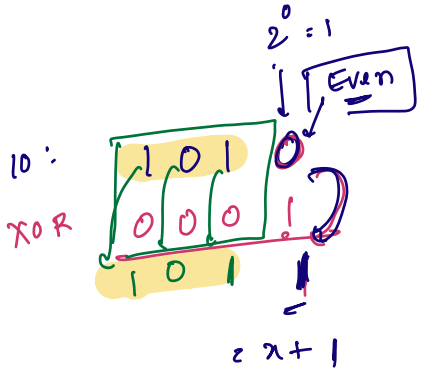


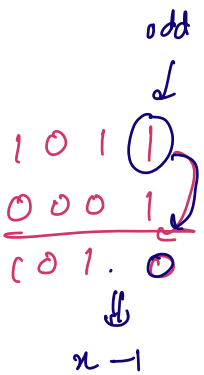












Question: Single Number ✓

Given an array where all the numbers appear even no. of times except one number which appears odd no. of times.
Find the odd number!

A = 2 8 3 1 2 1 2 3 2 8 1
0 1 2 3 4 5 6 7 8 9 10

'2' => 4 $\quad$ $2 \wedge 2 \wedge 2 \wedge 2 \Rightarrow 0$
'8' => 2 $\quad$ $8 \wedge 8 \Rightarrow 0$
'3' => 2 $\quad$ $3 \wedge 3 \Rightarrow 0$
'1' => 3 $\quad$ $1 \wedge 1 \wedge 1 \Rightarrow 1$

A = $2 \wedge 8 \wedge 3 \wedge 1 \wedge 2 \wedge 1 \wedge 2 \wedge 3 \wedge 2 \wedge 8 \wedge 1$

$\Rightarrow (2 \wedge 2 \wedge 2 \wedge 2) \wedge (8 \wedge 8) \wedge (3 \wedge 3) \wedge (1 \wedge 1 \wedge 1) = 1$
0 0 0 1

```
ans = 0;
for (i=0; i<N; i++){
    ans = ans ^ A[i];
}
return ans;
```

Quiz: Min no. of bits required to represent N in binary

N = 10 → 1010 (4 bits) → 00001010 → 8 bits

N = 25 → 11001 → 00011001 → 5 bits

O(logN) steps

2 | 25 | 1
2 | 12 | 0
2 | 6 | 0
2 | 3 | 1
2 | 1 | 1
0

Left Shift <<

a = 10

	7	6	5	4	3	2	1	0	
a = 10	0	0	0	0	1	0	1	0	=> $10 = 10 \times 2^0$
a<<1	0	0	0	1	0	1	0	0	=> $20 = 10 \times 2^1$
a<<2	0	0	1	0	1	0	0	0	=> $40 = 10 \times 2^2$
a<<3	0	1	0	1	0	0	0	0	=> $80 = 10 \times 2^3$

a << 4 : 0 0 1 0 0 0 0 0 => 160 = 10 × 2^4

Overflow

a << 5 : 0 1 0 0 0 0 0 0 => 64

$$a << i = a \times 2^i$$

Quiz:

$15 << 2 = 15 \times 2^2 = 15 \times 4 = 60$

1 << 1 => => 2^1

1 << 2 => => 2^2

1 << 3 => => 2^3

1 << 4 => => 2^4

1 << 5 => => 2^5

1 << 6 => => 2^6

1 << i => 2^i

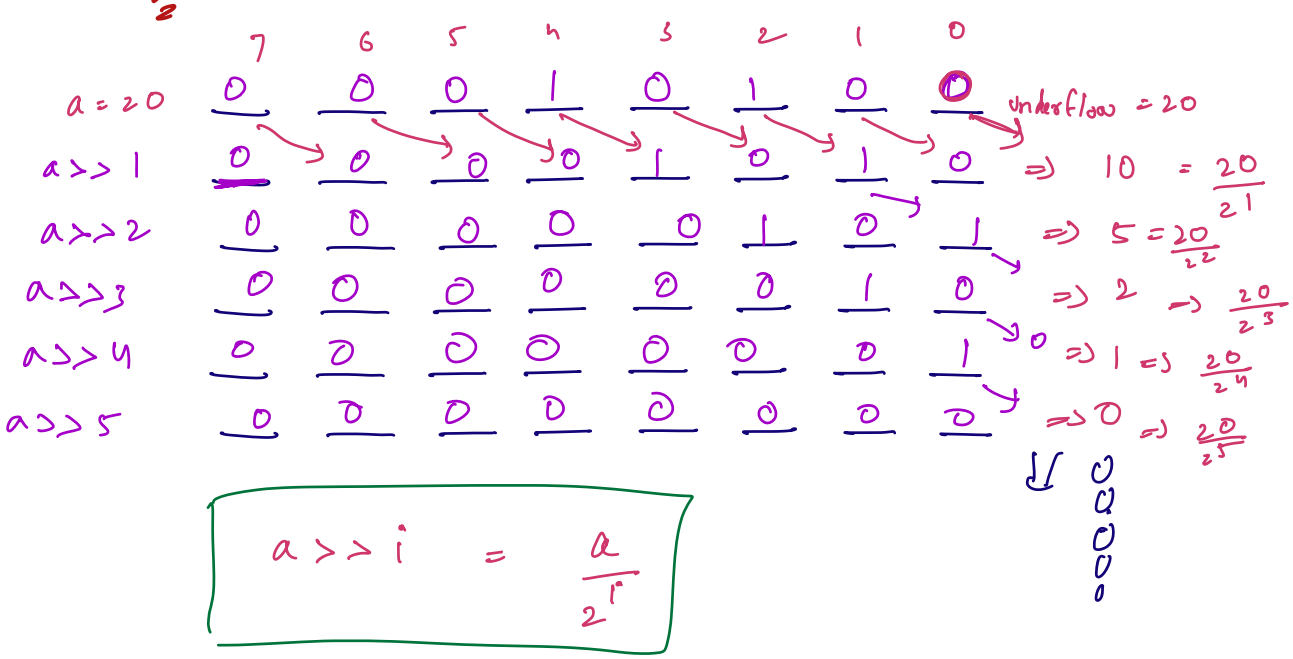
2^i using power function

T.C : O(1)

T.C : O(log i)

5^{10} → Binary

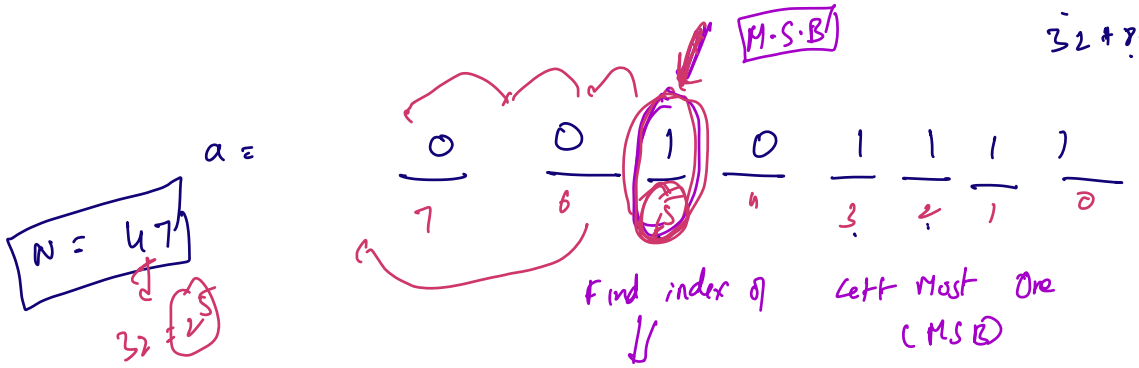
Right Shift



$$a >> i = \frac{a}{2^i}$$

$$29 >> 2 = \frac{29}{2^2} = \frac{29}{4} = 7$$

$$29 << 2 = 29 \times 2^2 = 29 \times 4 = 116$$

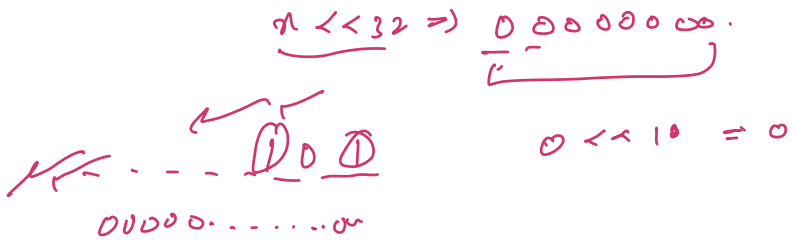


Task: Find Index of Left most One.

$47/2 = 23$

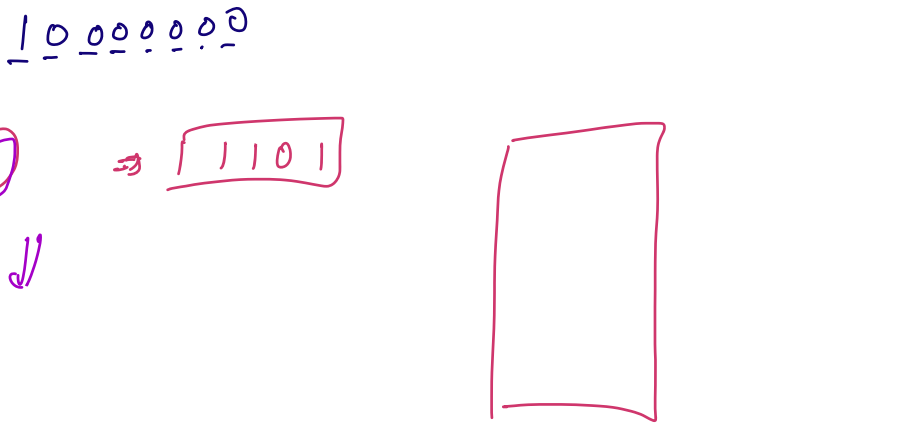
$0 << 10 \Rightarrow 0$

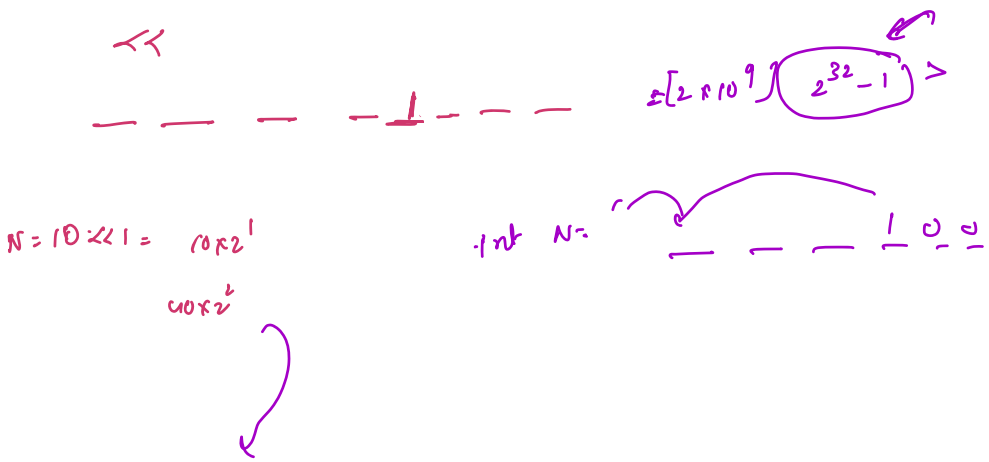
$1 << 1000 \Rightarrow 0$

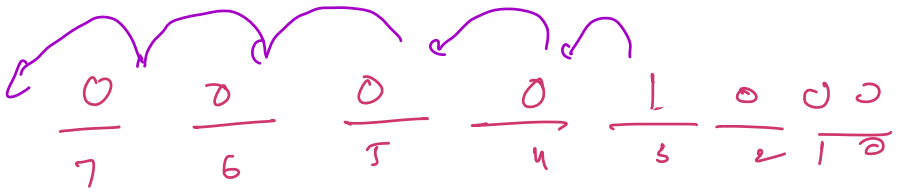


int : 4 bytes => 32 bits

long : 8 bytes => 64 bits







#bits = 8

index of leftmost 1 = 3

8 − 5

y − floor(log N)