"Heaven's light is our guide"

RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY

Department of Electrical & Computer Engineering

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Experiment No: 05

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5.1: Experiment Name: Study of

1. Z transformation of causal function
2. Determining ROC of causal function

5.2: Theory

The Z-transform is a mathematical tool used in digital signal processing and control systems theory to analyze discrete-time systems. It is an extension of the Laplace transform for continuous-time systems. The Z-transform converts a discrete-time signal into a complex function of a complex variable called the Z-transform variable.

A causal function refers to a function that depends only on past and present values of its input. In the context of the Z-transform, a causal function is one whose Z-transform has a region of convergence (ROC) that includes the unit circle in the complex plane. For a causal function, the Z-transform can be defined as:

X(z) = ∑[x(n) * z^(-n)], for n = 0 to ∞

where x(n) is the discrete-time input signal and X(z) is the Z-transform of x(n). The variable z^(-n) represents the delayed version of the signal x(n), with z^(-n) = 1/z^n. [1]

The Region of Convergence (ROC) in the Z-transform analysis of a causal function is the region in the complex plane where the Z-transform converges absolutely. For a causal function, the ROC is typically of the form |z| > R, where R is a positive real number. This means that the Z-transform converges for values of z that lie outside a circle centered at the origin with radius R. The ROC extends to infinity in the outer region of the circle. The specific choice of the radius R depends on the characteristics of the causal function, such as its decay rate or growth rate. It is essential to note that the stability of the system is determined by the inclusion of the unit circle (|z| = 1) within the ROC. If the unit circle is within the ROC, the system is considered stable. Let (where *u* is the Heaviside step function). Expanding *x*[*n*] on the interval (−∞, ∞) it becomes

$$x[n] = \left\{\ldots, 0, 0, 0, 1, 0.5, 0.5^2, 0.5^3, \ldots\right\}.$$

Looking at the sum

$$\sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} 0.5^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{0.5}{z}\right)^n = \frac{1}{1-0.5z^{-1}}.$$

The last equality arises from the infinite geometric series and the equality only holds if $|0.5z^{-1}| < 1$, which can be rewritten in terms of z as $|z| > 0.5$. Thus, the ROC is $|z| > 0.5$. In this case the ROC is the complex plane with a disc of radius 0.5 at the origin "punched out". [2]

5.3.1: Code

```
clc;
close all;
syms m;
x= [1 2 3 4];
l= length(x);
A=0;
z=sym('z');
for i=0:l-1
A=A+x(i+1).*z^(-i);
end
disp('ztransformation output of causal signal: ');
disp(A);
```

5.3.2: Output

```
Command Window
ztransformation output of causal signal:
2/z + 3/z^2 + 4/z^3 + 1
```

Figure 5.1: Output of causal function z-transformation

5.4.1: Code (ROC of causal function z transformation)

```
clc;
clear all;

b = [1, 2, 3]; % Define the coefficients of the causal function

% Compute the Z-transform
syms z;
%the poly2sym function is used to convert a polynomial
H = poly2sym(b, z);
X = ztrans(H);

% Find the ROC
syms r;
ROCs = solve(abs(subs(X, z, r)) < Inf, r);
% Plot the ROCs
figure;
hold on;
for i = 1:numel(ROCs)
r = ROCs(i);
plot(real(r), imag(r), 'bo');
end
plot(cos(0:0.01:2*pi), sin(0:0.01:2*pi), 'r--'); % Unit circle
xlabel('Real Axis');
ylabel('Imaginary Axis');
title('Region of Convergence (ROC)');
axis([-2 2 -2 2]);
axis equal;
grid on;
```

5.4.2: Output

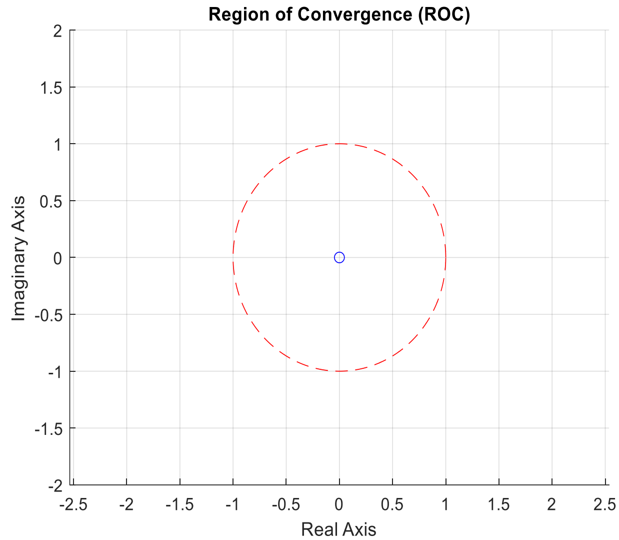

Figure 5.2: ROC of causal function

5.5: Conclusion & Discussion

The experiment was accomplished successfully and observed that the output of these codes was as same as the theory. By applying the Z-transform to a causal function, we can obtain its representation in the frequency domain and analyze its properties such as stability, impulse response, frequency response, and system behavior. Analyzing the ROC allows us to determine the convergence properties of the Z-transform and understand the behavior of the causal function in the frequency domain.

5.6: References

[1] https://www.tutorialspoint.com/digital_signal_processing/dsp_z_transform_existence.htm

[2] https://en.wikipedia.org/wiki/Z-transform#Example_2_(causal_ROC)