1) Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$by using induction.

- We can prove this using induction on the length |w| of w.
- For the base case, we have |w| = 0, which means that w = ϵ.
- From the definition of the reversal of a string, we have that ϵR = ϵ. So, (w^R)^R = (ϵ^R)^R=(ϵ^R)= ϵ=W and therefore our claim holds for |w| = 0.
- For the induction hypothesis, assume that (w^R)^R=W for every string w such that |w| = k, where k is an integer with k ≥ 0.
- Now, we can prove that (w^R)^R = w for every string w such that |w| = k +1. Since k +1 ≥ 1, we can write w = ua, where a ∈ Σ and u ∈ Σ∗. From the definition of reversal of a string, we know that w^R = (ua)^R = au^R. From item (b) in Problem (1), we also know that (w^R)^R = (au^R)^R = (u^R)^Ra. Since the length of u is k, we can use the induction hypothesis to conclude that (u^R)^R = u. So, (w^R)^R = (au^R)^R = (u^R)^Ra = ua = w, and our claim also holds for w ∈ Σ^∗ such that |w| = k+1. From the first principle of induction, our claim holds for every w ∈ ^∗.

2. Let L = {ab, aa, baa}. Which of the following strings are in L*: abaabaaabaa, aaaabaaaa,b aaaaabaaaab, baaaaabaa? Which strings are in L4?

- abaabaaabaa → ab aa baa ab aa (L5)
- aaaabaaaa → aa aa baa aa (L4)
- baaaaabaaaab→ baa aa ab aa aa b (Not a string of Ln) (∵ $b \in$ L)
- baaaaabaa → baa aa ab aa (L4)

3.Let ∑ = {a, b} and L = {aa, bb}. Use set notation to describe L'.

- ∑ - L(L' ={x: $x \notin L$ and $x \in \Sigma *$ })
- {λ, a, b, ab, ba} ∪ {w ∈ {a, b}* | |w| ≥ 3}

4.Let L be any language on a non-empty alphabet. Show that L and L' cannot both be finite.

- If we think a as a letter of the alphabet and both L and L' are finite, their union L+L' is finite. But L+L' contains all words a^n for natural n, as infinitely many, So there is a contradiction. So L and L' cannot both be finite.

5. Find grammars for ∑ = {a, b} that generate the sets of

(a) all strings with exactly one a.

- S->AaA , A->bA||λ

(b) all strings with at least one a.

- S →AaA , A→aA|bA|λ

(c) all strings with no more than three a's.

- S →A|AaA|AaAaA|AaAaAaA , A → bA|λ

(d) all strings with at least three a's. In each case, give convincing arguments that the grammar you give does indeed generate the indicated language

- S →AaAaAaA, A→aA|bA|λ

6. Give a simple description of the language generated by the grammar with productions

S ⟶ aA

A ⟶ bS

S ⟶ λ

- S--> λ
- S-->aA-->abS-->ab
- S-->aA-->abS- ->abaA-->abab
- This string in this language has empty strings and it starts with a and ends with b. It is a pattern of ab. L(G)=(ab)*

7. What language does the grammar with these productions generate?

S⟶ Aa

A ⟶ B

B ⟶ Aa

- S-->Aa-->Ba-->Aaa-->Baa-->Aaaa-->…………….
- It gets non terminal values. So it is not a valid string.

8. Are the two grammars with respective productions S ⟶ aSb |ab|λ And S ⟶ aAb |ab A ⟶ aAb |λ equivalent? Assume that S is the start symbol in both cases.

S ⟶ aSb |ab|λ

S--> λ

S ⟶ aAb |ab

A ⟶ aAb |λ

- These two grammars are not equivalent, because λ is generated from the first grammar at the start but λ does not generate from the second at the start. So the grammars are not equivalent.

9. Show that the grammar S ⟶ aSb | bSa | SS| a and S ⟶ aSb | bSa | a are not equivalent

- S-->aSb|bSa|SS|a
- S-->SS-->aa

- S ⟶ aSb | bSa | a
- S-->a

- From the grammar 2 , we can not get two 'a's in sequence order. So these are not equivalent.