Theorem 2.1.4

Let u, v, w $\in \Sigma^*$, then (uv)w = u(vw).

Proof : The proof is by induction on the length of the string w. The string w was chosen for compatibility with the recursive definition of strings, which builds on the right-hand side of an existing string.

Basis : length(w) = 0. Then w = λ, and (uv)w = uv by the definition of concatenation. On the other hand, u(vw) = u(v) = uv.

Inductive hypothesis : Assume that (uv)w = u(vw) for all strings w of length n or less.

Inductive step : We need to prove that (uv)w = u(vw) for all strings w of length n +1. Let w be such a string. Then w = xa for some string x of length n and a $\in \Sigma$ and

(uv)w = (uv)(xa) (substitution, w=xa)
= ((uv)x)a (definition of concatenation)
= (u(vx))a (inductive hypothesis)
= u((vx)a) (definition of concatenation)
= u(v(xa)) (definition of concatenation)
= u(vw) (substitution, xa = w)

Definition of reversal

Let u be a string in Σ^*. The reversal of u, denoting u^R, is defined as follows,
i) Basis : If length (u) = 0, then u = λ and $\lambda^R = \lambda$

ii) Recursive step : if length (u) = n > 0, thn u = wa for some string w with length n-1 and some a $\in \Sigma$, and $u^R = aw^R$

Theorem 2.1.6

Let u, v $\in \Sigma^*$,Then $(uv)^R = v^R u^R$.

Proof : the proof is by induction on the length of the string w.

Basis : If length (v) = 0, then v = λ, and $(uv)^R = u^R$. Similarly, $v^R u^R = \lambda^R u^R = u^R$.

Inductive Hypothesis : Assume $(uv)^R = v^R u^R$ for all strings v f length n or less.
Inductive Step : We must prove that, for any string v of length $(uv)^R = v^R u^R$. Let v be a string of length n+1. Then v = wa, where w is a string of length n and a $\in \Sigma$. The inductive step is established by

$(uv)^R = (u(wa))^R$
$= ((uw)a)^R$(associativity of concatenation)
$=a(uw)^R$(definition of reversal)
$=a(w^R u^R)$ (inductive hypothesis)
$=(aw^R)u^R$(associativity of concatenation)
$=(wa)^R u^R$(definition of reversal)
$= v^R u^R$