

## GRAPH :

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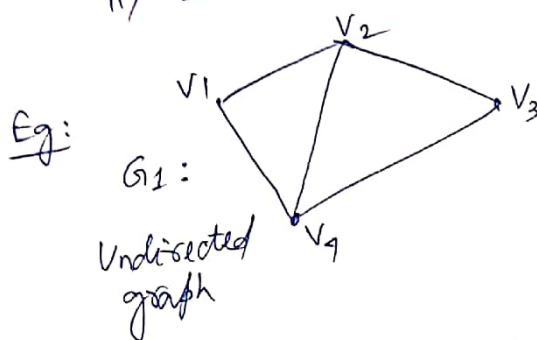
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- \* It is a non-linear data structure.
- \* In tree, there is a relationship: parent  $\rightarrow$  child.
- \* In graph the relationship is less restricted.

## GRAPH TERMINOLOGIES

1) Graph  $G = (V, E)$  consists of two sets

- $V$ : is the set of vertices
- $E$ : is the set of edges.



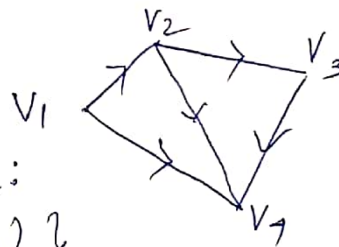
$$V = \{V_1, V_2, V_3, V_4\}$$

$$E = \{(V_1, V_2), (V_2, V_3), (V_3, V_4), (V_4, V_1), (V_2, V_4)\}$$

2) Diagraph: It is a directed graph s.t  $G = (V, E)$  where  $V$  is the set of all vertices and  $E$  is the set of ordered pairs of vertices from  $V$ .

$$V = \{V_1, V_2, V_3, V_4\}$$

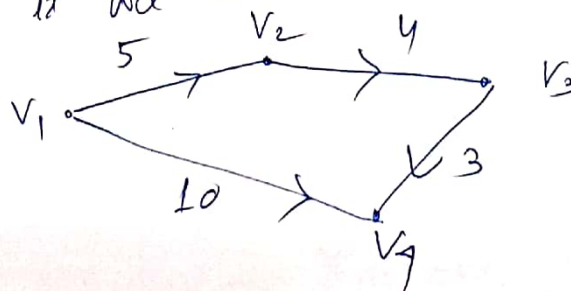
$$E = \{(V_1, V_2), (V_2, V_3), (V_3, V_4), (V_4, V_1), (V_2, V_4)\}$$



ie in a diagraph  $(V_i, V_j) \neq (V_j, V_i)$

## 3) WEIGHTED GRAPH :

A graph (or diagraph) is termed as weighted graph if all the edges in it are labeled with some weights.



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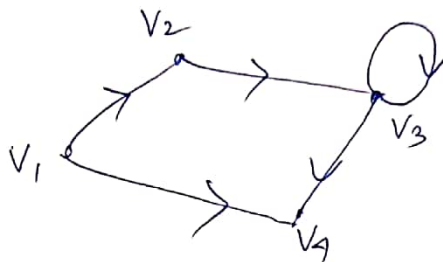
4) Adjacent Vertices:

Two vertices  $V_i$  and  $V_j$  are adjacent to each other if  $\exists$  an edge  $e_{ij} = (V_i, V_j)$

$e_{ij} : V_i \rightarrow V_j$

5) Self Loop:

If there is an edge whose start and end vertices are same that is  $(V_i, V_i)$  is an edge, then it is called a self loop (or simply a loop).



Here loop is at  $V_3$ .

6) Parallel Edges

If there ~~are~~ exist two or more edges between two vertices then those edges are parallel edges.

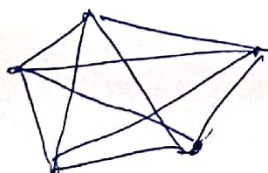
Note: A graph which has either self loop or parallel edges or both is called a MULTIGRAPH.

7) Simple Graph/Diagraph

A graph/diagraph having no self loop and parallel edges is called a simple graph/diagraph.

8) Complete Graph:

A graph/diagraph  $G$  is said to be complete if each vertex  $V_i$  is adjacent to every other vertex  $V_j$  in  $G$ .  
i.e.  $\exists$  edges from any vertex to all vertices.



## 9) Acyclic Graph:

If a graph/diagraph does not have any cycle then it is called acyclic graph.

Eg: Tree.

## 10) Isolated Vertex

A vertex  $v_i$  of a graph  $G$  is said to be isolated if there is no edge which connects any vertex of  $G$  to  $v_i$  or vice-versa.

## 11) Degree of a Vertex

Degree of  $(v_i) =$  No. of edges connected with the vertex.

Indegree  $(v_i) =$  no. of edges incident into  $v_i$  (i.e. ending at  $v_i$ )

Outdegree  $(v_i) =$  no. of edges emanating from  $v_i$  (Edges beginning at  $v_i$ )

## 12) Pendant Vertex

A vertex  $v_i$  is pendant if  $\text{indegree}(v_i) = 1$ , and  $\text{outdegree}(v_i) = 0$ .

Eg: Leafnode in a tree.

## 13) Connected Graph:

In a graph  $G$ , two vertices  $v_i$  and  $v_j$  are said to be connected if there is a path in  $G$  from  $v_i \rightarrow v_j$  or  $v_j \rightarrow v_i$ .

A graph is said to be connected if there is at least one path for every pair of distinct vertices  $v_i, v_j$  in  $G$ .

\* Strongly Connected: A graph/diagraph is strongly connected if  $\exists$  path  $v_i \rightarrow v_j$  and  $v_j \rightarrow v_i$ ,  $\forall i=1$  to  $n$  in  $G$ .

\* Weakly Connected: If a diagraph is not strongly connected but underlying graph is connected  $\Rightarrow$  the graph is weakly connected.



## Representation of Graphs:

- The ADT to represent a graph are
- 1) Sequential or Matrix Representation
  - 2) Linked List Representation.

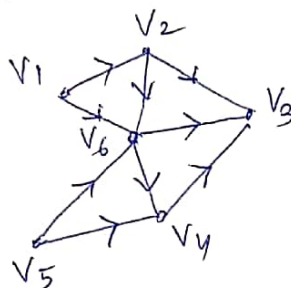
### #1) MATRIX REPRESENTATION OF A GRAPH

- i) Adjacency Matrix
- ii) Incidence Matrix.

#### #i) Adjacency Matrix

$$A = [a_{ij}] = \begin{cases} 1, & \text{if } v_i \text{ is adjacent to } v_j \\ 0, & \text{if there is no edge between } v_i \text{ and } v_j \end{cases}$$

Eg: Satellite Network  
Cellular Network  
WSN  
WiFi Network



$$A = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Note: i) Adjacency matrix does not depend on the ordering of the vertices of a graph  $G$ . We can obtain same matrix by interchanging rows and columns.

2) If graph  $G$  is undirected then the adjacency matrix of  $G$  will be symmetric.

i.e.  $[a_{ij}] = [a_{ji}]$  for every  $i$  and  $j$ .

3) No. of 1's in an adjacency matrix = No. of edges in the graph.

4) This matrix is called a bit matrix or Boolean Matrix.

# #ii INCIDENCE MATRIX:

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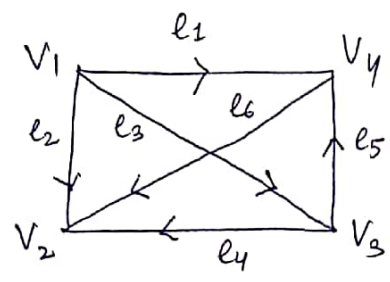
$$I = \begin{matrix} & e_1 & e_2 & e_3 & \dots & e_k \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_i \end{matrix} & \begin{bmatrix} | & | & | & & | \\ | & | & | & & | \\ | & | & | & & | \\ | & | & | & & | \\ | & | & | & & | \end{bmatrix} \end{matrix}$$

→ A row for a vertex  
→ A column for an edge.

$$I[i,j] = \begin{cases} -1, \\ 0 \\ 1 \end{cases}$$

Imp if  $e_k: v_i \rightarrow v_j \Rightarrow$  the column has the value = 1, in  $i$ th row = -1, in  $j$ th row = 0, otherwise

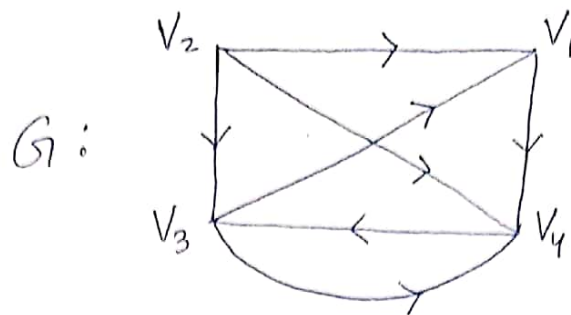
Eg:



$$A = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$I = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{matrix}$$

Example: -



The adjacency matrix  $A$  of graph  $G$  is:

$$A = \begin{matrix} & \begin{matrix} V_1 & V_2 & V_3 & V_4 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

Consider the powers  $A, A^2, A^3, \dots$  of the adjacency matrix  $A$  of  $G$ .

$$A^2 = A \cdot A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$A^4 = A^3 \cdot A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Hence  $a_k(i, j) = (i, j)$ th entry in the matrix  $A^k$ .  
 $\rightarrow$  It gives the no. of paths of length " $k$ " from node  $V_i \rightarrow V_j$ .

$$\det B_k = A + A^2 + A^3 + \dots + A^k$$

$b(i,j) = m$ . i.e. there are  $m$  number of paths of length " $k$ " are less from node  $v_i \rightarrow v_j$ .

### (iii) PATH MATRIX

Let  $G$  be a simple directed graph with  $n$ -nodes  $v_1, v_2, v_3, \dots, v_n$ . The path matrix or reachability matrix of graph  $G$  is the  $n$ -square matrix  $P = p_{ij}$  is defined as

$$p_{ij} = \begin{cases} 1, & \text{if } \exists \text{ a path from } v_i \rightarrow v_j \\ 0, & \text{otherwise.} \end{cases}$$

Now from the above diagram, we have

$$B_4 = A + A^2 + A^3 + A^4$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 & 3 \\ 5 & 0 & 6 & 8 \\ 3 & 0 & 3 & 5 \\ 2 & 0 & 3 & 3 \end{bmatrix}$$

$$\Rightarrow P = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

$\Rightarrow v_2$  is not reachable from any of the nodes.

# SHORTEST PATH ALGORITHM

$G$ : Directed graph with  $n$ -vertices,  $v_1, v_2, \dots, v_n$ .

Warshall's algorithm is more efficient than calculating the powers of the adjacency matrix  $A$ .

Let us define  $n$ -square logical matrices  $P_0, P_1, P_2, \dots, P_n$  as follows

Let  $P_k[i][j] = (i, j)^{\text{th}}$  entry of the matrix  $P_k$ .

i.e.  $P_k[i][j] = \begin{cases} 1, & \text{if there is a path from } v_i \rightarrow v_j \text{ which doesn't use any other nodes except possibly } v_1, v_2, \dots, v_k \\ 0, & \text{otherwise.} \end{cases}$

i.e.  $P_0[i][j] = 1$ , if there is an edge from  $v_i \rightarrow v_j$  (direct).

$P_1[i][j] = 1$ , there is a path from  $v_i \rightarrow v_j$  which doesn't pass through any other node except node  $v_1$ .

$P_2[i][j] = 1$ , if there is a path  $v_i \xrightarrow{v_1, v_2} v_j$

$\vdots$

$P_k[i][j] = 1$  can occur only if one of the following two cases occurs.

**Case-i**

There is a path from node  $v_i \rightarrow v_j$  which doesn't use any other nodes except possibly  $v_1, v_2, v_3, \dots, v_{k-1}$ ;

hence  $P_{k-1}[i][j] = 1$

**Case-ii**

There is a path from  $v_i \rightarrow v_k$  and  $v_k \rightarrow v_j$  where each path doesn't use any other nodes except possibly  $v_1, v_2, \dots, v_{k-1}$

i.e.  $P_{k-1}[i][k] = 1$  &  $P_{k-1}[k][j] = 1$



i.e. Elements of matrix  $P_k$  can be computed as:

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$$P_k[i][j] = P_{k-1}[i][j] \vee (P_{k-1}[i][k] \wedge P_{k-1}[k][j])$$

$\wedge$  → logical AND  
 $\vee$  → logical OR

## WARSHALL'S ALGORITHM (TO FIND PATH MATRIX)

```

1> for i = 0 to n-1
2>   for j = 0 to n-1
3>     if (a[i][j] != 0)
4>       P[i][j] = 0;
5>     else
6>       P[i][j] = 1;
7> for k = 0 to n-1
8>   for i = 0 to n-1
9>     for j = 0 to n-1
10>      P[i][j] = P[i][j] ∨ (P[i][k] ∧ P[k][j]);
11> Exit
  
```

} Initializing path matrix P.

## WARSHALL'S SHORTEST-PATH PROBLEM

Let  $G$  be a directed weighted graph with  $n$  nodes:  $v_1, v_2, \dots, v_n$ .

$e$ : edge,

$w(e) \geq 0$ : is the weight of an edge " $e$ ".

Weight of the graph  $G$  is  $W = [w_{ij}]$

where  $w_{ij} = \begin{cases} w(e), & \text{if there is an edge from } v_i \rightarrow v_j \\ 0, & \text{if there is no edge from } v_i \rightarrow v_j \end{cases}$

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Let  $D$  : is the distance matrix representing shortest paths between the nodes.

$$D = [d_{ij}]$$

$d_{ij}$  = length of a shortest path from  $V_i \rightarrow V_j$

Now we can define sequence of matrices  $D_0, D_1, D_2, D_3, \dots, D_n$  whose entries are defined as follows:

$$D_k[i][j] = \text{Min} \left( \text{Length of preceding path } V_i \rightarrow V_j, \text{ OR } \sum (\text{Length of preceding paths from } V_i \rightarrow V_k + V_k \rightarrow V_j) \right)$$

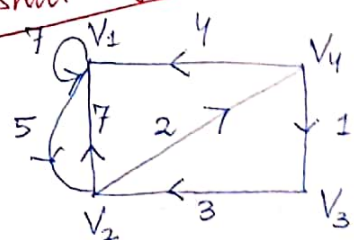
$$D_k[i][j] = \text{Min} (D_{k-1}[i][j], D_{k-1}[i][k] + D_{k-1}[k][j])$$

The initial matrix  $D_0 = W$  (where each "0"  $\leftarrow \infty$  (very large number))

The final matrix " $D_n$ " will be the desired matrix  $D$ .

Example: (Warshall's Algorithm) / Floyd-Warshall Algorithm

$$W = \begin{bmatrix} 7 & 5 & 0 & 0 \\ 7 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \end{bmatrix}$$



$$D_0 = \begin{bmatrix} 7 & 5 & \infty & \infty \\ 7 & \infty & \infty & 2 \\ \infty & 3 & \infty & \infty \\ 4 & \infty & 1 & \infty \end{bmatrix}$$

// No intermediate vertex between any two vertices

$$D_0 = \begin{bmatrix} 7 & 5 & \infty & \infty \\ 7 & \infty & \infty & 2 \\ \infty & 3 & \infty & \infty \\ 4 & \infty & 1 & \infty \end{bmatrix}$$

$$\Pi = \begin{bmatrix} V_1 V_1 & V_1 V_2 & - & - \\ V_2 V_1 & - & - & V_2 V_4 \\ - & V_3 V_2 & - & - \\ V_4 V_1 & - & V_4 V_3 & - \end{bmatrix}$$

$$D_1[2][2] = \min(D_0[2][2], D_0[2][1] + D_0[1][2]) = \min(\infty, 7+5) = 12$$

$$D_1[4][2] = \min(D_0[4][2], D_0[4][1] + D_0[1][2]) = \min(\infty, 4+5) = 9$$

$$D_1 = \begin{bmatrix} 7 & 5 & \infty & \infty \\ 7 & 12 & \infty & 2 \\ \infty & 3 & \infty & \infty \\ 4 & 9 & 1 & \infty \end{bmatrix}$$

$$\Pi = \begin{bmatrix} V_1 V_1 & V_1 V_2 & - & - \\ V_2 V_1 & V_2 V_1 V_2 & - & V_2 V_4 \\ - & V_3 V_2 & - & - \\ V_4 V_1 & V_4 V_1 V_2 & V_4 V_3 & - \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 7 & 5 & \infty & 7 \\ 7 & 12 & \infty & 2 \\ 10 & 3 & \infty & 5 \\ 4 & 9 & 1 & 4 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} V_1 V_1 & V_1 V_2 & - & V_1 V_2 V_4 \\ V_2 V_1 & V_2 V_1 V_2 & - & V_2 V_4 \\ V_3 V_2 V_1 & V_3 V_2 & - & V_3 V_2 V_4 \\ V_4 V_1 & V_4 V_1 V_2 & V_4 V_3 & V_4 V_3 V_2 V_4 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 7 & 5 & \infty & 7 \\ 7 & 12 & \infty & 2 \\ 10 & 3 & \infty & 5 \\ 4 & 4 & 1 & 6 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} & & & \\ & & & \\ & & & \\ V_4 V_3 V_2 & & & V_4 V_3 V_2 V_4 \end{bmatrix}$$

$$D_4 = \begin{bmatrix} 7 & 5 & 3 & 7 \\ 6 & 6 & 3 & 2 \\ 9 & 3 & 6 & 5 \\ 4 & 4 & 1 & 6 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} V_1 V_1 & V_1 V_2 & V_1 V_2 V_4 V_3 & V_1 V_2 V_4 \\ V_2 V_4 V_1 & V_2 V_4 V_3 V_2 & V_2 V_4 V_3 & V_2 V_4 \\ V_3 V_2 V_4 V_1 & V_3 V_2 & V_3 V_2 V_4 V_3 & V_3 V_2 V_4 \\ V_4 V_1 & V_4 V_3 V_2 & V_4 V_3 & V_4 V_3 V_2 V_4 \end{bmatrix}$$

(Ans.)

# Warshall's All-Pairs Shortest Path Algorithm $\Theta(n^3)$

Let  $W$ : Weight matrix.

$G$ : graph

$n$ : No. of nodes in the graph.

o/p:  $D$ : is a matrix to represent shortest path from  $V_i \rightarrow V_j$

1. for  $i=0; i \leq n-1; i++$

2. for  $j=0$  to  $n-1$   
 3. if  $(W[i][j] == 0)$  then  
 4.  $D[i][j] = 0$

5. else  
 6.  $D[i][j] = W[i][j]$

7. for  $k=0$  to  $n-1$

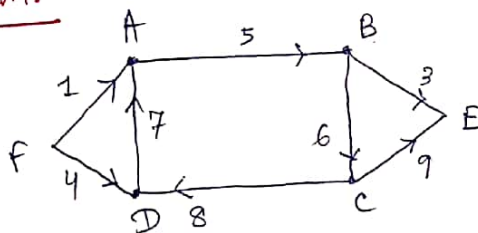
8. for  $i=0$  to  $n-1$

9. for  $j=0$  to  $n-1$

10.  $D[i][j] = \min(D[i][j], D[i][k] + D[k][j])$

11. Exit.

## Problem:





# SINGLE - SOURCE SHORTEST PATH

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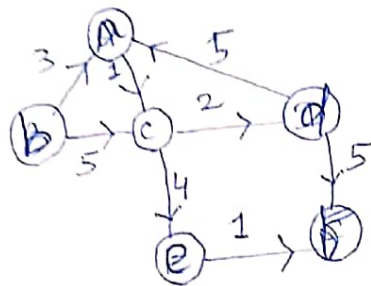
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## Background

1) In all-pairs shortest path problem there is a restriction of moving through certain vertices. Hence shortest path between any two pairs (via) restricted vertices may not be shortest.

There may exist shorter path than this.

Example:

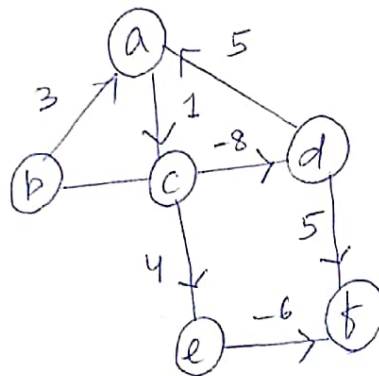


#i  $b \rightsquigarrow f$ :  $b \rightarrow a \rightarrow c \rightarrow e \rightarrow f$   
 weight  $(b \rightsquigarrow f) = 3 + 1 + 4 + 1 = 9$   
 $\Rightarrow$  length  $(b \rightsquigarrow f) = 4$  hops

#ii  $b \rightsquigarrow f$ :  $b \rightarrow c \rightarrow e \rightarrow f$   
 length  $(b \rightsquigarrow f) = 3$  hops

2) When there exist negative edge weights this weighted edge for finding shortest path doesn't work properly.

Example:-



i)  $d \rightsquigarrow f$ :  $d \rightarrow a \rightarrow c \rightarrow e \rightarrow f$   
 weight  $(d \rightsquigarrow f) = 4$ , length  $(d \rightsquigarrow f) = 4$   
 weight  $(d \rightarrow f) = 5$   
 length  $(d \rightarrow f) = 1$  hop.

Note:- 1) Shortest path problem is not defined for graphs that contain negative cost cycles.

2) However  $\exists$  soln for graphs that contain both positive and negative edge weights.

# DIJKSTRA'S ALGORITHM (single source shortest path problem)

/x Non-negative weights \*/

Input: -

Output: -

For each vertex  $v$ , Dijkstra's algorithm keeps track of three pieces of information,  $K_u$ ,  $d_u$ , and  $p_u$ .

$K_u$ : False/True: shortest path to  $v$  is not known/known.

$K_u \leftarrow \text{false} \quad \forall v \in V$  initially.

$d_u$ : Length of shortest path from  $v \rightarrow$  Destination vertex.

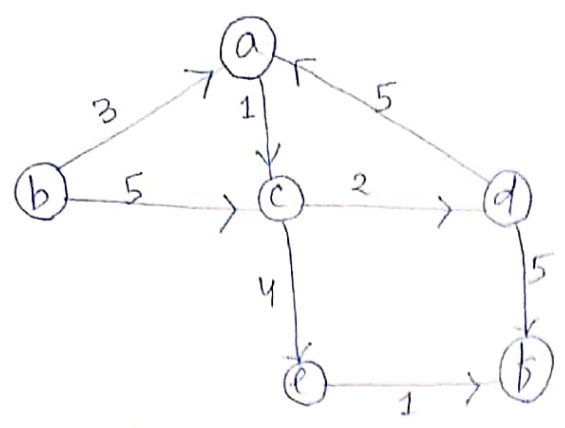
$d_u \leftarrow \infty, \quad \forall v \in V \text{ s.t. } v \neq v_s \text{ where } d_u = 0$   
 $\rightarrow$  source

$p_u$ : Predecessor of vertex  $v$  on the shortest path  $v_s \rightarrow v$   
 ie  $\{v_s, \dots, p_u, v\}$

Initially  $p_u$  is unknown for all  $v \in V$ .

Algorithm

- for  $i = 1$  to  $|V|$
1. Initialize  $K_u = \text{false} \quad \forall v \in V$
2. Select a vertex  $v$  having the smallest tentative distance ( $d_u$ )
3. Set  $K_u = \text{true}$
4. For each vertex  $w$  adjacent to  $v$  for which  $K_w \neq \text{true}$
5. if  $(d_w > d_u + c(v, w))$
6. then  $d_w = d_u + c(v, w)$
7. set  $p_w = v$
8. Exit



Initial

	a	b	c	d	e	f
$K_u$	F	F	F	F	F	F
$d_u$	$\infty$	0	$\infty$	$\infty$	$\infty$	$\infty$
$P_u$	-	-	-	-	-	-

Pass-1

	a	b	c	d	e	f
$K_u$	T	T	T	T	F	F
$d_u$	3	0	4	6	8	11
$P_u$	b	-	a	c	c	d

Pass-1

	a	b	c	d	e	f
$K_u$	F	T	F	F	F	F
$d_u$	3	0	5	$\infty$	$\infty$	$\infty$
$P_u$	b	-	b	-	-	-

Pass-2

	a	b	c	d	e	f
$K_u$	T	T	F	F	F	F
$d_u$	3	0	4	$\infty$	$\infty$	$\infty$
$P_u$	b	-	a	-	-	-

Pass-5

	a	b	c	d	e	f
$K_u$	T	T	T	T	T	T
$d_u$	3	0	4	6	8	9
$P_u$	b	-	a	c	c	e

Pass-3

	a	b	c	d	e	f
$K_u$	T	T	T	F	F	F
$d_u$	3	0	4	6	8	$\infty$
$P_u$	b	-	a	c	c	-

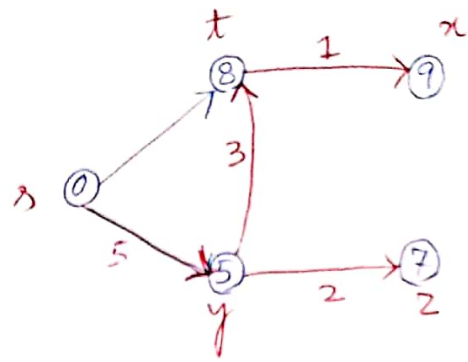
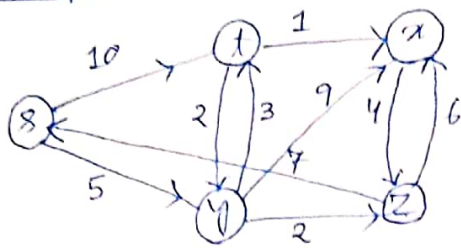
Pass-6

	a	b	c	d	e	f
$K_u$	T	T	T	T	T	T
$d_u$	3	0	4	6	8	9
$P_u$	b	-	a	c	c	e

(Ans)

152

Example :





# TRAVERSING A GRAPH :

DS

40

Objective: Systematically examine the nodes and edges of a graph  $G$ .

Methods:- Two standard methods are:

— 1> Depth First Search (DFS)

↳ It uses STACK to hold nodes for future processing

— 2> Breadth First Search (BFS)

↳ It uses QUEUE as an auxiliary structure to hold nodes for future processing

## Assumptions

Each node "n" of  $G$  will be in one of the 3-states,

that are

flag = 1 : (Ready state) : The initial state of the node "n".

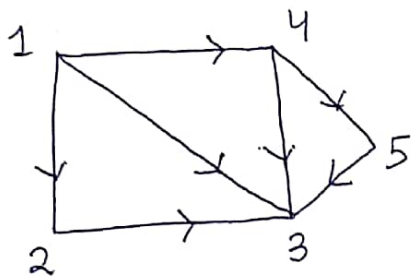
flag = 2 : (Waiting state) : The node "n" is in STACK/QUEUE waiting to be processed.

flag = 3 : (Processed state) : The node "n" has been processed.

(158)

Algorithm (DFS)

- 1) Set  $\boxed{\text{flag} = 1}$  for all nodes // Initialize all nodes to Ready state
- 2) Push (node 1 into the stack)
- 3) Set  $\boxed{\text{flag} = 2}$  for node 1 // change the status of node 1 to Waiting state
- 4) while (stack  $\neq$  NULL)
  - 5)  $x = \text{Pop}()$
  - 6) Process (x)
  - 7) Set  $\text{flag} = 3$  for node x // change status to process state
  - 8) for all  $y = \text{Adj}(x)$ 
    - 9) if ( $\text{flag}(y) == 1$ ) then
      - 10) Push(y) into the stack
      - 11)  $\text{flag} = 2$  for y.
- 12) Exit

Example (DFS):

Node	Adjacent List
1	2, 3, 4
2	3
3	
4	3, 5
5	3

Ans: -

1) Ready state 

1	1	1	1	1
---	---	---	---	---

 Status.

2) Push(1) onto the stack 

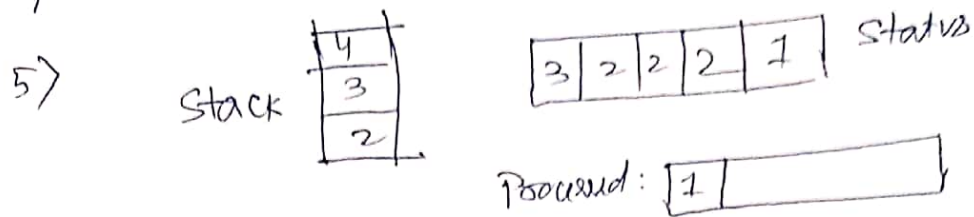
2	1	1	1	1
---	---	---	---	---

 Status.

- 3)  $x = \text{Pop}()$  status 

3	1	1	1	1
---	---	---	---	---

  
 $x = 1$   
 4) Push(2), Push(3), Push(4) // All adjacent of 1.



- 6)  $x = \text{Pop}()$   
 $x = 4$   
 Process (4): Processed 

1	4			
---	---	--	--	--

 status 

3	2	2	3	1
---	---	---	---	---

- 7) Push Adj(4) i.e. Push(5) into stack. 

5
3
2

, status 

3	2	2	3	2
---	---	---	---	---

- 8)  $x = \text{Pop}()$   
 $x = 5$   
 Processed 

1	4	5		
---	---	---	--	--

- 9)  $x = \text{Pop}()$   
 $x = 3$   

1	4	5	3	
---	---	---	---	--

- 10)  $x = \text{Pop}()$   
 $x = 2$   
 Processed 

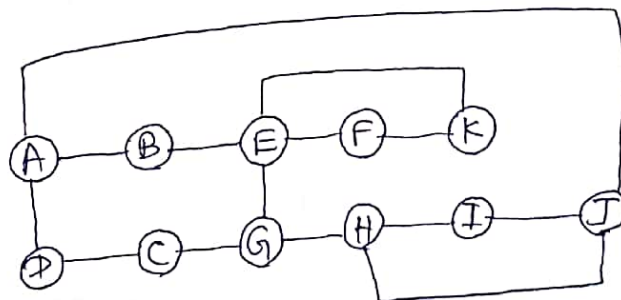
1	4	5	3	2
---	---	---	---	---

Now stack is Empty.

$\therefore$  DFS is completed

The nodes are processed in the order 1, 4, 5, 3, 2. (Ans)

Problem: -

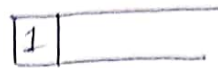


# Algorithm (BFS) : Queue

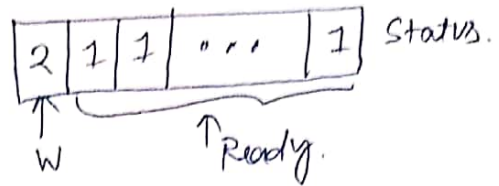
1> Set flag=1 for all nodes



2> Insert (node 1) into the queue



3> flag=2 for node 1

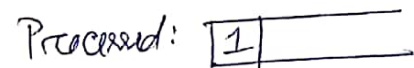


4> while (queue != Empty)

5> { x = Delete(); // Remove front node from queue

6> Process (x)

flag=3 for x



7> for all y in Adj(x)

{ if (flag == 1) then

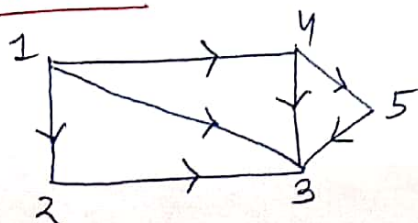
{ Insert(y)

flag=2 for y

Node	Adjacent List
1	2, 3, 4
2	3
3	
4	3, 5
5	5

14> Exit

Problem:-



BFS: Traversal: 1, 2, 3, 4, 5