

AVL TREE : [HEIGHT BALANCED]

DS

31

Defⁿ: For every node in a BST
 $\left| \text{Height of left subtree} - \text{Height of Right subtree} \right| \leq 1$
 \Rightarrow It is a height balanced tree

Known as AVL tree.
 2 - Russian Mathematicians { $\begin{cases} \rightarrow \text{E.M. LANDIS} \\ \rightarrow \text{G.M. ADE'SON - VEL'SKI} \end{cases}$

Characteristics:

- 1) Insertion of a node in an AVL tree: $T(n) = O(\lg n)$
- 2) Deletion " $T(n) = O(\lg n)$
- 3) Searching an element " $T(n) = O(\lg n)$
- 4) Height of a tree with n nodes $\leq 1.44 \lg n$

Defⁿ (Formal): An empty tree is height balanced. If T is a non-empty binary tree with T_L and T_R are the left and right subtree; then T is height balanced iff

- i) T_L and T_R are height balanced and
- ii) $|h_L - h_R| \leq 1$; where $h_L = \text{Height}(T_L)$
 $h_R = \text{Height}(T_R)$.

Note:- The Balance factor of any node "N" = $BF(N) = h_L - h_R$

$$BF(N) = \begin{cases} -1 & , \text{ if } h_L < h_R \\ 0 & , \text{ if } h_L = h_R \\ 1 & , \text{ if } h_L > h_R \end{cases}$$

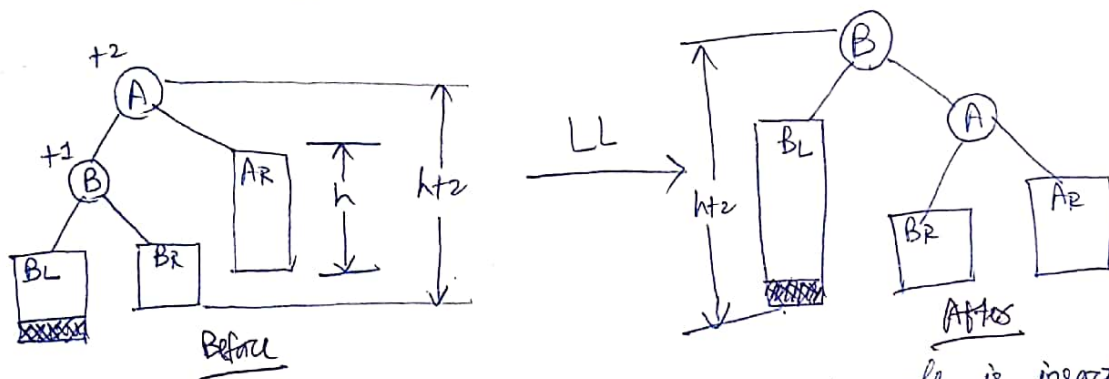
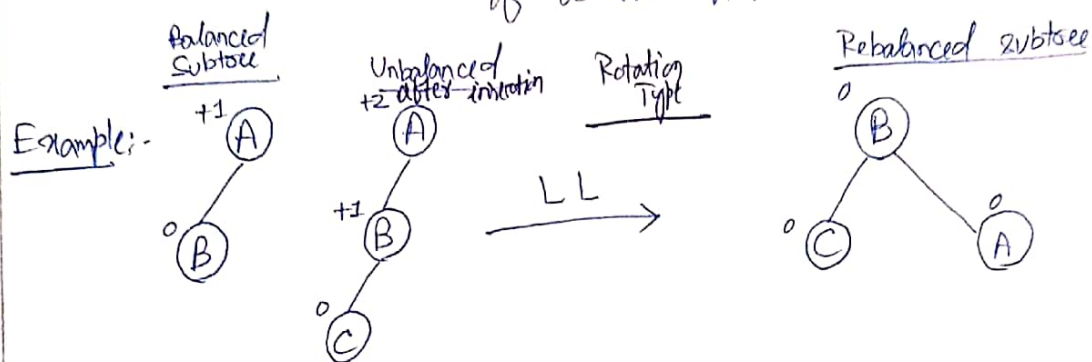
Rotations are characterized by the nearest ancestors of the newly inserted node whose balance factor = ± 2

Note:- if $(BF(N) > 1) \Rightarrow$ Rebalancing of the tree is required
 It is carried out by ROTATION

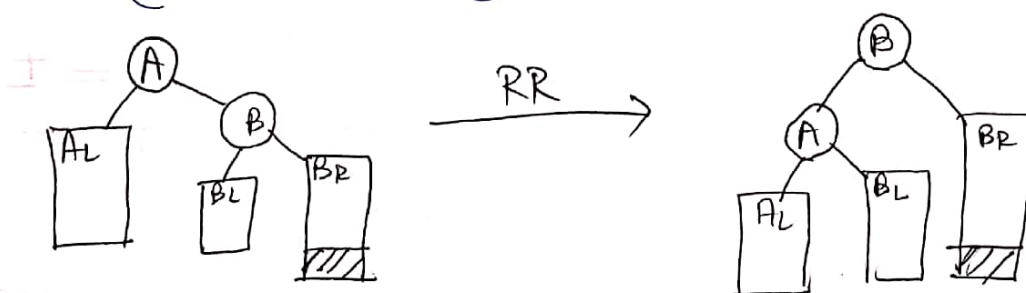
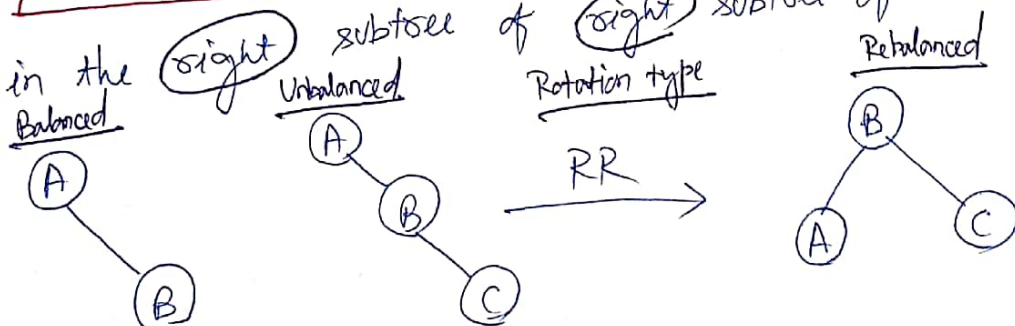
4 - Types of Rotation:

- 1) LL Rotation
- 2) RR "
- 3) LR "
- 4) RL "

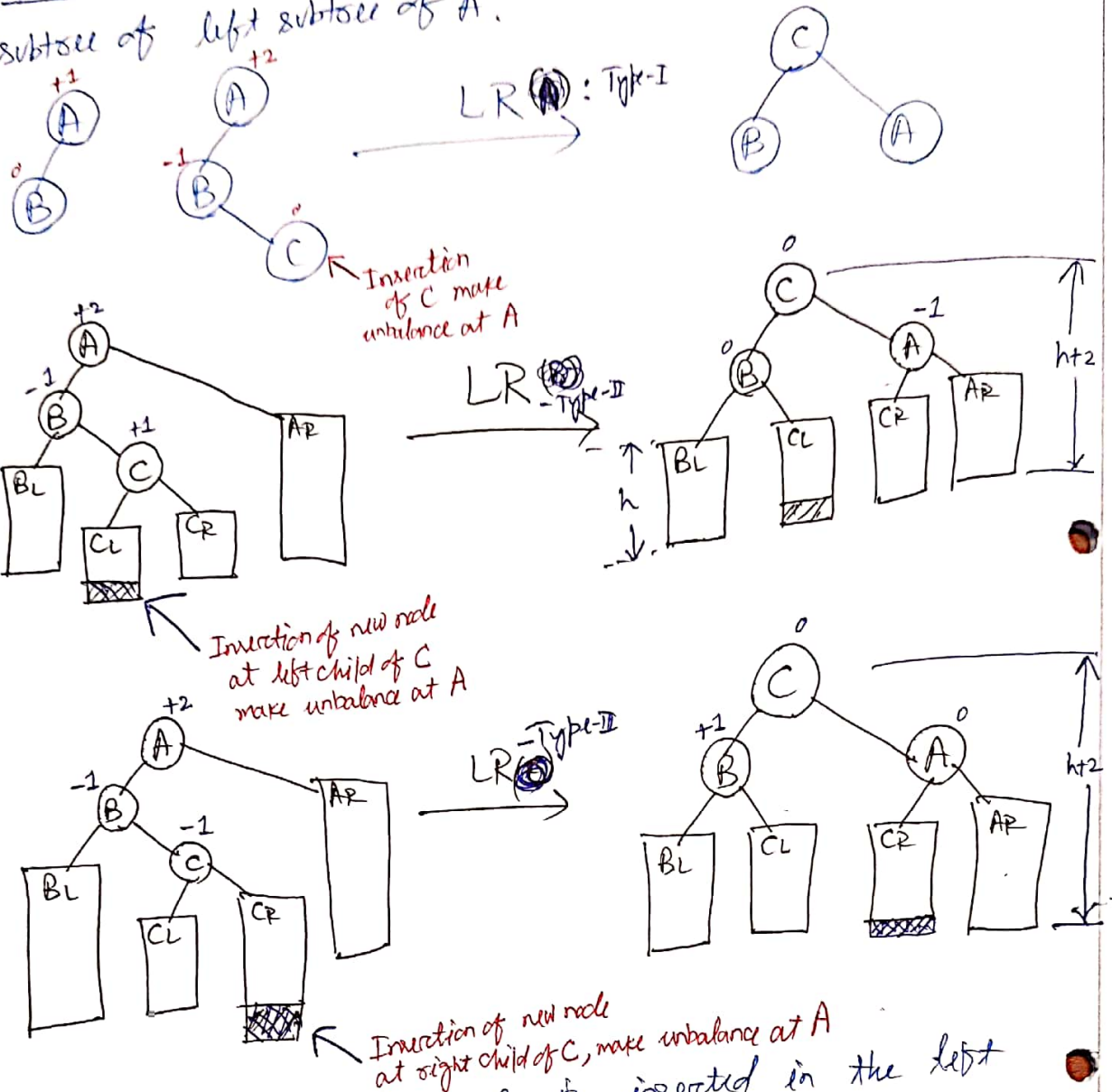
#1 LL Rotation: - It is applied when a new node is inserted in the left subtree of left subtree of a node "A".



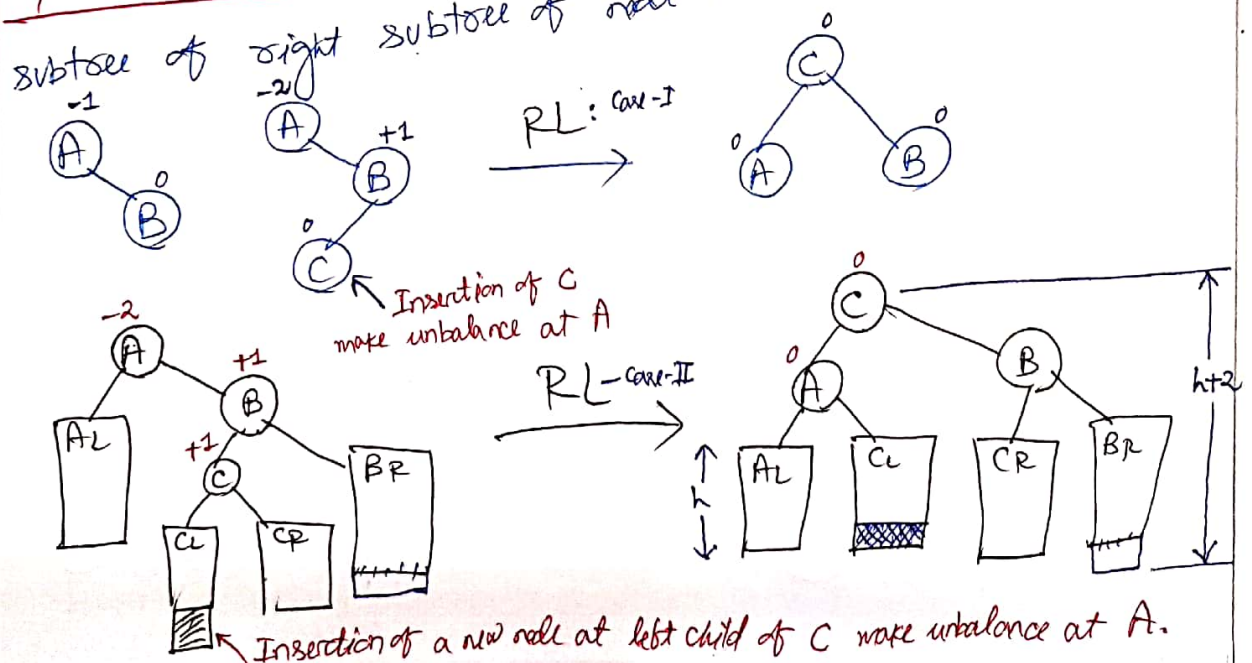
ii) RR Rotation: It is used when a new node is inserted in the right subtree of right subtree of node A.



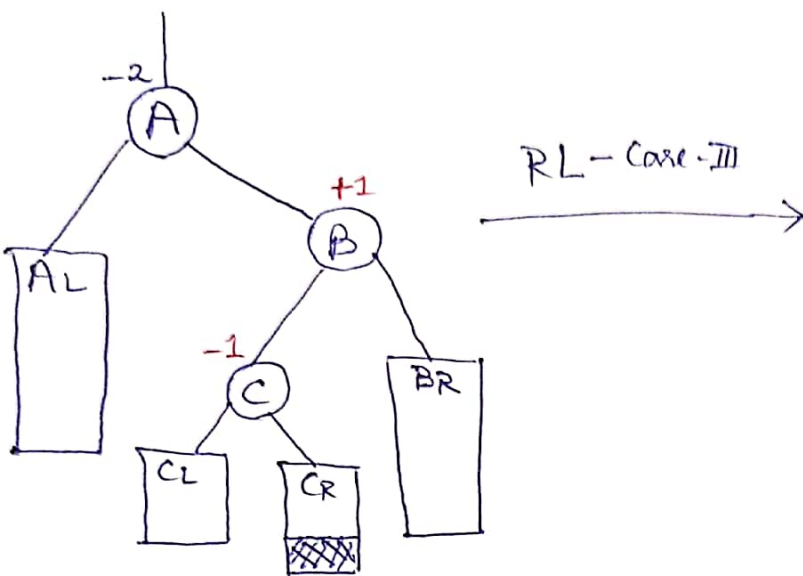
ii) LR Rotation: New node is inserted in the right subtree of left subtree of A.



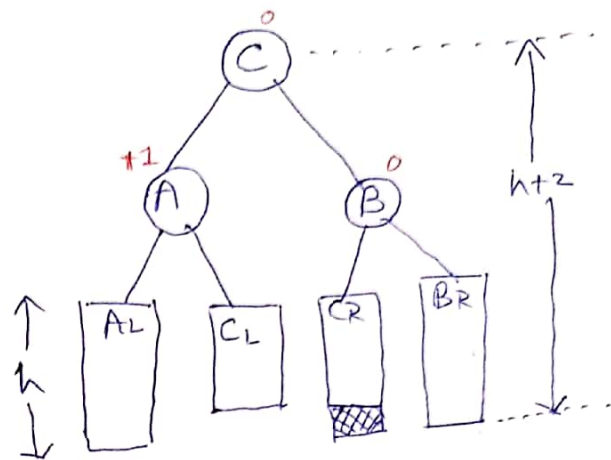
iii) RL Rotation: New node is inserted in the left subtree of right subtree of node A.



RL - Rotation



Insertion of a new node at right child of C makes tree unbalance at A



Example:- 55, 66, 77, 15, 11, 33, 22, 35, 25, 44, 88, 99

New Identifiers	After Insertion	Rotation	After Rebalancing
i) 55		—	NA
ii) 66		—	NA
iii) 77		RR →	
iv) 15		—	NA
v) 11		LL →	
vi) 33		LR →	
vii) 22		—	—
viii) 35		—	—
ix) 25		RL →	

Problem #1

Construct an AVL tree for the data set
jan, feb, mar, apr, may, jun, jul, aug, sep, oct, nov, dec.

DS

32

Problem #2

Draw the AVL tree resulting from the insertion of the following
integer keys.
50, 72, 96, 94, 107, 26, 12, 11, 9, 2.

Problem #3

Construct the AVL tree for the set of countries.
IND, USA, UZR, PAK, MYN, BAG, SRL, INS, AUS, CAN, DEN, CHN